2.1 The Extensive Form

Sunday, August 23, 2020 10:36 PM

- Dotted line means Player doesn't know where they are in the Game
- Simultaneous first moves doesn't matter who is written first

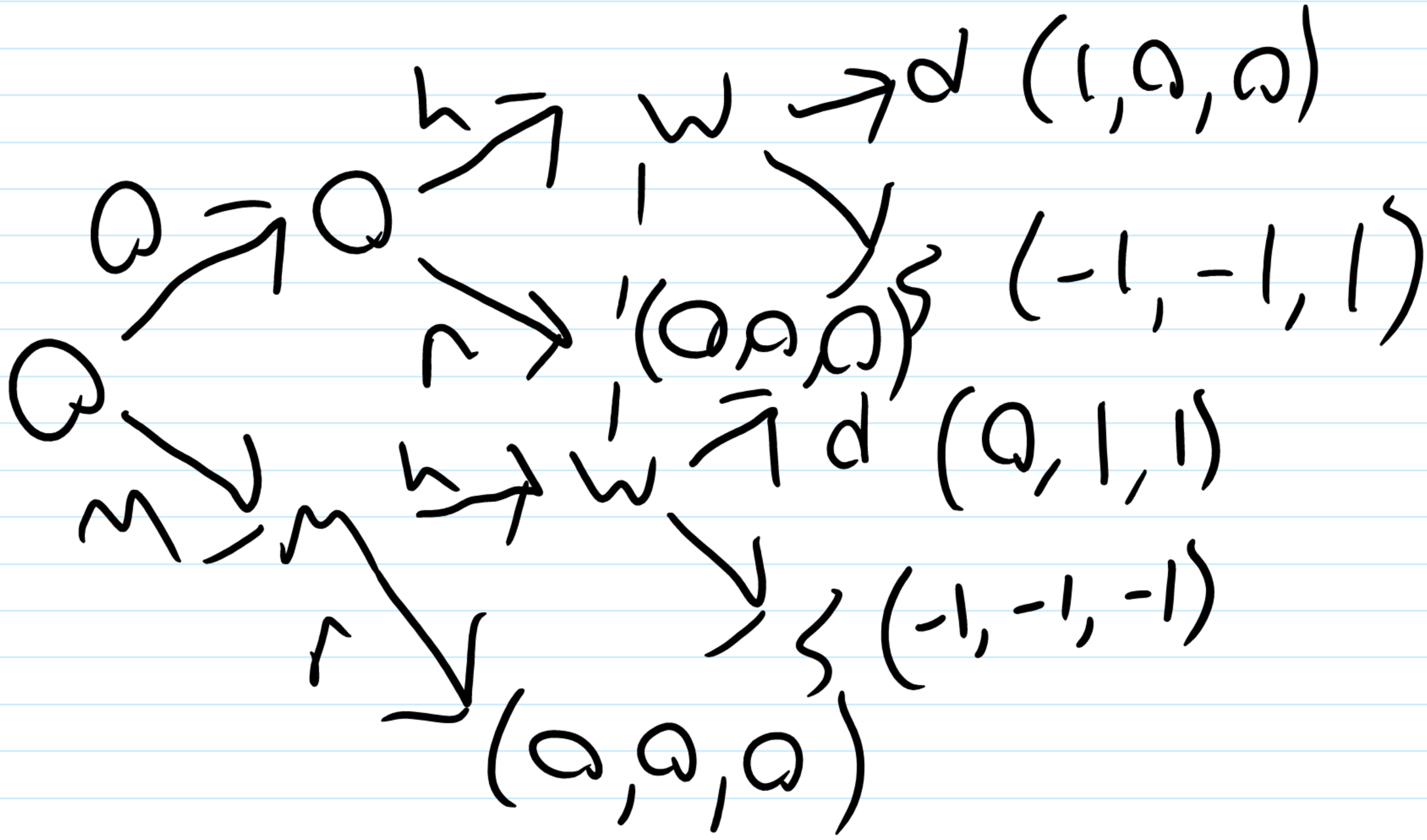
HW 2.2

Tuesday, August 25, 2020 6:05 PM

Passed Solution review

Owner of firm is hiring. They can hire, not hire, or let the manager decide. Manager can hire or not hire. If hired, the worker can work diligently or slack off. Worker doesn't know if O or M hired. Worker not hired all 0, W hired and shirks, O and M get -1 and W gets 1. If O hired and W is diligent, O gets 1, M and W get 0. If M hired, flip O and M

(O, M, W)

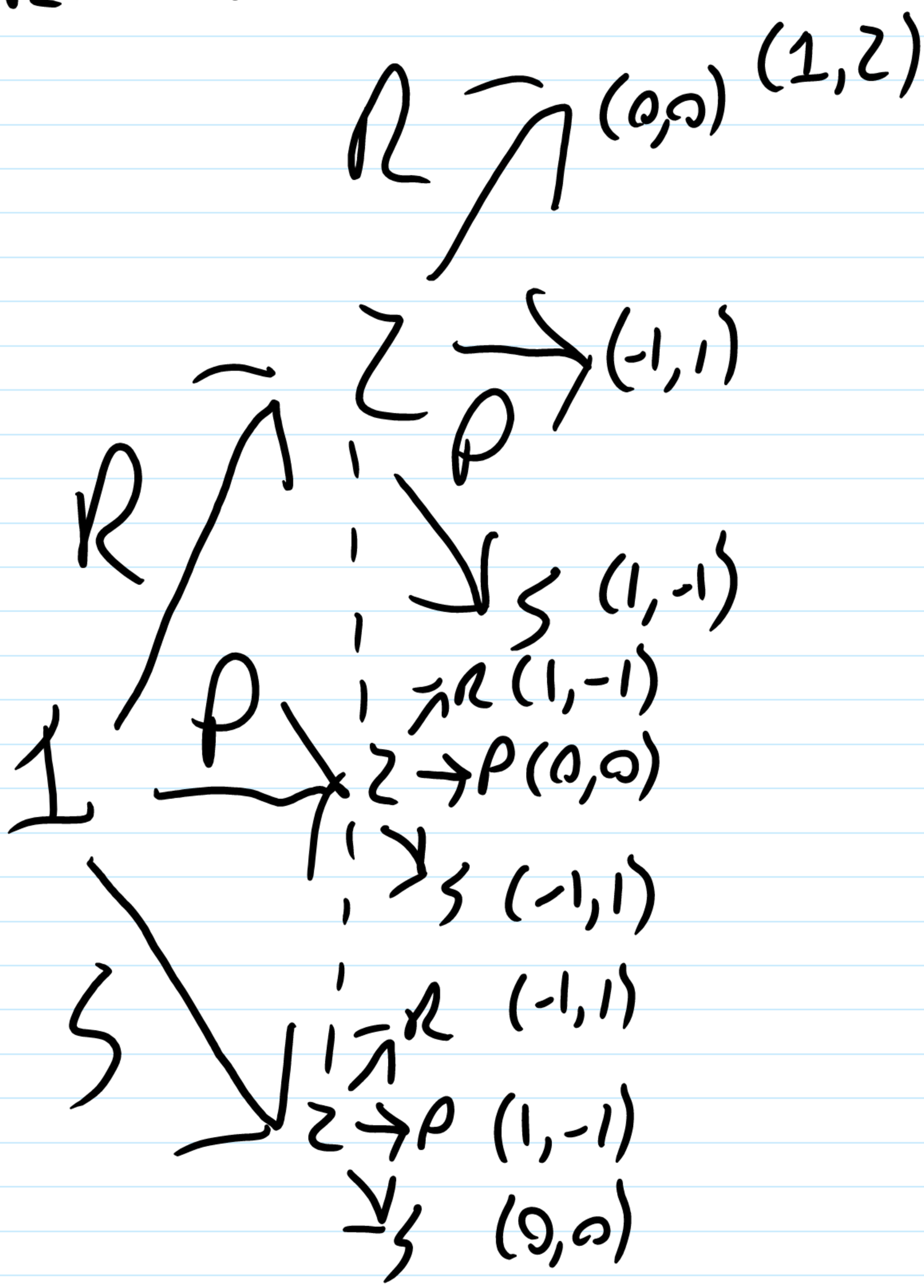


HW 2.4

Tuesday, August 25, 2020 6:19 PM

Passed Solution review

Rock paper scissors!



Because play is simultaneous, player order doesn't matter except for writing payoff orders

If the game was sequential, player 2 would always win

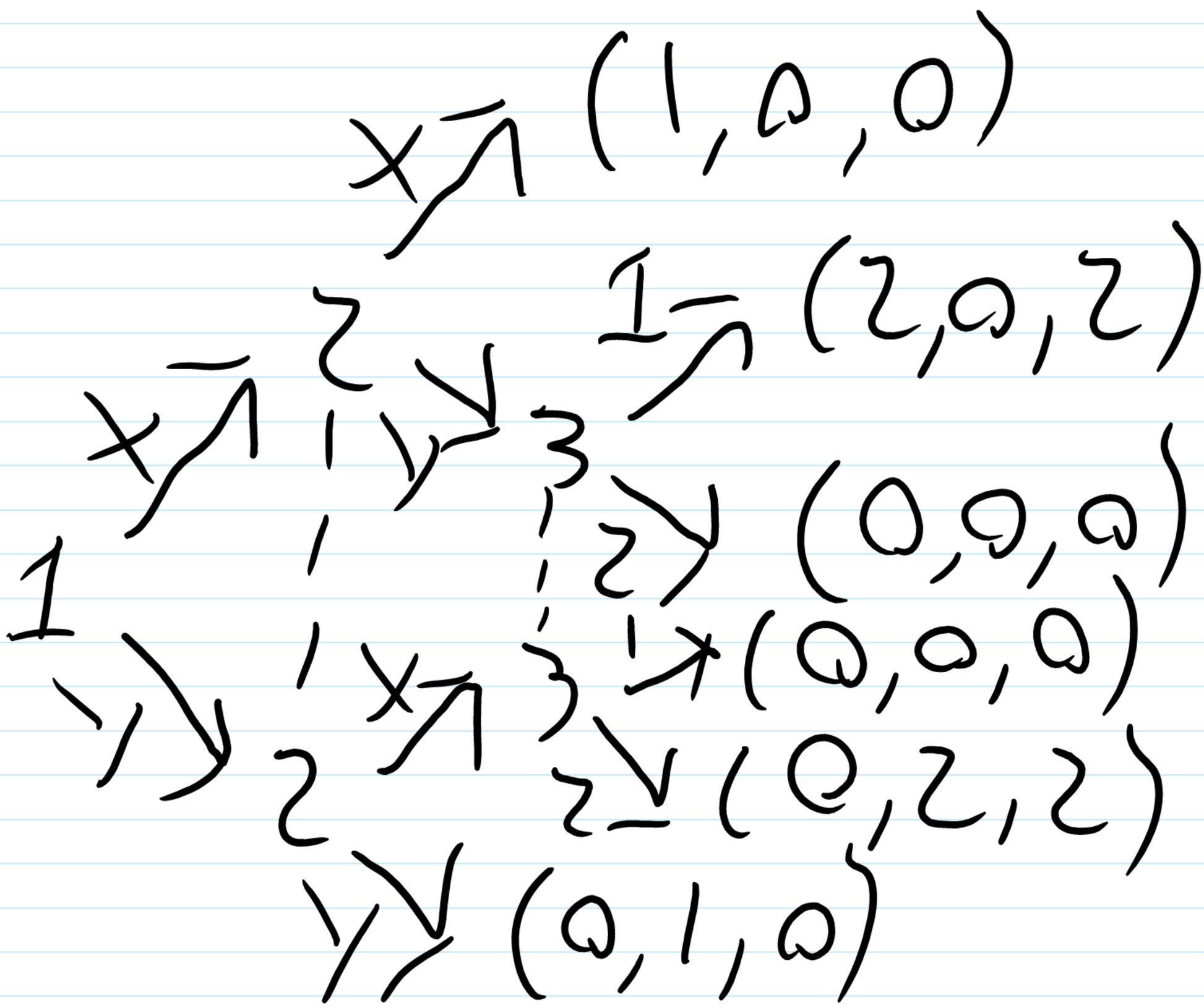
HW 2.6

Tuesday, August 25, 2020 6:25 PM

Passed Solution Review

Players 1,2,3. At first 1 and 2 move simultaneously between X and Y. Game ends if both choose X and Payoff is (1,0,0). If both Y, Game over w/ (0,1,0). Player 3 guesses who said what.

(1,2,3)



3.1 Strategies

Tuesday, August 25, 2020 10:09 AM

$S_2 = \{H, L\}$

$S_c = \{HH', HL', LH', LL'\}$ ← Cheryl + τ_{ii}^c from previous example on high/low effort

(H, LH') ← profile

$S = S_2 \times S_c$

↳ cartesian product (matrix multiplication)

$s_i \in S_i \quad S = s_i \times s_2 \times s_3 \times s_n$ ← All possibilities

S_{-i} is all strategies except player i (think vectors in $\emptyset$

Player i utility $= U_i(S)$ or $U_i(S_i, S_{-i})$

$i = 7$ then $S_{-7} = (S_i \ldots S_6, S_8 \ldots S_n)$ or $S(S_7, S_{-7})$

Every profile maps to payoffs

$U_i : S \to R$

Normal form?

3.2 Normal Form Representation

Tuesday, August 25, 2020 10:20 AM

Strategic form representation

Cheryl + Zeke

		C		
	HH'	HL'	LH'	LL'
Z H	-1,1	-1,1	0,1	1,0
Z L	0,1	0,0	0,1	0,0

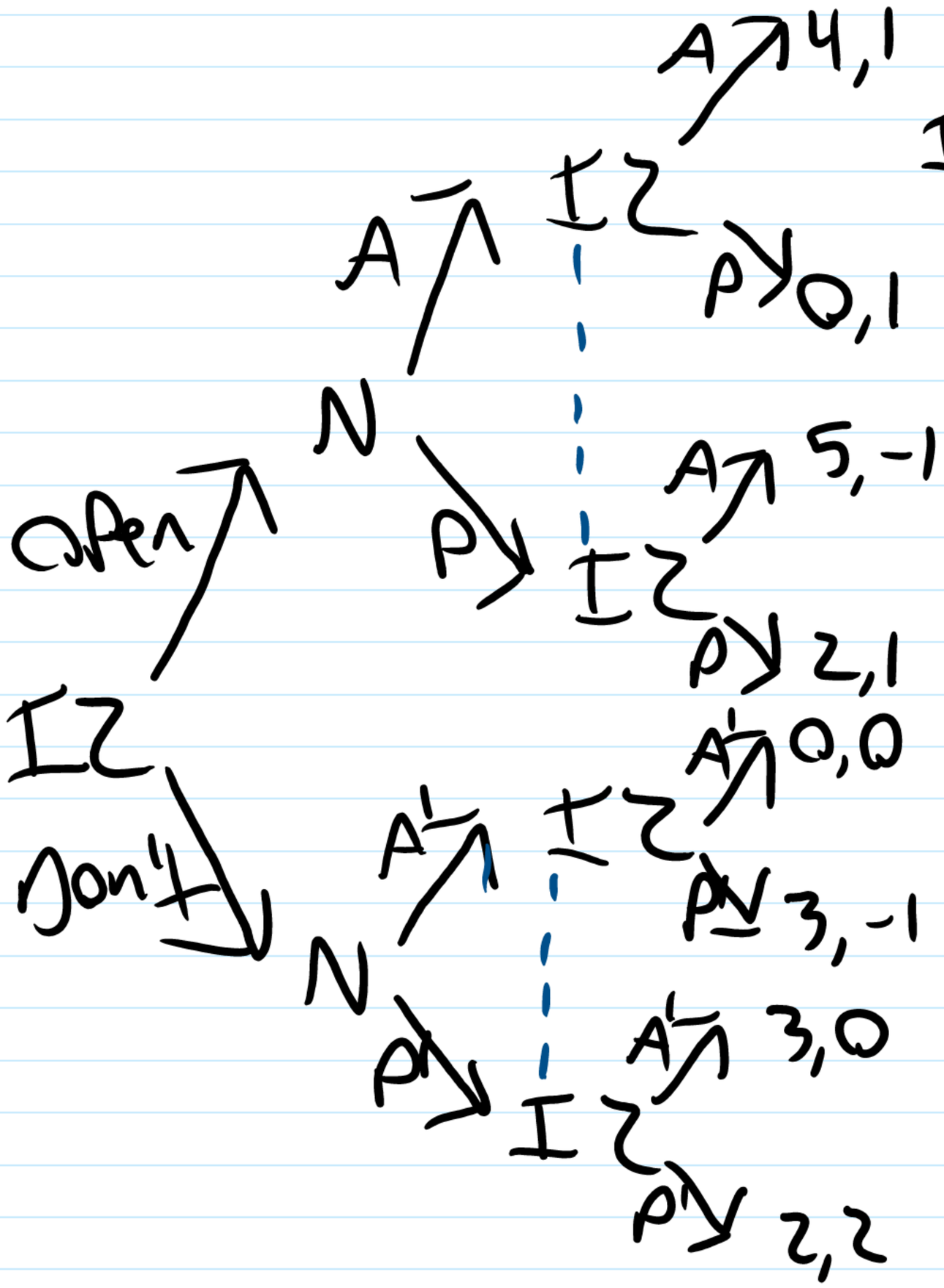
8 strategy profiles

$x \in \mathbb{R}$ $y \in \mathbb{R}$ $\rightarrow u_1(x,y)$ $u_2(x,y)$

real number

3.3 Example Strategies

Tuesday, August 25, 2020 8:52 PM

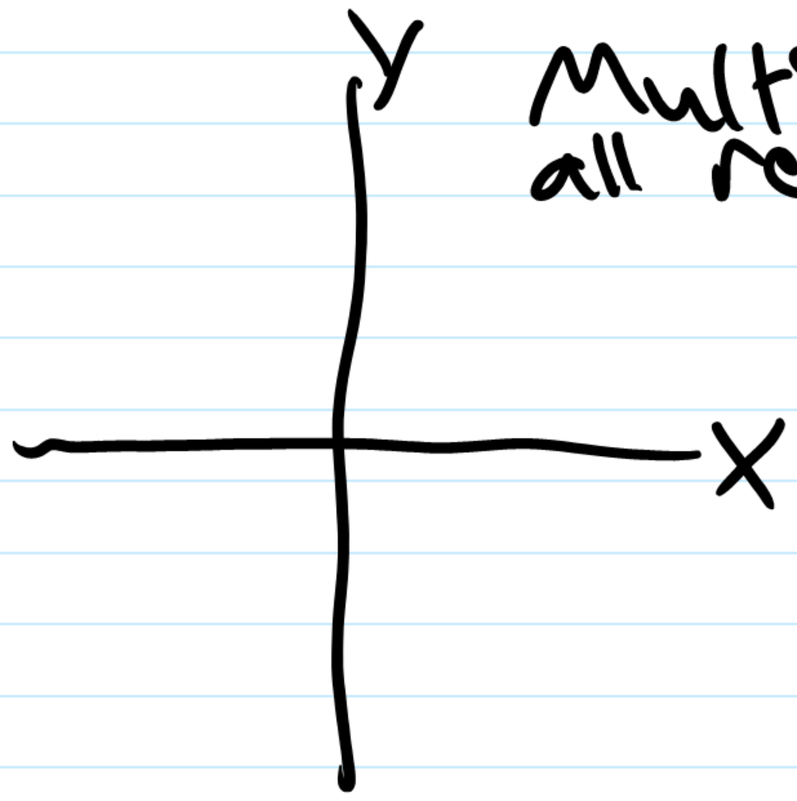


IZ:	N: AA'	AP'	PA'	PP'
OAA'	4,1	4,1		
OPA'				
OAA'			0,0	
OPA'	0,0		0,0	
OAP'	4,1	4,1		
OPP'				
OAP'				
OPP'		2,2		2,2

3 Extra Notes

Wednesday, August 26, 2020 10:07 AM

Cartesian Product

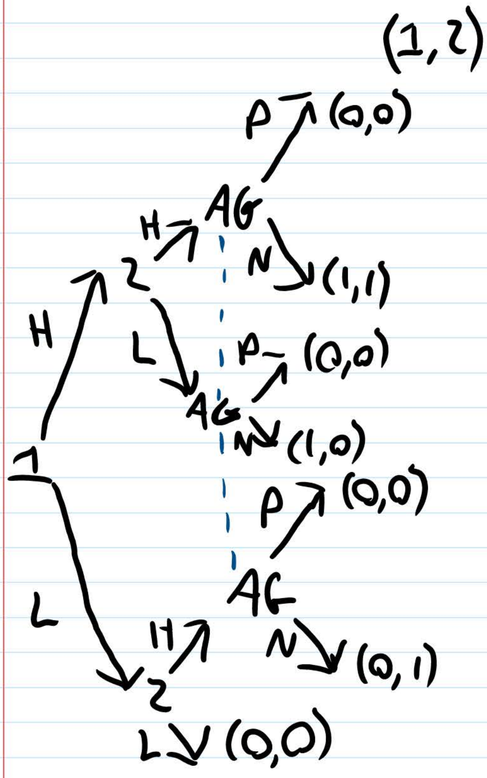


Multiplying x and y sets to get all resulting sets

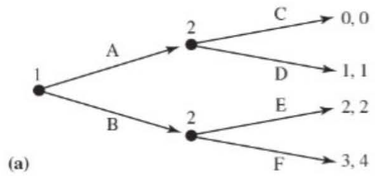
3 Example Problems

Friday, August 28, 2020 10:01 AM

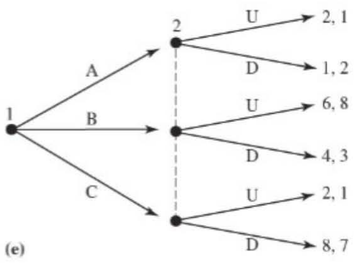
2.3. Draw the extensive form for the following game (invent your own payoff vectors, because I give you no payoff information). There is an industry in which two firms compete as follows: First, firm 1 decides whether to set a high price (H) or a low price (L). After seeing firm 1's price, firm 2 decides whether to set a high price (H) or a low price (L). If both firms selected the low price, then the game ends with no further interaction. If either or both firms selected the high price, then the attorney general decides whether to prosecute (P) or not (N) for anticompetitive behavior. In this case, the attorney general does not observe which firm selected the high price (or if both firms selected the high price).



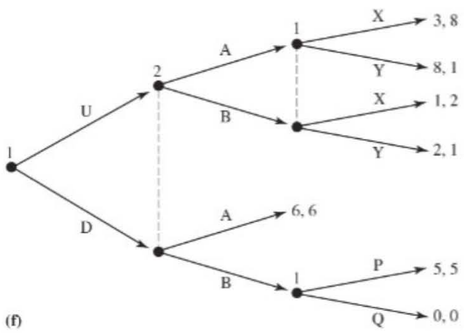
3.3. Draw the normal-form matrix of each of the following extensive-form games.



(a)



(e)



(f)

a)

	CE	DE	CF	DF
A	0,0	1,1	0,0	1,1
B	2,2	2,2	3,4	3,4

e)

	U	D
A		
B		
C		

f)

	A	B
UXP		
UXQ		
UYP		
UYQ		
DXP		
DXQ		
DYP		
DYQ		

=16 strategies

HW 3.2

Tuesday, August 25, 2020 6:33 PM

Passed Solution review

Manager decides whether or not to hire a worker. If M doesn't hire W game over. If hired W can do high or low effort. Based on effort, M can retain or fire W.

Not hire describes a strategy because not hire is the first move and a terminal point in the game.

No. A strategy must specify what to choose in all possible information sets, whether or not they will be reached if that strategy is played w/ fidelity.

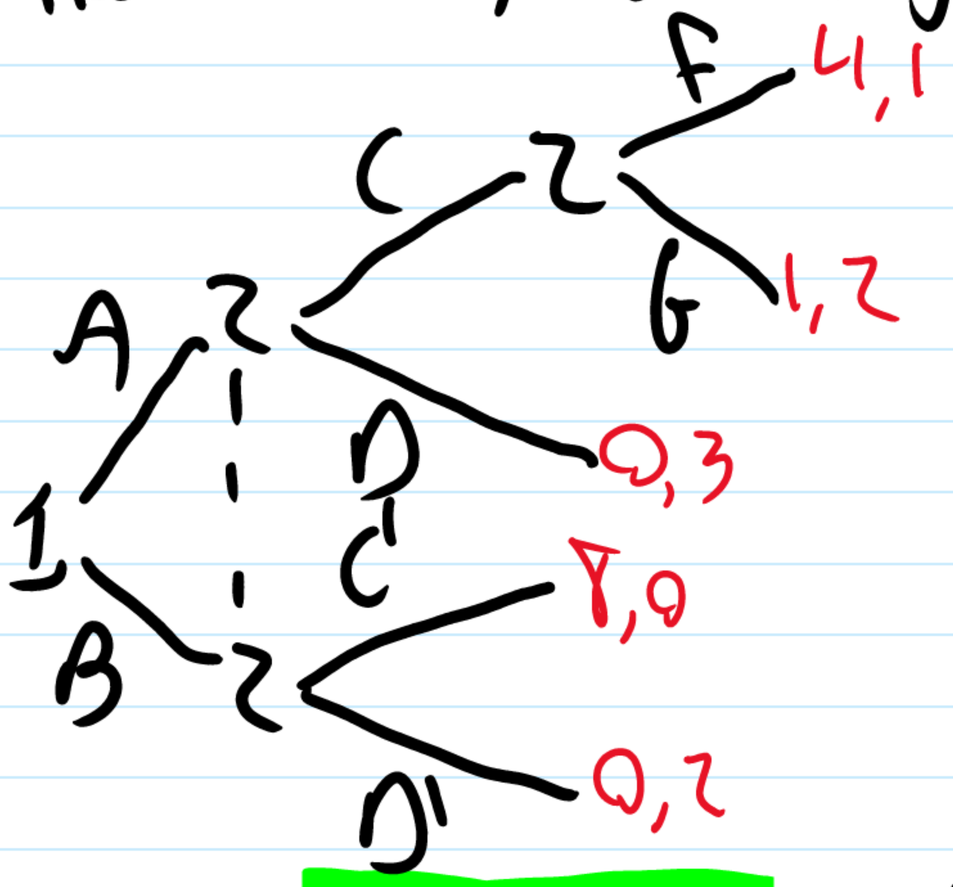
HW 3.4

Sunday, August 30, 2020 6:00 PM

Passed solution review

Worked w/ Hail + Isabel

How many strategies does Player 2 have?



Cf CG C'f C'G Df DG D'f D'G

A
B

Player 2 has 4 strategies.

HW 3.8

Sunday, August 30, 2020 6:20 PM

Passed solution review

Worked w/ Hail + Isabel

Cat = Baker Dog = Spike mouse = Cheesy

Baker wants to catch Cheesy + avoid Spike

Cheesy wants to tease baker + not get caught

Spike wants to rest and not be disturbed

morning: B + C simultaneously decide

B can Nap or Hunt

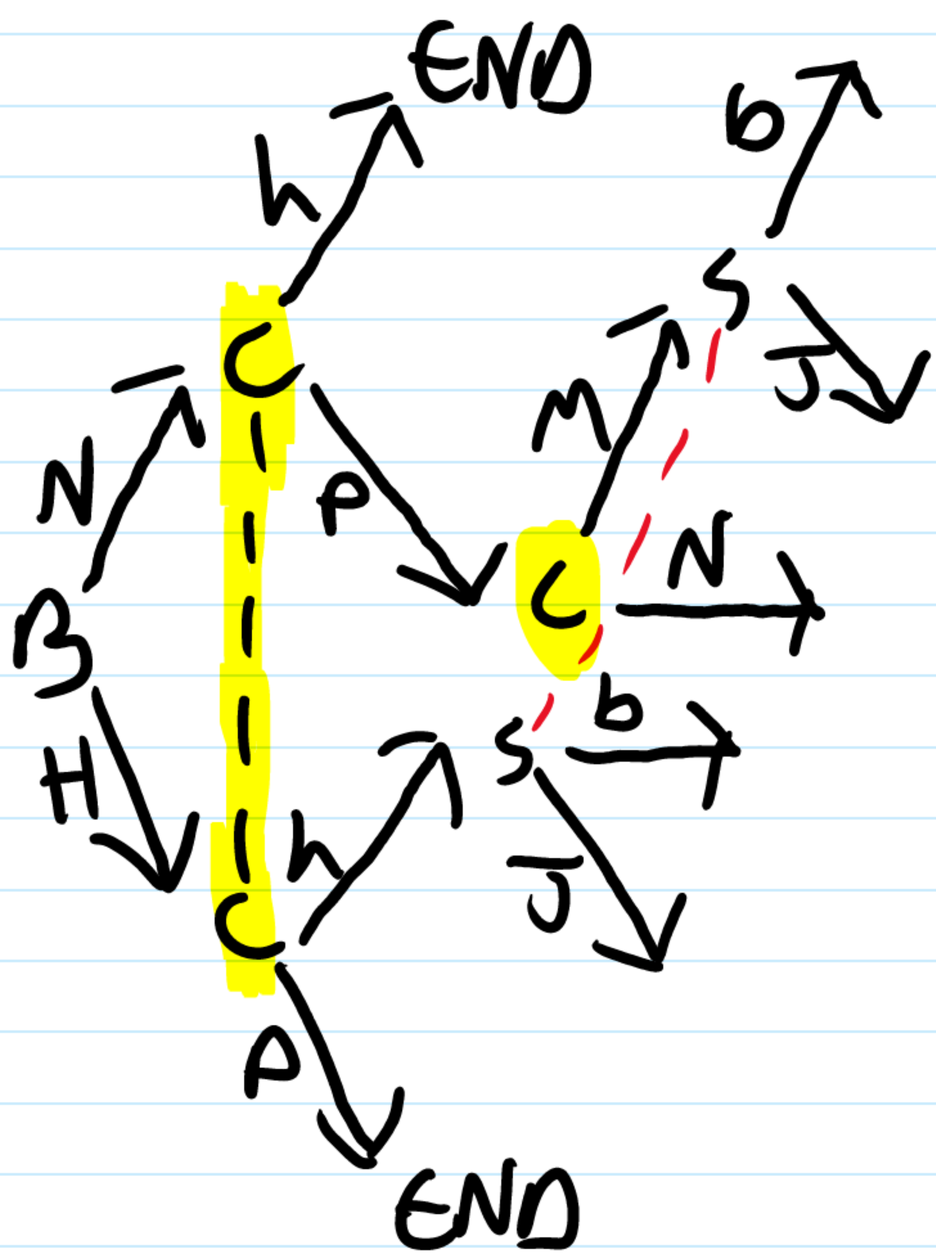
↳ moves S's bone

C can hide or Play

↳ Game ends if nap + hide or hunt + Play

If nap + Play, B must move or Not move the bone

If bone moved, S Punishes B or C then game ends



Cheezy has 2 information sets

B: N, H 2x

C: hM, PM, hN, PN 4x

S: b, J 2x

16?

$\boxed{\text{There are 16 strategy profiles}}$

4 Book Notes

Saturday, August 29, 2020 4:22 PM

Beliefs, Mixed Strategies, + expected Payoffs

belief is what a Player thinks will happen

Mixed Strategy is selecting a strategy according to a Probability distribution

regular Strategy = Pure Strategy

Mixed Strategy includes Pure Strategy

Payoff numbers can include preferences of Player Preferences over Probability distributions over outcomes

4.1 Beliefs and Expected Payoffs

Saturday, August 29, 2020 4:35 PM

Beliefs

Column

Row	C1	C2	C3
R_1	4,1	1,1	2,5
R_2	2,2	0,0	3,3
R_3	2,5	1,1	1,4
	P_1	P_2	$1-P_1-P_2$
	0.2	0.2	0.6

$\theta_{-R}, \theta_C \rightarrow \theta_{-i} \in \Delta S_{-i}$

Theta not row means theta column...

beliefs are represented by probability distributions

Expected Payoff/utility $\rightarrow E$

$\theta_C = (.2, .2, .6)$ $\qquad$ U = utility

$E(u_R \mid s_R = R_2, \theta_C) = .2 \cdot 2 + .2 \cdot 0 + .6 \cdot 3 = 2.2$

IF $\qquad$ and

$\theta_R = (.7, 0, .3) \rightarrow E(U_C \mid s_C = C_1, \theta_R) = .7 \cdot 1 + 0 \cdot 2 + .3 \cdot 5 = .85$

$$U_i(s_i, \theta_{-i}) = \sum_{s_{-i} \in S_{-i}} \theta_{-i}(s_{-i}) \cdot u(s_i, s_{-i})$$

4.2 More on Expected Utility

Saturday, August 29, 2020 4:53 PM

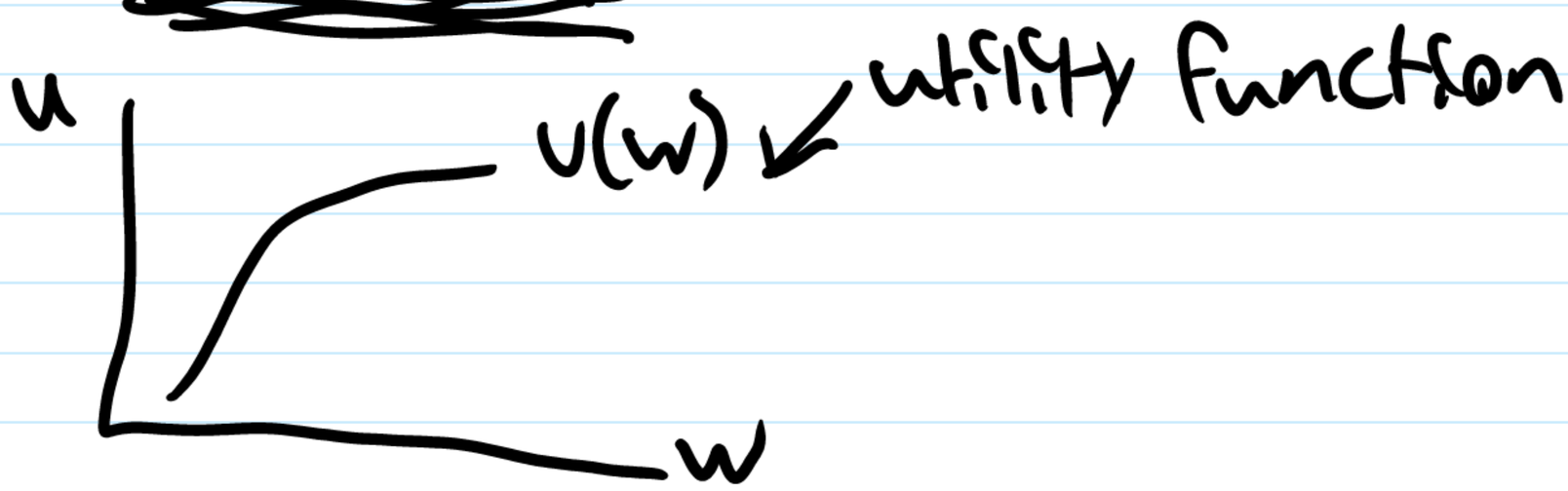
More on expected utility

Ranking = ordinal (can't multiply)

Risk neutral, money outcomes

↳ will take any fair gamble

Risk Aversion



Von-Neumann Morgenstern utility function

- Continuity
- More is better
- transitivity
- Independence of irrelevant outcomes

4.3 Mixed Strategies

Saturday, August 29, 2020 5:09 PM

Mixed Strategies

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	0,0	3,3
R_3	2,5	1,1	1,4
	p_1	p_2	$1-p_1-p_2$

$\sigma_i \in \Delta S_i$

Sigma

$\theta_c = (.2, .2, .6)$

$\sigma_R (.7, 0, .3)$

↳ NOT θ_R R belief

$$E(U_R)? = [\underbrace{.7 \cdot .2 \cdot 4}_{R_1C_1} + \underbrace{.7 \cdot .2 \cdot 1}_{R_1C_2} + \underbrace{.7 \cdot .6 \cdot 2}_{R_1C_3}] +$$

$$0[\underbrace{.2 \cdot 2}_{R_2C_1} + .2 \cdot 0 + .6 \cdot 3] +$$

$$.3[.2 \cdot 2 + .2 \cdot 1 + .6 \cdot 1]$$ = R Player expected utility given mixed strategy + their beliefs about C's strategy

$$U_i(\sigma_i, \theta_{-i}) = \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \theta_i(s_i) \cdot \theta_{-i}(s_{-i}) \cdot U_i(s_i, s_{-i})$$

4 Extra Notes

Monday, August 31, 2020 10:05 AM

mixed strategy is a way to obfuscate your goal

4 Extra Problems

Friday, September 4, 2020 10:08 AM

1)

1 \ 2	L	C	R
U	10,0	0,10	3,3
M	2,10	10,2	6,4
D	3,3	4,6	6,6

b) $U_2(M,R) = 4$

c) $(3 \cdot .25) + (.5 \cdot 6) + (.25 \cdot 6) = 5.25$

d) $(3 \cdot \frac{1}{3}) + (4 \cdot 2/3) + (6 \cdot 0) = 3\ 2/3$

h)

HW 4.2

Saturday, September 5, 2020 1:00 PM

Passed Solution review

$S_1 = \{H, L\}$ and $S_2 = \{X, Y\}$

If 1 plays H, Payoff = z. Player 1 Payoff = $U_1(L, X) = 0$
$U_1(L, Y) = 10$

Player 2 Payoff doesn't matter

a) Player 1

	Player 2: X	Player 2: Y
H	z, 0	z, 1
L	0, 1	10, 0

b) If 1 believes $\theta_2 = (.5, .5)$, Payoff of Playing H? Playing L? Value of z when H = L?

Payoff for H = .5 z
Payoff for L = 10 · .5 = 5

Player 1 is indifferent when z = 5

c) $\theta_2 = (\frac{1}{3}, 2/3)$. Payoff of Player 1 Playing L?

$= \frac{1}{3} \cdot 0 + 10 \cdot \frac{2}{3} = 0 + 20/3 = 20/3$

HW 4.3

Saturday, September 5, 2020 1:18 PM

Passed Solution review
worked w/ hail

	I	O
OA	2,2	2,2
OB	2,2	2,2
IA	4,2	1,3
IB	3,4	1,3

a) $U_1(\sigma_1, I)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$

$1/4(2+2+4+3) = 1/2 + 1/2 + 1 + 3/4 = 11/4$ or 2.75

b) $U_2(\sigma_1, O)$ for $\sigma_1 = (1/8, 1/4, 1/4, 3/8) = (1/8, 2/8, 2/8, 3/8)$

$2/8 + 4/8 + 6/8 + 9/8 = 21/8$

c) $U_1(\sigma_1, \sigma_2)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$, $\sigma_2 = (1/3, 2/3)$

	I	O
OA	1/12	2/12
OB	1/12	2/12
IA	1/12	2/12
IB	1/12	2/12

$2/12 + 4/12 + 4/12 + 3/12 + 4/12 + 4/12 + 2/12 + 2/12 = 23/12$

d) $U_1(\sigma_1, \sigma_2)$ for $\sigma_1 = (0, 1/3, 1/6, 1/2)$, $\sigma_2 = (2/3, 1/3)$

	I	O
OA	0	0
OB	4/18	2/18
IA	2/18	1/18
IB	6/18	3/18

$0 + 8/18 + 8/18 + 18/18 + 0 + 4/18 + 1/18 + 3/18 = 42/18 = 7/3$

HW 4.4

Saturday, September 5, 2020 2:07 PM

Worked w/ hail

Passed Solution review

FIGURE 3.4 Classic normal-form games.

1 \ 2	H	T
H	1, −1	−1, 1
T	−1, 1	1, −1

Matching Pennies

1 \ 2	C	D
C	2, 2	0, 3
D	3, 0	1, 1

Prisoners' Dilemma

1 \ 2	Opera	Movie
Opera	2, 1	0, 0
Movie	0, 0	1, 2

Battle of the Sexes

1 \ 2	H	D
H	0, 0	3, 1
D	1, 3	2, 2

Hawk-Dove/Chicken

1 \ 2	A	B
A	1, 1	0, 0
B	0, 0	1, 1

Coordination

1 \ 2	A	B
A	2, 2	0, 0
B	0, 0	1, 1

Pareto Coordination

D \ S	P	D
P	4, 0	2, −1
D	6, −1	0, 0

Pigs

For each game find:

$U_1(\sigma_1, \sigma_2)$ and $U_2(\sigma_1, \sigma_2)$ for

$\sigma_1 = (1/2, 1/2)$ and $\sigma_2 = (1/2, 1/2)$

Pennies: $U_1(\sigma_1, \sigma_2) = U_2(\sigma_1, \sigma_2) = .25 - .25 + .25 - .25 = 0$

Sexes: $U_1(\sigma_1, \sigma_2) = U_2(\sigma_1, \sigma_2)$

$= (2 \cdot 1/4) + 0 + 0 + (1 \cdot 1/4) = 3/4$

Chicken: $U_1(\sigma_1, \sigma_2) = U_2(\sigma_1, \sigma_2)$

$= 0 + (1 \cdot 1/4) + (3 \cdot 1/4) + (2 \cdot 1/4) = 6/4 = 1.5$

5 Book Notes

Saturday, August 29, 2020 5:30 PM

General Assumptions + Methodology

Must balance realism w/ manageable math

<u>Rationality</u>

Players select path to their preferred outcome

- maximizing one's payoff

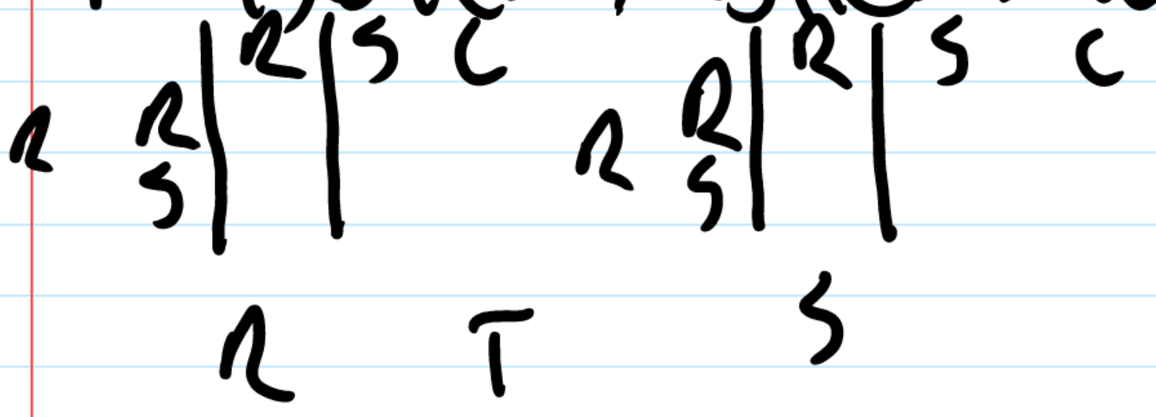
Assume players know how the game is played

Players are rational if they 1. form a belief about strategies and 2. given this belief they maximize their payoff

5 Extra Notes

Wednesday, September 2, 2020 10:06 AM

Prisoner's Dilemma w/ 3 Players



Players R, C, and T

6 Book Notes

Monday, August 31, 2020 3:17 PM

Dominance + Best Response

Contracts can bind players to strategies

6.1 Dominance and the Prisoner's Dilemma

Monday, August 31, 2020 2:46 PM

Dominance

Prisoner's Dilemma

		Hail	Silent	Rat
Gus	Silent	2,2	-1,3	
	Rat	3,-1	0,0	

Silence is dominated by Rat

↳ Always better

Weakly dominated = sometimes better

6.2 Dominance and Efficiency

Monday, August 31, 2020 2:54 PM

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	2,0	3,3
R_3	2,5	1,1	1,4

C_2 wd by C_1 $\quad$ r_3 wd by $r_1 + r_2$

C_2 sd by C_3

wd / sd = weakly/strongly dominated

Pure vs weak strategies

$\sigma_R = (.5, .5, 0)$ $\quad$ $\forall C_1$ $\quad 3 > 2$

$\forall C_2$ $\quad 1.5 > 1$

$\forall C_3$ $\quad 2.5 > 1$

$(.5, .5, 0)$ SD r_3

General Notation

s_i is dominated by σ_i if

$$u_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall \; s_{-i} \in S_{-i}$$

for all

Rational players <u>do NOT</u> play dominated strategies

PARETO EFFICIENCY

$\boxed{\text{Contracting}}$

$$u_i(q, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall \; s_{-i} \in S_{-i}$$

$$\& \; \forall \; s_i \in S_i \quad s_i \neq q$$

6.3 Best Responses

Saturday, September 5, 2020 12:00 PM

Best responses or Best replies

	C_1	C_2	C_3
R_1	4, 1	1, 1	2, 5
R_2	2, 2	2, 0	3, 3
R_3	2, 5	1, 1	1, 4

$BR_R(1,0,0) = R_1$

$(0,1,0)$ or $(0,0,1) = R_2$

$BR_C(1,0,0)$ or $(0,1,0) = C_3$

$(0,0,1) = C_1$

Column 2 is never BR because it's dominated

BR is $U_i(s_i, \theta_{-i}) \geq U_i(s_i', \theta_{-i})$ $\forall s_i \in S_i$. If true, strategy s is a BR to θ_{-i}

$B_C = \{C_1, C_3\}$

6.4 6.5 Establishing a Strategy is Never a Best Response

Saturday, September 5, 2020 12:12 PM

Establishing no set of beliefs make s_i a best response

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	2,0	3,3
R_3	2,5	1,1	1,4

$\Theta_c(p_1, p_2, 1-p_1-p_2)$

$U_R(R_3, \Theta_c) = 2 - p_1 + (1 \cdot p_2) + [1 \cdot (1 - p_1 - p_2)]$
$= 1 + p_1$

$U_R(R_2, \Theta_c) = 2p_1 + 2p_2 + 3(1 - p_1 - p_2)$
$= 3 - p_1 - p_2$

$U_R(R_3, \Theta_c) = 4p_1 + 1p_2 + 2(1 - p_1 - p_2)$
$= 2 + 2p_1 - p_2$

$U_R(R_3) > U_R(R_2) > 0$

$\hookrightarrow 1 + p_1 - 3 + p_1 + p_2 \geq 0$

$-2 + 2p_1 + p_2 \geq 0$

$p_2 \geq 2 - 2p_1$

$U_R(R_3) > U_R(R_1) \geq 0$

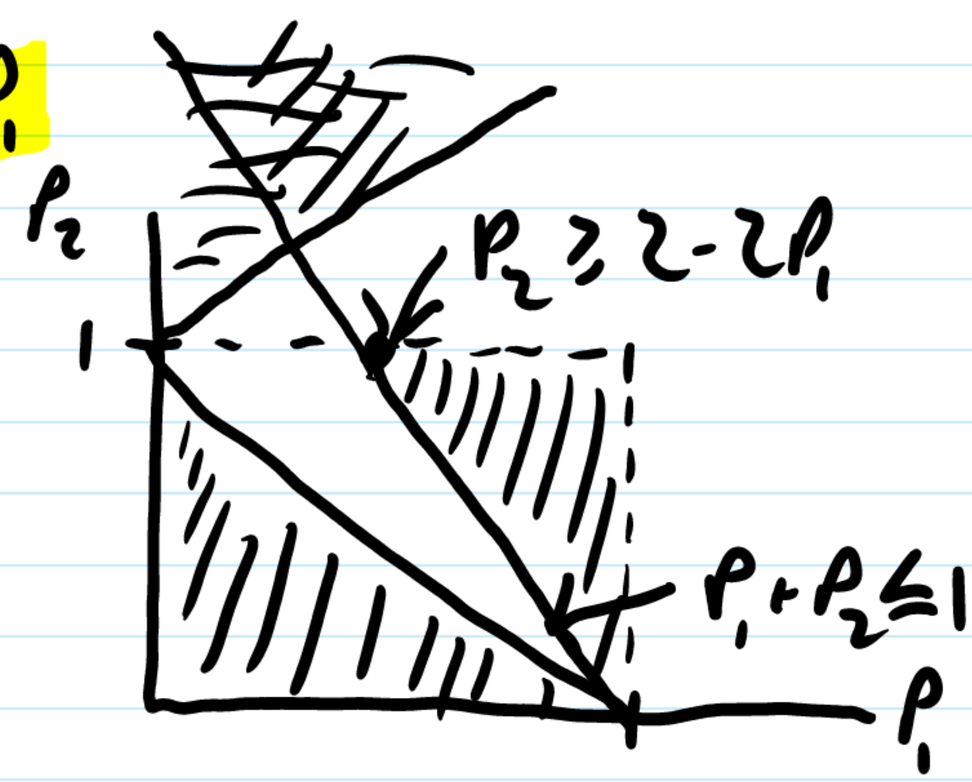
$\hookrightarrow 1 + p_1 - 2 - 2p_1 + p_2 \geq 0$

$-1 - p_1 + p_2 \geq 0$

$p_2 \geq 1 + p_1$

$p_1 + p_2 \leq 1$

Can't satisfy all 3!



R_3 is never best!

6.6 Undominated Strategies and Best Responses

Saturday, September 5, 2020 12:32 PM

Beliefs

	c_1	c_2	c_3
R_1	4,1	1,1	2,5
R_2	2,2	2,0	3,3
R_3	2,5	1,1	1,4

Strategic independence

Correlated beliefs

↳ $B_i^c = UD$.

6.7 Some Last Words on Weak Dominance

Saturday, September 5, 2020 12:49 PM

	c_1	c_2	c_3
r_1	4,1	1,1	3,1
r_2	2,2	0,3	3,2
	p_1	p_2	$1-p_1-p_2$

"fully mixed beliefs"

$B_i^{fc} = WVD_i$

6 Extra Problems

Friday, September 4, 2020 10:39 AM

a) B dom A, L dom R

b) L dom R, L weak dom C?

c) X dom Z, $2/3 V + 1/3 D$ dom M

6.1 Determine which strategies are dominated in the following normal-form games.

1 \ 2	L	R
A	3, 3	2, 0
B	4, 1	8, –1

(a)

1 \ 2	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

(b)

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

5. Represent in the normal form the rock–paper–scissors game (see Exercise 4 of Chapter 2 to refresh your memory) and determine the following best-response sets.

(a) $BR_1(\theta_2)$ for $\theta_2 = (1, 0, 0) = \{P\}$

(b) $BR_1(\theta_2)$ for $\theta_2 = (1/6, 1/3, 1/2) = \{R, S\}$

(c) $BR_1(\theta_2)$ for $\theta_2 = (1/2, 1/4, 1/4) = \{P, S\}$

(d) $BR_1(\theta_2)$ for $\theta_2 = (1/3, 1/3, 1/3) = \{R, P, S\}$

1 \ 2	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

b) $R = 0 + -1/3 + 1/2 = -2/6 + 3/6 = 1/6$

$P = 1/6 + 0 - 3/6 = -2/6$

$S = -1/6 + 2/6 + 0 = 1/6$

HW 6.2

Saturday, September 5, 2020 2:28 PM

Passed Solution review

1 \ 2	L	C	R
U	10, 0	0, 10	3, 3
M	2, 10	10, 2	6, 4
D	3, 3	4, 6	6, 6

2. For the game in Exercise 1 of Chapter 4, determine the following sets of best responses.

(a) $BR_1(\theta_2)$ for $\theta_2 = (1/3, 1/3, 1/3)$

(b) $BR_2(\theta_1)$ for $\theta_1 = (0, 1/3, 2/3)$

(c) $BR_1(\theta_2)$ for $\theta_2 = (5/9, 4/9, 0)$

(d) $BR_2(\theta_1)$ for $\theta_1 = (1/3, 1/6, 1/2)$

a) $U = 10/3 + 0 + 3/3 = 13/3$
$M = 2/3 + 10/3 + 6/3 = 18/3$ $\rightarrow BR_1(\theta_2) = \{M\}$
$D = 3/3 + 4/3 + 6/3 = 13/3$

b) $L = 0 + 10/3 + 6/3 = 16/3$
$C = 0 + 2/3 + 12/3 = 14/3$ $\rightarrow BR_2(\theta_1) = \{L, R\}$
$R = 0 + 4/3 + 12/3 = 16/3$

c) $U = 50/9 + 0 + 0 = 50/9$
$M = 10/9 + 40/9 + 0 = 50/9$ $\rightarrow BR_1(\theta_2) = \{U, M\}$
$D = 15/9 + 16/9 + 0 = 31/9$

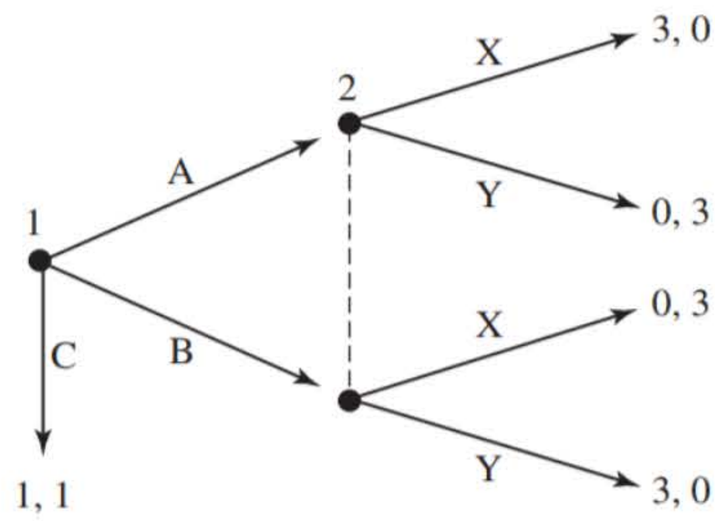
d) $L = 0 + 10/6 + 9/6 = 19/6$
$C = 20/6 + 2/6 + 18/6 = 40/6$ $\rightarrow BR_2(\theta_1) = \{C\}$
$R = 6/6 + 4/6 + 18/6 = 28/6$

HW 6.6

Saturday, September 5, 2020 2:39 PM

Passed Solution review

6. In the game pictured here, is it ever rational for player 1 to select strategy C? Why?



If Player 1 wants a guaranteed payoff, they would choose strategy C.

But, mathematically, they would choose A or B because the expected payoff value is 1.5

$AX = BY$ + $AY = BX$

By eliminating C, a mixed a+b generates 1.5

Let P denote Player 1's belief about probability of X

$u_1(C,P) \geq u_1(A,P)$
$1 \geq 3P + 0(1-P)$
$1/3 \geq P$

$u_1(C,P) \geq u_1(B,P)$
$1 \geq 0P + 3(1-P)$
$1 \geq 3 - 3P$
$P \geq 2/3$

No because P cannot satisfy both inequalities

HW 6.7

Saturday, September 5, 2020 2:41 PM

Passed Solution review

7. In the normal-form game pictured below, is player 1's strategy M dominated? If so, describe a strategy that dominates it. If not, describe a belief to which M is a best response.

1 \ 2	X	Y
K	9, 2	1, 0
L	1, 0	6, 1
M	3, 2	4, 2

M is dominated by mixed strategy $\{K, L\}$

M does not dominate Player 1's response because in X, K is better and in Y, L is better.

$$9p + 1(1-p) > 3$$
$$9p + 1 - p > 3$$
$$8p + 1 > 3$$
$$8p > 2$$
$$p > 1/4$$

$p > 5/20$

$$1p + 6(1-p) > 4$$
$$p + 6 - 6p > 4$$
$$6 - 5p > 4$$
$$-5p > -2$$
$$p < 2/5$$

$p < 8/20$

$$\sigma = (.3, .7, 0)$$

$p \quad 1-p \quad 0$

7 Book Notes

Saturday, September 5, 2020 3:52 PM

Rationalizability + Iterated dominance

Actions of one Player may affect another's Payoff

Players can eliminate (think Sudoku) strategies that don't make sense

↳ Iterative removal of (strictly) dominated strategies

↳ Iterative dominance

Strategies that survive iterative dominance are called rationizable strategies

The Second Strategic Tension

Strategic uncertainty

↳ Coordination problem:

7.1 Iterated Dominance and Rationalizability

Thursday, September 10, 2020 4:19 PM

		Bob			
		B_1	~~B_2~~	B_3	B_4
Alice	A_1	4,1	1,2	2,5	1,4
	A_2	2,3	3,1	1,0	1,2
	A_3	2,5	1,1	1,4	3,0

B_2 dom by mix $B_1 + B_3$

Eliminate B_2 to create R_1

Alice has no dominated strategies

R_1

		Bob			
		B_1	~~B_2~~	B_3	B_4
Alice	A_1	4,1		2,5	1,4
	~~A_2~~	2,3		1,0	1,2
	A_3	2,5		1,4	3,0

A_2 dom by $A_1 + A_3$

R_2

		Bob			
		B_1	~~B_2~~	B_3	~~B_4~~
Alice	A_1	4,1		2,5	1,4
	~~A_2~~				
	A_3	2,5		1,4	3,0

B_4 dom by B_3

R_3

		Bob			
		B_1	~~B_2~~	B_3	~~B_4~~
Alice	A_1	4,1		2,5	
	~~A_2~~				
	~~A_3~~	2,5		1,4	

A_3 dom by A_1

R_4

		Bob			
		B_1	~~B_2~~	B_3	~~B_4~~
Alice	A_1	4,1		2,5	
	~~A_2~~				
	~~A_3~~	2,5		1,4	

B_1 dom by B_3

R_5

		Bob			
		~~B_1~~	~~B_2~~	B_3	~~B_4~~
Alice	A_1			2,5	
	~~A_2~~				
	~~A_3~~				

You can eliminate dominated strategies

7.2 Strategic Uncertainty

Thursday, September 10, 2020 4:35 PM

Coordination Problem

↳ 2 people lose each other somewhere + look for each other in different national places

"Focal Points"

	Rabbit	Stag
Rabbit	1,1	1,0
Stag	0,1	4,4

7 Extra Problems

Monday, October 19, 2020 2:02 PM

Consider an analysis of changes in safety regulations for US automobiles.

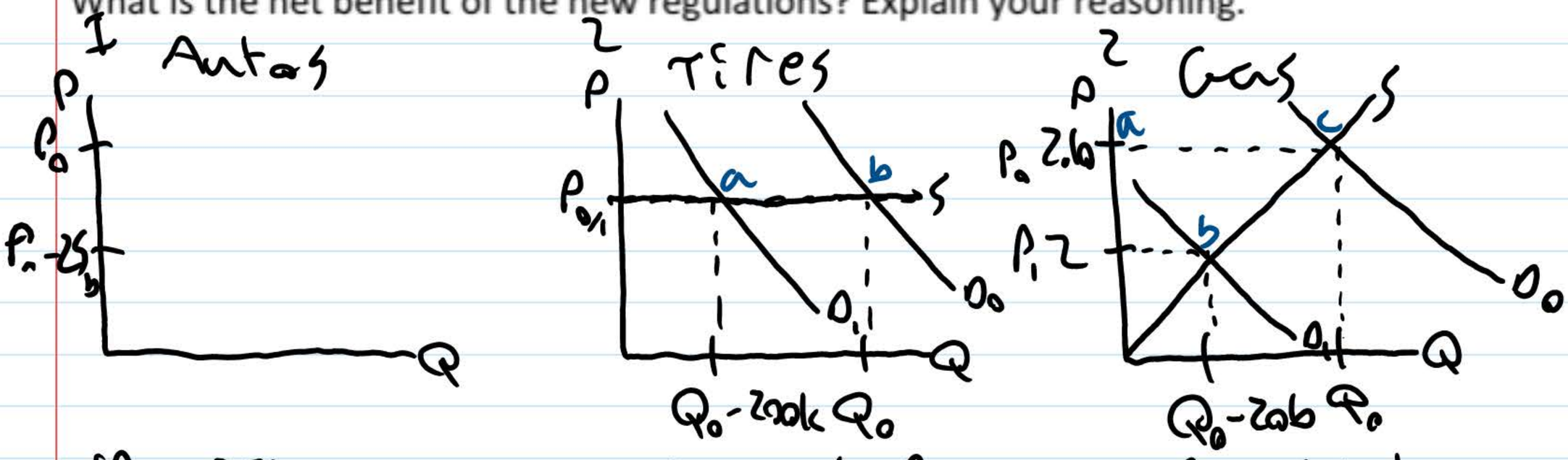
The net impact in the primary market is a loss of $25 billion per year, measured using actual prices and quantities before and after the change (i.e. not holding other prices constant).

The change in regulations effected the US tire market, reducing consumption by 200K, but price did not change appreciably.

The change in regulations effected the US gasoline market. The price of gasoline fell from $2.10 per gallon to $2.00, with both prices including a $0.5 per gallon tax. Annual gasoline consumption fell 20 billion gallons.

There is an external cost of $1 per gallon of gasoline.

What is the net benefit of the new regulations? Explain your reasoning.

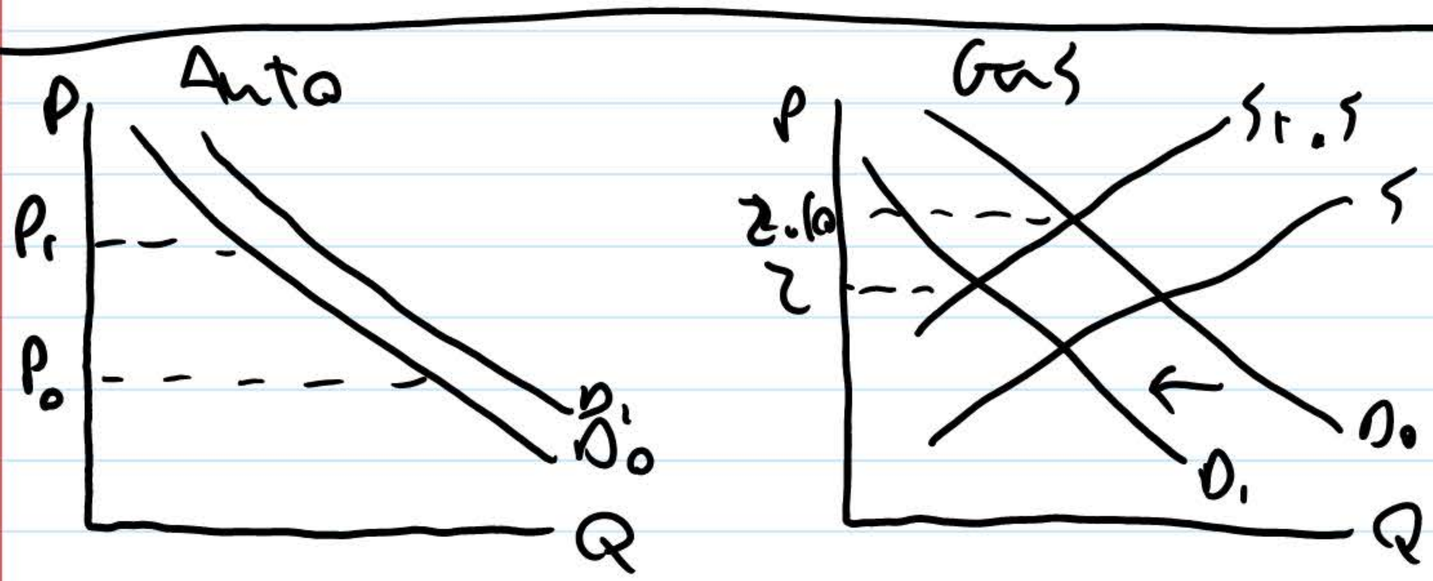


NB = -25b

NB = -200k · P

NB = -.1 · 20b

Total NB = -25b - (200k · P) - (.1 · 20b) - (.5 · 20b)



-25b + (1 · 20b) - (.5 · 20m · (1 + METB))

↳ Loss of tax rev

↳ savings on external costs

Ignore tires because P constant

gas price change

HW 7.2

Friday, September 18, 2020 9:17 AM

Passed Solution review

Suppose that you manage a firm and are engaged in a dispute with one of your employees. The process of dispute resolution is modeled by the following game, where your employee chooses either to "settle" or to "be tough in negotiation," and you choose either to "hire an attorney" or to "give in."

Employee \ You	Give in	Hire attorney
Settle	1, 2	0, 1
Be tough	3, 0	x, 1

In the cells of the matrix, your payoff is listed second; x is a number that both you and the employee know. Under what conditions can you rationalize selection of "give in"? Explain what you must believe for this to be the case.

If $x \geq 0$, be tough dom settle

If $x < 0$, no dominant strategies

If $p(\text{settle}) > .5$ and $x < 0$, then it makes sense to choose give in

(greater than)

HW 7.3

Friday, September 18, 2020 12:19 PM

Posted Solution review

Find the set of rationalizable strategies for the following game.

1 \ 2	a	b	c	d
w	5, 4	4, 4	4, 5	12, 2
x	3, 7	8, 7	5, 8	10, 6
y	2, 10	7, 6	4, 6	9, 5
z	4, 4	5, 9	4, 10	10, 9

Note that each player has more than one dominated strategy. Discuss why, in the iterative process of deleting dominated strategies, the order in which dominated strategies are deleted does not matter.

$R^0 = \{w, x, y\} \times \{a, b, c\}$

$R^1 = \{w, x\} \times \{c\}$

$R^2 = \{x\} \times \{c\}$

final strategy $= \boxed{\{x\} \times \{c\}}$

The order doesn't matter because a round isn't over until both players have moved.

x dom y

$2/3 w + 1/3 x$ dom z

c dom d

$9/10 c + 1/10 a$ dom b

$R^1 = \{w, x\} \times \{a, c\}$

c dom a

$R^2 = \{w, x\} \times \{c\}$

x dom w so $s = (x, c)$

In any iteration, each player considers all strategies not eliminated in the last iteration. Then, in the next iteration, all strategies eliminated for either player are eliminated. Since all are eliminated between iterations, "order" has little meaning within any given round.

HW 7.4

Friday, September 18, 2020 12:25 PM

Passed Solution review

Imagine that there are three major network-affiliate television stations in Turlock, California: RBC, CBC, and MBC. All three stations have the option of airing the evening network news program live at 6:00 P.M. or in a delayed broadcast at 7:00 P.M. Each station's objective is to maximize its viewing audience in order to maximize its advertising revenue. The following normal-form representation describes the share of Turlock's total population that is "captured" by each station as a function of the times at which the news programs are aired. The stations make their choices simultaneously. The payoffs are listed according to the order RBC, CBC, MBC. Find the set of rationalizable strategies in this game.

RBC \ CBC	6:00	7:00
6:00	~~14~~, 24, 32	~~8~~, 30, 27
7:00	30, 16, 24	13, ~~12~~, 50

RBC \ CBC	6:00	7:00
6:00	16, 24, ~~30~~	30, 16, ~~24~~
7:00	30, 23, ~~14~~	14, 24, ~~32~~

6:00 7:00

MBC

$R^0 = \{6, 7\} \times \{6, 7\} \times \{6pm\} = \{7pm, 6pm, 6pm\}$

$R^1 = \{7\} \times \{6, 7\} \times \{6\}$

$R^2 = \{7\} \times \{6\} \times \{6\}$

HW 7.6

Friday, September 18, 2020 1:11 PM

Passed Solution review

Suppose that in some two-player game, s_1 is a rationalizable strategy for player 1. If, in addition, you know that s_1 is a best response to s_2, can you conclude that s_2 is a rationalizable strategy for player 2? Explain.

If s_2 is a best response to the rationalizable s_1, then s_2 must also be rationalizable.

No. s_1 might be rationalizable because it is a best response to some other strategy, s_2 that is rationalizable, making s_1 rationalizable even if s_2 is not

HW 7.7

Friday, September 18, 2020 1:14 PM

Passed Solution review

Consider a guessing game with ten players, numbered 1 through 10. Simultaneously and independently, the players select integers between 0 and 10. Thus player i's strategy space is $S_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, for $i = 1, 2, \ldots, 10$. The payoffs are determined as follows: First, the average of the players' selections is calculated and denoted a. That is,

$$a = \frac{s_1 + s_2 + \cdots + s_{10}}{10},$$

where s_i denotes player i's selection, for $i = 1, 2, \ldots, 10$. Then, player i's payoff is given by $u_i = (a - i - 1)s_i$. What is the set of rationalizable strategies for each player in this game?

Max $a = 100/10 = 10$
Min $a = 0/10 = 0$

Max $U = a - 10 - 1 = a - 11 \rightarrow$ negative
Min $U = a - 1 - 1 = a - 2 \rightarrow$ maybe negative

If everyone chooses 0, final payoff is non-negative

$a \leq 10$ so $a - 10 - 1 \leq -1$ so $s_{10} = 0$ dominates
Now $a \leq 9$ so $s_9 = 0$ dominates
Now $a \leq 8$ so $s_8 = 0$ dominates

and so on...

$s = (0, 0, \ldots, 0)$

8 Book Notes

Thursday, September 10, 2020 3:59 PM

Location

Firm location is strategy

P and C sell soda at the beach and simultaneously + independently set up for the day
- 9 regions of = size
- 50 purchases in each region
- customers walk to nearest booth

	C	P
1		
2		
3		
4		
5		
6		
7		
8		
9		

Iterated dominance

$S_i = \{1,2,3,4,5,6,7,8,9\}$ reduced to
↳ $R_i^1 = \{2,3,4,5,6,7,8\}$
↳ $R_i^2 = \{3,4,5,6,7\}$
↳ $R_i^3 = \{4,5,6\} \rightarrow R_i^4 = \{5\}$

Criticisms of location model:
1) in context of market competition, doesn't include firms' specification of prices
2) IRL, agents may not move simultaneously
3) cannot apply the model with more than 2 products/firms
4) one-dimensional

Strategic Complementarities

Bob increases Alice's payoff but not his
Bob + Alice working together increases either or both

Contracts about effort can't necessarily be made

Complementarity

Rationalizability leads to unique predictions in 2 player games w/ 3 properties:
1) strategy spaces are intervals w/ lower + upper bounds
2) there are strategic complementarities
3) the slope of the best response functions is <1
↳ these are not required

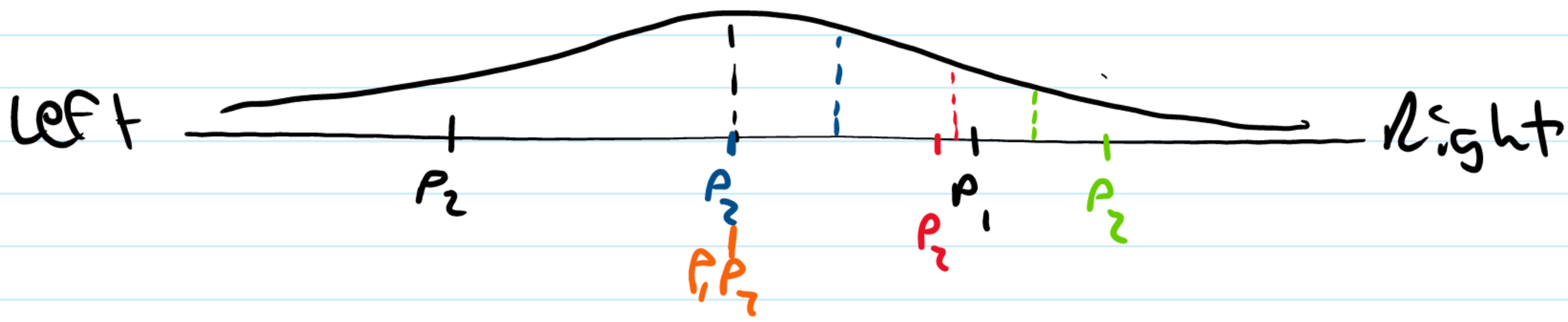
Social Unrest

Strength in numbers

8.1 Location - Median Voter Theorem - Informal

Thursday, September 10, 2020 6:47 PM

median voter theorem



8.2 Location - Median Voter Theorem - Formal

Thursday, September 10, 2020 6:57 PM

Location → median voter theorem
- Rationalizable platforms

A + B, max share

FL L CL C CR R FR

A chooses FL, B chooses FL VA = VB = .5
B chooses L VA = 1/7, VB = 6/7

A chooses L, B chooses FL VA = , VB = 1/7
L = .5
CL = 11/14

A chooses CL, B chooses FL = 3/14
L = 2/7
CL = 1/2

FL is dom for B
FL is dom for A → and also FR

If there are even positions, any combo of 2 middle are rationalizable

8.3 Partnership - Separable Activities

Tuesday, September 15, 2020 10:07 PM

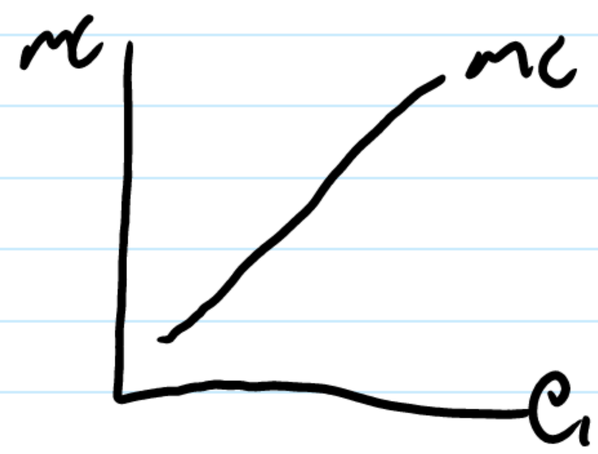
Partnership

Value $= V(e_1, e_2) = e_1 + e_2$

effort

$c_1(e_1) = \frac{1}{2}e_1^2$ $\quad$ $c_2(e_2) = \frac{1}{2}(e_2)^2$

$\frac{dc}{de_1} = e_1$



Each keeps $\frac{1}{2}V$

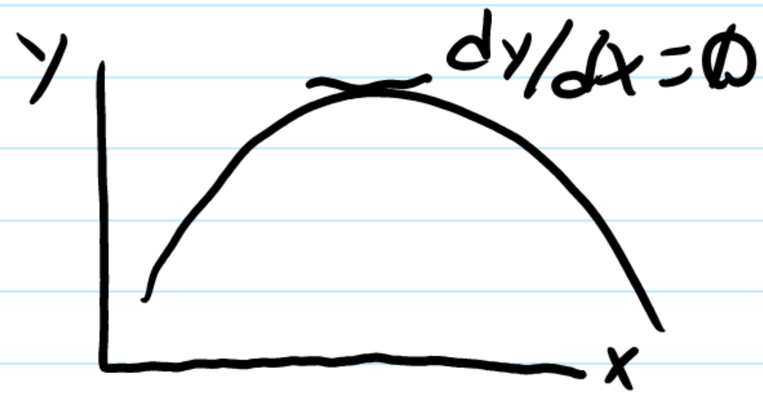
$\pi_1 = \frac{1}{2}V - c_1(e_1)$

$= \frac{1}{2}e_1 + \frac{1}{2}e_2 - \frac{1}{2}e_1^2$

$\frac{d\pi_1}{de_1} = \frac{1}{2} - 2(\frac{1}{2}) \cdot e_1^{(2-1)}$

$= \frac{1}{2} - e_1 = 0$

$e_1 = 1/2$



Same for P_2, $e_2 = 1/2$

$\pi_1 = \frac{1}{2}(\frac{1}{2} + \frac{1}{2}) - \frac{1}{2} \cdot \frac{1}{2}^2$

$= \frac{1}{2}(1) - \frac{1}{2}(\frac{1}{4})$

$= \frac{1}{2} - \frac{1}{8}$

$= 3/8$

$\pi_2 = 3/8$ $\quad \rightarrow \quad \pi = \pi_1 + \pi_2 = 3/4$

$\pi = e_1 + e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$

$d\pi/de_1 = 1 - e_1 = 0 \rightarrow e_1 = 1$

$d\pi/de_2 = 1 - e_2 = 0 \rightarrow e_2 = 1$

$\pi = 2 - \frac{1}{2} \cdot 1^2 - \frac{1}{2} \cdot 1^2 = 1$

profit

If independent...

$\pi = e_1 - \frac{1}{2}e_1^2$ $\quad$ $d\pi/de_1 = 1 - e_1 = 0 \rightarrow e_1 = 1$

same for P_2

8.4 Partnership - Inseparable Activities - Best Responses

Tuesday, September 15, 2020 10:24 PM

$$V = e_1 + e_2 + \tfrac{1}{4} e_1 e_2$$

$$c_1 = \tfrac{1}{2} e_1^2 \qquad c_2 = \tfrac{1}{2} e_2^2$$

$$\pi_1 = \tfrac{1}{2}(e_1 + e_2 + \tfrac{1}{4} e_1 e_2) - \tfrac{1}{2} e_1^2$$

$$\downarrow = \tfrac{1}{2}(e_1 + \bar{e}_2 + \tfrac{1}{4} e_1 \bar{e}_2) - \tfrac{1}{2} e_1^2$$

$$\sum_i P(X = x_i) \cdot x_i = E(x)$$

$$\int_0^\infty \Theta_2(v) \cdot v \, dv = E(e_2) = \bar{e}_2$$

from P_1 view

$$\pi_1 = \tfrac{1}{2} e_1 + \tfrac{1}{2} \bar{e}_2 + \tfrac{1}{8} e_1 \bar{e}_2 - \tfrac{1}{2} e_1^2$$

$$\frac{d\pi_1}{de_1} = \tfrac{1}{2} + \tfrac{1}{8} \bar{e}_2 - e_1 = 0 \rightarrow e_1 = \tfrac{1}{2} + \tfrac{1}{8} \bar{e}_2$$

$$e_2 = \tfrac{1}{2} + \tfrac{1}{8} \bar{e}_1$$

8.5 8.6 Partnership - Inseparable Activities

Tuesday, September 15, 2020 10:41 PM

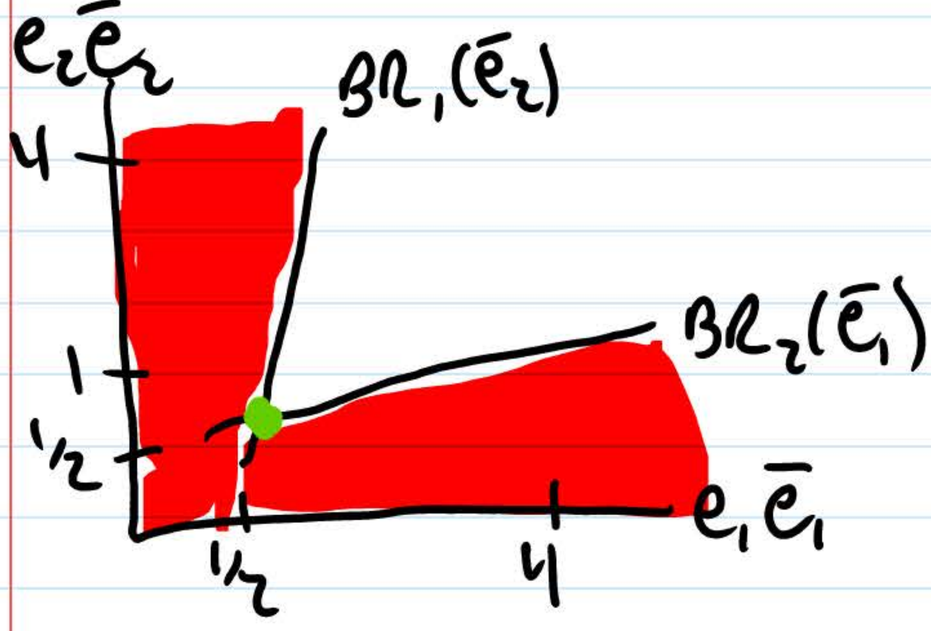
$e_1 < \frac{1}{2}$ is dom $\quad e_2 < 1/2$ is dom

$e_1 \le 4$

$e_2 \le 4$

R' $e_1 < 9/16$ $\quad e_2 < 9/16$ is dom $\rightarrow$ 4 as is > 1

$e_1 < \frac{1}{2} + \frac{1}{8}(9/16)$ is dom $\rightarrow$ same for e_2 $\rightarrow$ $> \frac{1}{2} + (\frac{1}{8} \cdot \frac{5}{8})$ is dom



$e_1 = \frac{1}{2} + \frac{1}{8} e_2$

$e_2 = \frac{1}{2} + \frac{1}{8} e_1$

$e = \frac{1}{2} + \frac{1}{8} e_1 \rightarrow 7/8 e_1 + 1/2 \rightarrow e_1 = 4/7 = e_2$

$e_2 = \frac{1}{2} + \frac{1}{8} \bar{e}_1 \, LL$ $\quad e_2 > LL$ when $\frac{1}{2} + \frac{1}{8} LL > LL \rightarrow 4/7 > LL$

lower limit

$4/7 < UL$

upper limit

I found a stylus that works

I found a stylus that works

8.7 8.8 Inefficiency in the Partner Game

Saturday, September 19, 2020 2:56 PM

A

$\pi = e_1 + e_2 + \frac{1}{4}e_1e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$

$\frac{d\pi}{de_1} = 1 + \frac{1}{4}e_2 - e_1 = 0$

$e_1 = 1 + 1/4\, e_2$

↳ same for e_2 (but flipped)

↳ $e_1 = 1 + 1/4\, e_1$

$e_1 = 4/3 = e_2$

$\pi = 4/3 + 4/3 + 1/4 \cdot 4/3 \cdot 4/3 - \frac{1}{2}(4/3)^2 - 1/2(4/3)^2 = 4/3$

$\pi_1 = \pi_2 = 2/3$

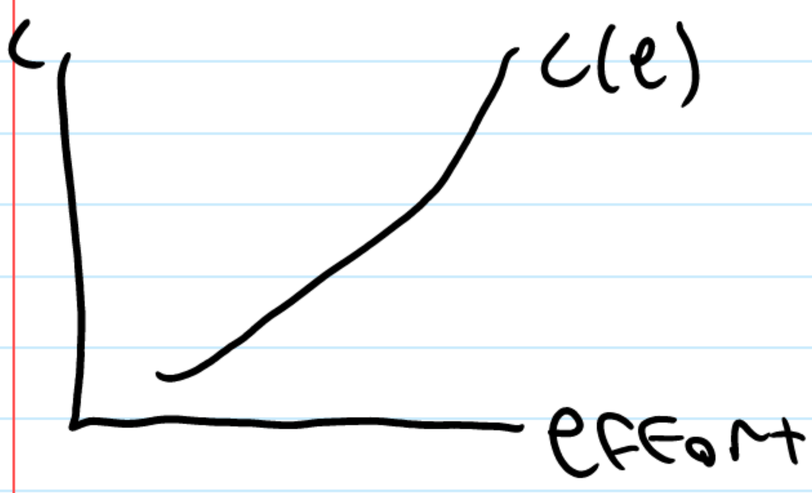
B

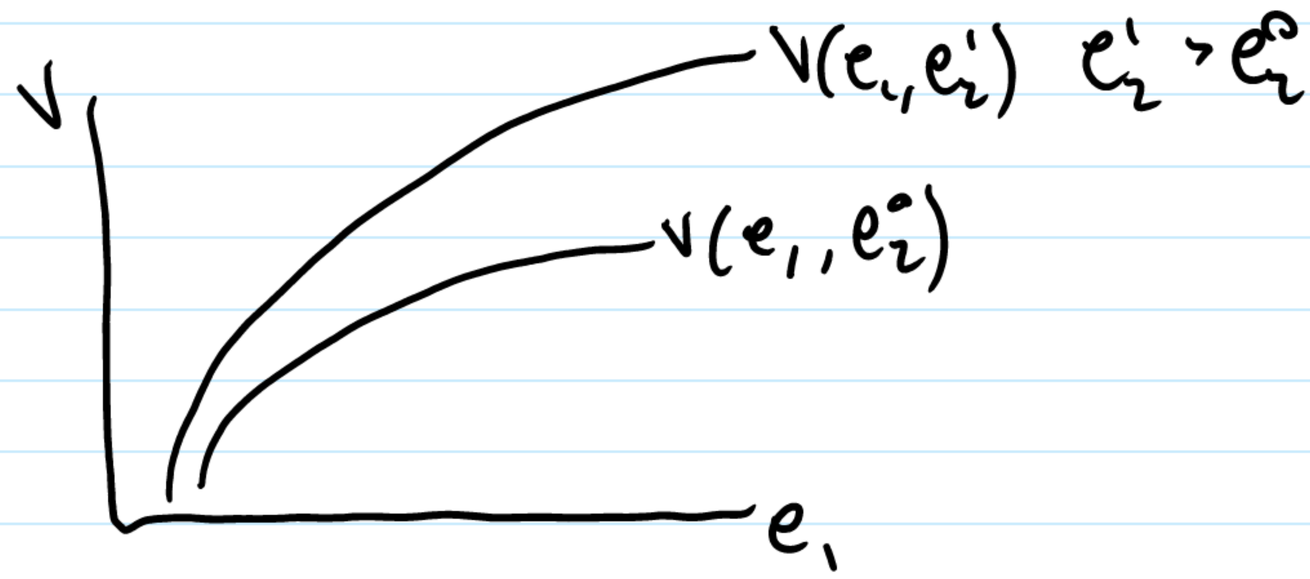
$\pi_1 = 1/2(4/7 + 4/7 + \frac{1}{4}(4/7 \cdot 4/7)) - \frac{1}{2}(4/7)^2 = 22/49$

↳ $< 2/3$

8.9 Joint Production and the Theory of the Firm

Saturday, September 19, 2020 3:08 PM





$v(e_1, e_2) - c_1(e_1) - c_2(e_2)$

$dv/de_1 - dc_1/de_1 = 0$

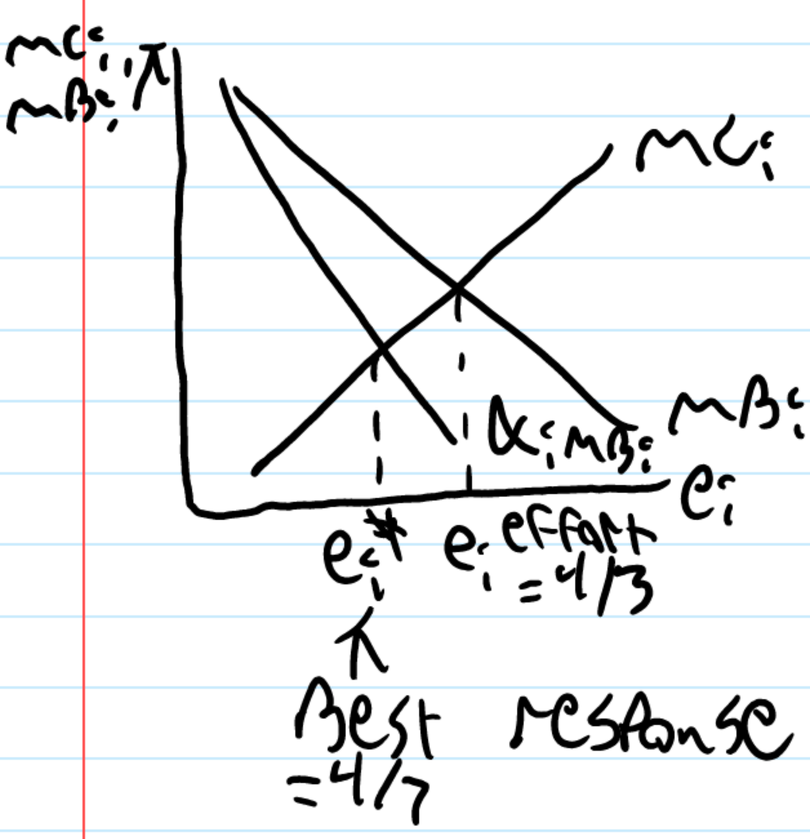
$MB_1 = MC_1$

$MB_2 = MC_2$

$\alpha_i v(e_i, e_{-i}) - c_i(e_i)$

$d\pi_i/de_i = \alpha_i \, dv/de_i - dc/de_i = 0$

$\alpha_i MB_i = MC_i \rightarrow \alpha < 1$



<u>Residual Claimant</u>

8.10 8.11 Social Unrest - Elements of the Model ~ alpha is 0

Saturday, September 19, 2020 3:58 PM

- everyone prefers unidirectional reform
- $i \in [0,1]$ index propensity to protest
  - expressing voice
  - maybe not peacefully
  uniformly distributed
- Payoffs α_i if protest, $\alpha > 0$
  - let x = fraction that protests
  - $\mu = E(x)$, beliefs
  - everyone gains $\beta(x-\lambda)$
  - Protesters gain $\delta(x-\lambda)$
  - non protestors gain $\gamma(\lambda-x)$

$u_i(P,x) = \alpha_i + \beta(x-\lambda) + \delta(x-\lambda) \rightarrow$ Protest

$u_i(H,x) = \gamma(\lambda-x) + \beta(x-\lambda) \rightarrow$ Stay home

$u_i(P,\mu) > u_i(H,\mu)$

$\alpha_i + \delta(\mu-\lambda) > \gamma(\lambda-\mu)$

$\alpha_i > (\gamma+\delta)(\lambda-\mu)$

$i > (\gamma+\delta)/\alpha \cdot (\lambda-\mu)$

$\alpha = 0$

$0 > (\gamma+\delta)(\lambda-\mu)$

suppose $\lambda > 1 \Rightarrow \mu = 0$

now $\lambda \downarrow$ $\lambda < 1$

History + current events coordinate

i protests if $i > (\delta+\gamma)/\alpha \cdot (\lambda-\mu)$

$a = (\delta+\gamma)/\alpha$

8.12 8.13 Analysis when Protest or Voice has value - alpha gt 0

Saturday, September 19, 2020 4:19 PM

$$a = \frac{(\delta + \gamma)}{\alpha}$$

assume $\lambda << 1$

↳ if λ big enough, most zealous won't protest

$1 \cdot \alpha > (\delta + \gamma)(1 - 0)$

$\alpha > (\delta + \gamma)$

$a < 1$

What beliefs are rationalizable in this world?

$\mu = 1$ $\quad i > (\lambda - \mu) a < 0 \quad 0 < a < 1, \lambda << 1$

$\mu = 0$ $\quad i > a\lambda$

$1 > a\lambda > 0$

$\mu = 1 - a\lambda$ $\quad i > a\lambda - a\mu$

$> a\lambda - a(1 - a\lambda)$

$x = 1 - a\lambda - a(1 - a\lambda) = (1 - a\lambda)(1 + a) > (1 - a\lambda)$

$\mu = (1 - a\lambda)(1 + a)$ $\quad i > a\lambda(1 - a\lambda)(1 + a)$

$x = 1 - a\lambda + a(1 - a\lambda)(1 + a)$

$= (1 - a\lambda)(1 + a + a^2)$

$\mu = (1 + a\lambda)(1 + a + a^2 + a^3 + \ldots)$

$a < 1$

8.14 Aside on the Geometric Series

Saturday, September 19, 2020 4:35 PM

Infinite sum of a^x, $0 < a < 1$

$$A = a^0 + a^1 + a^2 + \dots a^T + a^{T+1}$$

$$\downarrow$$

$$1$$

$$aA = a^1 + a^2 + \dots a^{T+1}$$

$$A - aA = 1 - a^{T+1}$$

$$A(1-a) = 1 - a^{T+1}$$

$$A = (1 - a^{T+1})(1/1-a)$$

$$\lim_{T \to \infty} A \qquad a^{T+1} \to 0 \qquad A = 1/1-a$$

8.15 Social Unrest - Conclusion

Saturday, September 19, 2020 4:45 PM

$$\mathcal{M} = (1-a\lambda)(1+a+a^2+\dots) \qquad \lambda \ll 1 \qquad a\lambda < a$$
$$= (1-a\lambda)/(1-a) > 1$$

$(\delta+\gamma) < \alpha \leftrightarrow$ even if $\lambda = 1$, $i = 1$ protests

$i = 1-\epsilon$ protests

tiny number

HW 8.2

Sunday, September 20, 2020 3:57 PM

Passed Solution review

Consider a location game with nine regions like the one discussed in this chapter. But instead of having the customers distributed uniformly across the nine regions, suppose that region 1 has a different number of customers than the other regions. Specifically, suppose that regions 2 though 9 each has ten customers, whereas region 1 has x customers. For what values of x does the strategy of locating in region 2 dominate locating in region 1?

x	10	10	10	10	10	10	10	10
1	2	3	4	5	6	7	8	9

Region 2 only makes sense if $x < 80$ because at that point region 1 is less than the rest of the board combined

Player 2 location

P1 Location	1	2	3	4	5	6	7	8	9
1	$\frac{x}{2}+40$	x	$x+5$	$x+10$	$x+15$	$x+20$	$x+25$	$x+30$	$x+35$
2	80	$\frac{x}{2}+40$	$x+10$	$x+15$	$x+20$	$x+25$	$x+30$	$x+35$	$x+40$
...									

only P1's payoffs

$80 > \frac{x}{2}+40$
$40 > x/2$
$80 > x$

$x/2+40 > x$
$40 > x/2$
$80 > x$

HW 8.6

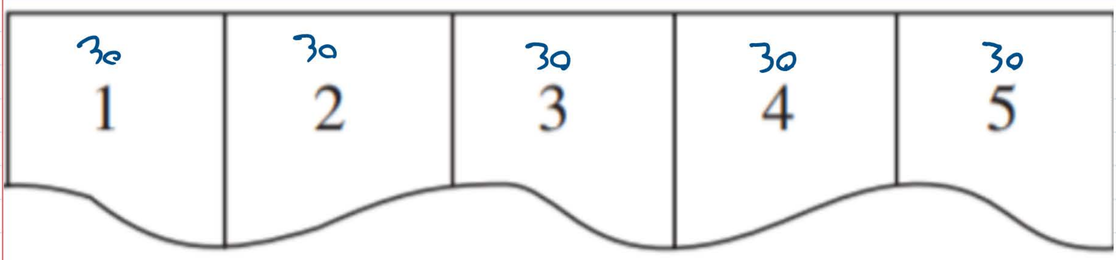
Sunday, September 20, 2020 4:28 PM

Passed solution review

Consider a location game with five regions on the beach in which a vendor can locate. The regions are arranged on a straight line, as in the original game discussed in the text. Instead of there being two vendors, as with the original game, suppose there are *three* vendors who simultaneously and independently select on which of the five regions to locate. There are thirty consumers in each of the five regions; each consumer will walk to the nearest vendor and purchase a soda, generating a $1.00 profit for the vendor. Assume that if some consumers are the same distance from the two or three nearest vendors, then these consumers are split equally between these vendors.

(a) Can you rationalize the strategy of locating in region 1?

(b) If your answer to part (a) is "yes," describe a belief that makes locating at region 1 a best response. If your answer is "no," find a strategy that strictly dominates playing strategy 1.



max rev = 30 · 5 · 1 = 150

a) It's never the best strategy to locate in region 1 unless it holds value in a different way (ATM, picnic tables, etc) or if it is guaranteed that all players choose region 1

b) No: location 3 would dominate because it is in the middle and can draw both sides

Yes: If there are other amenities that would draw more people to region 1, you would need to get at least 20 extra people

A mix of 2 & 3, $\sigma_i = (0, p, 1-p, 0, 0)$ dominates 1. Tedious to show

2 & 3 each weakly dominate 1. So, the mix strategy dominates 1.

3 chooses s = 1

	1	2	3	4	5
1	50	15	25	40	55
2	15	30	30	30	30
3	25	30	40	45	45
4	40	30	45	70	75
5	55	30	45	75	100

3 chooses s = 2

	1	2	3	4	5
1	120	60	30	45	75
2	60	50	30	40	55
3	30	30	60	60	60
4	45	40	60	70	75
5	75	35	60	75	100

3 chooses s = 3

	1	2	3	4	5
1	100	90	65	45	60
2	90	90	45	30	45
3	65	45	50	45	55
4	45	30	45	90	90
5	60	45	55	90	100

HW 8.7

Sunday, September 20, 2020 4:41 PM

Passed Solution review

Consider a game in which, simultaneously, player 1 selects a number $x \in [2, 8]$ and player 2 selects a number $y \in [2, 8]$. The payoffs are given by:

(2,3) (8,8)

$$u_1(x, y) = 2xy - x^2 \quad 28 \quad 8 \quad 32 \quad 64$$

$$u_2(x, y) = 4xy - y^2. \quad 0 \quad 15 \quad 60 \quad 192$$

(2,8) (4,6)

Calculate the rationalizable strategy profiles for this game.

$u_1\ d1 = 2y - 2x = -2(y-x)$

$u_1\ d2 = -2$

$u_2\ d1 = 4x - 2y$

$u_2\ d2 = -2$

$2 \cdot 8 \cdot 2 - 8^2 = 4 \cdot 8 - 64 = 32 - 64 = -32$

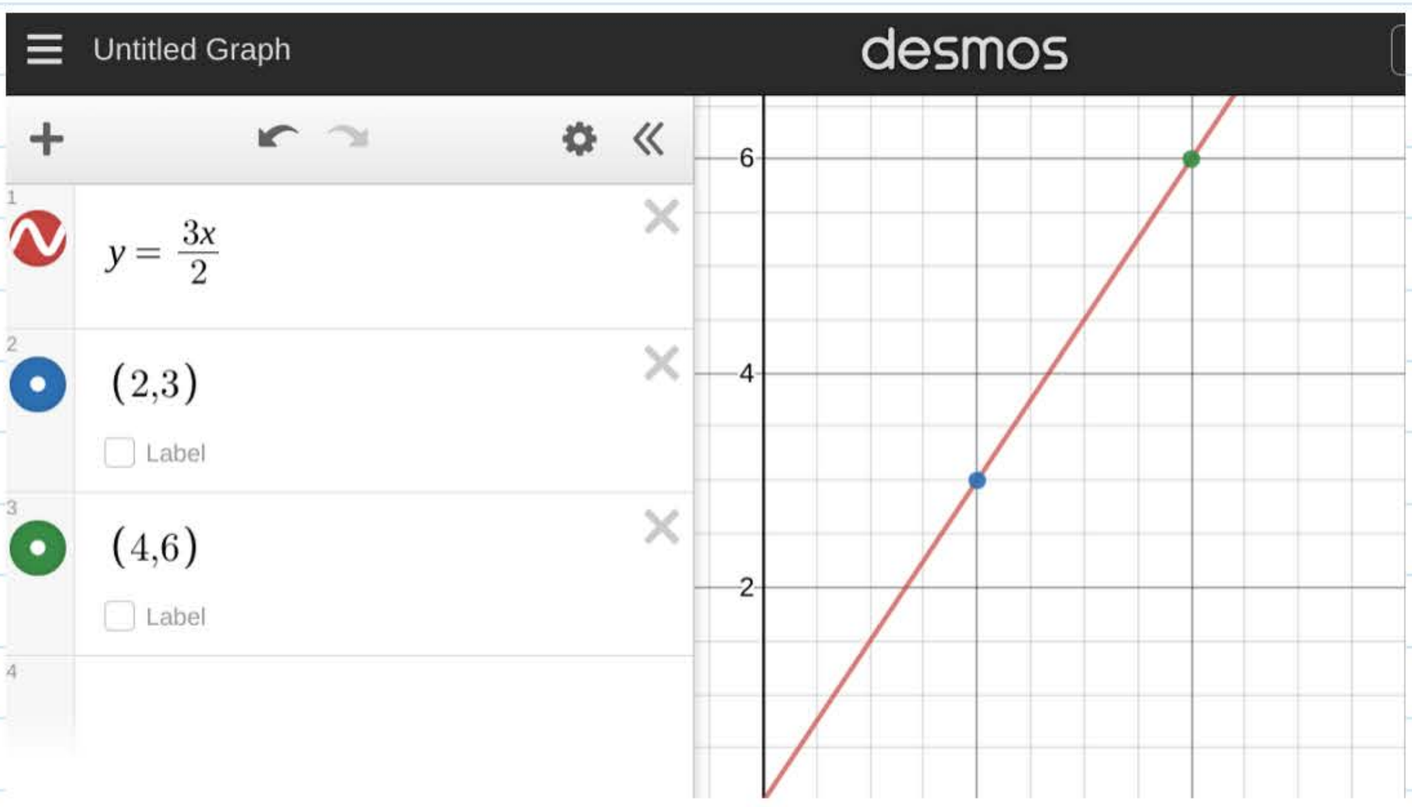
$4 \cdot 8 \cdot 2 - 2^2 = 64 - 4 = 60$

$2y - 2x = 0$

$4x - 2y = 0 \rightarrow y = \frac{3}{2}x$

↳ $x = 2, y = 3$ or $x = 4, y = 6$ are the only whole numbers between 2 and 8

But if x and y are both 8, the payoffs are $u_1 = 64$ and $u_2 = 192$



$$\frac{1}{2}\left[(2xy - x^2) + (4xy - y^2) + \frac{1}{4}(2xy - x^2)(4xy - y^2)\right] - \frac{1}{2}(2xy - x^2)^2$$

$du_1/dx = 2\bar{y} - 2x = 0 \rightarrow BR_1(\bar{y}) = \bar{y}$

$du_2/dx = 4\bar{x} - 2y = 0 \rightarrow BR_2(\bar{x}) = \begin{cases} 2\bar{x} & \bar{x} \le 4 \\ 8 & \bar{x} > 4 \end{cases}$

Suppose $\bar{x} = 2$ then $y = 4$

$\bar{y} = 4$, then $x = 4$

$\bar{x} = 4$, then $y = 8$

$\bar{y} = 8$, $x = 8$ → $s = (8, 8)$ is the only rationalizable strategy profile

HW 8.8

Sunday, September 27, 2020 3:30 PM

Passed solution review

8. Finish the analysis of the "social unrest" model by showing that for any $\alpha > 2$, the only rationalizable strategy profile is for all players to protest. Here is a helpful general step: Suppose that is it common knowledge that all players above y will protest, so $x \geq 1 - y$. Find a cutoff player number $f(y)$ with the property that given $x \geq 1 - y$, every player above $f(y)$ strictly prefers to protest.

Stay home — Protest

$u_i(H, x) = 4x - 2 \qquad u_i(P, x) = \gamma x - 4 + \alpha_i$

$i \in (y, 1]$

$\gamma(1-y) - 4 + \alpha_i > 4(1-y) - 2$

$\gamma - \gamma y - 4 + \alpha_i > 4 - 4y - 2$

$4 - \gamma y + \alpha_i > 2 - 4y$

$2 - \gamma y + \alpha_i > -4y$

$2 + \alpha_i > 4y$

$\alpha_i > 4y - 2$

$i > (4y-2)/\alpha$

if $\alpha = 2$ and $y \geq 1$, people will protest.

i protests if $\gamma x - 4 + \alpha_i > 4x - 2$

$\alpha_i > 2 - 4x$

Suppose $E(x) = \bar{x} = 0$

This becomes $i > 2/\alpha$

if $\alpha > 2$ $2/\alpha < 1$ so someone will protest even if $\bar{x} = 0$

therefore $\bar{x} = 0$ is not rationalizable

Suppose $\bar{x} = 1 - 2/\alpha$ which is the least rationalizable belief in R^1

Now i protests if $\alpha_i > 2 - 4(1 - 2/\alpha)$

$\alpha_i > -2 + 8/\alpha$

$i > -2/\alpha + 8/\alpha^2$

The least rationalizable belief is now

$\bar{x} = 1 + 2/\alpha - 8/\alpha^2$

$= 1 - 2/\alpha(4/\alpha - 1) > 1 - 2/\alpha$

each round x grows until

$\alpha(1-x) = 2 - 4x$

$\therefore x = \alpha - 2/\alpha - 4 > 1 \rightarrow$ so $x = 1$

9 Book Notes

Wednesday, September 23, 2020 9:04 AM

Nash Equilibrium

Rationalizability means:
1) Players form beliefs about each other's behavior
2) Players best respond to their beliefs
3) these facts are common knowledge among players

Behavior is **congruous** (coordinated) through social norms

Conguity:
1) games are repeated in society and player behavior "settles down" and the same strategies are repeated
2) Players meet before the game + agree on the strategies used. Players honor the agreement
3) Outside mediator recommends strategy profiles. Each player expects others to follow the recommendation + has incentive to do so themselves

Nash equilibrium: strategy profile $s \in S$ is a Nash equilibrium if and only if $s_i \in BR_i(s_{-i})$ for each player i. That is, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for every $s_i' \in S_i$ and each player i

Strict Nash equilibrium: strategy profile s is strict if and only if $\{s_i\} = BR_i(s_{-i})$ for each player i

To find: "Look for profiles such that each player's strategy is a best response to the strategy of the other"

Each Nash equilibrium is a rationalizable strategy

Equilibrium of the Partner Game

Coordination + Social Welfare

Nash equilibria don't always entail strategies that are preferred by the players as a group

↳ Nash of prisoner's dilemma (1,1) is inefficient

The third strategic tension

Coordinated inefficiency → QWERTY, VHS, etc.

Congruous Sets

Nash is only when players coordinate on a single profile

Set X is a congruous set because coordinating on X is consistent with common knowledge of BR

Congruous if X_i contains exactly those strategies that can be rationalized

Weakly congruous if each strategy in X_i can be rationalized with respect to X_{-i}

Experimental Game Theory

Strategic sophistication

9.1 Nash Equilibrium - General

Wednesday, September 23, 2020 11:54 AM

Solution Concepts So Far
1) Strongly dominant strategies
2) Weakly dominant strategies
3) Set of rationalizable strategies
   ↳ What if there's more than 1?
4) Nash Equilibrium
   ↳ $s \in S$ is a NE if $s_i \in BR_i(s_{-i})$ $\forall i$
   ↳ or $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ $\forall i$

Congruent

	B_1	B_2	B_3
A_1	$\boxed{3,1}$	2,3	4,0
A_2	0,6	0,0	2,0
A_3	3,2	1,4	$\boxed{3,1}$

Best replies given other player's strategy

(A_1, B_2) is only Pure strategy NE in game

1) Repeated observed play across society
2) Players talk before playing + honoring agreement

9.2 Nash Equilibrium - Examples

Wednesday, September 23, 2020 12:20 PM

A\B	Rat	Silent
Rat	0*, 0*	3*, -1
Silent	-1, 3*	2, 2

(Rat, Rat)

A\B	H	T
H	1*, -1	-1, 1*
T	-1, 1*	1*, -1

No Pure NE

A\B	Game	Show
Game	2, 3	0, 1
Show	1, 0	3, 2

2 Pure NE

↳ {G, S} x {G, S}

A\B	Stag	Hare
Stag	4, 4	0, 1
Hare	1, 0	1, 1

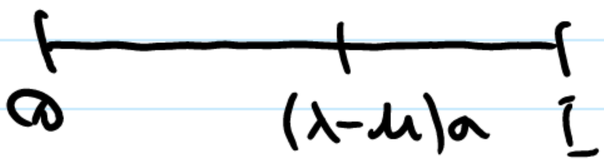
2 Pure NE

3rd strategic tension - Possibility of socially inefficient coordination

9.3 Nash Equilibrium - Social Unrest

Friday, September 25, 2020 7:17 PM

$i > (\lambda - \mu)(\sigma + \gamma/\alpha)$ ← Protest equation

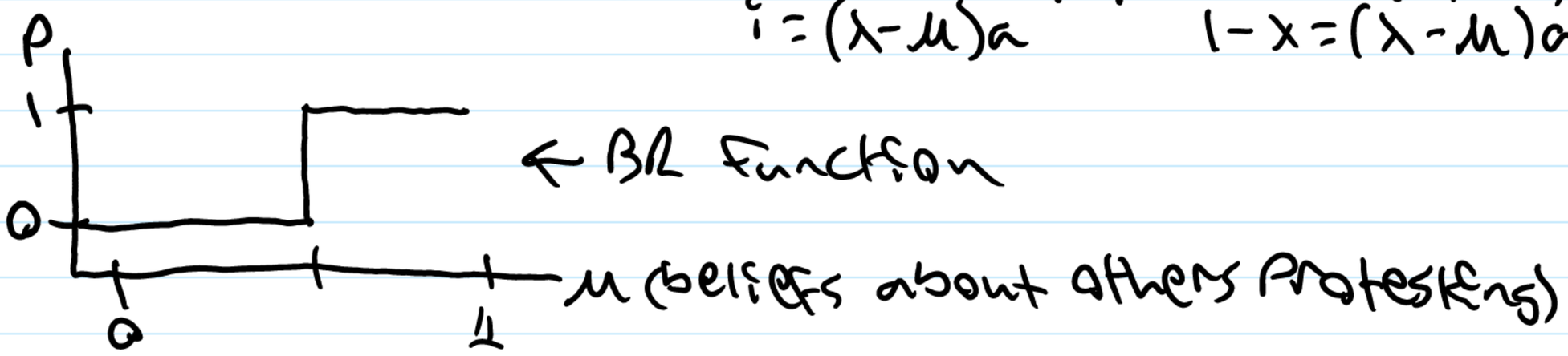


BR $P = 1$ $i > (\lambda - \mu)a$

$P = 0$ $i < (\lambda - \mu)a$

= indifferent to protest if $i = (\lambda - \mu)a$ or

$i = (\lambda - \mu)a$ $1 - x = (\lambda - \mu)a$



← BR function

Fraction that turns out $1 - x = (\lambda - \mu)a \rightarrow \mu = (1 - a\lambda)/(1 - a)$

where $\lambda < 1 \rightarrow \mu > 1$

if $\lambda \leq 1 \rightarrow \mu = 1$ $x = 1$ → everyone protests

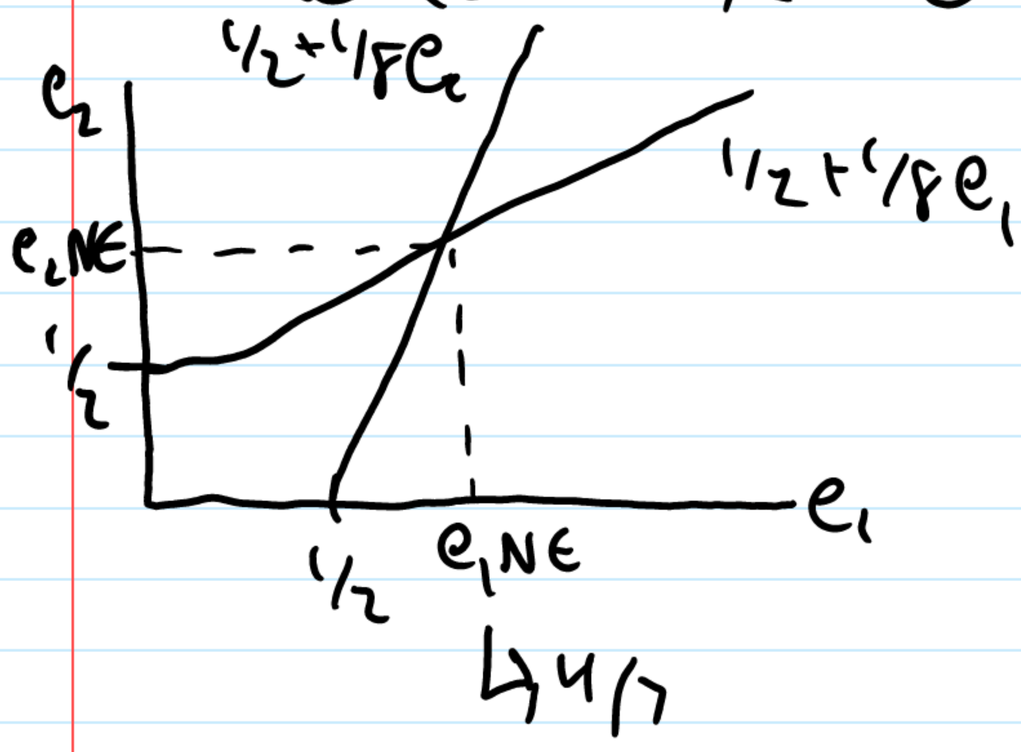
9.4 Nash Equilibrium - Partnership

Friday, September 25, 2020 7:26 PM

$$u_1 = \frac{1}{2}(e_1 + e_2 + e_1 e_2) - \frac{1}{2}e_1^2$$

$$\frac{du_1}{de_1} = \frac{1}{2}(1 + \frac{1}{4}\bar{e}_2) - e_1 = 0 \qquad BR_1(\bar{e}_2) = \frac{1}{2} + \frac{1}{8}\bar{e}_2 \leftarrow \text{2 equations - 2 unknown}$$

Same for Player 2



$$e_1 = e_2$$

HW 9.2

Thursday, October 1, 2020 10:11 AM

Passed Solution review

Find the Nash equilibria of and the set of rationalizable strategies for the games in Exercise 1 at the end of Chapter 6.

NE = (B,L) or S = (B,L)

1 \ 2	L	R
A	3, 3	2, 0
B	4, 1	8, −1

(a)

NE = (U,L) and (M,C)

L dom R

1 \ 2	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

(b)

UL makes more sense

NE = (U,X)

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

1 \ 2	L	R
U	1, 1	0, 0
D	0, 0	5, 5

(d)

All S are rationalizable

S = (U,X)

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

HW 9.4

Thursday, October 1, 2020 10:22 AM

Passed Solution review

Compute the Nash equilibria of the following location game. There are two people who simultaneously select numbers between zero and one. Suppose player 1 chooses s_1 and player 2 chooses s_2. If $s_i < s_j$, then player i gets a payoff of $(s_i + s_j)/2$ and player j obtains $1 - (s_i + s_j)/2$, for $i = 1, 2$. If $s_1 = s_2$, then both players get a payoff of $1/2$.

$s_i = x \quad s_j = y$

$$\frac{(x+y)}{2} = 1 - \frac{(x+y)}{2}$$

$$\frac{2(x+y)}{2} = 1$$

$$x+y = 1$$

if s_i and s_j both $= 1/2$, then they maximize and won't deviate

HW 9.6

Thursday, October 1, 2020 10:28 AM

Passed Solution review

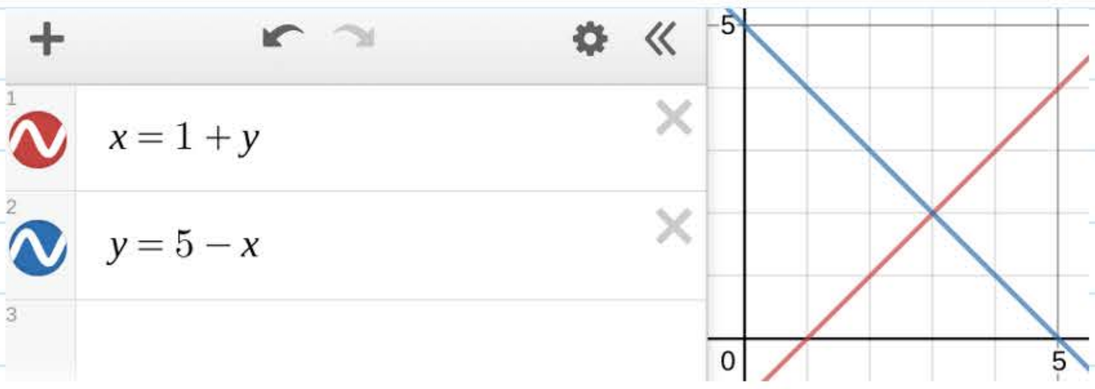
Consider a game in which, simultaneously, player 1 selects any real number x and player 2 selects any real number y. The payoffs are given by:

$$u_1(x, y) = 2x - x^2 + 2xy$$
$$u_2(x, y) = 10y - 2xy - y^2.$$

(a) Calculate and graph each player's best-response function as a function of the opposing player's pure strategy.

$du_1/dx = 2 - 2x + 2 \rightarrow x = 1 + y$

$du_2/dx = 10 - 2y - 2x \rightarrow y = 5 - x$



(b) Find and report the Nash equilibria of the game.

$x = 1 + 5 - x$

$x = 6 - x$

$2x = 6$

$x = 3 \rightarrow y = 5 - 3 = 2$

$NE = (3, 2)$

(c) Determine the rationalizable strategy profiles for this game.

$S = (3, 2)$

HW 9.10

Thursday, October 1, 2020 10:38 AM

Passed Solution review

Is the following statement true or false? If it is true, explain why. If it is false, provide a game that illustrates that it is false. "If a Nash equilibrium is not strict, then it is not efficient."

B	y	z
A w	1,1	1,2
x	2,1	4,4

False. (w,y) and (x,z) are both NE but (x,z) is better than (w,y)

HW 9.11

Sunday, October 4, 2020 4:18 PM

Passed Solution review

This exercise asks you to consider what happens when players choose their actions by a simple rule of thumb instead of by reasoning. Suppose that two players play a specific finite simultaneous-move game many times. The first time the game is played, each player selects a pure strategy at random. If player i has m_i strategies, then she plays each strategy s_i with probability $1/m_i$. At all subsequent times at which the game is played, however, each player i plays a best response to the pure strategy actually chosen by the other player the *previous* time the game was played. If player i has k strategies that are best responses, then she randomizes among them, playing each strategy with probability $1/k$.

(a) Suppose that the game being played is a prisoners' dilemma. Explain what will happen over time.

They confess forever because it's always the best response.

(b) Next suppose that the game being played is the battle of the sexes. In the long run, as the game is played over and over, does play always settle down to a Nash equilibrium? Explain.

	A	B
A	2,1	0,0
B	0,0	1,2

If (0,0) is chosen first, then (0,0) is the BR so they look for each other forever. If (2,1) or (1,2) is chosen first, it settles.

(c) What if, by chance, the players happen to play a strict Nash equilibrium the first time they play the game? What will be played in the future? Explain how the assumption of a strict Nash equilibrium, rather than a nonstrict Nash equilibrium, makes a difference here.

If it's strict, they'll never deviat because strict Nash is always better.

(d) Suppose that, for the game being played, a particular strategy s_i is not rationalizable. Is it possible that this strategy would be played in the long run? Explain carefully.

No. only best responses are played

HW 9.15

Sunday, October 4, 2020 4:43 PM

Worked w/ Dr Dewey's Solutions

Passed Solution review

15. Suppose you know the following for a particular three-player game: The space of strategy profiles S is finite. Also, for every $s \in S$, it is the case that $u_2(s) = 3u_1(s)$, $u_3(s) = [u_1(s)]^2$, and $u_1(s) \in [0, 1]$.

(a) Must this game have a Nash equilibrium? Explain your answer.

$u_2 = 3u_1$ $\quad u_3 = u_1^2 \rightarrow u_1 = \sqrt{u_3}$ $\quad u_2 = 3\sqrt{u_3}$

I'm trying to follow your solution because I don't understand.

Because u_2 and u_3 are quantities of u_1, they always try to maximize u_1. This means that u_1 is always an NE

(b) Must this game have an efficient Nash equilibrium? Explain your answer.

It is efficient because of reasons stated in Part A

(c) Suppose that in addition to the information given above, you know that s^* is a Nash equilibrium of the game. Must s^* be an efficient strategy profile? Explain your answer; if you answer "no," then provide a counterexample.

There could be an inefficient, non-strict NE because conditions don't dictate only having a strict NE

A B

	L	R
U	1,3,1	0,0,0
D	0,0,0	0,0,0

	L	R
U	3,9,9	0,0,0
D	0,0,0	0,0,0

u_1, u_2, u_3

ALU and BLU are both NE, but ALU is inefficient

10 Book Notes

Saturday, September 26, 2020 5:40 PM

Oligopoly, Tariffs, crime, and Voting

Cournot Duopoly model

- no product differentiation

$BR_1(q_2^*) = q_1^*$ and vice versa

- inefficient from firm's point of view
- firms select quantities not prices
- best suited to long production or short consumption cycles

Bertrand Duopoly model

- prices lower + quantities higher than Cournot
- undercut each other so long as price exceeds MC
- price setting

Tariff setting by 2 countries

A model of crime + police

$u_G = -xc^4 - y^2/x$

negative effect of crime

c^4 = cost of 1 police unit

$u_c = y^{1/2}/(1+xy)$

probability of escaping

criminal payoff

The median voter theorem

Strategic voting

10.1 Nash Equilibrium - Cournot Duopoly

Wednesday, September 30, 2020 6:56 PM

$P = 1 - Q$ $\qquad Q = \sum_{i=1}^{n} q_i \qquad Q_{-i} = Q - q_i$

$P = 1 - q_1 - q_2$

$c_i = .2q_i$

$\pi_i = P \cdot q_i - \text{Cost} = (1 - q_1 - q_2)q_i - .2q_i$

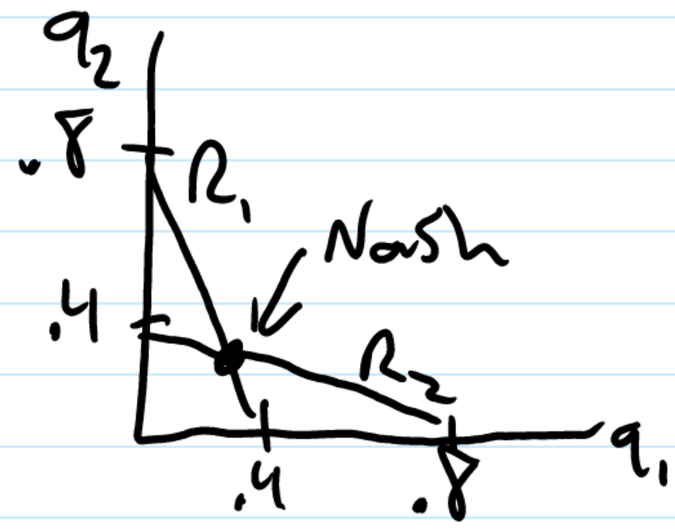
revenue

$\frac{d\pi_1}{dq_1} = -q_1 + (1 - q_1 - q_2) - .2 = 0$

$2q_1 = 1 - q_2 - .2 \rightarrow q_1 = .4 - \frac{1}{2}q_2$

$BR_1(q_2) = .4 - \frac{1}{2}q_2 \rightarrow 0 = .4 - 1/2 q_2$

$\hookrightarrow q_2 = .8$



$\bar{q}_2 \uparrow \rightarrow \bar{q}_1 \downarrow$

Different from partnership

$q_1 = .4 - 1/2 q_1 \rightarrow q_1 = 4/15$

10.2 Nash Equilibrium - Bertrand Duopoly

Wednesday, September 30, 2020 7:10 PM

Announce price P_i
Fill all demand at P_i
Consumers buy from lowest priced firm
If $P_1 = P_2$, split demand

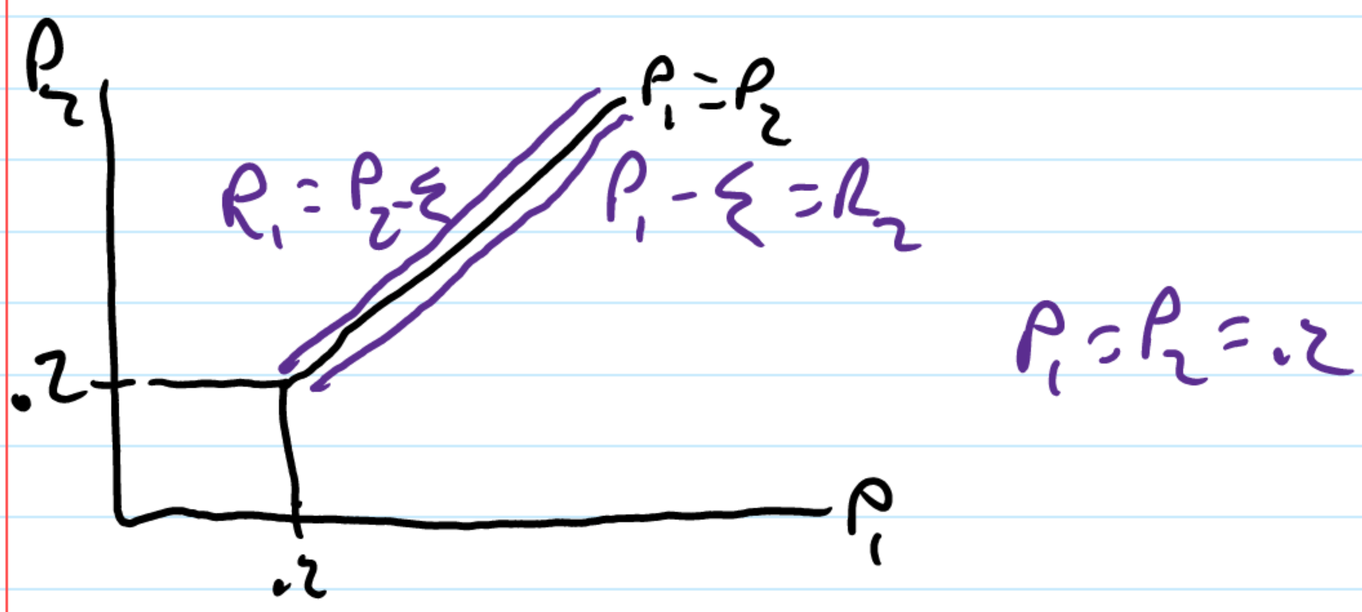
Mkt price $= \min(P_1, P_2) = P$

$Q = 1 - P$

$\pi_1 = (1-P_1)(P_1 - .2)$ if $P_1 < P_2$ $\quad \pi = (1-P_1)(P_1-.2)/2$
$= 0$ if $P_1 > P_2$

$BR = P_2 - \varepsilon \qquad P_2 > .2$

$P_2 = \begin{matrix} \geq .2 \\ \geq .2 \end{matrix} \qquad \begin{matrix} P_2 = .2 \\ P_2 < .2 \end{matrix}$



$P_1 = P_2 = .2$

Price competition is fiercer than quantities

10.3 Crime and Punishment 1

Monday, October 12, 2020 12:50 PM

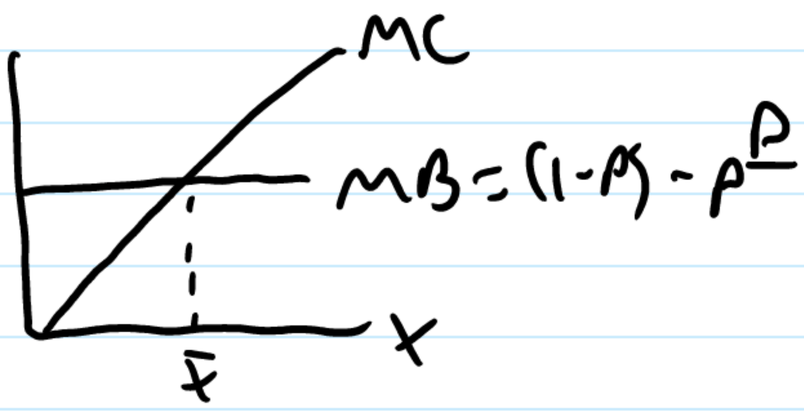
Perp goes free = 1
Perp gets caught =

Commit crime if opportunity cost < return
↳ $x \in [0,1]$

P = Punishment if caught
p = Prob. caught

Crime pays if $(1-p)\cdot 1 - p^P$

$\bar{x} = (1-p) - p^P$



10.4 Crime and Punishment 2

Monday, October 12, 2020 1:08 PM

Model of law enforcement

$\text{Cost} = \frac{1}{2}P^2$

α mitigation of cost of crime when captured

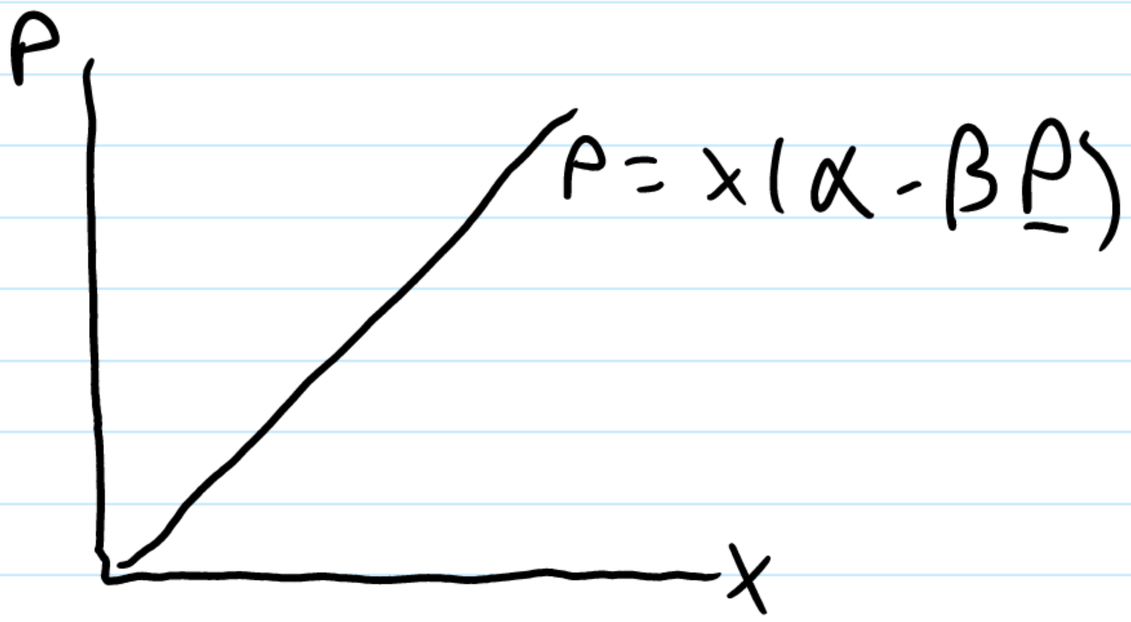
Cost of Punishment/unit $= \beta$

$\bar{x}$ = amount of crime

$u = 1 - (1-\alpha P)\bar{x} - \frac{1}{2}P^2 - P x \underline{P} \beta$

$\frac{du}{dP} = \alpha x - P - x\beta \underline{P} = 0$

$P = x(\alpha - \beta \underline{P})$ $\quad \alpha - \beta \underline{P} > 0$

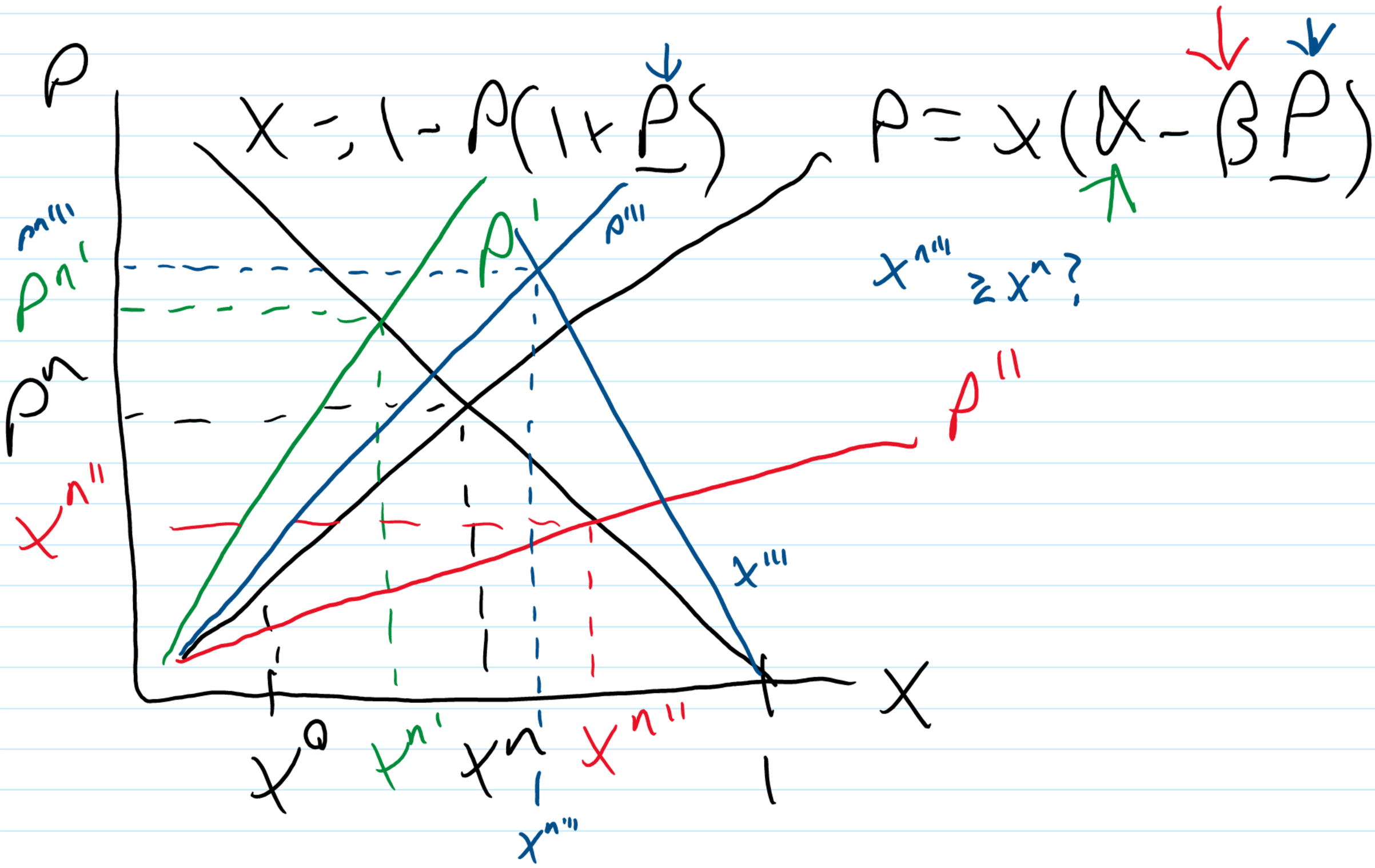


10.5 Crime and Punishment 3

Monday, October 12, 2020 1:23 PM

Equilibrium + comparative statistics in crime + punishment

$X = 1 - P(1+\underline{P})$
$P = X(\alpha - \beta \underline{P})$



$P = (1 - P(1+\underline{P}))(\alpha - \beta \underline{P}) \rightarrow P + P(1+\underline{P})(\alpha - \beta \underline{P}) = \alpha - \beta \underline{P}$

$\hookrightarrow P = (\alpha - \beta \underline{P}) / (1 + (1+\underline{P})(\alpha - \beta \underline{P}))$

$X^{n} = 1 / (1 + (1+\underline{P})(\alpha + \beta \underline{P}))$

10 Extra Problems

Monday, October 19, 2020 10:22 AM

10. Consider a game that has a continuum of players. In particular, the players are uniformly distributed on the interval [0, 1]. (See Appendix A for the definition of uniform distribution.) Each $x \in [0, 1]$ represents an individual player; that is, we can identify a player by her location on the interval [0, 1]. In the game, the players simultaneously and independently select either F or G. The story is that each player is choosing a type of music software to buy, where F and G are the competing brands. The players have different values of the two brands; they also have a preference for buying what other people are buying (either because they want to be in fashion or they find it easier to exchange music with others who use the same software). The following payoff function represents these preferences. If player x selects G, then her payoff is the constant g. If player x selects F, then her payoff is $2m - cx$, where c is a constant and m is the fraction of players who select F. Note that m is between 0 and 1.

(a) Consider the case in which $g = 1$ and $c = 0$. What are the rationalizable strategies for the players? Is there a symmetric Nash equilibrium, in which all of the players play the same strategy? If so, describe such an equilibrium.

(b) Next, consider the case in which $g = 1$ and $c = 2$. Calculate the rationalizable strategy profiles and show your steps. (Hint: Let $\overline{m}$ denote an upper bound on the fraction of players who rationally select F. Use this variable in your analysis.)

(c) Describe the rationalizable strategy profiles for the case in which $g = -1$ and $c = 4$. (Hint: Let $\overline{m}$ denote an upper bound on the fraction of players who rationally select F and let $\underline{m}$ denote a lower bound on the fraction of players who rationally select F.)

b) $g=1 \quad c=2$

$m=0$
fraction buying F

$x, \; G>F$
$x>x, \; G>F$
$\hat{x} \quad G \sim F$
$x>\hat{x} \quad G>F$
$x<\hat{x} \quad G<F$

$x \in [0,1]$
G, F
$g, 2m-cx$

$v(F) = 2m-2x$
$1 = 2m-2x$

c) $g=-1, \; c=4$

$-1 = 2m-4x$
$-1 = 2\hat{x}-4\hat{x}$
$\hat{x} = 1/2$

$m=0$
$-1 = 2\cdot 0 - 4x$
$x = 1/4$

$m = 1/4$
$-1 = 2\cdot 1/4 - 4x$
$4x = 3/2$
$m = 3/8$

$m=1$
$-1 = 2-4x$
$4x = 3$
$x = 3/4$
$m = 3/4$
$x = 5/8$

d) $g=1 \quad c=0 \quad m \in [0,1]$

$1>2m \quad G>F \quad NE \; m=0 \quad v(F) = 0<1$
$\quad \quad F>G \quad NE \; m=1 \quad v(F) = 2\cdot 1 = 2>1$

HW 10.1

Tuesday, October 13, 2020 6:48 PM

Passed Solution review

1. Consider a more general Cournot model than the one presented in this chapter. Suppose there are n firms. The firms simultaneously and independently select quantities to bring to the market. Firm i's quantity is denoted q_i, which is constrained to be greater than or equal to zero. All of the units of the good are sold, but the prevailing market price depends on the total quantity in the industry, which is $Q = \sum_{i=1}^{n} q_i$. Suppose the price is given by $p = a - bQ$ and suppose each firm produces with marginal cost c. There is no fixed cost for the firms. Assume $a > c > 0$ and $b > 0$. Note that firm i's profit is given by $u_i = p(Q)q_i - cq_i = (a - bQ)q_i - cq_i$. Defining Q_{-i} as the sum of the quantities produced by all firms except firm i, we have $u_i = (a - bq_i - bQ_{-i})q_i - cq_i$. Each firm maximizes its own profit.

(a) Represent this game in the normal form by describing the strategy spaces and payoff functions.

$P = a - bQ$ $\quad u_i = (a - bq_i - bQ_{-i})q_i - cq_i \rightarrow aq_i - bq_i^2 - bQ_{-i}q_i - cq_i$

Firm i \ Firm -i

$S_i = [0, \infty)$

$u_i = (a - bq_i - bQ_{-i})q_i - cq_i$

$du/dq_i = -2bq_i - bQ_{-i} - c = 0$

$2bq_i = -bQ_{-i} - c \rightarrow q_i = (-bQ_{-i} - c)/2b$

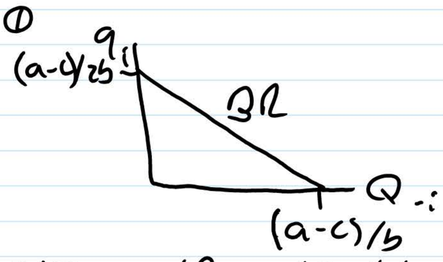
(b) Find firm i's best-response function as a function of Q_{-i}. Graph this function.

$du/dq_i = a - bQ_{-i} - c - 2bq_i = 0$

$2bq_i = a - bQ_{-i} - c$

$q_i = (a - bQ_{-i} - c)/2b$

$q_i = (a - 0 - c)/2b$



$Q = (a - bQ_{-i} - c)/2b \rightarrow$ WolframAlpha to solve

$c = a - bq$

$bq = a - c$

$q = (a - c)/b$

(c) Compute the Nash equilibrium of this game. Report the equilibrium quantities, price, and total output. (Hint: Summing the best-response functions over the different players will help.) What happens to the equilibrium price and the firm's profits as n becomes large?

$Q^* = nq^*$

$Q^*_{-i} = (n-1)q^*$

$q^* = (a-c)/(n+1)(b)$

$Q^* = (n)(a-c)/(n+1)(b)$

Plug in

$p^* = a - bn(a-c)/(n+1)(b) \rightarrow (a+cn)/(n+1)$

$u^* = p^*q^* - cq^* \rightarrow$ Plug in $\rightarrow 2(a-c)/(2b)(n+1)$

(d) Show that for the Cournot duopoly game ($n = 2$), the set of rationalizable strategies coincides with the Nash equilibrium.

$q^* = 2(a-c)/(2b)(3)$

$= 2(a-c)/6b$

$a - c/3b$

$R = (a-c)/3b$ for $i = 1, 2$

HW 10.4

Sunday, October 18, 2020 2:44 PM

Passed solution review

4. Consider the game between a criminal and the government described in this chapter.

Here is a game that illustrates how the government balances the social cost of crime with law-enforcement costs and how criminals balance the value of illegal activity with the probability of arrest. The game has two players: a criminal (C) and the government (G). The government selects a level of law enforcement, which is a number $x \geq 0$. The criminal selects a level of crime, $y \geq 0$. These choices are made simultaneously and independently. The government's payoff is given by $u_G = -xc^4 - y^2/x$ with the interpretation that $-y^2/x$ is the negative effect of crime on society (moderated by law enforcement) and c^4 is the cost of law enforcement, per unit of enforcement. The number c is a positive constant. The criminal's payoff is given by $u_C = y^{1/2}/(1 + xy)$, with the interpretation that $y^{1/2}$ is the value of criminal activity when the criminal is not caught, whereas $1/(1 + xy)$ is the probability that the criminal evades capture. Exercise 4 of this chapter asks you to compute the Nash equilibrium of this game.

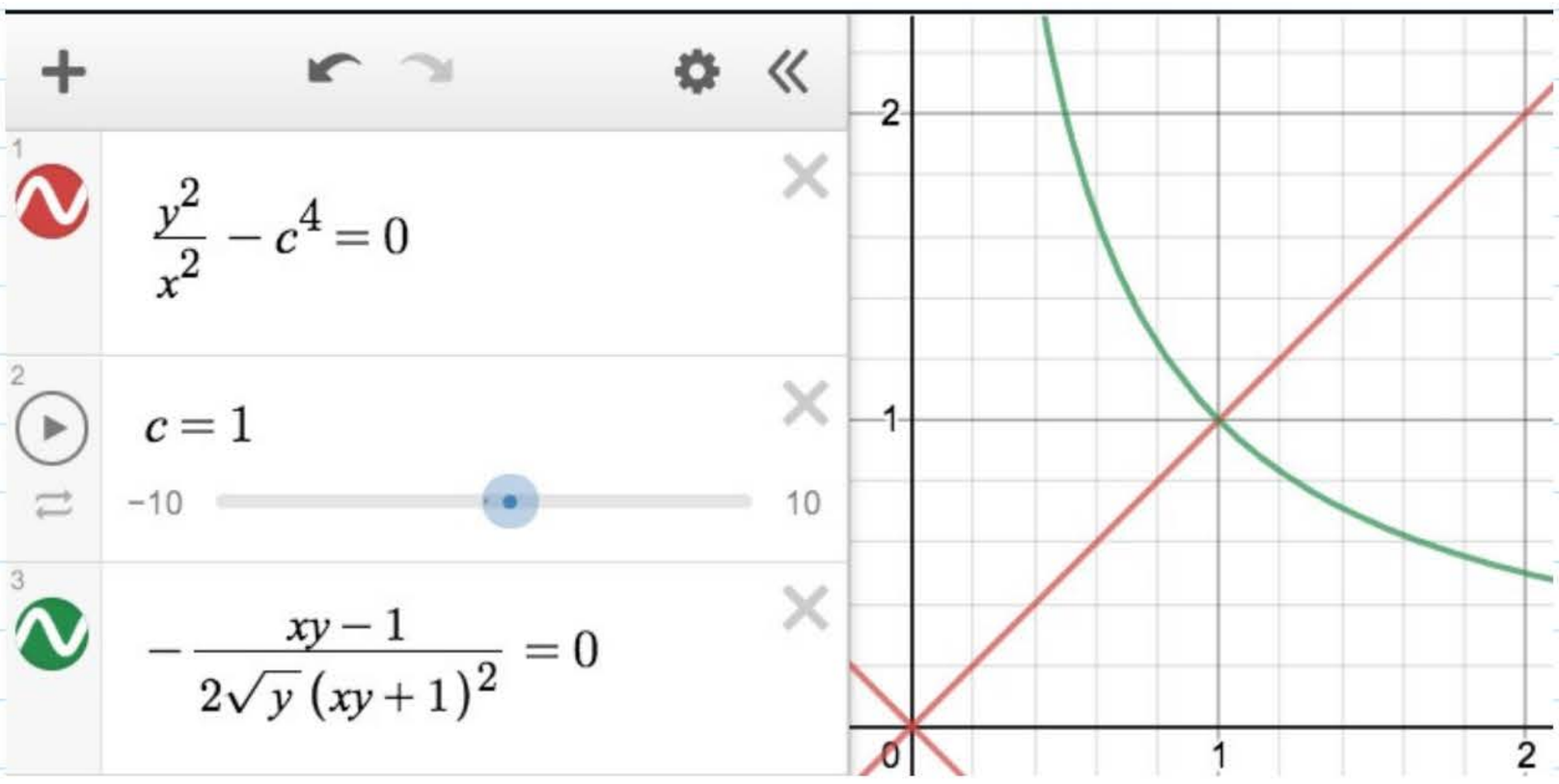
(a) Write the first-order conditions that define the players' best-response functions and solve them to find the best-response functions. Graph the best-response functions.

$G = -xc^4 - y^2/x$

$du/dG = y^2/x^2 - c^4 = 0$

$C = y^{1/2}/(1+xy)$

$du/dc = -xy^{-1}/[2\sqrt{y}(xy+1)^2] = 0$



(b) Compute the Nash equilibrium of this game.

$du/dG \rightarrow x = y/c^2$

$y = xc^2$

$y = 1/y\, c^2$

$x = 1/c$

$du/dc \rightarrow y = 1/x$

$x = 1/y$

$y = c$

(c) Explain how the equilibrium levels of crime and enforcement change as c increases.

As costs c increase, enforcement x decreases and criminal activity y increases

HW 10.5

Sunday, October 18, 2020 3:09 PM

Passed solution review

5. In the years 2000 and 2001, the bubble burst for many Internet and computer firms. As they closed shop, some of the firms had to liquidate sizable assets, such as inventories of products. Suppose eToys is going out of business and the company seeks a buyer for a truckload of Elmo dolls in its warehouse. Imagine that eToys holds an auction on eBay to sell the dolls and that two retailers (players 1 and 2) will bid for them. The rules of the auction are as follows: the retailers simultaneously and independently submit sealed bids and then eToys gives the merchandise to the highest bidder, who must pay his bid. It is common knowledge that the retailer who obtains the load of dolls can resell the load for a total of \$15,000. Thus, if player i wins the auction with bid b_i, then player i's payoff is \$15,000 $- b_i$. The losing retailer gets a payoff of \$0. If the retailers make the same bids ($b_1 = b_2$), then eToys declares each player the winner with probability 1/2, in which case player i obtains an expected payoff of $(1/2)(15{,}000 - b_i)$. What will be the winning bid in the Nash equilibrium of this auction game? If you can, describe the equilibrium strategies and briefly explain why this is an equilibrium. (Hint: This is similar to the Bertrand game.)

$b_1 = b_2 = 15k$

If they pay more, they lose money. If they pay less, they're not guaranteed a win.

This is mildly similar to the beach problem about how many customers you'd need to settle in section 1 of the beach.

forgot to say how to actually win

Win and profit $= 15000 - a - e$ where e is the smallest increment possible.

Never bid > 15000

HW 10.6

Sunday, October 18, 2020 3:13 PM

Passed Solution review

6. Imagine that a zealous prosecutor (P) has accused a defendant (D) of committing a crime. Suppose that the trial involves evidence production by both parties and that by producing evidence, a litigant increases the probability of winning the trial. Specifically, suppose that the probability that the defendant wins is given by $e_D/(e_D + e_P)$, where e_D is the expenditure on evidence production by the defendant and e_P is the expenditure on evidence production by the prosecutor. Assume that e_D and e_P are greater than or equal to 0. The defendant must pay 8 if he is found guilty, whereas he pays 0 if he is found innocent. The prosecutor receives 8 if she wins and 0 if she loses the case.

(a) Represent this game in normal form.

	Prosecutor	Wins	Loses
Defendant	Guilty	$-8, 8$	$-8, 0$
	Not Guilty	$0, 8$	$0, 0$

This doesn't happen

$u_P(e_P, e_D) = 8e_P/(e_P + e_D) - e_P$

$u_D(e_P, e_D) = 8e_D/(e_P + e_D) - e_D$

(b) Write the first-order condition and derive the best-response function for each player.

$du = 8/(e_P + e_D) - 8e_P(e_P + e_D)^2 - 1 = 0$

$8/(P + D) - 8P(P + D)^2 = 1$

$8(P + D) - 8P = (P + D)^2$ or $8D = (P + D)^2$

$P = 2\sqrt{2D} - D$

$D = 2\sqrt{2P} - P$

(c) Find the Nash equilibrium of this game. What is the probability that the defendant wins in equilibrium.

$P = 2\sqrt{2P} - P = D = 2$

Probability $= 1/2$

(d) Is this outcome efficient? Why?

NO Why? $e_D - e_P = .001$ gives the same probability but costs much less. This hinges on both sides spending nearly equal amounts.

HW 10.15

Sunday, October 18, 2020 4:58 PM

Passed Solution review

15. An island has two reefs that are suitable for fishing, and there are twenty fishers who simultaneously and independently choose at which of the two reefs (1 or 2) to fish. Each fisher can fish at only one reef. The total number of fish harvested at a single reef depends on the number of fishers who choose to fish there. The total catch is equally divided between the fishers at the reef. At reef 1, the total harvest is given by $f_1(r_1) = 8r_1 - \frac{r_1^2}{2}$, where r_1 is the number of fishers who select reef 1. For reef 2, the total catch is $f_2(r_2) = 4r_2$, where r_2 is the number of fishers who choose reef 2. Assume that each fisher wants to maximize the number of fish that he or she catches.

(a) Find the Nash equilibrium of this game. In equilibrium, what is the total number of fish caught?

$du/dR_1 = 8 - R_1/2$ $\qquad$ $du/dR_2 = 4$

$4 = 8 - R_1/2$
$8 - 4 = R_1/2$
$4 = R_1/2$
$8 = R_1$

$R_2 = 20 - 8$
$R_2 = 12$

$f_1(8) = 64 - 64/2 = 32$
$f_2(12) = 4 \cdot 12 = 48$

Total = 80

(b) The chief of the island asks his economics advisor whether this arrangement is efficient (i.e., whether the equilibrium allocation of fishers among reefs maximizes the number of fish caught). What is the answer to the chief's question? What is the efficient number of fishers at each reef?

No. You can maximize more

$8R_1 - R_1^2/2 + 4(20 - R_1)$

$du/dR_1 = 8 - R_1 - 4 \Rightarrow R_1 = 4 \Rightarrow R_2 = 16$

$8(4) - 16/2 = 32 - 8 = 24$
$4 \cdot 16 = 64$

$24 + 64 = 88$ fish

(c) The chief decides to require a fishing license for reef 1, which would require each fisher who fishes there to pay the chief x fish. Find the Nash equilibrium of the resulting location-choice game between the fishers. Is there a value of x such that the equilibrium choices of the fishers results in an efficient outcome? If so, what is this value of x?

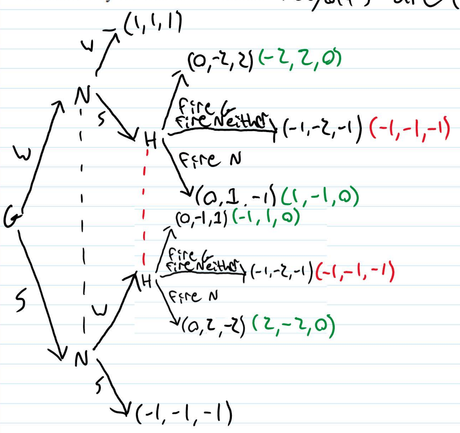
$R_1 \Rightarrow 8 - R_1/2 - x = 4$
$4 = R_1/2 + x$
$x = 2$

Midterm - Gus Lipkin

Friday, October 9, 2020 9:58 AM

1. Draw the extensive form for this game:

- Hailey (H) is the supervisor of Gus (G) and Nathan (N). Both Gus and Nathan choose to work hard (W) or slack off (S). Neither Gus not Nathan know if the other worked hard. Hailey can tell if anyone worked or not, but cannot tell who it was if only one does so.
- If both choose to work hard, the game ends, and all three get a payoff of 1.
- If both slack off, upper management sees the unit fail, all three get fired, and the game ends with all three receiving a payoff of -1. (You need not make upper management a player in the game—their only role was in determining the rule for payoffs.)
- If only one works hard, Hailey will see productivity is less than it should be and get a choice. Hailey can fire Nathan or Gus and keep her job, receiving a payoff of 0. If she fires neither, upper management fires the whole team and Hailey receives a payoff of -1.
- If Nathan is fired, if he worked hard his payoff is -2 and if he did not work hard his payoff is -1. If he is not fired, his payoff is 1 if he worked hard and 2 if he slacked off. Payoffs work the same way for Gus as for Nathan. Payoffs are (H,G,N)



2. For the game below, find the set of rationalizable strategies and any Nash Equilibria.

		Player 1			
		W	X	Y	Z
Player 2	A	~~1, 2~~	~~1, 2~~	~~0, 1~~	0, 1
	B	2, 1	0, 0	3, 4	0, 1
	C	2, 2	2, 3	2, 1	0, 2

0) x+y dom Z
1) C dom A
2) x+y dom W

forgot to do iterated dominance

NE are (C,X) and (B,Y)

The NE are (C,W), (C,X), (B,Y), and (0,2)

		Player 1			
		W	X	Y	Z
Player 2	A	~~1, 2~~	~~1, 2~~	~~0, 1~~	~~0, 1~~
	B	2, 1	0, 0	3, 4	0, 1
	C	2, 2	2, 3	~~2, 1~~	0, 2

(2/3 C + 1/3 B, Y) is the rationalizable strategy

Rationalizable profiles are {B,C} x {X,Y}

Twice Right!

3. Jackson (J) and Nick (N), partners, each decide to work hard or not, without observing the choice of the other. Let H_i be one if player $i \in \{J, N\}$ works hard and 0 otherwise. Payoffs for each are $3(1+H_N+H_J)-4H_i$. Represent the game in normal form. Find the rationalizable strategies and Nash equilibria of this game. Discuss the strategic tensions in the game.

Similar to Prisoner's dilemma

J \ N	H	N
H	5, 5	6, 2 (2,6)
N	2, 6 (6,2)	3, 3

HH = 3(1+1+1) - 4(1) = 3(3) - 4 = 9 - 4 = 5
NN = 3(1+0+0) - 4(0) = 3 - 0 = 3

HN = 3(1+1+0) - 4(1) = 3(2) - 4 = 6 - 4 = 2
NH = 3(1+0+1) - 4(0) = 6 - 0 = 6

← this mistake carried through

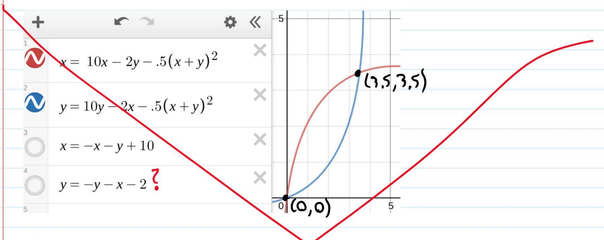
All strategies are rationalizable

(H,H) is the NE (N,N) is NE is strictly dominant

They will get a greater payout if they don't work hard and the other does. But if they both don't work hard, they both get less than if they cooperated + worked hard together.

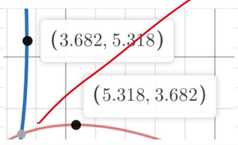
4. Isabel (I) and Raquel (R) each choose how many minutes of air time to purchase to advertise their product, with A_i denoting the number of minutes chosen by player $i \in \{I, R\}$. Isabel's payoff is $10A_I - 2A_R - 0.5(A_I+A_R)^2$. Raquel's payoffs are similarly defined. Find and graph the best response functions. Find the set of rationalizable strategies and the Nash Equilibria, if any.

Messed up variables for R's payoff



NE is (3.5,3.5)

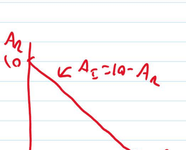
BR is in range



← maximum values for best responses
minimum is 0

$u_I = 10A_I - 2A_R - \frac{1}{2}(A_I + A_R)^2$

$du_I/dA_I = 10 - A_I + A_R = 0 \Rightarrow A_I = 10 - A_R$



$u_R = 10A_R - 2A_I - \frac{1}{2}(A_R + A_I)^2$

$du_R/dA_R = 10 - A_R - A_I = 0 \Rightarrow A_R = 10 - A_I$

The functions intersect at all values where $A_I + A_R = 10$

$[0,10] \times [10,0]$ are rationalizable

All combos that sum to 10 are NE

11 Book Notes

Sunday, October 11, 2020 4:45 PM

Mixed strategy Nash Equilibrium

Some games don't have an NE

Players can take advantage of the NE because it gives knowledge of the other player's strategy

Mixed strategy NE means no player can increase their payoff by switching to another strategy given the other's strategy

For a mix to be a BR, it must put positive probability only on pure strategies that are best responses

Randomization in sports

<u>Procedure</u>

1) Calculate rationalizable strategies with iterated dominance
2) Restrict to rationalizable and write equations for each player that neutralize each other's strategies
3) Solve the equations

11.1 Mixed Strategies 1

Wednesday, October 14, 2020 4:42 PM

Mixed strategy NE

Matching pennies

		H	T
			q
p	H	1,0	0,1
	T	0,1	1,0

p or q = Probability of choosing heads

$u_1(T|\theta_2) = u_1(H|\theta_2)$

$u_1(T|q) = u_1(H|q)$

$q \cdot 0 + (1-q) \cdot 1 = q \cdot 1 + (1-q) \cdot 0$

$1-q = q$

$q = 1/2$

Same for p_2

$u_1 = \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 0$

11.2 Mixed Strategies 2

Wednesday, October 14, 2020 4:50 PM

Penalty Kicks in Football

Players: Shooter and Keeper

Strategies: Shoot at Left or Right
anticipate Left or Right

k \ s	L	R
L	.7, .3	.8, .2
R	.9, .1	.5, .5

k \ s	L	R
L	12/45	8/45
R	15/45	10/45

$u_k(L) = u_k(R)$

$.3p + .9(1-p) = .8p + .5(1-p)$
$\hookrightarrow p = 4/9$

$u_s(L) = u_s(R)$

$.7q + .8(1-q) = .9q + .5(1-q)$
$\hookrightarrow q = 3/5$

Evidence?

11.3 Mixed Strategies 3

Wednesday, October 14, 2020 5:06 PM

Bob / Adam	L	C	R
U	3,6	6,4	0,0
M	6,4	5,6	0,0
D	0,0	0,0	2,2

$u_A(U) = u_A(M)$

$3q + 6(1-q) = 6q + 5(1-q)$

$\hookrightarrow q = 1/4$

$u_B(C) = u_B(C)$

$6p + 4(1-p) = 4p + 6(1-p)$

$\hookrightarrow p = 1/2$

$u_A = 5.25 > 2$

$u_B = 5 > 2$

Duopoly w/ capacity constraints

HW 11.3

Wednesday, October 21, 2020 7:00 PM

Passed Solution review

3. Consider another version of the lobbying game introduced in this chapter. Suppose the payoffs are the same as presented earlier, except in the case in which firm X lobbies and firm Y does not lobby. In this case, suppose the government's decision yields x to firm X and zero to firm Y. Assume that $x > 25$. The normal form of this game is pictured here.

X \ Y	L	N
L	−5, −5	x−15, 0
N	0, 15	10, 10

(a) Designate the (pure-strategy) Nash equilibria of this game (if it has any).

(N,L) and (L,N)

(b) Compute the mixed-strategy Nash equilibrium of the game.

$-5q + (x-15)(1-q) = 10 - 10q \rightarrow q = (25-x)/(20-x)$

$-5p + 15 - 15p = 10 - 10p$

$15 - 20p = 10 - 10p$

$5 = 10p$

$p = 1/2$

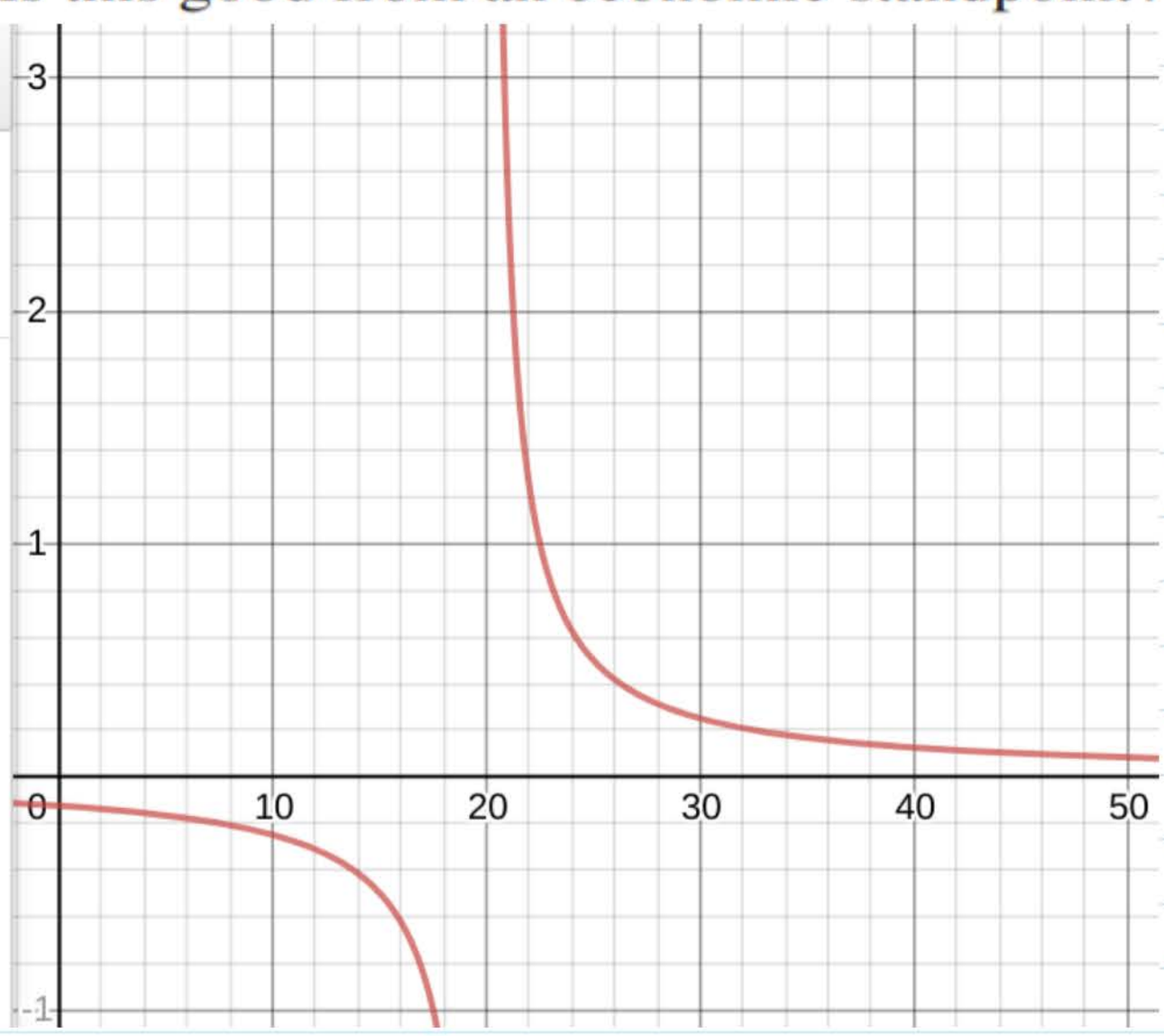
(c) Given the mixed-strategy equilibrium computed in part (b), what is the probability that the government makes a decision that favors firm X? (It is the probability that (L, N) occurs.)

$(L,N) = p(1-q) = \frac{1}{2}\left(1 - \frac{25-x}{20-x}\right) = \frac{1}{2} \cdot \frac{5}{x-20}$

(d) As x rises, does the probability that the government makes a decision favoring firm X rise or fall? Is this good from an economic standpoint?

$y = .5\left(\frac{5}{x-20}\right)$

as x increases, the probability decreases. After 20, (L,N) becomes better



HW 11.5

Wednesday, October 21, 2020 7:19 PM

Passed Solution Review

5. This exercise explores how, in a mixed-strategy equilibrium, players must put positive probability only on best responses. Consider the game in the following figure.

$>1 \to x=2$
$<1 \to x=.5$

1 \ 2	L	M	R
U	x, x	$x, 0$	$x, 0$
C	$0, x$	$2, 0$	$0, 2$
D	$0, x$	$0, 2$	$2, 0$

Compute the pure-strategy and mixed-strategy Nash equilibria for this game, and note how they depend on x. In particular, what is the difference between $x > 1$ and $x < 1$?

For both $x>1$ and $x<1$, NE $=(U,L)$. When $x<1$ there is also $\{(0,.5,.5),(0,.5,.5)\}$ unless $x>2$

This whole question is very confusing if $x \geq 2$ or $x \leq 0$

If all three played... $r_1 = r_2 = (1-x, 1/2, 1/2)$

symmetric game

If $x<0$, U,L is dominated which forces $r_1 = r_2 = (0, 1/2, 1/2)$

If $0 \leq x \leq 1$ U,L is Pure Eq and $(1-x, x/2, x/2)$ is mixed

I initially had the basics, but did not consider all cases

HW 11.6

Wednesday, October 21, 2020 7:43 PM

Passed Solution Review

6. Determine all of the Nash equilibria (pure-strategy and mixed-strategy equilibria) of the following games.

(a)

1 \ 2	H	T
H	1, −1	−1, 1
T	−1, 1	1, −1

(b)

1 \ 2	C	D
C	2, 2	0, 3
D	3, 0	1, 1

(c)

1 \ 2	H	D
H	2, 2	3, 1
D	3, 1	2, 2

(d)

1 \ 2	A	B	
A	1, 4	2, 0	p
B	0, 8	3, 9	1-p
	q	1-q	

(e)

1 \ 2	A	B	
A	2, 2	0, 0	p
B	0, 0	3, 4	1-p
	q	1-q	

(f)

1 \ 2	L	M	R	
U	8, 1	0, 2	4, 3	p
C	3, 1	4, 4	0, 0	1-p
D	5, 0	~~3, 3~~	~~1, 4~~	
	q	1-q		

d) 1: $1q + 2(1-q) = 0q + 3(1-q)$
$q + 2 - 2q = 3 - 3q$
$2 - q = 3 - 3q$
$-1 = -2q$
$q = .5$

2: $4p + 8(1-p) = 0p + 9(1-p)$
$4p + 8 - 8p = 9 - 9p$
$8 - 4p = 9 - 9p$
$1 = 5p$
$p = .2$

Pure NE: (A, A) and (B, B)
Mixed NE: $(\underset{p_1}{.2}A, .8B)$ and $(\underset{p_2}{.5}A, .5B)$

e) 1: $2q + 0 = 0 + 3(1-q)$
$2q = 3 - 3q$
$5q = 3$
$q = 3/5$

2: $2p + 0 = 0 + 4(1-p)$
$2p = 4 - 4p$
$6p = 4$
$p = 2/3$

Pure NE: (A, A) and (B, B)
Mixed NE: $(\underset{p_1}{2/3}A, 1/3B)$ and $(\underset{p_2}{3/5}A, 2/5B)$

f) M dom L Mix U/C dom D

1: $0 + 4(1-q) = 4q + 0$
$4 - 4q = 4q$
$4 = 8q$
$q = 1/2$

~~2: $2p + 4(1-p) = 3p + 0$~~
~~$2p + 4 - 4p = 3p$~~
~~$4 - 2p = 3p$~~
~~$4 = 5p$~~
~~$p = 4/5$~~

Pure NE: (C, M) and (U, R)
Mixed NE: $\sigma_1(0, 4/5, 1/5)$ and $\sigma_2(0, 1/2, 1/2)$

did dominance wrong

(f)

1 \ 2	L	M	R
U	8, 1	0, 2	4, 3
C	3, 1	4, 4	0, 0
D	5, 0	3, 3	1, 4

$2p + 4(1-p) = 4p + 0(1-p)$
$2p + 4 - 4p = 4p$
$4 = 6p$
$p = 2/3$

HW 11.8

Saturday, October 24, 2020 1:23 PM

Passed Solution review

8. Consider the following social problem.[5] A pedestrian is hit by a car and lies injured on the road. There are n people in the vicinity of the accident. The injured pedestrian requires immediate medical attention, which will be forthcoming if at least one of the n people calls for help. Simultaneously and independently, each of the n bystanders decides whether or not to call for help (by dialing 911 on a cell phone or pay phone). Each bystander obtains v units of utility if someone (anyone) calls for help. Those who call for help pay a personal cost of c. That is, if person i calls for help, then he obtains the payoff $v - c$; if person i does not call but at least one other person calls, then person i gets v; finally, if none of the n people calls for help, then person i obtains zero. Assume $v > c$.

(a) Find the symmetric Nash equilibrium of this n-player normal-form game. (Hint: The equilibrium is in mixed strategies. In your analysis, let p be the probability that a person does not call for help.)

Call $V-C$ | not call but other does V | no call 0

If symmetric, $p = 1-p$ so $(V-C) = (1-p^{n-1})\cdot V \rightarrow p = \frac{c}{v}\cdot\frac{1}{1-n}$

It's $p^{n-1} = c/v$ due to how probability works

$V - C = V(1-p^{n-1})$

$p^{n-1} = c/v$

$p = (c/v)^{(1/(n-1))}$

(b) Compute the probability that at least one person calls for help in equilibrium. (This is the probability that the injured pedestrian gets medical attention.) Note how this depends on n. Is this a perverse or intuitive result?

$1 - p^n = 1 - \frac{c}{v}\frac{n}{n-1} \rightarrow$

Forgot to do the whole problem?

$P(X>0) = 1 - P(0)$

$= 1 - (c/v)^{(n/(n-1))}$

$c/v < 1$ so as n grows, $P(X>0)$ decreases. This is perverse and intuitive. People will assume that someone else is calling in a large group. In smaller groups or individually it is apparent that you need to take charge

The limit is $P(X>0) \rightarrow 1 - c/v$

HW 11.13

Saturday, October 24, 2020 1:36 PM

Passed Solution review

13. Consider the following three-player team production problem. Simultaneously and independently, each player chooses between exerting effort (E) or not exerting effort (N). Exerting effort imposes a cost of 2 on the player who exerts effort. If two or more of the players exert effort, each player receives a benefit of 4 regardless of whether she herself exerted effort. Otherwise, each player receives zero benefit. The payoff to each player is her realized benefit less the cost of her effort (if she exerted effort). For instance, if player 1 selects N and players 2 and 3 both select E, then the payoff vector is (4, 2, 2). If player 1 selects E and players 2 and 3 both select N, then the payoff vector is $(-2, 0, 0)$.

(a) Is there a pure-strategy equilibrium in which all three players exert effort? Explain why or why not.

(0,0,0) = (N,N,N) is the pure NE. I'm having a hard time with words, but it makes sense given past problems

No. Any player could unilaterally improve from V=4-2 to V=4-0 by not exerting effort.

I'm dumb. This makes so much sense. There's no incentive for the third person to exert effort if they get 4 no matter what

(b) Find a symmetric mixed-strategy Nash equilibrium of this game. Let p denote the probability that an individual player selects N.

I'm honestly not sure and don't have the grasp to talk this through. I think it's (1/3, 1/3, 1/3)

$\cancel{p = 1 - 2p}$

$\cancel{p = 1/3}$

$U_i(E) = U_i(N)$

$4(1-p^2) + 0p^2 - 2 = 4(1-p)^2 + 0(1-(1-p)^2)$

$2 - 4p^2 = 4 + 4p^2 - 8p$

$8p^2 - 8p + 2 = 0$

$4p(1-p) = 1$

$p = 1/2$

Duh. Just needed to set equal then plug and chug

12 Book Notes

Tuesday, October 13, 2020 5:51 PM

Strictly competitive games and security strategies

Two player, strictly competitive games has the property that for every two strategy profiles $s, s' \in S$, $u_1(s) > u_1(s')$ if and only if $u_2(s) < u_2(s')$

↳ both players have opposite rankings of the outcomes

Strictly competitive = no room for joint game or compromise

Zero sum games

Security strategy (worst case) = $W_i s_i \equiv \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$

Security payoff level = $\max_{s_i \in S_i} W_i(s_i)$

Maxmin strategy → lowest average

12.1 Strictly Competitive Games

Saturday, October 17, 2020 5:42 PM

Zero Sum Game → all Payoffs add to zero

	L	R
L	0,0	1,-1
R	-1,1	0,0

Secure Strategy/Security Strategy = Maxmin

	L	R
L	-1,1	1,-1
R	1,-1	-1,1

Maxmin guarantees highest possible expected payoff as a worst outcome

Maxmin ↔ NE

	L	R	NE
U	2,3	1,1	sec
D	0,0	3,2	
		sec	

Non-strict comp

ss not a "solution"

	L	R	
U	2,3	0,1	
D	~~1,2~~	2,0	sec
	sec		

L dom R
U dom D

U,L is only rationalizable

HW 12.1

Sunday, October 25, 2020 2:01 PM

Passed Solution review

1. Determine which of the following games are strictly competitive.

1 \ 2	X	Y	Z
A	$^{3}2, 9^{1}$	$^{2}6, 5^{2}$	$^{1}7, 4^{2}$
B	$^{2}5, 6^{2}$	$^{1}8, 2^{3}$	$^{2}3, 8^{1}$
C	$^{1}9, 1^{3}$	$^{3}4, 7^{1}$	$^{1}7, 3^{3}$

(a)

1 \ 2	X	Y	Z
A	$^{1}8, 1^{2}$	$^{2}7, 2^{2}$	$^{3}3, 6^{1}$
B	$^{1}9, 0^{3}$	$^{3}2, 8^{1}$	$^{2}4, 5^{2}$
C	$^{2}7, 2^{1}$	$^{1}8, 1^{3}$	$^{3}6, 4^{3}$

(b)

1 \ 2	X	Y
A	$^{2}1, 1^{2}$	$^{1}2, 0^{2}$
B	$^{2}0, 2^{1}$	$^{1}1, 1^{1}$

(c)

1 \ 2	X	Y
U	$^{2}1, 1^{2}$	$^{1}2, 2^{2}$
D	$^{1}4, 4^{1}$	$^{2}3, 3^{1}$

(d)

a) No because $u_1(A,2) = u_1(C,2)$ but $u_2(A,2) > u_2(C,2)$

b) Yes

c) Yes

d) NO u_1 and u_2 $(D,x) > (D,y)$

HW 12.2

Sunday, October 25, 2020 2:07 PM

Passed Solution Review

2. Find the players' security strategies for the games pictured in Exercise 1.

1. Determine which of the following games are strictly competitive.

1 \ 2	X	Y	Z	
A	2, 9	6, 5	7, 4	2
B	5, 6	8, 2	3, 8	3
C	9, 1	4, 7	7, 3	1
	1	2	3	

(a)

1 \ 2	X	Y	Z	
A	8, 1	7, 2	3, 6	3
B	9, 0	2, 8	4, 5	1
C	7, 2	8, 1	6, 4	6
	0	1	4	

(b)

1 \ 2	X	Y	
A	1, 1	2, 0	1
B	0, 2	1, 1	0
	1	0	

(c)

1 \ 2	X	Y	
U	1, 1	2, 2	1
D	4, 4	3, 3	3
	1	2	

(d)

Start by finding lowest payoff in each row and column. Then the highest value found belongs to the row or column that is the security strategy.

HW 12.5

Sunday, October 25, 2020 2:20 PM

Passed Solution review

5. The guided exercise in this chapter demonstrates that not all security strategies are rationalizable. Find an example in which player 1's security strategy is dominated (in the first round of the rationalizability construction). For your example, what is the relation between player 1's security level and maxmin level?

1 \ 2	L	M	R	
U	2,9	8,5	7,4	2
C	9,6	4,2	3,8	3
D	9,1	8,7	7,3	7
	1	2	3	

Mix U,C dom D

D and R are secure strategies dominated by mix U+M and L+R.

Maxmin: $\sigma_1 = (1/2, 1/2, 0)$ + $\sigma_2 = (1/2, 1/2, 0)$

maxmin guarantees payoffs of 2 while secure strategy pays only 1

This all makes sense. I needed to develop my initial solution more

13 Book Notes

Wednesday, October 21, 2020 8:44 AM

Contract, law, and enforcement in static settings

Contract = agreement about behavior that is intended to be enforced

Contractual relationship

external Players = third Parties

Phases are contracting and implementation

Enforced by: self, external, automatically

Induced game comes from a self and externally enforced game

Complete contracting in discretionary environments

Complete contract

With full verifiability, there is an enforced contract yielding the efficient outcome (that which maximizes Players' total Pay/OFF)

limited verifiability

13.1 Contracting 1

Wednesday, October 21, 2020 9:10 AM

Preliminary Info

Contract?
Verifiability?

	Bob	I	N
Amy	Invest	Z_A, Z_B	Y_A, X_B
	Not	X_A, Y_B	$0, 0$

Z = both invest
X = other invests but you don't
Y = you invest, other doesn't

I, I is efficient

$$Z_A + Z_B > Y_A + X_B$$
$$> Y_B + X_A$$
$$> 0$$

When is I, I a NE?
$Z_A > X_A + Z_B > X_B$

If <u>Not</u> NE to choose I, can contracting help?

13.2 Contracting 2

Wednesday, October 21, 2020 9:20 AM

Complete contracts in discretionary environments

Discretionary? Penalties for breach of contract?
- full verifiability

Tranfers from A to B if not (I, I)
α, β, γ

		Bob: I	Bob: N
Amy	I	Z_A, Z_B	$Y_A+\beta, X_B-\beta$
	N	$X_A+\alpha, Y_B-\alpha$	$\gamma, -\gamma$

The one who gains from enforcement is the one who brings the suit → Plaintiff

1) If $Z_A > X_A$, $\alpha = 0$ else $Z_A > X_A + \alpha \rightarrow = 0 > Z_A - X_A > \alpha$
↳ Amy pays Bob

2) if $Z_B + X_B$, $\beta = 0$ else $Z_B > X_B - \beta \rightarrow \beta > X_B - Z_B > 0$

Certain external enforcement
↳ I, I is the induced NE

Results:
1) Full verifiability
2) discretionary, complete, contracting

↳ there is an externally enforced contract that yields I, I as the NE outcome of the induced game

13.3 Contracting 3

Wednesday, October 21, 2020 9:39 AM

Discretionary contracting w/ limited verifiability

Can only verify if something went wrong and not what

	Bob: I	N
Amy: I	Z_A, Z_B	$Y_A+\alpha, X_B-\alpha$
N	$X_A+\alpha, Y_B-\alpha$	$\alpha, -\alpha$

[I for NE?

$Z_A > X_A + \alpha \qquad \alpha Z_B > X_B - \alpha$

$Z_A - X_A > \alpha > X_B - Z_B \rightarrow Z_A + Z_B > X_A + X_B \rightarrow Z \gg X$

13.4 Contracting 4

Wednesday, October 21, 2020 9:47 AM

What if we specify a big penalty to both players if I,I does not happen

Residual claimant

13.5 Contracting 5

Saturday, October 24, 2020 4:12 PM

Court imposed breach remedies (not discretionary)

expectation Damages
↳ Court makes wronged party whole

Bob / Amy	I	N
I	Z_A, Z_B	$Z_A, X_B - (Z_A - Y_A)$
N	$X_A - (Z_B - Y_B), Z_B$	$-Z_B, -Z_A$

When is I, I the NE?

$Z_A > X_A - Z_B + Y_B$ $\quad$ $Z_B > X_B - Z_A + Y_A$

$Z_A + Z_B > X_A + Y_B$ $\quad$ $Z_B + Z_A > X_B + Y_A$

Result: expectation damages
↳ I, I is equilibrium of the induced game if I, I is efficient

Often Z is not known!
X not known!

13.6 Contracting 6

Saturday, October 24, 2020 4:51 PM

Reliance Damages

Z (and X) hard to quantify

Status quo = 0,0

$y < 0$

Bob / Amy	I	N
I	Z_A, Z_B	$0, X_B + Y_A$
N	$X_A + Y_A, 0$	$0, 0$

I, I is NE if $Z_B > X_B + Y_A$, $Z_A > X_A + Y_0$ might not hold

13.7 Contracting 7

Saturday, October 24, 2020 4:58 PM

Restitution Damages

Bob / Amy	I	N
I	Z_A, Z_B	$X_B + Y_A, 0$
N	$0, X_A + Y_B$	$0, 0$

I, I NE? → $Z_A > 0$, $Z_B > 0$, I, I efficient

x hard to measure

HW 13.1

Sunday, November 1, 2020 3:21 PM

Passed Solution review

1. Consider a contractual setting in which the technology of the relationship is given by the following underlying game:

1 \ 2	I	N
I	5, 5	−1, 1
N	7, −1	0, 0

Suppose an external enforcer will compel transfer α from player 2 to player 1 if (N, I) is played, transfer β from player 2 to player 1 if (I, N) is played, and transfer γ from player 2 to player 1 if (N, N) is played. The players wish to support the investment outcome (I, I).

(a) Suppose there is limited verifiability, so that $\alpha = \beta = \gamma$ is required. Assume that this number is set by the players' contract. Write the matrix representing the induced game and determine whether (I, I) can be enforced. Explain your answer.

	I	N
I	5,5	$-1+\beta, 1-\beta$
N	$7+\alpha, -1-\alpha$	$0+\gamma, 0-\gamma$

I, I can be enforced if the contract specifies a transfer

$5 > 7+\alpha$ ~~$5 > 1-\beta$~~ $5 > 1-\alpha$

$-2 > \alpha$ ~~$\beta > -4$~~ $\alpha > -4$

$\alpha = \beta$

$-4 < \alpha < -2 \rightarrow \alpha = -3$

(b) Suppose there is full verifiability, but that α, β, and γ represent reliance damages imposed by the court. Write the matrix representing the induced game and determine whether (I, I) can be enforced. Explain your answer.

	I	N
I	5,5	$-1+1, 1-1=0$
N	$7-1, -1+1$	0,0

I, I is not NE because N, I dom I, N for Player 1

Just plug in and solve

	I	N
I	5,5	0,0
N	6,0	0,0

$5 < 6$ so no

HW 13.2

Sunday, November 1, 2020 3:42 PM

Passed Solution review

2. Consider a contractual setting in which the technology of the relationship is given by the following partnership game:

1 \ 2	I	N
I	4, 4	−4, 9
N	2, −4	0, 0

Suppose the players contract in a setting of court-imposed breach remedies. The players can write a formal contract specifying the strategy profile they intend to play; the court observes their behavior in the underlying game and, if one or both of them cheated, imposes a breach transfer. The players wish to support the investment outcome (I, I).

(a) Write the matrix representing the induced game under the assumption that the court imposes *expectation damages*. Can a contract specifying (I, I) be enforced? Explain your answer.

(b) Write the matrix representing the induced game under the assumption that the court imposes *restitution damages*. Can a contract specifying (I, I) be enforced?

(c) Write the matrix representing the induced game under the assumption that the court imposes *reliance damages*. Can a contract specifying (I, I) be enforced with reliance transfers? Explain your answer.

(d) Suppose litigation is costly. When a contract is breached, each player has to pay a court fee of c in addition to the *reliance* transfer imposed by the court. What is the induced game in this case?

(e) Under what condition on c can (I, I) be enforced with reliance transfers and court costs?

(f) Continue to assume the setting of part (d). Suppose the court intervenes after a breach only if the plaintiff brings suit. For what values of c does the plaintiff have the incentive to sue?

(g) How does your answer to part (e) change if the court forces the losing party to pay all court costs?

a) Yes because there's no incentive to deviate from I, I

1\2	I	N
I	4, 4	4, 1
N	-6, 4	0, 0

b) Yes

1\2	I	N
I	4, 4	5, 0
N	0, -2	0, 0

c) No. Player 2 could still deviate

1\2	I	N
I	4, 4	0, 5
N	-2, 0	0, 0

d) OneNote sucks ass and won't let me insert a table into a list unless I type something first

1\2	I	N
I	4, 4	-c, 5 - c
N	-2 - c, -c	0, 0

e) c >= 1

f) Player 1 sues if c <= 4
Player 2 sues if c <= 4
Therefore a suit occurs if c <= 4

g) c >= .5

HW 13.12

Sunday, November 1, 2020 3:59 PM

Passed Solution review

12. Consider the following two-player team production problem. Each player i chooses a level of effort $a_i \geq 0$ at a personal cost of a_i^2. The players select their effort levels simultaneously and independently. Efforts a_1 and a_2 generate revenue of $r = 4(a_1 + a_2)$. There is limited verifiability in that the external enforcer (court) can verify only the revenue generated by the players, not the players' individual effort levels. Therefore, the players are limited to revenue-sharing contracts, which can be represented by two functions $f_1 : [0, \infty) \to [0, \infty)$ and $f_2 : [0, \infty) \to [0, \infty)$. For each player i, $f_i(r)$ is the monetary amount given to player i when the revenue is r. We require $f_1(r) + f_2(r) \leq r$ for every r.

Call a contract *balanced* if, for every revenue level r, it is the case that $f_1(r) + f_2(r) = r$. That is, the revenue is completely allocated between the players. A contract is *unbalanced* if $f_1(r) + f_2(r) < r$ for some value of r, which means that some of the revenue is destroyed or otherwise wasted.

(a) What are the efficient effort levels, which maximize the joint value $4(a_1 + a_2) - a_1^2 - a_2^2$?

$d1 = 4 - 2a_i \to 0 = 4 - 2a_i$ a_2 is the same

$a_1 = 2$

Total $= 4(2+2) = 4 \cdot 4 = 16$

(b) Suppose that the players have a revenue-sharing contract specifying that each player gets half of the revenue. That is, player i gets $(r/2) - a_i^2 = 2(a_1 + a_2) - a_i^2$. What is the Nash equilibrium of the effort-selection game?

$r/2 = 2a_1 + 2a_2 - a_i^2 + a_i^2$

$r/2 = 2a_1 + 2a_2$

$r = 4a_1 + 4a_2$?

max $2a_1 + 2a_2 - a_i^2$

$d1 = 2 - 2a_i = 0$

$2 = 2a_i$

$a_i = 1$ $a_1 = a_2$

(c) Next consider more general contracts. Can you find a balanced contract that would induce the efficient effort levels as a Nash equilibrium? If so, describe such a contract. If not, see if you can provide a proof of this result. $\frac{df_1}{dr} 4 = 2a_1$, $\frac{df_2}{dr} 4 = 2a_2$ for individual choice

It's symmetric so they should each get half. This is examined in part b and is not efficient

Efficiency: $4 = 2a_1$ and $4 = 2a_2$

Combining: $\frac{df_1}{dr} = 1$ and $\frac{df_2}{r} = 1$

Balancing: $f_1(r) + f_2(r) = r$, thus $\frac{df_1}{r} + \frac{df_2}{r} = 1$

You can't combine and balance

(d) Can you find an unbalanced contract that would induce the efficient effort levels as a Nash equilibrium? If so, describe such a contract. If not, provide a proof as best you can. to be efficient, $r = 16$

Players get $r - x$

max $r - x - a_i^2$ so $a_i = 2$

$f_i = \begin{cases} 8 & r = 16 \\ 0 & \text{otherwise} \end{cases}$

But no one would sue because they'd get zero

$2(r - x) < r \to r < 2x$

So long as someone gets more than half, you're good

(e) Would the issues of balanced transfers matter if the court could verify the players' effort levels? Explain.

If effort was verifiable, they could be punished accordingly

No because the contract would specify effort, not revenue

14 Book Notes

Saturday, October 24, 2020 5:25 PM

Details of the extensive form

Trees consist of nodes connected by branches

Tree rules:

1) Every node is a successor of the initial node, and the initial node is the only one with this property
2) Each node except the initial node has exactly one immediate predecessor. The initial node has no predecessors
3) Multiple branches extending from the same node have different action labels
4) Each information set contains decision nodes for only one of the players
5) All nodes in a given information set must have the same number of immediate successors and they must have the same set of action labels on the branches leading to these successors

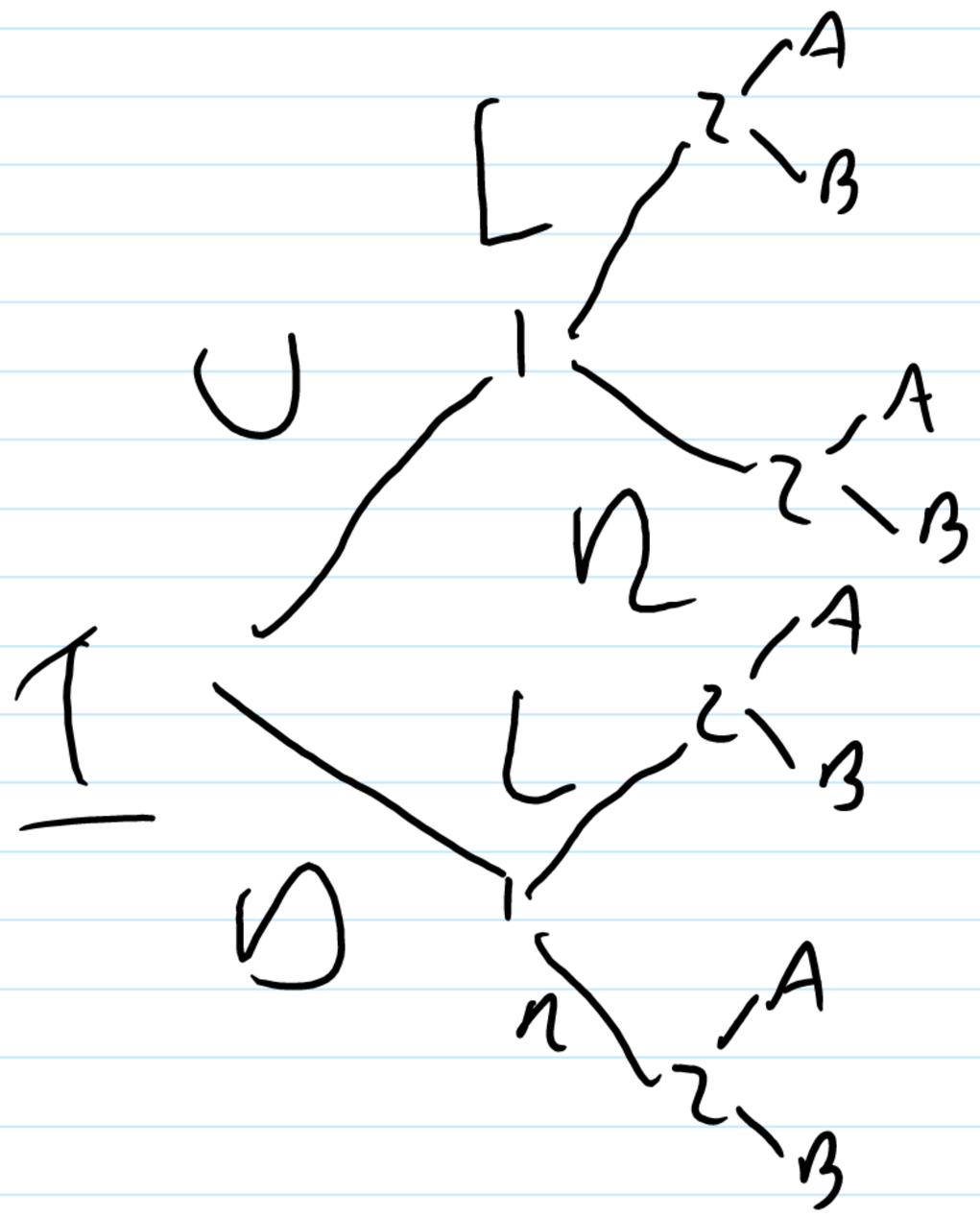
Perfect vs imperfect recall
Perfect vs imperfect information

14.1 Recall

Tuesday, October 27, 2020 6:15 PM

5 tree rules

Perfect vs imperfect recall → we always use perfect



→ imperfect player 1

HW 14.6

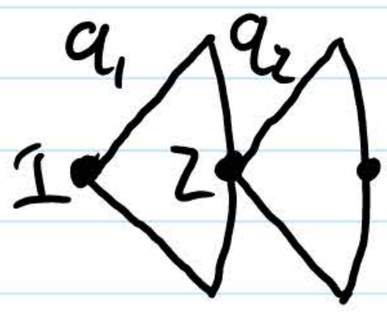
Sunday, November 1, 2020 4:34 PM

Passed Solution review

6. Consider a variant of the game described in Exercise 4. Suppose that the firms move sequentially rather than simultaneously. First, firm 1 selects its quantity q_1, and this is observed by firm 2. Then, firm 2 selects its quantity q_2, and the payoffs are determined as in Exercise 4, so that firm i's payoff is $(12 - q_i - q_j)q_i$. As noted in Exercise 6 of Chapter 3, this type of game is called the *Stackelberg duopoly model*. This exercise asks you to find some of the Nash equilibria of the game. Further analysis appears in Chapter 15.

Note that firm 1's strategy in this game is a single number q_1. Also note that firm 2's strategy can be expressed as a function that maps firm 1's quantity q_1 into firm 2's quantity q_2. That is, considering $q_1, q_2 \in [0, 12]$, we can write firm 2's strategy as a function $s_2 : [0, 12] \to [0, 12]$. After firm 1 selects a specific quantity q_1, firm 2 would select $q_2 = s_2(q_1)$.

(a) Draw the extensive form of this game.



$(12-q_1-q_2)q_1$, $(12-q_1-q_2)q_2$

(b) Consider the strategy profile (q_1, s_2), where $q_1 = 2$ and s_2 is defined as follows:

$$s_2(q_1) = \begin{cases} 5 & \text{if } q_1 = 2 \\ 12 - q_1 & \text{if } q_1 \neq 2 \end{cases}.$$

That is, firm 2 selects $q_2 = 5$ in the event that firm 1 chooses $q_1 = 2$; otherwise, firm 2 picks the quantity that drives the price to zero. Verify that these strategies form a Nash equilibrium of the game. Do this by describing the payoffs players would get from deviating.

$u_1 = 2(12-2-5) = 10 \to q_1(12-q_1-(12-q_1)) \to = 0$

$u_2 = 5(12-2-5) = 25 \to q_2(12-2-q_2) \to \max \to 10-2q_2 = 0$

$10 = 2q_2$

$q_2 = 5$

No gains from deviating so that is NE

(c) Show that for any $x \in [0, 12]$, there is a Nash equilibrium of the game in which $q_1 = x$ and $s_2(x) = (12 - x)/2$. Describe the equilibrium strategy profile (fully describe s_2) and explain why it is an equilibrium.

$$s = \begin{cases} (12-x)/2 & q_1 = x \\ 12-x & q_1 \neq x \end{cases}$$

$x(12-x)/2 = (12-x)^2/4 \to (12x-x^2)/2 = (12-x)^2/4$

No gains from deviating so it's NE

(d) Are there any Nash equilibria (q_1, s_2) for which $s_2(q_1) \neq (12 - q_1)/2$? Explain why or why not.

No. $(12-q_1)/2$ always maximizes P 2's payoff

15 Book Notes

Tuesday, October 27, 2020 6:21 PM

Sequential Rationality and Subgame Perfection

Sequential Rationality: optimal strategies should maximize payoffs, conditional on every information set at which this player has the move

↳ Plan ahead!

"look ahead"

Subgame perfect NE

Backward Induction

Procedure: process of analyzing a game from end to beginning. At each decision node, strike any dominated positions, given the terminal nodes that can be reached through the play of the actions identified at successor nodes

Result: Every finite game with perfect information has a pure strategy NE. Backward induction identifies the NE.

Subgame perfection

node x in a tree is said to **initiate a subgame** if neither x nor any of its successors are in an information set that contains nodes that are not successors of x. This is a **subgame**

Proper subgames don't start at the initial node

A strategy profile is called a **subgame perfect NE (SPE)** if it specifies a NE in every subgame of the original game

Subgames are a **refinement** of NE

Incredible threats in the Stackleberg Duopoly Game

The SPE of the Stackleberg Duopoly Game

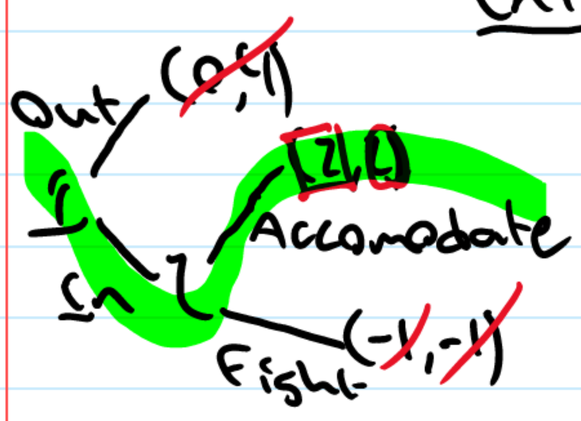
Technical Notes

For a given node x in a game, a player's **continuation value/payoff** is the payoff that this player will eventually get contingent on the path of play passing through node x.

net of past receipts

15.1 Dynamic Behavior - Sequential Rationality

Tuesday, October 27, 2020 7:05 PM



Entry to Predation

	A	P
I	2, 2	-1, -1
O	0, 4	0, 4

non-credible threats

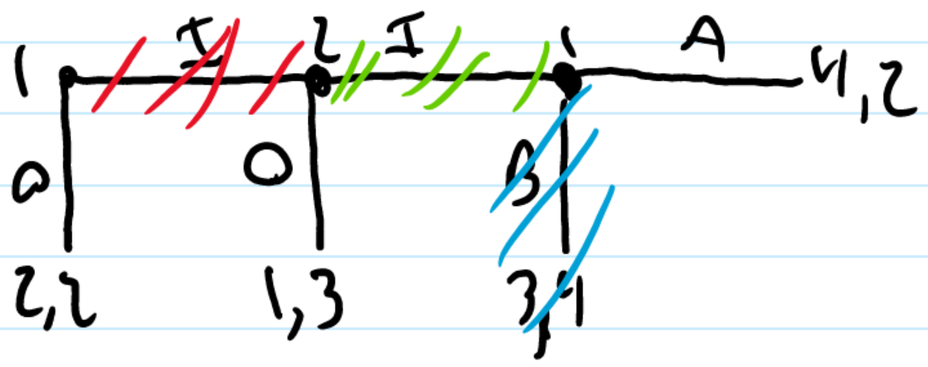
sequential rationality

Subgame perfection → SPE or SPNE

	A	P
I	2, 2	-1, -1

15.2 Backwards Induction

Tuesday, October 27, 2020 7:23 PM

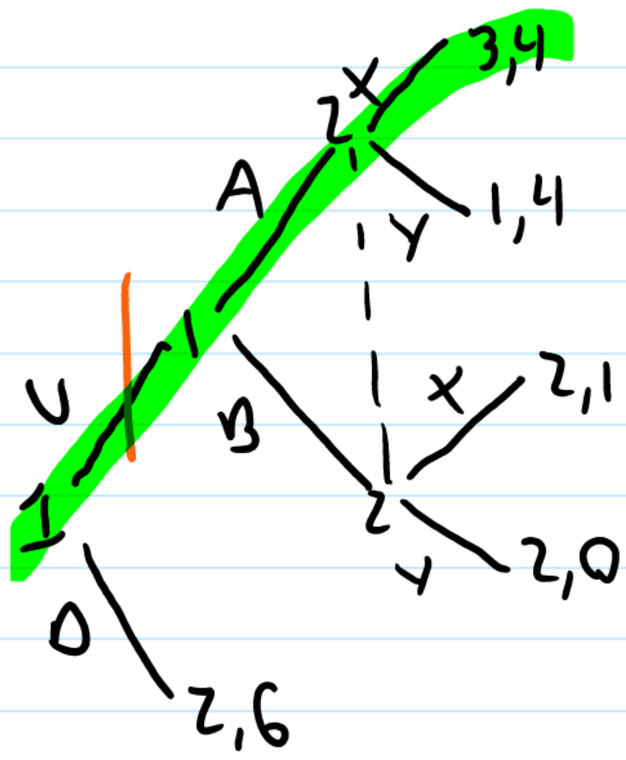


1, 2, 3

	T	Q
1A	4,2	1,3
1B	3,4	1,3
QA	2,2	2,2
QB	2,2	2,2

15.3 Subgame Perfection

Tuesday, October 27, 2020 7:34 PM



	X	Y
UA	3,4	1,4
UB	2,1	2,0
DA	2,6	2,6
DB	2,6	2,6

	X	Y
A	3,4	1,4
B	2,1	2,0

"Continuation Value"

"Single deviation principle"

"forward induction + conditional dominance"

15 Extra Problems

Friday, October 30, 2020 10:07 AM

Consider a game in which player 1 first selects between I and O. If player 1 selects O, then the game ends with the payoff vector (x, 1) (x for player 1), where x is some positive number. If player 1 selects I, then this selection is revealed to player 2 and then the players play the battle-of-the-sexes game in which they simultaneously and independently choose between A and B. If they coordinate on A, then the payoff vector is (3, 1). If they coordinate on B, then the payoff vector is (1, 3). If they fail to coordinate, then the payoff vector is (0, 0).

(a) Represent this game in the extensive and normal forms.

(b) Find the pure-strategy Nash equilibria of this game.

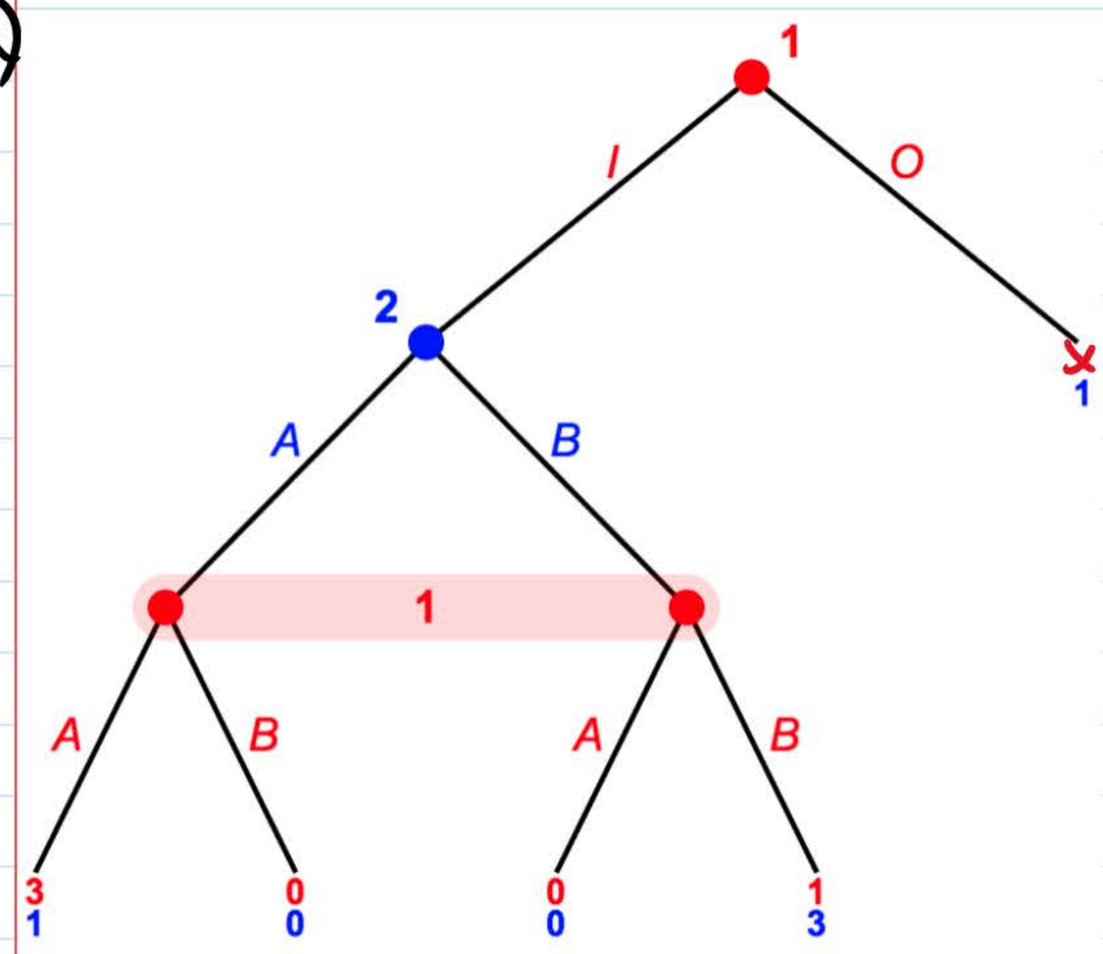
(c) Calculate the mixed-strategy Nash equilibria and note how they depend on x.

(d) Represent the proper subgame in the normal form and find its equilibria.

(e) What are the pure-strategy subgame perfect equilibria of the game? Can you find any Nash equilibria that are not subgame perfect?

(f) What are the mixed-strategy subgame perfect equilibria of the game?

a)



1 \ 2	A	B
I A	1, 3	0, 0
I B	0, 0	3, 1
O A	x, 1	x, 1
O B	x, 1	x, 1

b) if $x > 3$: (OA, A) (OA, B) (OB, A) (OB, B)
$x = 3$: $x > 3$ and (IA, A)
$1 < x < 3$: (IA, A) (OA, B) (OB, B)
$x = 1$: $1 < x < 3$ and (IB, B)
$x < 1$: (IA, A) and (IB, B)

c)

1\2	A	B	
A	3, 1	0, 0	p
B	0, 0	1, 3	$1-p$

Pure NE

Mixed NE

$\rightarrow 1p + 0(1-p) = 0p + 3(1-p) \rightarrow p = .75$
$q = .25$

$u_1(.75, .25) \rightarrow 3\frac{1}{4}\cdot\frac{3}{4} + 0 + 0 + 1\frac{1}{4}\cdot\frac{3}{4} = 9/16 + 3/16 = 12/16 = 3/4$

$u_2(.25, .75)$

B, B is subgame equilibria

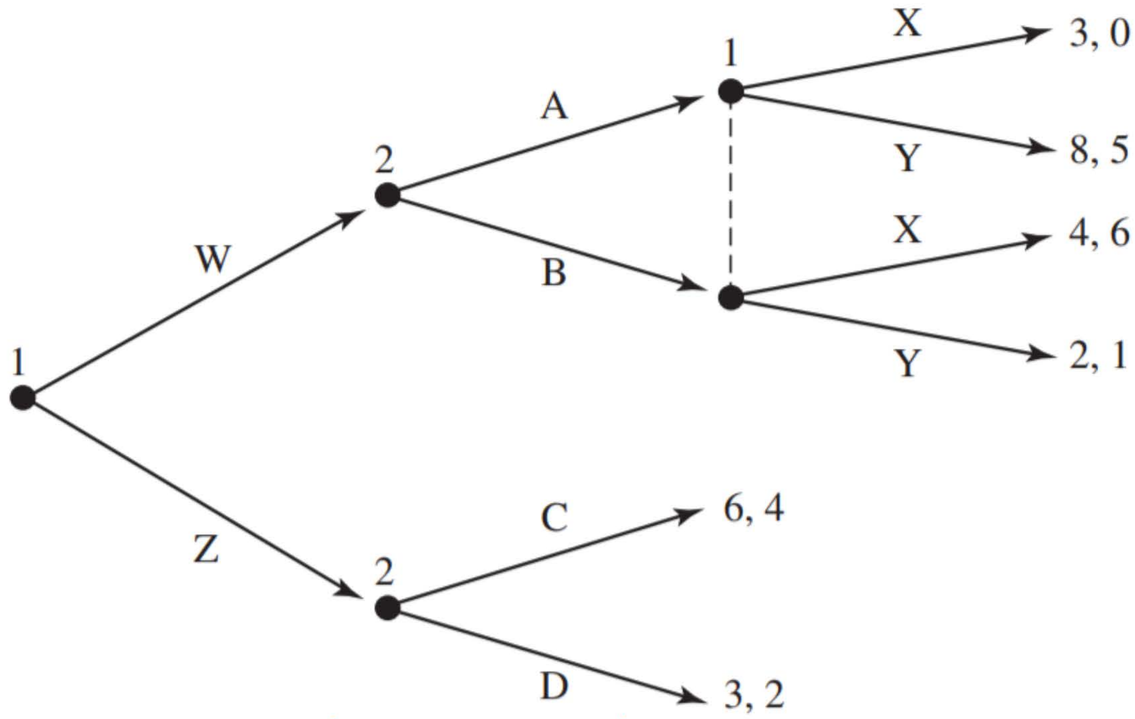
if $x < 1$, IB, B is eq
$x > 1$, OB, B is eq
$x < 3$, IA, A
$x > 3$, OA, A
$x < .75$, I $\sigma_1 = .75$, $\sigma_2 = .25$
$x > .75$, O $\sigma_1 = .75$, $\sigma_2 = .25$

HW 15.2

Sunday, November 8, 2020 3:39 PM

Passed Solution review

2. Compute the Nash equilibria and subgame perfect equilibria for the following games. Do so by writing the normal-form matrices for each game and its subgames. Which Nash equilibria are not subgame perfect?



(a)

1\2	AC	AD	BC	BD
WX	3,0	3,0	4,6	4,6
WY	8,5	8,5	2,1	2,1
ZX	6,4	3,2	6,4	3,2
ZY	6,4	3,2	6,4	3,2

Subgame

1\2	A	B
X	3,0	4,6
Y	8,5	2,1

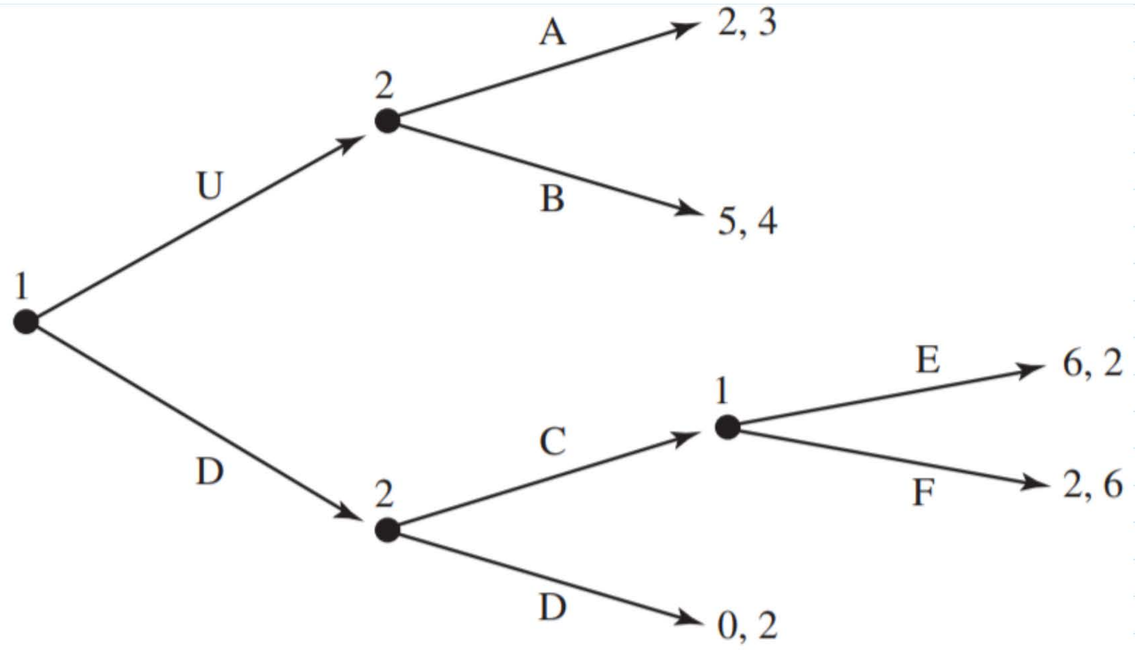
Subgame

1\2	C	D
	6,4	3,2

highlighted are NE

SPE: (WY, AC) and (ZX, BC)

NE: (WY, AC) (ZX, BC) (WY, AC) (ZY, BC) (WX, BD)



(b)

1\2	AC	AD	BC	BD
UE	2,3	2,3	5,4	5,4
UF	2,3	2,3	5,4	5,4
DE	6,2	0,2	6,2	0,2
DF	2,6	0,2	2,6	0,2

SPE: (UE, BD) and (DE, BC)

NE: (UE, BD) (DE, BC) (UF, BC) (DE, AC)

Subgames

1\2	A	B
	2,3	5,4

1\2	
E	6,2
F	2,6

1\2	C	D
E	6,2	0,2
F	2,6	0,2

HW 15.8

Sunday, November 8, 2020 4:34 PM

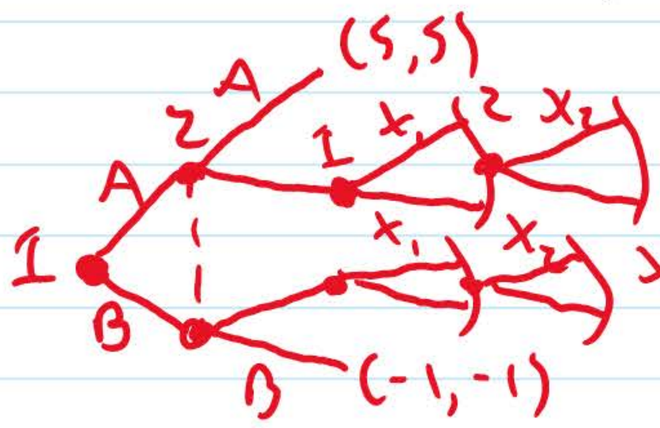
Passed Solution review

8. Imagine a game in which players 1 and 2 simultaneously and independently select A or B. If they both select A, then the game ends and the payoff vector is (5, 5). If they both select B, then the game ends with the payoff vector $(-1, -1)$. If one of the players chooses A while the other selects B, then the game continues and the players are required simultaneously and independently to select positive numbers. After these decisions, the game ends and each player receives the payoff $(x_1 + x_2)/(1 + x_1 + x_2)$, where x_1 is the positive number chosen by player 1 and x_2 is the positive number chosen by player 2.

(a) Describe the strategy spaces of the players.

$S_i = \{A, B\} \times (0, \infty) \times (0, \infty)$

A and B are separate



$x_1+x_2/1+x_1+x_2 = z_1 \quad 0<z_1<1$

$x_1+x_2/1+x_1+x_2 = z_2 \quad 0<z_2<1$

(b) Compute the Nash equilibria of this game.

(A, A) is a NE but so is every other positive number because $(x_1+x_2)/(1+x_1+x_2)$ will be positive and is the same for both

B is never part of NE because any defection yields a higher payoff.

(c) Determine the subgame perfect equilibria.

$dw/dx = 1/(1+x_1+x_2)^2 = 0$

↳ No solutions

According to b, they will choose the highest value possible

In the subgame each wants to select the largest possible value of x_i which isn't bounded. Thus any $A q_i$ with $q_i < \infty$ is not SPE

HW 15.12

Sunday, November 8, 2020 4:47 PM

Passed Solution review

12. Suppose players 1 and 2 will play the following prisoners' dilemma.

1 \ 2	C	D
C	5, 5	0, 8
D	7, 0	1, 1

Prior to interacting in the prisoners' dilemma, simultaneously each player i announces a binding penalty p_i that this player commits to pay the other player j in the event that player i defects and player j cooperates. Assume that these commitments are binding. Thus, after the announcements, the players effectively play the following induced game.[6]

1 \ 2	C	D
C	5, 5	$p_2, 8 - p_2$
D	$7 - p_1, p_1$	1, 1

(**a**) What values of p_1 and p_2 are needed to make (C, C) a Nash equilibrium of the induced game?

$5 = 7 - p_1 \rightarrow p_1 = 2$
$5 = 8 - p_2 \rightarrow p_2 = 3$ $\rightarrow p_1 \geq 2$ and $p_2 \geq 3$

(**b**) What values of p_1 and p_2 will induce play of (C, C) and would arise in a subgame perfect equilibrium of the entire game (penalty announcements followed by the prisoners' dilemma)? Explain.

p_i should be small for higher payoff, therefore...

$p_1 = 2$ and $p_2 = 3$

(**c**) Compare the unilateral commitments described here with contracts (as developed in Chapter 13).

To maximize joint profit, they agree to pick p_i by themselves

16 Book Notes

Saturday, October 31, 2020 11:59 AM

Topics in Industrial organization

Advertising and competition

Positive vs negative advertisement

A model of limit capacity

capacity constrained vs unconstrained

Dynamic Monopoly

total for all periods at/after purchase - P

	P_1	P_2
High	1200	500
Low	500	200

Regardless of P1, if at least one high customer and no lows purchase in P1, it is optimal for the seller to select $P_2 = 200$ in P_2

Scheme A: all customers in P1

$500 + 200 = 700$ $\quad$ $1700 - 700 = 1000$

$P_1 = 700$

Scheme B: all customers not in P_1

$P_1 > 1700$ so $P_2 = 500$

Scheme C: high in P_1, low in P_2

at $P_1 = 700$, highs buy but lows don't

max $1200 + 500 - P_1 \geq 500 - P_2 \rightarrow$ $\quad$ $P_2 = 200$

$P_1' \leq 1400$

$P_1 = 1400$, $P_2 = 200$

Price guarantees as a commitment to high prices

16.1 Stackelberg 1 - Monopoly Review

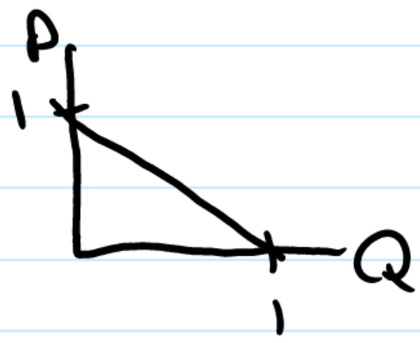
Saturday, October 31, 2020 12:29 PM

Stackelberg Monopoly

monopoly → $q = 1 - p$ $q = a - bp \rightarrow p = a/b - q/b$

↳ normalize to 1

↳ $p = 1 - q/b$



Cost = fixed cost + cq

↳ ignore frequently, esp. sunk costs

$u = p \cdot q - cq - f = (p - c) \cdot q = (1 - q - c) \cdot q$

$\frac{du}{dq} = 1 - q - c - q = 0 \rightarrow 2q = 1 - c \rightarrow q = 1 - c/2$ $p = 1 - \frac{1-c}{2}$

$u = \left[1 + c/2 - 2c/2\right](1 - c/2) = \left(\frac{1-c}{2}\right)^2$

Cournot Duopoly → 2 firms compete on quantity

16.2 Stackelberg 2 - Cournot Review Plus

Saturday, October 31, 2020 12:39 PM

Cournot Duopoly

$Q = q_1 + q_2 \rightarrow Q_{-i}, q_i \quad Q = q_i + Q_{-i}$

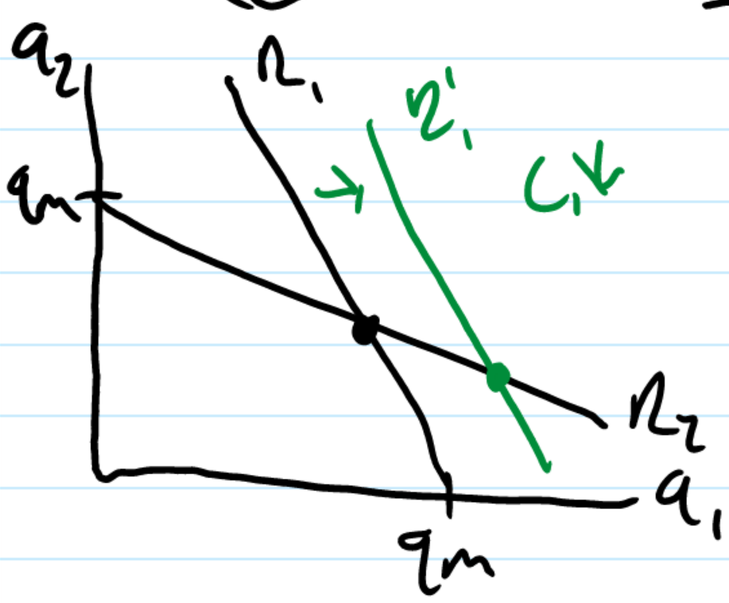
$u_i = (P - c_i)q_i - F_i$
$= (1 - q_i - q_{-i} - c_i) \cdot q_i$

$du/dq_i = 1 - q_i - q_{-i} - c_i - q_i = 0 \rightarrow q_i = 1 - c_i/2 - q_{-i}/2$

$q_i = 1 - c_i/2 - \frac{1}{2}(1 - c_i/2 - q_i/2) \rightarrow q_i = (1 - 2c + c_{-i})/3$

$Q = q_1 + q_2 = (1 - 2c_1 + c_2)/3 + (1 - 2c_2 + c_1)/3 \rightarrow Q = \frac{2 - c_1 - c_2}{3}$
$P = 1 + c_1 + c_2/3$

$u_i = ([(1 - c_i + c_{-i})/3] - c_i)([1 - 2c_i + c_{-i}]/3) = [(1 - 2c_i + c_{-i})/3]^2$



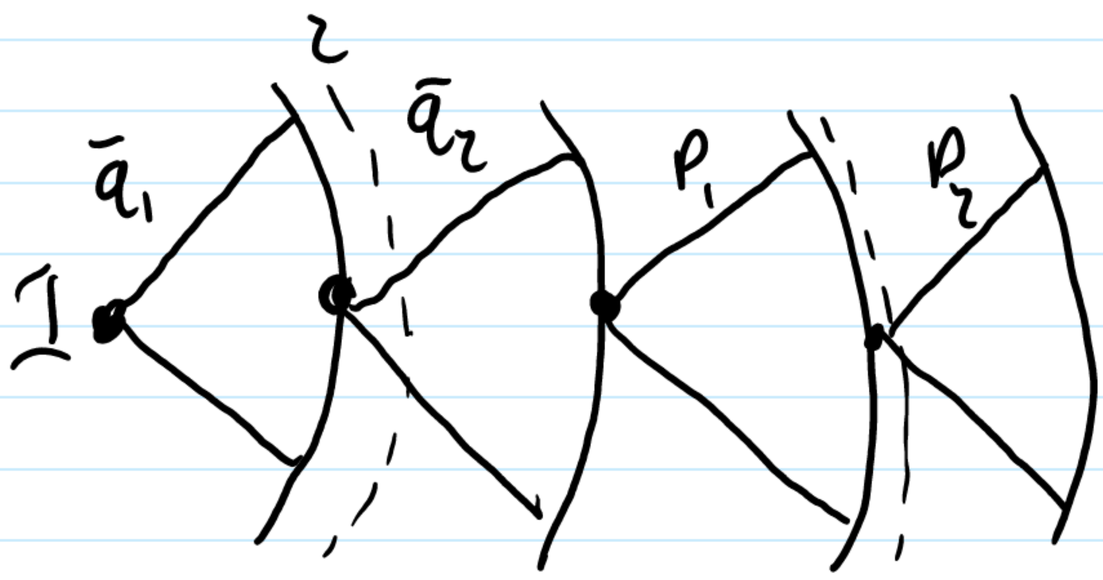
16.3 Stackelberg 3 - Background ~ Capacity ~ and Extensive Form

Saturday, October 31, 2020 12:53 PM

Stackelberg Duopoly

Capacity choice is the strategic variable

Capacity choice

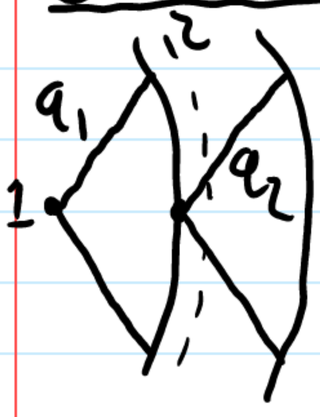


$q_1, q_2 \quad u_1, u_2$

16.4 Stackelberg 4 - Unreasonable Equilibria

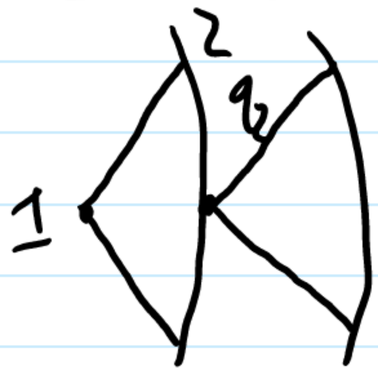
Saturday, October 31, 2020 1:04 PM

Cournot



u_1, u_2

Stackelberg



$P = 1 - q_1 - q_2$

$u_1 = (1 - q_1 - q_2 - c_1) q_1$

if $q_2 = (1 - c_1)$ then $u_1 = (-q_1) q_1 < 0$

$$s_2 = \begin{cases} q_2 = 1 - c_1 & \text{if } q_1 > 0 \\ q_2 = 1 - c_2/2 & \text{if } q_1 = 0 \end{cases}$$

$BR_1(s_2) = 0$

$BR_1(q_2) = 1 - c_1/2 - q_2/2 = 0$ → Non-credible Threat

16.5 Stackelberg 5 - SPE

Saturday, October 31, 2020 1:14 PM

$q_2 = 1 - c_2/2 - q_1/2$

$u_1 = (1 - q_1 - c_1 - q_2) q_1 = 1 - c_1 - 1 - c_2/2 - q_1/2) q_1$

$du/dq = 1 - c_1 - 1 - c_2/2 - q_1/2 - q_1/2 = 0 \rightarrow = (2 - 2c_1 + c_2)/2$

q_1 Stack > q_1 Cour

$q_2 S < q_2 C$

$Q S > Q C$

$P S < P C$

$U_1 S > U_1 C$

$U_2 S < U_2 C$

Strategic Subs

1st move advantage

2nd move disadvantage

16.6 16.7 Limit Capacity

Saturday, October 31, 2020 1:46 PM

ALCOA vs United States

Model

Firm 1	Firm 2
- In tech H	- In tech H
- In tech L	- In tech L
- Out	- Out

If 1 firm, monopoly
If 2 firms, Cournot

1 ⟶ 2 ⟶ 3

$c = 1/4$, $f = L$ or $c = 0$, $f = H$

Low tech $L < H$ High tech

monopoly $= u_i = (1 - c_i/2)^2$

Cournot $= u_i = [(1 - 2c_i - c_{-i})/3]^2$

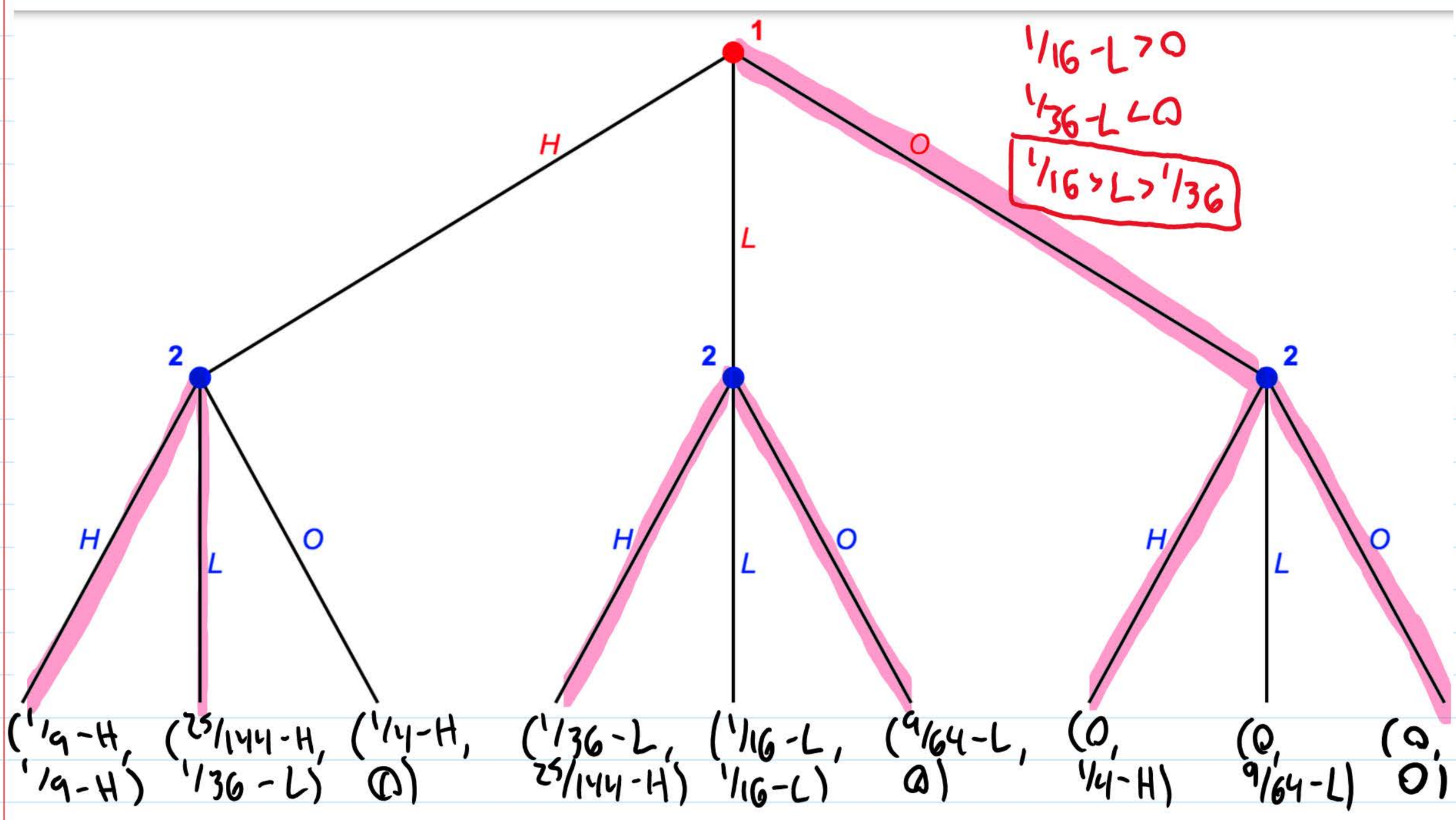
Assume monopolist prefers tech L

$(1 - .25/2)^2 - L > (\frac{1}{2})^2 - H$

$H - L > 1/4 - 9/64 = 7/64$

Extensive form

$u_1 = (\frac{1}{3})^2 - H = \frac{1}{9} - H \rightarrow \frac{1}{9} < H$ $7/63$



$1/16 - L > 0$

$1/36 - L < 0$

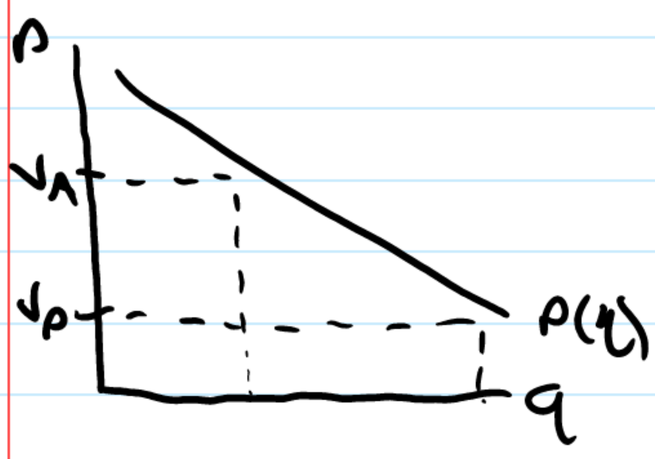
$\boxed{1/16 > L > 1/36}$

When does 1 choose H?

$1/4 - H > 1/16 - L \rightarrow 3/16 > H - L$

16.8 Dynamic Monopoly (Durable Goods Monopoly)

Saturday, November 7, 2020 3:43 PM



Today's monopolist is in competition with their future selves.

Time	1	2
High	5	3
Low	3	2

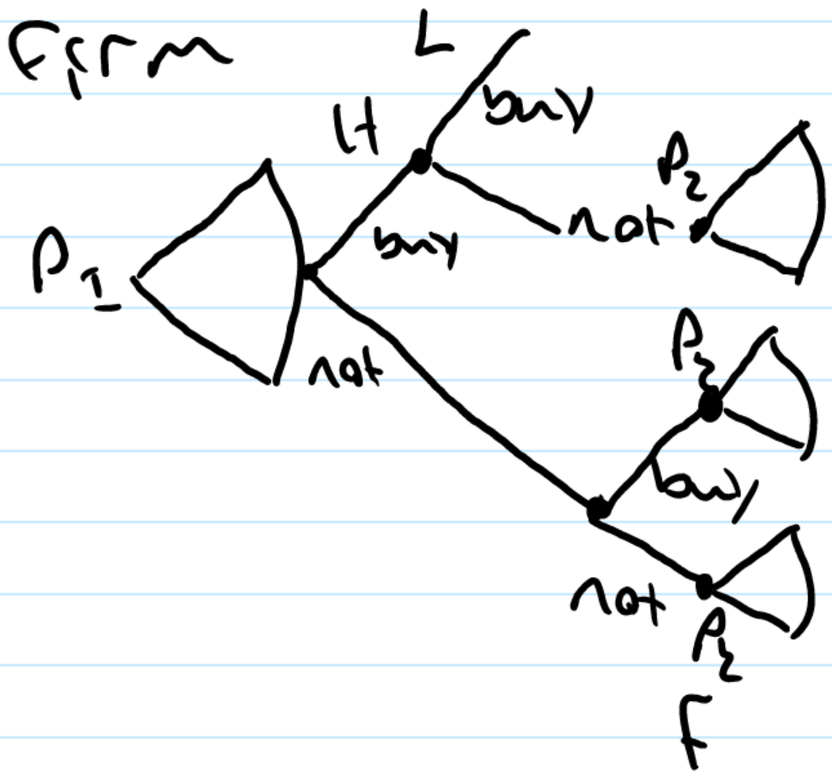
$P_2 = 2 \rightarrow 7 - P_1 = 3 - 2 \rightarrow P_1 = 7$

$L = 1 - h$

$U_{mon} = h7 + 2(1-h) = 2 + h$ so $P_1 = 5$

$\hookrightarrow V_{mon} = h5 + (1-h)5 = 5$

P_1, P_2 when is $P_1 = 7, P_2 = 2$; buy@1, buy@2 the SPE



$2 + 5h > 5 \Rightarrow h > 3/5$

$P_1 = 7$, $P_2 = 7 \rightarrow 7h > 2 + 5h \Rightarrow h > 2/3$

Price guarantee!

16 Extra Problems

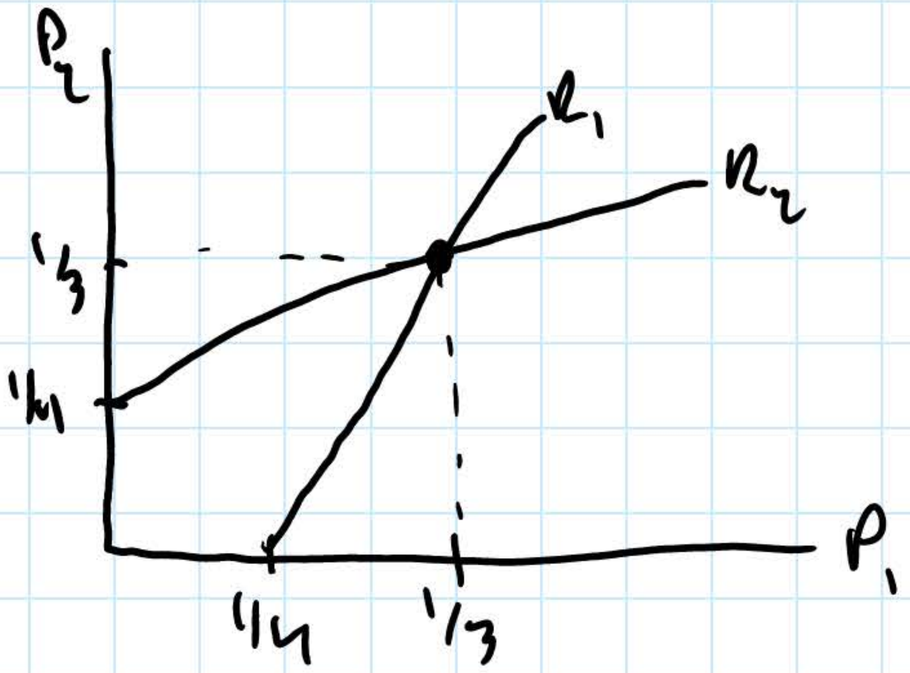
Friday, November 6, 2020 10:05 AM

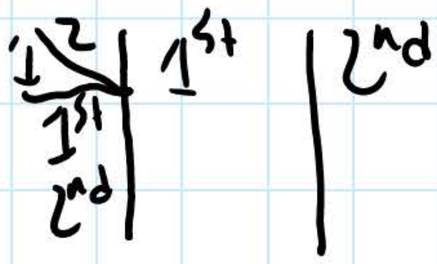
$c_1 = c_2 = 0$

$q_1 = \frac{1}{2} - p_1 + \frac{1}{2}p_2 \qquad q_2 = 1/2 - p_2 + \frac{1}{2}p_1$

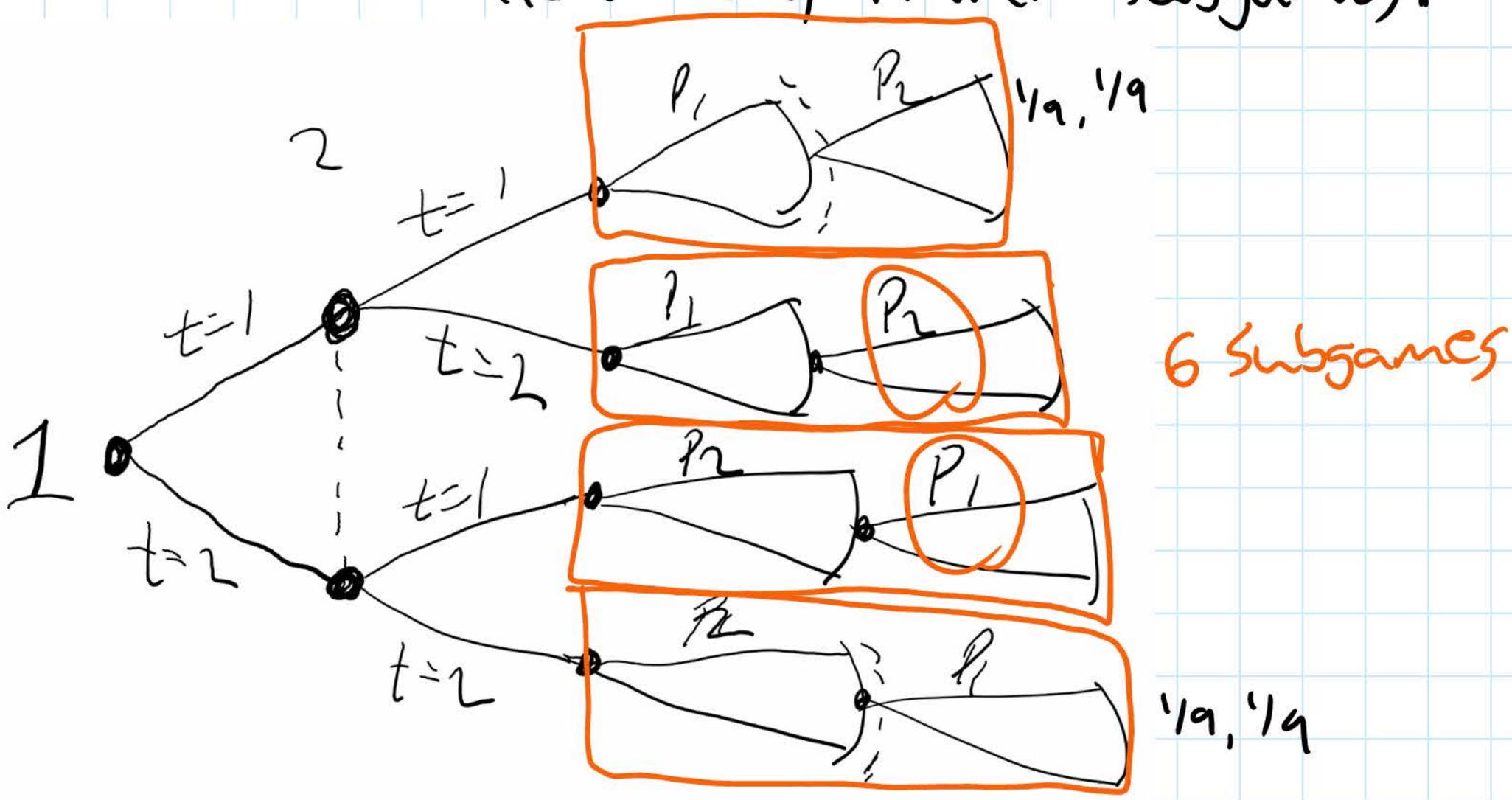
$u_1 = (p_1 - c_1)q_1 = p_1(\frac{1}{2} - p_1 + \frac{1}{2}p_1) \rightarrow \frac{du}{dp_1} = \frac{1}{2} - p_1 + \frac{1}{2}p_2 - p_1 = 0 \rightarrow p_1 = 1/4 + p_2/4$

$\hookrightarrow p_1 = 1/4$





How many proper subgames?



6 subgames

$u_1 = u_2 = \frac{1}{3}(\frac{1}{2} - \frac{1}{3} + \frac{1}{6}) = \frac{1}{9} \leftarrow$ if simultaneous moves

if P_1 goes first...

HW 16.5

Wednesday, November 11, 2020 7:13 PM

Passed Solution review

5. Consider a slight variation of the dynamic monopoly game analyzed in this chapter. Suppose there is only one high-type customer (Hal) and only one low-type customer (Laurie).

(a) Analyze this game and explain why $p_2 = 200$ is not optimal if Hal does not purchase a monitor in period 1. Find the optimal pricing scheme for Tony. Discuss whether Tony would gain from being able to commit to not selling monitors in period 2.

	Period 1	Period 2
Benefit to Hal	1200	500
Benefit to Laurie	500	200

$P_2 = 200$ is not optimal because it yields 400 while $P_2 = 500$ yields 500

If $P_1 = 1700$, $P_2 = 200$ $P_{total} = 1900$

No way to make enough \$ off Hal in P_1 to cover losses of not selling to Laurie

(b) Finally, analyze the game with one of each type of customer and ownership benefits given in the following figure. In this case, would Tony gain from being able to commit to not selling monitors in period 2?

	Period 1	Period 2
Benefit to Hal	1200	300
Benefit to Laurie	500	200

~~$P_1 = 1500$ $P_2 = 200$~~

~~No way to make enough \$ off Hal in P_1 to cover losses of not selling to Laurie~~

If Hal doesn't buy @ $t=1$ → $300 < 200 + 200$ so $P_2 = 200$

Hal buys @ $t=1$ if $1500 - P_1 \geq 300 - 200$ → $P_1 \leq 1400$

$\pi = 1600$. Tony would gain if he could commit to $P_1 = P_2$

HW 16.X1

Wednesday, November 11, 2020 6:05 PM

Passed Solution review

Two firms engage in price competition. They sell somewhat differentiated products (think Nike and adidas), which are substitutes but not perfect substitutes. The demand for firm $i \in \{1,2\}$ is $q_i = 0.5 - p_i + 0.5p_{-i}$. For simplicity, assume per unit operating costs are 0, or equivalently, that price is measured in terms of its difference from cost.

Here is the intuition for the strategic market situation captured by these demand curves. If both firms set prices of 0, total sales are 1, so we are scaling units so 1 is the largest reasonable number of sales. An increase in a firm's price leads customers to purchase less of the firm's product—some drop out of the market and others switch brands. In this case half switch and half drop out.

a) Find the Nash Equilibrium prices, and associated payoffs, assuming prices are chosen simultaneously and independently. Draw the reaction functions and label the equilibrium.

$c_1 = c_2 = 0$

$q_1 = \frac{1}{2} - p_1 + \frac{1}{2}p_2 \qquad q_2 = 1/2 - p_2 + \frac{1}{2}p_1$

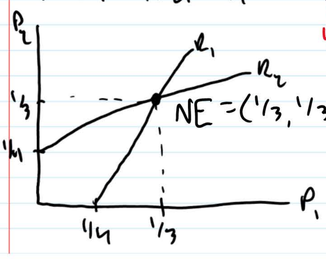
$u_1 = (p_1 - c_1)q_1 = p_1(\frac{1}{2} - p_1 + \frac{1}{2}p_1) \rightarrow \frac{du}{dp_1} = \frac{1}{2} - \cancel{p_1 + \frac{1}{2}p_2 - p_1} = 0 \rightarrow \cancel{p_1 = 1/4 + p_2/4}$

$\hookrightarrow \cancel{p_1 = 1/4}$

$p_i = (1 + p_{-i})/4$

$4p = 1 + p$

$3p = 1 \rightarrow p = 1/3$



$U_i = (1/2 - 1/6)1/3 = 1/9$

1 \ 2	1st	2nd
1st	1/9, 1/9	
2nd		1/9, 1/9

1 moves first, $p_1 = 1/3 + \varepsilon$

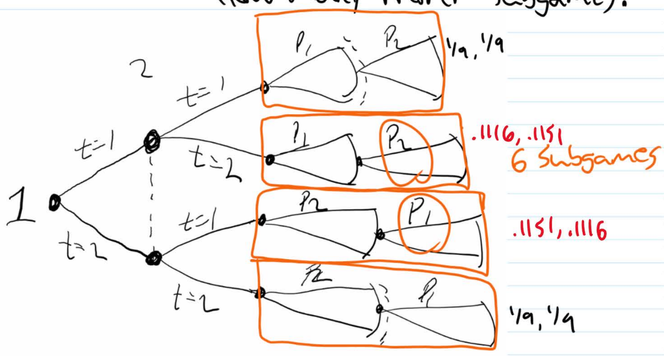
$p_2 = 1/3 + \varepsilon - \varepsilon$

ε = tiny #

b) Find the SPE prices, and associated payoffs, assuming firm 1 chooses price first in a sequential game. Label this outcome in the reaction function diagram, and, intuitively explain the nature of the difference form the simultaneous move game.

$u_1 = U_2 = \frac{1}{3}(\frac{1}{2} - \frac{1}{3} + \frac{1}{6}) = \frac{1}{9} \leftarrow$ if simultaneous moves

How many proper subgames?



1 \ 2	t=2	t=3
t=2	.1111, .1111	.1116, .1151
t=3	.1151, .1116	.1111, .1111

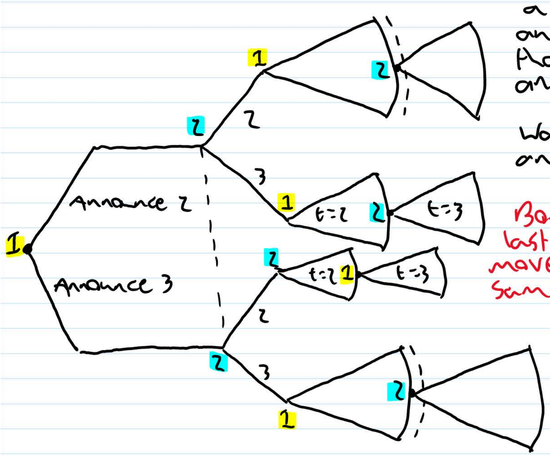
No NE where they announce simultaneously

$.1111p + .1116(1-p) = .1151p + .1111(1-p) \rightarrow p = 1/9$ and it's symmetric

$U_i = \frac{1}{81} \cdot .1111 + \frac{64}{81} \cdot .1111 \rightarrow U_i = .1115$

Symmetric mixed NE is worse than either of the 2 pure NE

c) Assume the game spans four discrete time periods. In the first period each firm simultaneously and independently announces whether they will post their price in period 2 or period 3. Once this announcement is made the firms are committed to that timing, as they are to their prices once posted. In the fourth period, sales and payoffs are realized at the posted prices. Draw the extensive form of this game and find the pure strategy SPE. Is there an equilibrium where players strategies regarding when to announce prices are mixed? If so, what is it? Comment on anything interesting about the timing in this game and how it related to first and second mover advantages.



Timing only matters if a player announces 2 and has a lower price than the other who announces at 3

Wait. Nvm. Prices are announced together

Both want to move last, but it's better to move first than at the same time

22 Book Notes

Saturday, November 7, 2020 4:01 PM

Repeated Games and Reputation

t = given time period
T = total # of periods

Each t game is a static stage game

A two-period repeated game

$T=2$ total payoff $= t_1 + t_2$

	Z	X	Y	Z
1	A	4,3	0,0	1,4
	B	0,0	2,1	0,0

Players must select NE in the second period

Preferences in subgames are the same as in t_1

In any repeated game, any sequence of stage Nash profiles can be supported as the outcome of a subgame perfect NE

Dewey says that so long as end game has multi NE, players have history to agree on end game

An Infinitely repeated game

δ = discount factor

$V \equiv 1 + \delta V \rightarrow V = 1/(1-\delta) = a/(1-\delta)$

Trigger strategy → cooperative and punishment profiles

Punishment is a stage NE profile

If a player deviates from cooperative, the others punish them

grim-trigger strategy

one-deviation property

The Equilibrium Payoff Set w/ Low Discounting

Cooperation is always eventually possible

22.1 Repeated Games - Finite Repetitions

Saturday, November 7, 2020 5:11 PM

1 \ 2 (B)	L	C	R
A: U	6,6	0,7	0,1
M	7,0	3,3	0,1
D	1,0	1,0	1,1

T reps

D,R for all T is a SPE

M,C all T is also

T MC
T-1 MC
T-2 MC
↓
unravels b/c Promise MC @T is not credible

Amy: t=1, U
t=2,...T-1, U if Bob has Played L up to then
t=T, M
else, D
Bob same
D,D T→(M,C)

SPE

Bob at T-1 → 6+1

T-1
6+3>7+1
9>8

T-2
6+6+3>7+1+1
15>9

22.2 Indefinitely Repeated Games 1

Monday, November 9, 2020 8:54 PM

Time value of money

r = interest rate

$\$1 \to (1+r)1 \to (1+r)^2$

$V_0 = V_0(1+r)^t$

now future

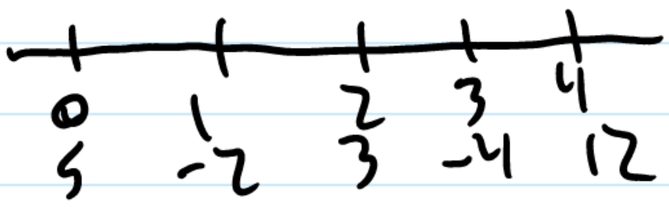
$V_1 = V_t/(1+r)^t$ $\delta = 1/(1+r)$ = discount factor < 1

$V_0 = V_t \delta^T$

f = Prob game ends after each round

$t = 0$ = now $\delta^0 = 1$ $\delta = \frac{1-f}{1+r}$

$V_0 = V_{10}[(1-f)^{10}/(1+r)^{10}] = V_{10}\delta^{10}$



$ENPV = 5 + \frac{1-f}{1+r}(-2) + \frac{(1-f)^2}{(1+r)^2} + \ldots = EPV$

Stage games and trigger strategies

$NPV = V + \delta V + \delta^2 V + \ldots = V(1 + \delta + \delta^2 + \ldots) = V\sum_{t=0}^{\infty} \delta^T$

22.3 Aside on Annuities and Perpetuities

Monday, November 9, 2020 9:10 PM

Paid V @ $t=1, 2, 3, \ldots, T$
Perpetuity V @ $t=1, 2, \ldots, \infty$

$V_A = \delta V + \delta^2 V + \ldots + \delta^T V = V(\delta + \delta^2 + \ldots \delta^T)$
$V_A = Va(\delta, T)$

$\hookrightarrow a(\delta, T) = \delta + \delta^2 + \ldots + \delta^T$
$a - \delta a = \delta - \delta^{T+1} = \delta(1-\delta^T)$
$a(1-\delta) = \delta(1-\delta)^T$
$a = (\delta/1-\delta)(1-\delta)^T$

$0 < \delta < 1 \quad T \to \infty, \delta^T \to 0 \to a \to \delta/1-\delta$

$V_{perpetuity} = \delta/1-\delta$

NPV of $V + \underbrace{\delta V + \delta^2 V + \ldots}_{\text{Perpetuity starts at } t=1} = V/(1-\delta)$

Payment now

22.4 Indefinitely Repeated Games 2

Monday, November 9, 2020 9:18 PM

	Coop	Defect
Coop	v^c, v^c	v^s, v^d
Defect	v^d, v^s	v^p, v^p

$v^d > v^c$
$v^c > v^p$
$v^p > v^s$

$\delta = 1-f/1+r$

Grim trigger: I'll be nice until you're mean, then I'm mean forever

Is grim a BR to grim?

Payoff of grim $= v^c/(1-\delta)$

Defect? $\rightarrow v^d + v^p\delta + v^p\delta^2 + \ldots = v^p(\delta/1-\delta) + v^d$

Grim is BR to Grim

IFF

$(1-\delta)v^c/1-\delta > v^p\delta/1-\delta + v^d)(1-\delta) \rightarrow \delta > v^d - v^c / v^d - v^p$

$\delta\uparrow$ more Coop
$v^d\uparrow$ less Coop
$v^p\uparrow$ less Coop
$v^c\uparrow$ more Coop

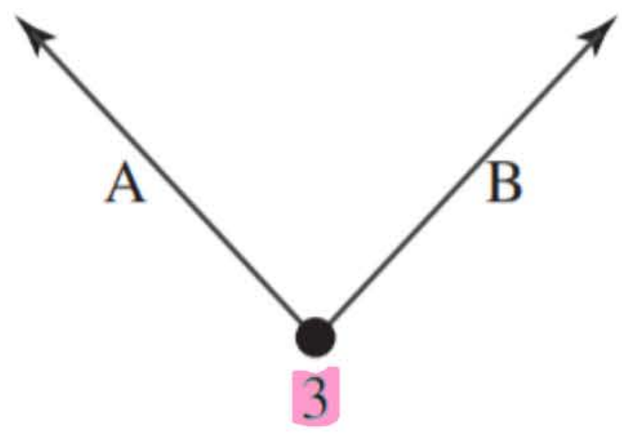
22 Extra Problems

Monday, November 9, 2020 10:25 AM

7. Consider the following three-player game.

1 \ 2	L	R
U	2, 2, 0	5, 5, 5
D	8, 6, 8	0, 7, 4

1 \ 2	L	R
U	4, 4, 1	4, 2, 8
D	0, 2, 9	4, 2, 5



The players make their choices simultaneously and independently. The payoffs are listed in order of the player numbers.

(a) Find the (pure-strategy) Nash equilibria of this game.

See highlights

(b) Consider the two-period repeated game in which this stage game is played twice and the repeated game payoffs are simply the sum of the payoffs in the two periods. Compute and report all of the subgame perfect equilibria of this repeated game. List the set of subgame perfect equilibrium payoffs.

DLA

Pure NE: (ULB) and (DRB)

1) D @ T=1
D @ T=2 if A@T

(more)

2) L @ T=1
R @ T=2 if U, A @ T=1 L otherwise

Is that a BR for Player 1?

4+8 = DLA + DRB then 2+4 = ULA + ULB

2?

6+2 or 7+4
8 or 11

3?

8+5 or 9+1
13 or 10

IF URA @ t=1 → 3
DRB @ t=1 →
else ULB @ t=2 →

HW 22.2

Wednesday, November 11, 2020 11:32 AM

Passed Solution review

2. Find conditions on the discount factor under which cooperation can be supported in the infinitely repeated games with the following stage games.

1 \ 2	C	D
C	2, 2	0, 4
D	4, 0	1, 1

(a)

1 \ 2	C	D
C	3, 4	0, 7
D	5, 0	1, 2

(b)

1 \ 2	C	D
C	3, 2	0, 1
D	7, 0	2, 1

(c)

Use the grim-trigger strategy profile.

a) $\frac{2}{1-\delta} \geq 4 + \frac{\delta}{1-\delta} \rightarrow \delta \geq 2/3$ Game is symmetric so δ is the same for both players

b) Player 1: $3/1-\delta \geq 5 + \delta/1-\delta \rightarrow \delta \geq 1/2$

Player 2: $4/1-\delta \geq (7-2) + 2/1-\delta \rightarrow \delta \geq 3/5$

$\rightarrow \delta \geq 3/5$

c) Player 1: $3/1-\delta \geq (7-2) + 2/1-\delta \rightarrow \delta \geq 4/5$

Player 2: C,C is already good, no reason to deviate

$\hookrightarrow \delta \geq 4/5$

HW 22.3

Wednesday, November 11, 2020 12:13 PM

Passed Solution review

3. Consider the following stage game.

1 \ 2	X	Y
A	5, 6	0, 0
B	8, 2	2, 2

(a) Find and report all of the (pure-strategy) Nash equilibria of this game.

(B, X) and (B, Y)

(b) Consider the two-period repeated game in which this stage game is played twice and the repeated-game payoffs are simply the sum of the payoffs in each of the two periods. Is there a subgame perfect equilibrium of this repeated game in which (A, X) is played in the first period? If so, fully describe the equilibrium. If not, explain why.

If (A,X) is played in round one, then they will stay there for round 2. If player one deviates, player two will play Y in round two. No matter what, player one will play A in round 1 and B in round 2.

HW 22.4

Wednesday, November 11, 2020 3:56 PM

Posted solution review

4. If its stage game has exactly one Nash equilibrium, how many subgame perfect equilibria does a two-period, repeated game have? Explain. Would your answer change if there were T periods, where T is any finite integer?

In the second period, you can't have subgame perfection without playing the NE of the stage game. If there's only one NE in the stage game, period two won't affect period one. The only SPE is the NE in both periods. This carries for T periods and doesn't change.

HW 22.10

Wednesday, November 11, 2020 4:07 PM

Posted solution review

10. Consider a repeated game between a supplier (player 1) and a buyer (player 2). These two parties interact over an infinite number of periods. In each period, player 1 chooses a quality level $q \in [0, 5]$ at cost q. Simultaneously, player 2 decides whether to purchase the good at a fixed price of 6. If player 2 purchases, then the stage-game payoffs are $6 - q$ for player 1 and $2q - 6$ for player 2. Here, player 2 is getting a benefit of $2q$. If player 2 does not purchase, then the stage-game payoffs are $-q$ for player 1 and 0 for player 2. Suppose that both players have discount factor δ.

(a) Calculate the efficient quality level under the assumption that transfers are possible (so you should look at the sum of payoffs).

$q = (6-q) + (2q-6)$ $\rightarrow U_1 + U_2 = 2q - q = q$

$q = 6 - q + 2q - 6$

$q = q$

max $q = 5$

(b) For sufficiently large δ, does this game have a subgame perfect Nash equilibrium that yields the efficient outcome in each period? If so, describe the equilibrium strategies and determine how large δ must be for this equilibrium to exist.

$\frac{6-q}{1-\delta} \geq 6 + \frac{0 \cdot \delta}{1-\delta}$ $\quad q = 5 \rightarrow \frac{1}{1-\delta} \geq 6 \rightarrow \delta \geq 5/6$

SPE if $\delta \geq 5/6$

↑ Not a complete answer

$q = 0$ is NE of stage game + payoffs $-, 0, 0$

If $q = 5$, $U_1 = 1$, $U_2 = 4$ ...

Seller: $1 \cdot 1/1-\delta > 6 + 0\,\delta/1-\delta$

$1 > 6 - 6\delta$

$6\delta > 5$

$\delta > 5/6$

Buyer: $6\,1/1-\delta > 0 + 0\,\delta/1-\delta$

Any δ works

If $\delta > 5/6$, buy at $t=1$ at t if $q_{t-k} = 5$ $\forall k$ else do not buy and $q = 5$ at $t=1$ & if the buyer has always purchased, else $q_t = 0$ is a NE

HW 22.11

Wednesday, November 11, 2020 4:22 PM

Passed Solution review

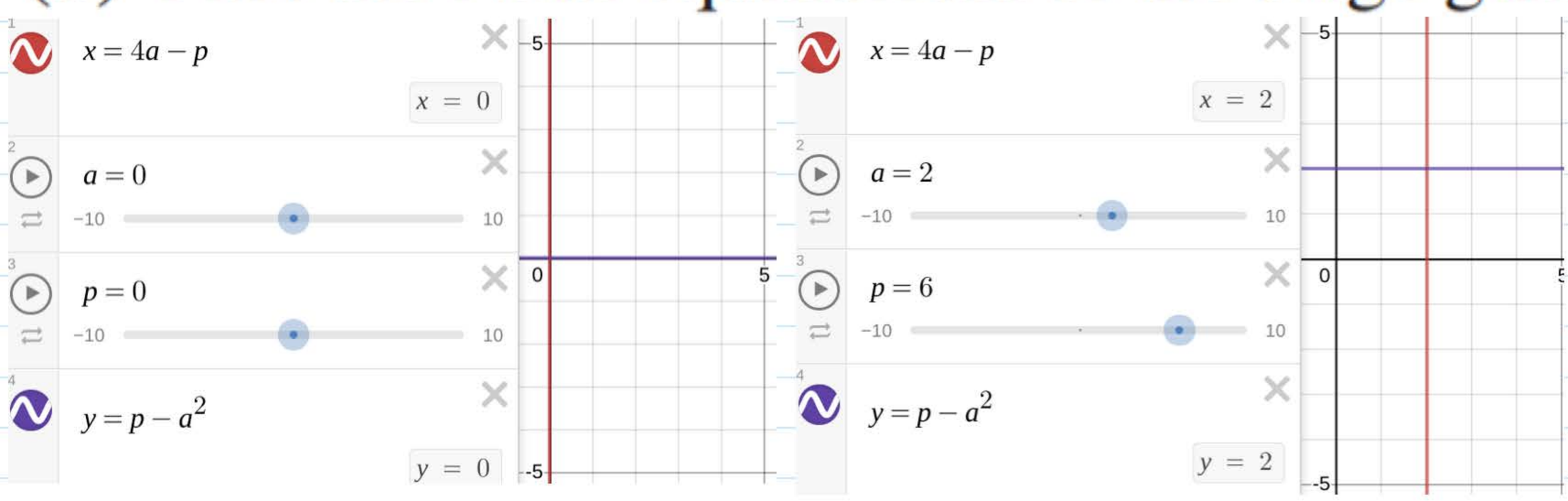
11. This is an extension of the previous exercise. Consider the following stage game between a manager (also called the "Principal") and a worker (the "Agent"). Let the manager be player 1 and the worker be player 2. Simultaneously, the manager chooses a bonus payment $p \in [0, \infty)$ and the worker chooses an effort level $a \in [0, \infty)$. The stage-game payoffs are $u_1(p, a) = 4a - p$ and $u_2(p, a) = p - a^2$.

(a) Determine the efficient effort level for the worker.

$\max = 4a - p + p - a^2 = 4a - a^2$

$dy/dx = 4 - 2a = 0 \rightarrow a = 2$

(b) Find the Nash equilibrium of the stage game.



NE is $a = p = 0$. otherwise input does not equal output

(c) Suppose the stage game is to be played twice (a two-period repeated game) and there is no discounting. Find all of the subgame perfect equilibria.

4. If its stage game has exactly one Nash equilibrium, how many subgame perfect equilibria does a two-period, repeated game have? Explain. Would your answer change if there were T periods, where T is any finite integer?

Because of this the subgame will also play $p = a = 0$

In the second period, you can't have subgame perfection without playing the NE of the stage game. If there's only one NE in the stage game, period two won't affect period one. The only SPE is the NE in both periods. This carries for T periods and doesn't change.

(d) Suppose the stage game is to be played infinitely many times in succession (an infinitely repeated game) and assume that the players share the discount factor $\delta < 1$. Find conditions on the discount factor under which there is a subgame perfect equilibrium featuring selection of the efficient effort level in each period (on the equilibrium path).

$a = 2 \quad p = 4$

$4 \le (4 \cdot 2 - 6)(\delta/1-\delta)$

$4 \le 2\delta/1-\delta$

$\delta \ge 2/3$

$\delta \ge 2/3$

$2^2 \le (6 - 2^2)(\delta/1-\delta)$

$4 \le 2\delta/1-\delta$

$\delta \ge 2/3$

Used wrong numbers

Principal: $(\delta - p)(1/1-\delta) \ge \delta + 0(\delta/1-\delta)$

$\delta - p \ge \delta - \delta\delta$

$\delta\delta \ge p$

Agent: $(p - 4)(1/1-\delta) \ge p + 0(\delta/1-\delta)$

$p - 4 \ge p - p\delta$

$p\delta \ge 4$

$p \ge 4/\delta$

$\delta\delta \ge p \ge 4/\delta$

$\delta^2 \ge 4/2$

$\delta \ge \sqrt{4/2}$

23 Book Notes

Monday, November 9, 2020 9:43 PM

Collusion, Trade agreements, and Goodwill

Dynamic Oligopoly and Collusion

Self-enforced contract

implicit contract

Enforcing International Trade Agreements

Goodwill and Trading a reputation

23.1 Collusion in Dynamic Oligopoly

Monday, November 9, 2020 10:07 PM

Repeated Games and Collusion

$V^c = V^{mon}/n$

$V^p = V^{NE}_{Bert}, V^{NE}_{Cour}$

V^d = Identical products, bertrand

V^d V_{mon}, $V^p = 0$

$V_i = [1 - q_i - (n-1)Q_{mon}/n - c_i]q_i$

$dV_i/dq_i =$ $= 0$

23 Extra Problems

Monday, November 16, 2020 10:18 AM

Proper Subgame

	C	D
C	f·10, f·10	, f·20
D	f·20,	

CC

everybody collaborates $g \cdot 10 + (1-g)\,0$

DD 0,0

CD $g(-1) + g(0)$, $(20) + (1-g)\,0$

Defection doesn't prove uncooperation

X: years punished

Grim w/ forgiveness

<u>Two Stages</u>

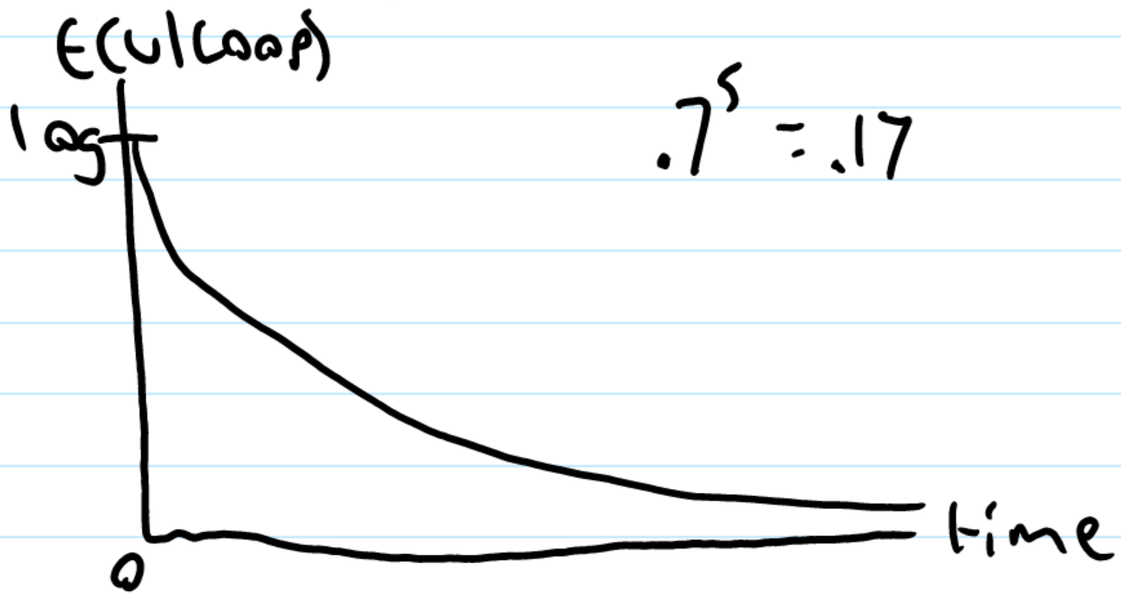
1) With no forgiveness, when is cooperation possible?

Prob $\pi_i = 10$ years up to t?

↳ If ever $\pi_{it} = 0$, $t < k$, what is π_{it}, $t \geq k$ = dd = 0

π_{it} $10g^{(t+1)}$

E(u|coop)



$.7^5 = .17$

HW 23.1

Wednesday, November 11, 2020 4:58 PM

Passed Solution review

1. Consider the Bertrand oligopoly model, where n firms simultaneously and independently select their prices, $p_1, p_2, \ldots, p_n$, in a market. (These prices are greater than or equal to 0.) Consumers observe these prices and only purchase from the firm (or firms) with the lowest price p, according to the demand curve $Q = 110 - p$, where $p = \min\{p_1, p_2, \ldots, p_n\}$. That is, the firm with the lowest price gets all of the sales. If the lowest price is offered by more than one firm, then these firms equally share the quantity demanded Q. Assume that firms must supply the quantities demanded of them and that production takes place at a constant cost of 10 per unit. (That is, the cost function for each firm is $c(q) = 10q$.) Determining the Nash equilibrium of this game was the subject of a previous exercise.

(a) Suppose that this game is infinitely repeated. (The firms play the game each period, for an infinite number of periods.) Define δ as the discount factor for the firms. Imagine that the firms wish to sustain a collusive arrangement in which they all select the monopoly price $p^M = 60$ each period. What strategies might support this behavior in equilibrium? (Do not solve for conditions under which equilibrium occurs. Just explain what the strategies are. Remember, this requires specifying how the firms punish each other. Use the Nash equilibrium price as punishment.)

$p = 60$ until someone defects, then $p = 10$

(b) Derive a condition on n and δ that guarantees that collusion can be sustained.

$q = (110-60)/n = 50/n$ $\quad$ $P = \frac{50}{n} \cdot 60 - \frac{50}{n} \cdot 10 = \frac{3000}{n} - \frac{500}{n} = 2500/n$

$\left(\frac{2500}{n}\right)\left(1/1-\delta\right) \geq 2500 \rightarrow \delta \geq 1 - 1/n$

(c) What does your answer to part (b) imply about the optimal size of cartels?

The smaller the cartel, the better

HW 23.6

Wednesday, November 11, 2020 5:29 PM

Passed Solution review

6. This exercise addresses the notion of goodwill between generations. Consider an "overlapping generations" environment, whereby an infinite-period game is played by successive generations of persons. Specifically, imagine a family comprising an infinite sequence of players, each of whom lives for two periods. At the beginning of period t, player t is born; he is young in period t, he is old in period $t+1$, and he dies at the end of period $t+1$. Thus, in any given period t, both player t and player $t-1$ are alive. Assume that the game starts in period 1 with an old player 0 and a young player 1.

When each player is young, he starts the period with one unit of wealth. An old player begins the period with no wealth. Wealth cannot be saved across periods, but each player t can, while young, give a share of his wealth to the next older player $t-1$. Thus, consumption of the old players depends on gifts from the young players. Player t consumes whatever he does not give to player $t-1$. Let x_t denote the share of wealth that player t gives to player $t-1$. Player t's payoff is given by $(1-x_t)+2x_{t+1}$, meaning that players prefer to consume when they are old.

(a) Show that there is a subgame perfect equilibrium in which each player consumes all of his wealth when he is young; that is, $x_t = 0$ for each t.

If a young player expects nothing when they're old, they'll leave nothing to the older generation

Suppose player k gives $q_k > 0$ to player k-1.
$U_k = (1-q) + 2(0) = 1-q$

Suppose player k gives 0 to player k-1
$U_k = 1 > 1-q \rightarrow$ this is NE

(b) Show that there is a subgame perfect equilibrium in which the young give all of their wealth to the old. In this equilibrium, how is a player punished if he deviates? mostly there but missing part of the complete answer

If t-1 gives $x_{t-1}=1$, then t gives $x_t=1$. If not, no one gets anything. Thus each young player gives \$1 to the old people

$x_t = 1$ if $x_{t-1}=1$ and $x_t = 0$ if $x_{t-1} > 1$
if coop: $U_t = 0+2$
if defect and $q<1$: $U_t = q+0 < 1 \rightarrow 2 > q$ this is an eq

(c) Compare the payoffs of the equilibria in parts (a) and (b).

Players get 1 in Part A and 2 in Part B. Full barlance of wealth is best.

$2 > 1$

HW 23.8

Sunday, November 15, 2020 3:55 PM

Passed Solution review

8. Consider the prisoners' dilemma stage game pictured here:

Row player \ Column player	C	D
C	3, 3	0, 5
D	5, 0	2, 2

Row player's payoff is listed first.

The following questions ask you to consider various discrete-time environments in which people meet and play the stage game. For each environment, you are to determine the extent to which cooperation can be sustained in a subgame perfect equilibrium.

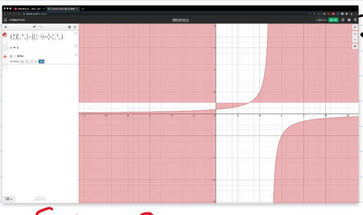
(a) Suppose that two players interact in an infinitely repeated game, with the stage game shown, and that the players share the discount factor $\delta \in (0, 1)$. Under what conditions on δ is there a subgame perfect equilibrium that supports both players selecting C each period?

$\frac{3}{1-\delta} = (5-2) + 2/1-\delta \rightarrow \delta = 2/3$

(b) Suppose that there is a society of k individuals. In each period, two people from the society are randomly selected to play the stage game. Individuals are equally likely to be selected. Furthermore, the random draw that determines who is selected in a given period is independent of the outcomes in previous periods. Thus, an individual has a probability $2/k$ of being selected to play the stage game in any given period. One of the people selected plays the role of Row player in the stage game, whereas the other person plays the role of Column player. (The stage game is symmetric, so the roles do not matter.) Those who are selected obtain the payoffs of the stage game, whereas the individuals who were not selected get 0 for this period. Everyone discounts the future using discount factor δ.

Assume that the individuals can identify each other. That is, individuals know the names and faces of everyone else in the society. Assume also that before the stage game is played in a given period, everyone observes who is selected to play. Further, everyone observes the outcome of each period's stage game.

Is there a sense in which it is more difficult to sustain cooperation in this random-matching setting than was the case in part (a)? Explain why or why not. Calculate the cutoff value of δ under which cooperation can be sustained.



This definitely isn't right and I do know why. I've gone too far ahead and don't know what's going on

Manual says the answer is $\delta \geq k/k+1$

$3 + [\delta/k(1-\delta)] \cdot 3 \geq 5 + 2[\delta/k(1-\delta)]$

$2\delta \geq 2k - 2k\delta$

$\delta(1+k) \geq k$

$\delta \geq k/1+k$

It is more difficult because there will be less to lose in the future since individuals play only 2/k of future days

(c) Consider the same setting as in part (b) *except assume that the individuals only observe their own history of play*. As before, individuals can identify each other. However, an individual only knows when he is selected to play the stage game; if he is not selected to play in a given period, he observes neither who was selected to play nor the outcome of the stage game. When an individual is selected to play the stage game, he learns whom he is matched with; then the stage game is played, and the two matched individuals observe the outcome.

Explain why cooperation is more difficult to sustain in this setting than in the setting of part (b). If you can, calculate the cutoff value of δ under which cooperation can be sustained.

Cooperation is more difficult to sustain because the players only know whether or not they have cooperated and whether or not the players they've faced cooperated when they faced them. I cannot calculate the cutoff value of delta because I don't know how. But the book says it's:

$\delta = 4(k-2)!/4(k-2)! + k!$

I don't understand this. Are the factorials there because you can only know a limited portion of the total history? It seems like a heavily modified version of the answer from part b. I graphed it and it gave me a slight bell curve which is too shallow to be a normal distribution.

Coop is much harder to sustain because you can only be punished if you are matched with someone you played against before

24 Book Notes

Tuesday, November 10, 2020 7:55 PM

Random events and incomplete information

nature's decision nodes, chance nodes, are open circles

players must know what else they could have done to be rational

Bayesian normal form: normal form game with nature

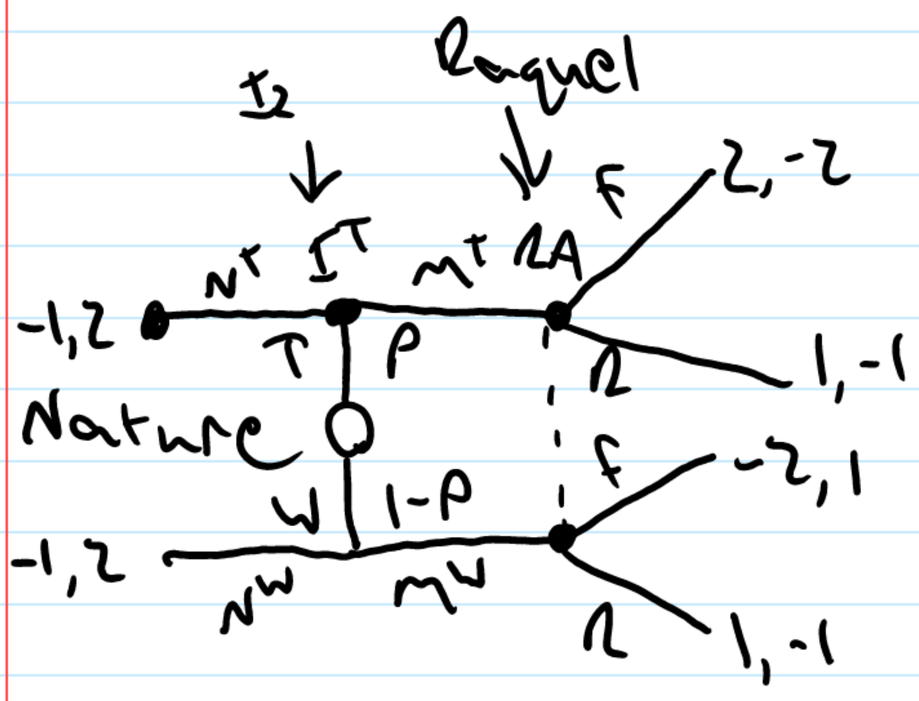
Payoffs are $xp + x(1-p)$ where x is expected payoff

24.1 Random Events and Incomplete Information

Sunday, November 15, 2020 5:57 PM

Part IV: Information!

O = nature or chance node → Nature doesn't get a Payoff



I2 \ Rachel	F	R
$M^T M^W$	$2P-2(1-P), -2P+(1-P)$	$1,-1$
$M^T N^W$	$2P-(1-P), -2P+2(1-P)$	$P-(1-P), -P+2(1-P)$
$N^T M^W$	$-P-2(1-P), 2P+(1-P)$	$-P-1\cdot(1-P), 2P-(1-P)$
$N^T N^W$	$-1,2$	$-1,2$

"Bayesian normal form"

HW 24.1

Thursday, November 19, 2020 1:08 PM

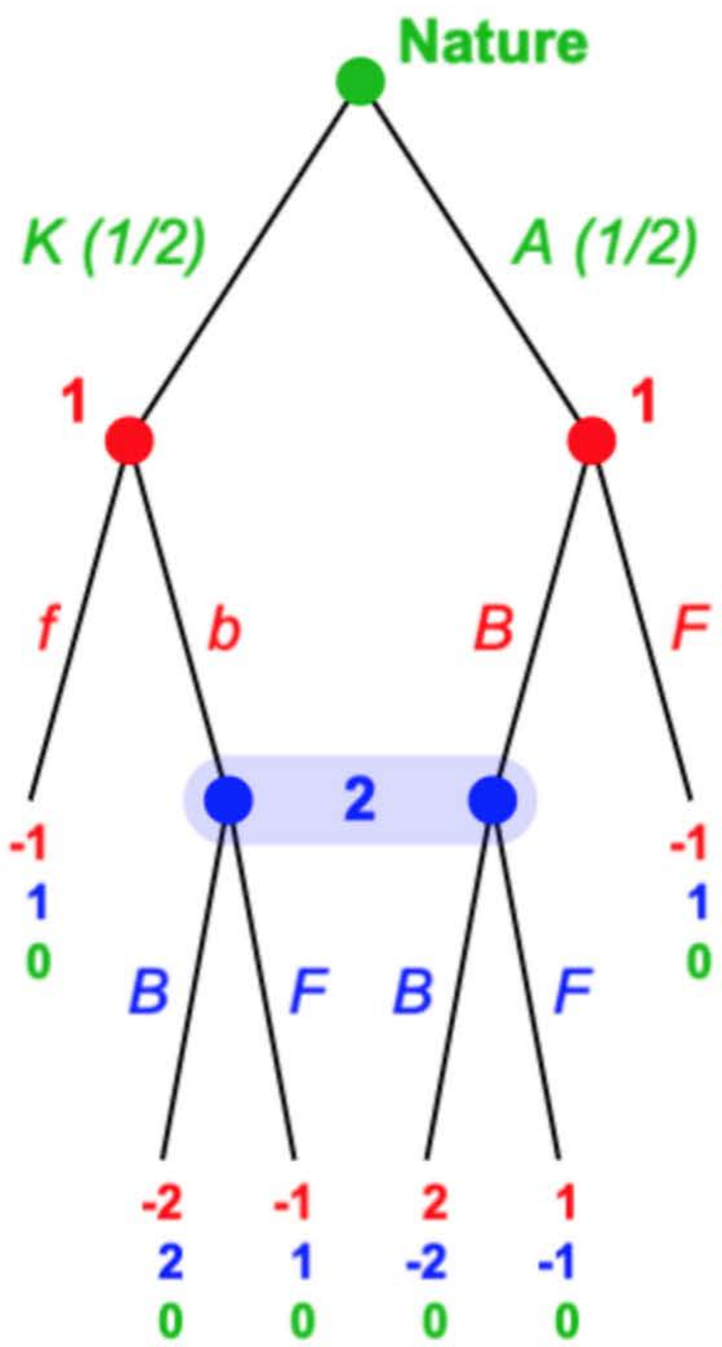
Passed Solution review

1. Here is a description of the simplest poker game. There are two players and only two cards in the deck, an Ace (A) and a King (K). First, the deck is shuffled and one of the two cards is dealt to player 1. That is, nature chooses the card for player 1. It is the Ace with probability 1/2 and the King with probability 1/2. *Player 2 does not receive a card.*

Player 1 observes his card and then chooses whether to bid (B) or fold (F). If he folds, then the game ends with player 1 getting a payoff of -1 and player 2 getting a payoff of 1 (that is, player 1 loses his ante to player 2). If player 1 bids, then player 2 must decide whether to bid or fold. When player 2 makes this decision, she knows that player 1 bid, but she has not observed player 1's card. The game ends after player 2's action. If player 2 folds, then the payoff vector is $(1, -1)$, meaning player 1 gets 1 and player 2 gets -1.

If player 2 bids, then the payoff depends on player 1's card; if player 1 holds the Ace, then the payoff vector is $(2, -2)$; if player 1 holds the King, then the payoff vector is $(-2, 2)$.

Represent this game in the extensive form and in the Bayesian normal form.



1 \ 2	B	F
Bb	0, 0	1, -1
Bf	1/2, 1/2	0, 0
Fb	-3/2, 3/2	0, 0
Ff	-1, 1	-1, 1

Bb, B and Bf, f and fb, f

$2p - 2(1-p)$

$2(1/2) - 2(1-1/2)$

$1 - 2(1/2)$

$1 - 1 = 0$

all Paths where Payoffs are equal + opposite

Fb, F and Ff, B

$-1/2 - 1(1-1/2)$

$-1/2 - 1(1/2)$

$-1 \rightarrow$ flip sign

Bb, F

full flip Ff, B to 1, -1

Fb, B

$-2p - 1(1-p)$

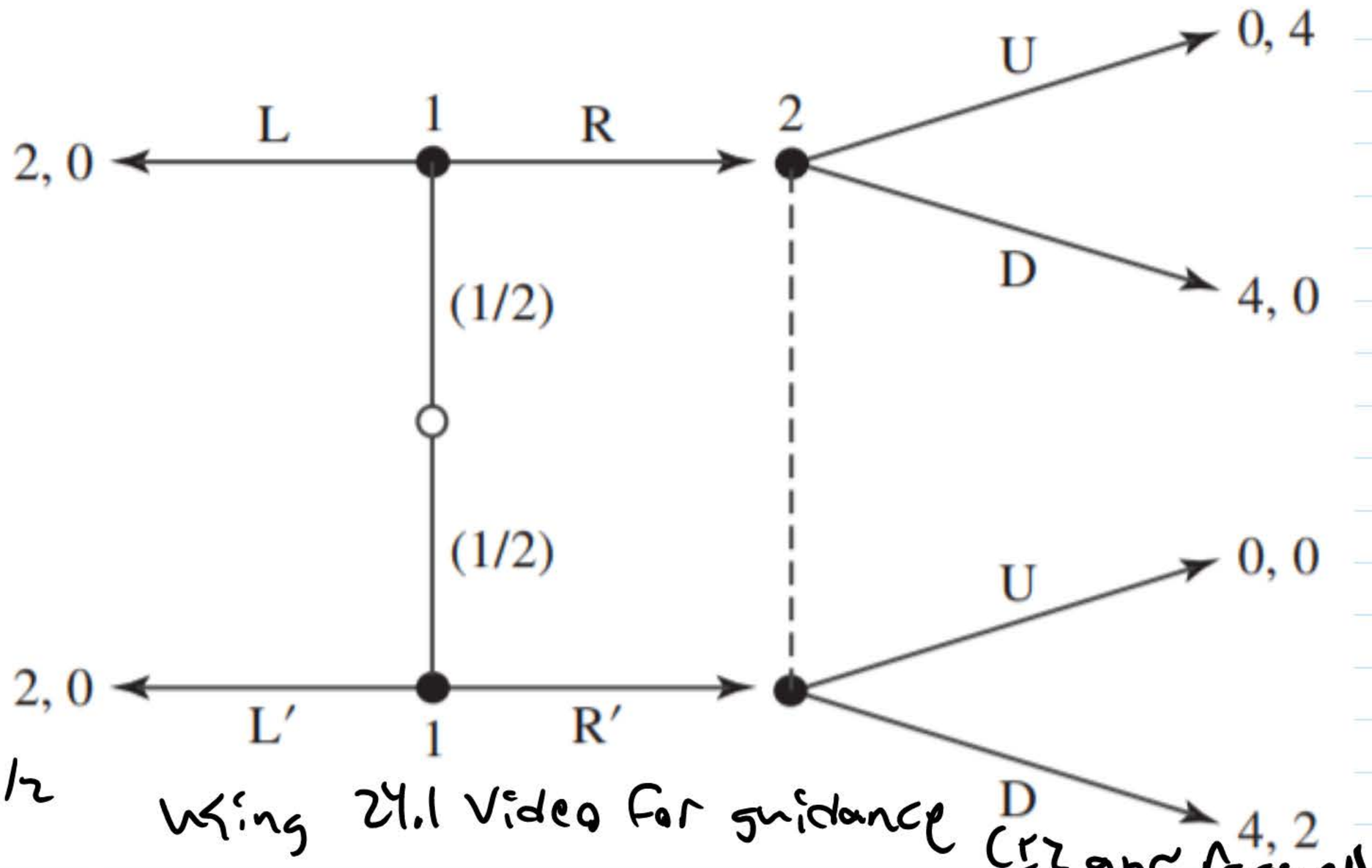
$-1 - 1/2 = -3/2 \rightarrow$ then flip sign $3/2$

HW 24.3

Thursday, November 19, 2020 1:47 PM

Passed Solution review

3. Represent the following game in the Bayesian normal form.



1 \ 2	U	D
LL'	2, 0	2, 0
LR'	1, 0	3, 1
RL'	1, 2	3, 0
RR'	0, 2	4, 1

$1-p = 1-1/2 = 1/2$

Using 24.1 Video for guidance (Liz and Raquell)

LL', U and LL', D
2, 0

LR', U
$2(1/2) - 0(1/2) = 1-0 = 1$ $\quad$ $0(1/2) - 0(1/2) = 0$

LR', D
$2(1/2) + 4(1/2) = 1+2 = 3$ $\quad$ $0(1/2) + 2(1/2) = 1$

RL', U
$0(1/2) + 2(1/2) = 1$ $\quad$ $0(1/2) + 4(1/2) = 2$

RL', D
$2(1/2) + 4(1/2) = 3$ $\quad$ $0(1/2) + 0(1/2) = 0$

RR', U
$0(1/2) + 0(1/2) = 0$ $\quad$ $4(1/2) + 0(1/2) = 2$

RR', D
$4(1/2) + 4(1/2) = 4$ $\quad$ $2(1/2) + 0(1/2) = 1$

26 Book Notes

Tuesday, November 10, 2020 8:07 PM

Bayesian Nash Equilibrium and Rationalizability

Two methods to evaluate bayesian games:
1) Compute rationalizability and NE on Bayesian normal form
2) Treat different types of players as separate players

Bayesian NE and **Bayesian rationalizability**

26.1 Bayesian Rationalizability and Bayesian Nash Equilibrium

Sunday, November 15, 2020 6:15 PM

Isabel \ Raquel	F	R
$M^T M^W$	1, -5/4	1, -1
~~$M^T N^W$~~	5/4, -1	1/2, -1/4
~~$N^T M^W$~~	-11/4, 3/4	1/2, 5/4
~~$N^P N^W$~~	-1, 2	-1, 2

→ $M^T M^W$, R → $P = 3/4$

← No proper subgames

→ $M^T M^W$, R

↑

Pooling Equilibrium: both I_T choose to fight

Another Approach

Consider $N^T N^S$ → I^T never choosen N

$BR_{RA} = \{F, R\}$ → Rational only if believe $P(T|M)$ is small

$$-2P(T|M) + 1P(W|M)$$
$$q \qquad 1-q$$
$$-2q + (1-q) > -1$$
$$2 > 3q$$
$$q < 2/3$$

~~Consider $M^T N^W$~~
~~BR_{RA} → run~~
~~I^W is mean in response to run~~

No separating equilibria

$M^T M^W$ $BR_{RA}(M^T M^W) = \text{run}$
$M^T M^W$ is BR_{I_T} + BR_{I_W} to run → Bayesian NE

If $P \neq 3/4$

26.3 More on Bayesian Nash Equilibrium

Sunday, November 15, 2020 6:40 PM

Cournot Raquel $c = .1$

Isabel $c = \begin{cases} 0 & p = 1/4 \\ .2 & 1-p = 3/4 \end{cases}$

$p = 1 - Q$

I^L $V_{IL} = (1 - q_R - q_{IL}) \cdot q_{IL}$ $\quad dv/dq_{IL} = 1 - q_R - 2q_{IL} \rightarrow q_{IL} = 1 - q_R/2$

I^H $V_{IH} = (1 - q_R - q_{IH} - .2)(q_{IH})$ $\quad dv/dq_{IH} = .8 - q_R - 2q_{IH} = 0 \rightarrow q_{IH} = .8 - q_R/2$

R $v_R = \frac{1}{4}(1 - q_R - q_{IL} - .1) q_R$ $\quad dv/dq_R = .9 - .25q_{IL} - .75q_{IH} - 2q_R$

$\hookrightarrow q_R = (.9 - .25q_{IL} - .75q_{IH})/2$

Plug back in

26 Extra Problems

Wednesday, November 18, 2020 10:27 AM

1. Consider the following game. Nature selects A with probability 1/2 and B with probability 1/2. If nature selects A, then players 1 and 2 interact according to matrix "A." If nature selects B, then the players interact according to matrix "B." These matrices are pictured here.

1 \ 2	V	W
X	6, 0	4, 1
Y	0, 0	0, 1
Z	5, 1	3, 0

A

1 \ 2	V	W
X	0, 0	0, 1
Y	6, 0	4, 1
Z	5, 1	3, 0

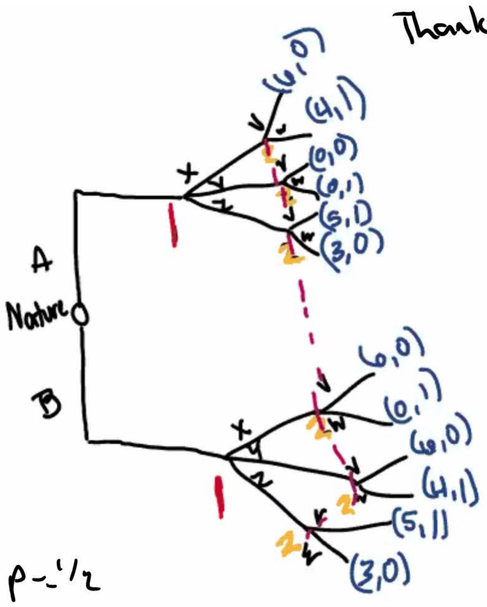
B

(a) Suppose that, when the players choose their actions, the players do not know which matrix they are playing. That is, they think that with probability 1/2 the payoffs are as in matrix A and that with probability 1/2 the payoffs are as in matrix B. Write the normal-form matrix that describes this Bayesian game. (This matrix is the "average" of matrices A and B.) Using rationalizability, what is the strategy profile that is played?

1 \ 2	V	W
X	3,0	2,1
Y	3,0	2,1
Z	5,1	3,0

(b) Now suppose that, before the players select their actions, player 1 observes nature's choice. (That is, player 1 knows which matrix is being played.) Player 2 does not observe nature's choice. Represent this game in the extensive form and in the Bayesian normal form. Using dominance, what is player 1's optimal strategy in this game? What is the set of rationalizable strategies in the game?

Thanks, Ez



	V	W
Xx	3,0	2,1
Xy	6,0	4,1
Xz	5.5, .5	3.5, .5
Yx	0,0	0,1
Yy	3,0	2,1
Yz	2.5, .5	1.5, .5
Zx	2.5, .5	1.5, .5
Zy	5.5, .5	3.5, .5
Zz	5,1	3,0

$p = 1/2$

XX, V and YY, V

$6(1/2) + 0(1-1/2) = 3+0 = 3$ $\qquad$ $0(1/2) + 0(1-1/2) = 0+0 = 0$

XY, V

$6(1/2) + 6(1/2) = 3+3 = 6$ $\qquad$ $0(1/2) + 0(1/2) = 0+0 = 0$

XZ, V and ZY, V

$6(1/2) + 5(1/2) = 3+2.5 = 5.5$ $\qquad$ $0(1/2) + 1(1/2) = 0+1/2 = 1/2$

HW 26.5

Thursday, November 19, 2020 3:02 PM

Posted Solution review

5. Consider a differentiated duopoly market in which firms compete by selecting prices and produce to fill orders. Let p_1 be the price chosen by firm 1 and let p_2 be the price of firm 2. Let q_1 and q_2 denote the quantities demanded (and produced) by the two firms. Suppose that the demand for firm 1 is given by $q_1 = 22 - 2p_1 + p_2$, and the demand for firm 2 is given by $q_2 = 22 - 2p_2 + p_1$. Firm 1 produces at a constant marginal cost of 10 and no fixed cost. Firm 2 produces at a constant marginal cost of c and no fixed cost. The payoffs are the firms' individual profits.

(a) The firms' strategies are their prices. Represent the normal form by writing the firms' payoff functions.

$\pi = (P - c)q$

$U_1 = (P_1 - 10)(22 - 2P_1 + P_2) = 42P_1 + P_1P_2 - 2P_1^2 - 220 - 10P_2$

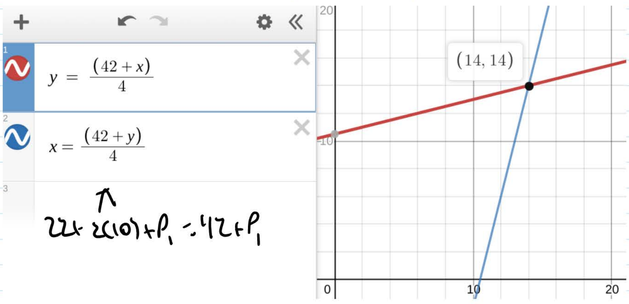
$U_2 = (P_2 - c)(22 - 2P_2 + P_1) = 22P_2 - 2P_2^2 + P_1P_2 - 22c + P_2c - P_1c$

(b) Calculate the firms' best-response functions.

$BR_1(P_2) = 42 + P_2/4$

$BR_2(P_1) = 22 + 2c + P_1/4$

(c) Suppose that $c = 10$ so the firms are identical (the game is symmetric). Calculate the Nash equilibrium prices.



$22 + 2(10) + P_1 = 42 + P_1$

(d) Now suppose that firm 1 does not know firm 2's marginal cost c. With probability 1/2 nature picks $c = 14$, and with probability 1/2 nature picks $c = 6$. Firm 2 knows its own cost (that is, it observes nature's move), but firm 1 only knows that firm 2's marginal cost is either 6 or 14 (with equal probabilities). Calculate the best-response functions of player 1 and the two types ($c = 6$ and $c = 14$) of player 2 and calculate the Bayesian Nash equilibrium quantities.

$\frac{1}{2}q_{2H} + \frac{1}{2}q_{2L}$

3 Players: P_1 P_{2L} P_{2H}

where 3 lines meet

BR_{P_1} = Plane, not line

$U_1 = (P_1 - 10)(22 - P_1 - \frac{1}{2}P_{26} - \frac{1}{2}P_{24})$

Solve: $\begin{cases} P_1 = 10.5 + P_{26}/8 + P_{24}/8 \\ P_{26} = 8.5 + P_1/4 \\ P_{24} = 12.5 + P_1/4 \end{cases}$

$\hookrightarrow P_1 = 14 \quad P_{26} = 12 \quad P_{24} = 16$

$V_1 = (P_1 - 10)(22 - 2P_1 + P_2) = 42P_1 + P_1P_2 - 2P_1^2 - 220 - 10P_2$

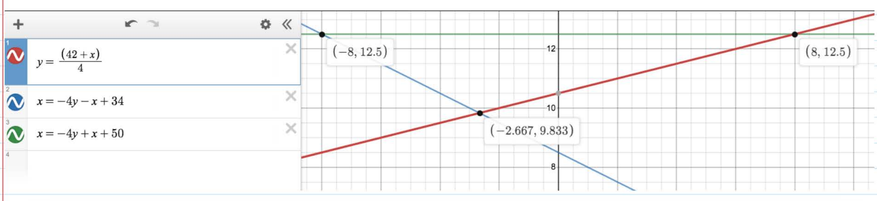
$V_{2L} = (P_2 - 6)(22 - 2P_2 + P_1) = -2P_2^2 + P_1P_2 + 34P_2 - 6P_1 - 132$

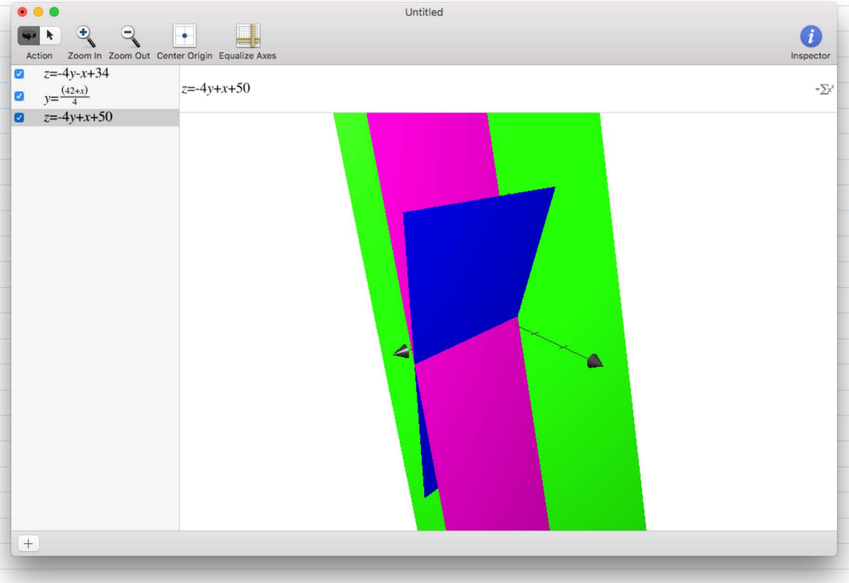
$V_{2H} = (P_2 - 14)(22 - 2P_2 + P_1) = -2P_2^2 + P_1P_2 + 50P_2 - 14P_1 - 308$

$BR_1 = 42 + P_2/4$

$BR_{2L} = -4P_2 + P_1 + 34$

$BR_{2H} = -4P_2 + P_1 + 50$



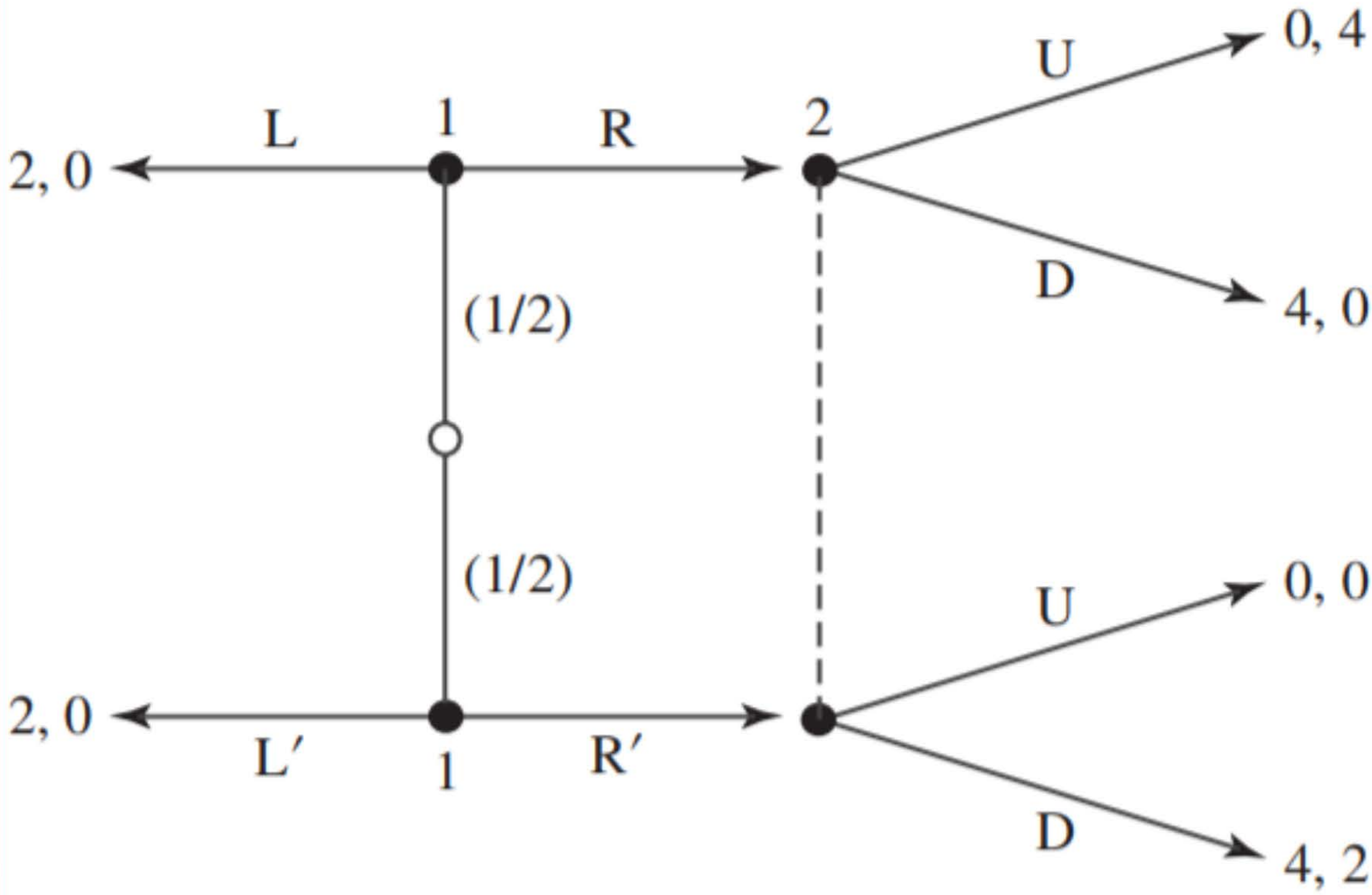


HW 26.6

Thursday, November 19, 2020 3:21 PM

Passed Solution review

6. Find the Bayesian Nash equilibrium of the game pictured here. Note that Exercise 3 of Chapter 24 asked you to convert this into the normal form.



	U	D
LL'	2, 0	2, 0
LR'	1, 0	3, 1
RL'	1, 2	3, 0
RR'	0, 2	4, 1

NE = LL', U

HW 26.9

Thursday, November 19, 2020 3:42 PM

Passed solution review

9. Consider the simple poker game described in Exercise 1 of Chapter 24, where there are just two cards in the deck and one card is dealt to player 1. This game has a single Nash equilibrium (perhaps in mixed strategies). Calculate and report the equilibrium strategy profile. Explain whether bluffing occurs in equilibrium.

1 \ 2	B	F
Bb	0, 0	1, -1
Bf	1/2, 1/2	0, 0
Fb	-3/2, 3/2	0, 0
Ff	-1, 1	-1, 1

Player 2 bluffs when they have a king because they always bid

NE = BF, B

See ex 24.1. The eq is in mixed strategies. P sometimes bids w/ a king (1/3 of the time)

27 Book Notes

Sunday, November 15, 2020 7:36 PM

Lemons, Auctions, and Information Aggregation

Markets and Lemons

buyers have a disadvantage

Asymmetric information can cause markets to malfunction

Auctions

buyers value things differently

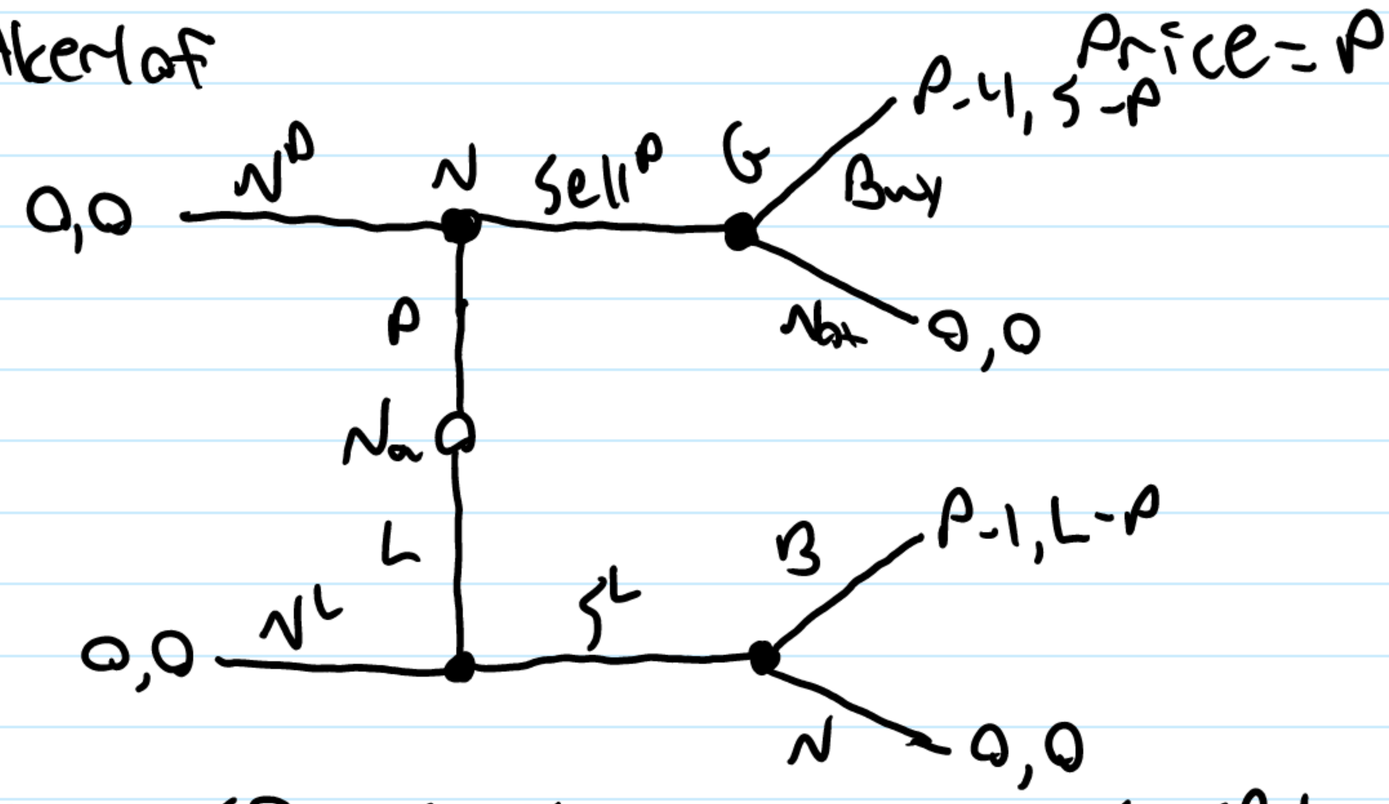
sellers want to sell high and can inflate prices w/ Auctions

Equilibrium varies by auction type

27.1 27.2 The Market for Lemons

Sunday, November 15, 2020 8:15 PM

George Akerlof



S^P $P>4$ $5f+2(1-f)>P$ → assumes $S^P S^L$ and Nick sells all cars

S^L $P>1$ $2+3f$ $>P$

$4>2+3f$

$2+3f>P>4 \rightarrow f>2/3$ N_I^P, S^L

$2>P>1$ $N_I^P S_I^L, B$ → separating eq

$f>2/3$ $S^P S^L, B$ → pooling eq

27.3 First Price Sealed Bid Auctions

Sunday, November 15, 2020 8:37 PM

2 buyers, v_i uniform on $[0,1]$

$U_1 = P(b_1 > b_2) \cdot (v_1 - b_1)$ $\quad b_i > v_i \quad b_i = v_i \quad b_i < v_i$

$= b_1/a \cdot (v_1 - b_1)$ $\quad b_2 > av_2 \quad b_1 > av_2 \quad b_1/a > v_2$

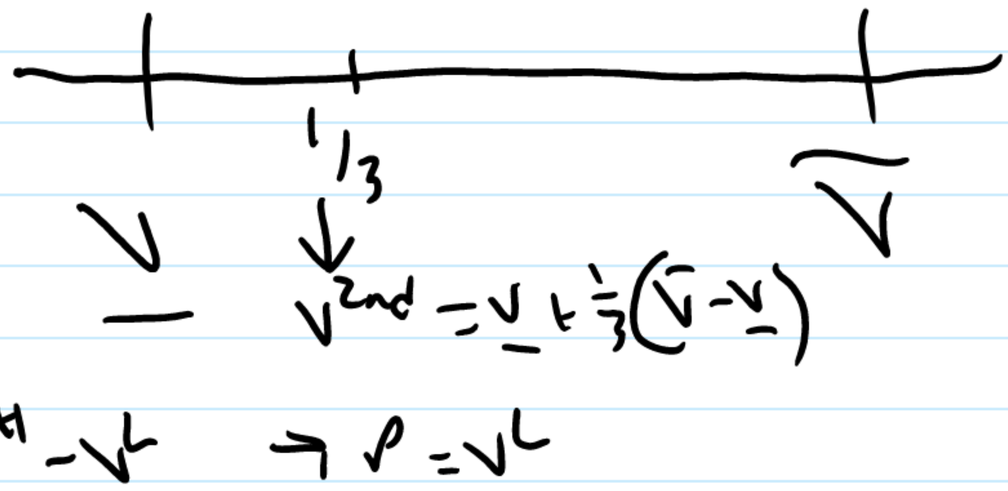
$dU/db_1 = v_1 - b_1/a - b_1/a \Rightarrow v_1 = 2b_1 \Rightarrow a = 1/2$ and $b_i = v_i/2$

27.4 More on Auctions with Private Values

Monday, November 16, 2020 8:32 PM

English Auction - First Price open bid

Highest value bidder wins by bidding 2nd highest value



$v^H - v^L \rightarrow P = v^L$

Second Price Auction (Sealed bid)

High bid wins + pays 2nd highest

$Prob(b_1 > b_2)(v_1 - b_2)$ weakly dom to bid v_1
$Prob(v_1 > v_2)(v_1 - v_2)$

Revenue Equivalence → sellers make the same

27.5 Common Values and Auction Wrap-up

Monday, November 16, 2020 8:55 PM

Common Values, Private information, + Winner's Curse

Book [10, 20] [10, 20] ← mechanic bids

15 15

Body observed by 1 c_B

mechanic observed by 2 c_m

$EV_1 = 70 - c_B - 15$ → $70 - c_B - c_m$

$EV_2 = 70 - 15 - c_m$

↳ EV_1 too high if $c_m > 15$ → "winner's curse"

↳ bid much less than EV_i

If c_B + c_m uniform on same interval,

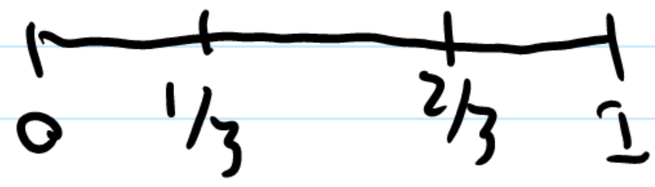
$b_1 = 70 - c_B - 20$

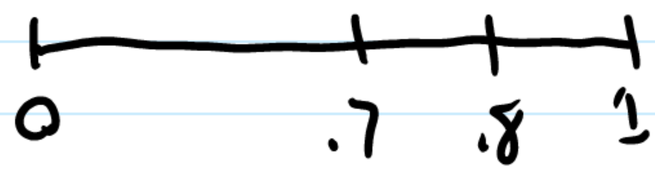
$b_2 = 70 - c_m - 20$

27 Extra Notes

Wednesday, November 18, 2020 10:14 AM

Revenue Equivalence





1st $b^w = .4$

2nd $pw = .7$

HW 27.1

Tuesday, November 24, 2020 11:48 AM

Passed Solution review

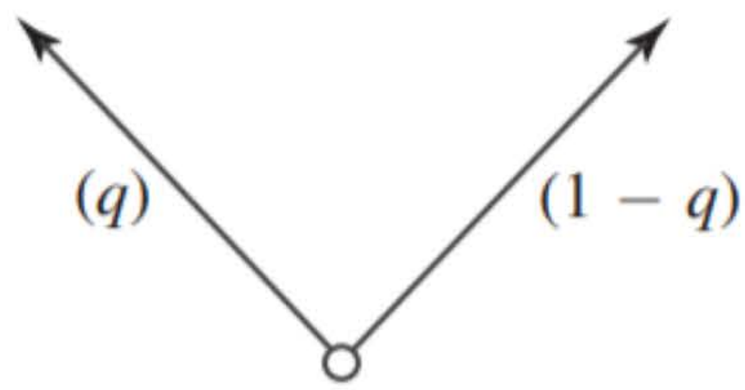
1. Regarding the trade game played by Jerry and Freddie that was analyzed in this chapter, are there values of p such that no equilibrium exists? Are there values of p such that the equilibrium entails *no* trade whatsoever?

Peach

J \ F	T^P	N^P
T	$3000 - p, p$	0, 2000
N	0, 2000	0, 2000

Lemon

J \ F	T^L	N^L
T	$1000 - p, p$	0, 0
N	0, 0	0, 0



There are no values of p that would result in no equilibrium. But there can be values for p where neither car is traded

no trade if $1000 < p \leq 2000$ or $p > 1000 + 2000q$

HW 27.4

Tuesday, November 24, 2020 11:54 AM

Posted Solution review

4. Suppose that a person (the "seller") wishes to sell a single desk. Ten people are interested in buying the desk: Ann, Bill, Colin, Dave, Ellen, Frank, Gale, Hal, Irwin, and Jim. Each of the potential buyers would derive some utility from owning the desk, and this utility is measured in dollar terms by the buyer's "value." The valuations of the 10 potential buyers are shown in the following table.

Ann	Bill	Colin	Dave	Ellen	Frank	Gale	Hal	Irwin	Jim
45	53	92	61	26	78	82	70	65	56

Each bidder knows his or her own valuation of owning the desk. Using the appropriate concepts of rationality, answer these questions:

(a) If the seller holds a second-price, sealed-bid auction, who will win the auction and how much will this person pay?

Colin wins and pays $82

(b) Suppose that the bidders' valuations are common knowledge among them. That is, it is common knowledge that each bidder knows the valuations of all of the other bidders. Suppose that the seller does not observe the bidders' valuations directly and knows only that they are all between 0 and 100. If the seller holds a first-price, sealed-bid auction, who will win the desk and how much will he or she have to pay? (Think about the Nash equilibrium in the bidding game. The analysis of this game is a bit different from the [more complicated] analysis of the first-price auction in this chapter because here the bidders know one another's valuations.)

Colin wins and pays $82 + c where c is the smallest denomination available

(c) Now suppose that the seller knows that the buyers' valuations are 45, 53, 92, 61, 26, 78, 82, 70, 65, and 56, but the seller does not know exactly which buyer has which valuation. The buyers know their own valuations but not one another's valuations. Suppose that the seller runs the following auction: She first announces a *reserve price* $\underline{p}$. Then simultaneously and independently the players select their bids; if a player bids below $\underline{p}$, then this player is disqualified from the auction and therefore cannot win. The highest bidder wins the desk and has to pay the amount of his or her bid. This is called a "sealed-bid, first price auction with a reserve price." What is the optimal reserve price $\underline{p}$ for the seller to announce? Who will win the auction? And what will the winning bid be?

If the seller knows how much people are willing to pay, they should set p = maximum WTP. Thus, Colin wins and pays $92

28 Book Notes

Monday, November 16, 2020 5:18 PM

Perfect Bayesian Equilibrium

Subgame Perfection does not capture sequential rationality

Conditional Beliefs about Types

Player type ≠ Player strategy

updated belief about player type

Sequential Rationality

Consistency of Beliefs

$q = Prob(F|G) = Prob(G|F)Prob(F) / Prob(G)$

Equilibrium Definition

Perfect Bayesian Equilibrium (PBE) if:

1) each player's strategy specifies optimal actions, given his beliefs and the strategies of other players
2) Beliefs are consistent w/ Bayes' rule whenever possible

separating vs pooling

Steps for calculating perfect Bayesian equilibria:

1. Start with a strategy for player 1 (pooling or separating).
2. If possible, calculate updated beliefs (q in the example) by using Bayes' rule. In the event that Bayes' rule cannot be used, you must arbitrarily select an updated belief; here you will generally have to check different potential values for the updated belief with the next steps of the procedure.
3. Given the updated beliefs, calculate player 2's optimal action.
4. Check whether player 1's strategy is a best response to player 2's strategy. If so, you have found a PBE.

28.1 Perfect Bayesian Equilibrium 1

Monday, November 16, 2020 9:42 PM

Bayes Rule

$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$

and

$P(B|A) = P(A|B) \cdot P(B) / P(A)$

$B, \sim B$?

$P(B), A$

Bayesian Updating

PBE

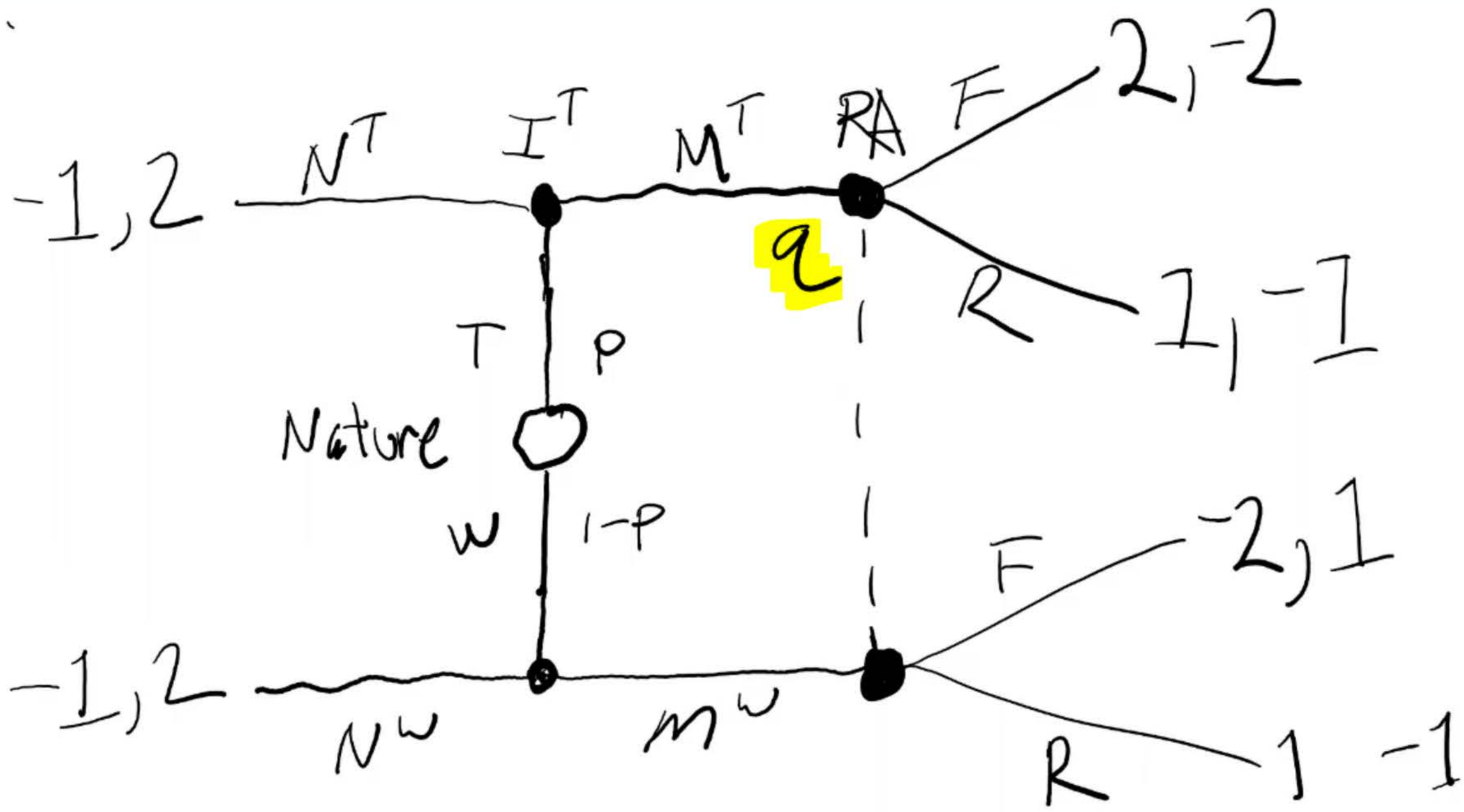
1) Play may reveal info about type subsequent rational should reflect that info
2) rationality calls for specifying beliefs about opponents type rationally at all information set

A strategy profile is a PBE if only if:

1) Each Player's actions are optimal given their beliefs and the other Players strategies
2) All beliefs are congruent w/ the equilibrium and are consistent w/ Bayes rule where it's applicable

28.2 Perfect Bayesian Equilibrium 2

Monday, November 16, 2020 10:03 PM



$q = P(T|M)$ $\quad p = 3/4$

given $\quad M^T M^W$, R q

$= P(M|T) \cdot P(T) / P(M)$

$= [1 \cdot 3/4] / 1 = 3/4$

<u>other values of p</u>

Consider $M^T N^W$

R is BR to M^T

Is M^T BR yes

Is N^W BR <u>NO</u> R runs if mean is played

Consider $M^T M^W$, R q

$-2p + 1(1-p) < -1$

$-2q + 1(1-q) > 2/3 < p$ $\rightarrow$ PBE if $p > 2/3$

what if $p < 2/3$?

28.3 Perfect Bayesian Equilibrium - 3

Wednesday, November 18, 2020 1:05 PM

Consider $M^T M^W$, R $\quad q = p \quad$ PBE IF $p > 2/3$

$-2p + 1(1-p) < -1$

$2/3 < p$

What if $p < 2/3$?

f is BR to $M^T M^W$ $\quad$ is M^W BR to F? → NO!

g Prob I^W is mean $= \frac{1}{2}(p/1-p)$

h Prob R fights if mean

q $P(T|M) = p/p + (1-p)g$

$U_{RA}(F) = U_{RA}(R)$

$-2q + 1(1-q) = -1$

$q = 2/3$ → $2/3 = p/p + (1-p)g$

$1.5p = p + (1-p)g$

$.5p = (1-p)g$

$g = (.5p)/(1-p)$

$U_{IW}(m) = U_{IW}(N)$

$-2h + 1(1-h) = -1$

$h = 2/3$

PBE

M^T, $\sigma_{IW} = 2/3$, $\sigma_{RA} = 2/3$, $q = P(T|M) = 2/3$

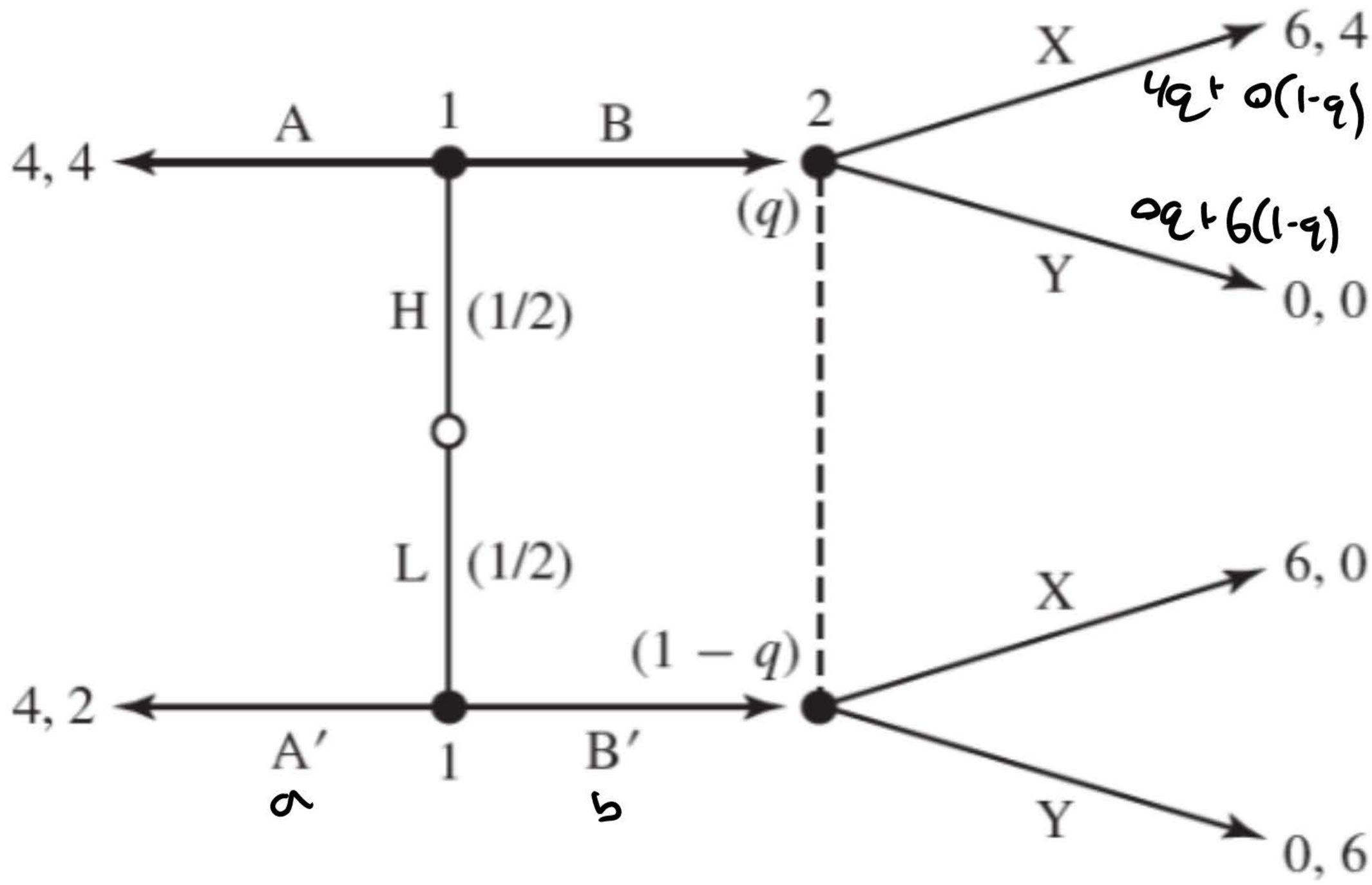
if $p \geq 2/3$

M^T, M^W, R

28 Extra Problems

Monday, November 23, 2020 10:10 AM

28.2. (Modified) Consider the following game:



a) Construct the normal form.

1 \ 2	X	Y
Aa	4, 3	4, 3
Ab	5, 2	2, 5
Ba	5, 3	2, 1
Bb	6, 2	0, 3

b) Find the PBE.

P(B) = 0 → No B in eq

q =

6 - 6q ≥ 4q → q ≤ 3/5

c) Instead of p=1/2, characterize the PBE for all values of p.

P high, q low

Ba, X q =

d) Suppose p=1/2 but the payoff for player 1 for B'X is 6-x. Characterize the PBE.

x > 2

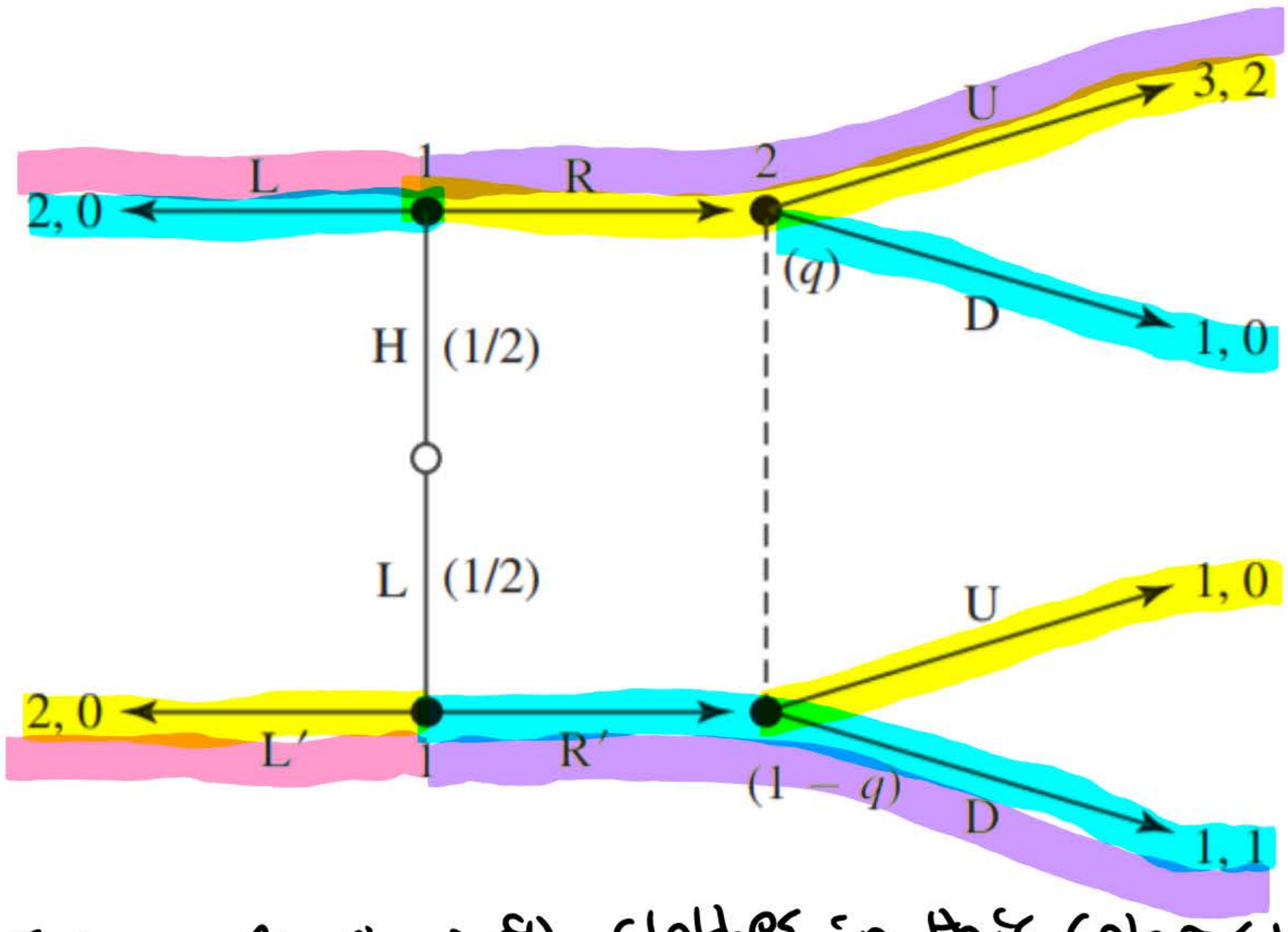
Ba, X q = 1

HW 28.3

Sunday, November 29, 2020 3:31 PM

Posted solution review

3. Consider the following game of incomplete information.



I know people with clothes in this color scheme

(a) Does this game have a *separating* perfect Bayesian equilibrium? If so, fully describe it.

RL', U

$q = .5 \cdot 1 / .5 \cdot 1 + .5 \cdot 0 = 1$

LR', D

$q = .5 \cdot 0 /$ doesn't matter $= 0$

(b) Does this game have a *pooling* perfect Bayesian equilibrium? If so, fully describe it.

$q = 0 \cdot 0 /$ doesn't matter $= 0$

LL', D $q \leq 1/3$ is Pooling PBE

$U_2(U|R) = 2q + 0(1-q) = 2q$

$U_2(D|R) = 0q + 1(1-q) = 1-q$

$2q > 1-q \rightarrow q > 1/3$

If $q < 1/3$, they tend towards L due to better payoffs

RR'

$q = .5/(2 \cdot .5) = 1/2$

$U_2(U|R) = 2(1/2) + 0(1-1/2) = 1$

$U_2(D|R) = 0(1/2) + 1(1-1/2) = 1/2$

$1 > 1/2$ so P_2 chooses U

RR' is not sustainable

HW 28.4

Sunday, November 29, 2020 4:24 PM

Passed Solution review

4. Consider an extensive-form game in which player 1 is one of two types: A and B. Suppose that types A and B have *exactly* the same preferences; the difference between these types has something to do with the payoff of another player. Is it possible for such a game to have a separating PBE, where A and B behave differently?

Yes, Player 1 may do something of interest to other Players if it is rational for them to do so

Their actions may signal something that makes a difference to the other Player

HW 28.5

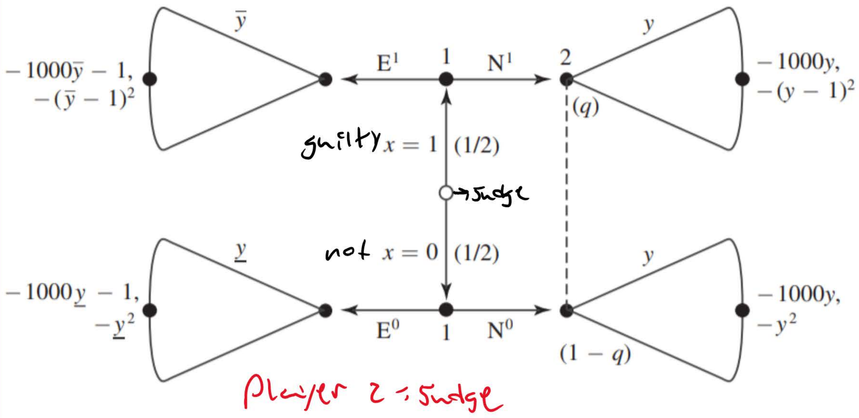
Sunday, November 29, 2020 3:34 PM

Passed Solution review

5. A defendant in a court case appears before the judge. Suppose the actual harm to the plaintiff caused by the defendant is equal to $1000x$ dollars, where either $x = 0$ or $x = 1$. That is, the defendant is either innocent ($x = 0$) or guilty of 1000 dollars of damage ($x = 1$). The defendant knows x and has evidence to prove it. The judge does not observe x directly; she only knows that $x = 1$ with probability 1/2 and $x = 0$ with probability 1/2.

The judge and defendant interact as follows: First, the defendant has an opportunity to provide his evidence of x. He freely chooses whether or not to provide the evidence; the court cannot force him to do it. Providing evidence to the court costs the defendant one dollar (for photocopying). If the defendant chooses to provide the evidence, then it reveals x to the judge. Whether or not evidence is provided, the judge then decides the level of damages y (in thousands of dollars) that the defendant must pay. The judge prefers to select y "fairly"; she would like y to be as close as possible to x.

The defendant wishes to minimize his monetary loss. These preferences and the players' interaction are summarized by the extensive-form diagram that follows. Note that "E" stands for "provide evidence" and N stands for "do not provide evidence."



(a) This game has a unique perfect Bayesian equilibrium. Find and report it. (Hint: Start by showing that it is optimal for the judge to set y equal to the expected value of x, given her belief.)

I \ 2	$x=1$
Ee	$[-1001-1]/2 = -501$,
En	
Ne	
Nn	

This doesn't make sense. All of Player 2's payoffs are negative. They should be $0 \le y \le$ whatever damages are awarded

No evidence given... $U_J = -q(y-1)^2 - (1-q)(y^2) \Rightarrow \frac{dU_J}{dy} = -2(y-1)q - 2(1-q)y = 0$
$= y = q$

$E(y) = 1q + 0(1-q) = q$

Consider E^0N^1; $\underline{x}=0$, $\bar{y}=1000$, $y=1000$, $q=1$
- given E^0N^1, $q=1$ so $y=1000$ is BR
- guilty pays 1000 either way and $-1000y - 1001$ so no evidence
- innocent always provides evidence

(b) In one or two sentences, explain why the result of part (a) is interesting from an economic standpoint.

Innocents provide evidence while guilties don't. Thus, the Judge can deduce the guilt

(c) Consider a version of the game in which x is an integer between 0 and K, inclusive, with each of these values equally likely. Compute the perfect Bayesian equilibrium of this game. (Hint: Use your intuition from part (a).)

Is guilt now a range? Is $x=2$ double guilty?

SPE is $E^0E^1 \ldots E^{k-1}N^k$, $y^i = 1000i$ $i<k$, $y = 1000k$ if no evidence given, $P(i=k|N) = 1$

All $i<k$ provide evidence to avoid $y=1000k$, k does not to save \$1, thus $P(i=k|N)=1$ is correct

$U_J = -q(y-1)^2 - (1-q)(y^2)$

$dU_J/dy = -2q(y-1) - 2(1-q)y = 0 \rightarrow y = q \cdot 1 = q \cdot 1000$

Min MSE

$\underline{x}=0$, $\bar{y}=1$, $q=1$, $y=1$

29 Book Notes

Thursday, November 26, 2020 6:57 PM

Job-Market Signalling + Reputation

- Workers can signal ability w/ costly schooling
- People may have incentives to lie about their identities

Jobs and School

Wisdom vs intelligence

Reputation and Incomplete Information

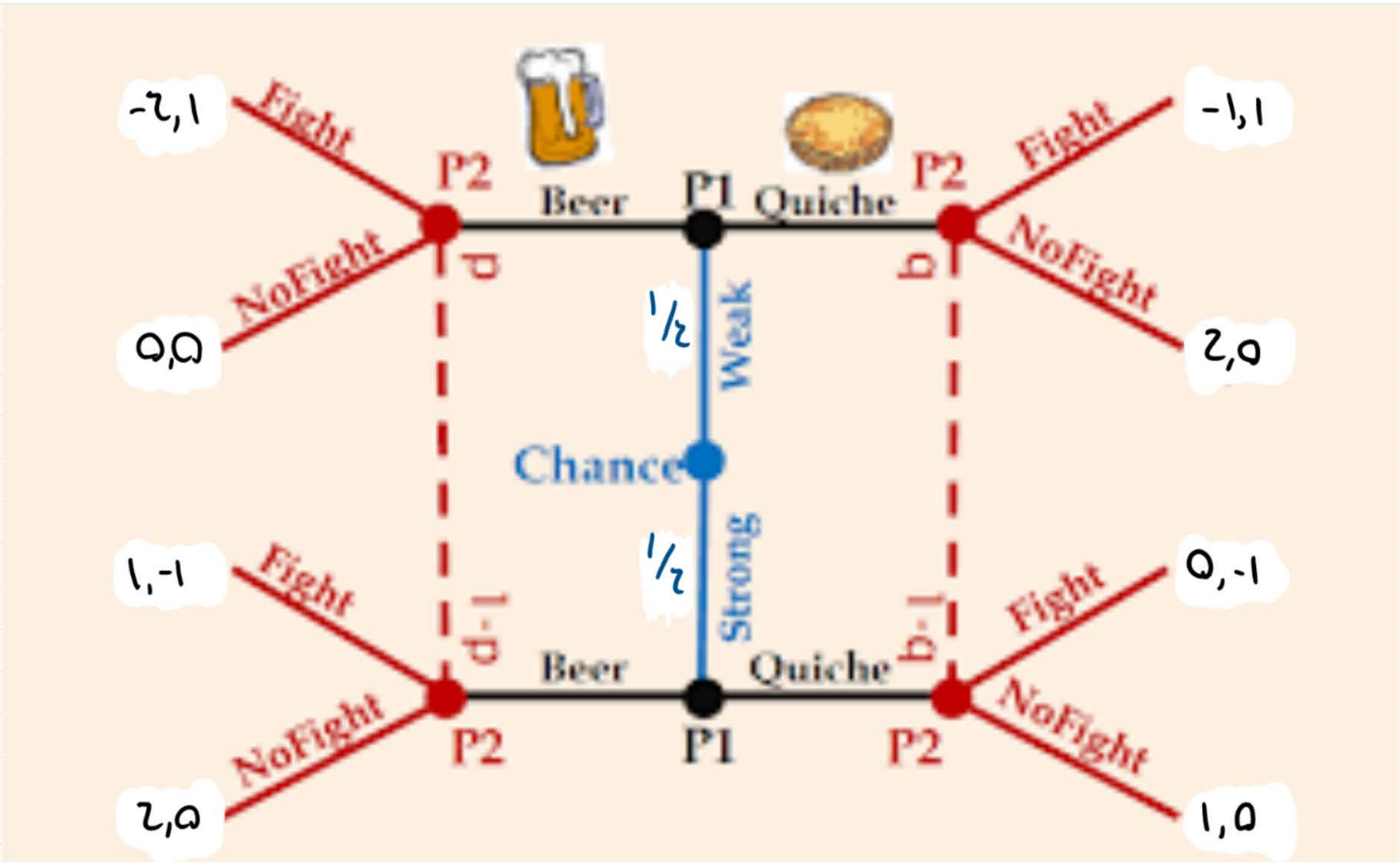
29.1 Signaling Games - Introduction

Thursday, November 26, 2020 7:09 PM

Senders and receivers → ability signaling → reputation game

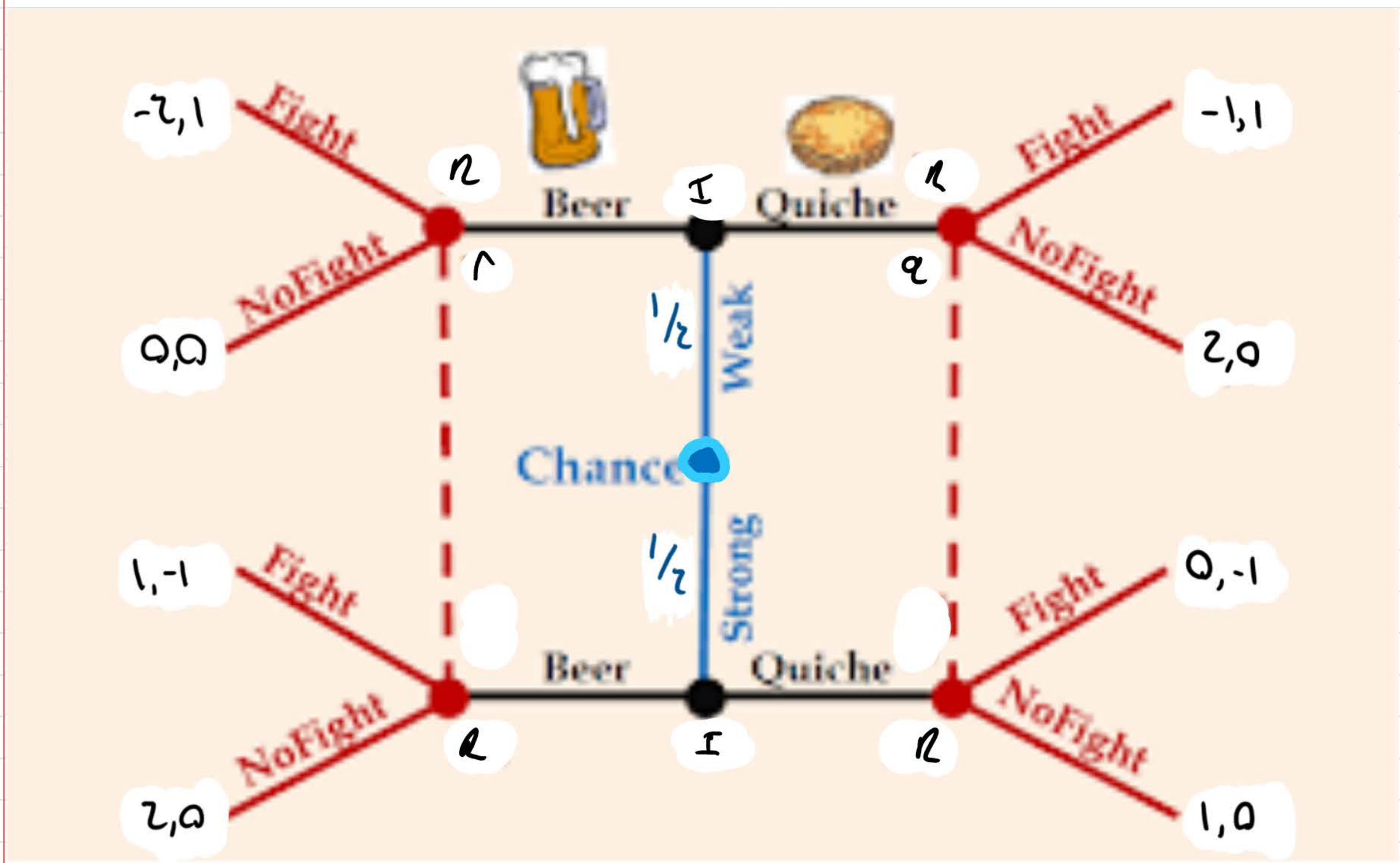
29.2 Beer-Quiche Game Introduction

Thursday, November 26, 2020 7:28 PM



29.3 Beer-Quiche Game with Pooling PBE

Thursday, November 26, 2020 7:34 PM



$B^r B^w, N^B F^Q$ $q = 1/2$ B^r is BR. So is B^w What about $N^B F^Q$?

If R sees beer, $V_R(F) = -1p + 1(1-p)$
$= -1/2 + 1/2 = 0$

$V_R(N) = 0 \longrightarrow N^B$ is a BR

Is F^Q a BR? $-1r + 1(1-r) \geq 0$
$r \leq 1/2$

PBE: $M^r M^w, N^B F^Q, q = 1/2, r \leq 1/2$

Consider
$B^r Q^w, N^B F^Q$

B^r is a BR, $q = 1$, N^B is a BR

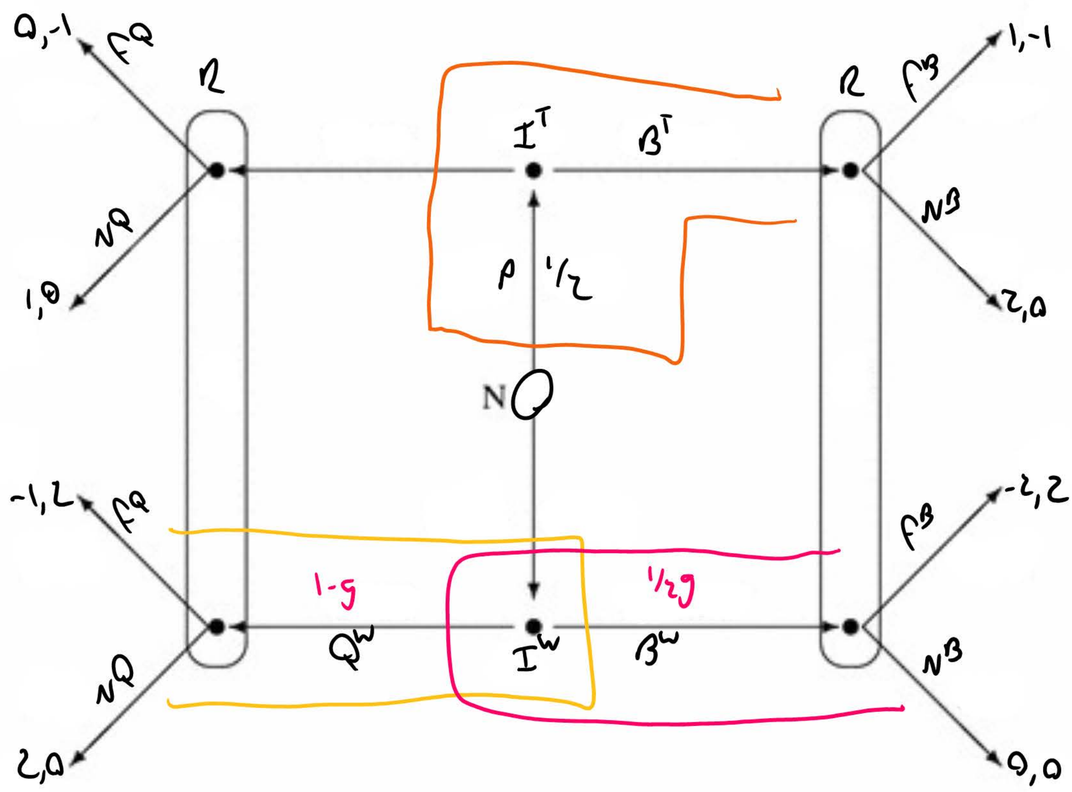
$r = 0$, F^Q is BR

Q^w? $V_{Iw}(B) = 0 > 1 = V_{Iw}(Q) \rightarrow$ Not a BR

Any Eq w/ Q^r?
$Q^r Q^w, F^B N^Q$
$Q^r B^w,$

29.4 29.5 Beer-Quiche Game with Partially Separating PBE

Thursday, November 26, 2020 7:50 PM



$B^T B^W$

$N^B F^Q$

$q = 1/2$ $-.5 + 1 > 0$

N^B NOT a BR

~~$F^B F^Q$~~ → I^W choose Quiche → B^W not BR

F^B ~~N^Q~~ I^W choose Q^W

N^B ~~N^Q~~ I^W choose Q^W

$B^T Q^W$ $N^B F^Q$

$q = 1$ → N^B is BR

$r = 0$ → F^Q is BR

B^T is BR

Q^W no B^W → 0 Q^W → -1

~~$N^B N^Q$~~ NO $r = 0$ → F^Q

F^B ~~N^Q~~ $r = 0$, $q = 1$ terrible

$F^B F^Q$ $q = 1$, F^B Not good Choice

No separating if I drinks beer

Q^T Never part of an equilibrium

Eq involves mixing for I

→ mixing for R

B^T, $\sigma_{IW} = g$, $\sigma_R = \{h, 1\}$, $q = 2/3$, $r = 0$ → $P(T|Q)$

$P(B|W)$ $P(F|B)$ $P(F|Q)$ $P(T|B)$

$-1q + 2(1-q) = 0$

$3q = 2$

$q = 2/3$

h: I^W indifferent if $-2h + 0(1-h) = -1$ → $h = 1/2$

$P(F|B) = 1/2 / (1/2 + 1/2g) = 1/(1+g) = 2/3$

$1.5 = 1 + g$

$g = 1/2$

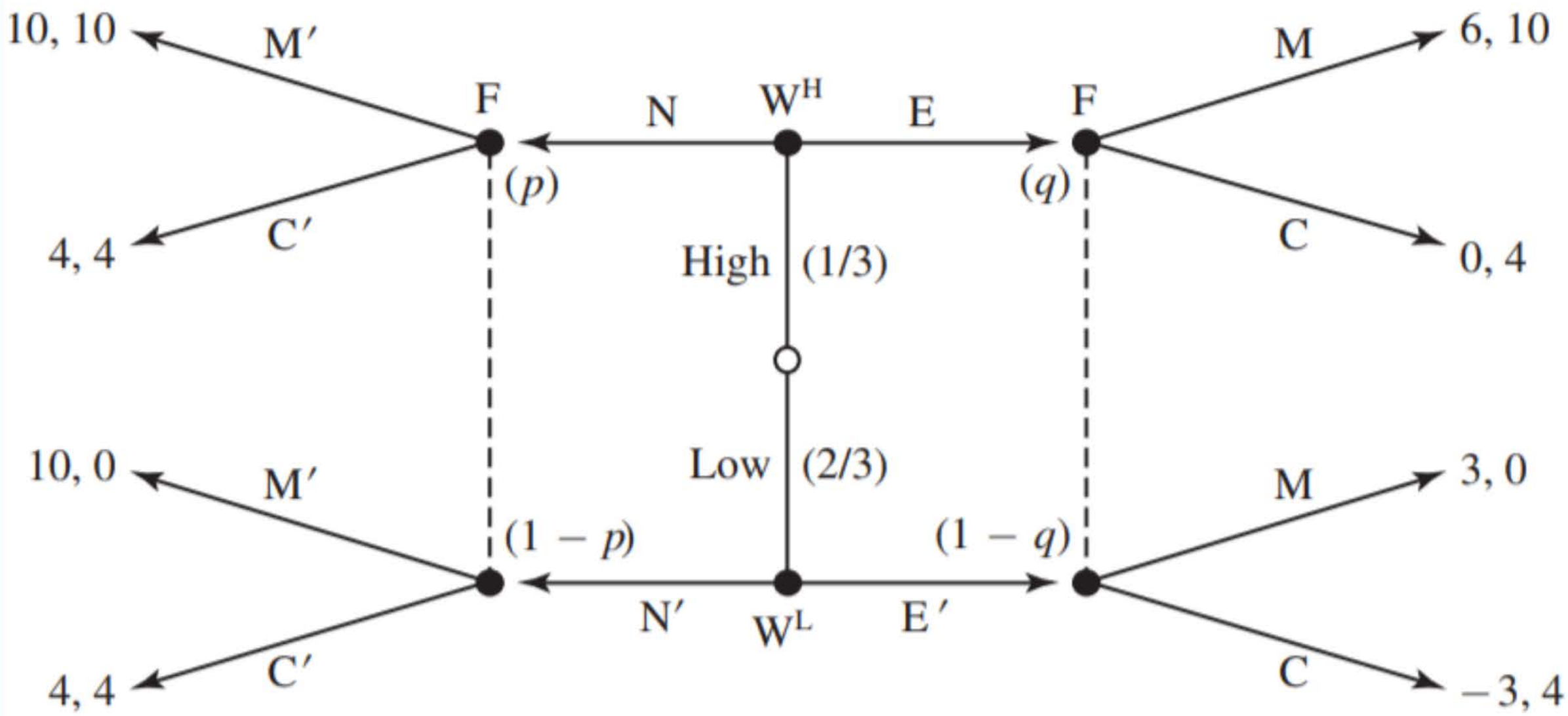
B^T, $\sigma_{IW}(1/2)$, $\sigma_R(1/2, 1)$, $q = 2/3$, $r = 0$ → Partially separating PBE

HW 29.1

Tuesday, December 1, 2020 3:08 PM

Passed Solution review

1. Consider the job-market signaling model analyzed in this chapter. Would an education be a useful signal of the worker's type if the types had the same education cost? Explain your answer.



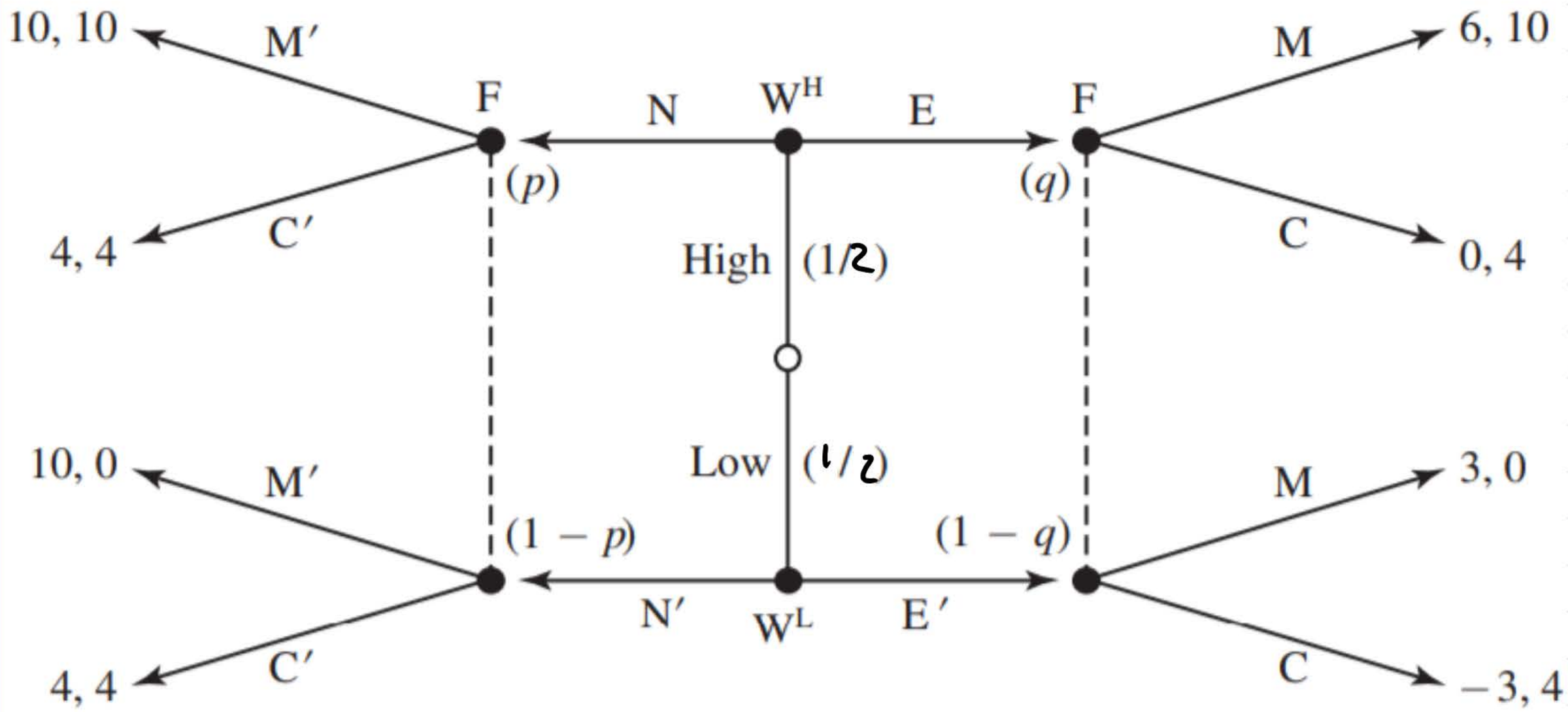
No, there would be no separation because BR would always be to get an education which makes education level a useless signal

HW 29.2

Tuesday, December 1, 2020 3:12 PM

Posted solution review

2. Compute the PBE of the job-market signaling model under the assumption that the worker is a high type with probability 1/2 and a low type with probability 1/2 (rather than probabilities of 1/3 and 2/3 as assumed in the text).



NE': $q=0$, $p=1$ → $CM'=3$ worker: 5, 3.5, −1.5, or 10 → worker deviates
No PBE

EN': $q=1$, $p=0$ → $MC'=7$ worker: 5, 3.5, 4.5, 4 → No incentive
PBE = (EN', MC') if $q=1$ and $p=0$

EE': $q=.5$ → M worker knows M and deviates to NN
No PBE

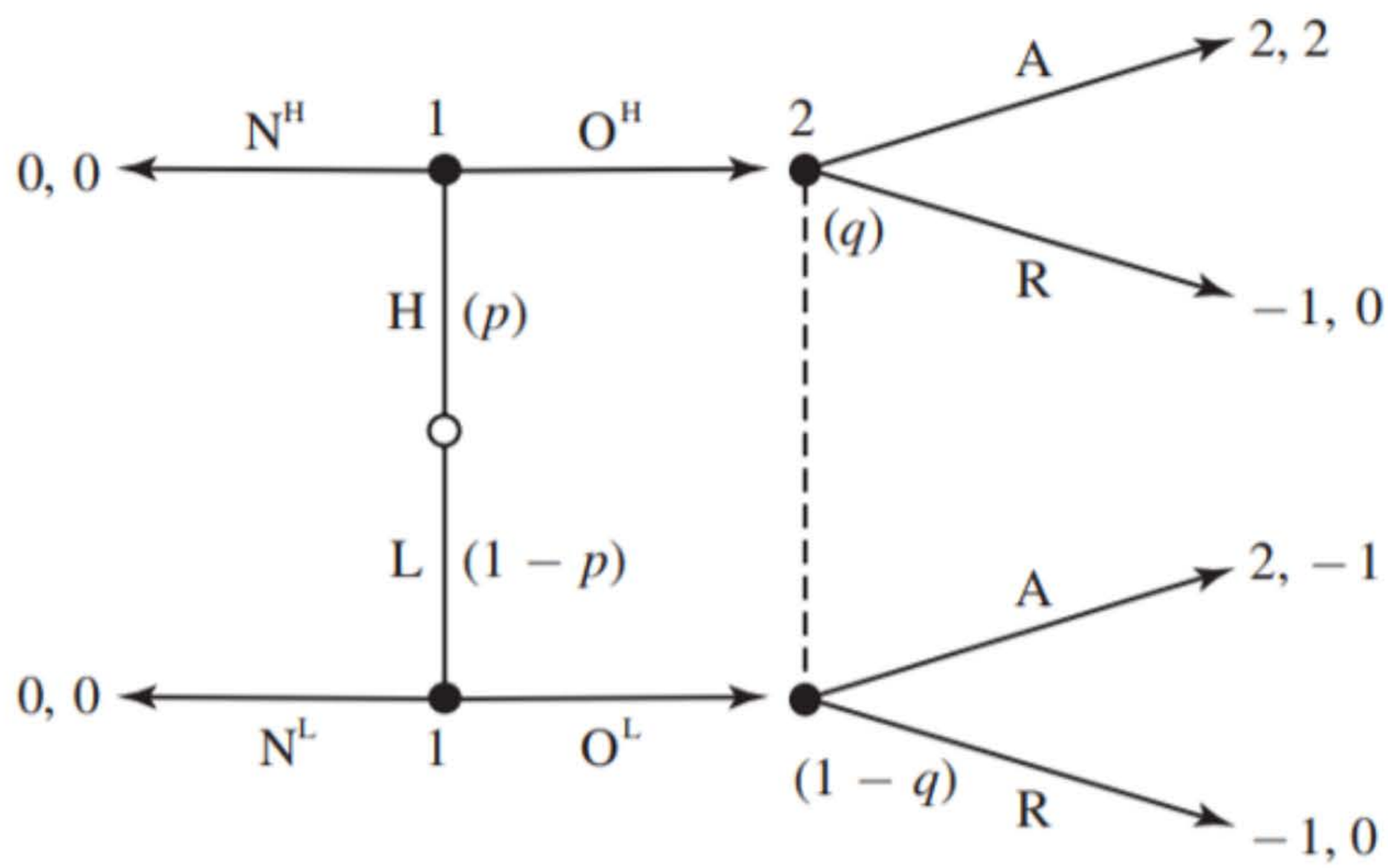
NN': $p=.5$ → M'C if $q<.4$ M'M otherwise → worker stays
PBE

HW 29.3

Tuesday, December 1, 2020 3:26 PM

Passed solution review

3. Consider the extensive-form game of incomplete information in the diagram that follows. There is a firm and a worker. In this game, nature first chooses the "type" of the firm (player 1). With probability p, the firm is of high quality (H) and, with probability $1 - p$, the firm is of low quality (L). The firm chooses either to offer a job to the worker (O) or not to offer a job (N). If no job is offered, the game ends and both parties receive 0. If the firm offers a job, then the worker either accepts (A) or rejects (R) the offer. The worker's effort on the job brings the firm a profit of 2. If the worker rejects an offer of employment, then the firm gets a payoff of -1 (associated with being jilted). Rejecting an offer yields a payoff of 0 to the worker. Accepting an offer yields the worker a payoff of 2 if the firm is of high quality and -1 if the firm is of low quality. The worker does not observe the quality of the firm directly.



(a) Is there a separating PBE in this game? If so, specify the equilibrium and explain under what conditions it exists. If not, briefly demonstrate why.

$O^H N^L$: P_2 plays A, P_1 deviates = no PBE
$N^H O^L$: P_2 plays R, P_1 deviates = no PBE

(b) Is there a pooling PBE in which both types of firms offer a job? If so, specify the equilibrium and explain under what conditions it exists. If not, briefly demonstrate why.

$O^H O^L$: P_2 plays A if $2q - 1(1-q) > 0 \Rightarrow q > 1/3 \Rightarrow P_1$ doesn't deviate
$(O^H O^L, A)$ is PBE when $q > 1/3$ and $q = p$

(c) Is there a pooling PBE in which neither type of firm offers a job? If so, specify the equilibrium and explain under what conditions it exists. If not, briefly demonstrate why.

$N^H N^L$ is BR to R. R is also BR to N^H and N^L. Thus, if $p \leq 1/3$
$N^H N^L$, R $p = q$ is pooling SPE

[Witty RPS/RBG Title]

Gus Lipkin
Florida Polytechnic University

Part I 3
- **Introduction** 3
- **Background** 3
- **Description of the Games** 4
  - Figure 1 4
  - Figure 2 5
  - Figure 3 5
- **How to Balance the Game** 6
- **Conclusion** 7

Author's note 8

Part II 8
- **Introduction** 8
- **Breaking Down the Initial Problem and Where I Went Wrong** 8
- **A Re-evaluation of the Game** 9
  - Figure 4 9
  - Figure 5 10
- **Remodeling the Game** 11
- **The Papal System: A Brief Description of the New Game** 14
- **Similarities Between the Two Systems in the News** 14
- **Flaws in the Plan** 14
- **Conclusion** 15

Author's Note II 15

Bibliography 16

Part I

Introduction

In elementary school, we are all taught that three branches of the federal government have checks and balances so that one branch never gains too much power. For almost 250 years, this system has functioned reasonably well with some minor power struggles and disagreements here and there, but no branch has managed to battle the other two and definitively come out on top. Even with the passing of Supreme Court Justice Ruth Bader Ginsburg, the Supreme Court will hold the same amount of power that it did before her passing. The issue right now is not that one branch has more power than another, but that the same people are in power in both the Executive and Legislative branches and stand a lot to gain by installing a new justice with whom they share a common lack of values and integrity. By applying game theory to the Supreme Court nomination process, we can learn where the system is failing and explore possible ways to improve it.

Background

Since the Circuit Judges Act of 1869, the United States Supreme Court has held space for one Chief Justice and eight Associate Justices (*Circuit Judges Act*, n.d.). When a vacancy in the Supreme Court opens, either by the death of a justice or their retirement, the President nominates a candidate who then goes before the Senate Judiciary Committee (Nguyen, 2015). If the committee approves of the nomination, they send their recommendation to the full Senate where the Senate then debates and ultimately decides the nomination with the Vice President stepping in as a tiebreaker (Nguyen, 2015). In order to properly understand the situation that Ruth Bader Ginsburg's passing creates, we must first understand the last time a Supreme Court Justice passed before retiring.

In February of 2016, just nine months before the presidential election in November and almost a full year before President Barack Obama's successor would take power, Justice Antonin Scalia passed away (Elving, 2018). Within hours of Justice Scalia's passing, Senate Majority Leader Mitch McConnell announced that the Republican senate would block the nomination of Merrick Garland by refusing to interview him because nine months was too close to the election of the next President (Elving, 2018). This allowed the seat to remain vacant until after the next President, Donald Trump, was elected. McConnell later eliminated the filibuster which would have allowed senators to block future nominees (Elving, 2018).

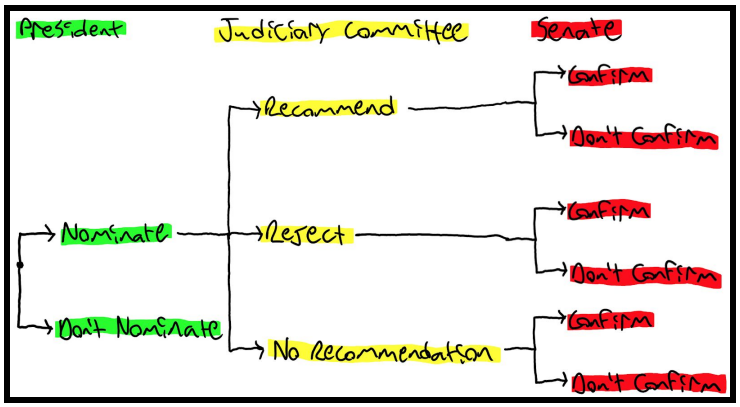
Having already nominated Justices Neil Gorsuch and Brett Kavanaugh to the Supreme Court, President Donald Trump has the opportunity to nominate a third Justice by the end of his first term (*Complete List of Donald Trump's Potential Nominees to the U.S. Supreme Court*, n.d.). Once again, within hours of a Supreme Court Justice's passing, Senate Majority Leader Mitch McConnell made an announcement on the nomination of a new Justice: "President Trump's nominee will receive a vote on the floor of the United States Senate" (Cochrane, 2020) & (Gambino, 2020). This is an extreme reversal of his announcement in 2016 that because it is an election year, the President should not be able to nominate anyone to the Supreme Court (Elving, 2018).

Description of the Games

Ideally, the nomination of a new Supreme Court Justice could be represented by the following extensive form game:

Figure 1

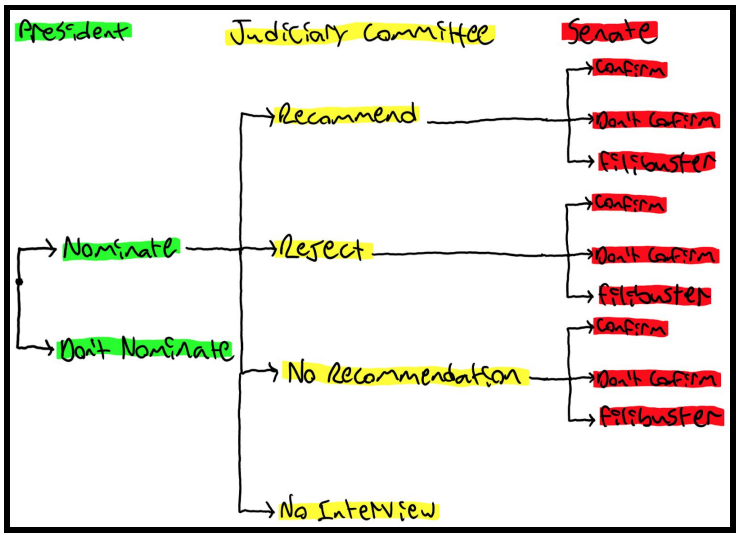
Extensive Form game representing the normal Supreme Court Nomination Process



While a President could choose not to nominate a Justice, the game only ends until the next President chooses to nominate a Justice. Once a President does nominate, the game does not end until the Senate either confirms or denies the nominee. However, if the Senate is unwilling to cooperate with the President or believes that the President should not be allowed to make that nomination, they are able to change the way the game is played and add more moves that they can make. The changed game gives the Judiciary Committee and the Senate a new move each. The former can choose to ignore the nomination and to not interview any nominees that the President puts forth while the latter can choose not to vote on the nominee regardless of the recommendation put forth by the Judiciary Committee.

Figure 2

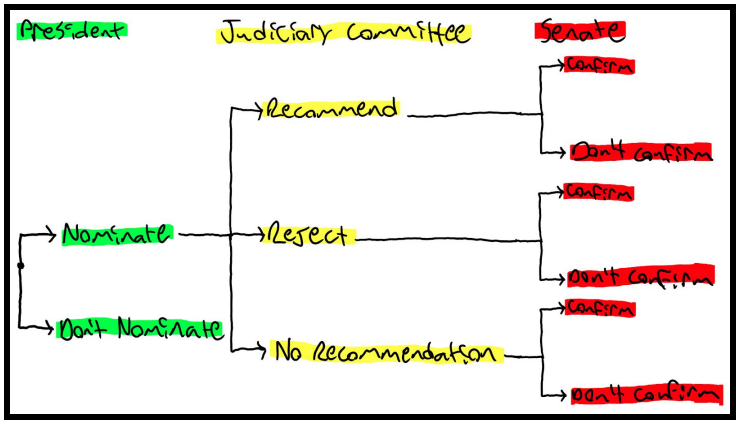
Extensive Form game representing the Supreme Court Nomination Process for Merrick Garland



In the game representing the nomination of Merrick Garland in 2016, the Republican controlled Judiciary Committee gave themselves a new move. By refusing to interview a nominee, they could effectively end the game. In reality, the game then returns to the President until they nominate someone new that the Judiciary Committee will interview or until a new President is sworn in and nominates someone new that the committee will approve of (Elving, 2018). Had Merrick Garland been interviewed by the Judiciary Committee, the Senate Republicans could have then filibustered as a way to block a vote because the Senate Democrats did not have enough members to force a vote (Elving, 2018). An attempted lawsuit to force the Judiciary Committee to interview Garland was thrown out by a federal judge (Elving, 2018).

Figure 3

Extensive Form game representing the current Supreme Court nomination process



While filibuster is still an option in the current Supreme Court nomination process, it has effectively been eliminated by changing the cloture rules, the rules that allow the senate to vote to end a debate, so that only a simple majority of fifty-one votes is needed (Smith, 2017). Because the Republicans control fifty-three of the 100 total Senate seats, they have the simple majority that they need to end any Democratic attempt at filibuster (Snell, 2018). Of the fifty-three Republican Senators, eleven are on the Judiciary Committee which has twenty-one total seats (*Members*, n.d.). This means that the Republicans can force any nominee through the committee and refusing to interview the nominee is not possible for the Democrats.

The current Supreme Court nomination process is relatively fair in a well-balanced government. However, because Senate Republicans have put their loyalty to the President over their loyalty to what few values they may have, the game is no longer balanced. No matter who the President nominates, the Judiciary Committee will interview them because the Democrats do not have the majority they need to refuse to interview them. Then, regardless of the recommendation the committee makes, the nominee goes before the full senate to either be confirmed or not confirmed. A filibuster from the Democrats is no longer an option because the Republicans only need fifty-one votes to end debate and control fifty-three seats and only two Republican Senators have said they would not vote for the nominee (Molly Reynolds, 2020). This leaves exactly the number of votes needed to confirm the new Justice (Smith, 2017). Nevertheless, Democrats press on and are trying to find ways to stall the Senate so that they cannot vote on the nominee (Molly Reynolds, 2020). One such tactic would be for the House to impeach Attorney General Bill Barr or President Trump because impeachment proceedings take precedent in the Senate (Molly Reynolds, 2020). But this is unlikely to work because impeachments require fifty-one votes as well, which the Republicans have ((Snell, 2018), (Molly Reynolds, 2020).

How to Balance the Game

Regardless of whether or not the Republicans or Democrats control the Presidency, Senate, and House, this is an unfair game and must be balanced for democracy to continue to function. This is extremely difficult. Do you change the game in its current form or create a new game? If you create a new game, how do you ensure it does not become imbalanced like the current one? Who is chosen to design the game? In my opinion, the ideal solution would be to put the confirmation vote in the hands of the people but this still leaves the position vulnerable to poor decisions by voters.

If voters were presented with a shortlist of candidates from the President and the current first past the post system was used, there would be no mechanism to vote against someone. If the President were to release a list of too many nominees, the Associate Justice position could be won with a very small portion of the vote such as when Jake Auchincloss won House seat MA-4 with only 22.41% of the vote (Newsroom, 2020).

Because the Judiciary Committee does not hold true power in the game besides refusing to interview the nominee, there is no substantial change that could be made at this point in the game besides flipping the majority in the committee between the Democrat and Republican parties. This leaves the Senate as a whole as the next step at which changes could be made, but there are issues that arise similar to those if the general population were to vote. So long as the President is the only one able to nominate candidates, the senate can only vote against the nominees and never put forward a candidate themselves. Even so, this leaves the nominee vulnerable to a blanket no vote from the opposing party so long as that party has fifty-one or more members.

It appears that without changing its entire structure, there is little room to balance the nomination process. As the hit movie *War Games* taught, sometimes the only way to win is to not play. While we have to nominate Supreme Court Justices, maybe we can play a different game that will allow us to influence the nomination process. By first instituting eighteen year term limits for Supreme Court Justices, each President would be able to appoint a Justice in the first and third year of each four year term (Newsroom, 2020). This creates a gap if there is a vacancy on the bench. To fill the gap, any retired Supreme Court Justice could be brought back in the order of retirement until it is time to appoint a new Justice (Newsroom, 2020). While these changes do not guarantee a certain quality of Justice appointed to the Supreme Court, it does provide a mechanism for Justices who become unpopular to have a set retirement date.

Conclusion

By studying the current and past nomination processes as games using game theory, we come to understand where the system is failing. We understand that something must change, but creating a new system by which a Supreme Court Justice is nominated is a daunting task. It must maintain the balance of power in the federal government and must be appealing to people of all political ideologies. Unfortunately, while there are people in power who have abandoned their duty to the people and have no moral compass, they will always seek to better themselves before the people and

that means confirming Justices that are not qualified and unable to separate church and state.

Author's note

Some parts of the original paper have been revised before the writing of Part II. The changes are minor and serve only to clarify the original language and any processes that may have been glossed over have some detail added.

Part II

Introduction

A lot has happened since the Part I of this series was written. One republican senator voted for Amy Coney Barrett after saying they would not, Amy Coney Barrett was confirmed and sworn in to the Supreme Court, and Mitch McConnell sent the Senate on break until November 9, without voting on a Coronavirus relief bill (Kim, 2020), (Johnson, n.d.). It would be easy to say that the game is over and therefore no longer worth consideration, but, as with any human institution, especially those where the term limits are defined by one's lifespan, there will come a time where the Supreme Court has another opening and it is important to make sure that the nomination process is not tampered with again.

Breaking Down the Initial Problem and Where I Went Wrong

When originally defining the problem, I tried to stay away from singling out the Democrats or Republicans as the players in the game while instead defining the players as the separate bodies involved in the process. This mischaracterizes the problem. The game is stable when examined at a process level. The President nominates a Justice, the Judiciary Committee interviews and chooses whether or not they want to recommend the candidate to the full Senate, and then the Senate confirms or does not confirm the candidate. However, the game quickly falls apart because the President and Senators are people and some are lizard people with no respect for human beings. But I digress. When the earlier described games are broken down to the personal level and party affiliations are noted, it becomes clear that one party follows the respected norms and the other party says "fuck your feelings" and does not respect the social norms (Wilson, 2020). These social norms were never written into law because in the past, people respected them and therefore never needed to write them down.

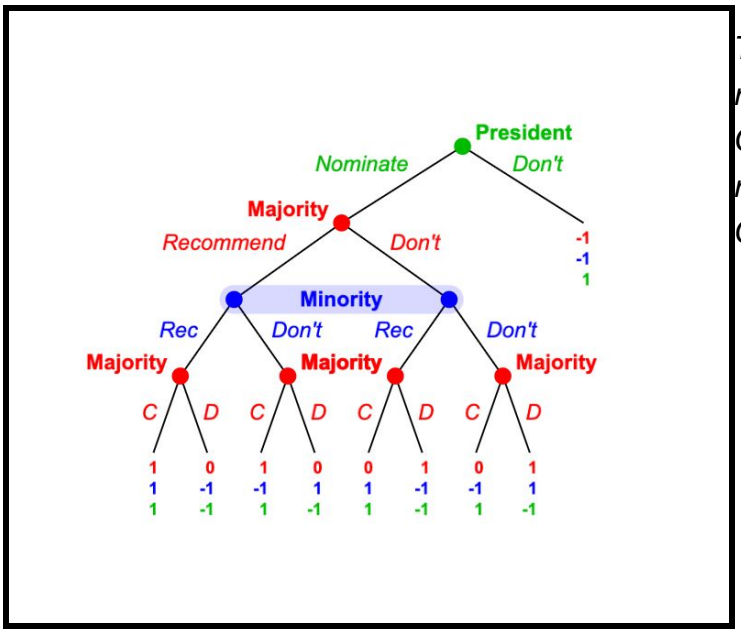
A Re-evaluation of the Game

When these norms are respected, the President nominates someone who is appropriately qualified for the position. The Judiciary Committee then interviews them and recommends or does not recommend them based on their qualifications and other parameters, and then the Senate votes to confirm them based again on their qualifications. The New York City Bar Association found her to be academically qualified to serve as a Supreme Court Justice but found her wanting on criteria which evaluated maturity, integrity and independence, temperament, and the court's role in protecting individual freedoms (*Reservations on Judge Amy Coney Barrett's Qualifications for Supreme Court*, n.d.).

When appropriate social norms are followed, the game presents as it does in Figure 1. When norms are not observed. The game changes significantly and presents as the following figure.

Figure 4

Extensive form game representing a nomination process where social norms are followed when the Judiciary Committee and Senate majority parties match.



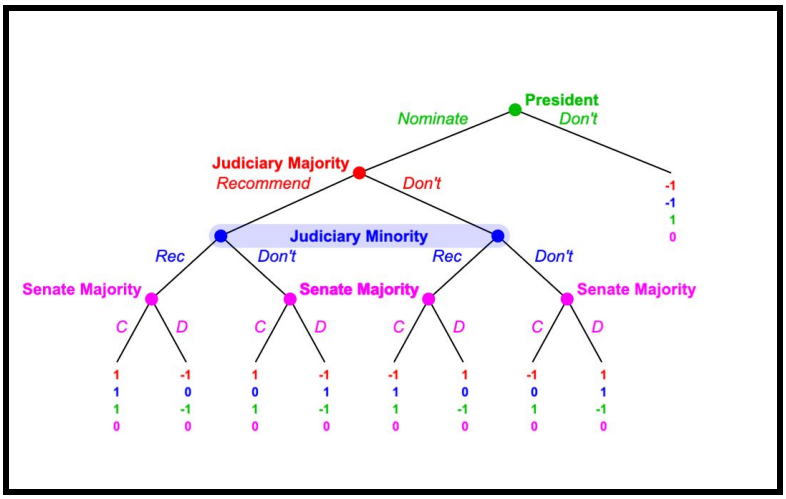
The second and third levels represent the Judiciary Committee. C and D on the fourth level represent the Senate's ability to Confirm or Deny a nomination.

Six assumptions were made in designing this game to ensure that proper social norms are properly demonstrated. First, everyone wants to nominate a new justice with the possible exception of the President. The possibility of the President not nominating a new Justice is extremely low. Second, the majority party is the same between the Judiciary Committee and full Senate. Third, payoffs for the general public are aligned

with those of their party. Fourth, if the majority party is in disagreement at any point in the process, their payoff is zero regardless of whether or not the Justice is confirmed. Fifth, while the minority party in the Judiciary Committee ultimately does not have any effect on the progress of the game assuming Senators vote on party lines at that stage, individual Senators in the majority for the full Senate may take into account those minority opinions and ultimately decide to cross the aisle and vote against their party's candidate. Sixth, if the Judiciary Committee majority and full Senate majority parties do not match, the game in Figure 5 is used instead. Finally, one acknowledgement must be made. Payoff values are not as cut and dry as they are laid out to be. In Figure 4 and Figure 5, the payoffs correspond to a general level of satisfaction for that party.

Figure 5

Extensive form game representing a nomination process where social norms are followed when the Judiciary Committee and Senate majority parties do not match.



The pink player has been added to make it easier to differentiate between parties. The blue and pink players are from the same party and share blue's payoffs. The payoffs listed for pink are irrelevant and are only there because the program forces them to be there.

Because only fifty-one votes are needed to confirm a Supreme Court Justice, the difference between a confirmation and no confirmation vote is just one Senator whereas if a super majority were required, sixty votes are needed to confirm (Molly Reynolds, 2020). In 2017, current Republican Senate Majority Leader Mitch McConnell initiated a change to Senate rules that allows Supreme Court Justices to be nominated with fifty-one votes after previously condemning a similar change in 2013 (Caldera, 2020). Unless any one party truly has super majority representation in the Senate, it is unlikely that a candidate will be confirmed without Senators deviating from party lines. Clearly the game cannot be balanced by shifting power around the Senate as

whichever party has majority power will be able to dictate the law of the land. Thus, the game must be changed in another way.

Let us, for a brief moment, return to Figure 4. While it is generally improper to simply add payoff values together, it is the easiest way to compare players' satisfaction with the outcome of any game following that form. The President has a total value of one because all outcomes when they nominate a candidate sum to zero, and they also have the option of not nominating a candidate. This is highly unlikely to happen, but if it did, the President would have a positive payoff because they are getting the result they wanted. The majority sums to a total of three while the minority sums to zero. Realistically, these numbers will be different due to the President choosing candidates that are more likely to be confirmed and playing out their own game of the median voter theorem. Payoffs change dramatically in Figure 5. The President still keeps their payoff of one which is unlikely to change no matter how much the game is changed. This time, however, because the Judiciary Committee and Senate majorities are not in agreement, the Judiciary Committee majority payoff changes to negative one and the minority, which is also the Senate majority, has a payoff total of three. This is not ideal. Ideally, no one party would have an advantage over the other. The only player with an advantage should be the President who can choose not to nominate a Justice at all. The only way to equalize the game is to spread the payoffs out between more players and aim for a zero sum game.

Remodeling the Game

As the United States has aged, its governmental structure has always been labelled a democracy where the people are able to elect their representatives in local, state, and federal government. In a piece for The Washington Post, columnist Paul Waldman points out that Amy Coney Barrett was confirmed by a Republican majority in the Senate which represents fewer voters than the Democrat minority and that President Trump himself was elected in a similar manner, but with the Electoral College taking the role of the Senate (Waldman, 2020). Waldman goes on to touch on the social norms mentioned before and says that Democrats will not take any actions that will cause Republicans to "raise a stink" in case the Republicans deem those actions inappropriate. Be that as it may, no matter how appropriately the Democrats conduct themselves, the Republicans will always "say that everything they do, no matter how by-the-book it might be, is an egregious violation of propriety and good conduct." He then provides a unique insight into the size of the court. Since the Circuit Judges Act of 1869, the Supreme Court has held nine Justices (*Circuit Judges Act*, n.d.). However, when a Justice retires or dies the court decreases in size until a new Justice is

confirmed (*What Happens When a Supreme Court Justice Dies? - FindLaw*, n.d.). In 2016, the Senate majority Republicans left the Supreme Court with eight seats after refusing to begin the process to confirm Merrick Garland after the death of Justice Antonin Scalia (Elving, 2018). Waldman concludes that the best way to balance the court is to expand it. Unfortunately, adding Justices does not balance the game, but it does correct the existing Supreme Court's direction and brings the majority views back into the center.

If adding more Justices to the bench does not balance the game, how can it be balanced? One of the purposes of appointing Federal judges for life is so that they are outside the influence of current politics and can focus solely on the cases presented to them (Berman, 2020). Another famous political structure that appoints for life or until retirement is that of the Roman Catholic Church (Rayman, 2015). The Pope is chosen from among a group of 120 Cardinals called the Cardinal Electors which make up the College of Cardinals, who, according to church law, must be under the age of eighty (*How Is a New Pope Chosen?*, n.d.). The Cardinals and Cardinal Electors are themselves chosen by the Pope (Reese & Religion News Service, 2020). Interestingly, if a Cardinal under the age of eighty is appointed, they also automatically become a Cardinal Elector, even if that means that there would be more than 120 Cardinals in the College (Reese & Religion News Service, 2020). How would the Supreme Court Justice appointment process game change if it was modelled after the appointment of a new Pope?

On 28 November 2020, thirteen new Cardinals will be appointed with nine of them being eligible Electors, thereby bringing the total number of Electors to 129 (Francis, 2020). Unlike the Cardinal Electors, there appears to be no limit on how many Cardinals there are because they are appointed for life and the Pope must aim to maintain 120 Cardinal Electors. As life expectancies have increased so has the number of Cardinals and the breakdown of their ages into groups has become more varied. Of the 218 appointed Cardinals at the time of this writing, 120 are under the age of eighty and make up 55.5% of all Cardinals. If the new Supreme Court Justice was chosen from the next highest court, that would mean they are chosen from the Court of Appeals (*Court Role and Structure*, n.d.). As of 1 June 2017, there were 160 active U.S. Circuit Court Judges (McMillion, 2017). The Congressional Research Service report that has the age bracket breakdown for all Federal Judges groups all ages seventy and above together so a direct comparison is not able to be made with the Cardinal Electors (McMillion, 2017). However, if the age bracket percentages are added to get as close as possible to the 55.5% of the Cardinal Electors, we find that seventy-nine Circuit Court Judges are under the age of sixty-five, representing just under fifty

percent of all Circuit Court Judges (McMillion, 2017). Performing a direct comparison is not ideal because there is no cap on the total number of Cardinals and while the number of Circuit Court Judges can change, it is much less volatile.

> *Authors note for the following paragraph: Please forgive a lack of sources. There are very few that are easy to find and most are in Latin. Recognizing that Wikipedia cites their sources, I am choosing to use Wikipedia instead because it is in English and I will not have to cite many different sources for one sentence each. (Contributors to Wikimedia projects, 2004)*

I much prefer round numbers, so let us say that the ideal number of Circuit Court Judge Electors is eighty, exactly half of the total number of Circuit Court Judges, and that they must be under the age of sixty-five to be an Elector. Without going too far into detail, when a new Pope needs to be chosen, the Cardinal Electors all meet all day, every day, and vote up to four times a day, until a Cardinal has received two-thirds of the vote. If a Pope is not chosen after two or three days, up to a full day's break is taken to pray and an address by the Senior Cardinal Deacon. If, after nine days of voting, a new Pope has not been elected, the two people with the most votes in any of the previous elections are eligible for a runoff election. The two people in the running are not allowed to vote and one must still receive two-thirds of the vote to become the Pope.

Some changes would need to be made to bring the process in line with what is expected of the government of the United States. First and foremost, the secret balloting would need to be replaced by public ballots. These need not be public to the Judge Electors at the time of voting, but it is important to make sure that they are published. In the 2007 movie *Pirates of the Caribbean: At World's End*, the pirate lords are tasked with electing a pirate king from among themselves. Each pirate lord votes for themselves and no king is crowned until Captain Jack Sparrow chooses to vote for Captain Elizabeth Swann instead. Even though they have devoted their life to their country, one should expect no better from the Judge Electors and thus must be barred from voting for themselves as the next Supreme Court Justice. By barring any Judge Electors over the age of sixty-five, the United States can be reassured that they will choose a Justice that they themselves would accept rulings from whereas if the Electors were very old and about to die, they may elect someone who would not be a good fit, simply because it would not affect them.

The Papal System: A Brief Description of the New Game

The President nominates and the Senate confirms new Federal judges when one dies, retires, or is promoted to the Supreme Court. When a Supreme Court Justice dies or retires, a meeting of the Judge Electors, members of the Circuit Court, meet and vote four times a day until a new Supreme Court Justice has been elected by two thirds of the vote. The newly accepted Justice must then accept the position.

Similarities Between the Two Systems in the News

At first glance the two systems do not have much in common besides that the Pope and Supreme Court Justices are appointed for life. However, in doing research for this paper, I have discovered further similarities. In 2001, then Pope John Paul II created forty-four new Cardinals which raised the number of Cardinal Electors to 135 (Cantwell, 2001). This caused a mild disturbance as it was perceived that the Pope wanted more influence in choosing his successor (Cantwell, 2001). In 2018, the National Catholic Reporter wrote how nearly half of the Cardinal Electors at the time had been appointed by Pope Francis (Reese & Religion News Service, 2018). The Reporter also drew attention to Pope Francis' unorthodox appointments and the changing demographics of the College of Cardinals (Reese & Religion News Service, 2018). This is incredibly similar to how President Donald Trump has appointed a third of the Supreme Court and critics draw attention to his nominees, in particular the sexual misconduct allegations against Justice Brett Kavanaugh and the cult to which Justice Amy Coney Barrett belongs (Cranley & Mark, 2018; Graham & LaFraniere, 2020; Trump, n.d.).

The Wikipedia article on the Papal Conclave makes an uncited claim that because the College of Cardinals is a small body, there have been proposals to expand the electorate. Similarly, as I have argued in Part II of this paper, the number of people who have a say in the choosing of the next Supreme Court Justice must be expanded.

Flaws in the Plan

Oddly enough, my proposal does not acknowledge the fact that the Senate has 100 members while in my proposal, there are only eighty Judge Electors. This decreases the number of people involved in choosing the next Supreme Court Justice. However, a super-majority of eighty votes is fifty-three and a third, rounded up to fifty four, which is three more than the current majority needed of fifty-one votes in the Senate.

As the National Catholic Reporter pointed out, it is possible that the President could install Circuit Court Judges that would choose a Justice that is preferred by the President, but there is no guarantee that there would be a Supreme Court opening

before those judges became too old to vote. This is also combated by the fact that unlike the College of Cardinals, there is a fixed number of Circuit Court Judges that cannot be exceeded.

Currently, there is no requirement that a Supreme Court Justice be a Judge before they are appointed. By forcing Judge Electors to choose, it is much more likely that they would choose a new Justice from themselves or from another Federal court rather than someone who may be uniquely qualified but has never been a Judge such as Justice Elena Kagan (*Elena Kagan*, 2015).

Conclusion

In Part I it was demonstrated that when Senators are not held accountable to the rules or social norms, the majority party has an obscene amount of power in choosing the next Supreme Court Justice. In re-evaluating the game in Part II, it was shown that this still holds true, even when all rules and social norms are followed, albeit to a lesser extent. By noting the similarities between the term limits for the Pope and Supreme Court Justices and Cardinals and Federal Judges, a new Supreme Court Justice appointment system was created in the image of the Papal Conclave. This decreases the ability of any one party to influence the highest court. Ultimately, the goal is to have the new supreme Court Justice be as representative of the people as possible without giving undue priority to any one party's politics.

Author's Note II

Part II has considerably less actual game theory than I was hoping for but at the level to which I needed to abstract, there was no other way. If I had gone into a deep dive I would have created ridiculous games that would be more fitting for a thesis of some kind. In order to save my sanity, I decided to keep it simple. In addition, I strayed from the goal because I found writing about the abstract concepts cathartic while writing about game theory increases my stress levels.

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