

The Extensive Form

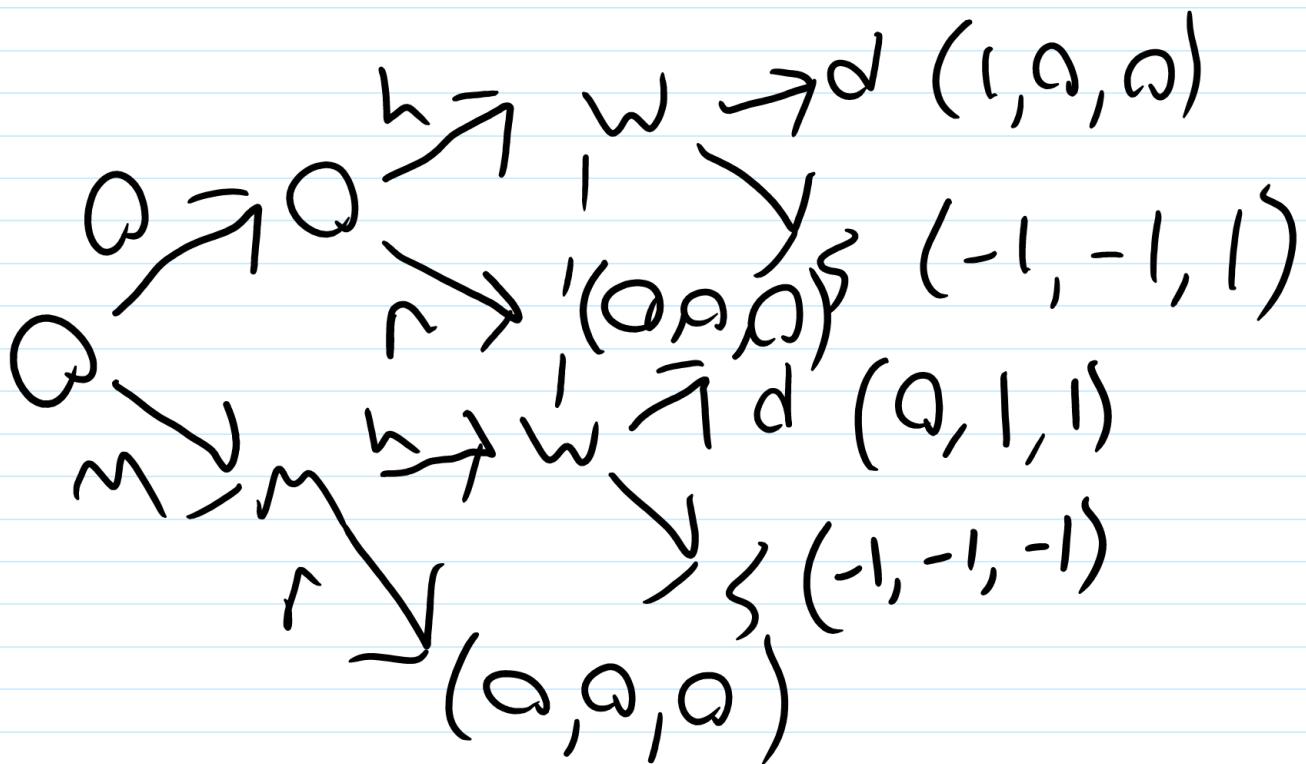
Sunday, August 23, 2020 10:36 PM

- Dotted line means player doesn't know where they are in the game
- Simultaneous first moves doesn't matter who is written first

Passed Solution Review

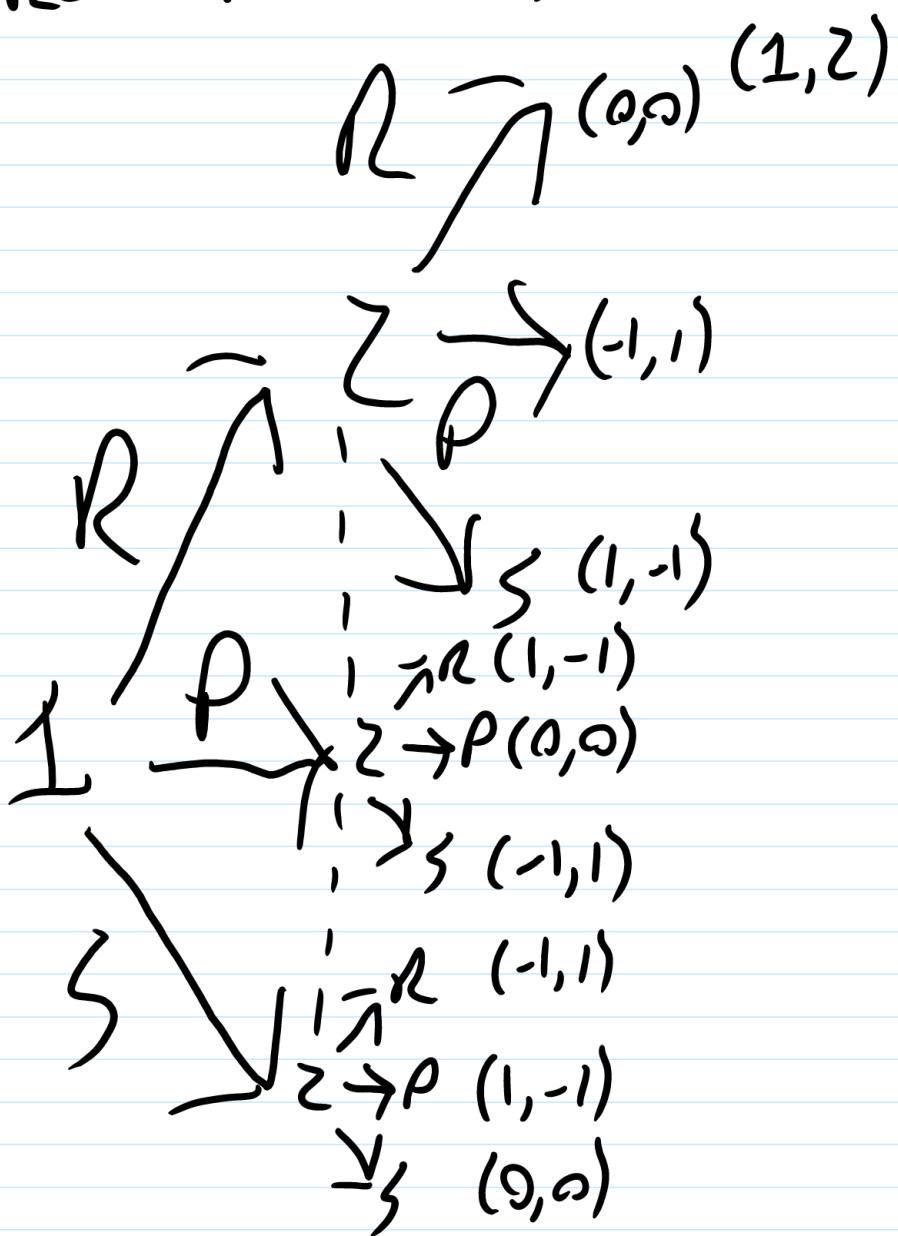
Owner of firm is hiring. They can hire, not hire, or let the manager decide. Manager can hire, or not hire. If hired, the worker can work diligently or slack off. Worker doesn't know if O or M hired. Worker not hired, all 0. W hired and shirks, Q and M get -1 and W gets +. If Q hired and W is diligent, Q gets 1, M and W get 0. If M hired, PIP Q and M

(Q, M, W)



Passed Solution Review

Rock Paper Scissors!



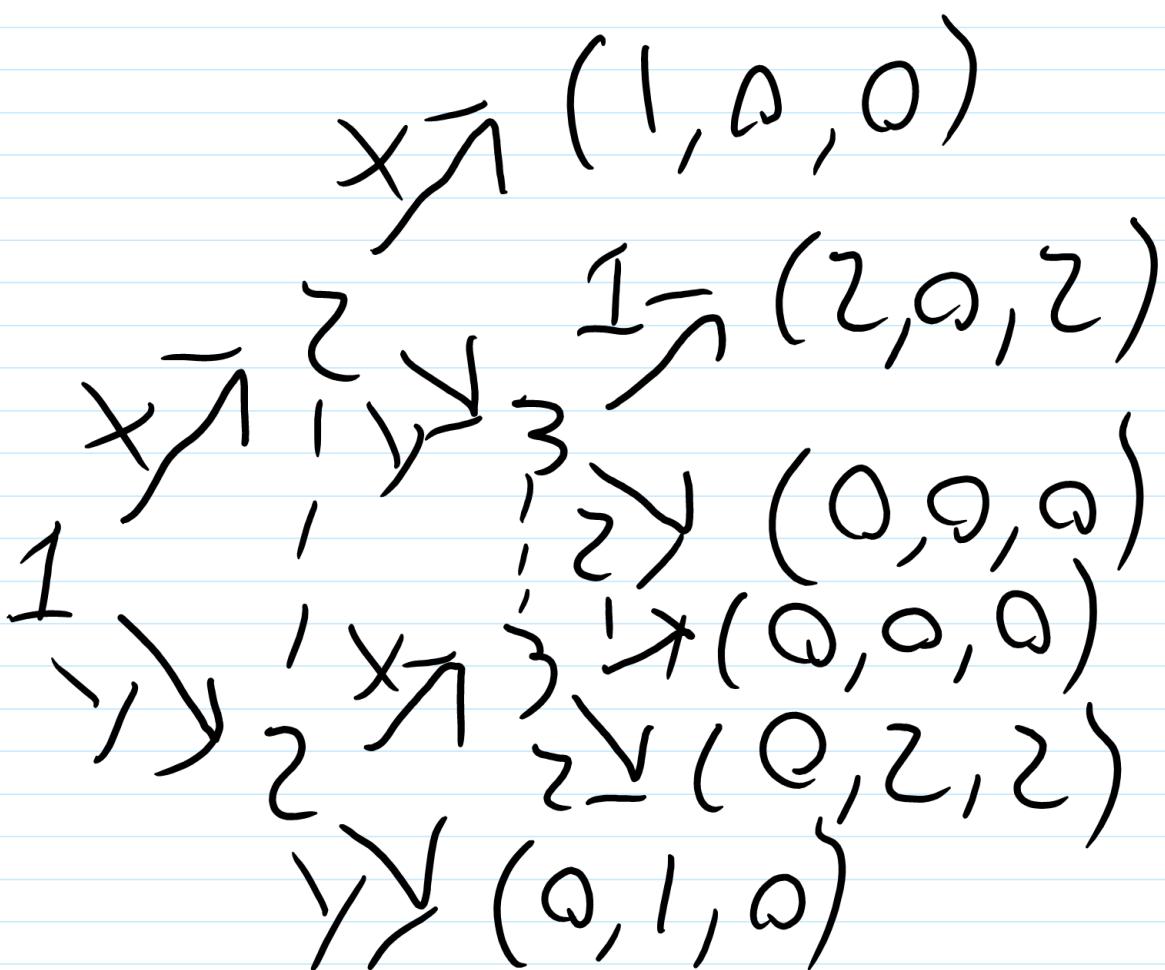
Because play is simultaneous, player order doesn't matter except for writing payoffs

If the game was sequential, Player 2 would always win

Passed Solution Review

Players 1, 2, 3. At first 1 and 2 move simultaneously between X and Y. Game ends if both choose X and Pay off is $(1, 0, 0)$. If both choose Y, game over w/ $(0, 1, 0)$. Player 3 guesses who said what.

$(1, 2, 3)$



3.1 Strategies

Tuesday, August 25, 2020 10:09 AM

$$S_2 = \{H, L\}$$

$$S_1 = \{HH', HL', LH', LL'\} \leftarrow \text{Cherry 1 + } \Sigma_i \text{ from previous example on High/Low Effort}$$

$(H, LH') \leftarrow \text{PROFILE}$

$$S = S_1 \times S_2$$

↳ Cartesian Product (matrix multiplication)

$$S_i(S) : S = S_1 \times S_2 \times S_3 \times \dots \times S_n \leftarrow \text{All Possibilities}$$

S_{-i} : S all strategies except Player i (think vectors in A)

Player i utility $= V_i(S)$ or $V_i(S_i, S_{-i})$

$i = 7$ then $S_{-7} = (S_1, \dots, S_6, S_8, \dots, S_n)$ or $S(S_7, S_{-7})$

Every profile maps to payoffs

$$U_i : S \rightarrow \mathbb{R}$$

Normal form?

Strategic Form Representation

Cheryl + Zell

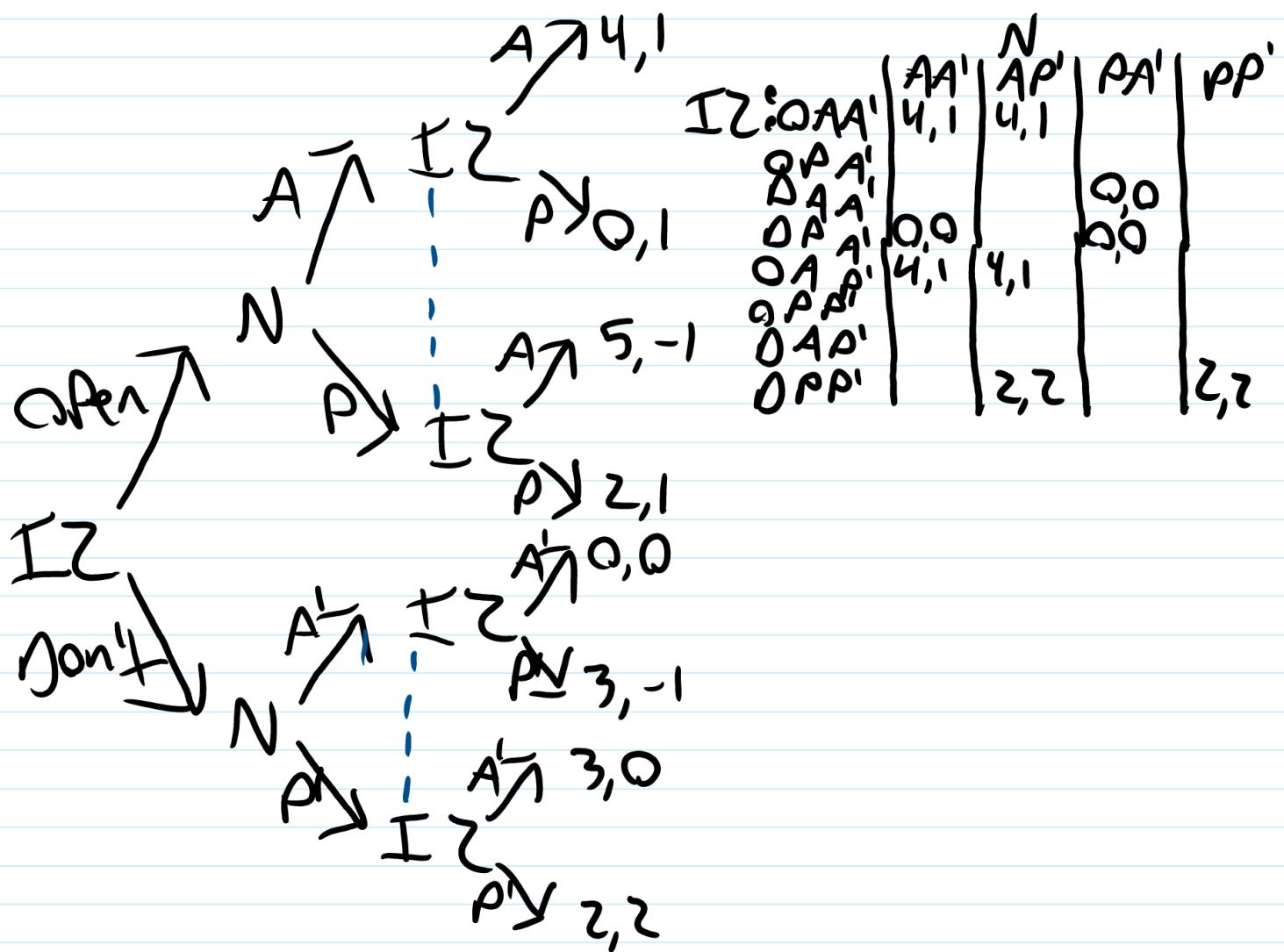
& Strategy Profiles

Zell	H	HH'	HL'	LH'	LL'
Cheryl	H	-1, 1	-1, 1	0, 1	1, 0
	L	0, 1	0, 0	0, 1	0, 0

$X \in \mathbb{R}$ $Y \in \mathbb{A} \rightarrow V_1(X, Y) \quad V_2(X, Y)$
by Real number

3.3 Example Strategies

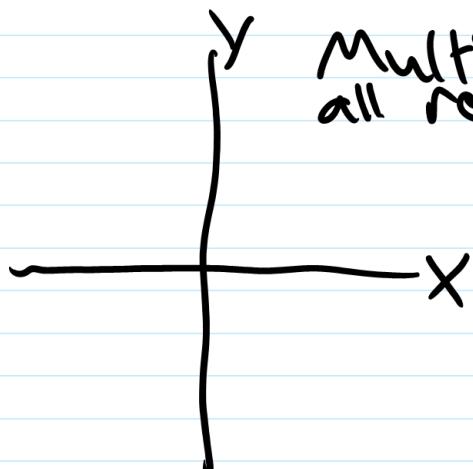
Tuesday, August 25, 2020 8:52 PM



3 Extra Notes

Wednesday, August 26, 2020 10:07 AM

Cartesian Product



Multiplying x and y sets to get
all resulting sets

Passed Solution review

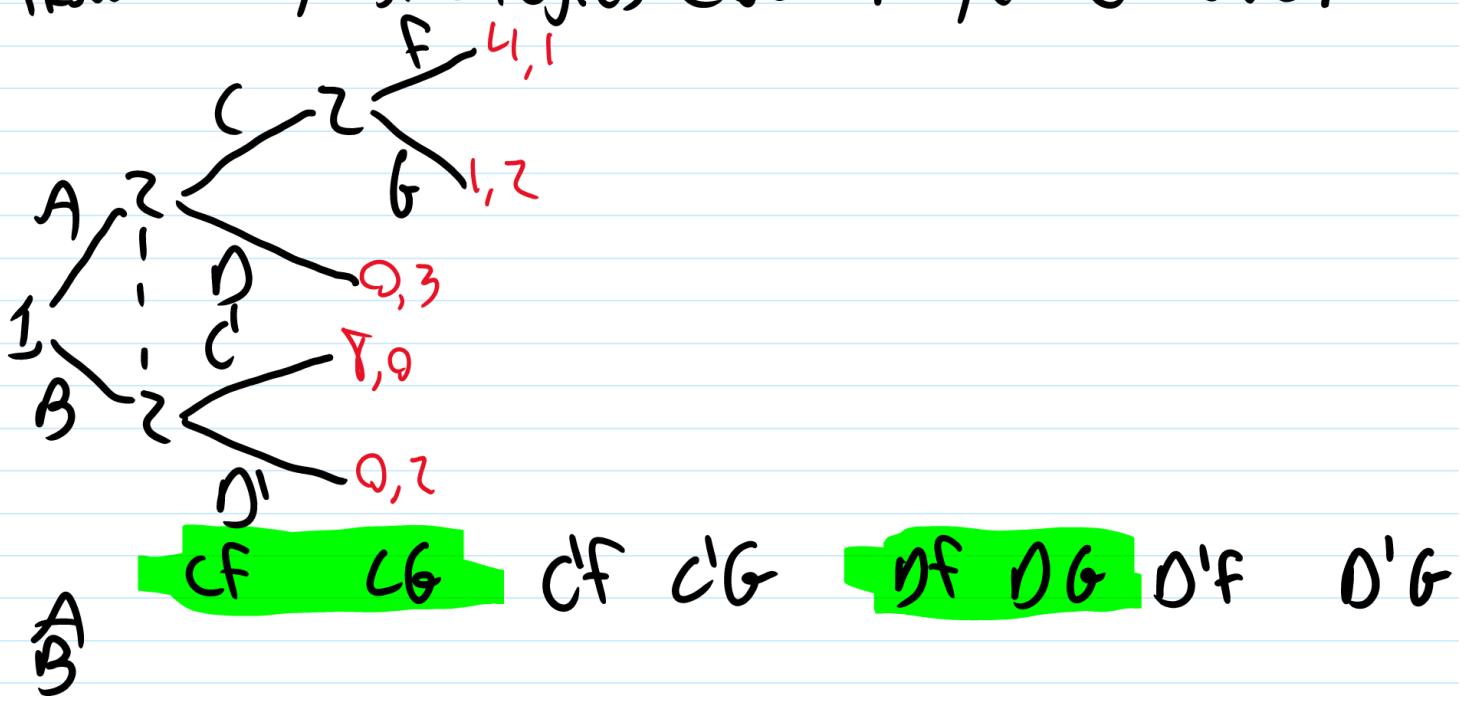
Manager decides whether or not to hire a worker. If M doesn't hire w game over. If hired v can do high or low effort. Based on effort, M can retain or fire w.

Not hire describes a strategy because not hire is the first move and a terminal point in the game.

No. A strategy must specify what to choose in all possible information sets, whether or not they will be reached if that strategy is played w/ fidelity.

Passed solution for review
Worked w/ Hall + Isabel

How many strategies does Player 2 have?



Player 2 has 4 strategies.

~~Possible Solution review~~
Worked w/ Hall + Isabel

Cat = Baker Dog = Spike mouse = Cheesy

Baker wants to catch Cheesy + avoid Spike

Cheesy wants to tease Baker + not get caught

Spike wants to rest and not be disturbed

Morning: B + C simultaneously decide

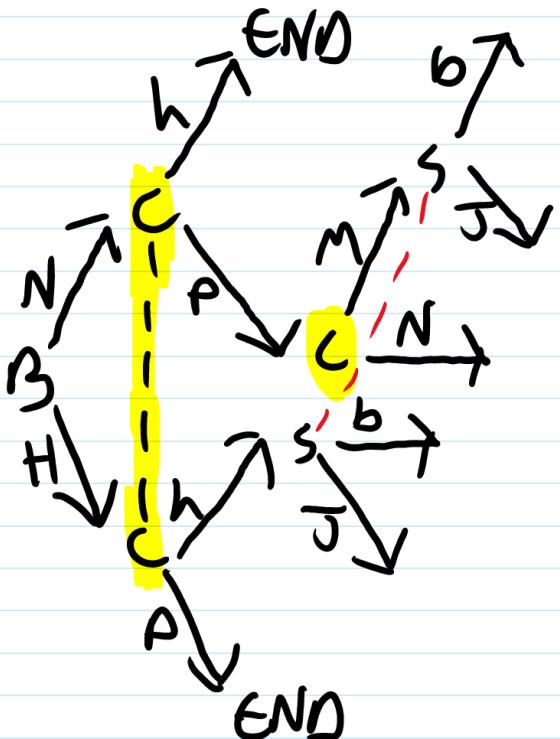
|
B can Nap or Hunt

C can hide or play
↳ moves S's bone

↳ game ends if nap+hide or hunt+play

If nap+play, B must move or not move the bone

If bone moved, S punishes B or C, then game ends



Cheesy has 2 information sets

B: N, H

C: hm, pm, hn, pn

S: b, j

2x

4x

2x
16?

There are 16 strategy profiles

Beliefs, Mixed Strategies, & expected Payoffs

belief is what a player thinks will happen

Mixed Strategy is selecting a strategy according to a probability distribution

regular strategy = pure strategy

Mixed Strategy includes Pure Strategy

Payoff numbers can include preferences of player preferences over probability distributions over outcomes

4.1 Beliefs and Expected Payoffs

Saturday, August 29, 2020 4:35 PM

Beliefs

		Column		
		C1	C2	C3
Row	R1	4,1	1,1	2,5
	R2	2,2	0,0	3,3
	R3	2,5	1,1	1,4

$$\begin{matrix} \rho_1 & \rho_2 & 1-\rho_1-\rho_2 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{matrix}$$

$$\Theta_{-R}, \Theta_C \rightarrow \Theta_i \in \Delta S_i$$

Theta not row means theta column ...

beliefs are represented by probability distributions

Expected Payoff/utility $\rightarrow E$

$$\Theta_C = (.2, .2, .6)$$

U=utility

$$E(U_R | S_R = R_2, \Theta_C) = .2 \cdot 2 + .2 \cdot 0 + .6 \cdot 3 = 2.2$$

\uparrow \uparrow
IF and

$$\Theta_R = (.7, .1, .2) \rightarrow E(U_C | S_C = C_1, \Theta_R) = .7 \cdot 1 + .1 \cdot 2 + .2 \cdot 5 = .85$$

$$U_i(S_i, \Theta_{-i}) = \sum_{S_{-i} \in S_{-i}} \Theta_{-i}(S_{-i}) \cdot U(S_i, S_{-i})$$

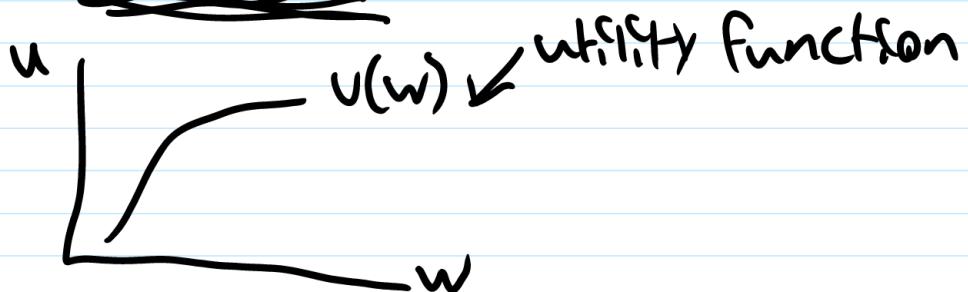
MORE ON EXPECTED UTILITY

Ranking = ordinal (can't multiply)

Risk neutral, money outcomes

U will take any fair gamble

Risk Aversion



Von-Neumann Morgenstern Utility Function

- Continuity
- More is better
- Transitivity
- Independence of irrelevant outcomes

4.3 Mixed Strategies

Saturday, August 29, 2020 5:09 PM

Mixed Strategies

$\sigma_i \in \Delta S_i$
Sigma

$$\Theta_C = (.2, .2, .6)$$

$$\sigma_R = (.7, 0, .3)$$

\hookrightarrow NOT Θ_1 R belief

$$E(V_R) = [0.7 \cdot 0.2 \cdot 4 + 0.7 \cdot 0.2 \cdot 1 + 0.7 \cdot 0.6 \cdot 2] +$$

$$0[0.2 \cdot 2 + 0.2 \cdot 0 + 0.6 \cdot 3] +$$

$$0.3[0.2 \cdot 2 + 0.2 \cdot 1 + 0.6 \cdot 1] = R \text{ Player expected utility given mixed strategy + their beliefs about C's strategy}$$

$$V_i(\sigma_i, \theta_{-i}) = \sum_{S_i \in S_i} \sum_{S_{-i} \in S_{-i}} \theta_i(S_i) \cdot \theta_{-i}(S_{-i}) \cdot V_i(S_i, S_{-i})$$

4 Extra Notes

Monday, August 31, 2020 10:05 AM

Mixed strategy is a way to obfuscate
your goal

4 Extra Problems

Friday, September 4, 2020 10:08 AM

$\frac{2}{3}$	L	C	R
U	$10, 0$	$0, 10$	$3, 3$
M	$2, 10$	$10, 2$	$6, 4$
D	$3, 3$	$4, 6$	$6, 6$

$$g) V_2(m, R) = 4$$

$$e) (3 \cdot .25) + (.5 \cdot 6) + (.25 \cdot 6) = 5.25$$

$$g) (3 \cdot \frac{1}{3}) + (4 \cdot \frac{1}{3}) + (6 \cdot \frac{1}{3}) = 3 \frac{2}{3}$$

h)

Passed Solution Review

$$S_1 = \{H, L\} \text{ and } S_2 = \{X, Y\}$$

IF 1 Plays H, Payoff = z. Player 1 Payoff: $V_1(L, X) = 0$
 $V_1(L, Y) = 10$
 Player 2 Payoff doesn't matter

		Player 2	
		X	Y
Player 1	H	z, 0	z, 1
	L	0, z	0, 0

b) IF 1 believes $\theta_2 = (0.5, 0.5)$, Payoff of Playing H? Playing L?

$$\begin{aligned} \text{Payoff for H} &= 0.5 \cdot z \\ \text{Payoff for L} &= 10 \cdot 0.5 = 5 \end{aligned}$$

Player 1 is indifferent when $z = 5$

c) $\theta_2 = (\frac{1}{3}, \frac{2}{3})$. Payoff of Player 1 Playing L?

$$= \frac{1}{3} \cdot 0 + 10 \cdot \frac{2}{3} = 0 + 20/3 = 20/3$$

Passed Solution review

Worked w/ hand

QA	$\begin{array}{ c } \hline 1 \\ \hline 2,2 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline 2,2 \\ \hline \end{array}$
QB	$\begin{array}{ c } \hline 2,2 \\ \hline 2,2 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2,2 \\ \hline 2,2 \\ \hline \end{array}$
IA	$\begin{array}{ c } \hline 4,2 \\ \hline 3,4 \\ \hline \end{array}$	$\begin{array}{ c } \hline 1,3 \\ \hline 1,3 \\ \hline \end{array}$
IB		

a) $V_1(G_3, I)$ for $G_1 = (1/4, 1/4, 1/4, 1/4)$

$$1/4(2+2+4+3) = 1/2 + 1/2 + 1 + 3/4 = 11/4 \text{ or } 2.75$$

b) $V_2(G_1, Q)$ for $G_1 = (1/8, 1/4, 1/4, 3/8) = (1/8, 2/8, 2/8, 3/8)$

$$2/8 + 4/8 + 6/8 + 9/8 = 21/8$$

c) $V_I(G_1, G_2)$ for $G_1 = (1/4, 1/4, 1/4, 1/4)$, $G_2 = (1/3, 2/3)$

QA	$\begin{array}{ c } \hline I \\ \hline 1/12 \\ \hline \end{array}$	$\begin{array}{ c } \hline Q \\ \hline 2/12 \\ \hline \end{array}$
QB	$\begin{array}{ c } \hline 1/12 \\ \hline 1/12 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2/12 \\ \hline 2/12 \\ \hline \end{array}$
IA	$\begin{array}{ c } \hline 1/12 \\ \hline 1/12 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2/12 \\ \hline 2/12 \\ \hline \end{array}$
IB	$\begin{array}{ c } \hline 1/12 \\ \hline 1/12 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2/12 \\ \hline 2/12 \\ \hline \end{array}$

$$2/12 + 2/12 + 4/12 + 3/12 + 4/12 + 4/12 + 2/12 + 2/12 = 23/12$$

d) $V_I(G_1, G_2)$ for $G_1 = (0, 1/3, 1/6, 1/2)$, $G_2 = (2/3, 1/3)$

QA	$\begin{array}{ c } \hline I \\ \hline 0 \\ \hline \end{array}$	$\begin{array}{ c } \hline Q \\ \hline 2/18 \\ \hline \end{array}$
QB	$\begin{array}{ c } \hline 2/18 \\ \hline 2/18 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2/18 \\ \hline 2/18 \\ \hline \end{array}$
IA	$\begin{array}{ c } \hline 6/18 \\ \hline 6/18 \\ \hline \end{array}$	$\begin{array}{ c } \hline 1/18 \\ \hline 1/18 \\ \hline \end{array}$
IB	$\begin{array}{ c } \hline 6/18 \\ \hline 6/18 \\ \hline \end{array}$	$\begin{array}{ c } \hline 3/18 \\ \hline 3/18 \\ \hline \end{array}$

$$0 + 2/18 + 2/18 + 18/18 + 0 + 4/18 + 1/18 + 3/18 = 42/18 = 7/3$$

FIGURE 3.4 Classic normal-form games.

	2	H	T
H	1, -1	-1, 1	
T	-1, 1	1, -1	

Matching Pennies

	2	C	D
1	2, 2	2, 2	2, 2
D	3, 1	1, 1	1, 1

Prisoner's Dilemma

	2	Opera	Movie
1	2, 1	0, 0	
Movie	0, 0	1, 2	

Battle of the Sexes

	2	H	D
1	0, 0	3, 1	
D	1, 3	2, 2	

Hawk-Dove/Chicken

	2	A	B
1	1, 1	0, 0	
B	0, 0	1, 1	

Coordination

	2	A	B
1	2, 2	0, 0	
B	0, 0	1, 1	

Pareto Coordination

	2	S	D
D	4, 4	2, 2	
P	2, 2	0, 0	
S	0, 0	3, 3	

Pigs

Worked w/ hair
Passed Solution review
for each game find:

$V_1(\delta_1, \delta_2)$ and $V_2(\delta_1, \delta_2)$ for
 $\delta_1 = (\frac{1}{2}, \frac{1}{2})$ and $\delta_2 = (\frac{1}{2}, \frac{1}{2})$

Pennies: $V_1(\delta_1, \delta_2) = V_2(\delta_1, \delta_2) = .25 - .25 + .25 - .25 = 0$

Sexes: $V_1(\delta_1, \delta_2) = V_2(\delta_1, \delta_2)$
 $= (2 \cdot \frac{1}{4}) + 0 + 0 + (1 \cdot \frac{1}{4}) = 3/4$

Chicken: $V_1(\delta_1, \delta_2) = V_2(\delta_1, \delta_2)$
 $= 0 + (1 \cdot \frac{1}{4}) + (3 \cdot \frac{1}{4}) + (2 \cdot \frac{1}{4}) = 6/4 = 1.5$

General Assumptions + Methodology

Must balance realism w/ manageable math

Rationality

Players select path to their preferred outcome
- maximizing one's payoff

Assume players know how the game is played

Players are rational if they 1. form a belief about strategies and 2. given this belief they maximize their payoff

5 Extra Notes

Wednesday, September 2, 2020 10:06 AM

PNG's Optimal w/ 3 Players

$\begin{array}{c} R \\ S \end{array} \left| \begin{array}{c} R \\ S \end{array} \right. \quad \begin{array}{c} R \\ S \end{array} \left| \begin{array}{c} R \\ S \end{array} \right. \quad \text{Players } R, C, \text{ and } T \\ R \quad T \quad S \end{array}$

Dominance + Best Response

Contracts can bind players to strategies

6.1 Dominance and the Prisoner's Dilemma

Monday, August 31, 2020 2:46 PM

Dominance

Prisoner's
Dilemma

	Hail	Silent	Rat
Gus	Silent 2, 2	Rat 3, -1	
Cass		0, 0	-1, 3

Silence is dominated by Rat
↳ Always better

Weakly dominated = sometimes better

R_1	$\left \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} \right $	$\left \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} \right $	C_1 WD by C_1	C_2 WD by C_1	C_3 WD by $C_1 + C_2$
R_2	$\left \begin{matrix} 1,1 \\ 2,2 \\ 2,5 \end{matrix} \right $	$\left \begin{matrix} 1,1 \\ 2,0 \\ 1,1 \end{matrix} \right $	C_1 SD by C_2	C_2 SD by C_3	
R_3	$\left \begin{matrix} 2,5 \\ 1,1 \end{matrix} \right $	$\left \begin{matrix} 3,3 \\ 1,4 \end{matrix} \right $			

WD / SD = weakly/strongly dominated

Pure vs weak strategies

$$\sigma_R = (.5, .5, 0) \quad \begin{matrix} v_{C1} & 3 & \geq 2 \\ v_{C2} & 1.5 & \geq 1 \\ v_{C3} & 2.5 & \geq 1 \end{matrix}$$

$(.5, .5, 0)$ SD C_3

General Notation

s_i is dominated by σ_i if

$$V_i(\sigma_i, s_{-i}) > V_i(s_i, s_{-i}) \quad \forall s_i \in S_i$$

↓ for all

Rational players do NOT play dominated strategies

PARETO EFFICIENCY

Contracting

$$V_i(q, s_{-i}) > V_i(s_i, s_{-i}) \quad \forall s_i \in S_i$$

$\wedge \forall s_i \in S_i \quad s_i \neq q$

6.3 Best Responses

Saturday, September 5, 2020 12:00 PM

Best responses or best replies

	C_1	C_2	C_3
R_1	(4, 1)	(1, 1)	(2, 5)
R_2	(2, 2)	(2, 0)	(3, 3)
	(2, 5)	(1, 1)	(1, 4)

$$BR_1(1, 0, 0) = R_1$$

$$(0, 1, 0) \text{ or } (0, 0, 1) = R_2$$

$$BR_2(1, 0, 0) \text{ or } (0, 1, 0) = C_3$$

$$(0, 0, 1) = C_1$$

Column 2, 3 never BR because it's dominated

$BR \Leftrightarrow V_i(s_i, \theta_{-i}) \geq V_i(s'_i, \theta_{-i}) \quad \forall s'_i \in S_i$. IF true,
strategy s is a BR to θ_{-i}

$$B_C = \{C_1, C_3\}$$

Establishing no set of beliefs make s_3 a best response

	C_1	C_2	C_3
R_1	4, 1	1, 1	2, 5
	2, 2	2, 0	3, 3
R_2	2, 5	1, 1	1, 4

$$\Theta_c(P_1, P_2, 1-P_1, 1-P_2)$$

$$V_R(R_3, \Theta_c) = \frac{2}{1+P_1} + (1-P_1) \cdot [1 \cdot (1-P_1 - P_2)]$$

$$V_R(R_2, \Theta_c) = \frac{2}{3} \cdot \frac{P_1}{1+P_1} + \frac{2}{3} (1-P_1 - P_2)$$

$$V_R(R_1, \Theta_c) = \frac{4}{2+2P_1} + \frac{1}{2} (1-P_1 - P_2)$$

$$V_R(R_3) > V_R(R_2) > 0$$

$$\begin{aligned} & 1+P_1 - 3 + P_1 + P_2 \geq 0 \\ & -2 + 2P_1 + P_2 \geq 0 \end{aligned}$$

$$P_2 \geq 2 - 2P_1$$

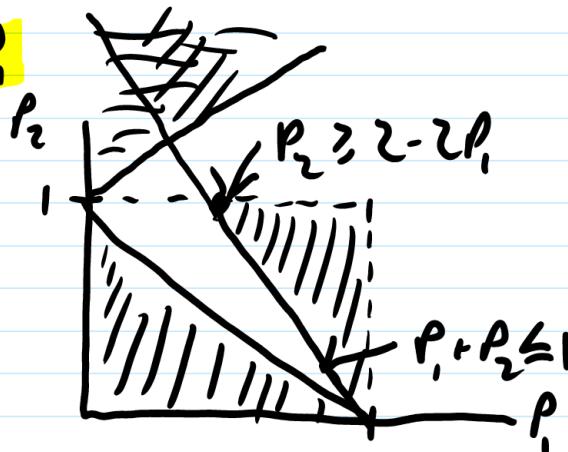
$$V_R(R_3) > V_R(R_1) \geq 0$$

$$\begin{aligned} & 1+P_1 - 2 - 2P_1 + P_2 \geq 0 \\ & -1 - P_1 + P_2 \geq 0 \end{aligned}$$

$$P_2 \geq 1 + P_1$$

$$P_1 + P_2 \leq 1$$

Can't satisfy all 3!



R_3 is never best!

6.6 Undominated Strategies and Best Responses

Saturday, September 5, 2020 12:32 PM

Beliefs

	C ₁	C ₂	C ₃
R ₁	4, 1	1, 1	2, 5
R ₂	2, 2	2, 0	3, 3
R ₃	2, 5	1, 1	1, 4

strategic independence

Correlated beliefs
 $\hookrightarrow \beta_f^C = VD:$

6.7 Some Last Words on Weak Dominance

Saturday, September 5, 2020 12:49 PM

$$\begin{array}{c|c|c|c} \ell_1 & C_1 & C_2 & C_3 \\ \hline \ell_2 & 4, i & 0, i & 3, i \\ \hline p_1 & p_2 & p_2 & 1-p_2 \end{array}$$

"fully mixed beliefs"

$$\beta_i^{FC} = WVD_i$$

6 Extra Problems

Friday, September 4, 2020 10:39 AM

g) B dom A, L dom R

g) L dom R, L weak dom C?

g) X dom Z, $\sqrt{5} + \sqrt{3}$ dom M

6.1 Determine which strategies are dominated in the following normal-form games.

	1	2
	L	R
A	3, 3	2, 0
B	4, 1	8, -1

(a)

	1	2	
	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

(b)

	1	2		
	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

5. Represent in the normal form the rock–paper–scissors game (see Exercise 4 of Chapter 2 to refresh your memory) and determine the following best-response sets.

(a) $BR_1(\theta_2) \text{ for } \theta_2 = (1, 0, 0) = \{P\}$

(b) $BR_1(\theta_2) \text{ for } \theta_2 = (1/6, 1/3, 1/2) = \{R, S\}$

(c) $BR_1(\theta_2) \text{ for } \theta_2 = (1/2, 1/4, 1/4) = \{P, S\}$

(d) $BR_1(\theta_2) \text{ for } \theta_2 = (1/3, 1/3, 1/3) = \{R, P, S\}$

R	P	S
P	Q, Q	-1, 1
S	1, -1	Q, Q
R	-1, 1	1, -1

b) $R = Q + -\frac{1}{3} + \frac{1}{2} = -\frac{2}{6} + \frac{3}{6} = \frac{1}{6}$
 $P = \frac{1}{6} + Q - \frac{3}{6} = -\frac{2}{6}$
 $S = -\frac{1}{6} + \frac{2}{6} + Q = \frac{1}{6}$

Passed Solution Review

	2	
1	L	C
U	10, 0	0, 10
M	2, 10	10, 2
D	3, 3	4, 6

2. For the game in Exercise 1 of Chapter 4, determine the following sets of best responses.

(a) $BR_1(\theta_2)$ for $\theta_2 = (1/3, 1/3, 1/3)$

(b) $BR_2(\theta_1)$ for $\theta_1 = (0, 1/3, 2/3)$

(c) $BR_1(\theta_2)$ for $\theta_2 = (5/9, 4/9, 0)$

(d) $BR_2(\theta_1)$ for $\theta_1 = (1/3, 1/6, 1/2)$

a) $V = \frac{10}{3} + 0 + \frac{3}{3} = \frac{13}{3}$
 $M = \frac{2}{3} + \frac{10}{3} + \frac{6}{3} = \frac{18}{3}$
 $D = \frac{3}{3} + \frac{4}{3} + \frac{6}{3} = \frac{13}{3}$ $\rightarrow BR_1(\theta_2) = \{M\}$

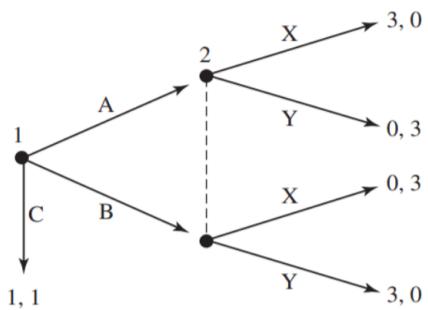
b) $L = 0 + \frac{10}{3} + \frac{6}{3} = \frac{16}{3}$
 $C = 0 + \frac{2}{3} + \frac{14}{3} = \frac{16}{3}$
 $R = 0 + \frac{4}{3} + \frac{12}{3} = \frac{16}{3}$ $\rightarrow BR_2(\theta_1) = \{L, R\}$

c) $V = \frac{50}{9} + 0 + 0 = \frac{50}{9}$
 $M = \frac{10}{9} + \frac{40}{9} + 0 = \frac{50}{9}$
 $D = \frac{15}{9} + \frac{16}{9} + 0 = \frac{31}{9}$ $\rightarrow BR_1(\theta_2) = \{V, M\}$

d) $L = 0 + \frac{10}{6} + \frac{9}{6} = \frac{19}{6}$
 $C = \frac{80}{6} + \frac{18}{6} + \frac{18}{6} = \frac{40}{6}$
 $R = \frac{6}{6} + \frac{4}{6} + \frac{18}{6} = \frac{28}{6}$ $\rightarrow BR_2(\theta_1) = \{C\}$

Passed Solution Review

6. In the game pictured here, is it ever rational for player 1 to select strategy C? Why?



If Player 1 wants a guaranteed payoff, they would choose strategy C.

But, mathematically, they would choose A or B because the expected payoff value is 1.5

$$AX = BY + AY = BX$$

By eliminating C, a mixed a+b generates 1.5

(Let P denote Player 1's belief about probability of X)

$$\begin{aligned} u_1(C, P) &\geq u_1(A, P) \\ 1 &\geq 3P + Q(1-P) \\ 1/3 &\geq P \end{aligned}$$

$$\begin{aligned} u_1(C, P) &\geq u_1(B, P) \\ 1 &\geq 0P + 3(1-P) \\ 1 &\geq 3 - 3P \\ P &\geq 2/3 \end{aligned}$$

NO because P cannot satisfy both inequalities

Passed Solution Review

7. In the normal-form game pictured below, is player 1's strategy M dominated? If so, describe a strategy that dominates it. If not, describe a belief to which M is a best response.

	X	Y
K	9, 2	1, 0
L	1, 0	6, 1
M	3, 2	4, 2

M is dominated
by mixed strategy
 $\{k, L\}$

M does not dominate Player 1's response because in X, K is better and in Y, L is better.

$$\begin{aligned} qP + 1(1-P) &> 3 \\ qP + \frac{1}{2} - P &> 3 \\ \frac{qP}{2} + 1 &> 3 \\ \frac{qP}{2} &> 2 \\ P &> 4/5 \end{aligned}$$

$$P > 5/20$$

$$\begin{aligned} 1P + 6(1-P) &> 4 \\ P + 6 - 6P &> 4 \\ 6 - 5P &> 4 \\ -5P &> -2 \\ P &< 2/5 \end{aligned}$$

$$P < 8/20$$

$$\sigma = (.3, .7, 0)$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ P & 1-P & 0 \end{matrix}$$

Rationalizability + Iterated dominance

Actions of one player may affect another's payoff.

Players can eliminate (think Sudoku) strategies that don't make sense

↳ Iterative removal of (strictly) dominated strategies

↳ Iterative dominance

Strategies that survive iterative dominance are called rationalizable strategies

The Second Strategic Tension

Strategic uncertainty

↳ Coordination problem:

7.1 Iterated Dominance and Rationalizability

Thursday, September 10, 2020 4:19 PM

		Bob			
		β_1	β_2	β_3	β_4
Alice	A_1	4,1	1,2	2,5	1,4
	A_2	2,3	3,1	1,0	1,2
	A_3	2,5	1,1	1,4	3,0

β_2 dom by mix $\beta_1 + \beta_3$,
Eliminate β_2 to create R_1

Alice has no dominated strategies

R_1

		Bob			
		β_1	β_2	β_3	β_4
Alice	A_1	4,1	1,2	2,5	1,4
	A_2	2,3		1,0	1,2
	A_3	2,5		1,4	3,0

A_2 dom by $A_1 + A_3$

R_2

		Bob			
		β_1	β_2	β_3	β_4
Alice	A_1	4,1	1,2	2,5	1,4
	A_2			1,0	1,2
	A_3	2,5		1,4	3,0

β_4 dom by β_3

R_3

		Bob			
		β_1	β_2	β_3	β_4
Alice	A_1	4,1	1,2	2,5	1,4
	A_2			2,5	
	A_3	2,5		1,4	

A_3 dom by A_1

R_{11}

		Bob			
		β_1	β_2	β_3	β_4
Alice	A_1	4,1	1,2	2,5	1,4
	A_2			2,5	
	A_3			1,4	

β_1 dom by β_3

R_5

		Bob			
		β_1	β_2	β_3	β_4
Alice	A_1	4,1	1,2	2,5	1,4
	A_2			2,5	
	A_3			1,4	

2,5

You can eliminate dominated strategies

7.2 Strategic Uncertainty

Thursday, September 10, 2020 4:35 PM

Coordination Problem

↳ 2 people lose each other somewhere + look for each other in different rational places

"focal points"

	Rabbit	Stag
Rabbit	1, 1	1, 0
Stag	0, 1	4, 4

7 Extra Problems

Friday, September 18, 2020 10:04 AM

Determine which strategy profiles are rationalizable for these games.

	2			
1	w	x	y	
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(e)

1	2			
w	a	b	c	
x	3, 4	4, 4	4, 5	10, 2
y	3, 7	8, 7	5, 8	10, 6
z	2, 10	7, 6	4, 6	9, 5
	4, 4	5, 9	4, 10	10, 9

Passed Solution Review

Suppose that you manage a firm and are engaged in a dispute with one of your employees. The process of dispute resolution is modeled by the following game, where your employee chooses either to “settle” or to “be tough in negotiation,” and you choose either to “hire an attorney” or to “give in.”

		You	
		Give in	Hire attorney
Employee	Settle	1, 2	0, 1
	Be tough	3, 0	$x, 1$

In the cells of the matrix, your payoff is listed second; x is a number that both you and the employee know. Under what conditions can you rationalize selection of “give in”? Explain what you must believe for this to be the case.

If $x \geq 0$, be tough dam Settle

If $x < 0$, no dominant strategies

+ greater than

If $p(\text{Settle}) > 0.5$ and $x < 0$, then it makes sense to choose give in

Rational Solution Review

Find the set of rationalizable strategies for the following game.

		a	b	c	d	
		w	5, 4	4, 4	4, 5	12, 2
		x	3, 7	8, 7	5, 8	10, 6
		y	2, 10	7, 6	4, 6	9, 5
		z	4, 4	5, 9	4, 10	10, 9

Note that each player has more than one dominated strategy. Discuss why, in the iterative process of deleting dominated strategies, the order in which dominated strategies are deleted does not matter.

$$R^0 = \{w, x, y\} \times \{a, b, c\}$$

$$R^1 = \{w, x\} \times \{c\}$$

$$R^2 = \{x\} \times \{c\}$$

Final strategy = $\boxed{\{x\} \times \{c\}}$

The order doesn't matter because a round isn't over until both players have moved.

$$\begin{aligned} x &\text{ dom } y \\ \frac{2}{3}w + \frac{1}{3}x &\text{ dom } z \\ c &\text{ dom } d \\ \frac{9}{10}c + \frac{1}{10}a &\text{ dom } b \end{aligned}$$

$$R^1 = \{w, x\} \times \{a, c\}$$

$$c \text{ dom } a$$

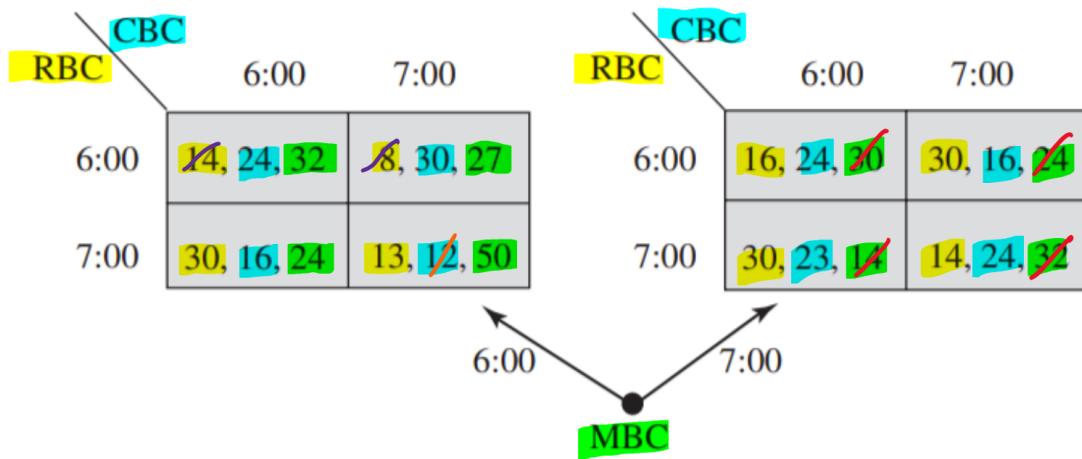
$$R^2 = \{w, x\} \times \{c\}$$

$$x \text{ dom } w \text{ so } S = (x, c)$$

In any iteration, each player considers all strategies not eliminated in the last iteration. Then, in the next iteration, all strategies eliminated for either player are eliminated. Since all are eliminated between iterations, "order" has little meaning within any given round.

Passed Solution Review

Imagine that there are three major network-affiliate television stations in Turlock, California: RBC, CBC, and MBC. All three stations have the option of airing the evening network news program live at 6:00 P.M. or in a delayed broadcast at 7:00 P.M. Each station's objective is to maximize its viewing audience in order to maximize its advertising revenue. The following normal-form representation describes the share of Turlock's total population that is "captured" by each station as a function of the times at which the news programs are aired. The stations make their choices simultaneously. The payoffs are listed according to the order **RBC**, **CBC**, **MBC**. Find the set of rationalizable strategies in this game.



$$L^0 = \{6, 7\} \times \{6, 7\} \times \{6\} = \{7pm, 6pm, 6pm\}$$

$$L^1 = \{7\} \times \{6, 7\} \times \{6\}$$

$$L^2 = \{7\} \times \{6\} \times \{6\}$$



Passed Solution Review

Suppose that in some two-player game, s_1 is a rationalizable strategy for player 1. If, in addition, you know that s_1 is a best response to s_2 , can you conclude that s_2 is a rationalizable strategy for player 2? Explain.

If s_2 is a best response to the rationalizable s_1 ,
then s_2 must also be rationalizable.

No, s_1 might be rationalizable because it is a best response to some other strategy, s_2 , that is rationalizable, making s_1 rationalizable even if s_2 is not

Passed Solution Review

Consider a guessing game with ten players, numbered 1 through 10. Simultaneously and independently, the players select integers between 0 and 10. Thus player i 's strategy space is $S_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, for $i = 1, 2, \dots, 10$. The payoffs are determined as follows: First, the average of the players' selections is calculated and denoted a . That is,

$$a = \frac{s_1 + s_2 + \cdots + s_{10}}{10},$$

where s_i denotes player i 's selection, for $i = 1, 2, \dots, 10$. Then, player i 's payoff is given by $u_i = (a - i - 1)s_i$. What is the set of rationalizable strategies for each player in this game?

$$\begin{aligned} \max a &= \frac{100}{10} = 10 \\ \min a &= 0/10 = 0 \end{aligned}$$

$$\begin{aligned} \max u &= a - 10 - 1 = a - 11 \rightarrow \text{negative} \\ \min u &= a - 1 - 1 = a - 2 \rightarrow \text{maybe negative} \end{aligned}$$

If everyone chooses 0, final payoff is non-negative

$$\begin{aligned} a \leq 10 &\quad \text{so } a - 10 - 1 \leq -1 \quad \text{so } s_{10} = 0 \text{ dominates} \\ \text{Now } a \leq 9 &\quad \text{so } s_9 = 0 \text{ dominates} \\ \text{Now } a \leq 8 &\quad \text{so } s_8 = 0 \text{ dominates} \end{aligned}$$

and so on...

$$S = (0, 0, \dots, 0)$$

Location

Firm location is strategy

- P and C sell soda at the beach and simultaneously independently set up for the day
- 9 regions of size
- 50 purchases in each region
- customers walk to nearest booth

Iterated dominance

$$S_i = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \text{ reduced to}$$

$$\downarrow R_i^1 = \{2, 3, 4, 5, 6, 7, 8\}$$

$$\downarrow R_i^2 = \{3, 4, 5, 6, 7\}$$

$$\downarrow R_i^3 = \{4, 5, 6\} \rightarrow R_i^4 = \{5\}$$

Criticisms of location model:

- 1) In context of market competition, doesn't include firms' specification of prices
- 2) IRL, agents may not move simultaneously
- 3) Cannot apply the model with more than 2 products/firms
- 4) One-dimensional

Strategic Complementarities

Bob increases Alice's payoff but not his
Bob + Alice working together increases either or both

Contracts about effort can't necessarily be made

Complementarity

Nonrationalizability leads to unique predictions in 2 player games w/ 3 properties:

- 1) Strategy spaces are intervals w/ lower + upper bounds
- 2) There are strategic complementarities
- 3) The slope of the best response functions is < 1

\hookrightarrow these are not required

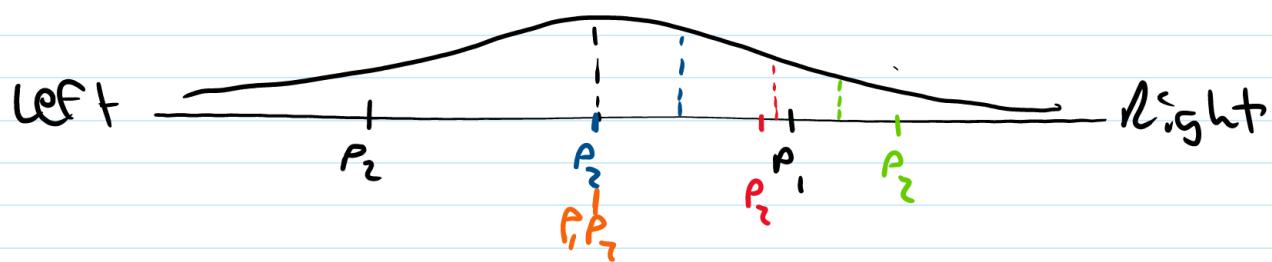
Social Unrest

Strength in numbers

8.1 Location - Median Voter Theorem - Informal

Thursday, September 10, 2020 6:47 PM

median voter theorem



8.2 Location - Median Voter Theorem - Formal

Thursday, September 10, 2020 6:57 PM

Location \rightarrow median Voter theorem
- Rationalizable platforms

A + B, max share

FL L CL C CR R FR

A chooses FL, B chooses FL $VA = VB = .5$
B chooses L $VA = \frac{1}{7}, VB = \frac{6}{7}$

A chooses L, B chooses FL $VA = .5, VB = \frac{1}{7}$

$$= .5$$

A chooses CL, B chooses FL $= \frac{11}{14}$
L $= \frac{3}{14}$
CL $= \frac{2}{7}$
CL $= \frac{1}{2}$

FL is dom for B \rightarrow and also FR
FL is dom for A

IF there are even positions, any combo of 2 middle
are rationalizable

Partnership

$$\text{Value} = V(e_1, e_2) = e_1 + e_2$$

↑ Effort

$$C_1(e_1) = \frac{1}{2}e_1^2 \quad C_2(e_2) = \frac{1}{2}(e_2)^2$$

Each keeps $\frac{1}{2}V$

$$\begin{aligned}\Pi_1 &= \frac{1}{2}V - C_1(e_1) \\ &= \frac{1}{2}e_1 + \frac{1}{2}e_2 - \frac{1}{2}e_1^2 \\ \frac{d\Pi_1}{de_1} &= \frac{1}{2} - 2\left(\frac{1}{2}\right) \cdot e_1^{(2-1)} \\ &= \frac{1}{2} - e_1 = 0 \\ e_1 &= 1/2\end{aligned}$$

Same for Π_2 , $e_2 = 1/2$

$$\begin{aligned}\Pi_1 &\leq \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\right) - \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{2}(1) - \frac{1}{2}\left(\frac{1}{4}\right) \\ &= \frac{1}{2} - \frac{1}{8} \\ &= 3/8\end{aligned}$$

$$\Pi_2 = 3/8 \rightarrow \Pi = \Pi_1 + \Pi_2 = 3/4$$

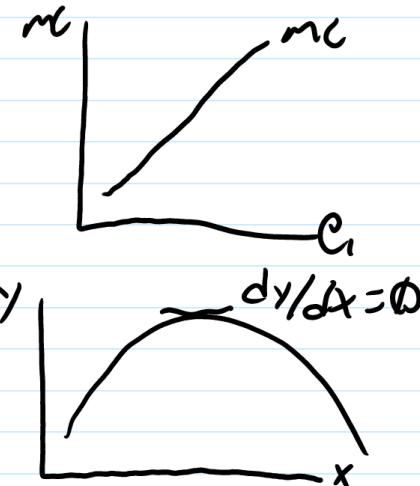
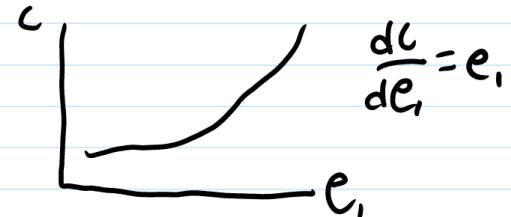
$$\Pi = e_1 + e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$$

$$\frac{d\Pi}{de_1} = 1 - e_1 = 0 \rightarrow e_1 = 1$$

$$\frac{d\Pi}{de_2} = 1 - e_2 = 0 \rightarrow e_2 = 1$$

$$\Pi = 1 - \frac{1}{2} \cdot 1^2 - \frac{1}{2} \cdot 1^2 = 1$$

PROFIT



If independent...

$$\Pi = e_1 - \frac{1}{2}e_1^2 \quad \frac{d\Pi}{de_1} = 1 - e_1 = 0 \rightarrow e_1 = 1$$

same for Π_2

8.4 Partnership - Inseparable Activities - Best Responses

Tuesday, September 15, 2020 10:24 PM

$$V = e_1 + e_2 + \frac{1}{4}e_1 e_2$$

$$c_1 = \frac{1}{2}e_1^2 \quad c_2 = \frac{1}{2}e_2^2$$

$$\pi_1 = \frac{1}{2}(e_1 + e_2 + \frac{1}{4}e_1 e_2) - \frac{1}{2}e_1^2$$

$$\downarrow \frac{1}{2}(e_1 + \bar{e}_2 + \frac{1}{4}e_1 \bar{e}_2) - \frac{1}{2}e_1^2$$

$$\sum_i p(x=x_i) \cdot x_i = E(x)$$

$$\int_0^\infty \Theta_2(v) \cdot v dv = E(e_2) = \bar{e}_2$$

↑
from P view



$$\pi_1 = \frac{1}{2}e_1 + \frac{1}{2}\bar{e}_2 + \frac{1}{8}e_1 \bar{e}_2 - \frac{1}{2}e_1^2$$

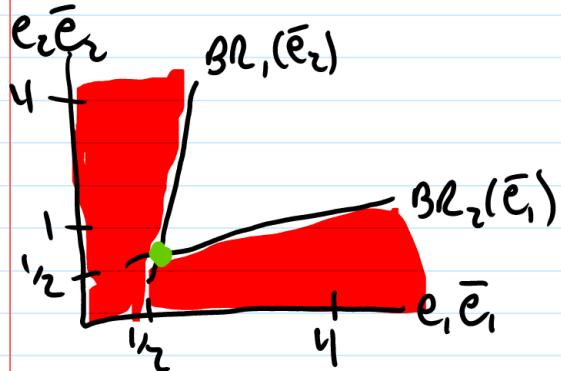
$$\frac{d\pi_1}{de_1} = \frac{1}{2} + \frac{1}{8}\bar{e}_2 - e_1 = 0 \rightarrow e_1 = \frac{1}{2} + \frac{1}{8}\bar{e}_2$$

$$e_2 = \frac{1}{2} + \frac{1}{8}\bar{e}_1$$

8.5 8.6 Partnership - Inseparable Activities

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$e_1 < \frac{1}{2}$ is dam $e_2 < \frac{1}{2}$ is dam
 $R' C_1 < \frac{9}{16}$ $e_2 < \frac{9}{16}$ is \uparrow^4 as ≤ 1 $e_1 \leq 4$
 $e_1 < \frac{1}{2} + \frac{1}{8}(\frac{9}{16})$ is dam \rightarrow same for $e_2 \rightarrow \frac{1}{2} + (\frac{1}{8} \cdot \frac{5}{4})$ is dam



$$e_1 = \frac{1}{2} + \frac{1}{8} e_2$$

$$e_2 = \frac{1}{2} + \frac{1}{8} e_1$$

$$e = \frac{1}{2} + \frac{1}{8} e_1 \rightarrow \frac{7}{8} e_1 + \frac{1}{2} \rightarrow e_1 = \frac{4}{7} = e_2$$

$$e_2 = \frac{1}{2} + \frac{1}{8} \bar{e}_1 \text{ LL } C_2 > \text{LL} \text{ when } \frac{1}{2} + \frac{1}{8} \text{ LL} > \text{LL} \rightarrow \frac{4}{7} > \text{LL}$$

↑ lower limit ↑ upper limit

$\frac{4}{7} < \text{UL}$

I found a stylus that works

I found a stylus that works

8.7 8.8 Inefficiency in the Partner Game

Saturday, September 19, 2020 2:56 PM

A

$$\Pi = e_1 + e_2 + \frac{1}{4}e_1e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$$

$$\frac{d\Pi}{de_1} = 1 + \frac{1}{4}e_2 - e_1 = 0$$

$$e_1 = 1 + \frac{1}{4}e_2$$

↳ same for e_2 (but flipped)

$$\begin{aligned} e_1 &= 1 + \frac{1}{4}e_2 \\ e_1 &= \frac{4}{3} = e_2 \end{aligned}$$

$$\Pi = \frac{4}{3} + \frac{4}{3} + \frac{1}{4} \cdot \frac{4}{3} \cdot \frac{4}{3} - \frac{1}{2} \left(\frac{4}{3}\right)^2 - \frac{1}{2} \left(\frac{4}{3}\right)^2 = \frac{4}{3}$$

$$\Pi_1 = \Pi_2 = \frac{2}{3}$$

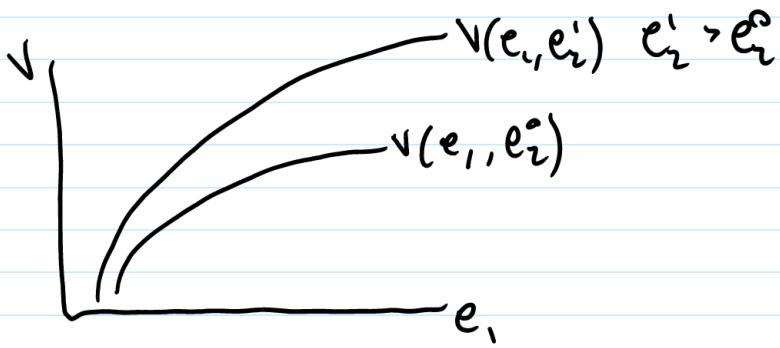
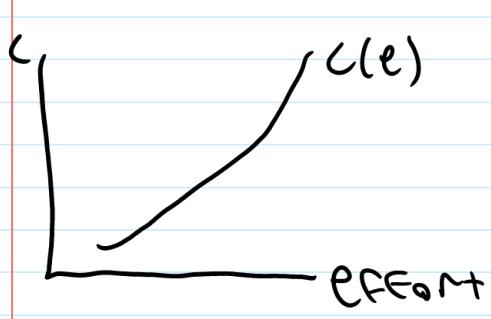
B

$$\Pi_1 = \frac{1}{2} \left(\frac{4}{7} + \frac{4}{7} + \frac{1}{4} \left(\frac{4}{7} \cdot \frac{4}{7} \right) \right) - \frac{1}{2} \left(\frac{4}{7} \right)^2 = \frac{22}{49}$$

↳ < $\frac{4}{3}$

8.9 Joint Production and the Theory of the Firm

Saturday, September 19, 2020 3:08 PM



$$v(e_i, e_j) - c_i(e_i) - c_j(e_j)$$

$$\frac{\partial v}{\partial e_i} = \frac{\partial c_i}{\partial e_i} = 0$$

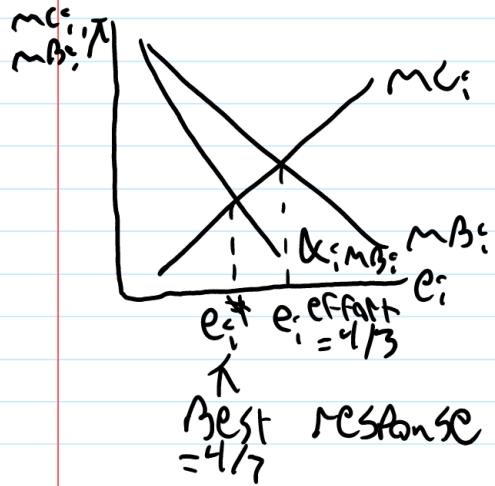
\uparrow

$$\begin{aligned} MB_1 &= MC_1 \\ MB_2 &= MC_2 \end{aligned}$$

$$\lambda_i v(e_i, e_{-i}) - c_i(e_i)$$

$$\frac{\partial \lambda_i v}{\partial e_i} = \lambda_i \frac{\partial v}{\partial e_i} - \frac{\partial c}{\partial e_i} = 0$$

$$\lambda_i MB_i = MC_i \rightarrow \lambda < 1$$



Residual Claimant

- everyone prefers unidirectional norm
- $\epsilon \in [0, 1]$ index propensity to protest
- expressing voice
- maybe not peacefully
- uniformly distributed
- Payoffs α_i if protest, $\alpha > 0$
- let $x = \text{fraction that protests}$
- $\mu = E(x)$, beliefs
- Everyone gains $\beta(x - \mu)$
- Protesters gain $\delta(x - \mu)$
- non protesters gain $\gamma(\lambda - x)$

$$U_i(P, x) = X_i + \beta(x - \mu) + \delta(x - \mu) \rightarrow \text{Protest}$$

$$U_i(H, x) = \gamma(\lambda - x) + \beta(x - \mu) \rightarrow \text{Stay home}$$

$$U_i(P, \mu) > U_i(H, \mu)$$

$$X_i + \delta(\mu - \mu) > \gamma(\lambda - \mu)$$

$$X_i > (\gamma + \delta)(\lambda - \mu)$$

$$\xi > (\gamma + \delta) / \alpha \cdot (\lambda - \mu)$$

$$\hookrightarrow x = \emptyset$$

$$0 > (\gamma + \delta)(\lambda - \mu)$$

Suppose $\lambda > 1 \rightarrow \mu = 0$

now $\lambda \downarrow \lambda < 1$

History + current events coordinate

? Protests if $\xi > (\delta + \gamma) / \alpha \cdot (\lambda - \mu)$

$$\alpha = (\delta + \gamma) / \alpha$$

8.12 8.13 Analysis when Protest or Voice has value - alpha > 0

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$$\alpha = \frac{(\delta + \gamma)}{\lambda}$$

assume $\lambda < 1$

↳ if λ big enough, most zealots won't protest

$$1 - \alpha > (\delta + \gamma)(1 - \theta)$$

$$\alpha > (\delta + \gamma)$$

$$\alpha < 1$$

What beliefs are rationalizable in this world?

$$M = 1 \quad i > (\lambda - M) \alpha < \theta \quad 0 < \alpha < 1, \lambda < 1$$

$$M = \theta \quad i > \alpha \theta \\ i > \alpha \lambda > \theta$$

$$M = 1 - \alpha \lambda \quad i > \alpha \lambda - \alpha \theta$$

$$x = 1 - \alpha \lambda - \alpha(1 - \alpha \lambda) \quad (1 - \alpha \lambda)(1 + \alpha) > (1 - \alpha \lambda)$$

$$M = (1 - \alpha \lambda)(1 + \alpha) \quad i > \alpha \lambda(1 - \alpha \lambda)(1 + \alpha)$$

$$x = 1 - \alpha \lambda + \alpha(1 - \alpha \lambda)(1 + \alpha)$$

$$= (1 - \alpha \lambda)(1 + \alpha + \alpha^2)$$

$$M = (1 + \alpha \lambda)(1 + \alpha + \alpha^2 + \alpha^3 + \dots)$$

$$\alpha < 1$$

8.14 Aside on the Geometric Series

Saturday, September 19, 2020 4:35 PM

Infinite sum of a^x , $|a| < 1$

$$A = a^0 + a^1 + a^{2+} \dots a^T + a^{T+1}$$

\downarrow
 $\underline{1}$

$$aA = a^1 + a^{2+} \dots a^{T+1}$$

$$A - aA = 1 - a^{T+1}$$

$$A(1-a) = 1 - a^{T+1}$$

$$A = \frac{(1-a^{T+1})}{1-a}$$

$$\lim_{T \rightarrow \infty} A = \frac{1}{1-a}$$

8.15 Social Unrest - Conclusion

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$$M = (1 - \alpha\lambda)(1 + \alpha\lambda + \alpha^2\lambda^2 + \dots) \quad \lambda \ll 1 \quad \alpha \ll \lambda$$
$$= (1 - \alpha\lambda)/(1 - \alpha) > 1$$

$(\delta + \gamma) < \alpha \leftrightarrow$ even if $\lambda = 1$, $\xi_i = 1$ protests
 $\xi_i = 1 - \epsilon$ protests
tiny number

Passed Solution Review

Consider a location game with nine regions like the one discussed in this chapter. But instead of having the customers distributed uniformly across the nine regions, suppose that region 1 has a different number of customers than the other regions. Specifically, suppose that regions 2 through 9 each has ten customers, whereas region 1 has x customers. For what values of x does the strategy of locating in region 2 dominate locating in region 1?

X 1	10 2	10 3	10 4	10 5	10 6	10 7	10 8	10 9
--------	---------	---------	---------	---------	---------	---------	---------	---------

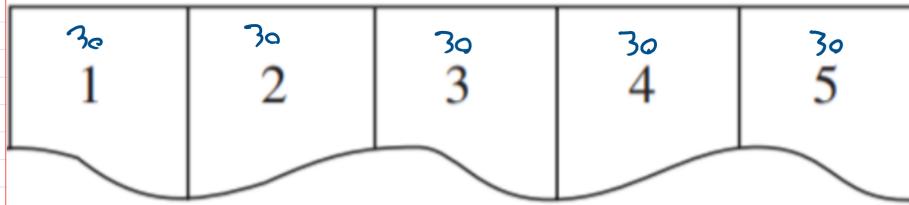
Region 2 only makes sense if $x < p_0$ because at that point region 1 is less than the rest of the grand combined player 2 location

$$\begin{aligned} y_0 &> \frac{x}{2} + 40 \\ 40 &> x/2 \\ y_0 &> x \end{aligned}$$

Passed Solution Review

Consider a location game with five regions on the beach in which a vendor can locate. The regions are arranged on a straight line, as in the original game discussed in the text. Instead of there being two vendors, as with the original game, suppose there are *three* vendors who simultaneously and independently select on which of the five regions to locate. There are thirty consumers in each of the five regions; each consumer will walk to the nearest vendor and purchase a soda, generating a \$1.00 profit for the vendor. Assume that if some consumers are the same distance from the two or three nearest vendors, then these consumers are split equally between these vendors.

- Can you rationalize the strategy of locating in region 1?
- If your answer to part (a) is "yes," describe a belief that makes locating at region 1 a best response. If your answer is "no," find a strategy that strictly dominates playing strategy 1.



$$\text{Max Rev} = 30 \cdot 5 \cdot 1 = 150$$

- It's never the best strategy to locate in region 1 unless it holds value in a different way (ATM, Picnic tables, etc) or if it is guaranteed that all players choose region 1
- No: location 3 would dominate because it is in the middle and can draw both sides
Yes: If there are other amenities that would draw more people to region 1, you would need to get at least 20 extra people

A mix of 2 & 3, $\delta_i(0, P, 1-P, 0, 0)$ dominates 1. Tedious to show

$2 \& 3$ each weakly dominate 1. So, the mix strategy dominates 1.

		3 chooses $s=1$				
		1	2	3	4	5
1	1	50	15	25	40	55
	2	15	30	30	30	30
3	25	30	40	45	45	
4	40	30	45	70	75	
5	55	30	45	75	100	

		3 chooses $s=2$				
		1	2	3	4	5
1	1	120	60	30	45	25
	2	60	50	30	40	55
3	30	30	60	60	60	
4	45	40	60	70	75	
5	75	35	60	75	100	

		3 chooses $s=3$				
		1	2	3	4	5
1	1	100	70	55	45	60
	2	70	90	45	30	45
3	65	45	50	45	55	
4	45	30	40	90	10	
5	60	45	55	90	00	

Passed Solution Review

Consider a game in which, simultaneously, player 1 selects a number $x \in [2, 8]$ and player 2 selects a number $y \in [2, 8]$. The payoffs are given by:

$$\begin{array}{c} u_1(x, y) = 2xy - x^2 \\ u_2(x, y) = 4xy - y^2 \end{array}$$

	$(2, 3)$	$(8, 7)$	
$(2, 3)$	18	32	64
$(8, 7)$	15	60	192
	$(2, 8)$	$(4, 6)$	

Calculate the rationalizable strategy profiles for this game.

$$\begin{aligned} u_1 d_1 &= 2y - 2x = 2(y-x) \\ u_1 d_2 &= -2 \end{aligned}$$

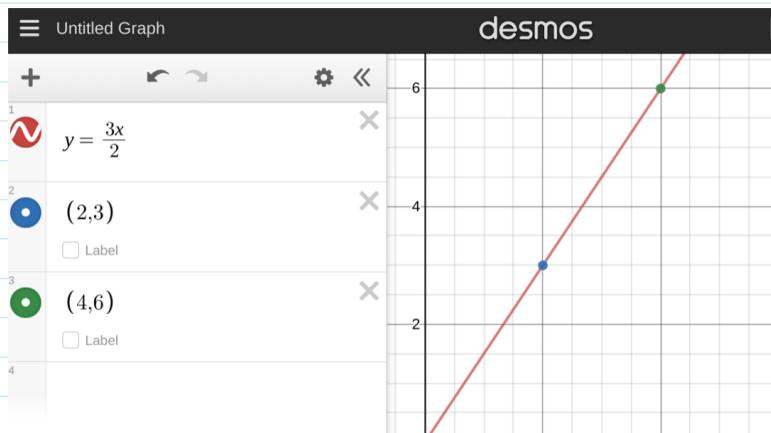
$$\begin{aligned} u_2 d_1 &= 4x - 2y \\ u_2 d_2 &= -2 \end{aligned}$$

$$\begin{aligned} 2 \cdot 8 \cdot 2 - 8^2 &= 4 \cdot 8 - 64 = 32 - 64 = -32 \\ 4 \cdot 8 \cdot 2 - 2^2 &= 64 - 4 = 60 \end{aligned}$$

$$\begin{cases} 2y - 2x = 0 \\ 4x - 2y = 0 \end{cases} \rightarrow y = \frac{3}{2}x$$

$\hookrightarrow x = 2, y = 3$ or $x = 4, y = 6$ are the only whole numbers between 2 and 8

But if x and y are both 8, the payoffs are $u_1 = 64$ and $u_2 = 192$



$$\frac{1}{2}[(2xy - x^2) + (4xy - y^2) + \frac{1}{4}(2xy - x^2)(4xy - y^2)] - \frac{1}{2}(2xy - x^2)^2$$

$$\begin{aligned} \frac{du_1}{dx} &= 2y - 2x = 0 \rightarrow BR_1(y) = \bar{y} \\ \frac{du_2}{dy} &= 4x - 2y = 0 \rightarrow BR_2(x) = \begin{cases} 2\bar{x} & \bar{x} \leq 4 \\ \bar{x} & \bar{x} > 4 \end{cases} \end{aligned}$$

Suppose $\bar{x} = 2$, then $y = 4$

$y = 4$, then $x = 4$

$\bar{x} = 4$, then $y = 8$

$y = 8$, $x = 8$

$\rightarrow (4, 4)$ is the only rationalizable strategy profile

Passed Solution review

8. Finish the analysis of the “social unrest” model by showing that for any $\alpha > 2$, the only rationalizable strategy profile is for all players to protest. Here is a helpful general step: Suppose that it is common knowledge that all players above y will protest, so $x \geq 1 - y$. Find a cutoff player number $f(y)$ with the property that given $x \geq 1 - y$, every player above $f(y)$ strictly prefers to protest.

$$\begin{array}{ll} \text{Stay home} & \text{Protest} \\ \downarrow & \downarrow \\ U_i(H, x) = 4x - 2 & U_i(P, x) = 8x - 4 + \alpha_i \end{array}$$

$i \in [y, 1]$

$$\begin{aligned} 8(1-y) - 4 + \alpha_i &> 4(1-y) - 2 \\ 8 - 8y - 4 + \alpha_i &> 4 - 4y - 2 \\ 4 - 8y + \alpha_i &> 2 - 4y \\ 2 - 8y + \alpha_i &> -4y \\ 2 + \alpha_i &> 4y \\ \alpha_i &> 4y - 2 \\ i &> (4y - 2)/\alpha \end{aligned}$$

If $\alpha = 2$ and $y \geq 1$, people will protest.

∴ Protests if $8x - 4 + \alpha_i > 4x - 2$
 $\alpha_i > 2 - 4x$

Suppose $\epsilon(x) = \bar{x} = 0$

This becomes $i > 2/\alpha$

If $x > 2/\alpha \leq 1$ so someone will protest even if $\bar{x} = 0$

Therefore $\bar{x} = 0$ is not rationalizable

Suppose $\bar{x} = 1 - 2/\alpha$ which is the least rationalizable belief in R'

Now ∴ protests if $\alpha_i > 2 - 4(1 - 2/\alpha)$
 $\alpha_i > -2 + 8/\alpha$
 $i > -2/\alpha + 8/\alpha^2$

The least rationalizable belief is now

$$\begin{aligned} \bar{x} &= 1 + 2/\alpha - 8/\alpha^2 \\ &= 1 - 2/\alpha(4/\alpha - 1) > 1 - 2/\alpha \end{aligned}$$

Each round \bar{x} grows until
 $\alpha(1-\bar{x}) = 2 - 4\bar{x}$

$$\bar{x} = \frac{\alpha - 2}{\alpha - 4} > 1 \rightarrow \text{so } \bar{x} = 1$$

Nash Equilibrium

Rationalizability means:

- 1) Players form beliefs about each other's behavior
- 2) Players best respond to their beliefs
- 3) these facts are common knowledge among players

Behavior is **congruous** (coordinated) through social norms

Congruity:

- 1) games are repeated in society and player behavior "settles down" and the same strategies are repeated
- 2) players meet before the game + agree on the strategies used, players honor the agreement
- 3) outside mediator recommends strategy profiles, each player expects others to follow the recommendation + has incentive to do so themselves

Nash equilibrium: strategy profile $s \in S$ is a **Nash equilibrium** if and only if $s_i \in BR_i(s_{-i})$ for each player i . That is, $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$ for every $s'_i \in S_i$ and each player i .

Strict Nash equilibrium: strategy profile s is strict if and only if $\{s_i\} = BR_i(s_{-i})$ for each player i .

To find: "Look for profiles such that each player's strategy is a best response to the strategy of the others"

Each Nash equilibrium is a rationalizable strategy

Equilibrium of the Partner Game

Coordination + Social Welfare

Nash equilibria don't always entail strategies that are preferred by the players as a group

↳ Nash of Prisoner's Dilemma (1,1) is inefficient

The third strategic tension

Coordinated inefficiency \rightarrow QWERTY, VHS, etc.

Congruous Sets

Nash is only when players coordinate on a single profile

Set X is a congruous set because coordinating on X is consistent with common knowledge of BR

Congruous if X contains exactly those strategies that can be rationalized

weakly congruous if each strategy in X can be rationalized with respect to X_{-i}

Experimental Game Theory

Strategic sophistication

9.1 Nash Equilibrium - General

Wednesday, September 23, 2020 11:54 AM

Solution Concepts So Far

- 1) Strongly dominant strategies
- 2) Weakly dominant strategies
- 3) Set of rationalizable strategies
↳ what if there's more than 1?
- 4) Nash Equilibrium
↳ $s_i^* \in S_i$ is a NE if $s_i^* \in BR_i(s_{-i}^*)$ & if
 $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i^*, s_{-i}) \forall i$

Concurrent

	β_1	β_2
α_1	3, 1	0, 3
α_2	0, 6	0, 0
α_3	7, 2	1, 4

Best replies given other player's strategy

(α_1, β_2) is only pure strategy NE of game

- 1) Repeated observed play across society
- 2) Players talk before playing + honoring agreement

9.2 Nash Equilibrium - Examples

Wednesday, September 23, 2020 12:20 PM

A\B	Rat	Silent
Rat	0, 0	3, -1
Silent	-1, 3	2, 2
(Rat, Rat)		

A\B	H	T
H	1, -1	-1, 1
T	-1, 1	0, 0
No Pure NE		

A\B	Game	Show
Game	2, 3	0, 1
Show	1, 0	3, 2
2 Pure NE		

$$\hookrightarrow \{0, 5\} \times \{0, 5\}$$

A\B	Stag	Hare
Stag	4, 4	0, 1
Hare	1, 0	1, 1
2 Pure NE		

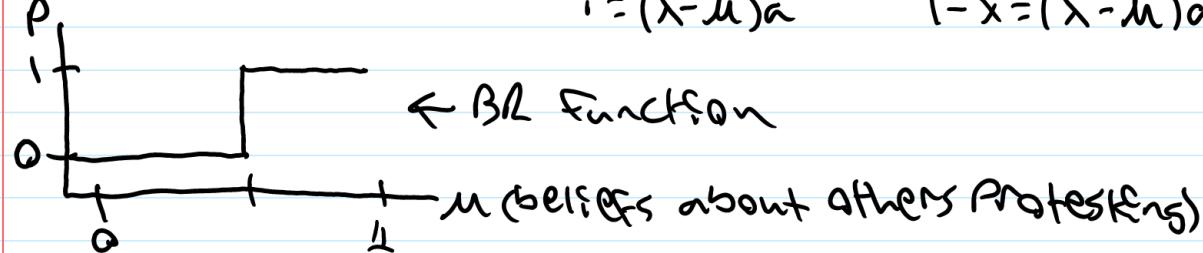
3rd Strategic tension - Possibility of socially inefficient coordination

9.3 Nash Equilibrium - Social Unrest

Friday, September 25, 2020 7:17 PM

$$i > (\lambda - \mu)(\delta + \theta/\alpha) \Leftarrow \text{protest equation}$$

↗ βR $P=1 : i > (\lambda - \mu)\alpha$
 ↘ $P=0 : i < (\lambda - \mu)\alpha$
 ← $1-x \rightarrow$ = indifferent to protest if
 $i = (\lambda - \mu)\alpha$ $i = (\lambda - \mu)\alpha$ or
 $1-x = (\lambda - \mu)\alpha$



Fraction that turns out $1-x = (\lambda - \mu)\alpha \rightarrow \mu = (1-\alpha)x/(1-\alpha)$
 where $\lambda < 1 \rightarrow \mu > 1$

if $\lambda \leq 1 \rightarrow \mu = 1 \quad x = 1 \rightarrow$ everyone protests

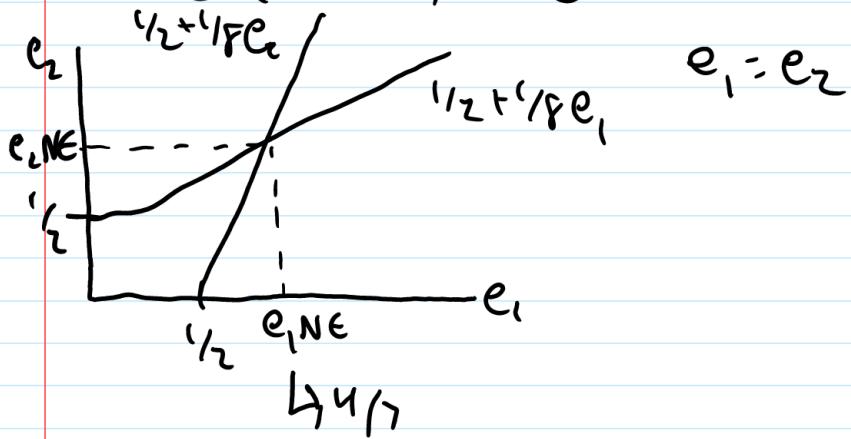
9.4 Nash Equilibrium - Partnership

Friday, September 25, 2020 7:26 PM

$$U_1 = \frac{1}{2}(e_1 + e_2 + e_1 e_2) - \frac{1}{2}e_1^2$$

$$\frac{\partial U_1}{\partial e_1} = \frac{1}{2}(1 + \frac{1}{4}\bar{e}_2) - e_1 = 0 \quad BR_1(\bar{e}_2) = \frac{1}{2} + \frac{1}{8}\bar{e}_2 \leftarrow 2 \text{ equations} - 2 \text{ unknowns}$$

Same for Player 2



$$e_1 = e_2$$

ΔU_1