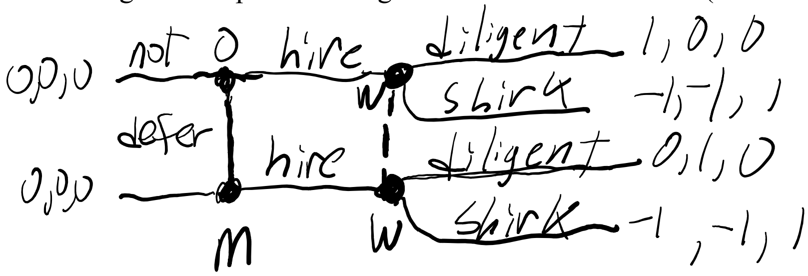
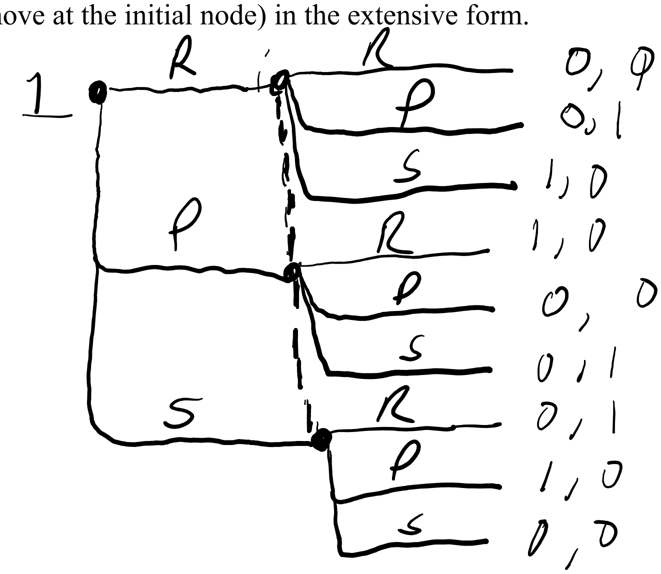
Game Theory Solutions – Ch 2 – Ch 6

2.2. Consider the following strategic situation concerning the owner of a firm (O), the manager of the firm (M), and a potential worker (W). The owner first decides whether to hire the worker, to refuse to hire the worker, or to let the manager make the decision. If the owner lets the manager make the decision, then the manager must choose between hiring the worker or not hiring the worker. If the worker is hired, then he or she chooses between working diligently and shirking. Assume that the worker does not know whether he or she was hired by the manager or the owner when he or she makes this decision. If the worker is not hired, then all three players get a payoff of 0. If the worker is hired and shirks, then the owner and manager each get a payoff of −1, whereas the worker gets 1. If the worker is hired by the owner and works diligently, then the owner gets a payoff of 1, the manager gets 0, and the worker gets 0. If the worker is hired by the manager and works diligently, then the owner gets 0, the manager gets 1, and the worker gets 1. Represent this game in the extensive form (draw the game tree).

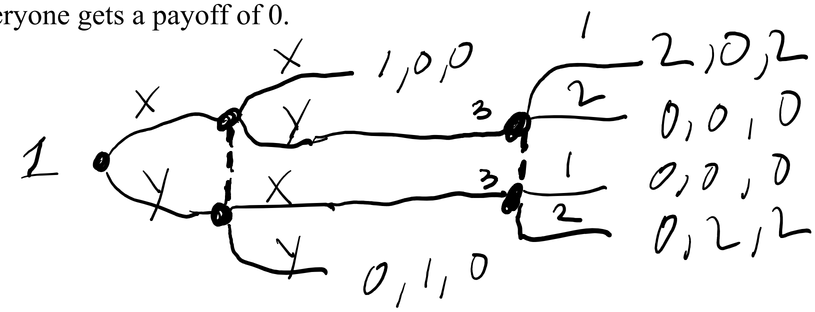


2.4. The following game is routinely played by youngsters—and adults as well—throughout the world. Two players simultaneously throw their right arms up and down to the count of "one, two, three." (Nothing strategic happens as they do this.) On the count of three, each player quickly forms his or her hand into the shape of either a rock, a piece of paper, or a pair of scissors. Abbreviate these shapes as R, P, and S, respectively. The players make this choice at the same time. If the players pick the same shape, then the game ends in a tie. Otherwise, one of the players wins and the other loses. The winner is determined by the following rule: rock beats scissors, scissors beats paper, and paper beats rock. Each player obtains a payoff of 1 if he or she wins, −1 if he or she loses, and 0 if he or she ties. Represent this game in the extensive form. Also discuss the relevance of the order of play (which of the players has the move at the initial node) in the extensive form.



If the game was sequential, the second mover would always win.

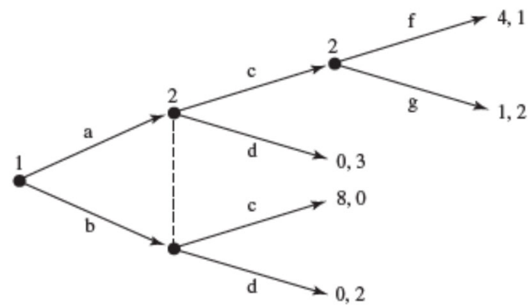
2.6 Represent the following game in the extensive form. There are three players, numbered 1, 2, and 3. At the beginning of the game, players 1 and 2 simultaneously make decisions, each choosing between “X” and “Y.” If they both choose “X,” then the game ends and the payoff vector is (1, 0, 0); that is, player 1 gets 1, player 2 gets 0, and player 3 gets 0. If they both choose “Y,” then the game ends and the payoff vector is (0, 1, 0); that is, player 2 gets 1 and the other players get 0. If one player chooses “X” while the other chooses “Y,” then player 3 must guess which of the players selected “X”; that is, player 3 must choose between “1” and “2.” Player 3 makes his selection knowing only that the game did not end after the choices of players 1 and 2. If player 3 guesses correctly, then he and the player who selected “X” each obtains a payoff of 2, and the player who selected “Y” gets 0. If player 3 guesses incorrectly, then everyone gets a payoff of 0.



3.2 Suppose a manager and a worker interact as follows. The manager decides whether to hire or not hire the worker. If the manager does not hire the worker, then the game ends. When hired, the worker chooses to exert either high effort or low effort. On observing the worker’s effort, the manager chooses to retain or fire the worker. In this game, does “not hire” describe a strategy for the manager? Explain.

No: A strategy must specify what to choose in all possible information sets, wheter or not they will be reached if that strategy is played with fidelity.

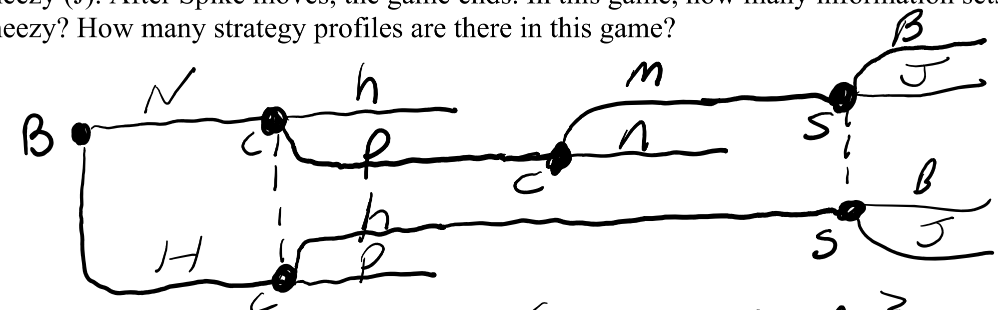
3.4 In the extensive-form game that follows, how many strategies does player 2 have?



cf, cg, df dg

Four.

3.8 Consider the following strategic setting involving a cat named Baker, a mouse named Cheezy, and a dog named Spike. Baker's objective is to catch Cheezy while avoiding Spike; Cheezy wants to tease Baker but avoid getting caught; Spike wants to rest and is unhappy when he is disturbed. In the morning, Baker and Cheezy simultaneously decide what activity to engage in. Baker can either nap (N) or hunt (H), where hunting involves moving Spike's bone. Cheezy can either hide (h) or play (p). If nap and hide are chosen, then the game ends. The game also will end immediately if hunt and play are chosen, in which case Baker captures Cheezy. On the other hand, if nap and play are chosen, then Cheezy observes that Baker is napping and must decide whether to move Spike's bone (m) or not (n). If he chooses to not move the bone, then the game ends. Finally, in the event that Spike's bone was moved (either by Baker choosing to hunt or by Cheezy moving it later), then Spike learns that his bone was moved but does not observe who moved it; in this contingency, Spike must choose whether to punish Baker (B) or punish Cheezy (J). After Spike moves, the game ends. In this game, how many information sets are there for Cheezy? How many strategy profiles are there in this game?



$\{N, H\} \times \{B, J\} \times \{hm, hn, pm, pn\}$

$2 \times 2 \times 4$

Cheesy has 2 information sets.
There are 16 strategy profiles.

4.2 Suppose we have a game where $S_1 = \{H, L\}$ and $S_2 = \{X, Y\}$. If player 1 plays H, then her payoff is z regardless of player 2's choice of strategy; player 1's other payoff numbers are $u_1(L, X) = 0$ and $u_1(L, Y) = 10$. You may choose any payoff numbers you like for player 2 because we will only be concerned with player l's payoff.

(a) Draw the normal form of this game.

1 \ 2	X	Y
H	Z,	Z,
L	0,	10,

(b) If player 1's belief is $\theta_2 = (1/2, 1/2)$, what is player 1's expected payoff of playing H? What is his expected payoff of playing L? For what value of z is player 1 indifferent between playing H and L?

$u_1(H|\theta_2) = Z \qquad U_1(L|\theta_2) = 1/2 \cdot 0 + 1/2 \cdot 10 = 5$

Indifferent if $Z = 5$

(c) Suppose $\theta_2 = (1/3, 2/3)$. Find player l's expected payoff of playing L.

$u_1(L|\theta_2) = 1/3 \cdot 0 + 2/3 \cdot 10 = 20/3$

4.3 Evaluate the following payoffs for the game pictured here:

1 \ 2	I	O
OA	2, 2	2, 2
OB	2, 2	2, 2
IA	4, 2	1, 3
IB	3, 4	1, 3

(a) $u_1(\sigma_1, I)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$

$1/4 \cdot 2 + 1/4 \cdot 2 + 1/4 \cdot 4 + 1/4 \cdot 3 = 11/4$

(b) $u_2(\sigma_1, O)$ for $\sigma_1 = (1/8, 1/4, 1/4, 3/8)$

$1/8 \cdot 2 + 1/4 \cdot 2 + 1/4 \cdot 3 + 3/8 \cdot 3 = 21/8$

(c) $u_1(\sigma_1, \sigma_2)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$, $\sigma_2 = (1/3, 2/3)$

$1/4(1/3 \cdot 2 + 2/3 \cdot 2) + 1/4(1/3 \cdot 2 + 2/3 \cdot 2) + 1/4(1/3 \cdot 4 + \frac{2}{3} \cdot 1) + 1/4(\frac{1}{3} \cdot 3 + \frac{2}{3} \cdot 1)$

$= 23/12$

(d) $u_1(\sigma_1, \sigma_2)$ for $\sigma_1 = (0, 1/3, 1/6, 1/2)$, $\sigma_2 = (2/3, 1/3)$

$0 + 1/3(2/3 \cdot 2 + 1/3 \cdot 2) + 1/6(2/3 \cdot 4 + 1/3 \cdot 1) + \frac{1}{2}(2/3 \cdot 3 + 1/3 \cdot 1) = 7/3$

4.4. For each of the classic normal-form games (see Figure 3.4), find $u1(\sigma 1, \sigma 2)$ and $u2(\sigma 1, s2)$ for $\sigma 1 = (1/2, 1/2)$ and $\sigma 2 = (1/2, 1/2)$.

Note: Only do Matching pennies, Battle of the Sexes, and Hawk Dove.

1 \ 2	H	T
H	1, −1	−1, 1
T	−1, 1	1, −1

Matching Pennies

1 \ 2	Opera	Movie
Opera	2, 1	0, 0
Movie	0, 0	1, 2

Battle of the Sexes

1 \ 2	H	D
H	0, 0	3, 1
D	1, 3	2, 2

Hawk-Dove/Chicken

$u_1 = \frac{1}{4}(1-1+1-1) = 0$

$u_2 = \frac{1}{4}(-1+1-1+1) = 0$

$u_1 = \frac{1}{4}(2+0+0+1) = 3/4$

$u_2 = \frac{1}{4}(1+0+0+2) = 3/4$

$u_1 = \frac{1}{4}(0+3+1+2) = 3/2$

$u_2 = \frac{1}{4}(0+1+3+2) = 3/2$

6.2 For the game in Exercise 1 of Chapter 4, determine the following sets of best responses.

1 \ 2	L	C	R
U	10, 0	0, 10	3, 3
M	2, 10	10, 2	6, 4
D	3, 3	4, 6	6, 6

(a) $BR1(\theta 2)$ for $\theta 2 = (1/3, 1/3, 1/3)$

$U_1(U) = 13/3 \quad U_1(M) = 18/3$

$U_1(D) = 13/3 \quad BR(\theta_2)\{M\}$

(b) $BR2(\theta 1)$ for $\theta 1 = (0, 1/3, 2/3)$

$U_2(L) = 16/3 \quad U_2(C) = 14/3 \quad U_2(R) = 16/3$

$BR_2(\theta_1) = \{L, R\}$

(c) $BR1(\theta 2)$ for $\theta 2 = (5/9, 4/9, 0)$

$U_1(U) = 50/9 \quad U_1(M) = 50/9 \quad U_1(R) = 39/9$

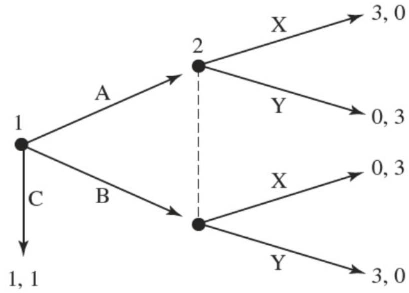
$BR_1(\theta_2) = \{U, M\}$

(d) $BR2(\theta 1)$ for $\theta 1 = (1/3, 1/6, 1/2)$

$U_2(L) = 19/6 \quad U_2(C) = 40/6 \quad U_2(R) = 28/6$

$BR_2(\theta_1) = \{C\}$

6.6 In the game pictured here, is it ever rational for player 1 to select strategy C? Why?



Let p denote player 1's belief about the probability of X.

$u_1(C,p) \geq u_1(A,p)$

$1 \geq 3p + 0(1-p)$

$1/3 \geq p$

$u_1(C,p) \geq u_1(B,p)$

$1 \geq 0p + 3(1-p)$

$1 \geq 3 - 3p$

$p \geq 2/3$

No, because p cannot satisfy both inequalities.

6.7 In the normal-form game pictured below, is player 1's strategy M dominated? If so, describe a strategy that dominates it. If not, describe a belief to which M is a best response.

1 \ 2	X	Y
K	9, 2	1, 0
L	1, 0	6, 1
M	3, 2	4, 2

$\sigma_1 = (1/3, 2/3, 0)$

$u_1(\sigma_1, X) = \frac{1}{3}\cdot 9 + \frac{2}{3}\cdot 1 = 11/3 > 3$

$u_1(\sigma_1, Y) = \frac{1}{3}\cdot 1 + \frac{2}{3}\cdot 6 = 13/3 > 4$

M is Dominated by $\sigma_1 = (1/3, 2/3, 0)$

7.2 Suppose that you manage a firm and are engaged in a dispute with one of your employees. The process of dispute resolution is modeled by the following game, where your employee chooses either to “settle” or to “be tough in negotiation,” and you choose either to “hire an attorney” or to “give in.” In the cells of the matrix, your payoff is listed second; x is a number that both you and the employee know. Under what conditions can you rationalize selection of “give in”? Explain what you must believe for this to be the case.

Employee \ You	Give in	Hire attorney
Settle	1, 2	0, 1
Be tough	3, 0	x, 1

$\theta = (p, 1-p)$ $u_2(G) = 2p$ $u_2(A) = 1$ $p \geq 1/2$ is required for the player 2 to give in.
x must not exceed 0 for this to be true, or "tough" dominates, and you hire the attorney.

7.3 Find the set of rationalizable strategies for the following game. Note that each player has more than one dominated strategy. Discuss why, in the iterative process of deleting dominated strategies, the order in which dominated strategies are deleted does not matter.

1 \ 2	a	b	c	d
w	5, 4	4, 4	4, 5	12, 2
x	3, 7	8, 7	5, 8	10, 6
y	2, 10	7, 6	4, 6	9, 5
z	4, 4	5, 9	4, 10	10, 9

x dominates y

$\frac{2}{3}w + \frac{1}{3}x$ dominates z

c dominates d

$\frac{9}{10}c + \frac{1}{10}a$ dominated b

$R^1 = \{w, x\} \times \{a, c\}$

c dominates a

$R^2 = \{w, x\} \times \{c\}$

x dominates w so $s = (x, c)$

In any given iteration, each player considers all strategies not eliminated in the last iteration. Then, in the next iteration, all strategies eliminated for either player are eliminated. Since all are eliminated between iterations, "order" has little meaning within any given round.

7.4. Imagine that there are three major network-affiliate television stations in Turlock, California: RBC, CBC, and MBC. All three stations have the option of airing the evening network news program live at 6:00 p.m. or in a delayed broadcast at 7:00 p.m. Each station's objective is to maximize its viewing audience in order to maximize its advertising revenue. The following normal-form representation describes the share of Turlock's total population that is "captured" by each station as a function of the times at which the news programs are aired. The stations make their choices simultaneously. The payoffs are listed according to the order RBC, CBC, MBC. Find the set of rationalizable strategies in this game.

MBC: 6:00

RBC \ CBC	6:00	7:00
6:00	14, 24, 32	8, 30, 27
7:00	30, 16, 24	13, 12, 50

MBC: 7:00

RBC \ CBC	6:00	7:00
6:00	16, 24, 30	30, 16, 24
7:00	30, 23, 14	14, 24, 32

6 dominates 7 for MBC

$R^1 = \{6,7\} \times \{6,7\} \times \{6\}$

7 dominates 6 for RBC

$R^2 = \{7\} \times \{6,7\} \times \{6\}$

6 dominates 7 for CBC

$R^3 = \{7\} \times \{6\} \times \{6\}$

$s = (7, 7, 6)$

7.6 Suppose that in some two-player game, s_1 is a rationalizable strategy for player 1. If, in addition, you know that s_1 is a best response to s_2, can you conclude that s_2 is a rationalizable strategy for player 2? Explain.

No. S_1 might be rationalizable because it is a best response to some other strategy, s_2', that is rationalizable, making s_1 rationalizable even if s_2 is not.

7.7. Consider a guessing game with ten players, numbered 1 through 10. Simultaneously and independently, the players select integers between 0 and 10. Thus player i's strategy space is $S_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, for $i = 1, 2, \ldots, 10$. The payoffs are determined as follows: First, the average of the players' selections is calculated and denoted a. That is, $a = (s_1 + s_2 + \ldots + s_{10})/10$ where s_i denotes player i's selection, for $i = 1, 2, \ldots, 10$. Then, player i's payoff is given by $u_i = (a - i - 1)s_i$. What is the set of rationalizable strategies for each player in this game?

$a \le 10$, so $a - 10 - 1 \le -1$, so $s_{10} = 0$ dominates.

Now $a \le 9$, so $s_9 = 0$ dominates

Now $a \le 8$, so $s_8 = 0$ dominates

⋮

so $s_1 = 0$ dominates

Now $a \le 1$,

$s = (0, 0, \ldots, 0)$

8.2. Consider a location game with nine regions like the one discussed in this chapter. But instead of having the customers distributed uniformly across the nine regions, suppose that region 1 has a different number of customers than the other regions. Specifically, suppose that regions 2 though 9 each has ten customers, whereas region 1 has *x* customers. For what values of *x* does the strategy of locating in region 2 dominate locating in region 1?

Player 2 Location

Player 1 Location	1	2	3	4	5	6	7	8	9
1	$\frac{x}{2}+40$	x	$x+5$	$x+10$	$x+15$	$x+20$	$x+25$	$x+30$	$x+35$
2	$80.$	$\frac{x}{2}+40$	$x+10$	$x+15$	$x+20$	$x+25$	$x+30$	$x+25$	$x+40$

only player 1's payoffs shown

$$80 > \frac{x}{2} + 40 \qquad x/2 + 40 > x$$

$$40 > x/2 \qquad 40 > x/2$$

$$80 > x \qquad 80 > x$$

8.6. Consider a location game with five regions on the beach in which a vendor can locate. The regions are arranged on a straight line, as in the original game discussed in the text. Instead of there being two vendors, as with the original game, suppose there are *three* vendors who simultaneously and independently select on which of the five regions to locate. There are thirty consumers in each of the five regions; each consumer will walk to the nearest vendor and purchase a soda, generating a $1.00 profit for the vendor. Assume that if some consumers are the same distance from the two or three nearest vendors, then these consumers are split equally between these vendors.

(a) Can you rationalize the strategy of locating in region 1?

(b) If your answer to part (a) is "yes," describe a belief that makes locating at region 1 a best response. If your answer is "no," find a strategy that strictly dominates playing strategy 1.

A mix of 2 & 3, $\sigma_i = (0, p, 1-p, 0, 0)$ dominates 1. It's tedious to show. You must write Player i's payoffs for every possible choice of the opponents. Below I show player 3's payoffs.

2 & 3 each weakly dominate 1. So, the mix strictly dominates 1.

The ties for player 3 are highlighted below.

3 choose s = 1

1 \ 2	1	2	3	4	5
1	50	15	25	40	55
2	15	30	30	30	30
3	25	30	40	45	45
4	40	30	45	70	75
5	55	30	45	75	100

3 choose s = 2

1 \ 2	1	2	3	4	5
1	120	60	30	45	75
2	60	50	30	40	55
3	30	30	60	60	60
4	45	40	60	70	75
5	75	35	60	75	100

3 choose S = 3

1 \ 2	1	2	3	4	5
1	100	90	55	45	60
2	90	90	45	30	45
3	65	45	50	45	55
4	45	30	45	90	70
5	60	45	55	90	00

8.7. Consider a game in which, simultaneously, player 1 selects a number $x \in [2, 8]$ and player 2 selects a number $y \in [2, 8]$. The payoffs are given by:
$u_1(x, y) = 2xy - x^2$
$u_2(x, y) = 4xy - y^2$.
Calculate the rationalizable strategy profiles for this game.

$\partial u_1/\partial x = 2\bar{y} - 2x = 0 \rightarrow BR_1(\bar{y}) = \bar{y}$

$\partial u_2/\partial y = 4\bar{x} - 2y = 0 \rightarrow BR_2(\bar{x}) = \begin{cases} 2\bar{x} & \bar{x} \leq 4 \\ 8 & \bar{x} > 4 \end{cases}$

- Suppose $\bar{x} = 2$, then $y = 4$
- $\bar{y} = 4$, then $x = 4$
- $\bar{x} = 4$, $y = 8$
- $\bar{y} = 8$ $x = 8$

So $s = (8, 8)$ is the only rationalizable strategy profile.

8.8 Finish the analysis of the "social unrest" model by showing that for any $a > 2$, the only rationalizable strategy profile is for all players to protest. Here is a helpful general step: Suppose that is it common knowledge that all players above y will protest, so $x \geq 1 - y$. Find a cutoff player number $f(y)$ with the property that given $x \geq 1 - y$, every player above $f(y)$ strictly prefers to protest.

i protests if

$8x - 4 + \alpha i > 4x - 2$

$\alpha i > 2 - 4x$

Suppose $E(x) = \bar{x} = 0$

This becomes

$i > 2/\alpha$

If $\alpha > 2$, $2/\alpha < 1$

So, some will protest even if $\bar{x} = 0$.

Therefore $\bar{x} = 0$ is not rationalizable.

Suppose now $\bar{x} = 1 - 2/\alpha$, which is the least rationalizable belief in R'.

Now i protests if

$\alpha i > 2 - 4(1 - 2/\alpha)$

$\alpha i > -2 + 8/\alpha$

$i > -2/\alpha + 8/\alpha^2$

The least rationalizable belief is now

$\bar{x} = 1 + 2/\alpha - 8/\alpha^2$

$= 1 - \frac{2}{\alpha}\left(\frac{4}{\alpha} - 1\right) > 1 - 2/\alpha$

This continues, with x growing each round until

$\alpha(1 - x) = 2 - 4x$

$\alpha = 2 + \alpha x - 4x$

$\alpha - 2 = x(\alpha - 4)$

$x = \frac{\alpha - 2}{\alpha - 4} > 1$. So, $x = 1$.

9.2 Find the Nash equilibria of and the set of rationalizable strategies for the games in Exercise 1 at the end of Chapter 6.

1 \ 2	L	R
A	3, 3	2, 0
B	4, 1	8, –1

(a)

1 \ 2	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

(b)

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

1 \ 2	L	R
U	1, 1	0, 0
D	0, 0	5, 5

(d)

a) B dominates A and L dominates R. So (B, L) is the only NE and the only rationalizable strategy profile.

b) L dominates R. Nothing else is dominated, so all else is rationalizable. With R eliminated, U weakly dominates M and D, so (U, L) is a reasonable prediction. Both (U L) and (M, C) are NE, but (M, C) is not a very sensible NE.

c) $\sigma_1 = (2/3, 0, 1/3)$ dominates M, X dominates Z

R^1 U dominates D, X dominates W.

R^2 X dominates Y. (U, X) is the only rationalizable strategy profile, and the only N.E.

d) Any profile is rationalizable. (U, L) & (D, R) are both N.E.

9.4. Compute the Nash equilibria of the following location game. There are two people who simultaneously select numbers between zero and one. Suppose player 1 chooses s_1 and player 2 chooses s_2. If $s_i < s_j$, then player i gets a payoff of $(s_i + s_j)/2$ and player j obtains $1 - (s_i + s_j)/2$, for $i = 1, 2$. If $s_1 = s_2$, then both players get a payoff of 1/2.

If $s_1 < s_2$, u_1 is increasing in s_1, so $BR_1(s_2)$ is never less than s_2.

If $s_1 > s_2$, u_1 is decreasing in s_1, so $BR(s_2)$ is never more than s_2.

Consider $s_1 = s_2 < 1/2$, so that $u_1 = u_2 = 1/2$

$s_1 = s_2 + \varepsilon$ (ε is tiny) $\Rightarrow u_1 \approx 1 - \frac{2s_2}{2} > 1/2$

so $s_1 = s_2 < 1/2$ is not a NE

Consider $s_1 = s_2 > 1/2$, so that $u_1 = u_2 = 1/2$.

$s_1 = s_2 - \varepsilon \Rightarrow u_1 \approx 2s_2/2 > 1/2$.

So $s_1 = s_2 > 1/2$ is not a NE.

Consider $s_1 = s_2 = 1/2$, so that $u_1 = u_2 = 1/2$

Now, $s_1 = s_2 + \varepsilon \Rightarrow u_1 = 1 - \frac{1}{2} - \frac{\varepsilon}{2} < 1/2$

$s_1 = s_2 - \varepsilon \Rightarrow u_1 = 1/2 - \frac{\varepsilon}{2} < 1/2$

So $s_1 = s_2 = 1/2$ is the N.E.

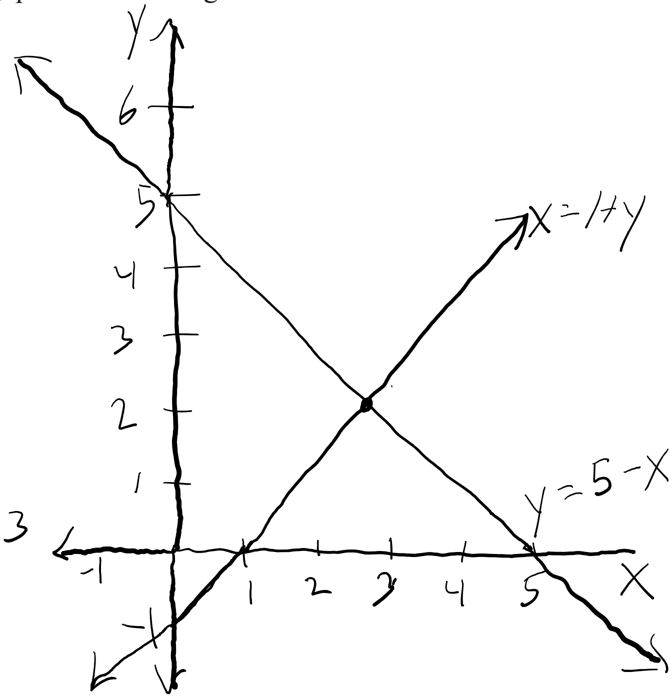
9.6. Consider a game in which, simultaneously, player 1 selects any real number x and player 2 selects any real number y. The payoffs are given by:
$u_1(x, y) = 2x - x^2 + 2xy$ $u_2(x, y) = 10y - 2xy - y^2$.

a. Calculate and graph each player's best-response function as a function of the opposing player's pure strategy.
b. Find and report the Nash equilibria of the game.
c. Determine the rationalizable strategy profiles for this game.

$$\frac{\partial u_1}{\partial x} = 2 - 2x + 2y = 0$$
$$x = 1 + y$$

$$\frac{\partial u_2}{\partial y} = 10 - 2y - 2x = 0$$
$$y = 5 - x$$

$x = 1 + (5 - x)$ $\quad$ $y = 5 - 3$
$2x = 6$ $\quad$ $y = 2$
$x = 3$



Note first there are no known bounds to start from. Setting that aside, suppose x_L is a known lower bound for x. Then y below $5 - x_L$ is eliminated. So, x above $1 + 5 - x_L$ can be eliminated. Then y below $x_L - 1$ is eliminated. Then x below $1 + x_L - 1 = x_L$ is eliminated. This elimination did not alter the lower bound of player 1's rationalizable strategies.

9.10. Is the following statement true or false? If it is true, explain why. If it is false, provide a game that illustrates that it is false. "If a Nash equilibrium is not strict, then it is not efficient."

A \ B	L	R
U	2, 2	0, 2
D	2, 0	1, 1

False. The profile (U, B) is a N.E. but not a strict N.E., and it is efficient.

9.11. This exercise asks you to consider what happens when players choose their actions by a simple rule of thumb instead of by reasoning. Suppose that two players play a specific finite simultaneous-move game many times. The first time the game is played, each player selects a pure strategy at random. If player *i* has m_i strategies, then she plays each strategy s_i with probability $1/m_i$. At all subsequent times at which the game is played, however, each player *i* plays a best response to the pure strategy actually chosen by the other player the *previous* time the game was played. If player *i* has *k* strategies that are best responses, then she randomizes among them, playing each strategy with probability $1/k$.

a. Suppose that the game being played is a prisoners' dilemma. Explain what will happen over time.

In the second round, each plays their dominant strategy, confess, because that is the only best response. This continues forever.

b. Next suppose that the game being played is the battle of the sexes. In the long run, as the game is played over and over, does play always settle down to a Nash equilibrium? Explain.

Not if one of the 2 NE profiles is not played the first time. They forever go where the other was last time. So if they were in different places the first time, they spend forever looking for eachother.

c. What if, by chance, the players happen to play a strict Nash equilibrium the first time they play the game? What will be played in the future? Explain how the assumption of a strict Nash equilibrium, rather than a nonstrict Nash equilibrium, makes a difference here.

The same profile. Because for each player, every other strategy s_i' yields a lower payoff against s_{-i} : $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}) \quad \forall\, s_i' \neq s_i \in S_i$. The strict inequality is what makes it so no one would randomly deviate to a different best response.

d. Suppose that, for the game being played, a particular strategy s_i is not rationalizable. Is it possible that this strategy would be played in the long run? Explain carefully.

No. Only strategies that are best responses to something will be selected by any player.

9.15. Suppose you know the following for a particular three-player game: The space of strategy profiles S is finite. Also, for every $s \in S$, it is the case that $u_2(s)=3u_1(s)$, $u_3(s)=[u_1(s)]^2$, and $u_1(s) \in [0,1]$.

a. Must this game have a Nash equilibrium? Explain your answer.

b. Must this game have an efficient Nash equilibrium? Explain your answer.

c. Suppose that in addition to the information given above, you know that s^* is a Nash equilibrium of the game. Must s^* be an efficient strategy profile? Explain your answer; if you answer "no," then provide a counterexample.

a) Yes. Each player chooses what is best for player 1 given the choices of the other players. The profile that maximizes u_1 is therefore always a NE.

b) The equilibrium in (a) is efficient.

c) If there are only strict N.E., the one in (a) is efficient and the only NE.
There may be inefficient non-strict NE.

	L	R
U	1, 3, 1	0, 0, 0
D	0, 0, 0	0, 0, 0

	L	R
U	0, 0, 0	0, 0, 0
D	0, 0, 0	1/2, 1, 1/4

(D, R, B) is an inefficient N.E.

A B

3