The Extensive Form

Sunday, August 23, 2020 10:36 PM

- Dotted line means player doesn't know where they are in the Game
- Simultaneous first moves doesn't matter who is written first

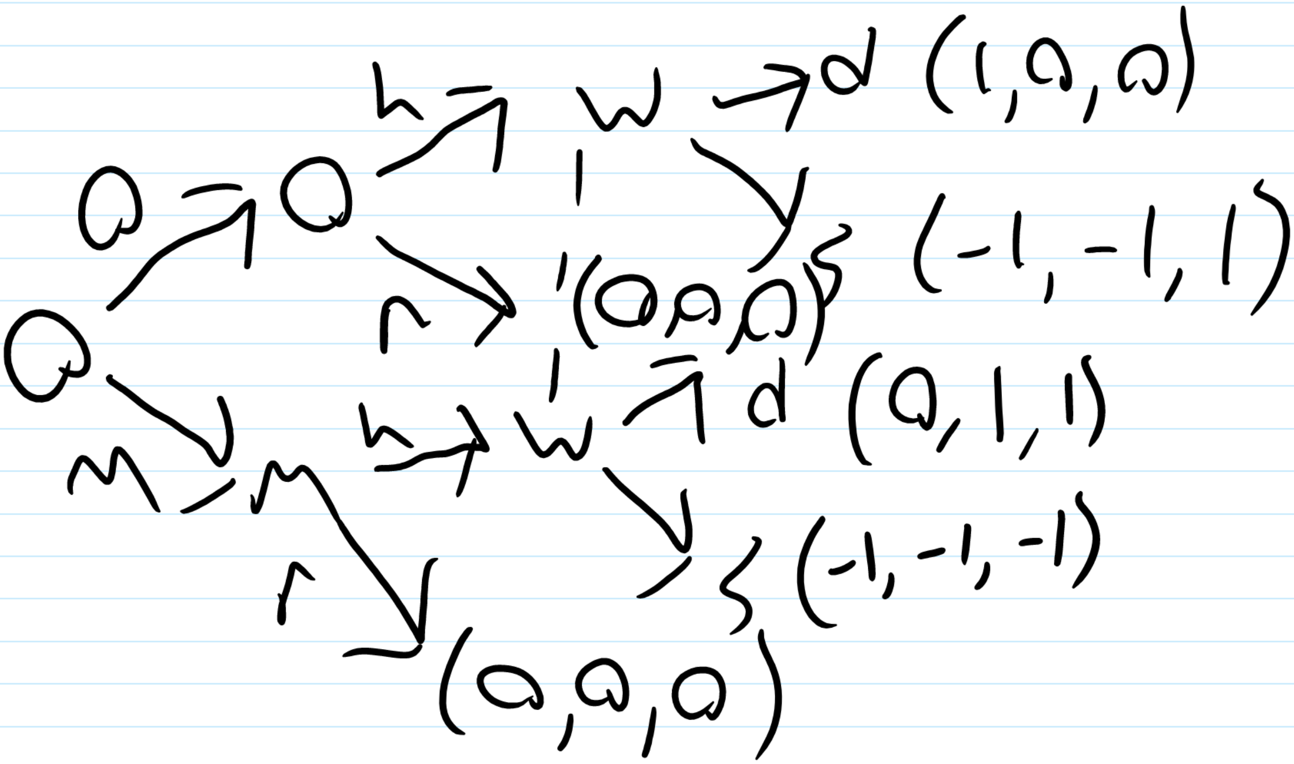
HW 2.2

Tuesday, August 25, 2020 6:05 PM

Passed Solution review

Owner of firm is hiring. They can hire, not hire, or let the manager decide. Manager can hire or not hire. If hired, the worker can work diligently or slack off. Worker doesn't know if O or m hired. Worker not hired, all 0, w hired and shirks, O and m get -1 and w gets 1. If O hired and w is diligent, O gets 1, m and w get 0. If m hired, flip O and m

(O, m, w)

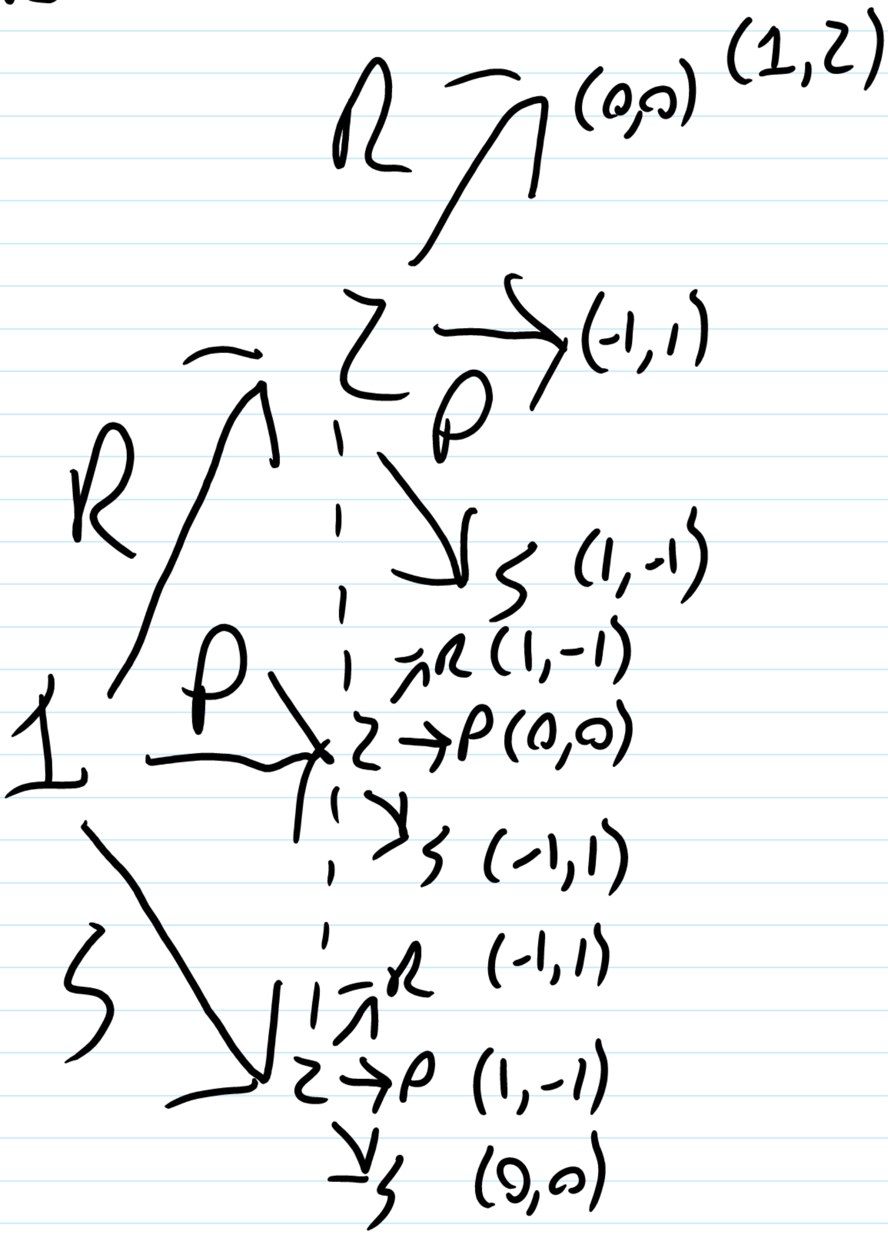


HW 2.4

Tuesday, August 25, 2020 6:19 PM

Passed Solution review

Rock paper scissors!



Because play is simultaneous, player order doesn't matter except for writing payoff orders

If the game was sequential, player 2 would always win

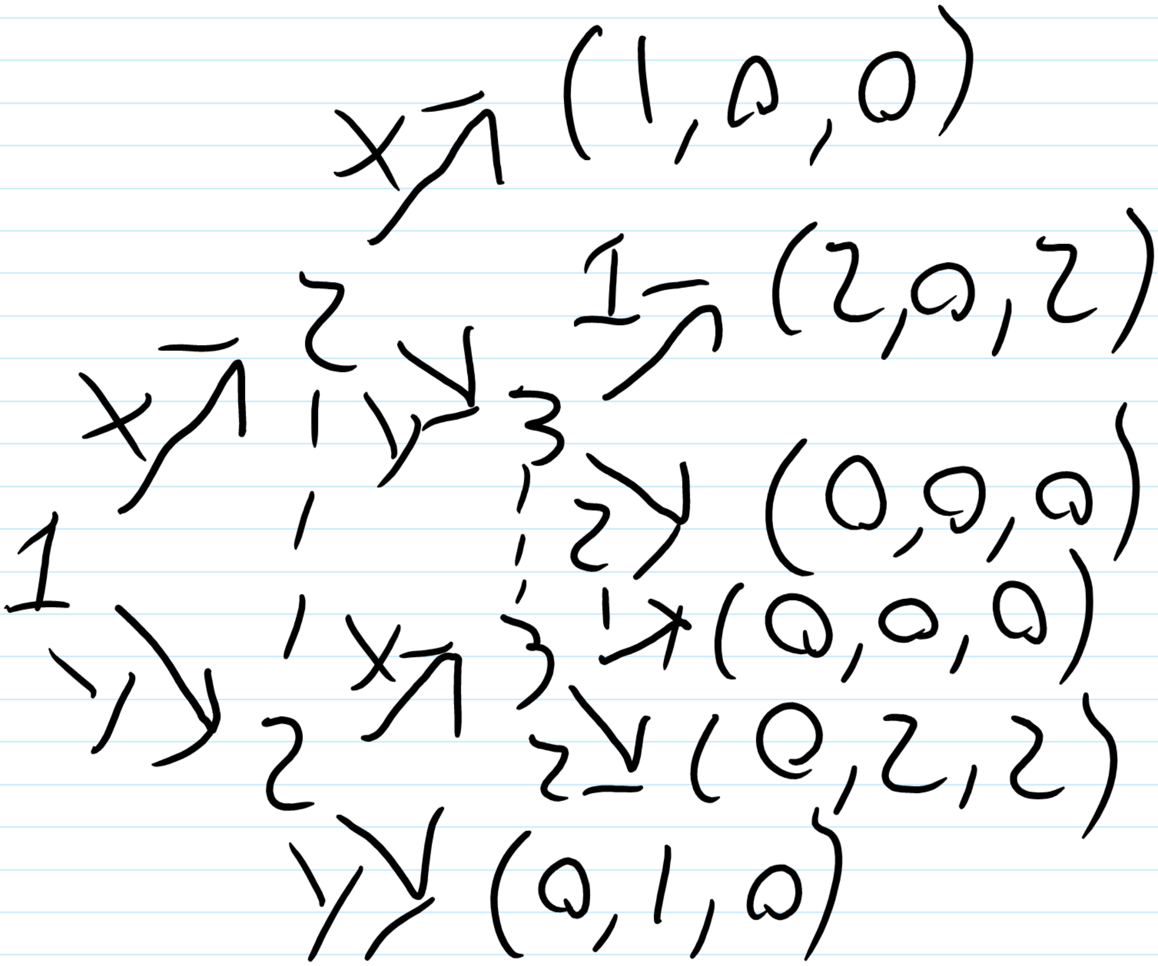
HW 2.6

Tuesday, August 25, 2020 6:25 PM

Passed Solution Review

Players 1,2,3. At first 1 and 2 move simultaneously between X and Y. Game ends if both choose X and Payoff is (1,0,0). If both Y, Game over w/ (0,1,0). Player 3 guesses who said what.

(1,2,3)



3.1 Strategies

Tuesday, August 25, 2020 10:09 AM

$S_2 = \{H, L\}$

$S_c = \{HH', HL', LH', LL'\}$ ← Check it out from previous example on high/low effort

(H, LH') ← Profile

$S = S_2 \times S_c$

↳ cartesian product (matrix multiplication)

$s_i \in S_i \quad S = s_1 \times s_2 \times s_3 \times s_n$ ← All possibilities

S_{-i} is all strategies except player i (think vectors in $\mathbb{R}$

Player i utility $= V_i(s)$ or $V_i(s_i, s_{-i})$

$i = 7$ then $s_{-7} = (s_1 \ldots s_6, s_8 \ldots s_n)$ or $s(s_7, s_{-7})$

Every profile maps to payoffs

$u_i : S \to R$

Normal form?

3.2 Normal Form Representation

Tuesday, August 25, 2020 10:20 AM

Strategic Form Representation

Cheryl + Zi

8 strategy profiles

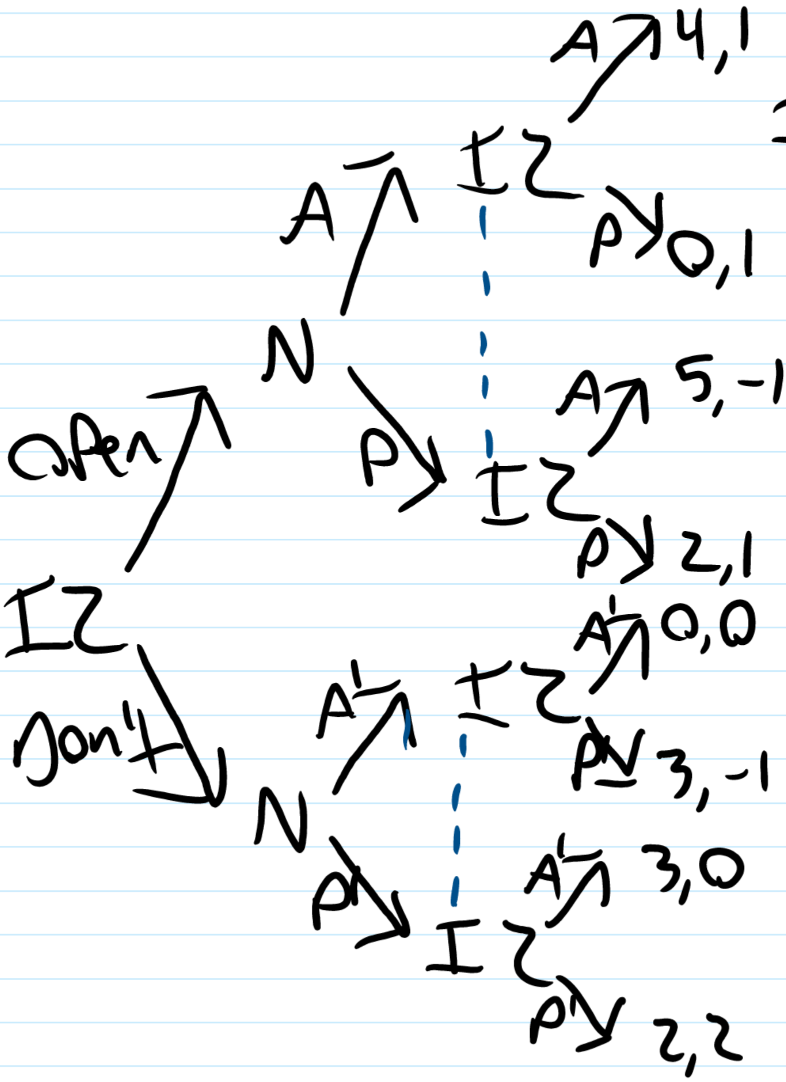
		C: HH'	HL'	LH'	LL'
Z	H	-1,1	-1,1	0,1	1,0
	L	0,1	0,0	0,1	0,0

$x \in R$ $\quad y \in R \quad \rightarrow v_1(x,y) \quad v_2(x,y)$

↳ Real number

3.3 Example Strategies

Tuesday, August 25, 2020 8:52 PM



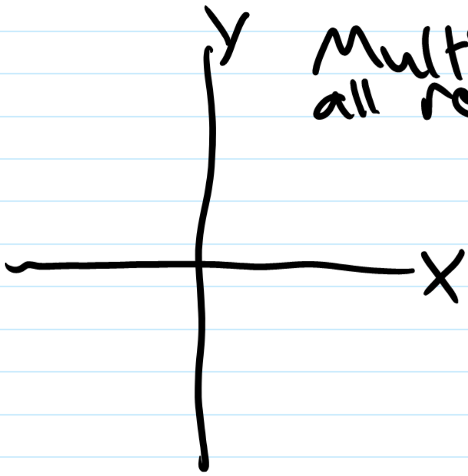
N

IZ:	AA'	AP'	PA'	PP'
OAA'	4,1	4,1		
OPA'				
DAA'			0,0	
DPA'	0,0		0,0	
OAP'	4,1	4,1		
OPP'				
DAP'				
DPP'		2,2		2,2

3 Extra Notes

Wednesday, August 26, 2020 10:07 AM

Cartesian Product



Multiplying x and y sets to get all resulting sets

HW 3.2

Tuesday, August 25, 2020 6:33 PM

Passed Solution review

Manager decides whether or not to hire a worker. If M doesn't hire W game over. If hired W can do high or low effort. Based on effort, M can retain or fire W.

Not hire describes a strategy because not hire is the first move and a terminal point in the game.

No. A strategy must specify what to choose in all possible information sets, whether or not they will be reached if that strategy is played w/ fidelity.

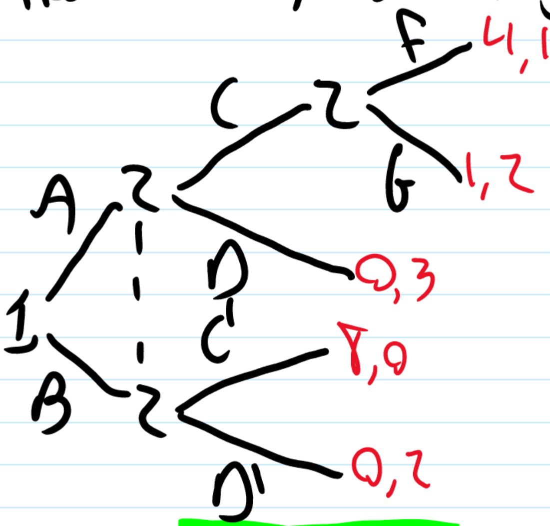
HW 3.4

Sunday, August 30, 2020 6:00 PM

Passed solution review

Worked w/ Hail + Isabel

How many strategies does Player 2 have?



CF CG C'F C'G DF DG D'F D'G

A
B

Player 2 has 4 strategies.

HW 3.8

Sunday, August 30, 2020 6:20 PM

Passed solution review

Worked w/ Hail + Isabel

Cat = Baker Dog = Spike mouse = cheesy

Baker wants to catch cheesy + avoid spike

cheesy wants to tease baker + not get caught

spike wants to rest and not be disturbed

morning: B+C simultaneously decide

B can Nap or Hunt

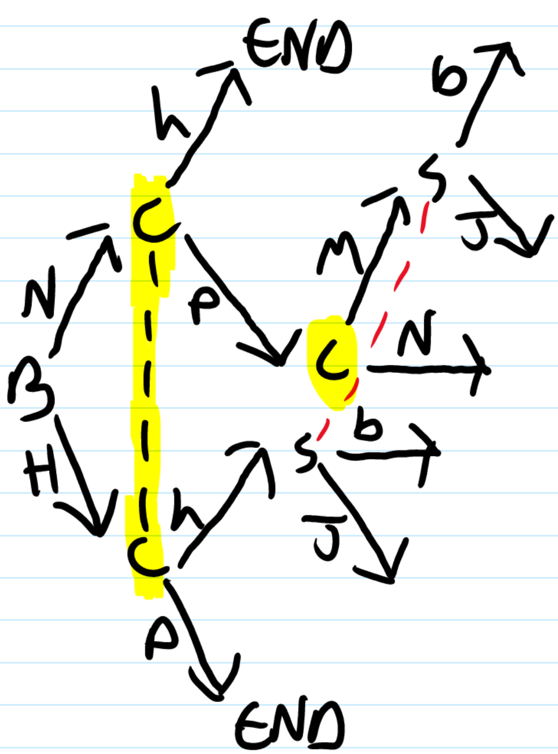
↳ moves S's bone

C can hide or Play

↳ Game ends if nap+hide or hunt+Play

If nap+Play, B must move or Not move the bone

If bone moved, S punishes B or C then game ends



Cheezy has 2 information sets

B: N, H 2x

C: hM, PM, hN, PN 4x

S: b, J 2x

16?

There are 16 strategy profiles

4 Book Notes

Saturday, August 29, 2020 4:22 PM

Beliefs, Mixed Strategies, + expected Payoffs

belief is what a player thinks will happen

Mixed strategy is selecting a strategy according to a probability distribution

regular strategy = pure strategy

Mixed strategy includes pure strategy

Payoff numbers can include preferences of player preferences over probability distributions over outcomes

4.1 Beliefs and Expected Payoffs

Saturday, August 29, 2020 4:35 PM

Beliefs

		Column		
		C1	C2	C3
Row	R_1	4,1	1,1	2,5
	R_2	2,2	0,0	3,3
	R_3	2,5	1,1	1,4
		P_1	P_2	$1-P_1-P_2$
		0.2	0.2	0.6

$\theta_{-R}, \theta_c \rightarrow \theta_{-i} \in \Delta S_{-i}$

Theta not row means theta column...

beliefs are represented by probability distributions

Expected Payoff/utility $\rightarrow E$

U=utility

$\theta_c = (.2, .2, .6)$

$E(u_R \mid S_R = R_2, \theta_c) = .2 \cdot 2 + .2 \cdot 0 + .6 \cdot 3 = 2.2$

$\mid$ = IF, $,$ = and

$\theta_R = (.7, 0, .3) \rightarrow E(U_c \mid S_c = C_1, \theta_R) = .7 \cdot 1 + 0 \cdot 2 + .3 \cdot 5 = .85$

$$U_i(S_i, \theta_{-i}) = \sum_{S_{-i} \in S_{-i}} \theta_{-i}(S_{-i}) \cdot U(S_i, S_{-i})$$

4.2 More on Expected Utility

Saturday, August 29, 2020 4:53 PM

More on expected utility

Ranking = ordinal (can't multiply)

Risk neutral, money outcomes

↳ will take any fair gamble

<u>Risk Aversion</u>

utility function

<u>Von-Neumann Morgenstern Utility function</u>

- Continuity
- More is better
- transitivity
- Independence of irrelevant outcomes

4.3 Mixed Strategies

Saturday, August 29, 2020 5:09 PM

Mixed Strategies

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	0,0	3,3
R_3	2,5	1,1	1,4
	p_1	p_2	$1-p_1-p_2$

$\sigma_i \in \Delta S_i$

sigma

$\theta_C = (.2, .2, .6)$

$\sigma_R (.7, 0, .3)$

↳ NOT θ_R R belief

$$E(U_R)? = \underset{R_1C_1}{[.7 \cdot .2 \cdot 4} + \underset{R_1C_2}{.7 \cdot .2 \cdot 1} + \underset{R_1C_3}{.7 \cdot .6 \cdot 2]} +$$

$$0\,\underset{R_2C_1}{[.2 \cdot 2} + .2 \cdot 0 + .6 \cdot 3] +$$

$$.3[.2 \cdot 2 + .2 \cdot 1 + .6 \cdot 1]$$ = R player expected utility given mixed strategy + their beliefs about C's strategy

$$U_i(\sigma_i, \theta_{-i}) = \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \theta_i(s_i) \cdot \theta_{-i}(s_{-i}) \cdot U_i(s_i, s_{-i})$$

4 Extra Notes

Monday, August 31, 2020 10:05 AM

Mixed strategy is a way to obfuscate your goal

4 Extra Problems

Friday, September 4, 2020 10:08 AM

1 \ 2	L	C	R
U	10,0	0,10	3,3
M	2,10	10,2	6,4
D	3,3	4,6	6,6

d) $u_2(M,R) = 4$

e) $(3 \cdot .25) + (.5 \cdot 6) + (.25 \cdot 6) = 5.25$

g) $(3 \cdot \frac{1}{3}) + (4 \cdot \frac{2}{3}) + (6 \cdot 0) = 3\ \frac{2}{3}$

h)

HW 4.2 Passed Solution review

Saturday, September 5, 2020 1:00 PM

$S_1 = \{H, L\}$ and $S_2 = \{X, Y\}$

If 1 Plays H, Payoff = z. Player 1 Payoff = $V_1(L,X) = 0$
$V_1(L,Y) = 10$

Player 2 Payoff doesn't matter

a) Player 1

	Player 2 X	Y
H	z, 0	z, 1
L	0, 1	10, 0

b) If 1 believes $\theta_2 = (.5, .5)$, Payoff of Playing H? Playing L? Value of z when H = L?

Payoff for H = .5 z
Payoff for L = 10 · .5 = 5
Player 1 is indifferent when z = 5

c) $\theta_2 = (\frac{1}{3}, \frac{2}{3})$. Payoff of Player 1 Playing L?
$= \frac{1}{3} \cdot 0 + 10 \cdot \frac{2}{3} = 0 + 20/3 = 20/3$

HW 4.3

Saturday, September 5, 2020 1:18 PM

Passed Solution review

Worked w/ hail

	I	Q
QA	2,2	2,2
QB	2,2	2,2
IA	4,2	1,3
IB	3,4	1,3

a) $U_1(\sigma_1, I)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$

$1/4(2+2+4+3) = 1/2 + 1/2 + 1 + 3/4 = 11/4$ or 2.75

b) $U_2(\sigma_1, Q)$ for $\sigma_1 = (1/8, 1/4, 1/4, 3/8) = (1/8, 2/8, 2/8, 3/8)$

$2/8 + 4/8 + 6/8 + 9/8 = 21/8$

c) $U_1(\sigma_1, \sigma_2)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$, $\sigma_2 = (1/3, 2/3)$

	I	Q
QA	1/12	2/12
QB	1/12	2/12
IA	1/12	2/12
IB	1/12	2/12

$2/12 + 4/12 + 4/12 + 3/12 + 4/12 + 4/12 + 2/12 + 2/12 = 23/12$

d) $U_1(\sigma_1, \sigma_2)$ for $\sigma_1 = (0, 1/3, 1/6, 1/2)$, $\sigma_2 = (2/3, 1/3)$

	I	Q
QA	0	0
QB	4/18	2/18
IA	2/18	1/18
IB	6/18	3/18

$0 + 8/18 + 8/18 + 18/18 + 0 + 4/18 + 1/18 + 3/18 = 42/18 = 7/3$

HW 4.4

Saturday, September 5, 2020 2:07 PM

Worked w/ hail

Passed Solution review

FIGURE 3.4 Classic normal-form games.

Matching Pennies

1 \ 2	H	T
H	1, −1	−1, 1
T	−1, 1	1, −1

~~**Prisoners' Dilemma**~~

1 \ 2	[illegible]	D
C	[illegible], 2	[illegible], 3
D	3, [illegible]	1, 1

Battle of the Sexes

1 \ 2	Opera	Movie
Opera	2, 1	0, 0
Movie	0, 0	1, 2

Hawk-Dove/Chicken

1 \ 2	H	D
H	0, 0	3, 1
D	1, 3	2, 2

~~**Coordination**~~

1 \ 2	A	B
A	[illegible], 1	0, [illegible]
B	0, 0	1, 1

~~**Pareto Coordination**~~

1 \ 2	A	B
A	2, 2	[illegible]
B	0, [illegible]	1, 1

~~**Pigs**~~

D \ S	P	D
P	4, [illegible]	2, [illegible]
D	6, 1	0, 0

For each game find:

$u_1(\sigma_1, \sigma_2)$ and $u_2(\sigma_1, \sigma_2)$ for $\sigma_1 = (1/2, 1/2)$ and $\sigma_2 = (1/2, 1/2)$

Pennies: $u_1(\sigma_1, \sigma_2) = u_2(\sigma_1, \sigma_2) = .25 - .25 + .25 - .25 = 0$

Sexes: $u_1(\sigma_1, \sigma_2) = u_2(\sigma_1, \sigma_2)$

$= (2 \cdot 1/4) + 0 + 0 + (1 \cdot 1/4) = 3/4$

Chicken: $u_1(\sigma_1, \sigma_2) = u_2(\sigma_1, \sigma_2)$

$= 0 + (1 \cdot 1/4) + (3 \cdot 1/4) + (2 \cdot 1/4) = 6/4 = 1.5$

5 Book Notes

Saturday, August 29, 2020 5:30 PM

General Assumptions + Methodology

Must balance realism w/ managable math

Rationality

Players select path to their preferred outcome

- Maximizing one's payoff

Assume players know how the game is played

Players are rational if they 1. form a belief about strategies and 2. given this belief they maximize their payoff

5 Extra Notes

Wednesday, September 2, 2020 10:06 AM

Prisoner's Dilemma w/ 3 Players

Players R, C, and T

R \ C	R	S
R		
S		

R T

R \ C	R	S
R		
S		

S

6 Book Notes

Monday, August 31, 2020 3:17 PM

Dominance + Best Response

Contracts can bind players to strategies

6.1 Dominance and the Prisoner's Dilemma

Monday, August 31, 2020 2:46 PM

Dominance

Prisoner's Dilemma

		Hail: Silent	Hail: Rat
Gus	Silent	2, 2	-1, 3
	Rat	3, -1	0, 0

Silence is dominated by rat

↳ Always better

Weakly dominated = sometimes better

6.2 Dominance and Efficiency

Monday, August 31, 2020 2:54 PM

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	2,0	3,3
R_3	2,5	1,1	1,4

C_2 Wd by C_1 r_3 Wd by $r_1 + r_2$

C_2 Sd by C_3

Wd / Sd = weakly/strongly dominated

Pure vs weak strategies

$\sigma_R = (.5, .5, 0)$ vC_1 3 > 2
vC_2 1.5 > 1
vC_3 2.5 > 1

$(.5, .5, 0)$ SD r_3

General Notation

S_i is dominated by σ_i if

$V_i(\sigma_i, S_{-i}) > V_i(S_i, S_{-i}) \;\forall\; S_{-i} \in S_{-i}$

$\forall$ → for all

Rational players <u>do NOT</u> play dominated strategies

PARETO EFFICIENCY

$\boxed{\text{Contracting}}$

$V_i(q, S_{-i}) > V_i(S_i, S_{-i}) \;\forall\; S_{-i} \in S_{-i}$
$\& \;\forall\; S_i \in S_{-i} \;\; S_i \neq q$

6.3 Best Responses

Saturday, September 5, 2020 12:00 PM

Best responses or Best replies

	C_1	C_2	C_3
R_1	4, 1	1, 1	2, 5
R_2	2, 2	2, 0	3, 3
R_3	2, 5	1, 1	1, 4

$BR_1(1,0,0) = R_1$

$(0,1,0)$ or $(0,0,1) = R_2$

$BR_2(1,0,0)$ or $(0,1,0) = C_3$

$(0,0,1) = C_1$

Column 2 is never BR because it's dominated

BR is $u_i(s_i, \theta_{-i}) \geq u_i(s_i', \theta_{-i})\ \forall s_i' \in S_i$. If true, strategy s is a BR to θ_{-i}

$B_c = \{C_1, C_3\}$

6.4 6.5 Establishing a Strategy is Never a Best Response

Saturday, September 5, 2020 12:12 PM

Establishing no set of beliefs make s_i a best response

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	2,0	3,3
R_3	2,5	1,1	1,4

$\theta_c(p_1, p_2, 1-p_1-p_2)$

$V_R(R_3, \theta_c) = 2 \cdot p_1 + (1 \cdot p_2) + [1 \cdot (1-p_1-p_2)]$
$= 1 + p_1$

$V_R(R_2, \theta_c) = 2p_1 + 2p_2 + 3(1-p_1-p_2)$
$= 3 - p_1 - p_2$

$V_R(R_3, \theta_c) = 4p_1 + 1p_2 + 2(1-p_1-p_2)$
$= 2 + 2p_1 - p_2$

$V_R(R_3) > V_R(R_2) > 0$

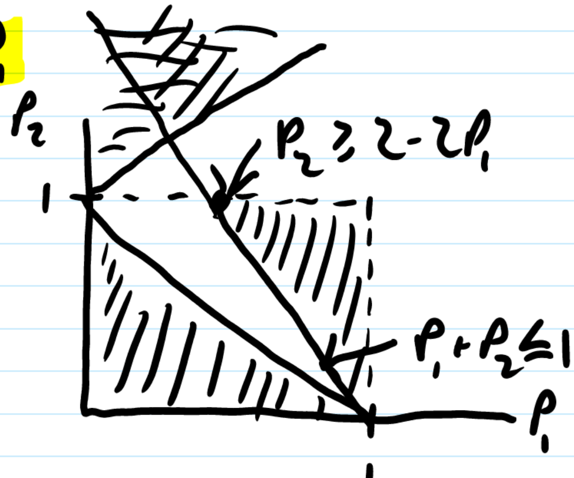
$\hookrightarrow 1 + p_1 - 3 + p_1 + p_2 \geq 0$
$-2 + 2p_1 + p_2 > 0$
$p_2 \geq 2 - 2p_1$

$V_R(R_3) > V_R(R_1) \geq 0$

$\hookrightarrow 1 + p_1 - 2 - 2p_1 + p_2 \geq 0$
$-1 - p_1 + p_2 \geq 0$
$p_2 \geq 1 + p_1$

$p_1 + p_2 \leq 1$

Can't Satisfy all 3!



R_3 is never best!

6.6 Undominated Strategies and Best Responses

Saturday, September 5, 2020 12:32 PM

Beliefs strategic independence

	C_1	C_2	C_3
R_1	4, 1	1, 1	2, 5
R_2	2, 2	2, 0	3, 3
R_3	2, 5	1, 1	1, 4

correlated beliefs

↳ $B_i^c = UD_i$

6.7 Some Last Words on Weak Dominance

Saturday, September 5, 2020 12:49 PM

	C_1	C_2	C_3
R_1	4, 1	1, 1	3, 1
R_2	2, 2	0, 3	3, 2
	p_1	p_2	$1 - p_1 - p_2$

"Fully mixed beliefs"

$B_i^{FC} = WUD_i$

6 Extra Problems

Friday, September 4, 2020 10:39 AM

a) B dom A, L dom R

b) L dom R, L weak dom C?

c) X dom Z, $\frac{2}{3}U + \frac{1}{3}D$ dom M

6.1 Determine which strategies are dominated in the following normal-form games.

(a)

1 \ 2	L	R
A	3, 3	2, 0
B	4, 1	8, –1

(b)

1 \ 2	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

(c)

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

5. Represent in the normal form the rock–paper–scissors game (see Exercise 4 of Chapter 2 to refresh your memory) and determine the following best-response sets.

(a) $BR_1(\theta_2)$ for $\theta_2 = (1, 0, 0)$ $= \{P\}$

(b) $BR_1(\theta_2)$ for $\theta_2 = (1/6, 1/3, 1/2)$ $= \{R, S\}$

(c) $BR_1(\theta_2)$ for $\theta_2 = (1/2, 1/4, 1/4)$ $= \{P, S\}$

(d) $BR_1(\theta_2)$ for $\theta_2 = (1/3, 1/3, 1/3)$ $= \{R, P, S\}$

1 \ 2	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

b) $R = 0 + -\frac{1}{3} + \frac{1}{2} = -\frac{2}{6} + \frac{3}{6} = \frac{1}{6}$

$P = \frac{1}{6} + 0 - \frac{3}{6} = \quad -\frac{2}{6}$

$S = -\frac{1}{6} + \frac{2}{6} + 0 = \quad \frac{1}{6}$

HW 6.2

Saturday, September 5, 2020 2:28 PM

Passed Solution Review

1 \ 2	L	C	R
U	10, 0	0, 10	3, 3
M	2, 10	10, 2	6, 4
D	3, 3	4, 6	6, 6

2. For the game in Exercise 1 of Chapter 4, determine the following sets of best responses.

(a) $BR_1(\theta_2)$ for $\theta_2 = (1/3, 1/3, 1/3)$

(b) $BR_2(\theta_1)$ for $\theta_1 = (0, 1/3, 2/3)$

(c) $BR_1(\theta_2)$ for $\theta_2 = (5/9, 4/9, 0)$

(d) $BR_2(\theta_1)$ for $\theta_1 = (1/3, 1/6, 1/2)$

a) $U = 10/3 + 0 + 3/3 = 13/3$
$M = 2/3 + 10/3 + 6/3 = 18/3$
$D = 3/3 + 4/3 + 6/3 = 13/3$
$\rightarrow BR_1(\theta_2) = \{M\}$

b) $L = 0 + 10/3 + 6/3 = 16/3$
$C = 0 + 2/3 + 12/3 = 14/3$
$R = 0 + 4/3 + 12/3 = 16/3$
$\rightarrow BR_2(\theta_1) = \{L, R\}$

c) $U = 50/9 + 0 + 0 = 50/9$
$M = 10/9 + 40/9 + 0 = 50/9$
$D = 15/9 + 16/9 + 0 = 31/9$
$\rightarrow BR_1(\theta_2) = \{U, M\}$

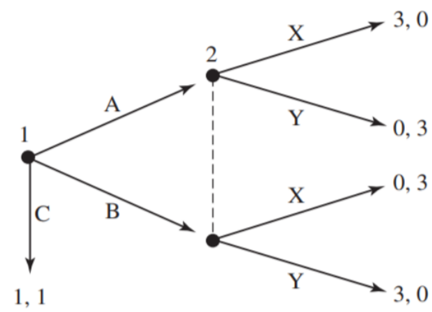
d) $L = 0 + 10/6 + 9/6 = 19/6$
$C = 20/6 + 2/6 + 18/6 = 40/6$
$R = 6/6 + 4/6 + 18/6 = 28/6$
$\rightarrow BR_2(\theta_1) = \{C\}$

HW 6.6

Saturday, September 5, 2020 2:39 PM

Passed Solution review

6. In the game pictured here, is it ever rational for player 1 to select strategy C? Why?



If Player 1 wants a guaranteed payoff, they would choose strategy C.

But, mathematically, they would choose A or B because the expected payoff value is 1.5

AX = BY + AY = BX

By eliminating C, a mixed a+b generates 1.5

Let P denote Player 1's belief about probability of X

$u_1(C,P) \geq u_1(A,P)$
$1 \geq 3P + 0(1-P)$
$1/3 \geq P$

$u_1(C,P) \geq u_1(B,P)$
$1 \geq 0P + 3(1-P)$
$1 \geq 3 - 3P$
$P \geq 2/3$

No because P cannot satisfy both inequalities

Passed Solution review

7. In the normal-form game pictured below, is player 1's strategy M dominated? If so, describe a strategy that dominates it. If not, describe a belief to which M is a best response.

1 \ 2	X	Y
K	9, 2	1, 0
L	1, 0	6, 1
M	3, 2	4, 2

M is dominated by mixed strategy {K, L}

M does not dominate player 1's response because in X, K is better and in Y, L is better.

$$\boxed{\begin{aligned} 9p + 1(1-p) &> 3 \\ 9p + 1 - p &> 3 \\ 8p + 1 &> 3 \\ 8p &> 2 \\ p &> 1/4 \end{aligned}}$$

$p > 5/20$

$$\boxed{\begin{aligned} 1p + 6(1-p) &> 4 \\ p + 6 - 6p &> 4 \\ 6 - 5p &> 4 \\ -5p &> -2 \\ p &< 2/5 \end{aligned}}$$

$p < 8/20$

$$\boxed{\sigma = (\underset{p}{.3}, \underset{1-p}{.7}, \underset{0}{0})}$$

Rationalizability + Iterated dominance

Actions of one player may affect another's payoff

Players can eliminate (think Sudoku) strategies that don't make sense

↳ Iterative removal of (strictly) dominated strategies

↳ Iterative dominance

Strategies that survive iterative dominance are called rationalizable strategies

The Second Strategic Tension

Strategic uncertainty

↳ Coordination problem:

7.1 Iterated Dominance and Rationalizability

Thursday, September 10, 2020 4:19 PM

Bob

Alice	B_1	~~B_2~~	B_3	B_4
A_1	4,1	1,2	2,5	1,4
A_2	2,3	3,1	1,0	1,2
A_3	2,5	1,1	1,4	3,0

B_2 dom by mix $B_1 + B_3$

Eliminate B_2 to create R_1

Alice has no dominated strategies

R_1

Bob

Alice	B_1	~~B_2~~	B_3	B_4
A_1	4,1		2,5	1,4
~~A_2~~	2,3		1,0	1,2
A_3	2,5		1,4	3,0

A_2 dom by $A_1 + A_3$

R_2

Bob

Alice	B_1	~~B_2~~	B_3	~~B_4~~
A_1	4,1		2,5	1,4
~~A_2~~				
A_3	2,5		1,4	3,0

B_4 dom by B_3

R_3

Bob

Alice	B_1	~~B_2~~	B_3	~~B_4~~
A_1	4,1		2,5	
~~A_2~~				
~~A_3~~	2,5		1,4	

A_3 dom by A_1

R_4

Bob

Alice	~~B_1~~	~~B_2~~	B_3	~~B_4~~
A_1	4,1		2,5	
~~A_2~~				
~~A_3~~				

B_1 dom by B_3

R_5

Bob

Alice	~~B_1~~	~~B_2~~	B_3	~~B_4~~
A_1			2,5	
~~A_2~~				
~~A_3~~				

You can eliminate dominated strategies

7.1 Iterated Dominance and Rationalizability

Thursday, September 10, 2020 4:19 PM

Bob

Alice	B_1	$\cancel{B_2}$	B_3	B_4
A_1	4,1	1,2	2,5	1,4
A_2	2,3	3,1	1,0	1,2
A_3	2,5	1,1	1,4	3,0

B_2 dom by mix $B_1 + B_3$

Eliminate B_2 to create R_1

Alice has no dominated strategies

R_1

Bob

Alice	B_1	$\cancel{B_2}$	B_3	B_4
A_1	4,1		2,5	1,4
$\cancel{A_2}$	2,3		1,0	1,2
A_3	2,5		1,4	3,0

A_2 dom by $A_1 + A_3$

R_2

Bob

Alice	B_1	$\cancel{B_2}$	B_3	$\cancel{B_4}$
A_1	4,1		2,5	1,4
$\cancel{A_2}$				
A_3	2,5		1,4	3,0

B_4 dom by B_3

R_3

Bob

Alice	B_1	$\cancel{B_2}$	B_3	$\cancel{B_4}$
A_1	4,1		2,5	
$\cancel{A_2}$				
$\cancel{A_3}$	2,5		1,4	

A_3 dom by A_1

R_4

Bob

Alice	$\cancel{B_1}$	$\cancel{B_2}$	B_3	$\cancel{B_4}$
A_1	4,1		2,5	
$\cancel{A_2}$				
$\cancel{A_3}$				

B_1 dom by B_3

R_5

Bob

Alice	$\cancel{B_1}$	$\cancel{B_2}$	B_3	$\cancel{B_4}$
A_1			2,5	
$\cancel{A_2}$				
$\cancel{A_3}$				

You can eliminate dominated strategies

7.2 Strategic Uncertainty

Thursday, September 10, 2020 4:35 PM

Coordination Problem

↳ 2 people lose each other somewhere + look for each other in different rational places

"Focal Points"

	Rabbit	Stag
Rabbit	1,1	1,0
Stag	0,1	4,4

Determine which strategy profiles are rationalizable for these games.

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

1 \ 2	a	b	c	d
w	5, 4	4, 4	4, 5	12, 2
x	3, 7	8, 7	5, 8	10, 6
y	2, 10	7, 6	4, 6	9, 5
z	4, 4	5, 9	4, 10	10, 9

HW 7.2

Friday, September 18, 2020 9:17 AM

Passed Solution review

Suppose that you manage a firm and are engaged in a dispute with one of your employees. The process of dispute resolution is modeled by the following game, where your employee chooses either to "settle" or to "be tough in negotiation," and you choose either to "hire an attorney" or to "give in."

Employee \ You	Give in	Hire attorney
Settle	1, 2	0, 1
Be tough	3, 0	x, 1

In the cells of the matrix, your payoff is listed second; x is a number that both you and the employee know. Under what conditions can you rationalize selection of "give in"? Explain what you must believe for this to be the case.

If $X \geq 0$, be tough dom settle
If $X < 0$, no dominant strategies
greater than
If p(settle) > .5 and $X < 0$, then it makes sense to choose give in

HW 7.3

Friday, September 18, 2020 12:19 PM

Passed Solution review

Find the set of rationalizable strategies for the following game.

1 \ 2	a	b	c	d
w	5, 4	4, 4	4, 5	12, 2
x	3, 7	8, 7	$\boxed{5, 8}$	10, 6
y	2, 10	7, 6	4, 6	9, 5
z	4, 4	5, 9	4, 10	10, 9

Note that each player has more than one dominated strategy. Discuss why, in the iterative process of deleting dominated strategies, the order in which dominated strategies are deleted does not matter.

$R^0 = \{w, x, y\} \times \{a, b, c\}$

$R^1 = \{w, x\} \times \{c\}$

$R^2 = \{x\} \times \{c\}$

final strategy $= \boxed{\{x\} \times \{c\}}$

The order doesn't matter because a round isn't over until both players have moved.

x dom y

$2/3w + 1/3x$ dom z

c dom d

$9/10c + 1/10a$ dom b

$R^1 = \{w, x\} \times \{a, c\}$

c dom a

$R^2 = \{w, x\} \times \{c\}$

x dom w so $S = (x, c)$

In any iteration, each player considers all strategies not eliminated in the last iteration. Then, in the next iteration, all strategies eliminated for either player are eliminated. Since all are eliminated between iterations, "order" has little meaning within any given round.

HW 7.4

Friday, September 18, 2020 12:25 PM

Passed Solution review

Imagine that there are three major network-affiliate television stations in Turlock, California: RBC, CBC, and MBC. All three stations have the option of airing the evening network news program live at 6:00 P.M. or in a delayed broadcast at 7:00 P.M. Each station's objective is to maximize its viewing audience in order to maximize its advertising revenue. The following normal-form representation describes the share of Turlock's total population that is "captured" by each station as a function of the times at which the news programs are aired. The stations make their choices simultaneously. The payoffs are listed according to the order RBC, CBC, MBC. Find the set of rationalizable strategies in this game.

MBC: 6:00

RBC \ CBC	6:00	7:00
6:00	~~14~~, 24, 32	~~8~~, 30, 27
7:00	30, 16, 24	13, ~~12~~, 50

MBC: 7:00

RBC \ CBC	6:00	7:00
6:00	16, 24, ~~30~~	30, 16, ~~24~~
7:00	30, 23, ~~14~~	14, 24, ~~32~~

$R^0 = \{6, 7\} \times \{6, 7\} \times \{6\text{PM}\}$ $= \{7\text{PM}, 6\text{PM}, 6\text{PM}\}$

$R^1 = \{7\} \times \{6, 7\} \times \{6\}$

$R^2 = \{7\} \times \{6\} \times \{6\}$

Passed Solution Review

Suppose that in some two-player game, s_1 is a rationalizable strategy for player 1. If, in addition, you know that s_1 is a best response to s_2, can you conclude that s_2 is a rationalizable strategy for player 2? Explain.

If s_2 is a best response to the rationalizable s_1,

then s_2 must also be rationalizable.

No. s_1 might be rationalizable because it is a best response to some other strategy, s_2, that is rationalizable, making s_1 rationalizable even if s_2 is not

Passed Solution review

Consider a guessing game with ten players, numbered 1 through 10. Simultaneously and independently, the players select integers between 0 and 10. Thus player i's strategy space is $S_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, for $i = 1, 2, \ldots, 10$. The payoffs are determined as follows: First, the average of the players' selections is calculated and denoted a. That is,

$$a = \frac{s_1 + s_2 + \cdots + s_{10}}{10},$$

where s_i denotes player i's selection, for $i = 1, 2, \ldots, 10$. Then, player i's payoff is given by $u_i = (a - i - 1)s_i$. What is the set of rationalizable strategies for each player in this game?

Max $a = 100/10 = 10$
Min $a = 0/10 = 0$

Max $U = a-10-1 = a-11 \rightarrow$ negative
Min $U = a-1-1 = a-2 \rightarrow$ maybe negative

If everyone chooses 0, final payoff is non-negative

$a \le 10$ so $a-10-1 \le -1$ so $s_{10} = 0$ dominates
Now $a \le 9$ so $s_9 = 0$ dominates
Now $a \le 8$ so $s_8 = 0$ dominates

and so on...

$s = (0, 0, \ldots, 0)$

8 Book Notes

Thursday, September 10, 2020 3:59 PM

Location

Firm location is strategy

P and C sell soda at the beach and simultaneously + independently set up for the day
- 9 regions of = size
- 50 purchases in each region
- customers walk to nearest booth

	C	P
1		
2		
3		
4		
5		
6		
7		
8		
9		

Iterated dominance

$S_i = \{1,2,3,4,5,6,7,8,9\}$ reduced to

↳ $R_i^1 = \{2,3,4,5,6,7,8\}$

↳ $R_i^2 = \{3,4,5,6,7\}$

↳ $R_i^3 = \{4,5,6\} \rightarrow R_i^4 = \{5\}$

Criticisms of location model:
1) in context of market competition, doesn't include firms' specification of prices
2) IRL, agents may not move simultaneously
3) Cannot apply the model with more than 2 products/firms
4) One-dimensional

Strategic Complementarities

Bob increases Alice's payoff but not his
Bob + Alice working together increases either or both

Contracts about effort can't necessarily be made

Complementarity

Rationalizability leads to a unique predictions in 2 player games w/ 3 properties:
1) Strategy spaces are intervals w/ lower + upper bounds
2) there are strategic complementarities
3) the slope of the best response functions is <1

↳ these are not required

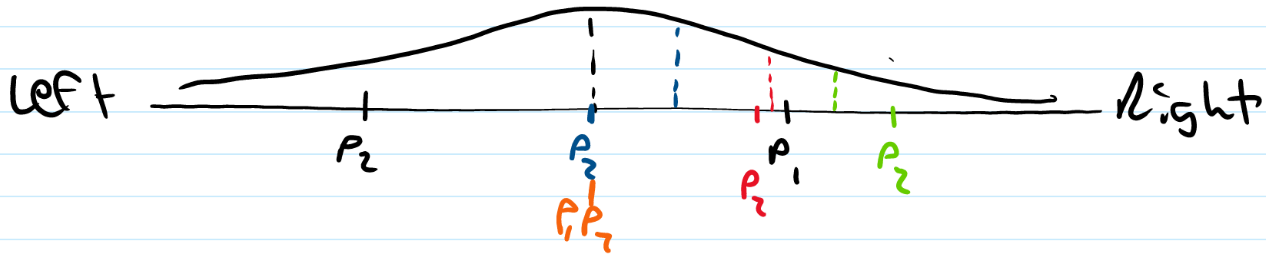
Social Unrest

Strength in numbers

8.1 Location - Median Voter Theorem - Informal

Thursday, September 10, 2020 6:47 PM

Median voter theorem



8.2 Location - Median Voter Theorem - Formal

Thursday, September 10, 2020 6:57 PM

Location -> Median Voter Theorem
-Rationalizable platforms

A + B, max share

FL L CL C CR R FR

A chooses FL, B chooses FL VA = VB = .5
B chooses L VA = 1/7, VB = 6/7
A chooses L, B chooses FL VA = , VB = 1/7
L = .5
CL = 11/14
A chooses CL, B chooses FL = 3/14
L = 2/7
CL = 1/2

FL is dom for B
FL is dom for A -> and also FR

IF there are even positions, any combo of 2 middle are rationalizable

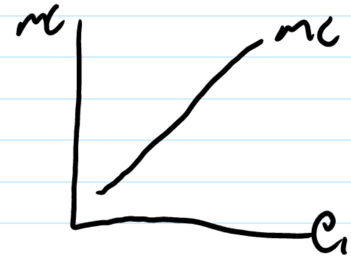
Partnership

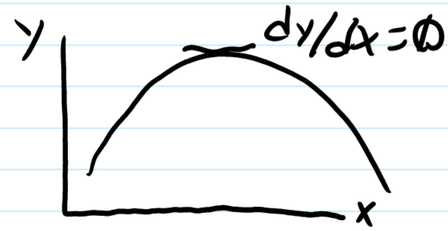
Value $= V(e_1, e_2) = e_1 + e_2$

effort

$C_1(e_1) = \frac{1}{2}e_1^2 \qquad C_2(e_2) = \frac{1}{2}(e_2)^2$

$\frac{dc}{de_1} = e_1$





each keeps $\frac{1}{2}V$

$\pi_1 = \frac{1}{2}V - c_1(e_1)$

$= \frac{1}{2}e_1 + \frac{1}{2}e_2 - \frac{1}{2}e_1^2$

$\frac{d\pi_1}{de_1} = \frac{1}{2} - 2(\frac{1}{2}) \cdot e_1^{(2-1)}$

$= \frac{1}{2} - e_1 = 0$

$e_1 = 1/2$

Same for P_2, $e_2 = 1/2$

$\pi_1 = \frac{1}{2}(\frac{1}{2} + \frac{1}{2}) - \frac{1}{2} \cdot \frac{1}{2}^2$

$= \frac{1}{2}(1) - \frac{1}{2}(\frac{1}{4})$

$= \frac{1}{2} - \frac{1}{8}$

$= 3/8$

$\pi_2 = 3/8 \qquad \rightarrow \quad \pi = \pi_1 + \pi_2 = 3/4$

$\pi = e_1 + e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$

$d\pi/de_1 = 1 - e_1 = 0 \quad \rightarrow e_1 = 1$

$d\pi/de_2 = 1 - e_2 = 0 \quad \rightarrow e_2 = 1$

$\pi = 2 - \frac{1}{2}\cdot 1^2 - \frac{1}{2}\cdot 1^2 = 1$

profit

If independent...

$\pi = e_1 - \frac{1}{2}e_1^2 \qquad d\pi/de_1 = 1 - e_1 = 0 \rightarrow e_1 = 1$

same for P_2

8.4 Partnership - Inseparable Activities - Best Responses

Tuesday, September 15, 2020 10:24 PM

$V = e_1 + e_2 + \frac{1}{4} e_1 e_2$

$c_1 = \frac{1}{2} e_1^2 \qquad c_2 = \frac{1}{2} e_2^2$

$\pi_1 = \frac{1}{2}(e_1 + e_2 + \frac{1}{4} e_1 e_2) - \frac{1}{2} e_1^2$

$\downarrow = \frac{1}{2}(e_1 + \bar{e}_2 + \frac{1}{4} e_1 \bar{e}_2) - \frac{1}{2} e_1^2$

$\sum_i P(X = X_i) \cdot X_i = E(X)$

$\int_0^\infty \Theta_2(v) \cdot v \, dv = \underset{\uparrow}{E}(e_2) = \bar{e}_2$

from P_1 view

$\pi_1 = \frac{1}{2} e_1 + \frac{1}{2} \bar{e}_2 + \frac{1}{8} e_1 \bar{e}_2 - \frac{1}{2} e_1^2$

$\frac{d\pi_1}{de_1} = \frac{1}{2} + \frac{1}{8} \bar{e}_2 - e_1 = 0 \rightarrow e_1 = \frac{1}{2} + \frac{1}{8} \bar{e}_2$

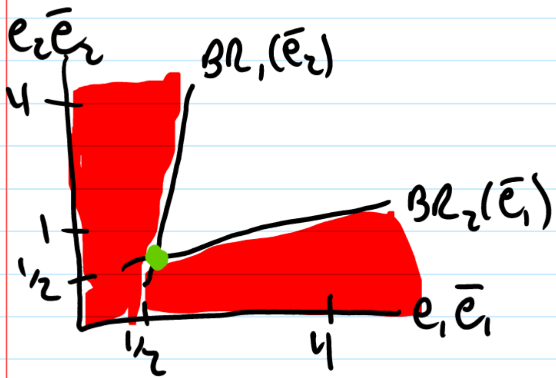
$e_2 = \frac{1}{2} + \frac{1}{8} \bar{e}_1$

$e_1 < \frac{1}{2}$ is dom $e_2 < 1/2$ is dom

$\to 4$ as is > 1 $e_1 \le 4$

R^1 $e_1 < 9/16$ $e_2 < 9/16$ is dom $e_2 \le 4$

$e_1 < \frac{1}{2} + \frac{1}{8}(9/16)$ is dom $\to$ same for e_2 $\to$ $> \frac{1}{2} + (\frac{1}{8} \cdot \frac{5}{8})$ is dom



$e_1 = \frac{1}{2} + \frac{1}{8} e_2$

$e_2 = \frac{1}{2} + \frac{1}{8} e_1$

$e = \frac{1}{2} + \frac{1}{8} e_1 \to \frac{7}{8} e_1 = 1/2 \to e_1 = 4/7 = e_2$

$e_2 = \frac{1}{2} + \frac{1}{8} \bar{e}_1 < LL$ $e_2 > LL$ when $\frac{1}{2} + \frac{1}{8} LL > LL \to 4/7 > LL$

Lower limit

$4/7 < UL$

upper limit

I found a stylus that works

I found a stylus that works

8.7 8.8 Inefficiency in the Partner Game

Saturday, September 19, 2020 2:56 PM

A

$\pi = e_1 + e_2 + \frac{1}{4}e_1e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$

$\frac{d\pi}{de_1} = 1 + \frac{1}{4}e_2 - e_1 = 0$

$e_1 = 1 + 1/4\, e_2$

↳ same for e_2 (but flipped)

$e_1 = 1 + 1/4\, e_1$

$e_1 = 4/3 = e_2$

$\pi = 4/3 + 4/3 + 1/4 \cdot 4/3 \cdot 4/3 - \frac{1}{2}(4/3)^2 - 1/2(4/3)^2 = 4/3$

$\pi_1 = \pi_2 = 2/3$

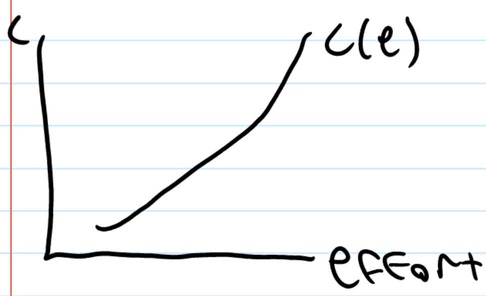
B

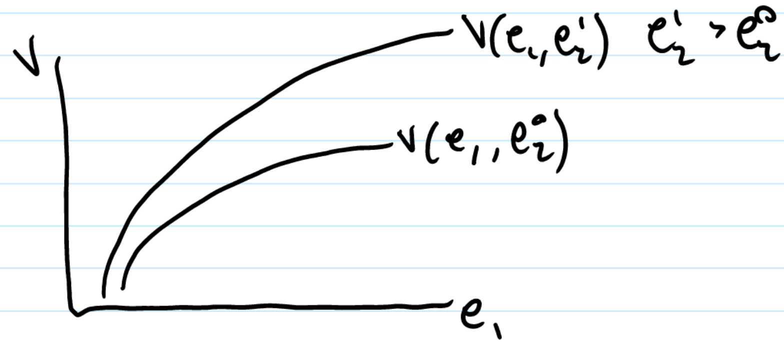
$\pi_1 = 1/2(4/7 + 4/7 + \frac{1}{4}(4/7 \cdot 4/7)) - \frac{1}{2}(4/7)^2 = 22/49$

↳ $< 2/3$

8.9 Joint Production and the Theory of the Firm

Saturday, September 19, 2020 3:08 PM





$V(e_1, e_2) - c_1(e_1) - c_2(e_2)$

$dV/de_1 - dc_1/de_1 = 0$

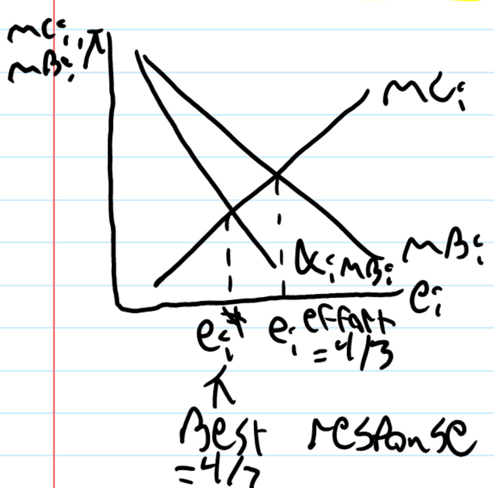
$MB_1 = MC_1$

$MB_2 = MC_2$

$\alpha_i V(e_i, e_{-i}) - c_i(e_i)$

$d\pi/de_i = \alpha_i \, dV/de_i - dc/de_i = 0$

$\alpha_i MB_i = MC_i$ $\rightarrow \alpha < 1$



Residual Claimant

8.10 8.11 Social Unrest - Elements of the Model ~ alpha is 0

Saturday, September 19, 2020 3:58 PM

- everyone prefers unidirectional reform
- $i \in [0,1]$ index propensity to protest
  - expressing voice
  - maybe not peacefully

  uniformly distributed
- Payoffs αi if protest, $\alpha > 0$
  - let x = fraction that protests
  - $\mu = E(x)$, beliefs
  - everyone gains $\beta(x - \lambda)$
  - Protesters gain $\delta(x - \lambda)$
  - non protestors gain $\gamma(\lambda - x)$

$U_i(P, x) = \alpha_i + \beta(x - \lambda) + \delta(x - \lambda) \rightarrow$ Protest

$U_i(H, x) = \gamma(\lambda - x) + \beta(x - \lambda) \longrightarrow$ Stay home

$U_i(P, \mu) > U_i(H, \mu)$

$\alpha_i + \delta(\mu - \lambda) > \gamma(\lambda - \mu)$

$\alpha_i > (\gamma + \delta)(\lambda - \mu)$

$i > (\gamma + \delta)/\alpha \cdot (\lambda - \mu)$

$\rightarrow \alpha = 0$

$0 > (\gamma + \delta)(\lambda - \mu)$

suppose $\lambda > 1 \Rightarrow \mu = 0$

now $\lambda\downarrow$ $\lambda < 1$

History + current events coordinate

i protests if $i > (\delta + \gamma)/\alpha \cdot (\lambda - \mu)$

$a = (\delta + \gamma)/\alpha$

8.12 8.13 Analysis when Protest or Voice has value - alpha gt 0

Saturday, September 19, 2020 4:19 PM

$a = \frac{(\delta + \gamma)}{\alpha}$

assume $\lambda << 1$

↳ if λ big enough, most zealous won't protest

$1 \cdot \alpha > (\delta + \gamma)(1 - 0)$

$\alpha > (\delta + \gamma)$

$a < 1$

What beliefs are rationalizable in this world?

$\mu = 1$ $\quad i > (\lambda - \mu)a < 0 \quad 0 < a < 1, \lambda << 1$

$\mu = 0$ $\quad i > a\lambda$

$1 > a\lambda > 0$

$\mu = 1 - a\lambda \quad i > a\lambda - a\mu$

$> a\lambda - a(1 - a\lambda)$

$x = 1 - a\lambda + a(1 - a\lambda) = (1 - a\lambda)(1 + a) > (1 - a\lambda)$

$\mu = (1 - a\lambda)(1 + a) \quad i > a\lambda(1 - a\lambda)(1 + a)$

$x = 1 - a\lambda + a(1 - a\lambda)(1 + a)$

$= (1 - a\lambda)(1 + a + a^2)$

$\mu = (1 + a\lambda)(1 + a + a^2 + a^3 + \ldots)$

$a < 1$

8.14 Aside on the Geometric Series

Saturday, September 19, 2020 4:35 PM

Infinite sum of a^t, $0 < a < 1$

$A = a^0 + a^1 + a^2 + \dots a^T + a^{T+1}$

$a^0 \downarrow 1$

$aA = a^1 + a^2 + \dots a^{T+1}$

$A - aA = 1 - a^{T+1}$

$A(1-a) = 1 - a^{T+1}$

$A = (1-a^{T+1})(1/1-a)$

$\lim A_{T \to \infty}$ $a^{T+1} \to 0$ $A = 1/1-a$

8.15 Social Unrest - Conclusion

Saturday, September 19, 2020 4:45 PM

$\mu = (1-a\lambda)(1+a+a^2+\dots)$ $\lambda < 1$ $a\lambda < a$

$= (1-a\lambda)/(1-a) > 1$

$(\delta+\gamma) < \alpha \leftrightarrow$ even if $\lambda = 1$, $\varsigma_i = 1$ protests

$\varsigma_i = 1 - \epsilon$ protests

tiny number

HW 8.2

Sunday, September 20, 2020 3:57 PM

Passed Solution Review

Consider a location game with nine regions like the one discussed in this chapter. But instead of having the customers distributed uniformly across the nine regions, suppose that region 1 has a different number of customers than the other regions. Specifically, suppose that regions 2 though 9 each has ten customers, whereas region 1 has x customers. For what values of x does the strategy of locating in region 2 dominate locating in region 1?

x	10	10	10	10	10	10	10	10
1	2	3	4	5	6	7	8	9

Region 2 only makes sense if $x < 80$ because at that point region 1 is less than the rest of the board combined

Player 2 location

P₁ Location	1	2	3	4	5	6	7	8	9
1	$\frac{x}{2}+40$	x	$x+5$	$x+10$	$x+15$	$x+20$	$x+25$	$x+30$	$x+35$
2	80	$\frac{x}{2}+40$	$x+10$	$x+15$	$x+20$	$x+25$	$x+30$	$x+35$	$x+40$
...									

only P₁'s payoffs

$$80 > \frac{x}{2}+40$$
$$40 > x/2$$
$$80 > x$$

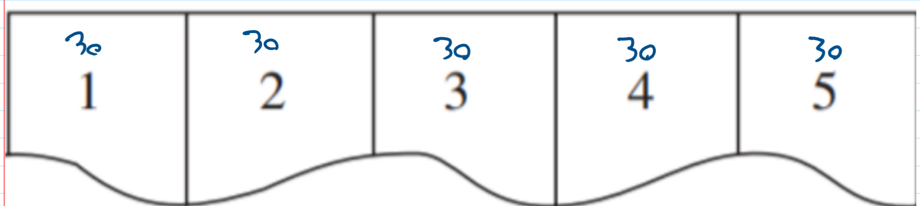
$$x/2+40 > x$$
$$40 > x/2$$
$$80 > x$$

Passed Solution review

Consider a location game with five regions on the beach in which a vendor can locate. The regions are arranged on a straight line, as in the original game discussed in the text. Instead of there being two vendors, as with the original game, suppose there are *three* vendors who simultaneously and independently select on which of the five regions to locate. There are thirty consumers in each of the five regions; each consumer will walk to the nearest vendor and purchase a soda, generating a \$1.00 profit for the vendor. Assume that if some consumers are the same distance from the two or three nearest vendors, then these consumers are split equally between these vendors.

(a) Can you rationalize the strategy of locating in region 1?

(b) If your answer to part (a) is "yes," describe a belief that makes locating at region 1 a best response. If your answer is "no," find a strategy that strictly dominates playing strategy 1.



max rev = 30 · 5 · 1 = 150

a) It's never the best strategy to locate in region 1 unless it holds value in a different way (ATM, picnic tables, etc) or if it is guaranteed that all players choose region 1

b) No: location 3 would dominate because it is in the middle and can draw both sides
Yes: If there are other amenities that would draw more people to region 1, you would need to get at least 20 extra people

A mix of 2 & 3, σ: (0, p, 1-p, 0, 0) dominates 1. Tedious to show

2 & 3 each weakly dominate 1. So, the mix strategy dominates 1.

3 chose s = 1

1 \ 2	1	2	3	4	5
1	50	15	25	40	55
2	15	30	30	30	30
3	25	30	40	45	45
4	40	30	45	70	75
5	55	30	45	75	100

3 chose s = 2

1 \ 2	1	2	3	4	5
1	120	60	30	45	75
2	60	50	30	40	55
3	30	30	60	60	60
4	45	40	60	70	75
5	75	35	60	75	100

0

3 chose s = 3

1 \ 2	1	2	3	4	5
1	100	90	55	45	60
2	90	90	45	30	45
3	55	45	50	45	55
4	45	30	45	90	70
5	60	45	55	90	00

HW 8.7

Sunday, September 20, 2020 4:41 PM

Passed Solution review

Consider a game in which, simultaneously, player 1 selects a number $x \in [2, 8]$ and player 2 selects a number $y \in [2, 8]$. The payoffs are given by:

(2,3) (8,8)

$$u_1(x, y) = 2xy - x^2 \quad 28 \quad 8 \quad 32 \quad 64$$

$$u_2(x, y) = 4xy - y^2. \quad 0 \quad 15 \quad 60 \quad 192$$

(2,8) (4,6)

Calculate the rationalizable strategy profiles for this game.

$u_1 \; d1 = 2y - 2x = 2(y-x)$ $\qquad$ $u_2 \; d1 = 4x - 2y$

$u_1 \; d2 = -2$ $\qquad$ $u_2 \; d2 = -2$

$2 \cdot 8 \cdot 2 - 8^2 = 4 \cdot 8 - 64 = 32 - 64 = -32$

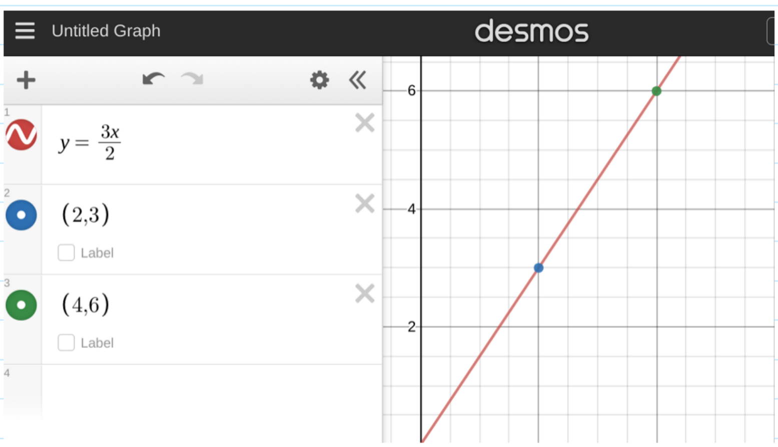
$4 \cdot 8 \cdot 2 - 2^2 = 64 - 4 = 60$

$2y - 2x = 0$

$4x - 2y = 0 \rightarrow y = \frac{3}{2}x$

↳ $x = 2, y = 3$ or $x = 4, y = 6$ are the only whole numbers between 2 and 8

But if x and y are both 8, the payoffs are $u_1 = 64$ and $u_2 = 192$



$$\frac{1}{2}\left[(2xy - x^2) + (4xy - y^2) + \frac{1}{4}(2xy - x^2)(4xy - y^2)\right] - \frac{1}{2}(2xy - x^2)^2$$

$du_1/dx = 2\bar{y} - 2x = 0 \rightarrow BR_1(\bar{y}) = \bar{y}$

$du_2/dx = 4\bar{x} - 2y = 0 \rightarrow BR_2(\bar{x}) = \begin{cases} 2\bar{x} & \bar{x} \leq 4 \\ 8 & \bar{x} > 4 \end{cases}$

Suppose $\bar{x} = 2$ then $y = 4$

$\bar{y} = 4$, then $x = 4$

$\bar{x} = 4$, then $y = 8$

$\bar{y} = 8$, $x = 8$ $\rightarrow$ $S = (8,8)$ is the only rationalizable strategy profile

Passed solution review

8. Finish the analysis of the "social unrest" model by showing that for any $\alpha > 2$, the only rationalizable strategy profile is for all players to protest. Here is a helpful general step: Suppose that is it common knowledge that all players above y will protest, so $x \geq 1 - y$. Find a cutoff player number $f(y)$ with the property that given $x \geq 1 - y$, every player above $f(y)$ strictly prefers to protest.

Stay home $\quad$ Protest

$u_i(H, x) = 4x - 2 \qquad u_i(P, x) = 8x - 4 + \alpha i$

$i \in (y, 1]$

$8(1-y) - 4 + \alpha i > 4(1-y) - 2$
$8 - 8y - 4 + \alpha i > 4 - 4y - 2$
$4 - 8y + \alpha i > 2 - 4y$
$2 - 8y + \alpha i > -4y$
$2 + \alpha i > 4y$
$\alpha i > 4y - 2$
$i > (4y-2)/\alpha$

if $\alpha = 2$ and $y \geq 1$, people will protest.

i protests if $8x - 4 + \alpha i > 4x - 2$
$\alpha i > 2 - 4x$

suppose $E(x) = \bar{x} = 0$
this becomes $i > 2/\alpha$
if $\alpha > 2$ $2/\alpha < 1$ so someone will protest even if $\bar{x} = 0$
therefore $\bar{x} = 0$ is not rationalizable

suppose $\bar{x} = 1 - 2/\alpha$ which is the least rationalizable belief in R'
Now i protests if $\alpha i > 2 - 4(1 - 2/\alpha)$
$\alpha i > -2 + 8/\alpha$
$i > -2/\alpha + 8/\alpha^2$

the least rationalizable belief is now
$\bar{x} = 1 + 2/\alpha - 8/\alpha^2$
$= 1 - 2/\alpha(4/\alpha - 1) > 1 - 2/\alpha$

each round x grows until
$\alpha(1 - x) = 2 - 4x$

$\therefore x = \alpha - 2/\alpha - 4 > 1 \rightarrow$ so $x = 1$

9 Book Notes

Wednesday, September 23, 2020 9:04 AM

Nash Equilibrium

Rationalizability means:
1) Players form beliefs about each other's behavior
2) Players best respond to their beliefs
3) these facts are common knowledge among players

Behavior is congruous (coordinated) through social norms

Congruity:
1) games are repeated in society and player behavior "settles down" and the same strategies are repeated
2) Players meet before the game + agree on the strategies used. Players honor the agreement
3) Outside mediator recommends strategy profiles. Each player expects others to follow the recommendation + has incentive to do so themselves

Nash equilibrium: strategy profile $s \in S$ is a Nash equilibrium if and only if $s_i \in BR_i(s_{-i})$ for each player i. That is, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for every $s_i' \in S_i$ and each player i

Strict Nash equilibrium: strategy profile s is strict if and only if $\{s_i\} = BR_i(s_{-i})$ for each player i

To find: "Look for profiles such that each player's strategy is a best response to the strategy of the other"

Each Nash equilibrium is a rationalizable strategy

Equilibrium of the Partner Game

Coordination + Social Welfare

Nash equilibria don't always entail strategies that are preferred by the players as a group

↳ Nash of prisoner's dilemma (1,1) is inefficient

The third strategic tension

Coordinated inefficiency → QWERTY, VHS, etc.

Congruous Sets

Nash is only when players coordinate on a single profile

Set X is a congruous set because coordinating on X is consistent with common knowledge of BR

Congruous if X_i contains exactly those strategies that can be rationalized

Weakly congruous if each strategy in X_i can be rationalized with respect to X_{-i}

Experimental Game Theory

Strategic sophistication

Solution Concepts so far...
1) Strongly dominant strategies
2) Weakly dominant strategies
3) Set of rationalizable strategies
↳ what if there's more than 1?
4) Nash Equilibrium
↳ $s \in S$ is a NE if $s_i \in BR_i(s_{-i})\ \forall i$
↳ or $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})\ \forall i$

Congruent

	B_1	B_2	B_3
A_1	$\boxed{3,1}$	2,3	4,0
A_2	0,6	0,0	2,0
A_3	5,2	1,4	$\boxed{3,1}$

Best replies given other player's strategy

(A_1, B_2) is only pure strategy NE in game

1) Repeated observed play across society
2) Players talk before playing + honoring agreement

9.2 Nash Equilibrium - Examples

Wednesday, September 23, 2020 12:20 PM

A\B	Rat	Silent
Rat	0*, 0*	3*, -1
Silent	-1, 3*	2, 2

(Rat, Rat)

A\B	H	T
H	1*, -1	-1, 1*
T	-1, 1*	1*, -1

No Pure NE

A\B	Game	Show
Game	2, 3	0, 1
Show	1, 0	3, 2

2 Pure NE

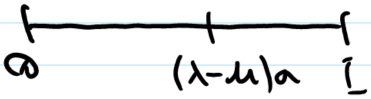
↳ $\{G, S\} \times \{G, S\}$

A\B	Stag	Hare
Stag	4, 4	0, 1
Hare	1, 0	1, 1

2 Pure NE

3rd strategic tension - Possibility of socially inefficient coordination

$i > (\lambda-\mu)(6+\delta/\alpha)$ ← Protest equation

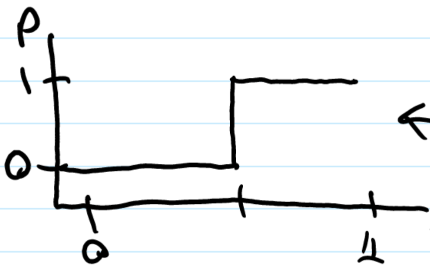


BR $P=1$ $i>(\lambda-\mu)a$

$P=0$ $i<(\lambda-\mu)a$

$\vdash$ 1-x $\dashv$ = indifferent to protest if $i=(\lambda-\mu)a$ or

$i=(\lambda-\mu)a$ $1-x=(\lambda-\mu)a$



← BR function

μ (beliefs about others protesting)

Fraction that turns out $1-x=(\lambda-\mu)a \rightarrow \mu=(1-a\lambda)/(1-a)$

where $\lambda<1 \rightarrow \mu>1$

if $\lambda\leq 1 \rightarrow \mu=1$ $x=1$ → everyone protests

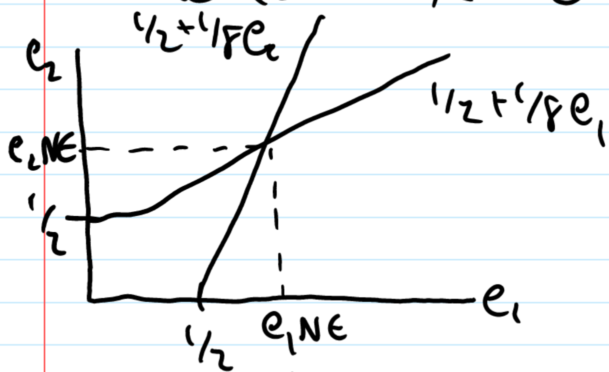
9.4 Nash Equilibrium - Partnership

Friday, September 25, 2020 7:26 PM

$u_1 = \frac{1}{2}(e_1 + e_2 + e_1 e_2) - \frac{1}{2}e_1^2$

$\frac{du_1}{de_1} = \frac{1}{2}(1 + \frac{1}{4}\bar{e}_2) - e_1 = 0$ $BR_1(\bar{e}_2) = \frac{1}{2} + \frac{1}{8}\bar{e}_2$ ← 2 equations - 2 unknown

Same for player 2



$e_1 = e_2$

↳ $4/7$

9.5 Nash Equilibrium - Cournot Duopoly

Wednesday, September 30, 2020 6:56 PM

$P = 1 - Q$ $\quad Q = \sum_{i=1}^{n} q_i \quad Q_{-i} = Q - q_i$

$P = 1 - q_1 - q_2$

$C_i = .2q_i$

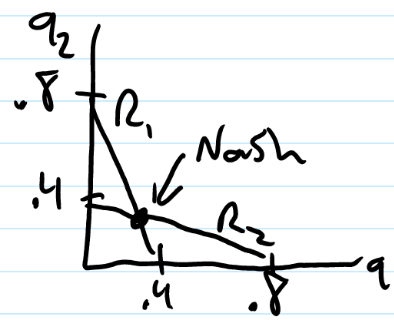
$\pi_i = \underbrace{P \cdot q_i}_{\text{revenue}} - \text{Cost} = (1 - q_1 - q_2)q_i - .2q_i$

$\frac{d\pi_1}{dq_1} = -q_1 + (1 - q_1 - q_2) - .2 = 0$

$2q_1 = 1 - q_2 - .2 \rightarrow q_1 = .4 - \frac{1}{2}q_2$

$BR_1(q_2) = .4 - \frac{1}{2}q_2 \rightarrow 0 = .4 - 1/2\, q_2$

$\hookrightarrow q_2 = .8$



$\bar{q}_2 \uparrow \rightarrow \bar{q}_1 \downarrow$

Different from Partnership

$q_1 = .4 - 1/2\, q_1 \rightarrow q_1 = 4/15$

9.6 Nash Equilibrium - Bertrand Duopoly

Wednesday, September 30, 2020 7:10 PM

Announce price P_i
Fill all demand at P_i
Consumers buy from lowest priced firm
If $P_1 = P_2$, split demand

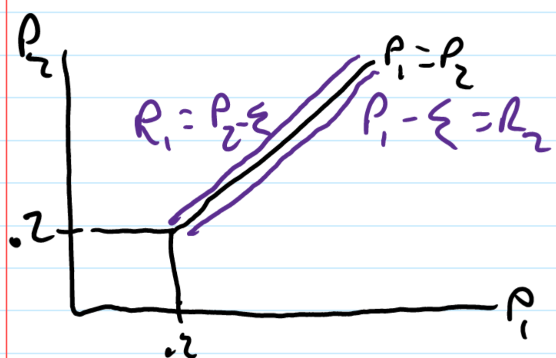
Mkt price $= \min(P_1, P_2) = P$

$Q = 1 - P$

$\Pi_1 = (1-P_1)(P_1 - .2)$ if $P_1 < P_2$ $\quad \Pi = (1-P_1)(P_1 - .2)/2$
$= 0$ if $P_1 > P_2$

$BR = P_2 - \varepsilon \quad P_2 > .2$

$P_2 = \; ? .2 \quad P_2 = .2$
$\geq .2 \quad P_2 < .2$



$P_1 = P_2 = .2$

Price competition is fiercer than quantities

HW 9.2

Thursday, October 1, 2020 10:11 AM

Find the Nash equilibria of and the set of rationalizable strategies for the games in Exercise 1 at the end of Chapter 6.

NE = (B,L) on S = (B,L)

1 \ 2	L	R
A	3, 3	2, 0
B	4, 1	8, −1

(a)

NE = (U,L) and (M,C)

L dom R

1 \ 2	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

UL makes more sense

(b)

NE = (U,X)

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

S = (U,X)

1 \ 2	L	R
U	1, 1	0, 0
D	0, 0	5, 5

(d)

All s are rationalizable

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

HW 9.4

Thursday, October 1, 2020 10:22 AM

Compute the Nash equilibria of the following location game. There are two people who simultaneously select numbers between zero and one. Suppose player 1 chooses s_1 and player 2 chooses s_2. If $s_i < s_j$, then player i gets a payoff of $(s_i + s_j)/2$ and player j obtains $1 - (s_i + s_j)/2$, for $i = 1, 2$. If $s_1 = s_2$, then both players get a payoff of $1/2$.

$s_i = x \quad s_j = y$

$$\frac{(x+y)}{2} = 1 - \frac{(x+y)}{2}$$

$$\frac{2(x+y)}{2} = 1$$

$$x + y = 1$$

if s_i and s_j both $= 1/2$, then they maximize and won't deviate

HW 9.6

Thursday, October 1, 2020 10:28 AM

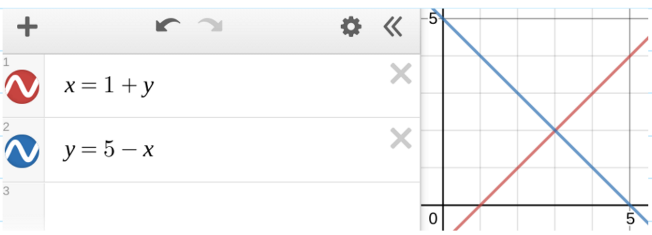
Consider a game in which, simultaneously, player 1 selects any real number x and player 2 selects any real number y. The payoffs are given by:

$$u_1(x, y) = 2x - x^2 + 2xy$$
$$u_2(x, y) = 10y - 2xy - y^2.$$

(a) Calculate and graph each player's best-response function as a function of the opposing player's pure strategy.

$du_1/dx = 2 - 2x + 2 \Rightarrow x = 1 + y$

$du_2/dx = 10 - 2y - 2x \rightarrow y = 5 - x$



(b) Find and report the Nash equilibria of the game.

$x = 1 + 5 - x$

$x = 6 - x$

$2x = 6$

$x = 3 \rightarrow y = 5 - 3 = 2$

$NE = (3, 2)$

(c) Determine the rationalizable strategy profiles for this game.

$s = (3, 2)$

HW 9.10

Thursday, October 1, 2020 10:38 AM

Is the following statement true or false? If it is true, explain why. If it is false, provide a game that illustrates that it is false. "If a Nash equilibrium is not strict, then it is not efficient."

A \ B	y	z
w	1,1	1,2
x	2,1	4,4

False. (w, y) and (x, z) are both NE but (x, z) is better than (w, y)

HW 9.11

Sunday, October 4, 2020 4:18 PM

This exercise asks you to consider what happens when players choose their actions by a simple rule of thumb instead of by reasoning. Suppose that two players play a specific finite simultaneous-move game many times. The first time the game is played, each player selects a pure strategy at random. If player i has m_i strategies, then she plays each strategy s_i with probability $1/m_i$. At all subsequent times at which the game is played, however, each player i plays a best response to the pure strategy actually chosen by the other player the *previous* time the game was played. If player i has k strategies that are best responses, then she randomizes among them, playing each strategy with probability $1/k$.

(a) Suppose that the game being played is a prisoners' dilemma. Explain what will happen over time.

They confess forever because it's always the best response.

(b) Next suppose that the game being played is the battle of the sexes. In the long run, as the game is played over and over, does play always settle down to a Nash equilibrium? Explain.

	A	B
A	2,1	0,0
B	0,0	1,2

If (0,0) is chosen first, then (0,0) is the BR so they look for each other forever. If (2,1) or (1,2) is chosen first, it settles.

(c) What if, by chance, the players happen to play a strict Nash equilibrium the first time they play the game? What will be played in the future? Explain how the assumption of a strict Nash equilibrium, rather than a nonstrict Nash equilibrium, makes a difference here.

If it's strict, they'll never deviat because strict Nash is always better.

(d) Suppose that, for the game being played, a particular strategy s_i is not rationalizable. Is it possible that this strategy would be played in the long run? Explain carefully.

No. only best responses are played