The Extensive Form

Sunday, August 23, 2020 10:36 PM

- Dotted line means player doesn't know where they are in the game
- Simultaneous first moves doesn't matter who is written first

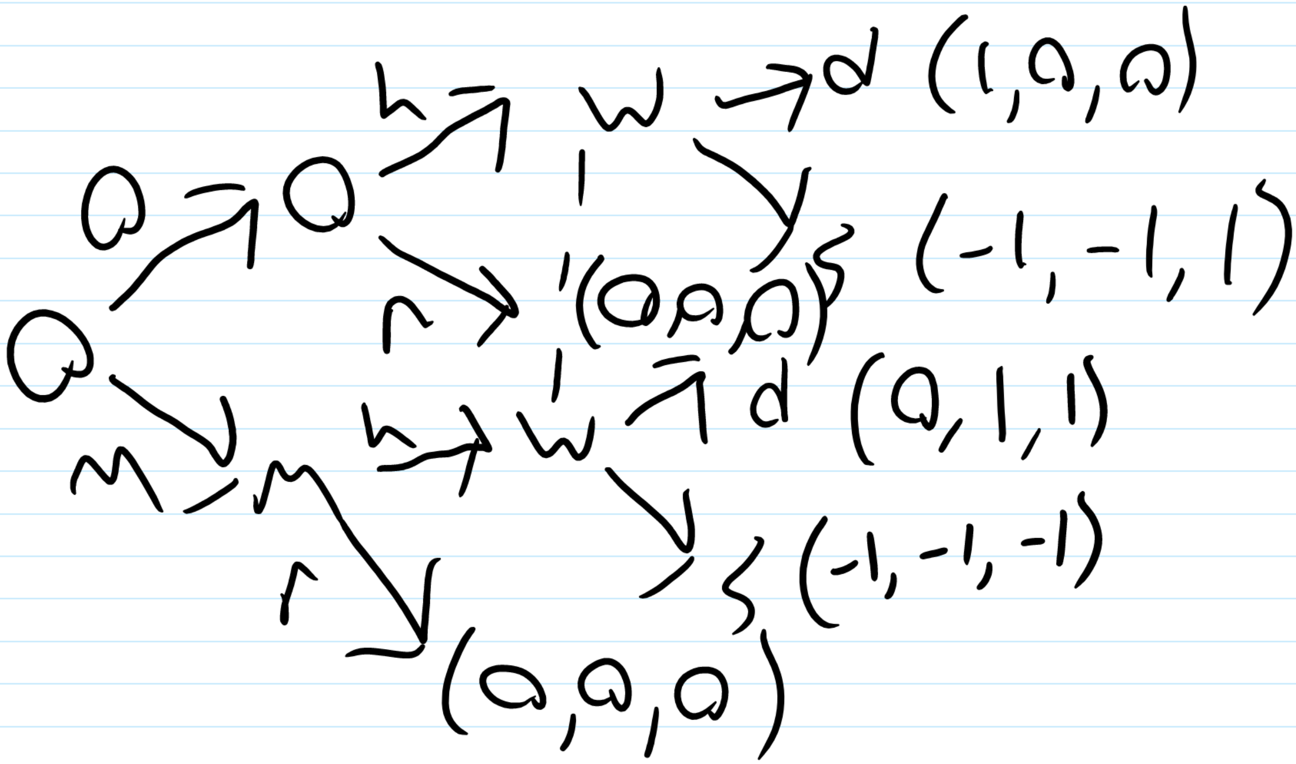
HW 2.2

Tuesday, August 25, 2020 6:05 PM

Passed Salution review

Owner of firm is hiring. They can hire, not hire, or let the manager decide. Manager can hire or not hire. If hired, the worker can work diligently or slack off. Worker doesn't know if O or m hired. Worker not hired, all 0, w hired and shirks, O and m get -1 and w gets 1. If O hired and w is diligent, O gets 1, m and w get 0. If m hired, flip O and m

(O, m, w)

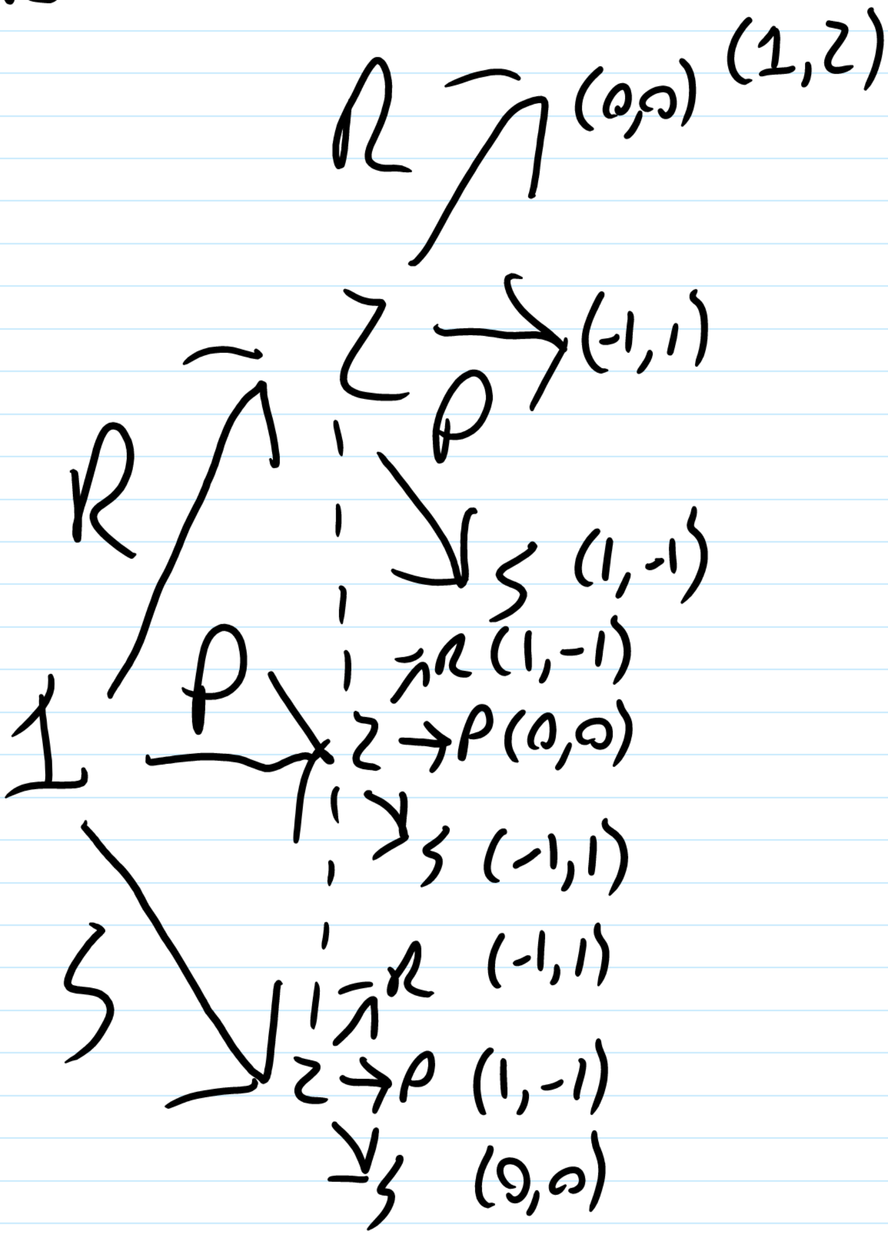


HW 2.4

Tuesday, August 25, 2020 6:19 PM

Passed Solution review

Rock paper scissors!



Because play is simultaneous, player order doesn't matter except for writing payoff orders

If the game was sequential, player 2 would always win

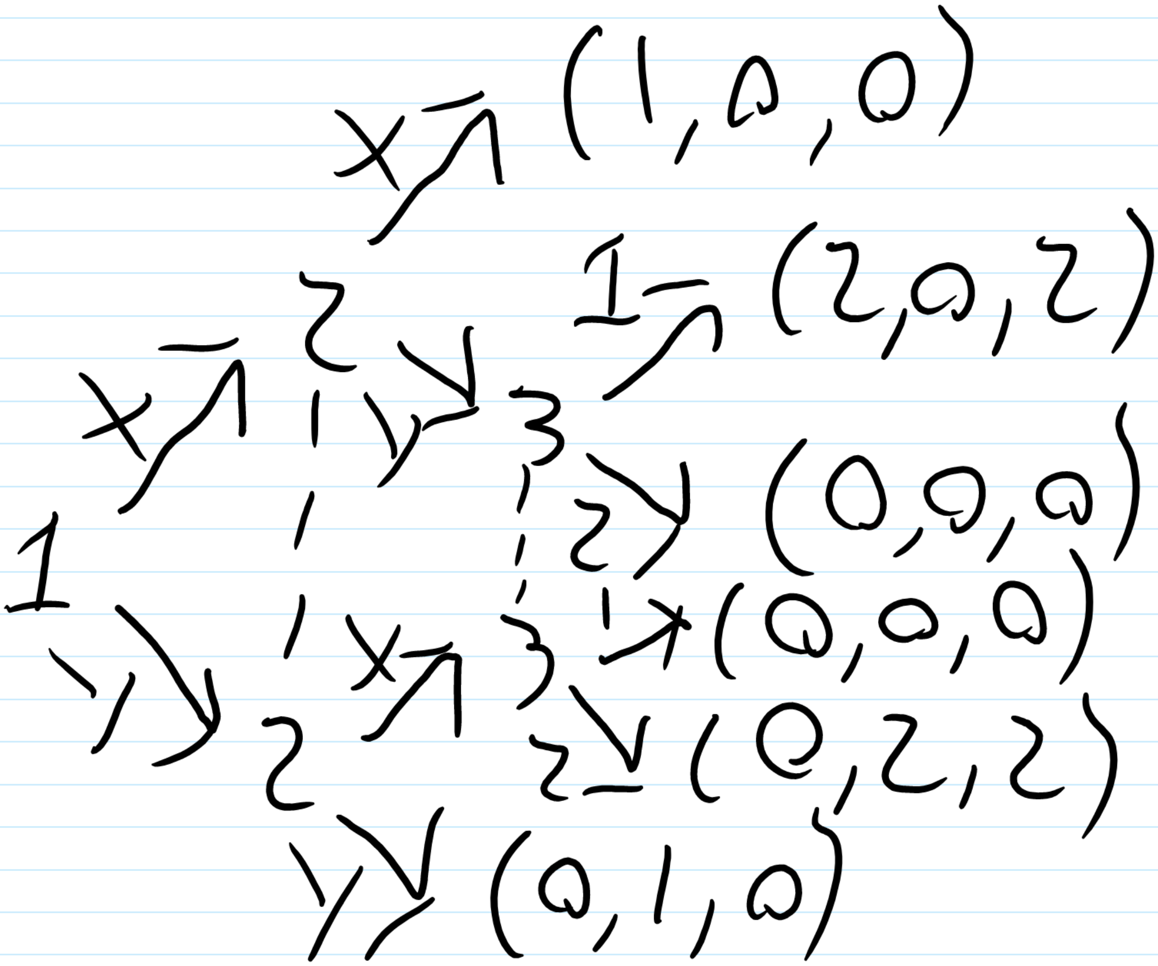
HW 2.6

Tuesday, August 25, 2020 6:25 PM

Passed Solution Review

Players 1,2,3. At first 1 and 2 move simultaneously between X and Y. Game ends if both choose X and Payoff is (1,0,0). If both Y, Game over w/ (0,1,0). Player 3 guesses who said what.

(1,2,3)



3.1 Strategies

Tuesday, August 25, 2020 10:09 AM

$S_2 = \{H, L\}$

$S_c = \{HH', HL', LH', LL'\}$ ← Cheryl + Zeii from previous example on high/low effort

(H, LH') ← Profile

$S = S_2 \times S_c$

↳ cartesian product (matrix multiplication)

$s_i \in S_i$ $S = s_i \times s_2 \times s_3 \times s_n$ ← All Possibilities

S_{-i} is all strategies except player i (think vectors in d

Player i utility $= V_i(s)$ or $V_i(s_i, s_{-i})$

$i = 7$ then $s_{-7} = (s_i \ldots s_6, s_8 \ldots s_n)$ or $s(s_7, s_{-7})$

Every profile maps to payoffs

$U_i : S \to R$

Normal form?

3.2 Normal Form Representation

Tuesday, August 25, 2020 10:20 AM

Strategic form representation

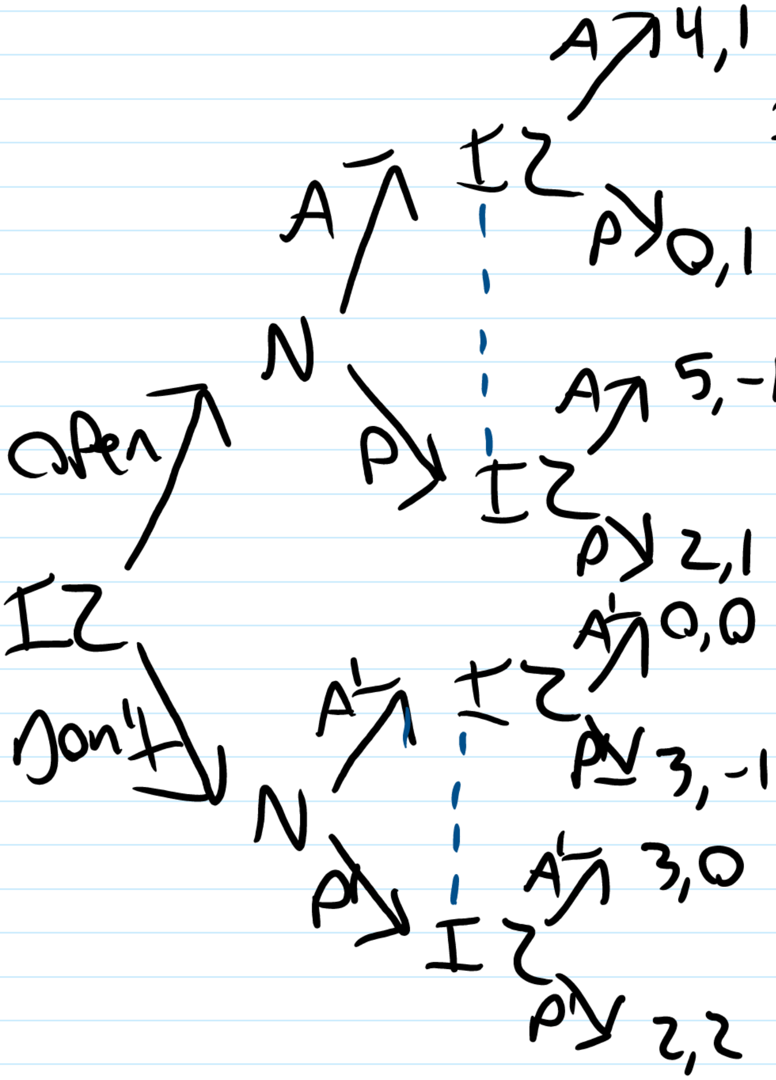
Cheryl + Zii

8 strategy profiles

	C: HH'	HL'	LH'	LL'
Z: H	-1,1	-1,1	0,1	1,0
L	0,1	0,0	0,1	0,0

$X \in \mathbb{R}$ $Y \in \mathbb{R}$ $\rightarrow V_1(X,Y)$ $V_2(X,Y)$

↳ Real number



N

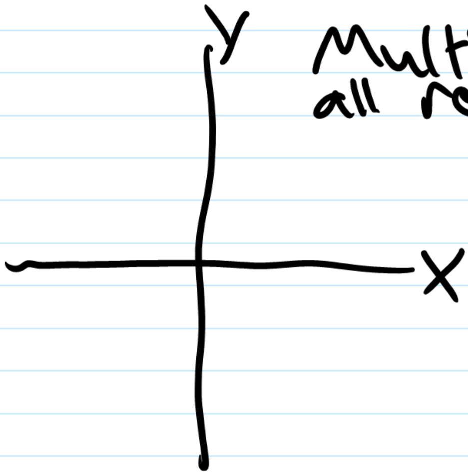
IZ:	AA'	AP'	PA'	PP'
OAA'	4,1	4,1		
OPA'				
DAA'			0,0	
DPA'	0,0		0,0	
OAP'	4,1	4,1		
OPP'				
DAP'				
DPP'		2,2		2,2

3 Extra Notes

Wednesday, August 26, 2020 10:07 AM

Cartesian Product

Multiplying x and y sets to get all resulting sets



HW 3.2

Tuesday, August 25, 2020 6:33 PM

Passed Solution review

Manager decides whether or not to hire a worker. If M doesn't hire W game over. If hired W can do high or low effort. Based on effort, M can retain or fire W.

Not hire describes a strategy because not hire is the first move and a terminal point in the game.

No. A strategy must specify what to choose in all possible information sets, whether or not they will be reached if that strategy is played w/ fidelity.

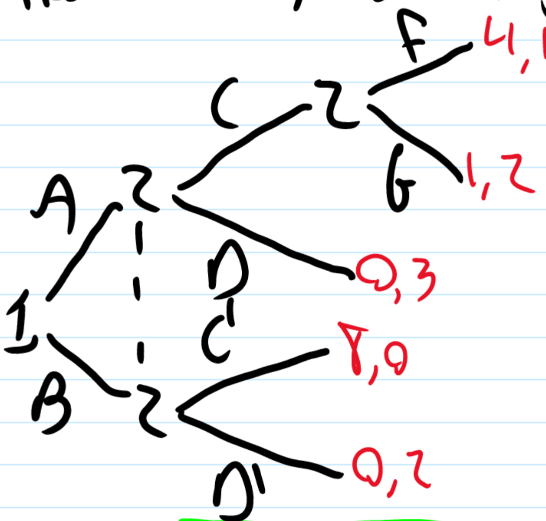
HW 3.4

Sunday, August 30, 2020 6:00 PM

Passed solution review

Worked w/ Hail + Isabel

How many strategies does Player 2 have?



CF CG C'F C'G DF DG D'F D'G

$\frac{A}{B}$

Player 2 has 4 strategies.

HW 3.8

Sunday, August 30, 2020 6:20 PM

Passed solution review

Worked w/ Hail + Isabel

Cat = Baker Dog = Spike mouse = cheesy

Baker wants to catch cheesy + avoid spike

cheesy wants to tease baker + not get caught

spike wants to rest and not be disturbed

morning: B + C simultaneously decide

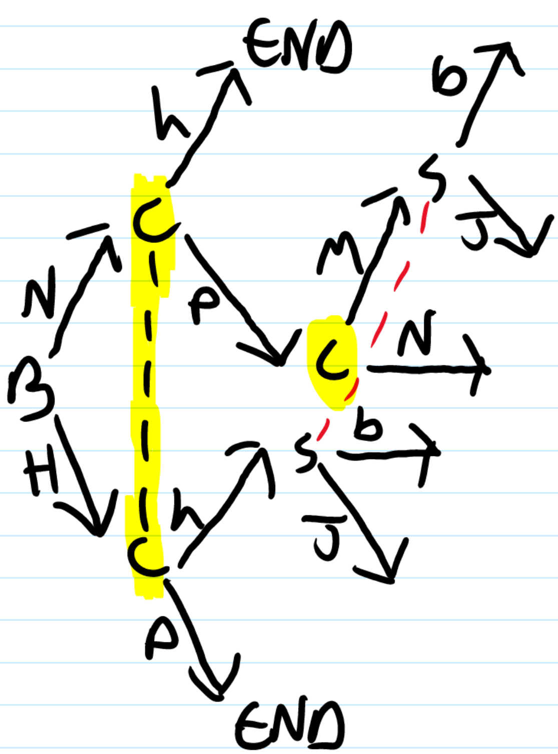
B can Nap or Hunt ↳ moves S's bone

C can hide or Play

↳ Game ends if nap + hide or hunt + Play

If nap + Play, B must move or Not move the bone

If bone moved, S punishes B or C then game ends



Cheezy has 2 information sets

B: N, H 2x

C: hM, PM, hN, PN 4x

S: b, J 2x

16?

$\boxed{\text{There are 16 strategy profiles}}$

4 Book Notes

Saturday, August 29, 2020 4:22 PM

Beliefs, mixed strategies, + expected payoffs

belief is what a player thinks will happen

mixed strategy is selecting a strategy according to a probability distribution

regular strategy = pure strategy

mixed strategy includes pure strategy

payoff numbers can include preferences of player preferences over probability distributions over outcomes

4.1 Beliefs and Expected Payoffs

Saturday, August 29, 2020 4:35 PM

Beliefs

		Column		
		C1	C2	C3
Row	R_1	4,1	1,1	2,5
	R_2	2,2	0,0	3,3
	R_3	2,5	1,1	1,4
		P_1	P_2	$1-P_1-P_2$
		0.2	0.2	0.6

$\theta_{-R}, \theta_c \rightarrow \theta_{-i} \in \Delta S_{-i}$

Theta not row means theta column...

beliefs are represented by probability distributions

Expected Payoff/utility $\rightarrow E$

$\theta_c = (.2, .2, .6)$ $\quad$ U=utility

$E(U_R \mid S_R = R_2, \theta_c) = .2 \cdot 2 + .2 \cdot 0 + .6 \cdot 3 = 2.2$

IF $\quad$ and

$\theta_R = (.7, 0, .3) \rightarrow E(U_c \mid S_c = C_1, \theta_R) = .7 \cdot 1 + 0 \cdot 2 + .3 \cdot 5$
$= .85$

$$U_i(S_i, \theta_{-i}) = \sum_{S_{-i} \in S_{-i}} \theta_{-i}(S_{-i}) \cdot U(S_i, S_{-i})$$

4.2 More on Expected Utility

Saturday, August 29, 2020 4:53 PM

More on expected utility

Ranking = ordinal (can't multiply)

Risk neutral, money outcomes

↳ will take any fair gamble

Risk Aversion

utility function

Von-Neumann Morgenstern utility function

- Continuity
- More is better
- Transitivity
- Independence of irrelevant outcomes

4.3 Mixed Strategies

Saturday, August 29, 2020 5:09 PM

Mixed Strategies

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	0,0	3,3
R_3	2,5	1,1	1,4
	p_1	p_2	$1-p_1-p_2$

$\sigma_i \in \Delta S_i$

sigma

$\theta_C = (.2, .2, .6)$

$\sigma_R (.7, 0, .3)$

↳ NOT θ_R R belief

$$E(U_R)? = [\underbrace{.7 \cdot .2 \cdot 4}_{R_1C_1} + \underbrace{.7 \cdot .2 \cdot 1}_{R_1C_2} + \underbrace{.7 \cdot .6 \cdot 2}_{R_1C_3}] +$$

$$0[\underbrace{.2 \cdot 2}_{R_2C_1} + .2 \cdot 0 + .6 \cdot 3] +$$

$$.3[.2 \cdot 2 + .2 \cdot 1 + .6 \cdot 1]$$ = R Player expected utilites given mixed strategy + their beliefs about C's strategy

$$U_i(\sigma_i, \theta_{-i}) = \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \theta_i(s_i) \cdot \theta_{-i}(s_{-i}) \cdot u_i(s_i, s_{-i})$$

4 Extra Notes

Monday, August 31, 2020 10:05 AM

Mixed Strategy is a way to obfuscate your goal

4 Extra Problems

Friday, September 4, 2020 10:08 AM

1 \ 2	L	C	R
U	10, 0	0, 10	3, 3
M	2, 10	10, 2	6, 4
D	3, 3	4, 6	6, 6

d) $u_2(M, R) = 4$

e) $(3 \cdot .25) + (.5 \cdot 6) + (.25 \cdot 6) = 5.25$

g) $(3 \cdot \frac{1}{3}) + (4 \cdot \frac{2}{3}) + (6 \cdot 0) = 3\ \frac{2}{3}$

h)

HW 4.2

Saturday, September 5, 2020 1:00 PM

Passed Solution review

$S_1 = \{H, L\}$ and $S_2 = \{X, Y\}$

If 1 plays H, Payoff = z. Player 1 Payoff = $V_1(L,X) = 0$
$V_1(L,Y) = 10$

Player 2 Payoff doesn't matter

a) Player 1, Player 2:

	X	Y
H	z, 0	z, 1
L	0, 1	10, 0

b) If 1 believes $\theta_2 = (.5, .5)$, Payoff of Playing H? Playing L? Value of z when H = L?

Payoff for H = .5 z
Payoff for L = 10 · .5 = 5

Player 1 is indifferent when z = 5

c) $\theta_2 = (\frac{1}{3}, \frac{2}{3})$. Payoff of Player 1 Playing L?

$= \frac{1}{3} \cdot 0 + 10 \cdot \frac{2}{3} = 0 + 20/3 = 20/3$

HW 4.3

Saturday, September 5, 2020 1:18 PM

Passed Solution review

worked w/ hail

	I	Q
QA	2,2	2,2
QB	2,2	2,2
IA	4,2	1,3
IB	3,4	1,3

a) $U_1(\sigma_1, I)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$

$1/4(2+2+4+3) = 1/2 + 1/2 + 1 + 3/4 = 11/4$ or 2.75

b) $U_2(\sigma_1, Q)$ for $\sigma_1 = (1/8, 1/4, 1/4, 3/8) = (1/8, 2/8, 2/8, 3/8)$

$2/8 + 4/8 + 6/8 + 9/8 = 21/8$

c) $U_1(\sigma_1, \sigma_2)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$, $\sigma_2 = (1/3, 2/3)$

	I	Q
QA	1/12	2/12
QB	1/12	2/12
IA	1/12	2/12
IB	1/12	2/12

$2/12 + 4/12 + 4/12 + 3/12 + 4/12 + 4/12 + 2/12 + 2/12 = 23/12$

d) $U_1(\sigma_1, \sigma_2)$ for $\sigma_1 = (0, 1/3, 1/6, 1/2)$, $\sigma_2 = (2/3, 1/3)$

	I	Q
QA	0	0
QB	4/18	2/18
IA	2/18	1/18
IB	6/18	3/18

$0 + 8/18 + 8/18 + 18/18 + 0 + 4/18 + 1/18 + 3/18 = 42/18 = 7/3$

HW 4.4

Saturday, September 5, 2020 2:07 PM

Worked w/ hail

Passed Solution review

For each game find:

$v_1(\sigma_1, \sigma_2)$ and $v_2(\sigma_1, \sigma_2)$ for $\sigma_1 = (1/2, 1/2)$ and $\sigma_2 = (1/2, 1/2)$

FIGURE 3.4 Classic normal-form games.

Matching Pennies

1 \ 2	H	T
H	1, −1	−1, 1
T	−1, 1	1, −1

~~Prisoners' Dilemma~~

1 \ 2	[illegible]	D
C	2, 2	[illegible] 3
D	3, [illegible]	1, 1

Battle of the Sexes

1 \ 2	Opera	Movie
Opera	2, 1	0, 0
Movie	0, 0	1, 2

Hawk-Dove/Chicken

1 \ 2	H	D
H	0, 0	3, 1
D	1, 3	2, 2

~~Coordination~~

1 \ 2	A	B
A	[illegible] 1	0, 0
B	0, 0	1, 1

~~Pareto Coordination~~

1 \ 2	A	B
A	2, 2	[illegible]
B	0, [illegible]	1, 1

~~Pigs~~

D \ S	P	D
P	4, [illegible]	2, [illegible]
D	6, 1	0, 0

Pennies: $v_1(\sigma_1, \sigma_2) = v_2(\sigma_1, \sigma_2) = .25 - .25 + .25 - .25 = 0$

Sexes: $v_1(\sigma_1, \sigma_2) = v_2(\sigma_1, \sigma_2)$

$= (2 \cdot 1/4) + 0 + 0 + (1 \cdot 1/4) = 3/4$

Chicken: $v_1(\sigma_1, \sigma_2) = v_2(\sigma_1, \sigma_2)$

$= 0 + (1 \cdot 1/4) + (3 \cdot 1/4) + (2 \cdot 1/4) = 6/4 = 1.5$

5 Book Notes

Saturday, August 29, 2020 5:30 PM

General Assumptions + Methodology

Must balance realism w/ managable math

Rationality

Players select path to their preferred outcome

- maximizing one's payoff

Assume players know how the game is played

Players are rational if they 1. form a belief about strategies and 2. given this belief they maximize their payoff

5 Extra Notes

Wednesday, September 2, 2020 10:06 AM

Prisoner's Dilemma w/ 3 Players

Players R, C, and T

R \ C	R	S
R		
S		

R \ C	R	S
R		
S		

R T S

6 Book Notes

Monday, August 31, 2020 3:17 PM

Dominance + Best Response

Contracts can bind players to strategies

6.1 Dominance and the Prisoner's Dilemma

Monday, August 31, 2020 2:46 PM

Dominance

Prisoner's Dilemma

		Hail: Silent	Hail: Rat
Gus	Silent	2,2	-1,3
Gus	Rat	3,-1	0,0

Silence is dominated by Rat
↳ Always better

Weakly dominated = sometimes better

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	2,0	3,3
R_3	2,5	1,1	1,4

C_2 Wd by C_1 r_3 Wd by $r_1 + r_2$

C_2 Sd by C_3

Wd / Sd = weakly/strongly dominated

Pure vs weak strategies

$\sigma_R = (.5, .5, 0)$ vc1 3 > 2
vc2 1.5 > 1
vc3 2.5 > 1

$(.5, .5, 0)$ SD r_3

General Notation

s_i is dominated by σ_i if

$V_i(\sigma_i, s_{-i}) > V_i(s_i, s_{-i})$ $\forall$ $s_{-i} \in S_{-i}$

$\forall$ → for all

Rational players <u>do NOT</u> play dominated strategies

PARETO EFFICIENCY

$\boxed{\text{Contracting}}$

$V_i(q, s_{-i}) > V_i(s_i, s_{-i})$ $\forall$ $s_{-i} \in S_{-i}$

& $\forall$ $s_i \in S_i$ $s_i \neq q$

6.3 Best Responses

Saturday, September 5, 2020 12:00 PM

Best responses or Best replies

	C_1	C_2	C_3
R_1	4, 1	1, 1	2, 5
R_2	2, 2	7, 0	3, 3
R_3	2, 5	1, 1	1, 4

$BR_1(1,0,0) = R_1$

$(0,1,0)$ or $(0,0,1) = R_2$

$BR_2(1,0,0)$ or $(0,1,0) = C_3$

$(0,0,1) = C_1$

Column 2 is never BR because it's dominated

BR is $U_i(s_i, \theta_{-i}) \geq U_i(s_i', \theta_{-i}) \; \forall s_i' \in S_i$. If true, strategy s is a BR to θ_{-i}

$B_c = \{C_1, C_3\}$

6.4 6.5 Establishing a Strategy is Never a Best Response

Saturday, September 5, 2020 12:12 PM

Establishing no set of beliefs make s_i a best response

	C_1	C_2	C_3
R_1	4,1	1,1	2,5
R_2	2,2	2,0	3,3
R_3	2,5	1,1	1,4

$\Theta_c(P_1, P_2, 1-P_1-P_2)$

$V_R(R_3, \Theta_c) = 2 \cdot P_1 + (1 \cdot P_2) + [1 \cdot (1-P_1-P_2)]$
$= 1 + P_1$

$V_R(R_2, \Theta_c) = 2P_1 + 2P_2 + 3(1-P_1-P_2)$
$= 3 - P_1 - P_2$

$V_R(R_3, \Theta_c) = 4P_1 + 1P_2 + 2(1-P_1-P_2)$
$= 2 + 2P_1 - P_2$

$V_R(R_3) > V_R(R_2) > 0$

$\hookrightarrow 1 + P_1 - 3 + P_1 + P_2 \geq 0$

$-2 + 2P_1 + P_2 \geq 0$

$P_2 \geq 2 - 2P_1$

$V_R(R_3) > V_R(R_1) \geq 0$

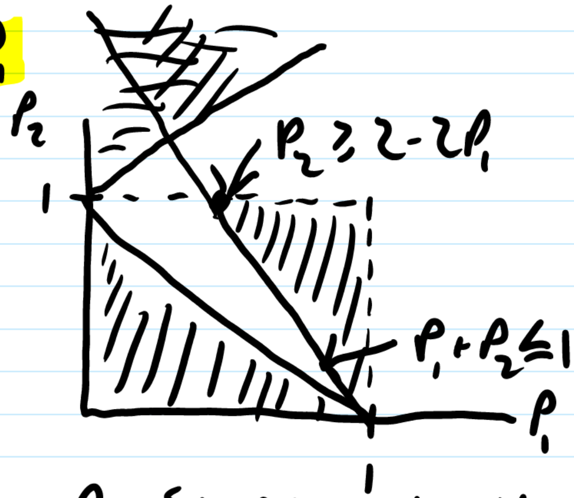
$\hookrightarrow 1 + P_1 - 2 - 2P_1 + P_2 \geq 0$

$-1 - P_1 + P_2 \geq 0$

$P_2 \geq 1 + P_1$

$P_1 + P_2 \leq 1$

Can't Satisfy all 3!



R_3 is never best!

6.6 Undominated Strategies and Best Responses

Saturday, September 5, 2020 12:32 PM

Beliefs

strategic independence

	C_1	C_2	C_3
R_1	4, 1	1, 1	2, 5
R_2	2, 2	2, 0	3, 3
R_3	2, 5	1, 1	1, 4

correlated beliefs

↳ $B_i^c = UD_i$

6.7 Some Last Words on Weak Dominance

Saturday, September 5, 2020 12:49 PM

	C_1	C_2	C_3
R_1	4, 1	1, 1	3, 1
R_2	2, 2	0, 3	3, 2
	p_1	p_2	$1 - p_1 - p_2$

"Fully mixed beliefs"

$B_i^{FC} = WUD_i$

a) B dom A, L dom R
b) L dom R, L weak dom C?
c) X dom Z, $\frac{2}{3}U + \frac{1}{3}D$ dom M

6.1 Determine which strategies are dominated in the following normal-form games.

(a)

1 \ 2	L	R
A	3, 3	2, 0
B	4, 1	8, −1

(b)

1 \ 2	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

(c)

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

5. Represent in the normal form the rock–paper–scissors game (see Exercise 4 of Chapter 2 to refresh your memory) and determine the following best-response sets.

(a) $BR_1(\theta_2)$ for $\theta_2 = (1, 0, 0)$ $= \{P\}$

(b) $BR_1(\theta_2)$ for $\theta_2 = (1/6, 1/3, 1/2)$ $= \{R, S\}$

(c) $BR_1(\theta_2)$ for $\theta_2 = (1/2, 1/4, 1/4)$ $= \{P, S\}$

(d) $BR_1(\theta_2)$ for $\theta_2 = (1/3, 1/3, 1/3)$ $= \{R, P, S\}$

1 \ 2	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

b) $R = 0 + -\frac{1}{3} + \frac{1}{2} = -\frac{2}{6} + \frac{3}{6} = \frac{1}{6}$
$P = \frac{1}{6} + 0 - \frac{3}{6} = \qquad -\frac{2}{6}$
$S = -\frac{1}{6} + \frac{2}{6} + 0 = \qquad \frac{1}{6}$

HW 6.2

Saturday, September 5, 2020 2:28 PM

Passed Solution Review

1 \ 2	L	C	R
U	10, 0	0, 10	3, 3
M	2, 10	10, 2	6, 4
D	3, 3	4, 6	6, 6

2. For the game in Exercise 1 of Chapter 4, determine the following sets of best responses.

(a) $BR_1(\theta_2)$ for $\theta_2 = (1/3, 1/3, 1/3)$
(b) $BR_2(\theta_1)$ for $\theta_1 = (0, 1/3, 2/3)$
(c) $BR_1(\theta_2)$ for $\theta_2 = (5/9, 4/9, 0)$
(d) $BR_2(\theta_1)$ for $\theta_1 = (1/3, 1/6, 1/2)$

a) $U = 10/3 + 0 + 3/3 = 13/3$
$M = 2/3 + 10/3 + 6/3 = 18/3$ $\rightarrow BR_1(\theta_2) = \{M\}$
$D = 3/3 + 4/3 + 6/3 = 13/3$

b) $L = 0 + 10/3 + 6/3 = 16/3$
$C = 0 + 2/3 + 12/3 = 14/3$ $\rightarrow BR_2(\theta_1) = \{L, R\}$
$R = 0 + 4/3 + 12/3 = 16/3$

c) $U = 50/9 + 0 + 0 = 50/9$
$M = 10/9 + 40/9 + 0 = 50/9$ $\rightarrow BR_1(\theta_2) = \{U, M\}$
$D = 15/9 + 16/9 + 0 = 31/9$

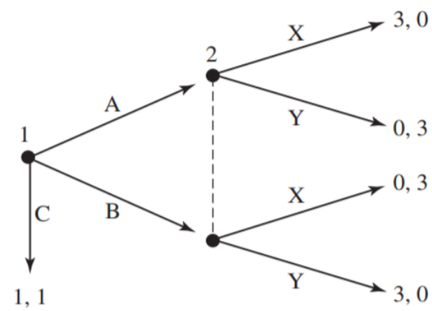
d) $L = 0 + 10/6 + 9/6 = 19/6$
$C = 20/6 + 2/6 + 18/6 = 40/6$ $\rightarrow BR_2(\theta_1) = \{C\}$
$R = 6/6 + 4/6 + 18/6 = 28/6$

HW 6.6

Saturday, September 5, 2020 2:39 PM

Passed Solution Review

6. In the game pictured here, is it ever rational for player 1 to select strategy C? Why?



If Player 1 wants a guaranteed payoff, they would choose strategy C.

But, mathematically, they would choose A or B because the expected payoff value is 1.5

AX = BY + AY = BX

By eliminating C, a mixed a+b generates 1.5

Let P denote Player 1's belief about probability of X

$u_1(C,P) \geq u_1(A,P)$

$1 \geq 3P + 0(1-P)$

$1/3 \geq P$

$u_1(C,P) \geq u_1(B,P)$

$1 \geq 0P + 3(1-P)$

$1 \geq 3-3P$

$P \geq 2/3$

No because P cannot satisfy both inequalities

HW 6.7

Saturday, September 5, 2020 2:41 PM

Passed Solution review

7. In the normal-form game pictured below, is player 1's strategy M dominated? If so, describe a strategy that dominates it. If not, describe a belief to which M is a best response.

1 \ 2	X	Y
K	9, 2	1, 0
L	1, 0	6, 1
M	3, 2	4, 2

M is dominated by mixed strategy {K, L}

M does not dominate player 1's response because in X, K is better and in Y, L is better.

$$\boxed{\begin{aligned} 9p + 1(1-p) &> 3 \\ 9p + 1 - p &> 3 \\ 8p + 1 &> 3 \\ 8p &> 2 \\ p &> 1/4 \end{aligned}}$$

$p > 5/20$

$$\boxed{\begin{aligned} 1p + 6(1-p) &> 4 \\ p + 6 - 6p &> 4 \\ 6 - 5p &> 4 \\ -5p &> -2 \\ p &< 2/5 \end{aligned}}$$

$p < 8/20$

$$\boxed{\sigma = (.3, .7, 0)}$$

p $1-p$ 0

7 Book Notes

Saturday, September 5, 2020 3:52 PM

Rationalizability + Iterated dominance

Actions of one player may affect another's payoff

Players can eliminate (think Sudoku) strategies that don't make sense

↳ Iterative removal of (strictly) dominated strategies

↳ Iterative dominance

Strategies that survive iterative dominance are called rationalizable strategies

The Second Strategic Tension

Strategic uncertainty

↳ Coordination problem:

7.1 Iterated Dominance and Rationalizability

Thursday, September 10, 2020 4:19 PM

	Bob			
Alice	B_1	~~B_2~~	B_3	B_4
A_1	4,1	1,2	2,5	1,4
A_2	2,3	3,1	1,0	1,2
A_3	2,5	1,1	1,4	3,0

B_2 dom by mix $B_1 + B_3$

Eliminate B_2 to create R_1

Alice has no dominated strategies

R_1

	Bob			
Alice	B_1	~~B_2~~	B_3	B_4
A_1	4,1		2,5	1,4
~~A_2~~	2,3		1,0	1,2
A_3	2,5		1,4	3,0

A_2 dom by $A_1 + A_3$

R_2

	Bob			
Alice	B_1	~~B_2~~	B_3	~~B_4~~
A_1	4,1		2,5	1,4
~~A_2~~				
A_3	2,5		1,4	3,0

B_4 dom by B_3

R_3

	Bob			
Alice	B_1	~~B_2~~	B_3	~~B_4~~
A_1	4,1		2,5	
~~A_2~~				
~~A_3~~	2,5		1,4	

A_3 dom by A_1

R_4

	Bob			
Alice	~~B_1~~	~~B_2~~	B_3	~~B_4~~
A_1	4,1		2,5	
~~A_2~~				
~~A_3~~				

B_1 dom by B_3

R_5

	Bob			
Alice	~~B_1~~	~~B_2~~	B_3	~~B_4~~
A_1			2,5	
~~A_2~~				
~~A_3~~				

You can eliminate dominated strategies

7.2 Strategic Uncertainty

Thursday, September 10, 2020 4:35 PM

Coordination Problem

↳ 2 people lose each other somewhere + look for each other in different rational places

"Focal Points"

	Rabbit	Stag
Rabbit	1,1	1,0
Stag	0,1	4,4

7 Extra Problems

Friday, September 18, 2020 10:04 AM

Determine which strategy profiles are rationalizable for these games.

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

1 \ 2	a	b	c	d
w	5, 4	4, 4	4, 5	12, 2
x	3, 7	8, 7	5, 8	10, 6
y	2, 10	7, 6	4, 6	9, 5
z	4, 4	5, 9	4, 10	10, 9

Passed Solution review

Suppose that you manage a firm and are engaged in a dispute with one of your employees. The process of dispute resolution is modeled by the following game, where your employee chooses either to "settle" or to "be tough in negotiation," and you choose either to "hire an attorney" or to "give in."

Employee \ You	Give in	Hire attorney
Settle	1, 2	0, 1
Be tough	3, 0	x, 1

In the cells of the matrix, your payoff is listed second; x is a number that both you and the employee know. Under what conditions can you rationalize selection of "give in"? Explain what you must believe for this to be the case.

If $X \geq 0$, be tough dom settle
If $X < 0$, no dominant strategies
If $p(\text{settle}) > .5$ (greater than) and $X < 0$, then it makes sense to choose give in

HW 7.3

Friday, September 18, 2020 12:19 PM

Passed Solution review

Find the set of rationalizable strategies for the following game.

1 \ 2	a	b	c	d
w	5, 4	4, 4	4, 5	12, 2
x	3, 7	8, 7	$\boxed{5, 8}$	10, 6
y	2, 10	7, 6	4, 6	9, 5
z	4, 4	5, 9	4, 10	10, 9

Note that each player has more than one dominated strategy. Discuss why, in the iterative process of deleting dominated strategies, the order in which dominated strategies are deleted does not matter.

$R^0 = \{w, x, y\} \times \{a, b, c\}$

$R^1 = \{w, x\} \times \{c\}$

$R^2 = \{x\} \times \{c\}$

final strategy $= \boxed{\{x\} \times \{c\}}$

The order doesn't matter because a round isn't over until both players have moved.

x dom y

$2/3w + 1/3x$ dom z

c dom d

$9/10c + 1/10a$ dom b

$R^1 = \{w, x\} \times \{a, c\}$

c dom a

$R^2 = \{w, x\} \times \{c\}$

x dom w so $S = (x, c)$

In any iteration, each player considers all strategies not eliminated in the last iteration. Then, in the next iteration, all strategies eliminated for either player are eliminated. Since all are eliminated between iterations, "order" has little meaning within any given round.

HW 7.4

Friday, September 18, 2020 12:25 PM

Passed Solution review

Imagine that there are three major network-affiliate television stations in Turlock, California: RBC, CBC, and MBC. All three stations have the option of airing the evening network news program live at 6:00 P.M. or in a delayed broadcast at 7:00 P.M. Each station's objective is to maximize its viewing audience in order to maximize its advertising revenue. The following normal-form representation describes the share of Turlock's total population that is "captured" by each station as a function of the times at which the news programs are aired. The stations make their choices simultaneously. The payoffs are listed according to the order RBC, CBC, MBC. Find the set of rationalizable strategies in this game.

MBC: 6:00

RBC \ CBC	6:00	7:00
6:00	~~14~~, 24, 32	~~8~~, 30, 27
7:00	30, 16, 24	13, ~~12~~, 50

MBC: 7:00

RBC \ CBC	6:00	7:00
6:00	16, 24, ~~30~~	30, 16, ~~24~~
7:00	30, 23, ~~14~~	14, 24, ~~32~~

$R^0 = \{6, 7\} \times \{6, 7\} \times \{6pm\}$ $= \{7pm, 6pm, 6pm\}$

$R^1 = \{7\} \times \{6, 7\} \times \{6\}$

$R^2 = \{7\} \times \{6\} \times \{6\}$

HW 7.6

Friday, September 18, 2020 1:11 PM

Passed Solution Review

Suppose that in some two-player game, s_1 is a rationalizable strategy for player 1. If, in addition, you know that s_1 is a best response to s_2, can you conclude that s_2 is a rationalizable strategy for player 2? Explain.

If s_2 is a best response to the rationalizable s_1, then s_2 must also be rationalizable.

No. s_1 might be rationalizable because it is a best response to some other strategy, s_2 that is rationalizable, making s_1 rationalizable even if s_2 is not

Passed Solution review

Consider a guessing game with ten players, numbered 1 through 10. Simultaneously and independently, the players select integers between 0 and 10. Thus player i's strategy space is $S_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, for $i = 1, 2, \ldots, 10$. The payoffs are determined as follows: First, the average of the players' selections is calculated and denoted a. That is,

$$a = \frac{s_1 + s_2 + \cdots + s_{10}}{10},$$

where s_i denotes player i's selection, for $i = 1, 2, \ldots, 10$. Then, player i's payoff is given by $u_i = (a - i - 1)s_i$. What is the set of rationalizable strategies for each player in this game?

Max $a = {}^{100}/_{10} = 10$
Min $a = {}^{0}/_{10} = 0$

Max $U = a-10-1 = a-11 \rightarrow$ negative
Min $U = a-1-1 = a-2 \rightarrow$ maybe negative

If everyone chooses 0, final payoff is non-negative

$a \leq 10$ so $a-10-1 \leq -1$ so $s_{10} = 0$ dominates
Now $a \leq 9$ so $s_9 = 0$ dominates
Now $a \leq 8$ so $s_8 = 0$ dominates

and so on...

$s = (0, 0, \ldots, 0)$

8 Book Notes

Thursday, September 10, 2020 3:59 PM

Location

Firm location is strategy

P and C sell soda at the beach and simultaneously + independently set up for the day
- 9 regions of = size
- 50 purchases in each region
- customers walk to nearest booth

	C	P
1		
2		
3		
4		
5		
6		
7		
8		
9		

Iterated dominance

$S_i = \{1,2,3,4,5,6,7,8,9\}$ reduced to

↳ $R_i^1 = \{2,3,4,5,6,7,8\}$

↳ $R_i^2 = \{3,4,5,6,7\}$

↳ $R_i^3 = \{4,5,6\} \rightarrow R_i^4 = \{5\}$

Criticisms of location model:
1) in context of market competition, doesn't include firms' specification of prices
2) IRL, agents may not move simultaneously
3) cannot apply the model with more than 2 products/firms
4) one-dimensional

Strategic Complementarities

Bob increases Alice's payoff but not his
Bob + Alice working together increases either or both

Contracts about effort can't necessarily be made

Complementarity

Rationalizability leads to a unique predictions in 2 player games w/ 3 properties:
1) Strategy spaces are intervals w/ lower + upper bounds
2) there are strategic complementarities
3) the slope of the best response functions is <1

↳ these are not required

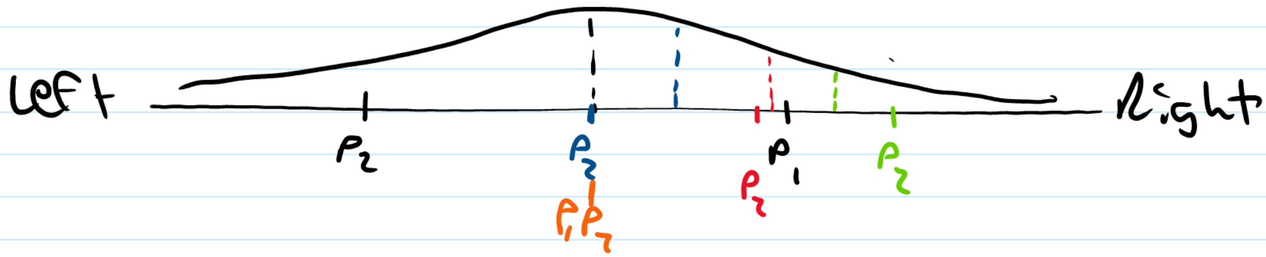
Social Unrest

Strength in numbers

8.1 Location - Median Voter Theorem - Informal

Thursday, September 10, 2020 6:47 PM

Median Voter theorem



8.2 Location - Median Voter Theorem - Formal

Thursday, September 10, 2020 6:57 PM

Location → Median Voter Theorem
- Rationalizable platforms

A & B, max share

FL L CL C CR R FR

A chooses FL, B chooses FL VA = VB = .5
B chooses L VA = 1/7, VB = 6/7
A chooses L, B chooses FL VA = , VB = 1/7
L = .5
CL = 11/14
A chooses CL, B chooses FL = 3/14
L = 2/7
CL = 1/2

FL is dom for B
FL is dom for A → and also FR

IF there are even positions, any combo of 2 middle are rationalizable

8.3 Partnership - Separable Activities

Tuesday, September 15, 2020 10:07 PM

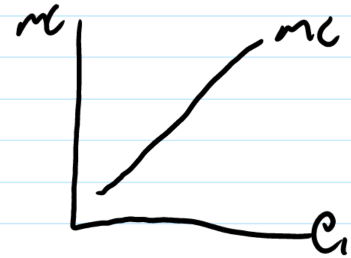
Partnership

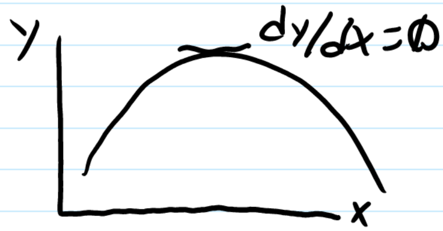
$Value = V(e_1, e_2) = e_1 + e_2$

↑ effort

$C_1(e_1) = \frac{1}{2}e_1^2 \qquad C_2(e_2) = \frac{1}{2}(e_2)^2$

$\frac{dc}{de_1} = e_1$





each keeps $\frac{1}{2}V$

$\pi_1 = \frac{1}{2}V - c_1(e_1)$

$= \frac{1}{2}e_1 + \frac{1}{2}e_2 - \frac{1}{2}e_1^2$

$\frac{d\pi_1}{de_1} = \frac{1}{2} - 2(\frac{1}{2}) \cdot e_1^{(2-1)}$

$= \frac{1}{2} - e_1 = 0$

$e_1 = 1/2$

Same for P_2, $e_2 = 1/2$

$\pi_1 = \frac{1}{2}(\frac{1}{2} + \frac{1}{2}) - \frac{1}{2} \cdot \frac{1}{2}^2$

$= \frac{1}{2}(1) - \frac{1}{2}(\frac{1}{4})$

$= \frac{1}{2} - \frac{1}{8}$

$= 3/8$

$\pi_2 = 3/8 \qquad \rightarrow \pi = \pi_1 + \pi_2 = 3/4$

$\pi = e_1 + e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$

$d\pi/de_1 = 1 - e_1 = 0 \rightarrow e_1 = 1$

$d\pi/de_2 = 1 - e_2 = 0 \rightarrow e_2 = 1$

$\pi = 2 - \frac{1}{2} \cdot 1^2 - \frac{1}{2} \cdot 1^2 = 1$

↑ Profit

If independent...

$\pi = e_1 - \frac{1}{2}e_1^2 \qquad d\pi/de_1 = 1 - e_1 = 0 \rightarrow e_1 = 1$

same for P_2

8.4 Partnership - Inseparable Activities - Best Responses

Tuesday, September 15, 2020 10:24 PM

$V = e_1 + e_2 + 1/4 e_1 e_2$

$c_1 = \frac{1}{2} e_1^2 \qquad c_2 = 1/2 e_2^2$

$\pi_1 = \frac{1}{2}(e_1 + e_2 + 1/4 e_1 e_2) - \frac{1}{2} e_1^2$

$\downarrow = \frac{1}{2}(e_1 + \bar{e}_2 + 1/4 e_1 \bar{e}_2) - \frac{1}{2} e_1^2$

$\sum_i P(X = X_i) \cdot X_i = E(X)$

$\int_0^{\infty} \Theta_2(v) \cdot v \, dv = E(e_2) = \bar{e}_2$

from P_1 view

$\pi_1 = \frac{1}{2} e_1 + \frac{1}{2}\bar{e}_2 + \frac{1}{8} e_1 \bar{e}_2 - \frac{1}{2} e_1^2$

$\frac{d\pi_1}{de_1} = \frac{1}{2} + \frac{1}{8}\bar{e}_2 - e_1 = 0 \rightarrow e_1 = \frac{1}{2} + \frac{1}{8}\bar{e}_2$

$e_2 = \frac{1}{2} + 1/8\,\bar{e}_1$

8.5 8.6 Partnership - Inseparable Activities

Tuesday, September 15, 2020 10:41 PM

$e_1 < \frac{1}{2}$ is dom $\quad e_2 < 1/2$ is dom

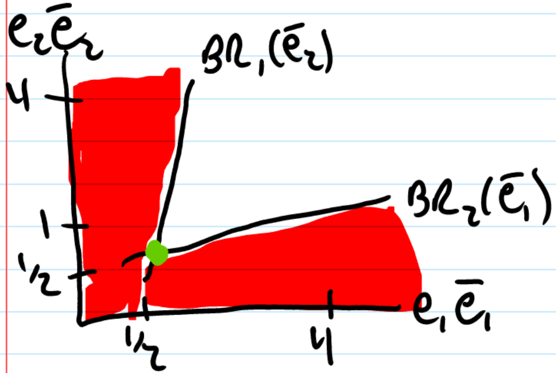
$R^1 \quad e_1 < 9/16 \quad e_2 < 9/16$ is dom

$\uparrow$ 4 as is > 1

$e_1 \leq 4$

$e_2 \leq 4$

$e_1 < \frac{1}{2} + \frac{1}{8}(9/16)$ is dom $\rightarrow$ same for e_2 $\rightarrow$ $> \frac{1}{2} + (\frac{1}{8} \cdot \frac{5}{8})$ is dom



$e_1 = \frac{1}{2} + \frac{1}{8} e_2$

$e_2 = \frac{1}{2} + \frac{1}{8} e_1$

$e = \frac{1}{2} + \frac{1}{8} e_1 \rightarrow 7/8 e_1 + 1/2 \rightarrow e_1 = 4/7 = e_2$

$e_2 = \frac{1}{2} + \frac{1}{8} \bar{e}_1 < LL \quad e_2 > LL$ when $\frac{1}{2} + \frac{1}{8} LL > LL \rightarrow 4/7 > LL$

$\uparrow$ Lower limit

$4/7 < UL$

$\uparrow$ upper limit

I found a stylus that works

I found a stylus that works

8.7 8.8 Inefficiency in the Partner Game

Saturday, September 19, 2020 2:56 PM

A

$\pi = e_1 + e_2 + \frac{1}{4}e_1e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$

$\frac{d\pi}{de_1} = 1 + \frac{1}{4}e_2 - e_1 = 0$

$e_1 = 1 + 1/4\, e_2$

↳ same for e_2 (but flipped)

$e_1 = 1 + 1/4\, e_1$

$e_1 = 4/3 = e_2$

$\pi = 4/3 + 4/3 + 1/4 \cdot 4/3 \cdot 4/3 - \frac{1}{2}(4/3)^2 - 1/2(4/3)^2 = 4/3$

$\pi_1 = \pi_2 = 2/3$

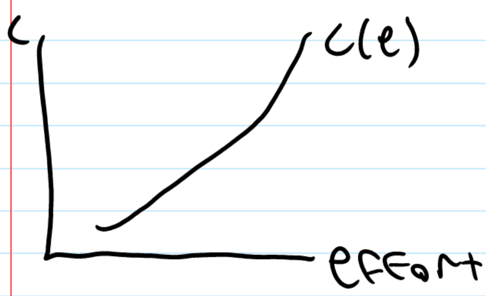
B

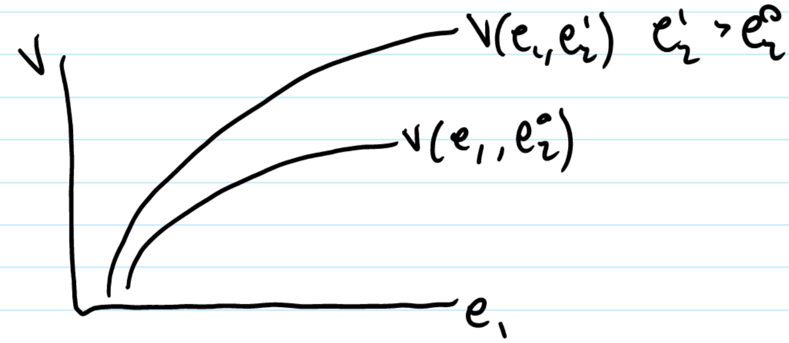
$\pi_1 = 1/2(4/7 + 4/7 + \frac{1}{4}(4/7 \cdot 4/7)) - \frac{1}{2}(4/7)^2 = 22/49$

↳ $< 2/3$

8.9 Joint Production and the Theory of the Firm

Saturday, September 19, 2020 3:08 PM





$$V(e_1, e_2) - c_1(e_1) - c_2(e_2)$$

$$dV/de_1 - dc_1/de_1 = 0$$

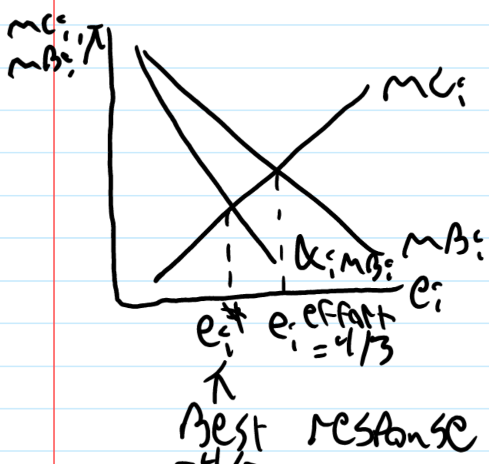
$$MB_1 = MC_1$$

$$MB_2 = MC_2$$

$$\alpha_i V(e_i, e_{-i}) - c_i(e_i)$$

$$d\pi/de_i = \alpha_i \, dV/de_i - dc/de_i = 0$$

$$\alpha_i MB_i = MC_i \rightarrow \alpha < 1$$



<u>Residual Claimant</u>

8.10 8.11 Social Unrest - Elements of the Model ~ alpha is 0

Saturday, September 19, 2020 3:58 PM

- everyone prefers unidirectional reform
- $i \in [0,1]$ index propensity to protest
  - expressing voice
  - maybe not peacefully

  uniformly distributed
- Payoffs αi if protest, $\alpha > 0$
  - let x = fraction that protests
  - $\mu = E(x)$, beliefs
  - everyone gains $\beta(x-\lambda)$
  - protesters gain $\delta(x-\lambda)$
  - non protestors gain $\gamma(\lambda - x)$

$u_i(P,x) = \alpha_i + \beta(x-\lambda) + \delta(x-\lambda) \rightarrow$ Protest

$u_i(H,x) = \gamma(\lambda - x) + \beta(x-\lambda) \longrightarrow$ Stay home

$u_i(P,\mu) > u_i(H,\mu)$

$\alpha_i + \delta(\mu-\lambda) > \gamma(\lambda-\mu)$

$\alpha_i > (\gamma+\delta)(\lambda-\mu)$

$i > (\gamma+\delta)/\alpha \cdot (\lambda-\mu)$

$\rightarrow \alpha = 0$

$0 > (\gamma+\delta)(\lambda-\mu)$

suppose $\lambda > 1 \Rightarrow \mu = 0$

now $\lambda\downarrow$ $\lambda < 1$

History + current events coordinate

i protests if $i > (\delta+\gamma)/\alpha \cdot (\lambda-\mu)$

$a = (\delta+\gamma)/\alpha$

8.12 8.13 Analysis when Protest or Voice has value - alpha gt 0

Saturday, September 19, 2020 4:19 PM

$a = \frac{(\delta + \gamma)}{\alpha}$

assume $\lambda < 1$

↳ if λ big enough, most zealous won't protest

$1 \cdot \alpha > (\delta + \gamma)(1 - 0)$

$\alpha > (\delta + \gamma)$

$a < 1$

What beliefs are rationalizable in this world?

$\mu = 1$ $\quad i > (\lambda - \mu)a < 0 \quad 0 < a < 1,\ \lambda < 1$

$\mu = 0$ $\quad i > a\lambda$

$1 > a\lambda > 0$

$\mu = 1 - a\lambda$ $\quad i > a\lambda - a\mu$

$> a\lambda - a(1 - a\lambda)$

$x = 1 - a\lambda + a(1 - a\lambda) = (1 - a\lambda)(1 + a) > (1 - a\lambda)$

$\mu = (1 - a\lambda)(1 + a)$ $\quad i > a\lambda(1 - a\lambda)(1 + a)$

$x = 1 - a\lambda + a(1 - a\lambda)(1 + a)$

$= (1 - a\lambda)(1 + a + a^2)$

$\mu = (1 + a\lambda)(1 + a + a^2 + a^3 + \ldots)$

$a < 1$

8.14 Aside on the Geometric Series

Saturday, September 19, 2020 4:35 PM

Infinite sum of a^t, $0 < a < 1$

$A = a^0 + a^1 + a^2 + \dots a^T + a^{T+1}$

$a^0 \downarrow 1$

$aA = a^1 + a^2 + \dots a^{T+1}$

$A - aA = 1 - a^{T+1}$

$A(1-a) = 1 - a^{T+1}$

$A = (1 - a^{T+1})(1/1-a)$

$\lim A_{T \to \infty}$ $a^{T+1} \to 0$ $A = 1/1-a$

8.15 Social Unrest - Conclusion

Saturday, September 19, 2020 4:45 PM

$M = (1 - a\lambda)(1 + a + a^2 + \ldots)$ $\lambda << 1$ $a\lambda < a$
$= (1 - a\lambda)/(1 - a) > 1$

$(\delta + \gamma) < \alpha \leftrightarrow$ even if $\lambda = 1$, $i = 1$ protests
$i = 1 - \epsilon$ protests
tiny number

Passed Solution review

Consider a location game with nine regions like the one discussed in this chapter. But instead of having the customers distributed uniformly across the nine regions, suppose that region 1 has a different number of customers than the other regions. Specifically, suppose that regions 2 though 9 each has ten customers, whereas region 1 has x customers. For what values of x does the strategy of locating in region 2 dominate locating in region 1?

x	10	10	10	10	10	10	10	10
1	2	3	4	5	6	7	8	9

Region 2 only makes sense if $x<80$ because at that point region 1 is less than the rest of the board combined

Player 2 location

P_1 Location	1	2	3	4	5	6	7	8	9
1	$\frac{x}{2}+40$	x	$x+5$	$x+10$	$x+15$	$x+20$	$x+25$	$x+30$	$x+35$
2	80	$\frac{x}{2}+40$	$x+10$	$x+15$	$x+20$	$x+25$	$x+30$	$x+35$	$x+40$
...									

only P_1's payoffs

$$80 > \frac{x}{2}+40$$
$$40 > x/2$$
$$80 > x$$

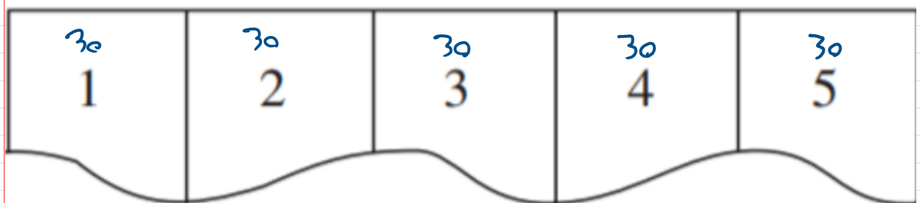
$$x/2+40 > x$$
$$40 > x/2$$
$$80 > x$$

Passed Solution review

Consider a location game with five regions on the beach in which a vendor can locate. The regions are arranged on a straight line, as in the original game discussed in the text. Instead of there being two vendors, as with the original game, suppose there are *three* vendors who simultaneously and independently select on which of the five regions to locate. There are thirty consumers in each of the five regions; each consumer will walk to the nearest vendor and purchase a soda, generating a \$1.00 profit for the vendor. Assume that if some consumers are the same distance from the two or three nearest vendors, then these consumers are split equally between these vendors.

(a) Can you rationalize the strategy of locating in region 1?

(b) If your answer to part (a) is "yes," describe a belief that makes locating at region 1 a best response. If your answer is "no," find a strategy that strictly dominates playing strategy 1.



max rev = 30 · 5 · 1 = 150

a) It's never the best strategy to locate in region 1 unless it holds value in a different way (ATM, picnic tables, etc) or if it is guaranteed that all players choose region 1

b) No: location 3 would dominate because it is in the middle and can draw both sides
Yes: If there are other amenities that would draw more people to region 1, you would need to get at least 20 extra people

A mix of 2 & 3, $\sigma_i (0, p, 1-p, 0, 0)$ dominates 1. Tedious to show

2 & 3 each weakly dominate 1. So, the mix strategy dominates 1.

3 chose s = 1

1 \ 2	1	2	3	4	5
1	50	15	25	40	55
2	15	30	30	30	30
3	25	30	40	45	45
4	40	30	45	70	75
5	55	30	45	75	100

3 chose s = 2

1 \ 2	1	2	3	4	5
1	120	60	30	45	75
2	60	50	30	40	55
3	30	30	60	60	60
4	45	40	60	70	75
5	75	35	60	75	100

0

3 chose s = 3

1 \ 2	1	2	3	4	5
1	100	90	55	45	60
2	90	90	45	30	45
3	55	45	50	45	55
4	45	30	45	90	70
5	60	45	55	90	00

HW 8.7

Sunday, September 20, 2020 4:41 PM

Passed Solution review

Consider a game in which, simultaneously, player 1 selects a number $x \in [2, 8]$ and player 2 selects a number $y \in [2, 8]$. The payoffs are given by:

(2,3) (8,8)

$$u_1(x, y) = 2xy - x^2 \quad 28 \quad 8 \quad 32 \quad 64$$

$$u_2(x, y) = 4xy - y^2. \quad 0 \quad 15 \quad 60 \quad 192$$

(2,8) (4,6)

Calculate the rationalizable strategy profiles for this game.

$u_1\ d1 = 2y - 2x = 2(y-x)$ $\qquad$ $u_2\ d1 = 4x - 2y$

$u_1\ d2 = -2$ $\qquad$ $u_2\ d2 = -2$

$2 \cdot 8 \cdot 2 - 8^2 = 4 \cdot 8 - 64 = 32 - 64 = -32$

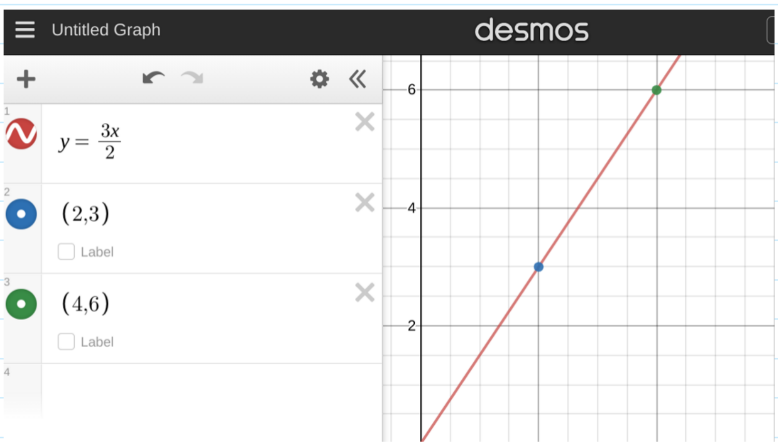
$4 \cdot 8 \cdot 2 - 2^2 = 64 - 4 = 60$

$2y - 2x = 0$

$4x - 2y = 0 \rightarrow y = \frac{3}{2}x$

↳ $x = 2, y = 3$ or $x = 4, y = 6$ are the only whole numbers between 2 and 8

But if x and y are both 8, the payoffs are $u_1 = 64$ and $u_2 = 192$



$\frac{1}{2}\left[(2xy - x^2) + (4xy - y^2) + \frac{1}{4}(2xy - x^2)(4xy - y^2)\right] - \frac{1}{2}(2xy - x^2)^2$

$du_1/dx = 2\bar{y} - 2x = 0 \rightarrow BR_1(\bar{y}) = \bar{y}$

$du_2/dx = 4\bar{x} - 2y = 0 \rightarrow BR_2(\bar{x}) = \begin{cases} 2\bar{x} & \bar{x} \leq 4 \\ 8 & \bar{x} > 4 \end{cases}$

Suppose $\bar{x} = 2$ then $y = 4$

$\bar{y} = 4$, then $x = 4$

$\bar{x} = 4$, then $y = 8$

$\bar{y} = 8$, $x = 8$ $\rightarrow$ $S = (8,8)$ is the only rationalizable strategy profile

HW 8.8

Sunday, September 27, 2020 3:30 PM

Passed solution review

8. Finish the analysis of the "social unrest" model by showing that for any $\alpha > 2$, the only rationalizable strategy profile is for all players to protest. Here is a helpful general step: Suppose that is it common knowledge that all players above y will protest, so $x \geq 1 - y$. Find a cutoff player number $f(y)$ with the property that given $x \geq 1 - y$, every player above $f(y)$ strictly prefers to protest.

Stay home → $u_i(H, x) = 4x - 2$

Protest → $u_i(P, x) = 8x - 4 + \alpha i$

$i \in (y, 1]$

$8(1-y) - 4 + \alpha i > 4(1-y) - 2$

$8 - 8y - 4 + \alpha i > 4 - 4y - 2$

$4 - 8y + \alpha i > 2 - 4y$

$2 - 8y + \alpha i > -4y$

$2 + \alpha i > 4y$

$\alpha i > 4y - 2$

$i > (4y-2)/\alpha$

if $\alpha = 2$ and $y \geq 1$, people will protest.

i protests if $8x - 4 + \alpha i > 4x - 2$

$\alpha i > 2 - 4x$

Suppose $E(x) = \bar{x} = 0$

This becomes $i > 2/\alpha$

if $\alpha > 2$ $2/\alpha < 1$ so someone will protest even if $\bar{x} = 0$

therefore $\bar{x} = 0$ is not rationalizable

Suppose $\bar{x} = 1 - 2/\alpha$ which is the least rationalizable belief in R'

Now i protests if $\alpha i > 2 - 4(1 - 2/\alpha)$

$\alpha i > -2 + 8/\alpha$

$i > -2/\alpha + 8/\alpha^2$

the least rationalizable belief is now

$\bar{x} = 1 + 2/\alpha - 8/\alpha^2$

$= 1 - 2/\alpha(4/\alpha - 1) > 1 - 2/\alpha$

each round x grows until

$\alpha(1-x) = 2 - 4x$

$\therefore x = \alpha - 2/\alpha - 4 > 1 \rightarrow$ so $x = 1$

9 Book Notes

Wednesday, September 23, 2020 9:04 AM

Nash Equilibrium

Rationalizability means:
1) Players form beliefs about each other's behavior
2) Players best respond to their beliefs
3) these facts are common knowledge among players

Behavior is congruous (coordinated) through social norms

Congruity:
1) games are repeated in society and player behavior "settles down" and the same strategies are repeated
2) Players meet before the game + agree on the strategies used. Players honor the agreement
3) Outside mediator recommends strategy profiles. Each player expects others to follow the recommendation + has incentive to do so themselves

Nash equilibrium: strategy profile $s \in S$ is a Nash equilibrium if and only if $s_i \in BR_i(s_{-i})$ for each player i. That is, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for every $s_i' \in S_i$ and each player i

Strict Nash equilibrium: Strategy profile s is strict if and only if $\{s_i\} = BR_i(s_{-i})$ for each player i

To find: "Look for profiles such that each player's strategy is a best response to the strategy of the other"

Each Nash equilibrium is a rationalizable strategy

Equilibrium of the Partner Game

Coordination + Social Welfare

Nash equilibria don't always entail strategies that are preferred by the players as a group

↳ Nash of prisoner's dilemma (1,1) is inefficient

The third strategic tension

Coordinated inefficiency → QWERTY, VHS, etc.

Congruous Sets

Nash is only when players coordinate on a single profile

Set X is a congruous set because coordinating on X is consistent with common knowledge of BR

Congruous if X_i contains exactly those strategies that can be rationalized
Weakly congruous if each strategy in X_i can be rationalized with respect to X_{-i}

Experimental Game Theory

Strategic sophistication

9.1 Nash Equilibrium - General

Wednesday, September 23, 2020 11:54 AM

Solution Concepts so far...

1) Strongly dominant strategies
2) Weakly dominant strategies
3) Set of rationalizable strategies
   ↳ what if there's more than 1?
4) Nash Equilibrium
   ↳ $s \in S$ is a NE if $s_i \in BR_i(s_{-i})\ \forall i$
   ↳ or $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})\ \forall i$

Congruent

	B_1	B_2	B_3
A_1	$\boxed{3,1}$	2,3	4,0
A_2	0,6	0,0	2,0
A_3	5,2	1,4	$\boxed{3,1}$

Best replies given other player's strategy

(A_1, B_2) is only pure strategy NE in game

1) Repeated observed play across society
2) Players talk before playing + honoring agreement

A\B	Rat	Silent
Rat	0*,0*	3*,-1
Silent	-1,3*	2,2

(Rat, Rat)

A\B	H	T
H	1*,-1	-1,1*
T	-1,1*	1*,-1

No Pure NE

A\B	Game	Show
Game	2,3	0,1
Show	1,0	3,2

2 Pure NE

↳ $\{G,S\} \times \{G,S\}$

A\B	Stag	Hare
Stag	4,4	0,1
Hare	1,0	1,1

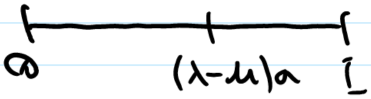
2 Pure NE

3rd strategic tension - Possibility of socially inefficient coordination

9.3 Nash Equilibrium - Social Unrest

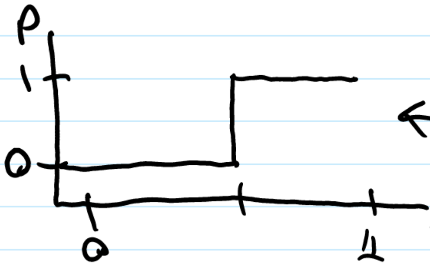
Friday, September 25, 2020 7:17 PM

$i > (\lambda-\mu)(\sigma+\delta/\alpha)$ ← Protest equation



BR $p=1$ $i>(\lambda-\mu)a$
$p=0$ $i<(\lambda-\mu)a$

$\vdash 1-x \dashv$ = indifferent to protest if $i=(\lambda-\mu)a$ or
$i=(\lambda-\mu)a$ $1-x=(\lambda-\mu)a$



← BR function

Fraction that turns out $1-x=(\lambda-\mu)a \to \mu=(1-a\lambda)/(1-a)$
where $\lambda<1 \to \mu>1$

if $\lambda\le 1 \to \mu=1$ $x=1$ → everyone protests

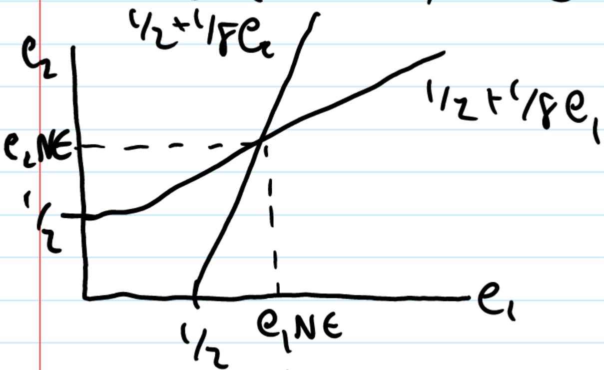
9.4 Nash Equilibrium - Partnership

Friday, September 25, 2020 7:26 PM

$u_1 = \frac{1}{2}(e_1 + e_2 + e_1 e_2) - \frac{1}{2}e_1^2$

$\frac{du_1}{de_1} = \frac{1}{2}(1 + \frac{1}{4}\bar{e}_2) - e_1 = 0$ $BR_1(\bar{e}_2) = \frac{1}{2} + \frac{1}{8}\bar{e}_2 \leftarrow$ 2 equations - 2 unknown

Same for player 2



$e_1 = e_2$

↳ $4/7$

HW 9.2

Thursday, October 1, 2020 10:11 AM

Passed solution review

Find the Nash equilibria of and the set of rationalizable strategies for the games in Exercise 1 at the end of Chapter 6.

NE = (B,L) and S = (B,L)

1 \ 2	L	R
A	3, 3	2, 0
B	4, 1	8, −1

(a)

NE = (U,L) and (M,C)

L dom R

1 \ 2	L	C	R
U	5, 9	0, 1	4, 3
M	3, 2	0, 9	1, 1
D	2, 8	0, 1	8, 4

UL makes more sense

(b)

NE = (U,X)

1 \ 2	W	X	Y	Z
U	3, 6	4, 10	5, 0	0, 8
M	2, 6	3, 3	4, 10	1, 1
D	1, 5	2, 9	3, 0	4, 6

(c)

1 \ 2	L	R
U	1, 1	0, 0
D	0, 0	5, 5

(d)

All S are rationalizable

S = (U,X)

1 \ 2	~~W~~	X	~~Y~~	~~Z~~
U	3, 6	4, 10	5, 0	0, 8
~~M~~	2, 6	3, 3	4, 10	1, 1
~~D~~	1, 5	2, 9	3, 0	4, 6

(c)

Passed solution review

Compute the Nash equilibria of the following location game. There are two people who simultaneously select numbers between zero and one. Suppose player 1 chooses s_1 and player 2 chooses s_2. If $s_i < s_j$, then player i gets a payoff of $(s_i + s_j)/2$ and player j obtains $1 - (s_i + s_j)/2$, for $i = 1, 2$. If $s_1 = s_2$, then both players get a payoff of $1/2$.

$s_i = x \quad s_j = y$

$\frac{(x+y)}{2} = 1 - \frac{(x+y)}{2}$

$\frac{2(x+y)}{2} = 1$

$x + y = 1$

if s_i and s_j both $= 1/2$, then they maximize and won't deviate

HW 9.6

Thursday, October 1, 2020 10:28 AM

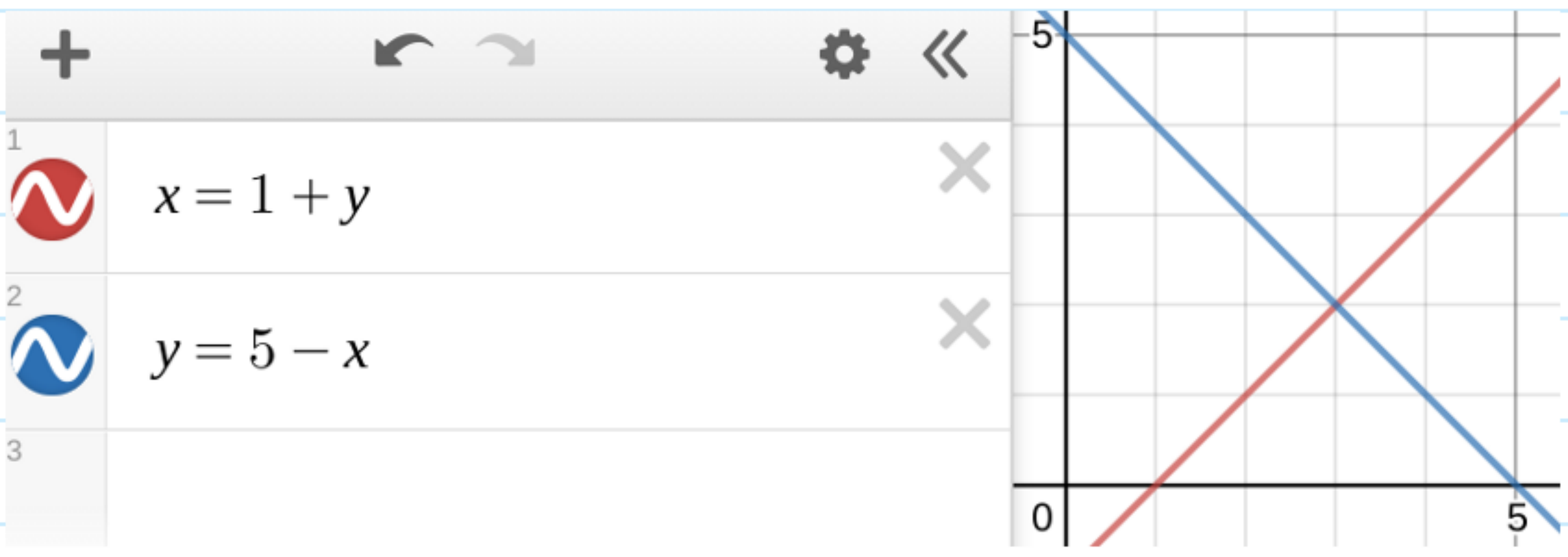
Passed Solution review

Consider a game in which, simultaneously, player 1 selects any real number x and player 2 selects any real number y. The payoffs are given by:

$$u_1(x, y) = 2x - x^2 + 2xy$$
$$u_2(x, y) = 10y - 2xy - y^2.$$

(a) Calculate and graph each player's best-response function as a function of the opposing player's pure strategy.

$du_1/dx = 2 - 2x + 2 \Rightarrow x = 1 + y$
$du_2/dx = 10 - 2y - 2x \rightarrow y = 5 - x$



(b) Find and report the Nash equilibria of the game.

$x = 1 + 5 - x$
$x = 6 - x$
$2x = 6$
$x = 3 \rightarrow y = 5 - 3 = 2$

$NE = (3, 2)$

(c) Determine the rationalizable strategy profiles for this game.

$s = (3, 2)$

HW 9.10

Thursday, October 1, 2020 10:38 AM

Passed solution review

Is the following statement true or false? If it is true, explain why. If it is false, provide a game that illustrates that it is false. "If a Nash equilibrium is not strict, then it is not efficient."

A \ B	y	z
w	1,1	1,2
x	2,1	4,4

False. (w,y) and (x,z) are both NE but (x,z) is better than (w,y)

HW 9.11

Sunday, October 4, 2020 4:18 PM

Passed Solution review

This exercise asks you to consider what happens when players choose their actions by a simple rule of thumb instead of by reasoning. Suppose that two players play a specific finite simultaneous-move game many times. The first time the game is played, each player selects a pure strategy at random. If player i has m_i strategies, then she plays each strategy s_i with probability $1/m_i$. At all subsequent times at which the game is played, however, each player i plays a best response to the pure strategy actually chosen by the other player the *previous* time the game was played. If player i has k strategies that are best responses, then she randomizes among them, playing each strategy with probability $1/k$.

(a) Suppose that the game being played is a prisoners' dilemma. Explain what will happen over time.

They confess forever because it's always the best response.

(b) Next suppose that the game being played is the battle of the sexes. In the long run, as the game is played over and over, does play always settle down to a Nash equilibrium? Explain.

	A	B
A	2,1	0,0
B	0,0	1,2

If (0,0) is chosen first, then (0,0) is the BR so they look for each other forever. If (2,1) or (1,2) is chosen first, it settles.

(c) What if, by chance, the players happen to play a strict Nash equilibrium the first time they play the game? What will be played in the future? Explain how the assumption of a strict Nash equilibrium, rather than a nonstrict Nash equilibrium, makes a difference here.

If it's strict, they'll never deviat because strict Nash is always better.

(d) Suppose that, for the game being played, a particular strategy s_i is not rationalizable. Is it possible that this strategy would be played in the long run? Explain carefully.

No. only best responses are played

HW 9.15

Sunday, October 4, 2020 4:43 PM

Worked w/ Dr Dewey's Solutions

Passed Solution review

15. Suppose you know the following for a particular three-player game: The space of strategy profiles S is finite. Also, for every $s \in S$, it is the case that $u_2(s) = 3u_1(s)$, $u_3(s) = [u_1(s)]^2$, and $u_1(s) \in [0, 1]$.

(a) Must this game have a Nash equilibrium? Explain your answer.

$u_2 = 3u_1$ $\quad u_3 = u_1^2 \rightarrow u_1 = \sqrt{u_3}$ $\quad u_2 = 3\sqrt{u_3}$

I'm trying to follow your solution because I don't understand.

Because u_2 and u_3 are quantities of u_1, they always try to maximize u_1. This means that u_1 is always an NE

(b) Must this game have an efficient Nash equilibrium? Explain your answer.

It is efficient because of reasons stated in Part A

(c) Suppose that in addition to the information given above, you know that s^* is a Nash equilibrium of the game. Must s^* be an efficient strategy profile? Explain your answer; if you answer "no," then provide a counterexample.

There could be an inefficient, non-strict NE because conditions don't dictate only having a strict NE

A / B

A:

	L	R
U	1,3,1	0,0,0
D	0,0,0	0,0,0

B:

	L	R
U	3,9,9	0,0,0
D	0,0,0	0,0,0

u_1, u_2, u_3

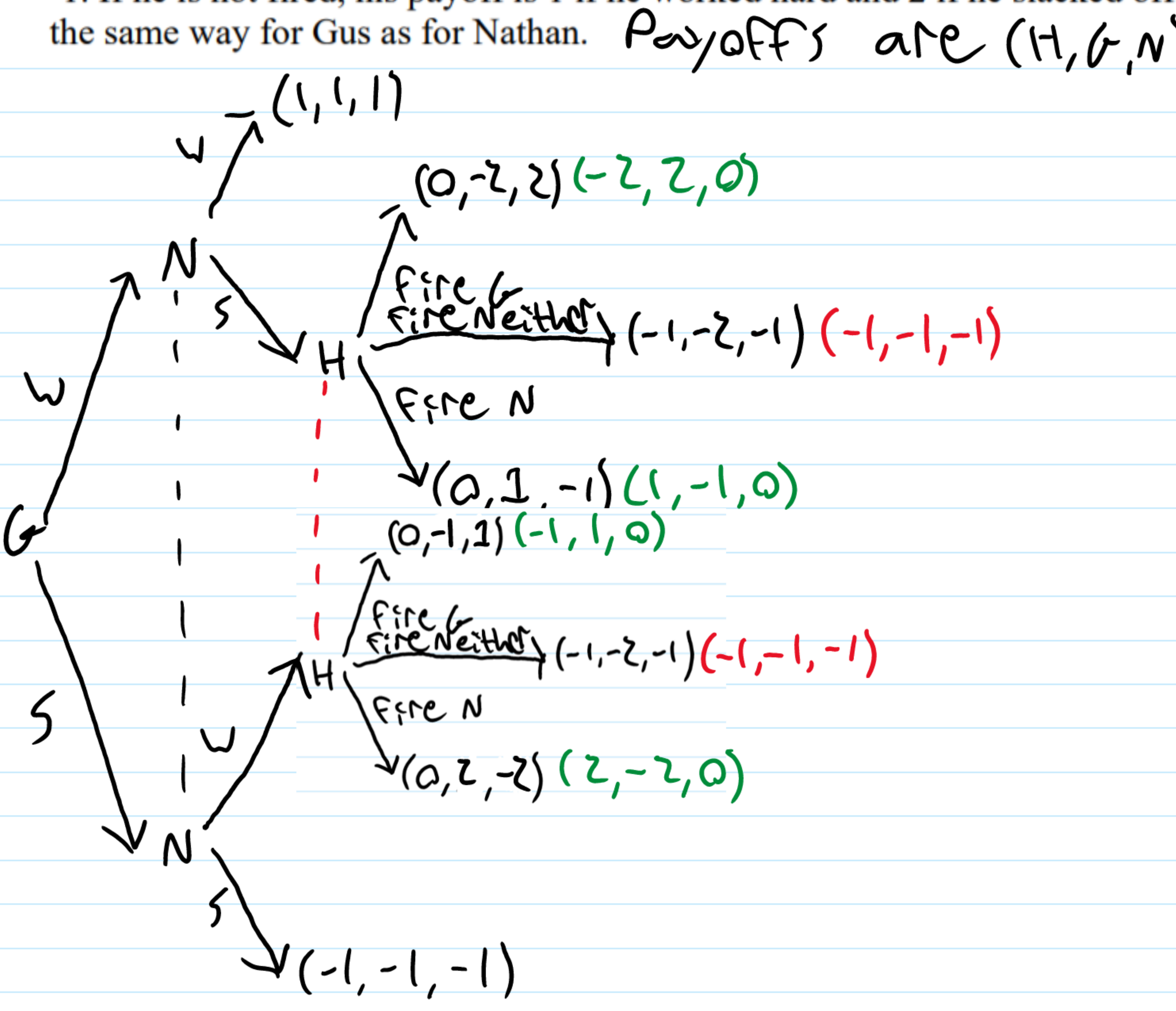
ALU and BLU are both NE, but ALU is inefficient

Midterm - Gus Lipkin

Friday, October 9, 2020 9:58 AM

1. Draw the extensive form for this game:

- Hailey (H) is the supervisor of Gus (G) and Nathan (N). Both Gus and Nathan choose to work hard (W) or slack off (S). Neither Gus not Nathan know if the other worked hard. Hailey can tell if anyone worked or not, but cannot tell who it was if only one does so.
- If both choose to work hard, the game ends, and all three get a payoff of 1.
- If both slack off, upper management sees the unit fail, all three get fired, and the game ends with all three receiving a payoff of -1. (You need not make upper management a player in the game—their only role was in determining the rule for payoffs.)
- If only one works hard, Hailey will see productivity is less than it should be and get a choice. Hailey can fire Nathan or Gus and keep her job, receiving a payoff of 0. If she fires neither, upper management fires the whole team and Hailey receives a payoff of -1.
- If Nathan is fired, if he worked hard his payoff is -2 and if he did not work hard his payoff is -1. If he is not fired, his payoff is 1 if he worked hard and 2 if he slacked off. Payoffs work the same way for Gus as for Nathan. Payoffs are (H,G,N)



2. For the game below, find the set of rationalizable strategies and any Nash Equilibria.

		Player 1			
		W	X	Y	Z
Player 2	A	~~1, 2~~	~~1, 2~~	~~0, 1~~	0, 1
	B	2, 1	0, 0	3, 4	0, 1
	C	2, 2	2, 3	2, 1	0, 2

0) X+Y dom Z
1) C dom A
2) X+Y dom W

forgot to do iterated dominance

NE are (C,X) and (B,Y)

The NE are (C,W), (C,X), (B,Y), and (0,Z)

		Player 1			
		W	X	Y	Z
Player 2	A	1, 2	1, 2	0, 1	0, 1
	B	2, 1	0, 0	3, 4	0, 1
	C	2, 2	2, 3	2, 1	0, 2

(2/3 C + 1/3 B, Y) is the rationalizable strategy

rationalizable profiles are {B,C} x {X,Y}

3. Jackson (J) and Nick (N), partners, each decide to work hard or not, without observing the choice of the other. Let H_i be one if player $i \in \{J, N\}$ works hard and 0 otherwise. Payoffs for each are $3(1+H_N+H_J)-4H_i$. Represent the game in normal form. Find the rationalizable strategies and Nash equilibria of this game. Discuss the strategic tensions in the game.

I was right!

Similar to Prisoner's dilemma

J \ N	H	N
H	5, 5	(2,6) 6, 2
N	2, 6 (6,2)	3, 3

HH = 3(1+1+1) - 4(1) = 3(3) - 4 = 9 - 4 = 5
NN = 3(1+0+0) - 4(0) = 3 - 0 = 3

HN = 3(1+1+0) - 4(1) = 3(2) - 4 = 6 - 4 = 2
NH = 3(1+0+1) - 4(0) = 6 - 0 = 6

← this mistake carried through

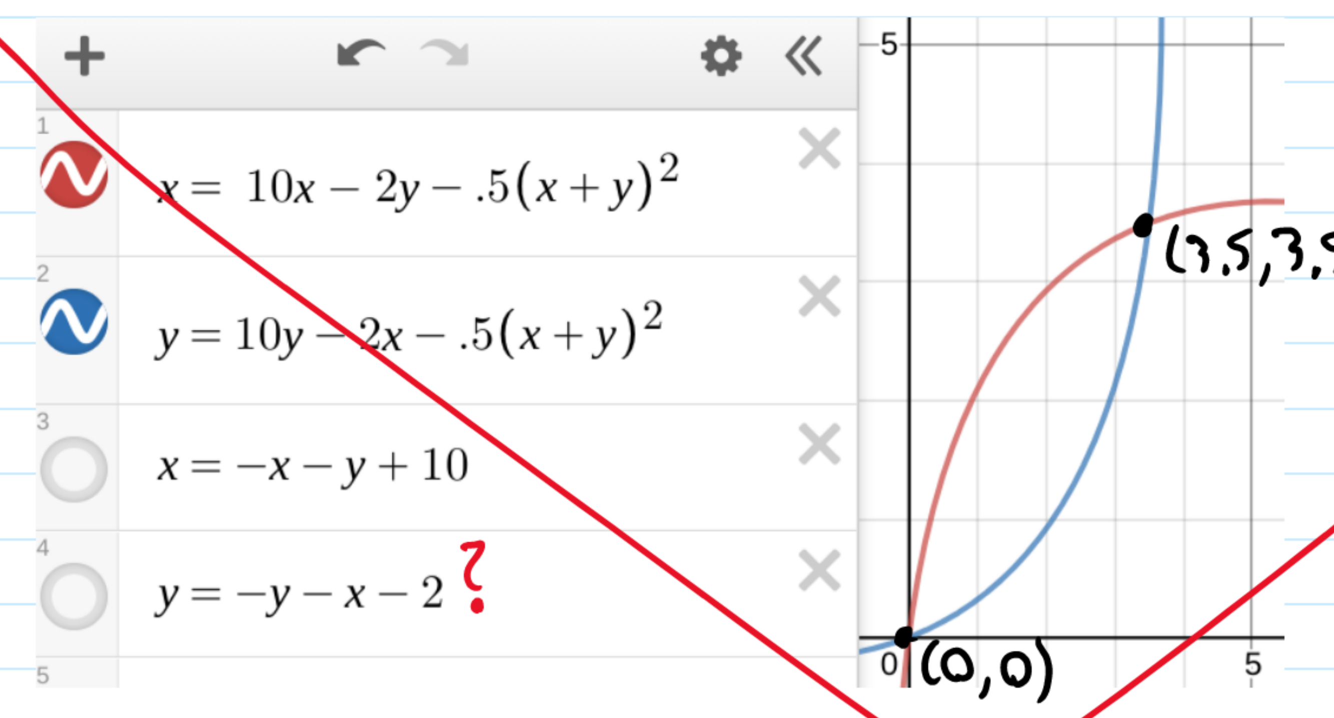
All strategies are rationalizable

(H,H) is the NE (N,N) is NE is strictly dominant

They will get a greater payout if they don't work hard and the other does. But if they both don't work hard, they both get less than if they cooperated & worked hard together.

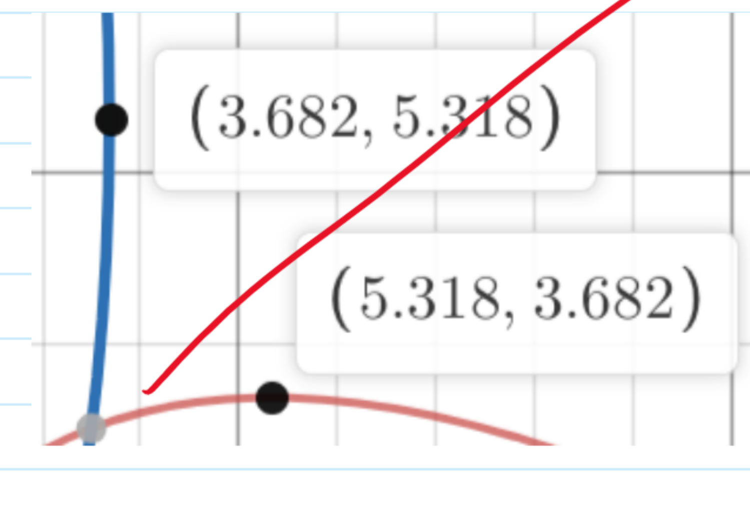
4. Isabel (I) and Raquel (R) each choose how many minutes of air time to purchase to advertise their product, with A_i denoting the number of minutes chosen by player $i \in \{I, R\}$. Isabel's payoff is $10A_I-2A_R-0.5(A_I+A_R)^2$. Raquel's payoffs are similarly defined. Find and graph the best response functions. Find the set of rationalizable strategies and the Nash Equilibria, if any.

messed up variables for R's payoff



NE is (3.5,3.5)

BR is in range



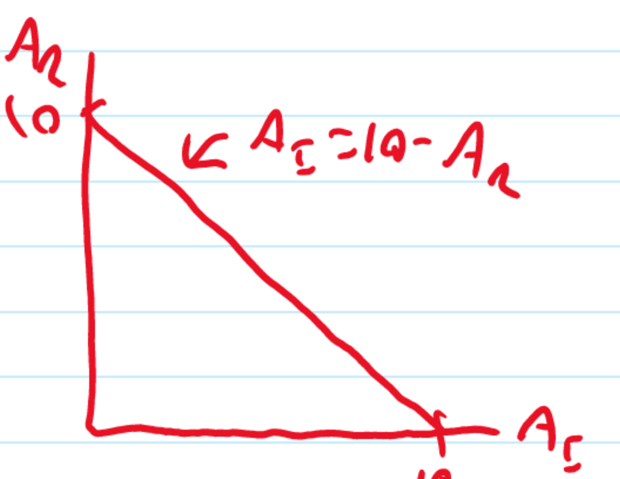
← maximum values for best responses
minimum is 0

$u_I = 10A_I - 2A_R - \frac{1}{2}(A_I + A_R)^2$

$du_I/dA_I = 10 - A_I + A_R = 0 \rightarrow A_I = 10 - A_R$

$u_R = 10A_R - 2A_I - \frac{1}{2}(A_R + A_I)^2$

$du_R/dA_R = 10 - A_R - A_I = 0 \rightarrow A_R = 10 - A_I$



The functions intersect at all values where $A_I + A_R = 10$

[0,10] x [10,0] are rationalizable

All combos that sum to 10 are NE

[Witty RPS/RBG Title]
Gus Lipkin
Florida Polytechnic University

Introduction

In elementary school, we are all taught that three branches of the federal government have checks and balances so that one branch never gains too much power. For almost 250 years, this system has functioned reasonably well with some minor power struggles and disagreements here and there, but no branch has managed to battle the other two and definitively come out on top. Even with the passing of Supreme Court Justice Ruth Bader Ginsburg, the Supreme Court will hold the same amount of power that it did before her passing. The issue right now is not that one branch has more power than another, but that the same people are in power in both the Executive and Legislative branches and stand a lot to gain by installing a new justice with whom they share a common lack of values and integrity. By applying game theory to the Supreme Court nomination process, we can learn where the system is failing and explore possible ways to improve it.

Background

Since the Circuit Judges Act of 1869, the United States Supreme Court has held space for one Chief Justice and eight Associate Justices (*Circuit Judges Act*, n.d.). When a vacancy in the Supreme Court opens, either by the death of a justice or their retirement, the President nominates a candidate who then goes before the Senate Judiciary Committee (Nguyen, 2015). If the committee approves of the nomination, they send their recommendation to the full Senate where the Senate then debates and ultimately decides the nomination with the Vice President stepping in as a tiebreaker (Nguyen, 2015). In order to properly understand the situation that Ruth Bader Ginsburg's passing creates, we must first understand the last time a Supreme Court Justice passed before retiring.

In February of 2016, just nine months before the presidential election in November and almost a full year before President Barack Obama's successor would take power, Justice Antonin Scalia passed away (Elving, 2018). Within hours of Justice Scalia's passing, Senate Majority Leader Mitch McConnell announced that the Republican senate would block the nomination of Merrick Garland because nine months was too close to the election of the next President (Elving, 2018). McConnell then removed one of the last tools available to Senators that would allow them to block a nomination vote by speaking until the vote could no longer be held, known as a filibuster (Elving, 2018). This allowed the seat to remain vacant until after the next President, Donald Trump, was elected.

Having already nominated Justices Neil Gorsuch and Brett Kavanaugh to the Supreme Court, President Donald Trump has the opportunity to nominate a third

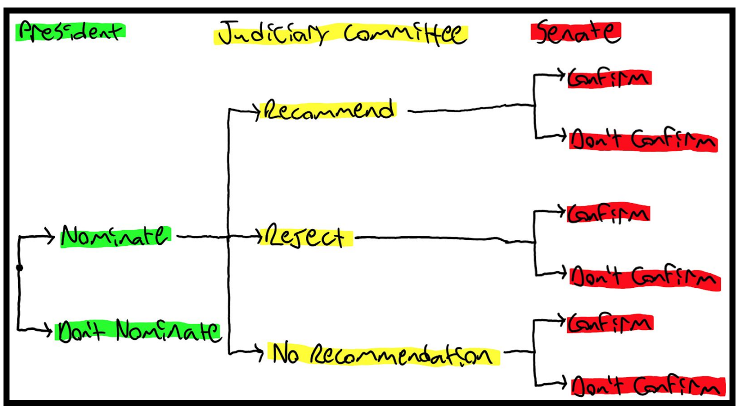
Justice by the end of his first term (*Complete List of Donald Trump's Potential Nominees to the U.S. Supreme Court*, n.d.). Once again, within hours of a Supreme Court Justice's passing, Senate Majority Leader Mitch McConnell made an announcement on the nomination of a new Justice: "President Trump's nominee will receive a vote on the floor of the United States Senate" (Cochrane, 2020) & (Gambino, 2020)). This is an extreme reversal of his announcement in 2016 that because it is an election year, the President should not be able to nominate anyone to the Supreme Court (Elving, 2018).

Description of the Games

Ideally, the nomination of a new Supreme Court Justice could be represented by the following extensive form game:

Figure 1

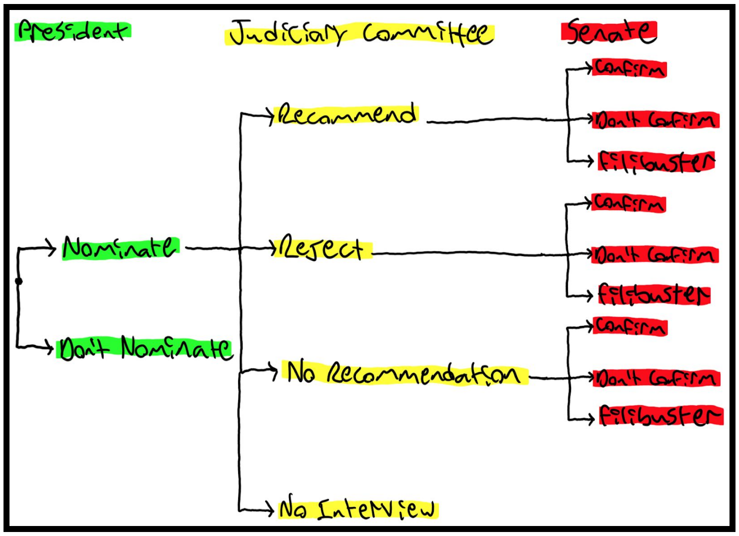
Extensive Form game representing the normal Supreme Court Nomination Process



While a President could choose not to nominate a Justice, the game only ends until the next President chooses to nominate a Justice. Once a President does nominate, the game does not end until the Senate either confirms or denies the nominee. However, if the Senate is unwilling to cooperate with the President or believes that the President should not be allowed to make that nomination, they are able to change the way the game is played and add more moves that they can make. The changed game gives the Judiciary Committee and the Senate a new move each. The former can choose to ignore the nomination and to not interview any nominees that the President puts forth while the latter can choose not to vote on the nominee regardless of the recommendation put forth by the Judiciary Committee.

Figure 2

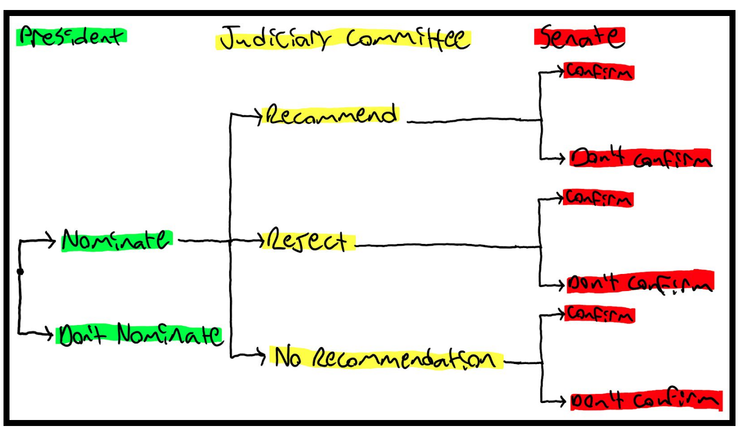
Extensive Form game representing the Supreme Court Nomination Process for Merrick Garland



In the game representing the nomination of Merrick Garland in 2016, the Republican controlled Judiciary Committee gave themselves a new move. By refusing to interview a nominee, they could effectively end the game. In reality, the game then returns to the President until they nominate someone new that the Judiciary Committee will interview or until a new President is sworn in and nominates someone new that the committee will approve of (Elving, 2018). Had Merrick Garland been interviewed by the Judiciary Committee, the Senate Republicans could have been filibustered as a way to block a vote because the Senate Democrats did not have enough members to force a vote (Elving, 2018).

Figure 3

Extensive Form game representing the current Supreme Court nomination process



While filibuster is still an option in the current Supreme Court nomination process, it has effectively been eliminated by changing the cloture rules, the rules that allow the senate to vote to end a debate, so that only a simple majority of fifty-one votes is needed (Smith, 2017). Because the Republicans control fifty-three of the 100 total Senate seats, they have the simple majority that they need to end any Democratic attempt at filibuster (Snell, 2018). Of the fifty-three Republican Senators, eleven are on the Judiciary Committee which has twenty-one total seats (*Members*, n.d.). This means that they can force any nominee through the committee and refusing to interview the nominee is no longer an option.

The current Supreme Court nomination process is relatively fair in a well-balanced government. However, because Senate Republicans have put their loyalty to the President over their loyalty to what few values they may have, the game is no longer balanced. No matter who the President nominates, the Judiciary Committee will interview them because the Democrats do not have the majority they need to refuse to interview them. Then, regardless of the recommendation the committee makes, the nominee goes before the full senate to either be confirmed or not confirmed. A filibuster from the Democrats is no longer an option because the Republicans only need fifty-one votes to end debate and control fifty-three seats and only two Republican Senators have said they would not vote for the nominee (Molly Reynolds, 2020). This leaves exactly the number of votes needed to confirm the new Justice (Smith, 2017). Nevertheless, Democrats press on and are trying to find ways to stall the Senate so that they cannot vote on the nominee (Molly Reynolds, 2020). One such tactic would be for the House to impeach Attorney General Bill Barr or President Trump because impeachment proceedings take precedent in the Senate (Molly Reynolds, 2020). But this is unlikely to work because impeachments require fifty-one votes as well, which the Republicans have ((Snell, 2018 & Molly Reynolds, 2020)).

How to Balance the Game

Regardless of whether or not the Republicans or Democrats control the Presidency, Senate, and House, this is an unfair game and must be balanced for democracy to continue to function. This is extremely difficult. Do you change the game in its current form or create a new game? If you create a new game, how do you ensure it does not become imbalanced like the current one? Who is chosen to design the game? In my opinion, the ideal solution would be to put the confirmation vote in the hands of the people but this still leaves the position vulnerable to poor decisions by voters.

If voters were presented with a shortlist of candidates from the President and the current first past the post system was used, there would be no mechanism to vote against someone. If the President were to release a list of too many nominees, the Associate Justice position could be won with a very small portion of the vote such as when Jake Auchincloss won House seat MA-4 with only 22.41% of the vote (Newsroom, 2020).

Because the Judiciary Committee does not hold true power in the game besides refusing to interview the nominee, there is no substantial change that could be made at this point in the game. This leaves the Senate as a whole as the next step at which changes could be made, but there are issues that arise similar to those if the general population were to vote. So long as the President is the only one able to nominate candidates, the senate can only vote against the nominees and never put forward a candidate themselves. Even so, this leaves the nominee vulnerable to a blanket no vote from the opposing party so long as that party has fifty-one or more members.

It appears that without changing its entire structure, there is little room to balance the nomination process. As the hit movie *War Games* taught, sometimes the only way to win is to not play. While we have to nominate Supreme Court Justices, maybe we can play a different game that will allow us to influence the nomination process. By first instituting eighteen year term limits for Supreme Court Justices, each President would be able to appoint a Justice in the first and third year of each four year term (Newsroom, 2020). This creates a gap if there is a vacancy on the bench. To fill the gap, any retired Supreme Court Justice could be brought back in the order of retirement until it is time to appoint a new Justice (Newsroom, 2020). While these changes do not guarantee a certain quality of Justice appointed to the Supreme Court, it does provide a mechanism for Justices who become unpopular to have a set retirement date.

Conclusion

By studying the current and past nomination processes as games using game theory, we come to understand where the system is failing. We understand that something must change, but creating a new system by which a Supreme Court Justice is nominated is a daunting task. It must maintain the balance of power in the federal government and must be appealing to people of all political ideologies. Unfortunately, while there are people in power who have abandoned their duty to the people and have no moral compass, they will always seek to better themselves before the people and that means confirming Justices that are not qualified and unable to separate church and state.

Bibliography

Circuit Judges Act. (n.d.). Retrieved September 24, 2020, from https://www.fjc.gov/history/timeline/circuit-judges-act

Cochrane, E. (2020, September 19). How Mitch McConnell Can Quickly Push Through Trump's Supreme Court Nominee. *The New York Times*. https://www.nytimes.com/2020/09/19/us/politics/mitch-mcconnell-trump-supreme-court.html

Complete list of Donald Trump's potential nominees to the U.S. Supreme Court. (n.d.). Retrieved September 24, 2020, from https://ballotpedia.org/Complete_list_of_Donald_Trump%27s_potential_nominees_to_the_U.S._Supreme_Court

Elving, R. (2018, June 29). *What Happened With Merrick Garland In 2016 And Why It Matters Now*. NPR. https://www.npr.org/2018/06/29/624467256/what-happened-with-merrick-garland-in-2016-and-why-it-matters-now

Gambino, L. (2020, September 19). *Mitch McConnell vows US Senate will push on with Trump's pick to replace Ginsburg*. http://www.theguardian.com/us-news/2020/sep/19/mitch-mcconnell-vows-us-senate-will-push-on-with-trumps-pick-to-replace-ginsburg

Members. (n.d.). Retrieved September 24, 2020, from https://www.judiciary.senate.gov/about/members

Molly Reynolds, B. W. (2020, September 22). *Can Democrats Stop the Nomination?* https://www.theatlantic.com/ideas/archive/2020/09/questions-watch-senate/616428/

Newsroom, W. (2020, September 1). *Live Election Results: 2020 Mass. Senate, Contested Congressional District Primaries*. WBUR. https://www.wbur.org/news/2020/09/01/massachusetts-senate-congress-primary-race-results

Nguyen, T. (2015). *Guides: Supreme Court Nominations Research Guide: Nomination & Confirmation Process*. https://guides.ll.georgetown.edu/c.php?g=365722&p=2471070

Smith, A. (2017, April 6). *SENATE GOES NUCLEAR: McConnell kills the filibuster for Supreme Court nominees to get Trump's court pick over the top*. Business Insider. https://www.businessinsider.com/nuclear-option-gorsuch-filibuster-senate-2017-4

Snell, K. (2018, November 6). *Election Results Give Split Decision: Democrats Win House & GOP Keeps Senate Majority*. NPR. https://www.npr.org/2018/11/06/664506915/republicans-keep-senate-majority-as-democrats-make-gains-in-the-house