HW 23.8

Sunday, November 15, 2020 3:55 PM

Passed Solution review

8. Consider the prisoners' dilemma stage game pictured here:

Row player \ Column player	C	D
C	3, 3	0, 5
D	5, 0	2, 2

Row player's payoff is listed first.

The following questions ask you to consider various discrete-time environments in which people meet and play the stage game. For each environment, you are to determine the extent to which cooperation can be sustained in a subgame perfect equilibrium.

(a) Suppose that two players interact in an infinitely repeated game, with the stage game shown, and that the players share the discount factor $\delta \in (0, 1)$. Under what conditions on δ is there a subgame perfect equilibrium that supports both players selecting C each period?

$\frac{3}{1-\delta} = (5-2) + 2/1-\delta \rightarrow \delta = 2/3$

(b) Suppose that there is a society of k individuals. In each period, two people from the society are randomly selected to play the stage game. Individuals are equally likely to be selected. Furthermore, the random draw that determines who is selected in a given period is independent of the outcomes in previous periods. Thus, an individual has a probability $2/k$ of being selected to play the stage game in any given period. One of the people selected plays the role of Row player in the stage game, whereas the other person plays the role of Column player. (The stage game is symmetric, so the roles do not matter.) Those who are selected obtain the payoffs of the stage game, whereas the individuals who were not selected get 0 for this period. Everyone discounts the future using discount factor δ.

Assume that the individuals can identify each other. That is, individuals know the names and faces of everyone else in the society. Assume also that before the stage game is played in a given period, everyone observes who is selected to play. Further, everyone observes the outcome of each period's stage game.

Is there a sense in which it is more difficult to sustain cooperation in this random-matching setting than was the case in part (a)? Explain why or why not. Calculate the cutoff value of δ under which cooperation can be sustained.

This definitely isn't right and I do know why. I've gone too far ahead and don't know what's going on

Manual says the answer is $\delta \geq k/k+1$

$3 + [\delta/k(1-\delta)] \cdot 3 \geq 5 + 2[\delta/k(1-\delta)]$

$2\delta \geq 2k - 2k\delta$

$\delta(1+k) \geq k$

$\delta \geq k/1+k$

It is more difficult because there will be less to lose in the future since individuals play only 2/k of future days

(c) Consider the same setting as in part (b) *except assume that the individuals only observe their own history of play*. As before, individuals can identify each other. However, an individual only knows when he is selected to play the stage game; if he is not selected to play in a given period, he observes neither who was selected to play nor the outcome of the stage game. When an individual is selected to play the stage game, he learns whom he is matched with; then the stage game is played, and the two matched individuals observe the outcome.

Explain why cooperation is more difficult to sustain in this setting than in the setting of part (b). If you can, calculate the cutoff value of δ under which cooperation can be sustained.

Cooperation is more difficult to sustain because the players only know whether or not they have cooperated and whether or not the players they've faced cooperated when they faced them.
I cannot calculate the cutoff value of delta because I don't know how. But the book says it's:

$\delta = 4(k-2)!/4(k-2)! + k!$

I don't understand this. Are the factorials there because you can only know a limited portion of the total history? It seems like a heavily modified version of the answer from part b. I graphed it and it gave me a slight bell curve which is too shallow to be a normal distribution.

Coop is much harder to sustain because you can only be punished if you are matched with someone you played against before