The Impact of Precalculus on Student Success at Florida Polytechnic University: A Study of the Initial Academic Year (2017-2018)

By

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ABSTRACT

Success rates in calculus 1 are imperative to student success and retention rate in Science, Technology, Engineering, and Mathematics focused institutions and programs. This study uses a fuzzy regression discontinuity design to estimate the effect of placing students in precalculus using the Assessment and Learning in Knowledge Spaces (ALEKS) Placement Test at Florida Polytechnic University in the 2017-2018 academic year. Placement into Precalculus reduced the probability of successfully completing Calculus 1 by the end of the second semester of enrollment by 0.5 relative to placement into Calculus. Two mechanisms potentially account for most of this effect. First peer effects reflected by higher high school GPAs in the Calculus sections populated by those not placed into Precalculus raised the success of those students while the opposite was true for those placed first into Precalculus. When applied to the whole cohort of students, one cohort's increase in success cancels the decrease in success of the other cohort. Second, the structure of the intervention itself may reduce the chance of success by introducing another hurdle in the form of the need to pass Precalculus before attempting Calculus and by reducing the possible number of attempts at Calculus prior to the end of the Spring semester.

ACKNOWLEDGEMENTS

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1. INTRODUCTION

Proper math placement is at the forefront of issues in Science, Technology, Engineering, and Mathematics (STEM) institutions and programs. Even among STEM institutions, Florida Poly, which opened in Fall 2014, exists in a unique situation. It is a 100% STEM institution in which all students are required to pass Calculus 1 (henceforth simply Calculus). Moreover, it faces pressure from state accountability metrics to graduate students in four years and regardless of degree program, failure to pass Calculus by the end of the first academic year puts students critically behind in their degree progression. Through its first three years, all entering students who did not transfer credit for Calculus were placed directly into it, to allow them to move expeditiously into courses for which Calculus was a prerequisite. This, however, meant Florida Poly relied entirely on mathematics preparation students received through high school.

Florida Polytechnic also relies heavily on admitting students from within the state of Florida due to the combination of low state tuition and generous Florida specific merit-based scholarships which, together, make Florida State University System (SUS) institutions far more attractive for in-state students than is the case in many other states. However, according to data from the Florida Department of Education, as shown in more detail in Table 1, only 90,122 of 991,018 student enrollments in mathematics classes in Florida high schools, 9.1%, were in courses considered appropriate preparation for STEM programs in 2017-2018 (Florida Department of Education, 2018) making it difficult to fill incoming classes with students completely ready to enter Calculus.

Table 1: Florida High School Students in Calculus Preparatory Course in 2017-2018.

Course	Students		Course	Students
MATH ANALYSIS HON	2017		AICE MATH&MECH 2 AL	1
ANALYSIS OF FUNC HON	4300		AICE MA&PR&ST 1 AS	382
CALCULUS HON	3618		IB PRECALCULUS	2580
AP CALCULUS AB	15482		IB MYP PRECALCULUS	273
AP CALCULUS BC	4461		IB CALCULUS/DESC STA	2045
PRECALCULUS HON	43601		IB FURTHER MATH 1	8
AICE MATH 1 AS	1523		IB ADV CALCULUS	362
AICE MATH&MECH 1 AS	40		TRIG HON	9429
			Total	**90122**

Over Florida Polytechnic's first three years of operation, low rates of success in Calculus significantly hindered student progression. For example, 42% of students that began in Fall 2016 and did not transfer Calculus credit failed to successfully complete it by the end of Spring 2017. To increase the percentage of students passing Calculus by the end of their first year, Florida Polytechnic university added Precalculus and Trigonometry (henceforth Precalculus) to its course offerings for Fall 2017, utilizing Assessment in Learning Knowledge Spaces (ALEKS) resources to place students appropriately. Previous research indicated ALEKS to be a better placement indicator than traditional measures such as ACT math scores (Ahlgren & Harper, 2011).

Of course, many other factors likely impact success in Calculus. “STEM course design and instruction are based not only on the assumption of students’ prior knowledge, but also on the assumption that students know how to be a student: what a syllabus is, and how to read and use it; how to navigate the rules of the complex college bureaucracy […] and how to effectively seek help—what type of help and from whom—when they are struggling” (Cromley, et. al., 2015). Providing adequate and appropriate help for students is essential. “Academic support centers, which can include course-specific tutoring by peers or professionals, workshops on time management and study strategies, testing and accommodation for learning disabilities, and other supports, are another university-level resource that can affect achievement and retention in STEM” (Cromley, et. al., 2015).

The purpose of this study is to estimate the causal impact of the addition of Precalculus, with ALEKS placement, on Calculus success rates at Florida Polytechnic University in the 2017-2018 academic year. Employing a Fuzzy Regression Discontinuity Design (RDD), placing a student in Precalculus was found to reduce the probability of successfully completing Calculus 1 by the end of Spring 2018 by 0.5 relative to placement directly in Calculus. This is consistent with previous research in the International Journal of Mathematical Education in Science and Technology in 2014 by Gerhard Sonnert and Philip M. Saddler. Through their analysis, using a series of regression discontinuities, they found no statistically significant indication that college pre-calculus helped student performance in college level calculus. Mathematics preparation heavily influenced the college calculus grade, whereas taking college pre-calculus had a negative impact that was not significant. Evidence is presented that two mechanisms may account for much of the observed difference. First, those placed in Precalculus in Fall will have at most one chance at passing Calculus by the end of Spring, not the two they would have had if they took Calculus in Fall, and must first clear Precalculus to get to that attempt. Second, they end up taking Calculus with a weaker peer group. Since those placed directly into Calculus therefore have a stronger peer group, their success rate would be higher, meaning that the fact that placement into precalculus reduces success for those may not alter the overall success rate.

2. DATA AND DESCRIPTION OF MAIN VARIABLES

The variables used in the analysis were directly provided by Florida Poly’s Office of Institutional Research (OIRE) or were calculated based on data from OIRE. The primary dependent variable is *SUCCESS*, which equals one if a student passed Calculus during the Fall semester in which they were admitted or in the following Spring, and otherwise is zero. The independent variable of primar[1]y interest, PRECALC, is one if the student took Precalculus in Fall 2017 and is otherwise zero. *HSGPA* is the student's high school GPA. *ACT* is the concordance score on the ACT. The concordance scores are determined by College Board and ACT.

Summary statistics for these variables are shown in Table 2, separately for students taking Calculus in Fall 2016 and those taking either Calculus or Precalculus in Fall 2017. Of the 288 students in Fall 2017, 170, or 59%, took Precalculus. Overall, the success rate increased from 0.58 to 0.62 from the Fall 2016 cohort to the Fall 2017 cohort. However, at the same time, both standard metrics of the academic preparedness of the incoming class, *HSGPA* and *ACT*, increased as well, reflecting a notable increase in selectivity taking place at Florida Poly at the time, which should have raised success rates on its own. We explore the potential impacts of increased selectivity further in Figures 1 and 2.

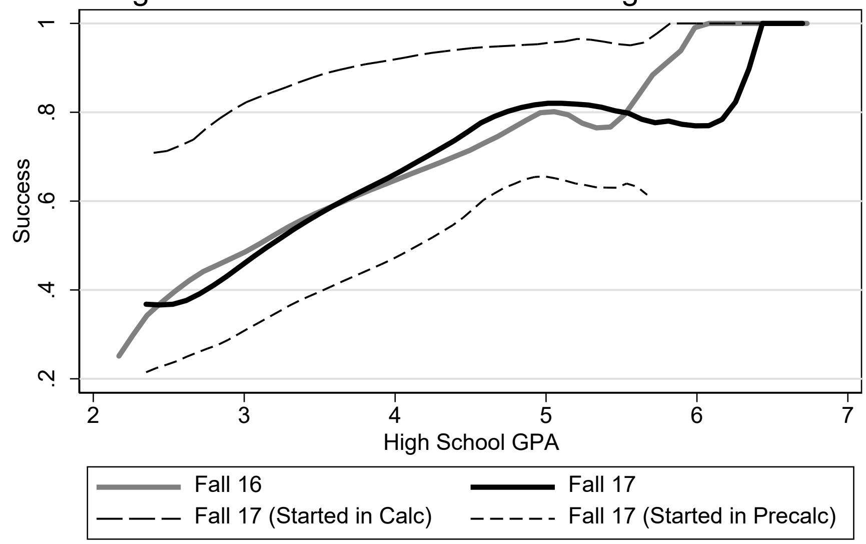
Table 2: Sample Means by Start Year and Fall Class

Start Year	2016	2017		
Fall Class	Calculus	Either	Calculus	Precalculus
SUCCESS	0.58	0.62	0.90	0.43
HSGPA	3.70	3.86	4.01	3.76
ACT	25.11	26.41	27.13	25.91
N	430	290	118	172

Figure 1 plots mean success rates against *HSGPA* by start year and by beginning math class for those starting in Fall 2017. There is no apparent increase in success from the Fall 16 to Fall 17 cohort conditional on *HSGPA*. From the figure, it simply appears Precalculus was populated with students with a lower probability of success, but that it did not improve the probability of success since the combined 2017 success probability closely tracks that of 2016. Figure 2 repeats the analysis of Figure 1 for ACT scores, with similar results—no apparent changes in success rates conditioned on *ACT*. It is also interesting to note that *HSGPA* was much more predictive of *SUCCESS* than *ACT* for both cohorts of students. As a result, we focus on *HSGPA* as the relevant control variable in our models.

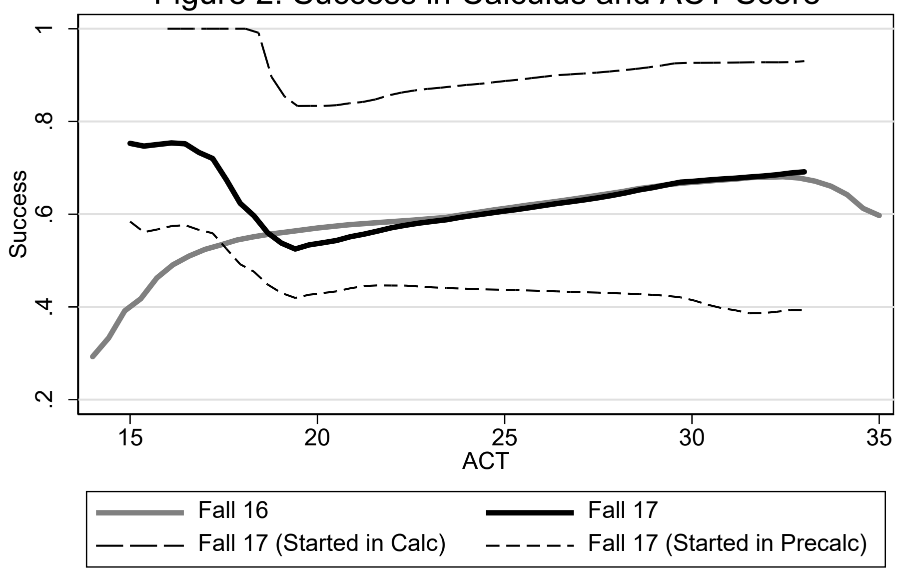
[1] The data to replicate the research is available at https://drive.google.com/drive/folders/1L2yEgMZhj8_0v80zEGdHIStHz9A_AUFf?usp=sharing.

Figure 1: Success in Calculus and High School GPA



Local mean smoothed at 50 points, smoothing window 1.

Figure 2: Success in Calculus and ACT Score



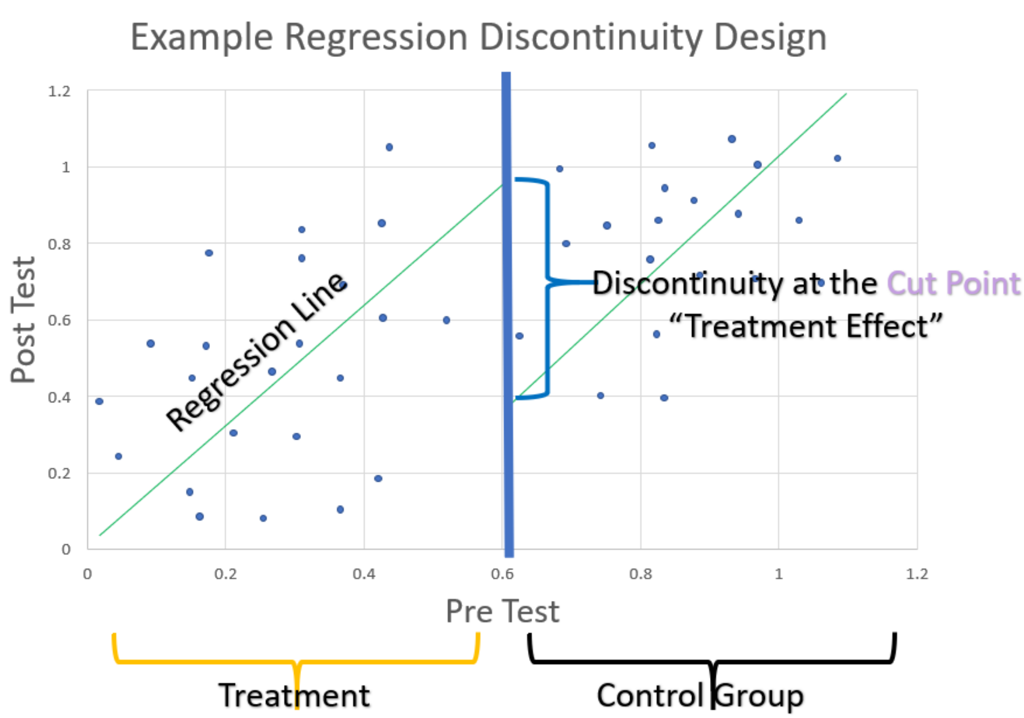
Local mean smoothed at 50 points, smoothing window 5.

Two additional variables are used in the primary analysis. *FEMALE* equals one if the student was female and zero otherwise. The final variable is *ALEKS*. In Fall 2017, incoming students either completed the ALEKS Prep for Calculus modules or else took the ALEKS PPL math placement test. *ALEKS* is a “superscore” in which, after rescaling the two scores to make comparable, the maximum score is used. *ALEKS* was used for placement, with the intent of placing those scoring 71 or higher in Calculus and those scoring below 71 in Precalculus.

3. PRIMARY ANALYSIS METHOD: FUZZY REGRESSION DISCONTINUITY DESIGN

This study uses regression discontinuity design (RDD), and more specifically, fuzzy RDD. RDD was introduced in 1960 with by Thistlethwaite and Campbell as a method for evaluating social programs (Thistlethwaite & Campbell, 1960). From there it expanded to evaluate other programs, and has been found particularly suited to evaluation of educational interventions in which placement into treatment status, for example taking Precalculus prior to taking Calculus, is assigned based on whether a score on a pre-test is above or below an established cut point. RDD exploits the fact that assignment at the cutoff for placement into treatment mimics random experimental assignment.

Figure 3:



The idea is illustrated in Figure 3. Students just above or below the cutoff are very similar, with the only non-negligible systematic difference being treatment status. The question thus becomes do students just below the cut point perform better in calculus having taken pre-calculus compared to their counterparts just on the other side of the cut point who did not take pre-calculus. Thus, any discontinuity at the cut point reflects the average local treatment effect, in this case, the effect of Precalculus (Jacob, Zhu, Somers, & Bloom, 2012).

The internal validity of the regression discontinuity design is compromised when there is noncompliance with treatment assignment based on the pretest. This can be addressed by using intention to treat, defined as one for those whose pretest scores indicate treatment and zero for those whose pretest scores indicate nontreatment, as an instrumental variable for treatment status. In this case, treatment is placement into Precalculus, which may be plagued by non-compliance. It is important to note that in this case, compliance was relatively high as seen in Table 3 below. However, intention to treat, determined by position relative to the predetermined ALEKS cutoff score, is a common and valid instrument with which to recover the causal effect of treatment (Jacob, Zhu, Somers, & Bloom, 2012).

Since the dependent variable is binary, we employ the probit model. Since we need to instrument for treatment we utilize Stata's biprobit command, which implements a bivariate probit model (Lewbel, Dong, and Yang, 2012). PRECALCULUS is modeled as dependent on intention to treat and the other exogenous predictors in the system, and SUCCESS is simultaneously modeled as dependent on PRECALCULUS and the exogenous predictors other than intention to treat.

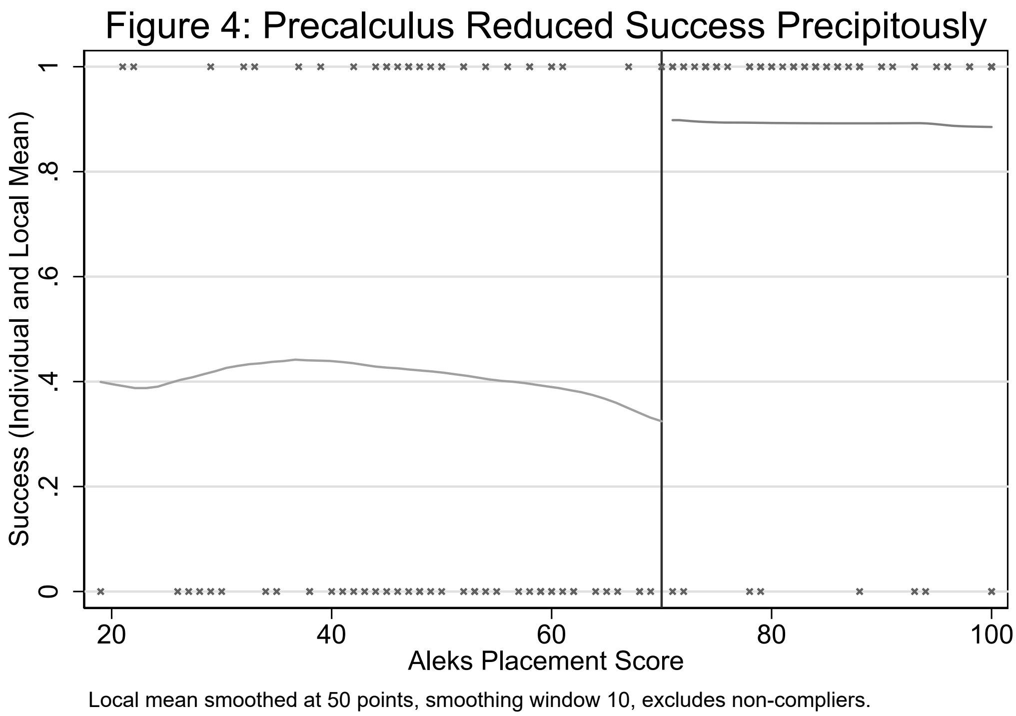
4. PRIMARY ANALYSIS

In Fall 2017, the cutoff score for placement into Calculus was 71—those scoring 71 or above on ALEKS were to be placed in Calculus and those scoring below 71 were to be placed in Precalculus . However, the cutoff was not hard. Students had the ability to opt out of the test or ignore a high placement and go directly into Precalculus. Further, with appropriate documentation of previous credit for Precalculus, students who tested into Precalculus could choose to take calculus consistent with established articulation agreements. Table 5 tabulates compliance with intention to treat as defined by the ALEKS cut score of 71 for the 173 students for whom ALEKS scores are available. A total of 18 total students were non-compliant: 12 took Precalculus when they placed into calculus while 6 took Calculus when they placed into Precalculus. Despite the somewhat voluntary nature of the program in 2017, compliance was relatively high.

Table 3: Compliance

		Took Precalculus		
		No	Yes	Total
Placed into Precalculus	No	80	12	92
	Yes	6	75	81
	Total	86	87	173

Ignoring noncompliers, Figure 4 presents an initial simple look at the discontinuity in *SUCCESS* at the *ALEKS* cutoff. The figure shows the local mean success rates as a function of *ALEKS* scores above and below the cutoff. Two phenomena are readily apparent, Precalculus appears to halve success rates, and conditional on being above or below the cutoff, *ALEKS* scores are not at all predictive of success.



The difference in SUCCESS for those at the ALEKS cutoff in Figure 4 is potentially biased as it does not deal appropriately with noncompliers, but simply excludes them. To correct for endogenous treatment due to noncompliance, we use intention to treat as an instrumental variable for treatment. Since there was relatively little crossover, and since a large portion of the class was placed into each treatment status, intention to treat is a very strong instrument in this case. The Pearson correlation coefficient is 0.793. However, the range of the Pearson correlation is artificially restricted by the binary nature of the variables. By contrast, according to the Stata Base Manual Description section, the tetrachoric correlation coefficient, which is based on the notion of continuous latent variables underlying the binary variables, has a range of -1 to 1 (Edwards & Edwards, 2020). The tetrachoric correlation coefficient is 0.950. Thus, not only is the fuzzy RDD technique appropriate to uncover the causal effect of *PRECALCULUS* on *SUCCESS*, but the effect is likely to be relatively precisely estimated.

Table 4: Models of the Causal Effect of Placement into Precalculus

Model	(1)	(2)
Estimation Method	Biprobit	2SLS
Marginal Effects or Coefficients	Marginal Effects	Coefficients
PRECALCULUS	-0.500***	-0.482*
(0 to 1)	(0.145)	(0.159)
FEMALE	0.129	0.119
(0 to 1)	(0.090)	(0.094)
HSGPA	0.246***	0.247***
(*dP/dx*)	(0.059)	(0.057)
ALEKS	-0.002	-0.001
(*dP/dx*)	(0.003)	(0.003)

1. N=170 students
2. Standard errors in parentheses
3. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Model 1 in Table 4 presents the results from a bivariate probit regression, expressed as marginal effects. For continuous regressors, effects are the derivative of the probability of success with respect to the regressor evaluated at the sample mean. For binary regressors, effects are the difference between the predicted probability with the regressor evaluated at one and the predicted probability evaluated at zero, with other variables at the sample mean. Model 2 is a linear probability model estimated using Two-Stage Least Squares (2SLS). Results presented for model 2 are simply the normal regression coefficients. While Linear Probability Models are ill-suited to estimating effects at all levels of predictor variables, they generally return appropriate estimated of average partial effects, and thus serve as a useful check on the bivariate probit results (Angrist & Pischke, 2014).

The marginal effect of *PRECALC* in Model 1 a binary variable, indicates taking precalculus reduced the probability of success by 0.500 (from 0.893 to 0.393 based on the predicted values from Model 1) and with a standard error of only 0.145, fully consistent with what was observed in Figure 4. The effects estimated in Model 2 are nearly identical. Placement in Precalculus in Fall 2017 caused the probability of success to fall by 0.5 compared to placement in Calculus for an otherwise average student at the ALEKS cut point. This is the primary finding of this study.

Before proceeding to the next section, which considers mechanisms that might account for the precipitous negative effect of placement into Precalculus on *SUCCESS*, it is worth briefly considering the effects of the control variables. FEMALE is associated with a notably higher probability of success, but with a relatively large standard error, so perhaps there is not much to be made of that. *HSGPA* on the other hand is strongly associated with SUCCESS, with a 1-point increase in GPA associated with an increase of 0.246 in the probability of *SUCCESS*. As suggested by Figure 4, the ALEKS placement score itself is not at all associated with *SUCCESS*.

5. SECONDARY ANALYSES

The primary analysis above showed that placement into Precalculus in Fall 2017 caused a precipitous decline in the probability of successfully completing Calculus by the end of Spring 2018. Surely additional preparation on mathematical material pertinent to calculus must boost the probability of success, purely on logical grounds. What might account for this effect?

Figure 1 and Table 2 suggest one possibility: peer effects. The average *HSGPA* of classmates was considerably higher for those placed into Calculus than for those placed into Precalculus. A large literature demonstrates that characteristics of peers may have strong associations with individual academic outcomes (Sacerdote, 2011). In this case, students with higher high school GPAs may model better study habits, may exert more positive peer pressure, or may simply make better and more reliable study partners, for example.

To quantify the potential role of peer effects, we estimate a probit model of the probability of passing calculus on the first attempt, *PASS CALC*, in which the predictor variables include FEMALE and HSGPA, but also two additional variables. *2017* is one if the Calculus class was offered in the 2017-2018 academic year. At that time, there was a concerted effort to increase academic standards at Florida Poly. Thus we expect this effect to be negative, and it turns out to be so. The variable of interest is *PEER HSGPA*, the average GPA of all of the other students in the students Calculus section. The sample includes those who started in Fall of 2016 and took calculus that semester, those who started Fall 2017 and took calculus that semester, and those who started Fall 2017, took Precalculus that semester, passed it, and took calculus in Spring 2018. Results are shown as Model 3 in Table 5.

Table 5: Models to Quantify Possible Mechanisms for the Precalculus Effect

Model	(3)	(4)
Dependent Variable	*PASS CALC*	*PROGRESS*
FEMALE	0.0868	-0.0523
(0 to 1)	(0.053)	(0.156)
HSGPA	0.256^{***}	0.397^{***}
(*dP/dx*)	(0.031)	(0.108)
ALEKS		0.002
(*dP/dx*)		(0.003)
PEER HSGPA	1.251^{***}	
(*dP/dx*)	(0.231)	
2017 COHORT	-0.275^{***}	
(0 to 1)	(0.064)	
N	603	85

1. Standard errors in parentheses
2. * p < 0.05, ** p < 0.01, *** p < 0.001

The marginal effect of *PEERHSGPA* in Model 3 sizeable and estimated with a relatively small standard error. It indicates an increase of 0.1 in *PEERHSGPA* increases the probability of success by about 0.125. We estimate a student from the Fall 2017 cohort with an average *HSGPA* of 3.86 would have a probability of passing calculus of 0.832 if *PEERHSGPA* were 4.01 (average for those taking calculus in Fall 2017) but only 0.483 if *PEERHSGPA* were 3.76 (the average for those taking Precalculus in Fall 2017). This decline of 0.349 is statistically significant at any conventional level (p<0.001). Thus, adverse peer effects might explain much of the negative effect of Precalculus found in Models 1 and 2. This negative effect comes from both increasing the probability of success for those placed directly in Calculus through better peer effects and decreasing the probability of success for those placed first into Precalculus–overall the whole cohort of students experience no change in success.

While peer effects appear able to account for much of the negative effect of *PRECALCULUS* on *SUCCESS*, they do not appear to account for it all. The very structure of the intervention under study, placement into a Precalculus class rather than Calculus 1, provides another strong candidate. To some degree, passing a class is a random event for all students, and this may be particularly true for marginal students intended to be helped by taking Precalculus. Thus, even in many cases where passing Precalculus is quite a bit more likely than passing Calculus, and where passing Precalculus substantially increases the chance of passing Calculus, placement into Precalculus will reduce the chance of success simply by creating the additional hurdle of passing Precalculus and reducing the number of chances to pass Calculus.

To illustrate, let f denote the probability a given student passes Calculus without first taking Precalculus. For simplicity assume it is independent across attempts and that the student takes both attempts if needed. The probability the student passes calculus by the end of Spring, if not first placed in Precalculus, is $1-(1-f)^2$. Suppose that for that same student, the probability of passing Precalculus is $g>f$. If they do not pass Precalculus, they do not get an attempt at Calculus in Spring. Suppose that if they do pass Precalculus, their chance of passing Calculus increases to $h>f$ due to studying the material in Precalculus. The probability the student passes Calculus in Spring, having been first placed in Precalculus, is then gh. This will be less than their probability of passing calculus by Spring having not been placed into Precalculus if $gh<(2-f)f$. This inequality might or might not hold in any particular case, since f is smaller than the smaller of g and h, but $2-f$ is larger than the larger of g or h. However, Precalculus may easily reduce the change of success even when it has a large positive direct effect on the chance of passing Calculus. To see that, consider a student for whom the chance of passing Calculus is 0.5, the chance of passing Precalculus is 0.8, and the chance of passing Calculus having taken Precalculus increases from 0.5 all the way to 0.8. The student's chance of success placed in Calculus is 0.75, while their chance of success placed in Precalculus is only 0.64.

To get a sense of the magnitude of the additional hurdle created by the chance of not progressing from Precalculus to calculus for a typical student, we estimate a probit model where the response variable, *PROGRESS*, takes the value one for students who passed precalculus in Fall 2017 and registered for Calculus in Spring 2018 and is zero for other students taking Precalculus in Fall 2017. Predictor variables are *FEMALE*, *HSGPA* and *ALEKS*. Results are shown as Model 4 in Table 4. This model predicts that a student with *HSGPA* equal to 3.86 at the *ALEKS* cut-score would have a probability of passing Precalculus of 0.784 with a standard error of 0.072. Together with the difference due to peer effects estimated from Model 3, this would reduce the chance of success for those taking Precalculus to 0.379 (0.784×0.483). We also need to estimate the probability the same student would have passed Calculus if not placed in Precalculus. In that case, Model 3 suggested the probability of passing Calculus in Fall 2017 would have been 0.832 and, had they not passed in Fall 2017, the probability would have been 0.483 in Spring 2018 due to the lower value of *PEERHSGPA*. Thus, the probability of this student passing Calculus in Fall 2017 or Spring 2018 had they not been placed in Precalculus would have been 1-(1-0.832)(1-0.483)=0.913.

Together this suggests these two mechanisms reduce the probability of success by 0.534 (0.913-0.379), very slightly larger than the estimates from Models 1 and 2. It is quite plausible that the lion's share of the negative effect of Precalculus is due to these two factors.

6. DISCUSSION AND CONCLUSION

The primary analysis in this study showed that that placing students in Precalculus based on their ALEKS score caused a large reduction in the probability of successfully completing Calculus by the end of a student's second semester compared to being placed directly into Calculus—the point estimate was a reduction of 0.5 with a standard error of 0.145.

The secondary analyses demonstrated that this impact can be largely accounted for by two mechanisms. The first is peer effects, the average high school GPA was notably higher in Calculus sections in the Fall 2017 semester which were populated by those not placed into Precalculus. This appears to have raised the success rates of those placed into Calculus and lowered the success rates of those placed into Precalculus. The second is the fact that adding Precalculus both adds an additional hurdle to clear before starting Calculus and reduces the number of chances to pass Calculus by the end of Spring from two for those placed directly into Calculus to one for those that pass Precalculus on their first attempt and to zero for those that fail their first attempt at Precalculus.

While adding Precalculus, with placement using ALEKS, did not increase the chance of passing Calculus by the end of the Spring semester of the first year at Poly, that does not mean it has no educational value, for two reasons. First, the two mechanisms hypothesized to drive the negative effect have nothing directly to do with the educational value of the material or its direct impact on mathematical ability. Second, consider the information presented in Table 5 below. Forty-two percent of Poly students in Fall 2017 likely lacked sufficient preparation to place into Precalculus. To serve the educational needs of these students, it would appear necessary to first provide College Algebra then Precalculus and only then for them to take Calculus. This may explain in part why ALEKS appeared to be so unpredictive of success in Precalculus—most of the students in it were not ready for it by ALEKS standards. Without a more wholistic intervention strategy, of which Precalculus is only one part and which will further delay student progress or else delay entry into Poly, Poly may not be able to meet these students' needs.

Table 5: ALEKS Corporation Recommended Placement and Fall 2017 Poly Students

Score	*Course Placement*	*% Students*
0-13	Basic Math/Pre-Algebra	
14-29	Beginning Algebra	46%
30-45	Intermediate Algebra	11.6%
46-60	College Algebra	22.5%
61-75	Precalculus /Business Calculus	19.1%
76-100	Calculus 1	42.2%

Florida Poly must identify appropriate measures of placing students into the appropriate course and providing adequate support for their success. It may be warranted to further research ALEKS Knowledge Spaces to gain deeper understanding of student scores in each section. For example, focus on the trigonometry scores. Florida Poly may not need only a placement mechanism, but also to raise the cut score for Calculus, to offer additional preparatory coursework, and to provide more support for students who struggle, or else to limit admissions only to students with a higher level of preparation for Calculus. For Fall 2018, the cut score was indeed raised and thus could help in creating better placement. But if many students were still not prepared for Pre-Calculus, it likely did not change matters much for the better. A follow-on study to include the students enrolling in Fall 2018 and 2019 would be useful additional work, both to see the impact of the increase in the cut-score, and also to confirm the results of this study on a larger sample.

APPENDIX A: ALEKS AND KNOWLEDGE SPACE THEORY

Dr. Jean-Claude Falmagne, an internationally renowned mathematician and Professor of Cognitive Sciences who is the chairman and founder of ALEKS Corporation, began research in Knowledge Space Theory (KST) in the early 1980s (Doignon and Falmagne 1985). Knowledge space theory was used to formally describe the infrastructure of any given domain of knowledge. The motivation behind KST research stems from the used of standardized tests as placement tests. Such test results place individuals in a lacking number of classifications. Doignon and Falmagne (1985) proposed "an assessment in a scholarly subject should uncover the individual's 'knowledge state', that is, the exact set of concepts mastered by the individual."

ALEKS employs the concepts of KST in its online educational resources, including ALEKS Prep for Calculus. According to the ALEKS website,

> ALEKS is a ground-breaking technology developed from research at New York University and the University of California, Irvine, by a team of software engineers, mathematicians, and cognitive scientists with the support of a multi-million-dollar grant from the National Science Foundation. ALEKS is fundamentally different from previous educational software. At the heart of ALEKS is an artificial intelligence engine that assesses each student individually and continuously.

ALEKS is based upon the original theoretical work in KST. ALEKS' preparation modules for students utilizes KST to identify holes in student understanding and adapts to individual student needs. The ALEKS Prep for Calculus includes 281 topics within calculus. The Prep for Calculus modules begins with a pre-assessment then adapts to student understanding to provide support for student learning. The ALEKS system as a remedial math preparation resource, "combines adaptive, diagnostic testing with an electronic learning and practical tutorial in several domains relevant for higher education" (Tempelaar, et al., 2006).

The ALEKS Placement, Preparation, and Learning, ALEKS PPL, combines research-based, accurate placement assessment while providing personalized learning tools to assess students while helping them refresh important knowledge (ALEKS, *2017).* According to the ALEKS website What is ALEKS PPL, "The key components that make ALEKS PPL unique [are]:

- Artificial intelligence efficiently assesses course readiness;
- Open-response, adaptive assessment covers 314 topics in 30 questions or less;
- Places students from Basic Math up to Calculus I in single assessment;
- Seamless transition from placement assessment(s) to individualized Prep and Learning Modules;
- Six months access to Prep and Learning Modules to refresh their knowledge, and 12 months to take up to four additional assessments to improve their course placement;
- Mastery-based learning motivates students to achieve higher placement results;
- Online flexibility allows for on- and off-campus testing and simple implementation."

ALEKS Corporation works with clients from around the United States. Within the state of Florida, Embry-Riddle Aeronautical University, Florida Agricultural and Mechanical University, Florida Atlantic University, Florida International University, Florida State University, University of Florida, University of Miami, and the University of North Florida all utilize ALEKS PPL in addition to Florida Polytechnic University. Each university, except Florida Polytechnic University, utilizes the ALEKS PPL as a means of placing students into a variety of math courses built in secession to prepare students for their degree requirements. Florida Polytechnic University uses ALEKS to only place into MAC 1147 of MAC 2311, there is no lower course offered.

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