# Mask Mandate Model and Analysis Assignment Part 2

In Part 1, you were to construct and analyze a model of the benefits and costs of mask mandates in order to draw insights on whether, and under what conditions, mask mandates may be a good idea socially. In Part 2, you will build on that based on the feedback from Part 1 and the information below. I suspect you found that in Part 1, even with the nudges from class, it was hard to know how to proceed with making a useful model. Welcome to the world of the analyst. If there was a precise algorithm for it, no one would need to hire you as an analyst, they could just buy the software. So, for Part 2, I am going to give you a bigger nudge, and hopefully as we iterate you will develop a better understanding of taking on an analysis from scratch by yourself.

A few general comments on this type of exercise. It may seem intuitively obvious what the conclusions from such a model should be. If so, what is the point of doing the "rigorous" modeling, when we are making the models as stupid simple as we can anyway? There are three. First, to check that the conclusions you think are obvious are logically consistent. That is, to help make sure you are not fooling yourself. Second, to help make it clear to others that your intuitive conclusions are, indeed, logically consistent, who might not think they are sensible. Third, often the exercise in thinking carefully about something will lead to new insights that are more subtle or that you simply would not otherwise have thought of.

The model you started for Part 1 may be totally different from the bits we sketched in class. You can also just complete and analyze that model based on the feedback you received if you want to. On the following two pages I sketch below something like the pieces we talked about in class, in a little more detail, and with a few improvements, in case seeing it laid out helps you move forward. Change what you like, as long as there is a reason to do so.

If you spot typos, need clarification of wording, or have other questions, please use the class discussion topic for this assignment on canvas.

Definitions

n: Number of people in the population under analysis, e.g. Poly students.

$s_0$: Initial fraction that are sick. Inherited from the previous time period

s: Fraction that get sick this period

m: Fraction that wear masks. Assume the probability an individual wears a mask is independent of health status (those infected do not know they are and are otherwise not different from those not infected)

$m_v$: Fraction that voluntarily wear a mask if asked to even if there is no enforcement effort.

E: Expenditure on enforcement needed to achieve any given level of compliance with a mandate. E is a function of n and m if $m>m_v$, $E(n,m-m_v)$, and otherwise is 0.

$c_m$: Cost of an individual wearing a mask

$c_s$: Cost of an individual being sick

$s(s_0,m,n)$: A function relating the fraction that get sick this period to the fraction sick, the fraction wearing masks, and population size.

Objective Function

With these definitions, the total cost to society is:

$$nc_m m+nc_s s(s_0,m,n)+E(n,m-m_v).$$

The idea is to minimize this cost. If you are using this model, take the derivative wrt m, set it to 0, and analyze the implications of the equation you get. The analysis takes effort, insight, and artful creativity.

General Analysis

Keep in mind there are three candidate solutions. The best might be a corner with m=0, or another corner with $m=m_v$. Both have 0 enforcement costs. Part of the analysis is to determine when you should ask for voluntary compliance and when people should go without a mask. The third is what you get by setting the derivative to 0. When is that better than the other two? In that solution, how to the various parameters effect the desired level of m? If you don't employ specific forms for E() and s(), you would probably do best to consider this graphically, or a mix of graphs and logical arguments about equations and inequalities. Alternatively, you might want to substitute some specific functions for E() and s(). If you do, they need to be reasonable for your purpose.

Form for E()

A simple version is E=ne(m) if $m>m_v$ and 0 otherwise, where e(m) has positive first and second derivatives. An example is $e(m)=a(m-m_v)/(1-m)$ where a>0.

We did not mention this in class, but you might also approximate e(m) as b+dm where d>0 as an approximation of e(m) in the neighborhood of the solution. Here d is the rate cost increases with m in the local area of the solution. This makes the math easier, but has other problems. If you use it (and I probably would), you need to think about what these problems might be.

Form for s()

1) You have to make a decision about whether the initial sick can be reinfected and stay sick longer. Henceforth, I assume not, but you can assume so. Better might be to assume some fraction are immune after being sick and some fraction are not; I am not going to add that complication, but you can. Since I am assuming not, the population that might get sick is $n(1-s_0)$.

2) Let f be the probability someone is infected from any given exposure to someone without a mask. Let g be the probability someone is infected from any given exposure to someone with a mask, g<f. (How much less it needs to be to matter is something to consider in the analysis). Let q be the fraction of n that each person interacts with. The probability an individual does not get sick is then $(1-f)^{q(1-m)ns_0}(1-g)^{qmns_0}=(1-f)^{qns_0}[(1-g)/(1-f)]^{mqns_0}$. If we define $k=(1-g)/(1-f)>1$, the probability an individual does not become sick can be written as $[(1-f)k^m]^{qns_0}$. The probability they do get sick is $1-[(1-f)k^m]^{qns_0}$, so:

$s(s_0,m,n)=n(1-s_0)[\{1-[(1-f)k^m]^{qns_0}\}$.

Methods of Analysis

If you use a specific function for E() and s(), you can proceed two ways. One is to use the simple version for E() and the version above for s, take the derivative, and actually solve analytically for m. It is ugly, but I think doable. Then you can use your favorite program to draw graphs of how m varies with all the various parameters of interest. the other is to