

1. What is the present value of the payments in the table to the right? Time is measured with the present being 0. Assume the riskless annual rate of return is 2.5%.

Time	1	2	3	4	5
Value	-50	-50	-100	-200	500

Answer:

$$V_0 = -50 \times 1.025^{-1} - 50 \times 1.025^{-2} - 100 \times 1.025^{-3} - 200 \times 1.025^{-4} + 500 \times 1.025^{-5} = 71.51.$$

2. Use the fact that the present value of a perpetuity paying \$X per period starting in one year is X/r , where r is the riskless rate of return, to determine the present value of annual payments of \$X accruing for 20 years, starting one year from now. Hint: Think of it as a perpetuity less the appropriately discounted value of a perpetuity starting 20 years from now.

Answer:

Think of this as owning the perpetuity through 20 payments and then losing the rights to the 21st payment on. From the point of view of $t=20$, the value of payments, starting at $t=21$ and continuing forever, is X/r . Converting that value (at $t=20$) to current value gives $\frac{1}{(1+r)^{20}} \frac{X}{r}$. So

$$V_0 = \frac{X}{r} - \frac{1}{(1+r)^{20}} \frac{X}{r} = \frac{X}{r} \left(1 - \frac{1}{(1+r)^{20}} \right).$$

3. What is the present value of the uncertain payment stream in the table to the right? Time is measured with the present being 0. P(End) is the probability the payment stream is permanently terminated before that period's payment is made, conditional on the previous period's payment having been made. So, you have to work out the probability the venture survives long enough for each payment to be made. The riskless annual rate of return is 4%.

Time	1	3	6	10
P(End)	0.1	0.1	0.4	0.7
Value	-10	-5	60	1000

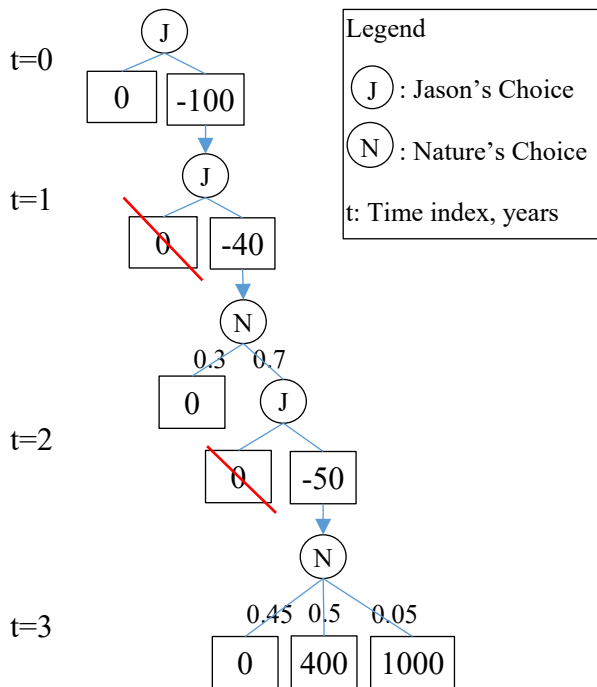
Answer:

$$\begin{aligned} EPV &= -10 \times 0.9 \times 1.04^{-1} - 5 \times 0.9 \times 0.9 \times 1.04^{-3} \\ &\quad + 60 \times 0.9 \times 0.9 \times 0.6 \times 1.04^{-6} + 1000 \times 0.9 \times 0.9 \times 0.6 \times 0.3 \times 1.04^{-10} \\ &= 109.29 \end{aligned}$$

4. Jason is considering developing a process innovation. It requires an initial investment of \$100, then another investment of \$40 after one year. Jason thinks the probability it will turn out to be feasible after two years is 0.7. If it is feasible, it will then take another expenditure of \$50 (2 years from the initial investment) to complete. It will then be ready to demonstrate 3 years from the initial investment. Jason thinks there is a 0.05 probability that with a successful demonstration he will sell his innovation for \$1,000 and a 0.5 probability he will sell it for \$400, and that otherwise there will be no interest. The annual discount rate (riskless rate of return) is 5%. There are no other costs and Jason is risk neutral.

a. Illustrate the decision(s) to be made with a decision tree.

Answer:



b. What is the present expected value of the project?

Answer: First, note that it would not make sense to not spend the \$40 at t=1 if it made sense to spend \$100 at t=0 knowing you would have to spend another \$40 at t=1 before you found anything new. You either spend both or neither. Similarly, it would not make sense to make the initial investments and not make the final \$50 if the project is feasible.

So, if Jason proceeds:

$$EPV = -100 - \frac{40}{1.05} + 0.7 \left(-\frac{50}{1.05^2} + \left(0.5 \frac{400}{1.05^3} + 0.05 \frac{1000}{1.05^3} \right) \right) = -18.67$$

Therefore, the EPV is 0, since he would not proceed.

c. What probability of selling the project for \$400 would make Jason indifferent between pursuing it or not, assuming $P(1000)$ stays the same?

Answer:

Working from the equation above, solve the following for f :

$$-100 - \frac{40}{1.05} + 0.7 \left(-\frac{50}{1.05^2} + \left(f \frac{400}{1.05^3} + 0.05 \frac{1000}{1.05^3} \right) \right) = 0, \text{ so } f \approx 0.577.$$

d. Jason may obtain an expert's opinion of the feasibility of his idea for a fee. Suppose the consultant's studied opinion is completely accurate and Jason thinks there is a 70% chance they will find the innovation feasible. How much is the opinion worth?

Answer:

Obviously, Jason would not proceed with bad news from the consultant. With good news from the consultant:

$$EPV = -100 - \frac{40}{1.05} - \frac{50}{1.05^2} + \left(0.5 \frac{400}{1.05^3} + 0.05 \frac{1000}{1.05^3} \right) = 32.51.$$

Therefore, if the information is purchased:

$$EPV = 0.7 \times 34.4 + 0.3 \times 0 = 22.76.$$

Since Jason would not proceed otherwise, the report is worth up to 22.76.

e. Suppose, having dealt with consultants on similar projects in the past, Jason guestimates there is a 0.7 probability the consultant will report the innovation is probably feasible and otherwise the consultant will report the idea is probably not feasible. Jason thinks that if the consultant says the idea is probably not feasible, the probability it is feasible is 0.19 and that if the consultant says the idea is probably feasible, the probability it is feasible is 0.92. How much is the opinion worth?

Answer:

Above, we showed that Jason would not proceed with a 70% chance of success. Therefore, if the consultant reports bad news, Jason would not proceed with only a 19% chance of success, and EPV is 0. If the consultant reports probably success:

$$EPV = -100 - \frac{40}{1.05} + 0.92 \left(-\frac{50}{1.05^2} + \left(0.5 \frac{400}{1.05^3} + 0.05 \frac{1000}{1.05^3} \right) \right) = 18.86.$$

Therefore, having purchased the information:

$$EPV = 0.7 \times 18.86 + 0.3 \times 0 = 13.2.$$

Since Jason would not pursue the project without the report, the report is worth up to \$13.2.

5. After incurring a cost of \$300 to set up a cafeteria for a day, each meal costs \$4 to prepare and serve. If the inverse demand for meals on Sunday is $p = 9 - 0.025q$, what price and quantity maximize profit, and what is maximum profit? Illustrate with a figure.

Answer: If the cafeteria opens, the highest possible profit is -50, shown below. So, it should not open, and the maximum profit is \$0 at $q=0$.

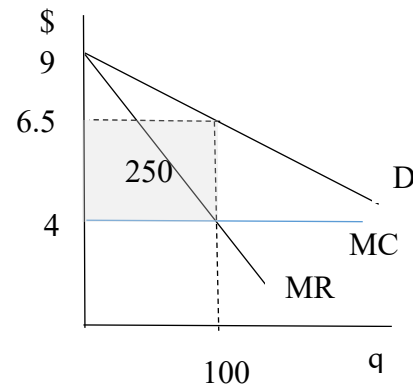
$$\pi = (9 - 0.025q)q - 4q - 300$$

$$\frac{d\pi}{dq} = 9 - 0.05q - 4 = 0$$

$$q^* = 100$$

$$p^* = 6.5$$

$$\pi = (6.5 - 4)100 - 300 = -50$$



6. Inverse demand is $p = -5 + 0.5m - 0.75q$, where m is per capita income. If the cost per unit is constant at \$5, calculate the profit maximizing price as a function of per capita income. How much does the profit maximizing price increase per \$1 increase in per capita income?

Answer:

$$\pi = (-5 + 0.5m - 0.75q)q - 5q$$

$$\frac{d\pi}{dq} = -5 + 0.5m - 1.5q - 5 = 0$$

$$q^* = (m - 20)/3$$

$$p^* = -5 + 0.5m - (3/4)(m - 20)/3$$

$$p^* = -5 + 0.5m - 0.25m + 5$$

$$p^* = 0.25m$$

$$\frac{dp^*}{dm} = 0.25$$

7. A firm sells q_B mugs of beer at price p_B , and q_P slices of pizza at price p_P . The inverse demand for mugs of beer is $p_B = 5 - 0.25q_B + 0.1q_P$ and the inverse demand for pizza slices is $p_P = 4 - 0.5q_P + 0.1q_B$. It costs \$1/mug to serve beer and \$2/slice to serve pizza. Find the prices and quantities that maximize profit and the maximum profit.

Answer:

First, set up the expression for the firm's profit, and set $MR=MC$ for each product:

$$\pi = (5 - 0.25q_B + 0.1q_P)q_B + (4 - 0.5q_P + 0.1q_B)q_P - q_B - 2q_P$$

$$\frac{\partial \pi}{\partial q_B} = (5 - 0.5q_B + 0.1q_P) + 0.1q_P - 1 = 0 \quad \frac{\partial \pi}{\partial q_P} = (4 - q_P + 0.1q_B) + 0.1q_P - 2 = 0$$

$$0.5q_B = 4 + 0.2q_P$$

$$q_P = 2 + 0.2q_B$$

$$q_B = 8 + 0.4q_P$$

In the FOC, the terms in the parentheses represent MR to each firm from selling another unit of their product. The interesting thing in these FOCs is that the MR for pizza is higher when more beer is sold, and vice versa. So, each firm's sales boost MR for the other firm—this is the piece left out of the parentheses in the FOC above. That means it is optimal to sell more of each product than would otherwise be the case. If the different items were sold by different firms, prices would be higher for each item, because neither firm would internalize the impact its sales have on the other firm's profits. Horizontal integration between firms with market power selling complementary products results in lower, not higher, prices.

$$q_B = 8 + 0.4(2 + 0.2q_B)$$

$$q_P = 2 + 0.2q_B$$

$$= 8 + 0.8 + 0.08q_B$$

$$q_P = 2 + 0.2 \times 9.57 = 3.91$$

$$0.96q_B = 8.8$$

$$p_B = 5 - 0.25 \times 9.57 + 0.1 \times 3.91 = 3$$

$$q_B = 8.8/0.92$$

$$p_P = 4 - 0.5 \times 3.91 + 0.1 \times 9.57 = 3$$

$$= 9.57$$

$$\pi = (3 - 1)9.57 + (3 - 2)3.91 = 23.04$$

8. Demand is given by $q = 400p^{-2}$. Cost per unit is \$10. What are the profit maximizing price and quantity and what is the maximum profit?

Answer:

$$p^* = \left(\frac{-2}{-2+1} \right) 10 = 20$$

$$q^* = \frac{400}{20^2} = 1$$

$$\pi^* = (20 - 10)1 = 10$$

9. At the current price of \$8, you sell 24 units. Cost is \$4/unit. Based on publically available estimates, you think the elasticity of demand is approximately -3. Estimate the profit maximizing price and the quantity sold and profit at that price.

Answer: $p^* = \left(\frac{-3}{-3+1} \right) 4 = 6$. As a rough approximation, that is a 25% decline, so quantity

would increase about $3 \cdot 25\%$, or 75%, to about 42. That makes profit about $(6-4)42=84$.

To be more precise, if demand is reasonably approximated by constant elasticity ($q = Ap^{-3}$), the

ratio of the new to old quantity is $\frac{q}{24} = \frac{6^{-3}}{8^{-3}}$ so $q = 24(4/3)^3 = 56.89$, and profit is \$113.78.

The approximations differ so much because percentage changes are a worse approximation of log point changes as the change becomes larger, and a 25% price change is large.

10. Inverse demand for movie tickets is $p_S = 16 - 0.01q_S$ for senior citizens and $p_A = 24 - 0.01q_A$ for others. The marginal cost of serving one more movie goer is \$4. Determine the profit maximizing prices for each type of customer.

$$\pi = (16 - 0.01q_S)q_S + (24 - 0.01q_A)q_A - 4(q_S + q_A)$$

$$\frac{d\pi}{dq_S} = 16 - 0.02q_S - 4 = 0 \quad \frac{d\pi}{dq_A} = 24 - 0.02q_A - 4 = 0$$

$$0.02q_S = 12$$

$$0.02q_A = 20$$

$$q_S = 600$$

$$q_A = 1000$$

$$p_S = 16 - 0.01 \cdot 600 = 10$$

$$p_A = 24 - 0.01 \cdot 1000 = 14$$

$$\pi = 10 \cdot 600 + 14 \cdot 1000 - 4 \cdot 1600 = 13600$$

11. The inverse demand for evening movie tickets is given by $p_E = 25 - 0.01q_E$ while the inverse demand for matinee tickets is $p_M = 15 - 0.01q_M$.

a. Assuming the marginal cost of serving one more customer is \$2 holding capacity constant, and that cost of adding capacity is \$2 per unit, determine profit maximizing capacity, prices and quantities.

Start by assuming evenings will sell out, but matinees will not. Then:

$$\pi = (25 - 0.01q_E)q_E + (15 - 0.01q_M)q_M - 2q_E - 2q_M - 2q_E$$

$$\frac{d\pi}{dq_E} = 25 - 0.02q_E - 2 - 2 = 0 \quad \frac{d\pi}{dq_M} = 15 - 0.02q_M - 2 = 0$$

$$0.02q_E = 21$$

$$0.02q_M = 13$$

$$q_E = 1050$$

$$q_M = 650$$

$$p_E = 25 - 0.01 \cdot 1050 = 14.50 \quad p_M = 15 - 0.01 \cdot 650 = 8.50$$

The assumption on which the problem set up was based holds, so the solution is fine.

$$\pi = (14.50 - 4) \cdot 1050 + (8.50 - 2) \cdot 650 = 15250$$

b. Find the value of per unit capacity cost, k , at which the constraint that matinee quantity is less than or equal to capacity is just binding. That is, at all lower values of k , matinee ticket sales will be less than capacity, and at k or higher, matinee and evening sales both equal capacity.

With k now a variable, profit is: $\pi = (25 - 0.01q_E)q_E + (15 - 0.01q_M)q_M - 2q_E - 2q_M - kq_E$.

At what value of k will the evening quantity just equal the matinee quantity from part a, or 650?

$$\frac{d\pi}{dq_E} = 25 - 0.02q_E - 2 - k = 0$$

$$1150 - 50k = 650$$

$$0.02q_E = 23 - k$$

$$50k = 500$$

$$q_E = 1150 - 50k$$

$$k = 10$$

So, if $k > \$10$ all capacity will be used for evening and matinee shows, and the problem would need to be reformulated to reflect that.

12. Inverse demand will be $p_H = 12 - 0.005q_H$ with probability 0.4, and otherwise $p_L = 12 - 0.01q_L$. Any product not sold must be disposed of at a cost of \$1 per unit. For parts a and b, production must take place *before* demand uncertainty is resolved and cost per unit is constant at \$3.

a. Find the profit maximizing prices and quantities for each state of demand.

First, assume more is sold when demand is high. That means high demand determines the production level (capacity) and that with probability 0.6, $q_H - q_L$ units will have to be disposed of.

$$E(\pi) = 0.4(12 - 0.005q_H)q_H + 0.6(12 - 0.01q_L)q_L - 3q_H - 0.6 \cdot 1(q_H - q_L)$$

$$\frac{d\pi}{dq_H} = 0.4(12 - 0.01q_H) - 3 - 0.6 = 0$$

$$\frac{d\pi}{dq_L} = 0.6(12 - 0.02q_L) + 0.6 = 0$$

$$0.01q_H = 3$$

$$0.02q_L = 13$$

$$q_H = 300$$

$$q_L = 650$$

$$p_H = 12 - 0.005 \cdot 300 = 10.5$$

$$p_L = 12 - 0.01 \cdot 650 = 5.50$$

The assumption on which the problem set up was based does not hold. The problem must be reformulated with $q_H = q_L$. There will then be no disposal needed.

$$E(\pi) = 0.4(12 - 0.005q)q + 0.6(12 - 0.01q)q - 3q$$

$$\frac{d\pi}{dq} = 0.4(12 - 0.01q) + 0.6(12 - 0.02q) - 3 = 0$$

$$p_H = 12 - 0.005 \cdot 562.5 = 9.19$$

$$p_L = 12 - 0.01 \cdot 562.5 = 6.38$$

$$12 - 0.004q - 0.012q - 3 = 0$$

$$E(\pi) = (0.4 \cdot 9.19 + 0.6 \cdot 6.38 - 3)562.5$$

$$0.016q = 9$$

$$E(\pi) = 2531.25$$

$$q = 562.5$$

b. Suppose you can set up an analytics program to obtain additional information on the probability of high demand. Your best guess is that with probability 0.3 they will tell you the probability of high demand is 0.89, and that otherwise they will tell you the probability of high demand is 0.19. What is the analytics program worth per period?

Since the low demand production constraint was binding when $P(H)$ was 0.4, it will surely bind when $P(H)$ is only 0.19. We don't know if it will bind if $P(H)$ is 0.89. So if $P(H)$ is 0.89, first try:

$$E(\pi) = 0.89(12 - 0.005q_H)q_H + 0.11(12 - 0.01q_L)q_L - 3q_H - 0.11 \cdot 1(q_H - q_L)$$

$$\frac{d\pi}{dq_H} = 0.89(12 - 0.01q_H) - 3 - 0.11 = 0$$

$$\frac{d\pi}{dq_L} = 0.11(12 - 0.02q_L) + 0.11 = 0$$

$$0.01q_H = 8.506$$

$$0.02q_L = 13$$

$$q_H = 850.6$$

$$q_L = 650$$

$$p_H = 12 - 0.005 \cdot 850.6 = 7.75$$

$$p_L = 12 - 0.01 \cdot 650 = 5.50$$

The $q_H > q_L$, so this is fine.

$$E(\pi) = 0.89 \cdot 7.75 \cdot 850.6 + 0.11 \cdot 5.5 \cdot 650 - 3 \cdot 850.6 - 0.11 \cdot 1(850.6 - 650) = 3686.4$$

If $P(H)$ is 0.19:

$$E(\pi) = 0.19(12 - 0.005q)q + 0.81(12 - 0.01q)q - 3q$$

$$\frac{d\pi}{dq} = 0.19(12 - 0.01q) + 0.81(12 - 0.02q) - 3 = 0 \quad p_H = 12 - 0.005 \cdot 497.24 = 9.51$$

$$12 - 0.0019q - 0.0162q - 3 = 0$$

$$p_L = 12 - 0.01 \cdot 497.24 = 7.03$$

$$0.0181q = 9$$

$$E(\pi) = (0.19 \cdot 9.51 + 0.81 \cdot 7.03 - 3)497.24$$

$$q = 497.24$$

$$E(\pi) = 2237.57$$

So, with the analytics program: $E(\pi) = 0.3 \cdot 3686.4 + 0.7 \cdot 2237.57 = 2672.22$. Without it, from a, $E(\pi) = 2531.25$. The value of the program is then 140.97.

c. Assume you do not have recourse to additional information as in (b). Instead, suppose that in addition to your current production line (that costs \$3 per unit) you could add a just in time production line with a cost of \$5 per unit. Find the maximum expected profit if you add this line, and therefore its value per production period. *Hint*: Since your base line cost is only \$3 per unit, you would always use it to produce any units you are certain to sell (low demand sales). The question is whether or not it saves money to use the just in time line for additional production when demand is high. The answer determines how you add the unit cost of units q_H through q_L , and the potential disposal cost, to the problem setup.

If you use the just in time line when demand is high, expected profit is:

$$E(\pi) = 0.4(12 - 0.005q_H)q_H + 0.6(12 - 0.01q_L)q_L - 3q_L - 0.4 \cdot 5(q_H - q_L)$$

$$\frac{d\pi}{dq_H} = 0.4(12 - 0.01q_H) - 0.4 \cdot 5 = 0 \quad \frac{d\pi}{dq_L} = 0.6(12 - 0.02q_L) - 3 + 0.4 \cdot 5 = 0$$

$$0.01q_H = 7$$

$$0.02q_L = 62/6$$

$$q_H = 700$$

$$q_L = 1550/3 = 516.67$$

$$p_H = 12 - 0.005 \cdot 700 = 8.50$$

$$p_L = 12 - 0.01 \cdot 516.67 = 6.83$$

$$E(\pi) = 0.4 \cdot 8.5 \cdot 700 + 0.6 \cdot 6.83 \cdot 516.67 - 3 \cdot 516.67 - 0.4 \cdot 5(700 - 516.67) = 2581.67$$

So, the value of the just in time line is $2581.67 - 2531.25 = 50.42$.

d. Continuing from (c), assume the safe rate of interest is 4% annually (so 0.04/12 monthly), and that you make one production run per month. Using the fact that the present value of a perpetual payment of \$V starting one period from the current period is V/r , calculate an upper bound of the expected present value of adding a just in time production line.

You make \$50.42 per month of additional profit. If the additional profit starts one month from the expenditure and continues forever, the present value is: $\sum_{t=1}^{\infty} \frac{50.42}{(1 + 0.04/12)^t}$. This converges to

$50.42 / (0.04/12)$, or \$15,125. This is an upper bound—the line is worth something less than this.

The real value would reflect costs of breakdowns and maintenance—though we could assume that was figured into the \$5 per unit cost. However, we also need to account for the possibility the product grows obsolete or an entirely new production process develops.

13. Chris' preferences are represented by $u = SB$, where S is the number of pizza slices he eats and B is the number of mugs of beer he drinks. Pizza costs \$2 per slice, beer costs \$3 per mug, and Chris has \$36 to spend on beer and pizza.

- Find the beer and pizza consumption bundle that maximizes his utility.
- Sketch the budget line and the indifference curve corresponding to Chris' choice.

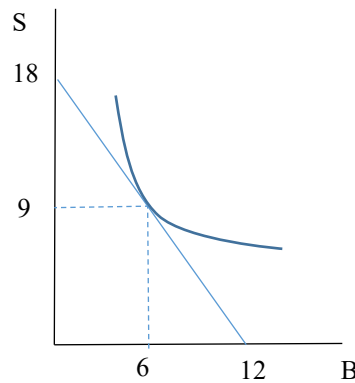
$$\frac{MU_B}{MU_S} = \frac{S}{B} = \frac{3}{2} = \frac{P_B}{P_S}$$

$$S = 1.5B$$

$$36 = 3B + 2 \cdot 1.5B = 6B$$

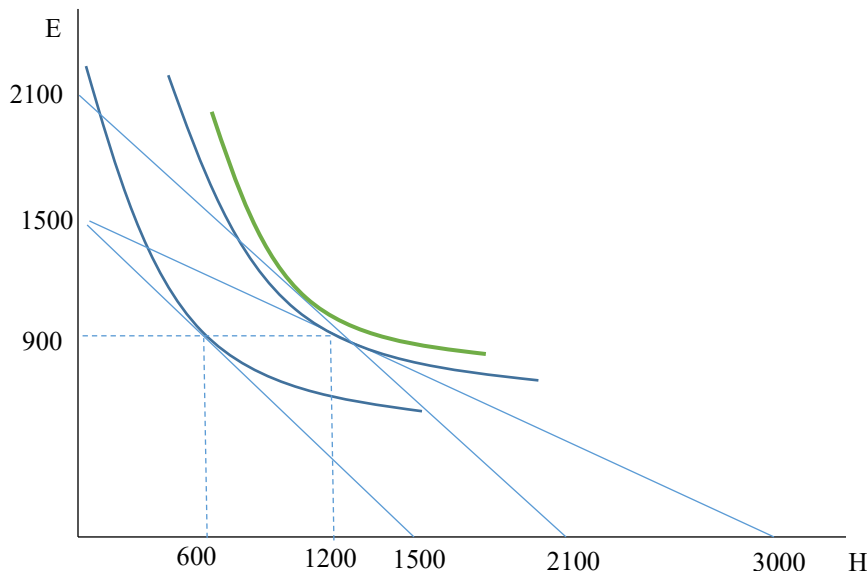
$$B = 6$$

$$S = 9$$



14. For purposes of this question, divide all things into housing, H , and money spent on everything else, E . Consider a household with a monthly income of \$1,500 facing a cost of \$1 per square foot to rent housing who chooses to live in a 600 square foot residence.

- Draw the household's budget line and an indifference curve appropriate to their choice.
- Consider a proposal to subsidize housing under which this household would face a price of \$0.50 per square foot. In that circumstance, suppose the household chooses to rent a 1,200 square foot residence. Show the new budget line and indifference curve.
- Now, for the analytical part... Use your figure to show that if the money spent subsidizing housing had simply been given to the household to spend as they saw fit (shifting their original budget line out but leaving the housing price at \$1 per square foot), the household would rent a smaller residence, spend more on other things, and reach a higher indifference curve, and so be better off. Explain why this is so. As part of your explanation, consider the rate at which housing can actually be changed into other things (given by market prices) compared to the rate the household is willing to trade housing for other things at their optimum choice given the artificial (subsidized) price.



- The budget line is $E=1500-H$.
- The new budget line is $E=1500-0.5H$.
- Paying \$0.50 per square foot on a 1200 square foot residence costs \$600. With the \$600, the budget line would be $E=2100-H$. Note that the indifference curve tangent to this budget line is higher than the indifference curve reached in part b. Why is this in intuitive terms? Because at the subsidized price, another unit of housing is only worth \$0.5 to the household, though it actually costs \$1, just half the cost is paid by taxpayers. Taking the last \$0.5 spent by taxpayers and the last \$0.5 spent on housing by the household, and instead spending it on other things (e.g. food or entertainment) costs the household housing worth only \$0.5 to them and gives them other things they would willingly pay \$1 for.

15. Ben's preferences are represented by $u = 0.3 \ln H + 0.7 \ln E + 0.1S$, where H is square feet of housing consumed monthly, E is the amount spent monthly on everything else, and $S=1$ if he lives somewhere sunny like Florida (no snow or sleet and little freezing weather) and 0 otherwise. He is considering 2 jobs, one in Tampa and one in Boston. The job in Boston pays \$7,000 per month. Housing costs \$4 per square foot monthly in Boston and \$1.5 per square foot monthly in Tampa. Calculate the salary in Tampa that would make Ben indifferent between the job in Tampa and the job in Boston. Illustrate with a figure.

The optimal choice in Boston:

$$\frac{MU_H}{MU_E} = \frac{0.3}{H} \frac{E}{0.7} = \frac{4}{1} = \frac{P_H}{P_E}$$

$$E = \frac{28}{3}H$$

$$7000 = 4H + \frac{28}{3}H = \frac{40}{3}H$$

$$H = \frac{21000}{40} = 525$$

$$E = 7000 - 4 \cdot 525 = 4900$$

$$u = 0.3 \ln 525 + 0.7 \ln 4900 = 7.827$$

The salary in Tampa, S , must be high enough to yield the same utility.

$$\frac{MU_H}{MU_E} = \frac{0.3}{H} \frac{E}{0.7} = 1.5 = \frac{P_H}{P_E}$$

$$E = 3.5H$$

$$Y = 1.5H + 3.5H = 5H$$

$$H = W/5$$

$$E = W - 3W/10 = 0.7W$$

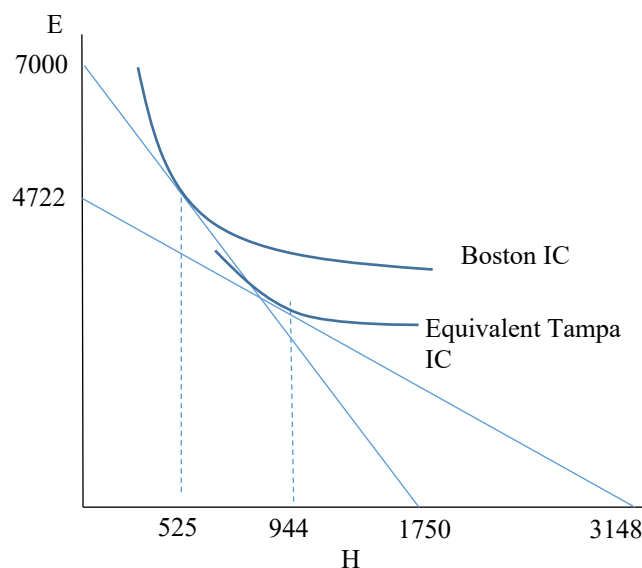
$$\begin{aligned} u &= 0.3 \ln 0.2 + 0.3 \ln W \\ &\quad + 0.7 \ln 0.7 + 0.7 \ln W + 0.1 \\ &= 7.827 \end{aligned}$$

$$\begin{aligned} \ln W &= 7.727 - 0.3 \ln 0.2 - 0.7 \ln 0.7 \\ &= 8.46 \end{aligned}$$

$$W = e^{8.46} = 4722$$

$$H = 944$$

$$E = 3306$$



16. An individual's inverse demand for a particular beer is $p = 3.5 - 0.25q$, where q is the number of bottles per period. The marginal cost is \$0.5 per bottle.

a. If bottles are sold at marginal cost, what is consumer surplus per consumer?

$$p = 0.5$$

$$0.25q = 3$$

$$q = 12$$

$$CS = 3.5q - 0.125q^2 - 0.5q = 18$$

b. If bottles must be sold one at a time and the firm maximizes profit, what are profit and consumer surplus per customer?

$$\pi = (3.5 - 0.25q)q - 0.5q$$

$$0.5q = 3$$

$$q = 6$$

$$p = 3.5 - 1.5 = 2$$

$$\pi = (2 - 0.5)6 = 9$$

$$CS = 0.5(3.5 - 2)6 = 4.5$$

c. Suppose the firm can sell packages of any number of bottles it chooses and resale is not possible. What number of bottles should be bundled together, and what price should the bundle be sold at, to maximize profit?

From a, 12 bottles per pack maximizes value added. The price for the pack is full willingness to pay, the area under the demand curve. That is 0.5×12 plus the consumer surplus of 18, or \$24.

d. Illustrate a-c with a figure.

