ECP 5007 – Economic Analysis for Technologists – Final Exam – Spring 2016 - Solution

1. Alice (a) and Bob (b) are in a winner take all R&D race to develop a solution to a problem. Let V_i represent the value to $i \in (a,b)$ if they win and let x_i represent i's R&D expenditure. The probability each wins, p_i, is their share of R&D expenditures. Each maximizes $u_i = V_i p_i - x_i$.

a) Solve for the reaction functions, which depend on the value of winning.

I'll do bit more here than required to make later parts of the question easier.

Maximizing: $$u_a = V_a \frac{x_a}{x_a + x_b} - x_a$$ $$\frac{du_a}{dx_a} = V_a \frac{x_b}{(x_a + x_b)^2} - 1 = 0$$. The reaction functions are: $$x_a = \sqrt{V_a x_b} - x_b$$ $$x_b = \sqrt{V_b x_a} - x_a$$.

To find the Nash equilibrium, note that from the FOC for the maximum:

$$V_a x_b = (x_a + x_b)^2 = V_b x_a$$ $$x_b = \frac{V_b}{V_a} x_a$$, so $$V_b x_a = \left(x_a + \frac{V_b}{V_a} x_a \right)^2$$ $$V_b x_a = x_a^2 \left(\frac{V_a + V_b}{V_a} \right)^2$$. Solving: $$x_a = \left(\frac{V_a}{V_a + V_b} \right)^2 V_b$$ $$x_b = \left(\frac{V_b}{V_a + V_b} \right)^2 V_a$$.

Note from these that the probability Alice wins is $\tilde{p}_a = \frac{V_a}{V_a + V_b}$. Alice's utility in equilibrium is

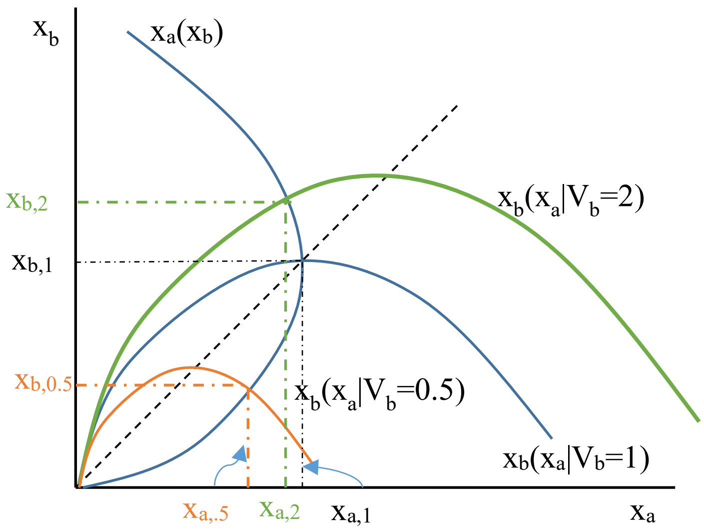
then $$\tilde{u}_a = V_a \frac{V_a}{V_a + V_b} - \left(\frac{V_a}{V_a + V_b} \right)^2 V_b = \frac{V_a^3}{(V_a + V_b)^2} = \tilde{p}_a^2 V_a$$ and Bob's is $\tilde{u}_b = \frac{V_b^3}{(V_a + V_b)^2} = (1 - \tilde{p}_a)^2 V_b$

b) Assuming V_a=1, solve for the Nash equilibrium expenditures and win probabilities in three scenarios: V_b=0.5, V_b=1, and V_b=2.

With the work above, this is just plugging things in:

Parameters		**Equilibrium Values**				
V_a	V_b	x_a	x_b	p_a	u_a	u_b
1	0.5	0.222	0.111	0.667	0.444	0.056
1	1	0.250	0.250	0.500	0.250	0.250
1	2	0.222	0.444	0.333	0.111	0.889

c) Sketch Alice's reaction function and Bob's reaction function for each of the three cases in b, labeling the equilibria.



d) Suppose Bob gets to commit to his R&D expenditure first. In the sequential move Nash equilibria, does Bob spend more or less than in the simultaneous move case if V_b=0.5? What about if V_b=2? Finally, what if V_b=1. In which case(s) is Alice better off if Bob moves first, and in which case(s) is she worse off? Just use what you know about first mover advantages, strategic complements, and strategic substitutes to explain. You don't have to solve for the actual values--you can if you want too, but it is not necessary.

This question is about the difference between outcomes depending on whether strategies are strategic substitutes or strategic complements. Recall that strategies are strategic complements when one player increasing their choice leads the other to do the same, so the reaction functions slope up. Strategic substitutes refers to the case where when one player chooses a higher level the other wants to choose a lower level, so the reaction functions slope down.

The difference from problems we have seen before is that each player's reaction function first slopes up then slopes down. So strategies will usually be strategic complements for one player and strategic substitutes for the other.

Start with the case where Bob values the price half as much as Alice. In this case, Alice's reaction function slopes up at the simultaneous solution shown in the figure. Bob wants Alice to spend less, so if he commits first, he spends less than he would if they moved at the same time in order to get Alice to spend less. Since Bob spends less, Alice is better off.

In the case where Bob values the price twice as much as Alice, Alice's reaction function slopes down at the simultaneous move equilibrium. Therefore, to get Alice to spend less, Bob spends more. As a result, Alice is worse off.

The case where both place the same value of the prize is tricky. At the simultaneous move solution, the slope of Alice's reaction function is 0. If Bob moves first, he can't do any better than just playing the same thing he would play if they moved at the same time. So, Alice is neither better nor worse off.

2. An argument for active local economic development policy is that there are increasing returns to scale in local labor markets. Therefore, there are local spillover benefits from attracting more jobs. (This applies particularly to high skill jobs, but let's abstract from that). A related argument is that urban transportation infrastructure investment is important for attracting jobs. Assume both are correct (FWIW, I think they are). It is common to use high congestion as an indicator that more investment in transportation infrastructure is needed. In this question, you will analyze whether using congestion as an indicator of the need for more transportation infrastructure investment is logically consistent with the two previous arguments.

Assumptions and Notation

1) The inverse supply of labor is $w = r + c + \theta n$, where n is the number of workers, w is the wage rate, r is the cost of a residence with access to the city, c is the disutility, or cost, of congestion, and θ is the slope of inverse labor supply. Higher wages are required to attract more workers to the city all else equal, so $\theta > 0$. Since the city is a tiny fraction of the entire world from which workers may be attracted, θ may not be much greater than 0. The higher rent or congestion, the higher wages must be to attract a given number of workers.

2) The rent gradient, or inverse housing supply, is $r = \lambda n$ where $\lambda > 0$ reflects an upward sloping supply of accessible housing. Rents accrue to non-workers—a development corporation.

3) The disutility of congestion is $c = \delta n$ where $\delta > 0$ reflects the contribution of another worker to congestion. Investments that lower congestion holding n constant reduce δ.

4) The demand for workers is perfectly elastic at $w = p$, where p is worker productivity.

5) Productivity is $p = \alpha + \beta n$. Increasing returns to labor market size means $\beta > 0$. Intuitively, $\alpha > 0$ is the city's productivity before the onset of limits to accessible housing, congestion, and increasing returns to city size. If that is not positive, no one comes to the city to begin with.

6) Increasing returns to scale don't overwhelm the collective forces of rising congestion, rising rents, and upward sloping labor supply, so wage increases needed to attract additional workers all else equal, so $\lambda + \delta + \theta > \beta$.

a) Solve for equilibrium city size and congestion, n_e and c_e, as functions of θ, λ, δ, α, and β.

Substituting into inverse labor supply for rent and congestion gives $w^S = (\lambda + \delta + \theta)n$. Substituting into inverse labor demand for productivity gives $w^D = \alpha + \beta n$. Equating these to find the equilibrium gives $n_e = \dfrac{\alpha}{\lambda + \delta + \theta - \beta}$. Congestion is then $c_e = \dfrac{\alpha\delta}{\lambda + \delta + \theta - \beta}$.

b) Suppose an increase in investment in infrastructure reduces δ. What happens to equilibrium city size and congestion? That is, what are the derivatives of n_e and c_e w.r.t. δ?

More efficient transportation infrastructure increases city size (obviously), so the derivative of n_e and n_e w.r.t. δ is negative, $\dfrac{\partial n_e}{\partial \delta} = \dfrac{-\alpha}{(\lambda + \delta + \theta - \beta)^2} < 0$. Then, the derivative of c_e w.r.t. δ is negative, $\dfrac{\partial c_e}{\partial \delta} = \dfrac{\alpha(\lambda + \theta - \beta)}{(\lambda + \delta + \theta - \beta)^2}$. This is positive only if $\lambda + \theta - \beta > 0$, or $\lambda + \theta > \beta$.

c) Consider the usefulness of congestion as a measure of the efficiency of transportation investment in light of the results of part b. Hint: $\lambda + \theta - \beta$ should be important in your answer. Recall it is entirely plausible for θ to be only slightly positive.

The direct impact of an improvement in transportation system efficiency that reduces δ is $-n\Delta\delta$, so a reduction in congestion. However, it is obvious from the derivative above that the effect on congestion in equilibrium, which includes not just the direct reduction in δ, but also the resulting increase in n, is not the same. More than not just being the same, it need not even move in the same direction. Assuming $\theta \approx 0$, an improvement in transportation system efficiency that lowers δ will only reduce congestion if $\lambda > \beta$. The larger the spillover benefits of increasing the number of workers in the city, as measured by β, the worse congestion is as a measure of the efficiency of the transportation infrastructure. ***If*** the spillover benefits increase productivity ***faster*** than the increase in the number of workers drives up rents, congestion actually gets worse in equilibrium the better the transportation system.

EXTRA, not required. If you are interested, the area above inverse labor supply and below the equilibrium wage, or $\int_0^{n_e} (w_e - r_e - c_e - \theta x)\,dx$, is producer surplus accruing to workers (PS_W). The area above inverse housing supply and below equilibrium rent, or $\int_0^{n_e} (r_e - \lambda x)\,dx$, is producer surplus accruing to the development corporation (PS_D). Maximizing the sum of these may be a potentially rational objective of local economic development policy. How are these impacted by a transportation infrastructure investment that reduces δ? How does this relate to the value of congestion as a measure of the quality of transportation infrastructure?

Integrating gives:

$$PS_W = (w_e - r_e - c_e)n_e - \frac{1}{2}\theta n_e^2 = \frac{1}{2}\theta n_e^2 = \frac{1}{2}\theta\left(\frac{\alpha}{\lambda+\delta+\theta-\beta}\right)^2$$

and

$$PS_D = r_e n_e - \frac{1}{2}\lambda n_e^2 = \frac{1}{2}\lambda n_e^2 = \frac{1}{2}\lambda\left(\frac{\alpha}{\lambda+\delta+\theta-\beta}\right)^2.$$

BOTH workers and the development corporation are better off when the transportation system is more efficient, whether congestion is better or worse. Formally, both fall when δ increases:

$$\frac{\partial PS_D}{\partial \delta} = -\theta\alpha^2(\lambda+\delta+\theta-\beta)^{-3} < 0 \quad \text{and} \quad \frac{\partial PS_D}{\partial \delta} = -\lambda\alpha^2(\lambda+\delta+\theta-\beta)^{-3} < 0.$$

Moreover, these effects are LARGER the LARGER the local benefits to attracting more jobs. Formally, the fall in both resulting from an increase δ is larger when β is larger:

$\dfrac{\partial^2 PS_D}{\partial\delta\partial\beta} = -3\theta\alpha^2(\lambda+\delta+\theta-\beta)^{-4} < 0$ and $\dfrac{\partial PS_D}{\partial \delta} = -3\lambda\alpha^2(\lambda+\delta+\theta-\beta)^{-4} < 0$. So, the larger the spillover effects, the stronger the case for investing in transportation infrastructure, but the worse congestion is as an indicator of the return from additional transportation investment.