1. What is the present value of the payments in the table to the right? Time is measured with the present being 0. Assume the riskless annual rate of return is 2.5%.

Time	1	2	3	4	5
Value	-50	-50	-100	-200	500

Answer:

$V_0 = -50 \times 1.025^{-1} - 50 \times 1.025^{-2} - 100 \times 1.025^{-3} - 200 \times 1.025^{-4} + 500 \times 1.025^{-5} = 71.51$.

2. Use the fact that the present value of a perpetuity paying \$X per period starting in one year is X/r, where r is the riskless rate of return, to determine the present value of annual payments of \$X accruing for 20 years, starting one year from now. Hint: Think of it as a perpetuity less the appropriately discounted value of a perpetuity starting 20 years from now.

Answer:

Think of this as owning the perpetuity through 20 payments and then losing the rights to the 21st payment on. From the point of view of t=20, the value of payments, starting at t=21 and continuing forever, is X/r. Converting that value (at t=20) to current value gives $\frac{1}{(1+r)^{20}}\frac{X}{r}$. So

$$V_0 = \frac{X}{r} - \frac{1}{(1+r)^{20}}\frac{X}{r} = \frac{X}{r}\left(1 - \frac{1}{(1+r)^{20}}\right).$$

3. What is the present value of the uncertain payment stream in the table to the right? Time is measured with the present being 0. P(End) is the probability the payment stream is permanently terminated before that period's payment is made, conditional on the previous period's payment having been made. So, you have to work out the probability the venture survives long enough for each payment to be made. The riskless annual rate of return is 4%.

Time	1	3	6	10
P(End)	0.1	0.1	0.4	0.7
Value	-10	-5	60	1000

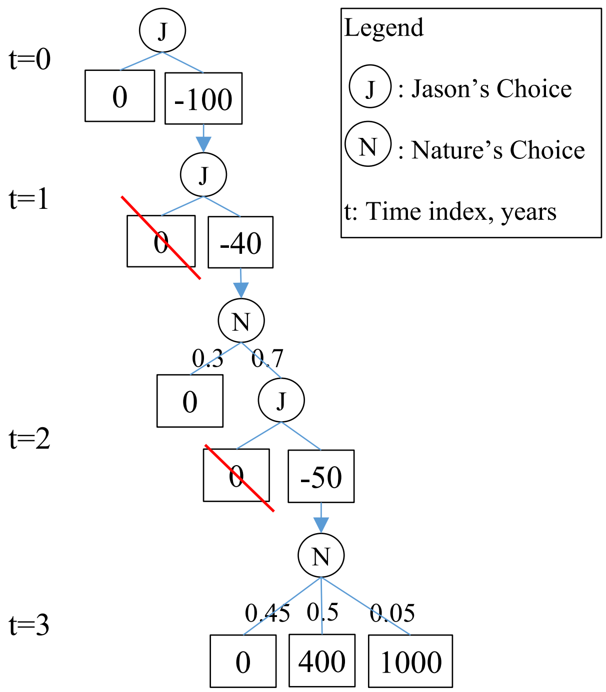
Answer:

$$\begin{aligned} EPV &= -10 \times 0.9 \times 1.04^{-1} - 5 \times 0.9 \times 0.9 \times 1.04^{-3} \\ &\quad + 60 \times 0.9 \times 0.9 \times 0.6 \times 1.04^{-6} + 1000 \times 0.9 \times 0.9 \times 0.6 \times 0.3 \times 1.04^{-10} \\ &= 109.29 \end{aligned}$$

4. Jason is considering developing a process innovation. It requires an initial investment of $100, then another investment of $40 after one year. Jason thinks the probability it will turn out to be feasible after two years is 0.7. If it is feasible, it will then take another expenditure of $50 (2 years from the initial investment) to complete. It will then be ready to demonstrate 3 years from the initial investment. Jason thinks there is a 0.05 probability that with a successful demonstration he will sell his innovation for $1,000 and a 0.5 probability he will sell it for $400, and that otherwise there will be no interest. The annual discount rate (riskless rate of return) is 5%. There are no other costs and Jason is risk neutral.

a. Illustrate the decision(s) to be made with a decision tree.

Answer:



b. What is the present expected value of the project?

Answer: First, note that it would not make sense to not spend the $40 at t=1 if it made sense to spend $100 at t=0 knowing you would have to spend another $40 at t=1 before you found anything new. You either spend both or neither. Similarly, it would not make sense to make the initial investments and not make the final $50 if the project is feasible.

So, if Jason proceeds:

$$EPV = -100 - \frac{40}{1.05} + 0.7\left(-\frac{50}{1.05^2} + \left(0.5\frac{400}{1.05^3} + 0.05\frac{1000}{1.05^3}\right)\right) = -18.67$$

Therefore, the EPV is 0, since he would not proceed.

c. What probability of selling the project for $400 would make Jason indifferent between pursuing it or not, assuming P(1000) stays the same?

Answer:

Working from the equation above, solve the following for f:

$$-100-\frac{40}{1.05}+0.7\left(-\frac{50}{1.05^2}+\left(f\frac{400}{1.05^3}+0.05\frac{1000}{1.05^3}\right)\right)=0$$, so f≈0.577.

d. Jason may obtain an expert's opinion of the feasibility of his idea for a fee. Suppose the consultant's studied opinion is completely accurate and Jason thinks there is a 70% chance they will find the innovation feasible. How much is the opinion worth?

Answer:

Obviously, Jason would not proceed with bad news from the consultant. With good news from the consultant:

$$EPV=-100-\frac{40}{1.05}-\frac{50}{1.05^2}+\left(0.5\frac{400}{1.05^3}+0.05\frac{1000}{1.05^3}\right)=32.51.$$

Therefore, if the information is purchased:

$$EPV=0.7\times 34.4+0.3\times 0=22.76.$$

Since Jason would not proceed otherwise, the report is worth up to 22.76.

e. Suppose, having dealt with consultants on similar projects in the past, Jason guestimates there is a 0.7 probability the consultant will report the innovation is probably feasible and otherwise the consultant will report the idea is probably not feasible. Jason thinks that if the consultant says the idea is probably not feasible, the probability it is feasible is 0.19 and that if the consultant says the idea is probably feasible, the probability it is feasible is 0.92. How much is the opinion worth?

Answer:

Above, we showed that Jason would not proceed with a 70% chance of success. Therefore, if the consultant reports bad news, Jason would not proceed with only a 19% chance of success, and EPV is 0. If the consultant reports probably success:

$$EPV=-100-\frac{40}{1.05}+0.92\left(-\frac{50}{1.05^2}+\left(0.5\frac{400}{1.05^3}+0.05\frac{1000}{1.05^3}\right)\right)=18.86.$$

Therefore, having purchased the information:

$$EPV=0.7\times 18.86+0.3\times 0=13.2.$$

Since Jason would not pursue the project without the report, the report is worth up to \$13.2.