1. What is the present value of the payments in the table to the right? Time is measured with the present being 0. Assume the riskless annual rate of return is 2.5%.

Time	1	2	3	4	5
Value	-50	-50	-100	-200	500

Answer:

$V_0 = -50 \times 1.025^{-1} - 50 \times 1.025^{-2} - 100 \times 1.025^{-3} - 200 \times 1.025^{-4} + 500 \times 1.025^{-5} = 71.51$.

2. Use the fact that the present value of a perpetuity paying \$X per period starting in one year is X/r, where r is the riskless rate of return, to determine the present value of annual payments of \$X accruing for 20 years, starting one year from now. Hint: Think of it as a perpetuity less the appropriately discounted value of a perpetuity starting 20 years from now.

Answer:

Think of this as owning the perpetuity through 20 payments and then losing the rights to the 21st payment on. From the point of view of t=20, the value of payments, starting at t=21 and continuing forever, is X/r. Converting that value (at t=20) to current value gives $\frac{1}{(1+r)^{20}}\frac{X}{r}$. So

$$V_0 = \frac{X}{r} - \frac{1}{(1+r)^{20}}\frac{X}{r} = \frac{X}{r}\left(1 - \frac{1}{(1+r)^{20}}\right).$$

3. What is the present value of the uncertain payment stream in the table to the right? Time is measured with the present being 0. P(End) is the probability the payment stream is permanently terminated before that period's payment is made, conditional on the previous period's payment having been made. So, you have to work out the probability the venture survives long enough for each payment to be made. The riskless annual rate of return is 4%.

Time	1	3	6	10
P(End)	0.1	0.1	0.4	0.7
Value	-10	-5	60	1000

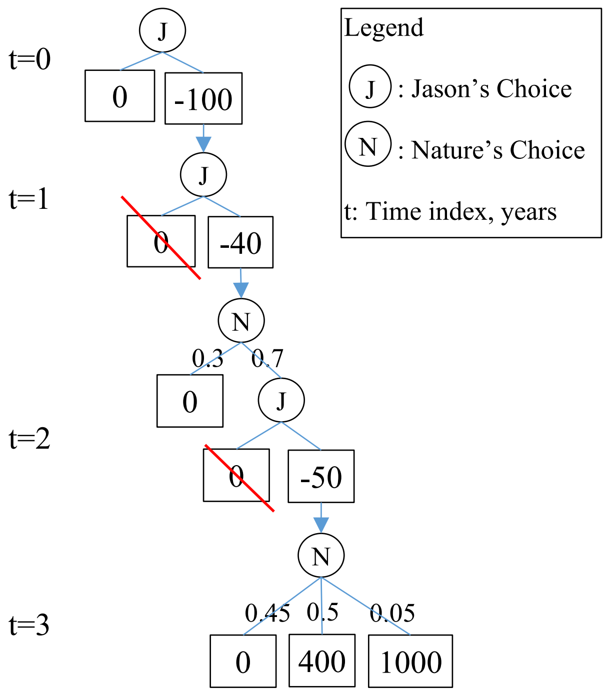
Answer:

$$\begin{aligned} EPV &= -10 \times 0.9 \times 1.04^{-1} - 5 \times 0.9 \times 0.9 \times 1.04^{-3} \\ &\quad + 60 \times 0.9 \times 0.9 \times 0.6 \times 1.04^{-6} + 1000 \times 0.9 \times 0.9 \times 0.6 \times 0.3 \times 1.04^{-10} \\ &= 109.29 \end{aligned}$$

4. Jason is considering developing a process innovation. It requires an initial investment of $100, then another investment of $40 after one year. Jason thinks the probability it will turn out to be feasible after two years is 0.7. If it is feasible, it will then take another expenditure of $50 (2 years from the initial investment) to complete. It will then be ready to demonstrate 3 years from the initial investment. Jason thinks there is a 0.05 probability that with a successful demonstration he will sell his innovation for $1,000 and a 0.5 probability he will sell it for $400, and that otherwise there will be no interest. The annual discount rate (riskless rate of return) is 5%. There are no other costs and Jason is risk neutral.

a. Illustrate the decision(s) to be made with a decision tree.

Answer:



b. What is the present expected value of the project?

Answer: First, note that it would not make sense to not spend the $40 at t=1 if it made sense to spend $100 at t=0 knowing you would have to spend another $40 at t=1 before you found anything new. You either spend both or neither. Similarly, it would not make sense to make the initial investments and not make the final $50 if the project is feasible.

So, if Jason proceeds:

$$EPV = -100 - \frac{40}{1.05} + 0.7\left(-\frac{50}{1.05^2} + \left(0.5\frac{400}{1.05^3} + 0.05\frac{1000}{1.05^3}\right)\right) = -18.67$$

Therefore, the EPV is 0, since he would not proceed.

c. What probability of selling the project for \$400 would make Jason indifferent between pursuing it or not, assuming P(1000) stays the same?

Answer:

Working from the equation above, solve the following for f:

$$-100-\frac{40}{1.05}+0.7\left(-\frac{50}{1.05^2}+\left(f\frac{400}{1.05^3}+0.05\frac{1000}{1.05^3}\right)\right)=0\text{, so f≈0.577.}$$

d. Jason may obtain an expert's opinion of the feasibility of his idea for a fee. Suppose the consultant's studied opinion is completely accurate and Jason thinks there is a 70% chance they will find the innovation feasible. How much is the opinion worth?

Answer:

Obviously, Jason would not proceed with bad news from the consultant. With good news from the consultant:

$$EPV=-100-\frac{40}{1.05}-\frac{50}{1.05^2}+\left(0.5\frac{400}{1.05^3}+0.05\frac{1000}{1.05^3}\right)=32.51.$$

Therefore, if the information is purchased:

$$EPV=0.7\times 34.4+0.3\times 0=22.76.$$

Since Jason would not proceed otherwise, the report is worth up to 22.76.

e. Suppose, having dealt with consultants on similar projects in the past, Jason guestimates there is a 0.7 probability the consultant will report the innovation is probably feasible and otherwise the consultant will report the idea is probably not feasible. Jason thinks that if the consultant says the idea is probably not feasible, the probability it is feasible is 0.19 and that if the consultant says the idea is probably feasible, the probability it is feasible is 0.92. How much is the opinion worth?

Answer:

Above, we showed that Jason would not proceed with a 70% chance of success. Therefore, if the consultant reports bad news, Jason would not proceed with only a 19% chance of success, and EPV is 0. If the consultant reports probably success:

$$EPV=-100-\frac{40}{1.05}+0.92\left(-\frac{50}{1.05^2}+\left(0.5\frac{400}{1.05^3}+0.05\frac{1000}{1.05^3}\right)\right)=18.86.$$

Therefore, having purchased the information:

$$EPV=0.7\times 18.86+0.3\times 0=13.2.$$

Since Jason would not pursue the project without the report, the report is worth up to \$13.2.

5. After incurring a cost of \$300 to set up a cafeteria for a day, each meal costs \$4 to prepare and serve. If the inverse demand for meals on Sunday is $p = 9 - 0.025q$, what price and quantity maximize profit, and what is maximum profit? Illustrate with a figure.

Answer: If the cafeteria opens, the highest possible profit is -50, shown below. So, it should not open, and the maximum profit is \$0 at q=0.

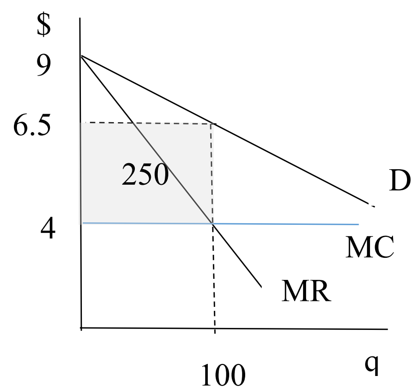
$$\pi = (9 - 0.025q)q - 4q - 300$$

$$\frac{d\pi}{dq} = 9 - 0.05q - 4 = 0$$

$$q^* = 100$$

$$p^* = 6.5$$

$$\pi = (6.5 - 4)100 - 300 = -50$$



6. Inverse demand is $p = -5 + 0.5m - 0.75q$, where m is per capita income. If the cost per unit is constant at \$5, calculate the profit maximizing price as a function of per capita income. How much does the profit maximizing price increase per \$1 increase in per capita income?

Answer:

$$\pi = (-5 + 0.5m - 0.75q)q - 5q$$

$$\frac{d\pi}{dq} = -5 + 0.5m - 1.5q - 5 = 0$$

$$q^* = (m - 20)/3$$

$$p^* = -5 + 0.5m - (3/4)(m - 20)/3$$

$$p^* = -5 + 0.5m - 0.25m + 5$$

$$p^* = 0.25m$$

$$\frac{dp^*}{dm} = 0.25$$

7. A firm sells q_B mugs of beer at price p_B, and q_P slices of pizza at price p_P. The inverse demand for mugs of beer is $p_B = 5 - 0.25q_B + 0.1q_P$ and the inverse demand for pizza slices is $p_P = 4 - 0.5q_P + 0.1q_B$. It costs \$1/mug to serve beer and \$2/slice to serve pizza. Find the prices and quantities that maximize profit and the maximum profit.

Answer:

First, set up the expression for the firm's profit, and set MR=MC for each product:

$$\pi = (5 - 0.25q_B + 0.1q_P)q_B + (4 - 0.5q_P + 0.1q_B)q_P - q_B - 2q_P$$

$$\frac{\partial \pi}{\partial q_B} = (5 - 0.5q_B + 0.1q_P) + 0.1q_P - 1 = 0 \qquad \frac{\partial \pi}{\partial q_B} = (4 - q_P + 0.1q_P) + 0.1q_P - 2 = 0$$

$$0.5q_B = 4 + 0.2q_P \qquad q_P = 2 + 0.2q_B$$

$$q_B = 8 + 0.4q_P$$

In the FOC, the terms in the parentheses represent MR to each firm from selling another unit of their product. The interesting thing in these FOCs is that the MR for pizza is higher when more beer is sold, and vice versa. So, each firm's sales boost MR for the other firm—this is the piece left out of the parentheses in the FOC above. That means it is optimal to sell more of each product than would otherwise be the case. If the different items were sold by different firms, prices would be higher for each item, because neither firm would internalize the impact its sales have on the other firm's profits. Horizontal integration between firms with market power selling complementary products results in lover, not higher, prices.

$$q_B = 8 + 0.4(2 + 0.2q_B) \qquad q_P = 2 + 0.2q_B$$

$$= 8 + 0.8 + 0.08q_B \qquad q_P = 2 + 0.2 \times 9.57 = 3.91$$

$$0.96q_B = 8.8 \qquad p_B = 5 - 0.25 \times 9.57 + 0.1 \times 3.91 = 3$$

$$q_B = 8.8/0.92 \qquad p_P = 4 - 0.5 \times 3.91 + 0.1 \times 9.57 = 3$$

$$= 9.57 \qquad \pi = (3-1)9.57 + (3-2)3.91 = 23.04$$

8. Demand is given by $q = 400p^{-2}$. Cost per unit is \$10. What are the profit maximizing price and quantity and what is the maximum profit?

Answer:

$$p^* = \left(\frac{-2}{-2+1}\right)10 = 20$$

$$q^* = \frac{400}{20^2} = 1$$

$$\pi^* = (20-10)1 = 10$$

9. At the current price of \$8, you sell 24 units. Cost is \$4/unit. Based on publically available estimates, you think the elasticity of demand is approximately -3. Estimate the profit maximizing price and the quantity sold and profit at that price.

Answer: $p^* = \left(\frac{-3}{-3+1}\right)4 = 6$. As a rough approximation, that is a 25% decline, so quantity would increase about 3*25%, or 75%, to about 42. That makes profit about (6-4)42=84.

To be more precise, if demand is reasonably approximated by constant elasticity ($q = Ap^{-3}$), the ratio of the new to old quantity is $\frac{q}{24} = \frac{6^{-3}}{8^{-3}}$ so $q = 24(4/3)^3 = 56.89$, and profit is \$113.78.

The approximations differ so much because percentage changes are a worse approximation of log point changes as the change becomes larger, and a 25% price change is large.

10. Inverse demand for movie tickets is $p_S = 16 - 0.01q_S$ for senior citizens and $p_A = 24 - 0.01q_A$ for others. The marginal cost of serving one more movie goer is \$4. Determine the profit maximizing prices for each type of customer.

$$\pi = (16 - 0.01q_S)q_S + (24 - 0.01q_A)q_A - 4(q_S + q_A)$$

$$\frac{d\pi}{dq_S} = 16 - 0.02q_S - 4 = 0 \qquad \frac{d\pi}{dq_A} = 24 - 0.02q_S - 4 = 0$$

$$0.02q_S = 12 \qquad 0.02q_A = 20$$

$$q_S = 600 \qquad q_A = 1000$$

$$p_S = 16 - 0.01 \cdot 600 = 10 \qquad p_A = 24 - 0.01 \cdot 1000 = 14$$

$$\pi = 10 \cdot 600 + 14 \cdot 1000 - 4 \cdot 1600 = 13600$$

11. The inverse demand for evening movie tickets is given by $p_E = 25 - 0.01q_E$ while the inverse demand for matinee tickets is $p_M = 15 - 0.01q_M$.

a. Assuming the marginal cost of serving one more customer is \$2 holding capacity constant, and that cost of adding capacity is \$2 per unit, determine profit maximizing capacity, prices and quantities.

Start by assuming evenings will sell out, but matinees will not. Then:

$$\pi = (25 - 0.01q_E)q_E + (15 - 0.01q_M)q_M - 2q_E - 2q_M - 2q_E$$

$$\frac{d\pi}{dq_E} = 25 - 0.02q_S - 2 - 2 = 0 \quad \frac{d\pi}{dq_M} = 15 - 0.02q_M - 2 = 0$$

$$0.02q_E = 21 \qquad 0.02q_M = 13$$

$$q_E = 1050 \qquad q_M = 650$$

$$p_E = 25 - 0.01 \cdot 1050 = 14.50 \quad p_M = 15 - 0.01 \cdot 6.50 = 8.50$$

The assumption on which the problem set up was based holds, so the solution is fine.

$$\pi = (14.50 - 4) \cdot 1050 + (8.50 - 2) \cdot 650 = 15250$$

b. Find the value of per unit capacity cost, k, at which the constraint that matinee quantity is less than or equal to capacity is just binding. That is, at all lower values of k, matinee ticket sales will be less than capacity, and at k or higher, matinee and evening sales both equal capacity.

With k now a variable, profit is: $\pi = (25 - 0.01q_E)q_E + (15 - 0.01q_M)q_M - 2q_E - 2q_M - kq_E$.

At what value of k will the evening quantity just equal the matinee quantity from part a, or 650?

$$\frac{d\pi}{dq_E} = 25 - 0.02q_S - 2 - k = 0 \qquad 1150 - 50k = 650$$

$$0.02q_E = 23 - k \qquad 50k = 500$$

$$q_E = 1150 - 50k \qquad k = 10$$

So, if k>\$10 all capacity will be used for evening and matinee shows, and the problem would need to be reformulated to reflect that.

12. Inverse demand will be $p_H = 12 - 0.005q_H$ with probability 0.4, and otherwise $p_L = 12 - 0.01q_L$. Any product not sold must be disposed of at a cost of $1 per unit. For parts a and b, production must take place *before* demand uncertainty is resolved and cost per unit is constant at $3.

a. Find the profit maximizing prices and quantities for each state of demand.

First, assume more is sold when demand is high. That means high demand determines the production level (capacity) and that with probability o.6, q_H-q_L units will have to be disposed of.

$$E(\pi) = 0.4(12 - 0.005q_H)q_H + 0.6(12 - 0.01q_L)q_L - 3q_H - 0.6 \cdot 1(q_H - q_L)$$

$$\frac{d\pi}{dq_E} = 0.4(12 - 0.01q_H) - 3 - 0.6 = 0 \qquad \frac{d\pi}{dq_L} = 0.6(12 - 0.02q_L) + 0.6 = 0$$

$$0.01q_H = 3 \qquad 0.02q_L = 13$$

$$q_H = 300 \qquad q_L = 650$$

$$p_H = 12 - 0.005 \cdot 300 = 10.5 \qquad p_L = 12 - 0.01 \cdot 650 = 5.50$$

The assumption on which the problem set up was based does not hold. The problem must be reformulated with q_H=q_L. There will then be no disposal needed.

$$E(\pi) = 0.4(12 - 0.005q)q + 0.6(12 - 0.01q)q - 3q$$

$$\frac{d\pi}{dq} = 0.4(12 - 0.01q) + 0.6(12 - 0.02q) - 3 = 0$$

$$12 - 0.004q - 0.012q - 3 = 0$$

$$0.016q = 9$$

$$q = 562.5$$

$$p_H = 12 - 0.005 \cdot 562.5 = 9.19$$

$$p_L = 12 - 0.01 \cdot 562.5 = 6.38$$

$$E(\pi) = (0.4 \cdot 9.19 + 0.6 \cdot 6.38 - 3)562.5$$

$$E(\pi) = 2531.25$$

b. Suppose you can set up an analytics program to obtain additional information on the probability of high demand. Your best guess is that with probability 0.3 they will tell you the probability of high demand is 0.89, and that otherwise they will tell you the probability of high demand is 0.19. What is the analytics program worth per period?

Since the low demand production constraint was binding when P(H) was 0.4, it will surely bind when P(H) is only 0.19. We don't know if it will bind if P(H) is 0.89. So if P(H) is 0.89, first try:

$$E(\pi) = 0.89(12 - 0.005q_H)q_H + 0.11(12 - 0.01q_L)q_L - 3q_H - 0.11 \cdot 1(q_H - q_L)$$

$$\frac{d\pi}{dq_H} = 0.89(12 - 0.01q_H) - 3 - 0.11 = 0 \qquad \frac{d\pi}{dq_L} = 0.11(12 - 0.02q_L) + 0.11 = 0$$

$$0.01q_H = 8.506 \qquad 0.02q_L = 13$$

$$q_H = 850.6 \qquad q_L = 650$$

$$p_H = 12 - 0.005 \cdot 850.6 = 7.75 \qquad p_L = 12 - 0.01 \cdot 650 = 5.50$$

The q_H>q_L, so this is fine.

$$E(\pi) = 0.89 \cdot 7.75 \cdot 850.6 + 0.11 \cdot 5.5 \cdot 650 - 3 \cdot 850.6 - 0.11 \cdot 1(850.6 - 650) = 3686.4$$

If P(H) is 0.19:

$$E(\pi)=0.19(12-0.005q)q+0.81(12-0.01q)q-3q$$

$$\frac{d\pi}{dq}=0.19(12-0.01q)+0.81(12-0.02q)-3=0$$

$$12-0.0019q-0.0162q-3=0$$

$$0.0181q=9$$

$$q=497.24$$

$$p_H=12-0.005\cdot 497.24=9.51$$

$$p_L=12-0.01\cdot 497.24=7.03$$

$$E(\pi)=(0.19\cdot 9.51+0.81\cdot 7.03-3)497.24$$

$$E(\pi)=2237.57$$

So, with the analytics program: $E(\pi)=0.3\cdot 3686.4+0.7\cdot 2237.57=2672.22$. Without it, from a, E(π)=2531.25. The value of the program is then 140.97.

c. Assume you do not have recourse to additional information as in (b). Instead, suppose that in addition to your current production line (that costs \$3 per unit) you could add a just in time production line with a cost of \$5 per unit. Find the maximum expected profit if you add this line, and therefore its value per production period. *Hint*: Since your base line cost is only \$3 per unit, you would always use it to produce any units you are certain to sell (low demand sales). The question is whether or not it saves money to use the just in time line for additional production when demand is high. The answer determines how you add the unit cost of units q_H through q_L, and the potential disposal cost, to the problem setup.

If you use the just in time line when demand is high, expected profit is:

$$E(\pi)=0.4(12-0.005q_H)q_H+0.6(12-0.01q_L)q_L-3q_L-0.4\cdot 5(q_H-q_L)$$

$$\frac{d\pi}{dq_E}=0.4(12-0.01q_H)-0.4\cdot 5=0$$

$$0.01q_H=7$$

$$q_H=700$$

$$p_H=12-0.005\cdot 700=8.50$$

$$\frac{d\pi}{dq_L}=0.6(12-0.02q_L)-3+0.4\cdot 5=0$$

$$0.02q_L=62/6$$

$$q_L=1550/3=516.67$$

$$p_L=12-0.01\cdot 516.67=6.83$$

$$E(\pi)=0.4\cdot 8.5\cdot 700+0.6\cdot 6.83\cdot 516.67-3\cdot 516.67\cdot -0.4\cdot 5(700-516.67)=2581.67$$

So, the value of the just in time line is 2581.67-2531.25=50.42.

d. Continuing from (c), assume the safe rate of interest is 4% annually (so 0.04/12 monthly), and that you make one production run per month. Using the fact that the present value of a perpetual payment of \$V starting one period from the current period is \$V/r, calculate an upper bound of the expected present value of adding a just in time production line.

You make \$50.42 per month of additional profit. If the additional profit starts one month from the expenditure and continues forever, the present value is: $\sum_{t=1}^{\infty}\frac{50.42}{(1+0.04/12)^t}$. This converges to $50.42/(0.04/12)$, or \$15,125. This is an upper bound—the line is worth something less than this. The real value would reflect costs of breakdowns and maintenance—though we could assume that was figured into the \$5 per unit cost. However, we also need to account for the possibility the product grows obsolete or an entirely new production process develops.

13. Chris' preferences are represented by $u = SB$, where S is the number of pizza slices he eats and B is the number of mugs of beer he drinks. Pizza costs \$2 per slice, beer costs \$3 per mug, and Chris has \$36 to spend on beer and pizza.

a. Find the beer and pizza consumption bundle that maximizes his utility.

b. Sketch the budget line and the indifference curve corresponding to Chris' choice.

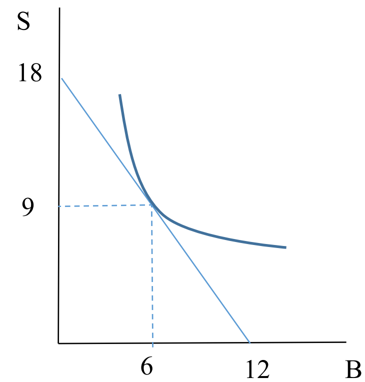
$$\frac{MU_B}{MU_S} = \frac{S}{B} = \frac{3}{2} = \frac{P_B}{P_S}$$

$$S = 1.5B$$

$$36 = 3B + 2 \cdot 1.5B = 6B$$

$$B = 6$$

$$S = 9$$

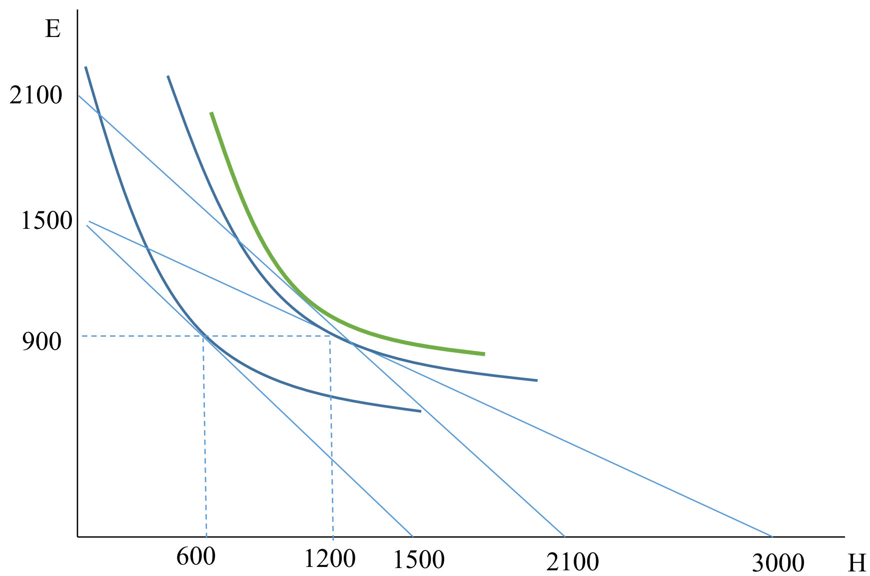


14. For purposes of this question, divide all things into housing, *H*, and money spent on everything else, *E*. Consider a household with a monthly income of $1,500 facing a cost of $1 per square foot to rent housing who chooses to live in a 600 square foot residence.

a. Draw the household's budget line and an indifference curve appropriate to their choice.

b. Consider a proposal to subsidize housing under which this household would face a price of $0.50 per square foot. In that circumstance, suppose the household chooses to rent a 1,200 square foot residence. Show the new budget line and indifference curve.

c. Now, for the analytical part... Use your figure to show that if the money spent subsidizing housing had simply been given to the household to spend as they saw fit (shifting their original budget line out but leaving the housing price at $1 per square foot), the household would rent a smaller residence, spend more on other things, and reach a higher indifference curve, and so be better off. Explain why this is so. As part of your explanation, consider the rate at which housing can actually be changed into other things (given by market prices) compared to the rate the household is willing to trade housing for other things at their optimum choice given the artificial (subsidized) price.



a. The budget line is E=1500-H.

b. The new budget line is E=1500-0.5H.

c. Paying $0.50 per square foot on a 1200 square foot residence costs $600. With the $600, the budget line would be E=2100-H. Note that the indifference curve tangent to this budget line is higher than the indifference curve reached in part b. Why is this in intuitive terms? Because at the subsidized price, another unit of housing is only worth $0.5 to the household, though it actually costs $1, just half the cost is paid by taxpayers. Taking the last $0.5 spent by taxpayers and the last $0.5 spent on housing by the household, and instead spending it on other things (e.g. food or entertainment) costs the household housing worth only $0.5 to them an gives them other things they would willingly pay $1 for.

15. Ben's preferences are represented by $u = 0.3\ln H + 0.7\ln E + 0.1S$, where H is square feet of housing consumed monthly, E is the amount spent monthly on everything else, and S=1 if he lives somewhere sunny like Florida (no snow or sleet and little freezing weather) and 0 otherwise. He is considering 2 jobs, one in Tampa and one in Boston. The job in Boston pays \$7,000 per month. Housing costs \$4 per square foot monthly in Boston and \$1.5 per square foot monthly in Tampa. Calculate the salary in Tampa that would make Ben indifferent between the job in Tampa and the job in Boston. Illustrate with a figure.

The optimal choice in Boston:

$$\frac{MU_H}{MU_E} = \frac{0.3}{H}\frac{E}{0.7} = \frac{4}{1} = \frac{P_H}{P_E}$$

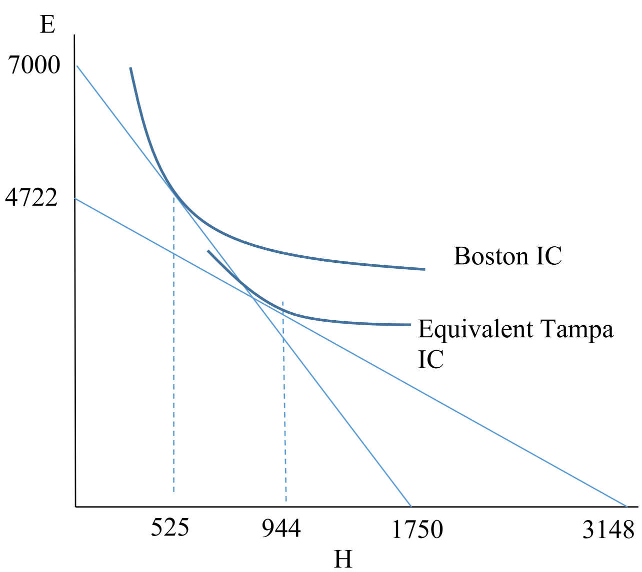
$$E = \frac{28}{3}H$$

$$7000 = 4H + \frac{28}{3}H = \frac{40}{3}H$$

$$H = \frac{21000}{40} = 525$$

$$E = 7000 - 4\cdot 525 = 4900$$

$$u = 0.3\ln 525 + 0.7\ln 4900 = 7.827$$



The salary in Tampa, S, must be high enough to yield the same utility.

$$\frac{MU_H}{MU_E} = \frac{0.3}{H}\frac{E}{0.7} = 1.5 = \frac{P_H}{P_E}$$

$$E = 3.5H$$

$$Y = 1.5H + 3.5H = 5H$$

$$H = W/5$$

$$E = W - 3W/10 = 0.7W$$

$$\begin{aligned} u &= 0.3\ln 0.2 + 0.3\ln W \\ &\quad +0.7\ln 0.7 + 0.7\ln W + 0.1 \\ &= 7.827 \end{aligned}$$

$$\begin{aligned} \ln W &= 7.727 - 0.3\ln 0.2 - 0.7\ln 0.7 \\ &= 8.46 \end{aligned}$$

$$W = e^{8.46} = 4722$$

$$H = 944$$

$$E = 3306$$

16. An individual's inverse demand for a particular beer is $p = 3.5 - 0.25q$, where q is the number of bottles per period. The marginal cost is \$0.5 per bottle.

a. If bottles are sold at marginal cost, what is consumer surplus per consumer?

$p = 0.5$

$0.25q = 3$

$q = 12$

$CS = 3.5q - 0.125q^2 - 0.5q = 18$

b. If bottles must be sold one at a time and the firm maximizes profit, what are profit and consumer surplus per customer?

$\pi = (3.5 - 0.25q)q - 0.5q$

$0.5q = 3$

$q = 6$

$p = 3.5 - 1.5 = 2$

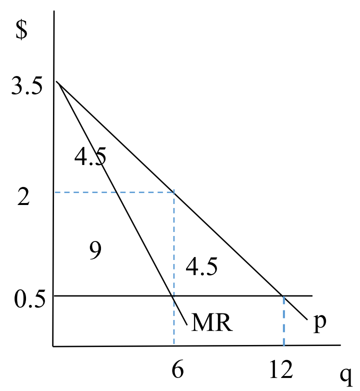
$\pi = (2 - 0.5)6 = 9$

$CS = 0.5(3.5 - 2)6 = 4.5$

c. Suppose the firm can sell packages of any number of bottles it chooses and resale is not possible. What number of bottles should be bundled together, and what price should the bundle be sold at, to maximize profit?

From a, 12 bottles per pack maximizes value added. The price for the pack is full willingness to pay, the area under the demand curve. That is 0.5*12 plus the consumer surplus of 18, or \$24.

d. Illustrate a-c with a figure.



17. There are two types of consumer's for a particular beer, with inverse demands given by $p_1 = 3.5 - 0.25q_1$ and $p_2 = 3.3 - 0.4q_2$, respectively, where q is the number of bottles per period. Marginal cost is \$0.5 per bottle.

a. Assuming 40% of customers are type 1 and the remainder are type 2, set up the optimization problem with all 4 constraints, though only 2 will bind.

$$Max \quad \pi = 0.4\left(P_1 - 0.5q_1\right) + 0.6\left(P_2 - 0.5q_2\right)$$

$$st \quad \begin{aligned} &3.5q_1 - 0.125q_1^2 \geq P_1 \\ &3.3q_1 - 0.2q_2^2 \geq P_2 \\ &3.5q_1 - 0.125q_1^2 - P_1 \geq 3.5q_2 - 0.125q_2^2 - P_2 \\ &3.3q_2 - 0.2q_2^2 - P_2 \geq 3.3q_1 - 0.2q_1^2 - P_1 \end{aligned}$$

b. Solve the problem to find the optimal bundle for each consumer type, their prices, the firm's profits, and each consumer type's surplus.

The second and third constraints bind. From them:

$$P_2 = 3.3q_2 - 0.2q_2^2$$

and

$$3.5q_1 - 0.125q_1^2 - P_1 = 3.5q_2 - 0.125q_2^2 - P_2$$

$$P_1 = P_2 + \left(3.5q_1 - 0.125q_1^2\right) - \left(3.5q_2 - 0.125q_2^2\right)$$

$$P_1 = P_2 + \left(3.5q_1 - 0.125q_1^2\right) - \left(3.5q_2 - 0.125q_2^2\right)$$

$$P_1 = \left(3.3q_2 - 0.2q_2^2\right) + \left(3.5q_1 - 0.125q_1^2\right) - \left(3.5q_2 - 0.125q_2^2\right)$$

$$P_1 = 3.5q_1 - 0.125q_1^2 - 0.2q_2 - 0.075q_2^2$$

Substituting into the expression for profit and maximizing:

$$\pi = 0.4\left(3.5q_1 - 0.125q_1^2 - 0.2q_2 - 0.075q_2^2 - 0.5q_1\right) + 0.6\left(3.3q_2 - 0.2q_2^2 - 0.5q_2\right)$$

$$\frac{\partial \pi}{\partial q_1} = 0.4\left(3 - 0.25q_1\right) = 0$$

$$q_1 = 12$$

$$\frac{\partial \pi}{\partial q_2} = 0.6\left(2.8 - 0.4q_2\right) - 0.4\left(0.2 + 0.15q_2\right) = 0$$

$$0.3q_2 = 1.6$$

$$q_2 = 5.33$$

$$P_2 = 3.3 \cdot 5.33 - 0.2 \cdot 5.33^2 = 11.91$$

$$P_1 = 3.5 \cdot 12 - 0.125 \cdot 12^2 - 0.2 \cdot 5.33 - 0.075 \cdot 5.33^2 = 20.80$$

$$\pi = 0.4\left(20.80 - 0.5 \cdot 12\right) + 0.6\left(11.91 - 0.5 \cdot 5.33\right) = 11.47$$

c. How much additional profit could be made if the company could engage in perfect 1st degree price discrimination?

While 12 units maximizes surplus from type 1, a price above \$20.80 can be charged with perfect price discrimination: $P_1 = 3.5 \cdot 12 - 0.125 \cdot 12^2 = 24$.

To find the quantity that maximizes surplus from type 2:

$$p_2 = MC$$
$$3.3 - 0.4q_2 = 0.5$$
$$0.4q = 2.8$$
$$q = 7$$

So:

$$P_2 = 3.3 \cdot 7 - 0.2 \cdot 7^2 = 13.3$$

From there:

$$\pi = 0.4(24 - 0.5 \cdot 12) + 0.6(13.3 - 0.5 \cdot 7)$$
$$\pi = 0.4(18) + 0.6(9.8) = 13.08$$

So, profit is \$1.65 higher.

d. Let α represent the fraction of consumers that are type 1, while all remaining customers are type 2. Set up the optimization problem and solve for the optimal bundles and prices as a function of α. What happens to the bundles and prices as α increases? Explain why in intuitive terms.

Working from b, the binding constraints are the same. So:

$$\pi = \alpha\left(3.5q_1 - 0.125q_1^2 - 0.2q_2 - 0.075q_2^2 - 0.5q_1\right) + (1-\alpha)\left(3.3q_2 - 0.2q_2^2 - 0.5q_2\right)$$

$$\frac{\partial \pi}{\partial q_1} = 0.4(3 - 0.25q_1) = 0$$

$$q_1 = 12$$

$$\frac{\partial \pi}{\partial q_2} = -\alpha(0.2 + 0.15q_2) + (1-\alpha)(2.8 - 0.4q_2) = 0$$

$$(0.4 - 0.25\alpha)q_2 = 2.8 - 3\alpha$$

$$q_2 = \frac{2.8 - 3\alpha}{0.4 - 0.25\alpha}$$

$$\frac{dq_2}{d\alpha} = \frac{-3(0.4 - 0.25\alpha) + 0.25(2.8 - 3\alpha)}{(0.4 - 0.25\alpha)^2} = \frac{-0.5}{(0.4 - 0.25\alpha)^2} < 0$$

Type 1 quantity is unchanged, since it was calculated to generate the highest possible willingness to pay regardless of alpha. However, as the share of type 1 customers goes up, the size of the type 2 bundle goes down. This is to make type 2's package less attractive to type 1, so more profit can be extracted from type 1 by increasing the price paid by type 1, since there are more of them. As type 2's quantity falls and as all surplus is extracted from them, the type 2 price falls.

18. This one requires going beyond what we talked about explicitly in class. As in the previous problem, assume there are two types of beer consumers with inverse demands of $p_1 = 3.5 - 0.25q_1$ and $p_2 = 3.3 - 0.4q_2$ and marginal cost is \$0.5 per bottle. Let α represent the fraction of type 1 consumers. Suppose the firm can only offer one bundle at one price, either because of resale limits or other practical considerations.

a. Set up the profit maximization problem. Of the four constraints from 5a, two become irrelevant. Which two and why do they become irrelevant? Of the other two, assuming both types of customers are served, which is will be binding, and why?

Since there is only one type of bundle, the incentive compatibility constraints, which ensure each type chooses the intended bundle, become irrelevant.

The type 1 participation constraint does not bind, for the same reason it did not above—type 1 will get some consumer surplus anytime type 2 is willing to buy, since they place more value on the product.

b. Set up profit as a function of alpha assuming both types are served, and show that alpha drops out. Then, solve for the profit maximizing bundle and its price assuming both types are served.

The constraint is: $P = 3.3q - 0.2q^2$. So

$$\pi = \alpha\left(3.3q - 0.2q^2 - 0.5q\right) + (1-\alpha)\left(3.3q - 0.2q^2 - 0.5q\right)$$

or

$$\pi = 3.3q - 0.2q^2 - 0.5q$$

So, *assuming* both types are served, profit does not depend on alpha. Maximizing:

$$\frac{d\pi}{dq} = 3.3 - 0.4q - 0.5 = 0$$

$$0.4q = 2.8$$

$$q = 7$$

$$P = 3.3 \cdot 7 - 0.2 \cdot 7^7 = 13.3$$

$$\pi = 13.3 - 0.5 \cdot 7 = 9.8$$

c. At what value of α would it make sense to price one type of customer out of the market? Which type is priced out? Why? What is the profit maximizing bundle and its price if it is most profitable to serve only one customer type?

If there are many type 1 customers and few enough type 2 customers, it would make sense to target the type 1 customers and extract all surplus from them. That, though, would price type 2 customers out of the market. To find the exact level of alpha where this would occur, first find profit if type 2 is priced out:

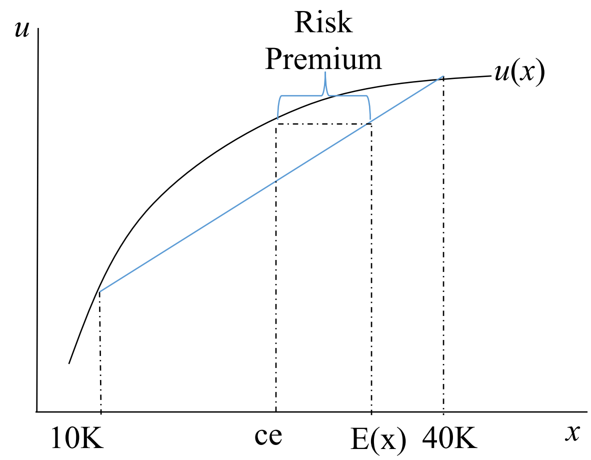
The constraint is: $P = 3.5q - 0.125q^2$. So $\pi = \alpha\left(3.5q - 0.125q^2 - 0.5q\right)$.

As shown above, this is maximized with q=12, for which type 1 is willing to pay \$24. Then: $\pi = \alpha\left(24 - 0.5 \cdot 12\right) = 18\alpha$. If this is higher than the profit in 6b, it is the solution. So, if: 18α≥9.8, or if α≥0.54, it is optimal to focus only on the large customers.

19. Risk Aversion and Insurance

Individual preferences for risky monetary outcomes (x) are well approximated by the von Neumann-Morgenstern utility function $u = \sqrt{x}$. Income is \$40,000. There is a 0.05 probability of a \$30,000 loss.

a. Calculate the certainty equivalent of an individual's lottery and the risk premium, and show them graphically.



$$\sqrt{ce} = 0.95\sqrt{40000} + 0.05\sqrt{10000}$$
$$= 190 + 5 = 195$$
$$ce = 195^2 = 38025$$
$$E(x) = 0.95 \cdot 40000 + 0.05 \cdot 10000$$
$$= 38500$$
$$RP = E(x) - ce$$
$$= 38500 - 38025$$
$$= 475$$

b. Suppose there are 2 identical individuals and losses are independent. Neither, one, or both may suffer a loss. If the two pool their risk, and so share any losses equally, what is the certainty equivalent of an individual's lottery? In terms of the certainty equivalent, how much value did they gain from pooling their risk?

No loss: $P(x=40000)=0.95^2=0.9025$

One loss: $P(x=40000-30000/2=25000)=2\cdot 0.950\cdot 0.05=0.095$

Two losses: $P(x=10000)=0.05^2=0.0025$

$$\sqrt{ce} = 0.9025\sqrt{40000} + 0.095\sqrt{25000} + 0.0025\sqrt{10000}$$
$$\sqrt{ce} = 195.77$$
$$ce = 195.77^2 = 38326.21$$

Compared to a, each gains \$301.21, for a total gain of \$602.42.

c. Suppose there are 5 identical individuals facing independent losses, with the total number of losses therefore following the binomial distribution. That is, the probability of exactly k losses in n trials if the chance of an individual loss is p is $P(k \mid n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$. If all 5 pool their risk, what is the certainty equivalent of each individual's lottery? How much value is created (in total) by pooling risk rather than each individual bearing their own risk?

# Losses	P	Total Loss	x	u(x)
0	0.7738	0	40000	200.00
1	0.2036	30000	34000	184.39
2	0.0214	60000	28000	167.33
3	0.0011	90000	22000	148.32
4	<0.0001	120000	16000	126.49
5	<0.0001	150000	10000	100.00

E(u(x))=196.06

ce=38439.85

Compared to a, each gains $414.85, for a total gain of $2074.27.

d. Suppose there are 100,000 identical individuals and risks are independent. A risk neutral insurance company insures everyone at the actuarially fair rate. What is that rate? What is each individual's now certain income? How much value is added by the insurance company?

The actuarially fair rate just covers expected losses, which are 0.05*30000=1500. Individual income is now 38500. Each gains 475 relative to a, for 47.5M in total.

e. For the binomial distribution, the mean number of occurrences is np and the standard deviation of the total number of occurrences is $\sqrt{np(1-p)}$. Suppose regulations require the insurance company in (d) to hold reserves so that there is only a 0.000001 probability they will experience losses over annual revenue plus reserves, which they could not cover. How much is needed in reserves? With large numbers of observations, the binomial distribution is very closely approximated by the normal distribution, with the appropriate mean and standard deviation. So, you can use the normal approximation with the implied mean and standard deviation, or, you can use any program that can calculate the inverse of the cumulative binomial distribution (excel will do it using the binom.inv() function).

The company must be able to cover 5331 losses (there is less than a one in one million chance losses will be higher), or $159.93M. The company's revenue from premiums is only $150M. So, the company needs reserves of $9.93M beyond annual revenue.

f. Interpret the role of reinsurance companies in light of (e).

To reduce their exposure to losses, insurance companies can essentially buy insurance against rare events when losses turn out to be very high. This reduces the capital (reserve) requirements needed to write insurance policies while maintaining solvency, thus the ability to meet obligations, with (near) certainty. When all losses at the individual company level are not independent, and they likely are not, this becomes all the more important.

20. Find any pure strategy nash equilibria of this game.

a3,b4

		Bob			
		b1	b2	b3	b4
Alice	a1	0,0	0,0	0,1	1,5
	a2	0,0	1,1	2,5	1,4
	a3	1,0	4,4	2,3	2,5
	a4	5,1	3,2	3,1	-2,-2

21. Find all pure strategy Nash equilibria of the following game in which Alice chooses the row (first payoff in each cell), Bob chooses the column (second payoff in each cell), and Charlie chooses the table (third payoff in each cell).

Best responses marked by bold. The Nash equilibriua are Down, Right, Table 1, and Up, Left, Table 2

Table 1

		Bob	
		Left	Right
Alice	Up	0,1,0	1,**1**,**2**
	Down	**1**,2,**3**	**2**,**3**,**2**

Table 2

		Bob	
		Left	Right
Alice	Up	**2**,**2**,**3**	**0**,**2**,1
	Down	1,**2**,**3**	**0**,1,0

22. Suppose the offense, trailing by 5 points, has the ball on the 3 yard line with 1 second left on the game clock. Neither team has any timeouts. The offense can choose personnel better suited to run or personnel better suited to pass. The defense can choose personnel better suited to defend the run or better suited to defend the pass.

If both choose run the probability of a touchdown is 0.25.
If the offense chooses pass and the defense chooses run, the probability of a touchdown is 0.75.
If the offense chooses pass and the defense chooses pass, the probability of a touchdown is 0.4.
If the offense chooses run and the defense chooses pass, the probability of a touchdown is 0.5.

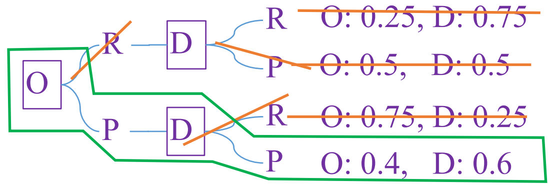
a) Assume moves are simultaneous. Show there is no pure strategy equilibrium and find the mixed strategy equilibrium. What is the probability of a touchdown in equilibrium?

		Defense	
		Run	Pass
Offense	Run	0.25,**0.75**	**0.5**,0.5
	Pass	**0.75**,0.25	0.4,**0.6**

Best replies in bold. There are no pure strategy Nash equilibria.
f: probability offence chooses run personnel
d: probability defense chooses run personnel
For the offense to mix, d must make them indifferent:
0.25d+0.5(1-d)=0.75d+0.4(1-d), d=1/6
For the defense to mix, f must make them indifferent:
0.75f+0.25(1-f)=0.5f+0.6(1-f), f=7/12

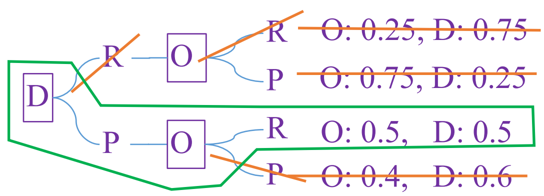
The probability of a touchdown is then:
(1/6)(7/12)(1/4)+(1/6)(5/12)(3/4)+(5/6)(7/12)(1/2)+(5/6)(5/12)(4/10)=0.46

b) Show the extensive form if the offense commits to its move first and find the backward induction solution.



At the second stage, the defense matches the offense's decision.
At the first stage, the offense therefore chooses pass.
The offense scores with probability 0.4.

c) Show the extensive form if the defense commits to its move first and find the backward induction solution.



At the second stage, the offense chooses opposite the defense's decision.
At the first stage, the defense therefore chooses pass.
The offense scores with probability 0.5.

23. Continuation of #3. Each team can choose to send in its package right away (early) or try to wait to see what the other chooses (late). If both choose early or both choose late, the result is a simultaneous move game. Find the Nash equilibria of this game in which each side chooses its personnel package and whether to send it in early or late.

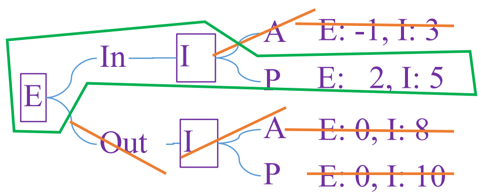
The dominant strategies are for each to move late.

		Defense	
		Early	Late
Offense	Early	0.46,0.54	0.4,**0.6**
	Late	**0.5**,0.5	**0.46,0.54**

24. An incumbent firm faces a potential entrant. The firms produce differentiated products. The entrant chooses In or Out. The incumbent may choose to offer a variety of its product that competes directly with the entrant's (Aggressive) or may continue with only its current product (Passive). Payoffs for possible action pairs (with the entrant listed first) are:

In, Aggressive → -1, 3
In, Passive → 2, 5
Out, Aggressive → 0, 8
Out, Passive → 0, 10

a) Analyze the game where the entrant moves first. Find all pure strategy equilibria and identify which, if any, are subgame perfect.

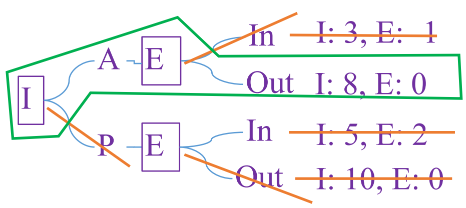


		Incumbent			
		A,A	A,P	P,A	P,P
Entrant	In	-1,3	-1,3	**2**,**5**	**2**,**5**
	Out	**0**,8	**0**,**10**	0,8	0,**10**

In the extensive form, at the second stage the incumbent always chooses passive. Anticipating this, the entrant enters. So, the backward induction solution (subgame perfect equilibrium) is for the entrant to enter and the incumbent to be passive.

Examining the normal form, there are two additional Nash equilibria, so three in total. One in which the incumbent would choose aggressive if the entrant stayed out, but still chooses passive in response to enter, and so the entrant enters. This arises only because it does not matter what the incumbent does when the entrant stays out if the entrant is going to enter. It is a nonsense equilibrium that is not sequentially rational. In the other, the incumbent is aggressive if the entrant enters and passive if not, so the entrant stays out. While not nonsense, because the incumbent would like to be able to make that threat to impact play, is not sequentially rational. If ever actually faced with entry, no rational incumbent delivers on the threat.

b) Analyze the game where the incumbent moves first. Find all pure strategy equilibria and identify which, if any, are subgame perfect.



		Entrant			
		I,I	I,O	O,I	O,O
Incumbent	A	3,-1	3,-1	**8**,**0**	8,**0**
	P	**5**, **2**	**10**,0	5, **2**	**10**,0

In the extensive form, at the second stage the entrant stays out if the incumbent is aggressive and enters if they are passive. Anticipating this, the incumbent is aggressive. The backward induction solution (subgame perfect) is for the incumbent to be aggressive and the entrant to stay out.

Examining the normal form, there is an additional Nash equilibria in which the entrant would enter no matter what, and this leads the incumbent to choose passive. But, if ever actually faced with an aggressive incumbent, no rational entrant would deliver on that threat.

25. Continuation of 24. The incumbent is indefinitely lived. The probability of continuing after any given round of play is *f* and they discount (expected) future values at rate *r*. Each entrant plays only one round and knows how the incumbent played in the past. We want to determine the value to the incumbent of maintaining the ability to introduce and withdraw new product varieties on short notice.

a) Suppose the incumbent has to move first each round because it takes time to introduce and position a new product. Calculate the expected present value of the incumbent's profit.

$$V = 8\sum_{t=0}^{\infty}\left(\frac{f}{1+r}\right) = 8\frac{1+r}{1+r-f}$$

b) Suppose the incumbent can maintain the capacity to introduce a new product on short notice, and similarly to withdraw it from the market whenever it chooses. Determine the values of *r* and *f* for which the following strategies constitute a sequentially rational or sub-game perfect Nash Equilibrium. (Sequential rational is closely akin to subgame perfection. It means the strategy is rational at every point, even those that will not actually be reached in the equilibrium.)

Incumbent: In each round, if the entrant enters, introduce the new product and withdraw it at the end of the round, and if the entrant does not enter do not introduce the new product.

Entrant: Enter if the incumbent has ever been passive and do not enter otherwise.

As long as the incumbent plays the strategy proposed, it is never in the entrants' interest to enter. So, the entrants' strategy is obviously a best reply to the incumbent's.

In this case, it is also obvious the incumbent will not want to deviate, because they would get a lower payoff from deviating and then, against this entrant strategy, would make 5 forever. This is obviously lower than making 10 forever, which is $V = 10\left(\frac{1+r}{1+r-f}\right)$.

The heart of the question is whether the incumbent's strategy is sub-game perfect. Suppose some entrant does enter at time 0. Given the incumbent's strategy, of punishing entry, their payoff is:

$$V = 3 + 10\sum_{t=1}^{\infty}\left(\frac{f}{1+r}\right)^t = 3 - 10 + 10\sum_{t=0}^{\infty}\left(\frac{f}{1+r}\right)^t = 10\left(\frac{1+r}{1+r-f}\right) - 7.$$

If they deviate, by being passive, the entrant will enter evermore, and the best reply by the incumbent is passive forever. The payoff is: $V_{Deviate} = 5\left(\frac{1+r}{1+r-f}\right)$.

The incumbent would punish entry if $10\left(\frac{1+r}{1+r-f}\right) - 7 \geq 5\left(\frac{1+r}{1+r-f}\right)$, or $f \geq (2/7)(1+r)$.

c) How much might it be worth to the incumbent to be able to introduce and withdraw new products on short notice, as a function of *r* and *f*?

At worst, by going first each round as in (a), the entrant reaches the equilibrium in (a). Being able to attain the equilibrium in (b) increases the their payoff by $(10-8)\left(\frac{1+r}{1+r-f}\right)$.

26. Alice and Bob repeatedly play the one shot game to the right, in which they sell differentiated products. Each can choose to price high as they would if there was no competition, lower, or at cost. Alice and Bob may each be normal or crazy, but do not know they type of their opponent. A crazy player prices high until their competitor does not, and charges at cost forever after their opponent charges a price below high. The probability of a crazy player is g. The discount rate is r. The number of repetitions is known to be T. Think of $1/2$-h as reflecting how much pricing at cost hurts an opponent charging lower prices, and $(1/2) > h > (1/10)$.

		Bob		
		High	Lower	At Cost
Alice	High	3/4,3/4	1/3,1	1/10,0
	Lower	1,1/3	1/2,1/2	h,0
	At Cost	0,1/10	0,h	0,0

a) What is the one shot Nash equilibrium if all players are rational?

Lower is a strongly dominant strategy for rational players, so the equilibrium is to choose prices above cost and lower than the monopoly level.

b) What price will rational players charge in the last round, and why?

Lower. First, that is what their opponent will charge if they are rational, to which lower is the best reply. If their opponent is crazy, there is no future in which that opponent can punish them for charging low by charging at cost, so their best reply is again lower prices.

c) For what values of r, g, and h is it a Nash equilibrium for sane players to charge high prices in all rounds before the last? Hint: work out if it is equilibrium play in the second to last round. Interpret your result.

Assume that if the opponent is rational, their strategy is to charge high all periods before T and lower in T, and consider how a rational player responds anticipating this behavior if their opponent is rational. If they would not deviate at time T-1, they would not deviate before then, so we focus on time T-1 moving on.

Expected profit charging high at T-1 and lower at T is: $3/4 + [\ (1-g)/2 + g\ ]/(1+r)$

Expected profit charging lower at T-1 and lower at T is: $1 + [\ (1-g)/2 + gh\ ]/(1+r)$

The first is higher than the second if: $-1 + g(1-h)3/(1+r) > 0$, or if $(1+r)/3 < g(1-h)$

Prices may remain high until the last play as long as the product of the probability of crazy opponent and the proportion of profit a crazy opponent can cost a rational player by playing low is notably larger than $(1+r)/3$. So, the more likely a crazy player, the more a crazy player can punish defectors, and the lower the discount rate, the more likely high prices will be sustained through period T-1.

27. Alice and Bob (A and B) produce differentiated products and compete in quantities. Inverse demand for Alice's product is $p_A = 1 - q_A - 0.5q_B$ and inverse demand for Bob's is $p_B = 1 - q_B - 0.5q_A$. Bob's cost is $C_B(q_B) = 0.05 + 0.2q_B$.

a) Assume Alice's cost is $C_A(q_A) = 0.025 + 0.15q_A$. Find each firm's reaction function and illustrate the Nash Equilibrium. Solve for the equilibrium prices, quantities, and profits.

$$\pi_A = (1 - q_A - 0.5q_B)q_A - 0.025 - 0.15q_A$$

$$\frac{d\pi_A}{dq_A} = 1 - 0.5q_B - 2q_A - 0.15 = 0$$

$$q_A = 0.425 - 0.25q_B$$

$$q_A = 0.425 - 0.25(0.4 - 0.25q_A)$$

$$q_A = 0.347$$

$$q_B = 0.4 - 0.25 \cdot 0.347 = 0.313$$

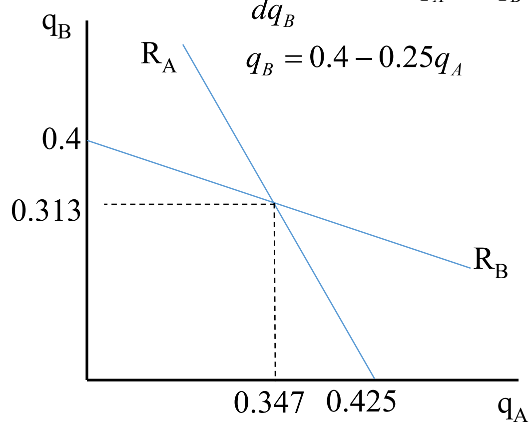
$$p_A = 0.497 \quad \pi_A = 0.095$$

$$p_B = 0.513 \quad \pi_B = 0.048$$

$$\pi_B = (1 - q_B - 0.5q_A)q_B - 0.05 - 0.2q_B$$

$$\frac{d\pi_B}{dq_B} = 1 - 0.5q_A - 2q_B - 0.2 = 0$$

$$q_B = 0.4 - 0.25q_A$$



b) The two firms engage in a bidding war for the exclusive rights to an innovation which will lower the winner's marginal cost by 0.05 per unit. Who wins? How much do they pay?

Hint: To answer those questions, you first have to determine what the equilibrium would be if Alice win's and what it would be if Bob wins. The winner need not bid their full willingness to pay, only to (slightly) exceed the most the opponent would pay.

If Alice wins: (R_B remains as in a):

$$\pi_A = (1 - q_A - 0.5q_B)q_A - 0.025 - 0.1q_A$$

$$\frac{d\pi_A}{dq_A} = 1 - 0.5q_B - 2q_A - 0.1 = 0$$

$$q_A = 0.45 - 0.25q_B$$

$$q_A = 0.45 - 0.25(0.4 - 0.25q_A)$$

$$q_A = 0.373 \quad q_B = 0.4 - 0.25 \cdot 0.372 = 0.307$$

$$p_A = 0.473 \quad \pi_A = 0.114$$

$$p_B = 0.507 \quad \pi_B = 0.044$$

If Bob wins: (R_A remains as in a):

$$\pi_B = (1 - q_B - 0.5q_A)q_B - 0.05 - 0.15q_B$$

$$\frac{d\pi_B}{dq_B} = 1 - 0.5q_A - 2q_B - 0.15 = 0$$

$$q_B = 0.425 - 0.25q_A$$

$$q_A = 0.425 - 0.25(0.425 - 0.25q_A)$$

$$q_A = 0.34 = q_B \quad \pi_A = 0.091$$

$$p_A = 0.49 = p_B \quad \pi_B = 0.066$$

Given the numbers above, Alice makes 0.114-0.091=0.023 more if she wins. Bob makes 0.066-0.044=0.022 more if he wins. Without rounding, gains are (just under) 0.0238 and 0.0216. Alice wins by bidding 0.0216. Compared to (a), Alice makes 0.0024 LESS once the cost of the auction is accounted for. But she can't not bid for it because then Bob would win and she would lose even more. Bob makes 0.004 LESS because Alice increases her production after winning.

28. Alice and Bob produce differentiated products and compete in prices. Demand for Alice's product is $q_A = \frac{2}{3}(1 + p_B - 2p_A)$ and demand for Bob's is $q_B = \frac{2}{3}(1 + p_A - 2p_B)$. (These are the demand functions corresponding to the inverse demand functions from problem 1.) Bob's cost function is $C_B(q_B) = 0.2 + 0.2q_B$.

a) Assume Alice's cost is $C_A(q_A) = 0.1 + 0.15q_A$. Find each firm's reaction function and illustrate the Nash Equilibrium. Solve for the equilibrium price, quantities, and profits.

$$\pi_A = \frac{2}{3}(p_A - 0.15)(1 + p_B - 2p_A) - 0.025 \qquad \pi_B = \frac{2}{3}(p_B - 0.2)(1 + p_A - 2p_B) - 0.05$$

$$\frac{d\pi_A}{dp_A} = \frac{2}{3}\left(-2(p_A - 0.15) + (1 + p_B - 2p_A)\right) = 0 \qquad \frac{d\pi_B}{dp_B} = \frac{2}{3}\left(-2(p_B - 0.2) + (1 + p_A - 2p_B)\right) = 0$$

$$p_A = 0.325 + 0.25p_B \qquad p_B = 0.35 + 0.25p_A$$

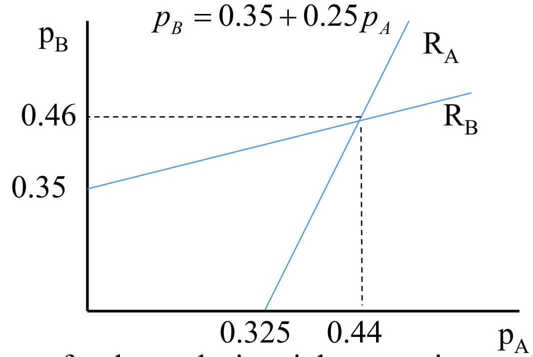
$$p_A = 0.325 + 0.25(0.35 + 0.25p_A)$$

$$p_A = 0.44$$

$$p_B = 0.35 + 0.25 \cdot 0.44 = 0.46$$

$$q_A = 0.387 \quad \pi_A = 0.139$$

$$q_B = 0.347 \quad \pi_B = 0.103$$



b) The two firms engage in a bidding war for the exclusive rights to an innovation which will lower the winner's marginal cost by 0.05 per unit. Who wins? How much do they pay?

If Alice wins: (R_B remains as in a):

$$\pi_A = \frac{2}{3}(p_A - 0.1)(1 + p_B - 2p_A) - 0.025$$

$$\frac{d\pi_A}{dp_A} = \frac{2}{3}\left(-2(p_A - 0.1) + (1 + p_B - 2p_A)\right) = 0$$

$$p_A = 0.3 + 0.25p_B$$

$$p_A = 0.3 + 0.25(0.35 + 0.25p_A)$$

$$p_A = 0.413$$

$$p_B = 0.35 + 0.25 \cdot 0.413 = 0.453$$

$$q_A = 0.418 \quad \pi_A = 0.141$$

$$q_B = 0.338 \quad \pi_B = 0.096$$

If Bob wins: (R_A remains as in a):

$$\pi_B = \frac{2}{3}(p_B - 0.15)(1 + p_A - 2p_B) - 0.05$$

$$\frac{d\pi_B}{dp_B} = \frac{2}{3}\left(-2(p_B - 0.15) + (1 + p_A - 2p_B)\right) = 0$$

$$p_B = 0.325 + 0.25p_A$$

$$p_A = 0.325 + 0.25(0.325 + 0.25p_A)$$

$$p_A = p_B = 0.433 \quad \pi_A = 0.133$$

$$q_A = q_B = 0.378 \quad \pi_B = 0.108$$

Without the rounding above, Alice makes just under 0.0084 more if she wins. Bob makes just under 0.0117 more if he wins. Bob wins by bidding 0.0084. Compared to (a), Bob makes 0.0029 LESS once the cost of winning the auction is accounted for. But he can't not bid for it because then Alice would win and he would lose even more. Alice makes 0.0063 LESS than in (a) because Bob lowers his price after winning.

29. Perfect Competition

a) There are 100 firms in a perfectly competitive industry. Each firm's cost function is $C(q) = 10 + 0.5q^2$. Demand is $Q = 800 - 100p$. Work out and illustrate the short run equilibrium.

$p = MC = q$

$q_S(p) = p$

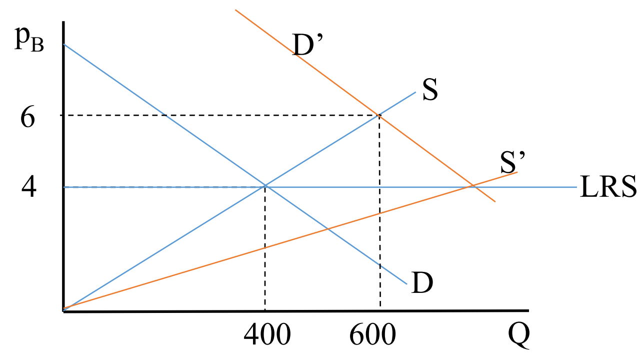
$Q_S(p) = 100p$

$Q_S = Q_D$

$100p = 800 - 100p$

$p^e = 4 = q(p^e)$

$Q^e = 400$



b) An increase in the price of a substitute product increases demand to $Q = 1200 - 100p$. Find the new equilibrium. Describe the adjustment process and illustrate.

$Q_S = Q_D$

$100p = 1200 - 100p$

$p^e = 6 = q(p^e)$

$Q^e = 600$

At the initial price of \$4, the increase in demand from D to D' creates a shortage. The shortage pushes price up, increasing the quantity supplied and decreasing the quantity demanded until equilibrium is retored.

c) Suppose the minimum long run average cost is \$4 and occurs at q=4. Describe the adjustment to the long run equilibrium. Illustrate.

The profit potential at p=6 attracts resources into the industry, increasing supply until price is reduced to \$4. The short run supply curve is S', and the long run supply curve, if it is a constant cost industry, is a horizontal line at p=4.

30. The current state sales tax rate in Florida is 6%. Ignore local sales taxes. Suppose the elasticity of demand for items subject to the sales tax is -2 and supply is perfectly elastic (at p=1). Current annual taxable sales are approximately $400 billion.

a) Estimate the excess burden (deadweight cost) of the current tax.

Roughly, if price were 6% lower, demand would be 12% higher, so taxable sales would be $448 billion. Using a constant elasticity approximation, this is perhaps better estimated as $Q(1)/400=(1/1.06)^{-2}$, so $Q=400(1.06)^{2}=449.44$. But, both are just good educated guesses. Then DWL=0.06*49.44/2, or $1.48 billion annually.

b) Estimate the tax rate needed to raise revenue to $30 billion, and the resulting incremental excess burden.

For reference, current tax revenue is 0.06*400=$24 billion. W need $R=tQ(1+t)=30$.

$$30 = t400\left(\frac{1.06}{1+t}\right)^2$$

$$(1+t)^2 = t\cdot 40\cdot 1.06^2$$

$$t^2 - 12.98t + 1 = 0$$

$$t = 0.5\left(12.98 - \sqrt{12.98^2 - 4}\right) \approx 0.077$$

At that tax rate, taxable sales are:

$$Q(1+t) = 400\left(\frac{1.06}{1.077}\right)^2 = 387.12$$

The excess burden is now (0.077)(449.44-387.12)/2, or $2.4 billion. An increase of 0.92 billion, or 15 cents per dollar of additional revenue raised.

c) Illustrate.

In the figure, the initial DWL is area a. The incremental DWL is b+c.