12.6-7

Friday, February 19, 2021 2:37 PM

$$Var(\hat{\beta}) = (X^T X)^{-1} X^T E(rr^T) X (X^T X)^{-1}$$

$$\Sigma = \begin{bmatrix} E r_1 r_1 & & r_1 r_n \\ & \ddots & \\ r_n r_1 & & E r_n r_n \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{11} & & \\ & & \\ & & \sigma_{nn} \end{bmatrix}$$

$$\hat{\Sigma} = \begin{bmatrix} \hat{r}_1 \hat{r}_1 & & 0 \\ & \hat{r}_2 \hat{r}_2 & \\ 0 & & \hat{r}_n \hat{r}_n \end{bmatrix} \quad E(r_1 r_2) \neq 0 \qquad n^2 \to \frac{n(n+1)}{2}$$

$\downarrow$

or fill it all in

$$\hat{\Sigma} = \hat{r}\hat{r}^T \quad \begin{bmatrix} \hat{r}_1 \\ \hat{r}_2 \\ \vdots \\ \hat{r}_n \end{bmatrix} \begin{bmatrix} \hat{r}_1 & \hat{r}_2 & \cdots & \hat{r}_n \end{bmatrix}$$

$$\widehat{Var(\hat{\beta})} = (X^T X)^{-1} (X^T \hat{r})(\hat{r}^T X)(X^T X)^{-1}$$

$$X^T \hat{r} = \begin{bmatrix} \sum \hat{r}_t \\ \sum x_{t1} \hat{r}_t \\ \vdots \\ \sum x_{tk} \hat{r}_t \end{bmatrix} = 0 \to \text{Normal}$$

$$\hat{\Sigma} = \begin{bmatrix} \hat{r}_1 \hat{r}_1 & \hat{r}_1 \hat{r}_2 & \hat{r}_1 \hat{r}_n \\ \hat{r}_2 \hat{r}_1 & & \\ & & \end{bmatrix}$$ $\to$ weight based on diagonal distance

Noted as w

w_g — g is lag

$w_0 = 1$

$w_g = 1 \quad g \le G$

$w_g = 0 \quad g > G$

Newey-West in 1984

$\hookrightarrow w_g = 1 - \frac{g}{G+1}$

Small sample correction

$\left(\frac{n}{n-k-1}\right) \hat{\Sigma} \to w_g = 1 - \frac{g}{G+1}$

$\hookrightarrow$ Newey West

$G = ?$

$G = \frac{3}{4} T^{1/3}$