β = Slope of Population regression function

$\sum_i y_i x_{ij} = \sum \hat{y}_i x_{ij}$

$\sum_i y_i x_{ik} = \sum \hat{y}_i x_{ik}$

$\vdots$

$x'y = x'\hat{y}$

$\hat{y} = x\hat{\beta}$

$x' = x^{Transpose} = x^T$

$x'y = x'x\hat{\beta}$

$\begin{bmatrix} 4/10 & -1/10 \\ -1/5 & 3/10 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ → Identity matrix

$(x^T x)^{-1} x^T y = (x^T x)^{-1} x^T x \hat{\beta}$

$\hat{\beta} = (x^T x)^{-1} x^T y$

→ unknown Pop. Values

$E(\hat{\beta}) = ? = E[(x^T x)^{-1} x^T (x\beta + r)]$

$= E[\beta + (x^T x)^{-1} x^T r]$

$= \beta + (x^T x)^{-1} x^T E(r|x)$

↳ $= 0$ in population

$E(\hat{\beta}) = \beta$ if no spec error!

$Var(\hat{\beta}) \rightarrow E((\hat{\beta} - E(\hat{\beta}))^2)$

$E((\hat{\beta} - \beta)^2) = E(\cancel{\beta} + (x^T x)^{-1} x r - \cancel{\beta})^2$

$E[(x^T x^{-1}) x^T r r^T x (x^T x^{-1})]$

$Var(\hat{\beta}) = (x^T x)^{-1} x^T E(rr^T|x) x (x^T x)^{-1}$

$rr^T = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} [r_1 \, r_2 \ldots r_n]$ = $n \times n$ matrix

$E(rr^T|x) = ?$

Estimating the Sigma Matrix

$E(r_i r_j | x) = 0$

1) No spec error
2) random sample

$\Sigma = E[$diagonal matrix$]$

$\hat{\Sigma} = [$diagonal hat matrix$]$ → Assume homoskedasticity $E(r_i^2) = E(r_j^2)$ $\forall \, i=j$

$E(r_i^2) = \sigma_i^2 = \sigma_j^2 = \sigma^2$

I = identity matrix

Homoskedasticity

$Var(\hat{\beta}) = (x^T x)^{-1} x^T \sigma^2 I x (x^T x)^{-1}$

$= \sigma^2 (x^T x)^{-1} x^T x (x^T x)^{-1}$

$= \sigma^2 (x' x)^{-1}$ → default

= diagonal = Variance + off diagonal = Covariance

Sandwich Estimator

Eicker-Huber-White Estimator

$\widehat{Var}(\hat{\beta}) = (x^T x)^{-1} x^T \hat{\Sigma} x (x^T x)^{-1}$

hard to calculate

Corr factor $= \frac{n}{n-k-1}$

Heteroskedasticity Robust Variance Estimator

Assumptions:
1) Correct Specification
2) random sample