Economic Analysis for Technologists

Notes: 1-14-2020

0. What is "Economic Analysis for Technologists"?

- Show a Circular Flow diagram & relate Macro, Micro, OR, OM, Psychology
- This is a "Managerial Economics" course adapted to an Engineering and Technology School.
- Text and notes based on my Managerial Economics Class at UF

1. ANALYTICAL APPROACH AND BASIC TOOLS

- Why is a "Managerial Economics" course useful?
  - Economics: study of using limited resources to greatest advantage
  - Firm's goal: maximize shareholder wealth with limited resources
- Firm's value is the present value of profit (π)
- $Profit = Revenue - Cost \quad or \quad \pi = pq - C(q)$

Present Value

- Interest rate 10%, \$1 now is worth \$1.1 next period: Future Value = 1.1·\$1
- \$1 next year worth \$1/1.1 (<\$1) this year
- FV=(1+r)PV for 1 period
- FV=(1+r)(1+r)PV for 2 periods
- FV=PV$(1+r)^t$ for t periods
- PV=FV/$(1+r)^t$
- Firm value is the present value of profit (π)

$$V_0 = \pi_0 + \frac{\pi_1}{(1+r)} + \frac{\pi_2}{(1+r)^2} + \frac{\pi_3}{(1+r)^3} + \cdots$$

$$V_0 = \sum_t \frac{\pi_t}{(1+r)^t}$$

- But, future not known with certainty

Information Structure

- Complete or Perfect - Everyone knows everything
- Incomplete information
  - Risk vs Uncertainty
- Asymmetric Information
  - Some know more than others about quality, value, etc…
  - We will not get to this until much later

Expected Value and Attitudes toward Risk

- When facing uncertain future, attitudes toward uncertainty, not just amount of uncertainty, matter.
- Suppose the probability profit is \$40 is 0.8, otherwise profit is -\$100
  - $\Pr(\pi=40)=0.8$, or, $f=0.8$
  - $E(\pi) = 0.8(40) + 0.2(-100) = 32 - 20 = 12$
- Generally, add up probability (f) of each possible event (i) times its value (x) $EV = \sum_i f_i x_i$
- Now Suppose $\Pr(\pi=30)=0.1$, otherwise $\pi=10$
  - $E(x) = 0.1(30) + 0.9(10) = 3 + 9 = 12$
- Same EV, but this one is less risky
- Certainty Equivalent (CE)
- Risk Neutral, Risk Averse
- Assume firms behave as if they are risk neutral – why?

Expected Present Value and the Value of the Firm

Model firms as if they maximize the expected present value of profit:

$$V_0 = \sum_t \frac{E(\pi_t)}{(1+r)^t} = \sum_t \frac{\sum_i f_{it}\pi_{it}}{(1+r)^t} \text{ USE CAUTION!!!!}$$

Application - Value of Information with Yes or No Decisions

ex. Drill or not, open a franchise or not, etc....

Choose option with highest $E(\pi)$, proceed or not

$Pr(S)$ = Probability of Success

π_S = Profit if successful

π_F = Profit if fail <0

$Pr(S)\pi_S+[1-Pr(S)]\pi_F>0 \rightarrow$ proceed, $<0\rightarrow$ not proceed,

So, $E(\pi|NoInfo)=Max(Pr(S)\pi_S+[1-Pr(S)]\pi_F,0)$

Example: $Pr(S)=1/2$, $\pi_S=100$, $\pi_F=-60$

$E(\pi|NoInfo)=Max[(1/2/)100-(1/2)60),0]=Max(20,0)=20$

Suppose you can buy additional information

Information is a report of either "good news" (positive) or "bad news" (negative) on chance of success, not 100% accurate

Where does this come from? Experience, consultants, gut perception

ex. geologic studies, prob of oil given characteristics of location

Suppose you have access to "reports" on 20 similar projects in the past where you have drilled, with the following results:

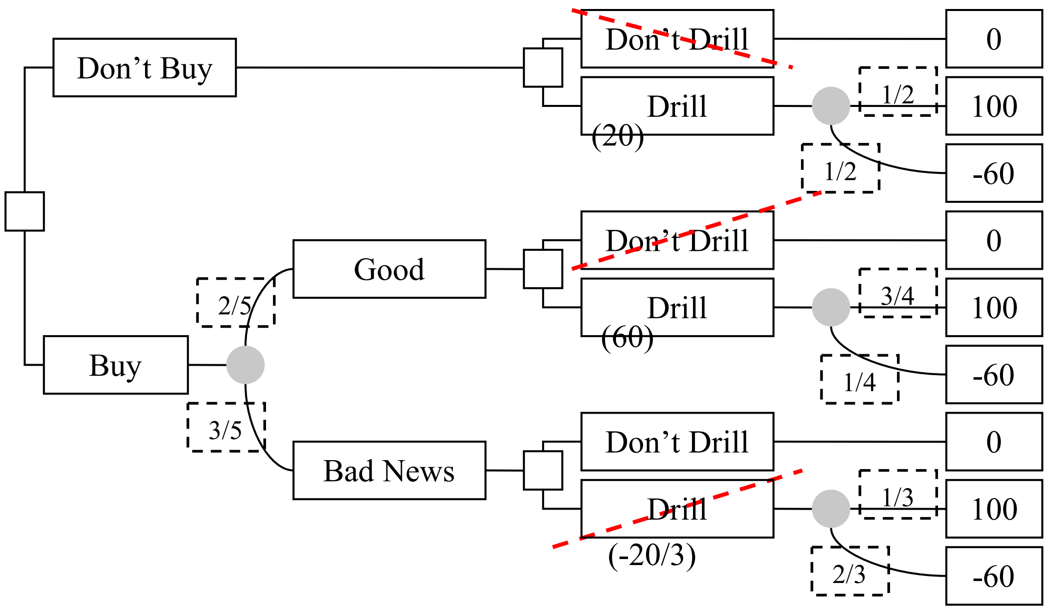
		Outcome	
		Success	**Failure**
News	**Good**	6	2
	Bad	4	8

Considering buying it in a similar situation and using it to decide whether to drill or not.

What is the "value" of this info?

The most you would be willing to pay to buy it before this venture.

Work through the decision tree



If proceed with a good report, expected profit is

(3/4)(100)-(1/4)(60)=75-15=60.

Generally:

$E(\pi|G)=\mathrm{Max}(\Pr(S|G)\pi_S+\Pr(F|G)\pi_F,0)$

Info only valued if >0 proceeding.

If proceed with a bad report, expected profit is

(1/3)(100)-(2/3)(60)=-20/3<0. (Already knew <0 from above).

Generally:

$E(\pi|B)=\mathrm{Max}(\Pr(S|B)\pi_S+\Pr(F|B)\pi_F,0)$.

If were >=0 proceeding, info is worthless.

In example (tree)

$E(\pi|\mathrm{Info})=(2/5)[(3/4)(100)-(1/4)(60)]=(2/5)(60)=24$

$E(\pi|\mathrm{NoInfo})=20$ (above)

Value of Info = 24-20 = 4

Generally, assuming proceed with good news and not with bad:

$E(\pi|\mathrm{Info})=\Pr(G)E(\pi|G)$

$E(\pi|\mathrm{Info})=\Pr(G)[\Pr(S|G)\pi_S+\Pr(F|G)\pi_F]$

Value of info is: $E(\pi|\mathrm{Info})-E(\pi|\mathrm{NoInfo})$

$\Pr(G)[\Pr(S|G)\pi_S+\Pr(F|G)\pi_F]-\mathrm{Max}(\Pr(S)\pi_S+[1-\Pr(S)]\pi_F,0)$

**Info only has value if it impacts your decision!

<u>A word of warning – uses and limits of simple models</u>

We will often make use of simple models to highlight the important aspects of a given situation, and, to check our reasoning.

Results should not be taken too literally. Sometimes best seen as "as-if" models.

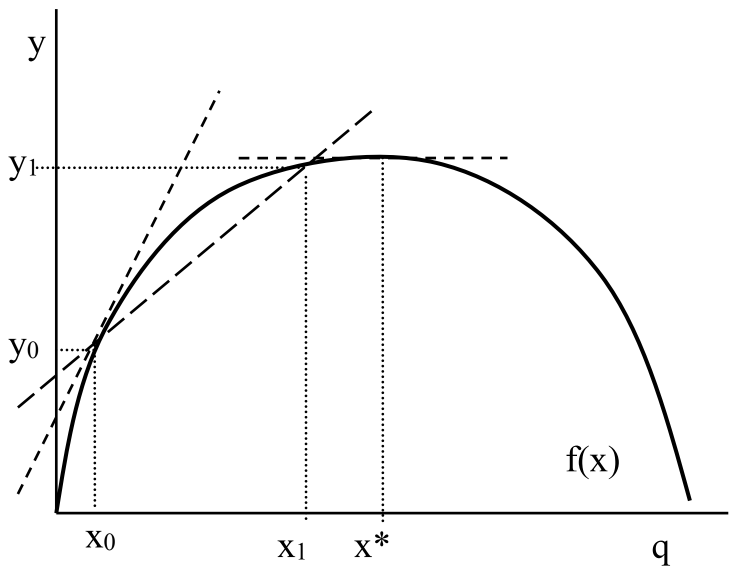
Generally - do not put much stock in any result or assertion as unless:

1) It is consistent with a simple theory that has a concise formal mathematical expression
2) It is consistent with a believable, intuitive, story
3) Multiple empirical studies are consistent with it

APPENDIX TO CHAPTER 1: CALCULUS, EXPONENTS, AND LOGS

Differential Calculus and Optimization

Suppose you are standing on a gentle and "smooth" hill in a dense fog. You are supposed to meet someone at the top. How can you tell if you are at the top?



Look at line through thru (x_1,y_1) and (x_0,y_0). Slope is "average" rate of change of f(x) with changes in between x_0 and x_1.

Slope of line thru (x_1,y_1) and (x_0,y_0) is "average" rate of change of f(x) with changes in x. Note, $\Delta x = x_1 - x_0$, so $x_1 = x_0 + \Delta x$. So:

$$\frac{\Delta f(x)}{\Delta x} = \frac{y_1 - y_0}{x_0 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

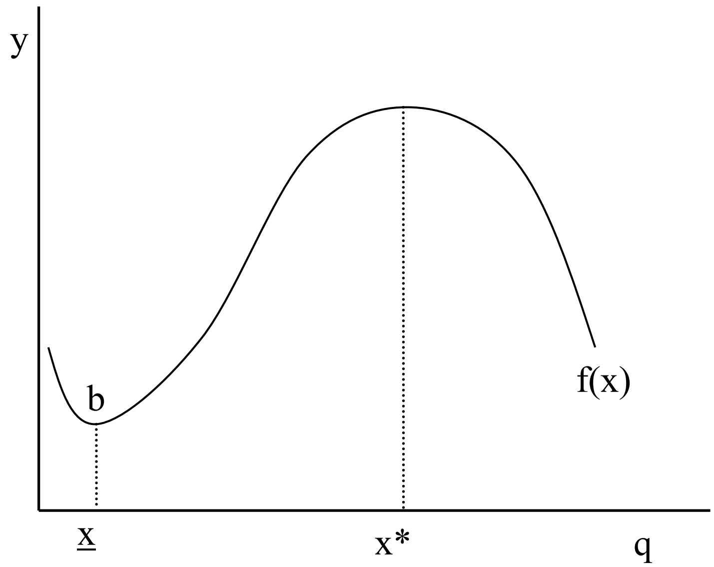
Imagine moving x_1 closer to x_0, so the change in x gets small. The line approaches tangent at (x_0,y_0). Slope of tangent is rate of change at x_0.

The derivative, f' or dy/dx, is this rate of change as Δx gets very small.

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

At the top of the hill, tangent is flat, derivative is 0.

What about point b in the figure below? Derivative is 0 there. Fails SOC.



Some Rules for Derivatives

Power Rule: $y = ax^b \Rightarrow \dfrac{dy}{dx} = bax^{b-1}$

Sum Rule: $y = f(x) + g(x) \Rightarrow \dfrac{dy}{dx} = f'(x) + g'(x)$

Product Rule: $y = f(x)g(x) \Rightarrow \dfrac{dy}{dx} = f'(x)g(x) + g'(x)f(x)$

Chain Rule: $y = g(f(x)) \Rightarrow \dfrac{dy}{dx} = g'\left(f(x)\right)f'(x)$

Exponents

What is x^a? a is just the number of times you multiply 1 by x.

x is the same as $1x^1$. x^2 is $1xx$. Etc…

x^0 means multiply 1 by 0 x's, so, $x^0=1$.

What is x^{-a}? Dividing is the opposite of multiplication, so, a is the number of times you divide 1 by x: x^{-1} is $1/x$. x^{-2} is $1/xx$. Etc…

All the standard "rules" for exponents follow from this definition, don't need to memorize them if you know the definition in an intuitive way

i. $x^{a+b} = x^a x^b$

ii. $(x^a)^b = x^{ab}$

iii. $x^{-a} = \dfrac{1}{x^a}$

<u>Exponential Function</u>

$f(x) = ae^{bx}$ where $e = 2.71828$

Derivative: $\dfrac{dy}{dx} = bae^{bx}$.

What is *e*? Base unit of continual growth processes.

Imagine a process that doubles itself every time period, r=1. At the end of one period, the future value of 1 unit growing for 1 period is $FV=(1+1)^1=2$. Of course, it is doubling, duh!

Imagine compounding that growth n times over the period. $FV = \left(1+\dfrac{1}{n}\right)^n$.

Suppose we compound monthly. FV=2.61304.

Daily? FV=2.71456.

Hourly? FV=2.71812.

By the minute? 2.71828.

With continuous growth, each little bit of growth starts growing as soon as it emerges. In the limit, as we approach continuous growth, the FV of one unit growing continuously for one period of time is *e*.

If we let it grow for 2 periods, we have $FV=ee=e^2$. For x periods, e^x.

Suppose it grows for one period at r=2, we could think of it as 100% growth occurring twice, so $FV=ee=e^2$. At rate x, e^x.

Generally, then, if it grows continuously at rate r for t periods, $FV=e^{rt}$.

If the initial amount is PV, instead of 1, we just have $FV=PVe^{rt}$.

So, *e* has tons of applications to growth processes. More important, you can't understand natural logs without *e*, and we will use natural logs often.

Natural Log

The log of x is the power to which $e = 2.71828$ must be raised to get x.

The natural log undoes the exponential function, if $x = e^y$, $y = \ln(x)$.

Since e^y is the amount one unit grows into after growing continuously for one period at rate y, y periods at rate r=1, or, t periods at rate r where rt=y, ln(x) is the combination of growth rate and growth time, rt, needed for one unit to grow continuously into x units.

Natural log functions have many useful properties that make them easy to manipulate algebraically.

v. $\ln(xy) = \ln(x) + \ln(y)$

vi. $\ln(x/y) = \ln(x) - \ln(y)$

vi. $\ln(x^a) = a\ln(x)$

All of follow readily from the definition of exponents and natural logs.

Derivative is easy to calculate: $y = a\ln(x) \Rightarrow \frac{dy}{dx} = \frac{a}{x}$

These make log-linear or constant elasticity approximations easy to work with.

2. COST, DEMAND, AND PROFIT MAXIMIZATION

Recall: $Profit = Revenue - Cost \quad or \quad \pi = R - C$

Cost – Briefly (Detail in Chapter 6).

Cost function, C(q), the minimum possible cost of producing q units.

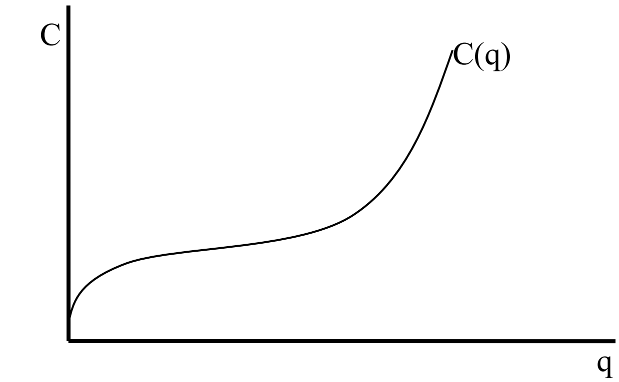
Assumes:

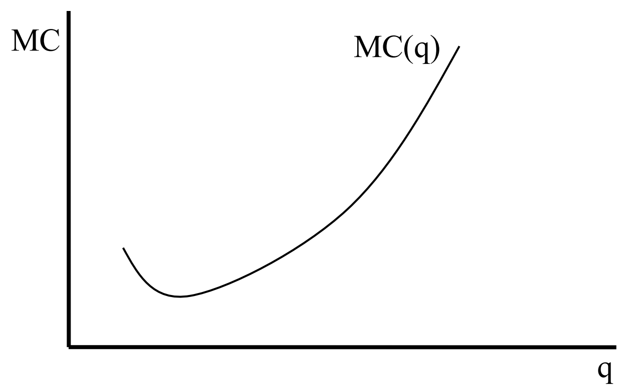
1) No simple waste of inputs

2) The most efficient production technique is chosen. (An Engineering/OR/OM problem)

Depends on input prices, (r capital, w labor), technology, time available for production, etc…

For notation, $C = C(q; w, r, z)$ where z is anything else that effects cost.





Marginal cost (MC), is the derivative of the cost function: $MC = \frac{dC}{dq}$.

Marginal cost ≈ incremental cost, the cost pf producing one more unit.

Common Approximations for Applications

Fixed cost of F>0 and constant per unit cost of c>0: $C(q) = F + cq$. $MC = c$

Fixed cost plus constant elasticity wrt q: $C(q) = F + cq^d$ (c>0, d>0)

$MC = cdq^{d-1}$ (*increasing* if d>1)

Quadratic $C(q) = F + aq + bq^2$ (a>0, b>0,)

$MC = a + 2bq$ (increasing throughout

Cubic $C(q) = F + aq + bq^2 + cq^3$ (a>0, b<0, c>0)

$MC = a + 2bq + 3cq^2$ (decreasing, then increasing, MC possible)

Demand

(Consumer Theory - theory behind demand - not until Ch 4)

The quantity consumers will purchase is a function of the price charged, $q=q(p)$.

[Q will denotes a "total" quantity, q an individual's quantity.]

Q also depends on: income (m), prices of substitutes and complements (p_s and p_c), size of the market or number of consumers (N), etc…

From principles, what is the effect of each of these variables on demand?

What is the difference between normal and inferior goods?

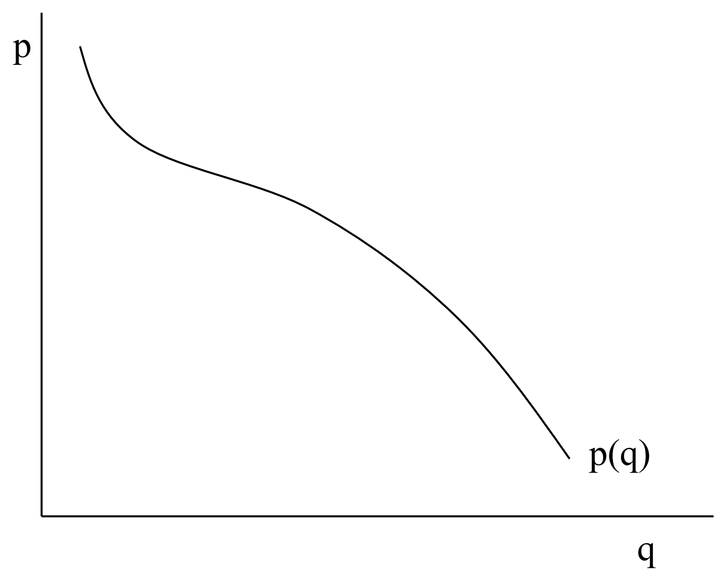
Notation, $q=q(p;m,p_s,p_c,N,x)$, x represents other things that might effect demand.

We could say the price the firm could charge depends on how much they want to sell. That is inverse demand, which is often EASIER to deal with.

Notation, $p=p(q;M,p_s,p_c,N,x)$ or $p=p(q;M,q_s,q_c,N,x)$ represents inverse demand.

We draw "inverse" demand and just call it demand. But p and q are both determined by market forces, neither is independent.

Show demand shifts graphically



Measuring the Sensitivity of Quantity Demanded to Price

The ***slope*** of a demand curve is change in q in response to a small change in p.

Derivative of q with respect to p: $\frac{dq(p)}{dp} < 0$.

Slope can be different at different prices, but, is always negative.

The slope of an inverse demand curve is the inverse of the slope of the demand curve, how fast p changes with q: $\frac{dp(q)}{dq} < 0$.

Slope depends on the units in which p and q are quoted: dollars, cents, euros, yen, … singles, dozens, hundreds, pounds, ounces, gallons, etc…

The ***elasticity*** of y with respect to x is the percentage change in y relative to the percentage change in x.

Elasticity is expressed in percentage terms, so unit free. Makes it very handy…

The elasticity of demand with respect to price, denoted $\eta<0$, is:

$$\eta = \frac{\%\Delta q}{\%\Delta p} = \frac{\Delta q/q}{\Delta p/p} = \frac{\Delta q}{\Delta p}\frac{p}{q}.$$

For small changes in price, this is just the slope times p/q: $\eta = \frac{dq}{dp}\frac{p}{q}$.

Elasticity is different at different prices

Usually near 0 at low enough prices

Becomes higher in absolute value (more negative) at higher prices

Demand Approximations

Theory says demand slopes down, says nothing about shapes

For applications (including profit maximization) need an approximation

Two types are most common:

1) linear – assumes slope is constant

2) log-linear or constant elasticity – assumes elasticity is constant

Linear Demand Approximations

$$q_D = b_0 + b_p p + b_M M + b_S p_S + b_C p_C + b_N N + b_Z Z + \varepsilon$$

where ε is a random error. Signs of slopes?

Make up and interpret an example

If we lump everything except price together and call it "a" and drop the subscript, since there is now only one b, this is just: $q_D = a + bp$

Note: Often we will speak of the slope as a positive number and write $q = a - bp$

Suppose demand is q=10000-250p. (Interpret)

Then inverse demand is p=10000/250-q/250 or p=40-0.004q.

For a linear demand curve, elasticity is $\eta = \frac{dq}{dp}\frac{p}{q} = b\frac{p}{q}$. This is higher (in absolute value) at high prices and approaches 0 as p does.

Example: For above demand curve, if p=10, q=7,500, elas=-250(10/10000)=-0.25

Log-Linear or Constant Elasticity Demand Approximations

$$q_D = e^{b_0+\varepsilon} p^{b_p} M^{b_M} p_S{}^{b_S} p_C{}^{b_C} N^{b_N} Z^{b_Z}$$

where ε is a random disturbance and b_p is negative.

Using the rules for logs, taking the log of both sides of this approximation gives:

$$\ln(q_D) = b_0 + b_p \ln(p) + \ln b_M (M) + \ln b_S (p_S) + \ln b_C (p_C) + b_N \ln(N) + b_Z \ln(Z) + \varepsilon$$

So, this approximation is "linear" in the logs of the variables.

If we lump everything except price together and call it "a" and drop the subscript, since there is now only one b, this is just:

$q_D = ap^b$ or $\ln(q_D) = \ln(a) + b\ln(p)$.

What is the slope? $\frac{dq}{dp} = bap^{b-1} = b\frac{ap^b}{p} = b\frac{q}{p}$.

So, what is the elasticity? $\eta = \frac{dq}{dp}\frac{p}{q} = b\frac{q}{p}\frac{p}{q} = b$.

The elasticity is *CONSTANT* and equal to the exponent on price.

Example: Suppose $q=20000p^{-2}$. Elas=-2. At p=20, q=20000/400=50. Slope at that price is -2(50/20)=-5. Inverse demand is $p = \sqrt{\frac{20000}{q}} \approx 141.42q^{-0.5}$.

Revenue and Marginal Revenue

Revenue = price × quantity, R=pq.

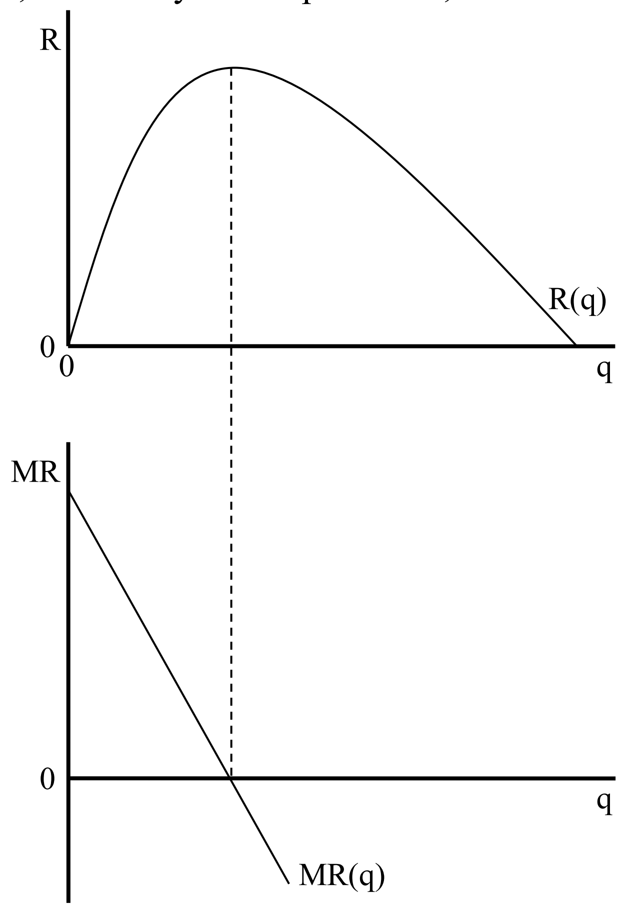
Both p and q cannot be independently chosen.

A *price taking* firm, so small it has no impact on market price, can only choose q.

A *price making* firm is still constrained by the demand. Choose p or q, not both.

So, revenue depends on how much is sold, **or** the price charged. If 0 are sold, R=0. If a price making firm sells too much, eventually drives price to 0, R=0.

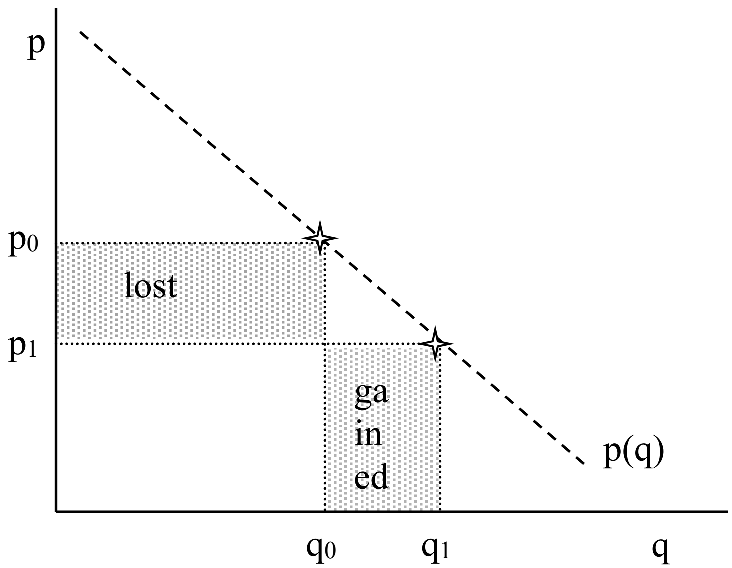
$R = p(q)q$ or $R = pq(p)$



The change in revenue from selling another unit, marginal revenue (MR) is the derivative of revenue: $MR = \frac{dR}{dq}$.

$$R = p(q)q$$
$$\frac{dR}{dq} = p + \frac{dp}{dq}q$$

So, MR is less than price for a price making firm! What is the intuition for that?



Example

$$p = 7 - 0.3q$$
$$R = (7 - 0.3q)q$$
$$\frac{dR}{dq} = 7 - 0.6q$$

Relate MR to Elasticity

$$\frac{dR}{dq} = p + \frac{dp}{dq} q \frac{p}{p} = p + \frac{dp}{dq} \frac{q}{p} p$$
$$\frac{dR}{dq} = p + \frac{1}{\eta} p = p\left(1 + \frac{1}{\eta}\right)$$

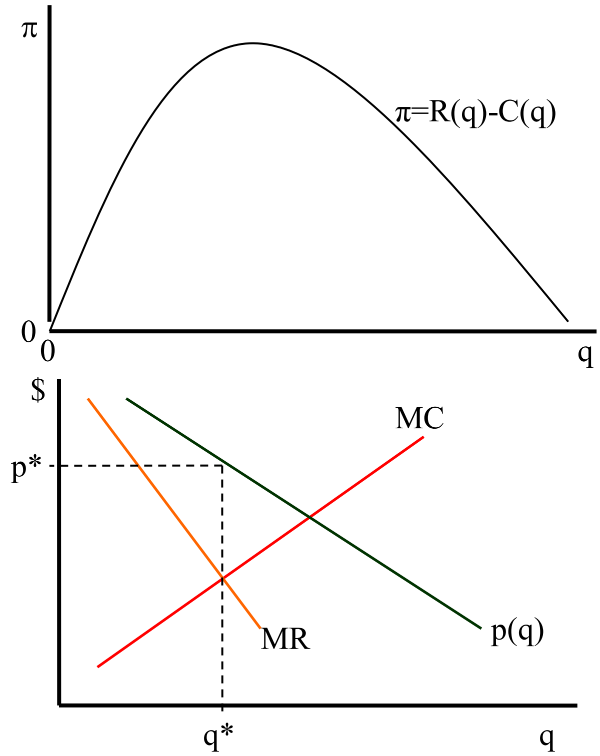
So, as elasticity gets bigger, MR gets bigger, closer to p. If elasticity is less than 1 in absolute value, MR is *NEGATIVE*.

Suppose η=-0.25. MR=p(1-1/0.25)=(1-4)p=-3p.

Lowering *p* cuts revenue (profit) if customers not price sensitive.

Maximizing Profit

For a price maker, if you charge a low enough price, and sell "too many", profit is 0. If you charge a high enough price, and sell "too few", profit is 0. Between those points, profit is higher. How do you find the quantity that gets you to the top of the profit hill?



Working from inverse demand, profit is: $\pi = p(q)q - C(q)$

To maximize something with calculus, set the derivative equal to 0:

$\frac{d\pi}{dq} = \left(p + \frac{dp}{dq}q\right) - \frac{dC}{dq} = 0$, this is MR=MC.

Why????? What should you do if MR>MC? What if MR<MC?

This basic idea is at the core of the class! We will apply it again and again in more complex situations!

Suppose instead revenue is maximized. Set MR=0. WHY??? Price too low, quantity too high, profit lower than max.

*** Important to remember profit, not dollar volume of sales, is the bottom line. BEWARE the incentives of commissioned sales staff or those that are paid bonuses based on sales or market share…

Example: $p = 7 - 0.3q$, $C(q) = 9 + 1.1q$

$$\pi = (7 - 0.3q)q - 9 - 1.1q$$
$$\frac{d\pi}{dq} = 7 - 0.6q - 1.1 = 0$$
$$q = 5.9/0.6$$

$$p = 7 - 0.3(5.9/0.6) = 4.05$$
$$\pi = 4.05(5.9/0.6) - 1.1(5.9/0.6) - 9 \approx 20$$
$$\pi = (4.05 - 1.1)(5.9/0.6) - 9 \approx 20$$

Could do the same thing with demand instead of inverse demand.

$$p = 7 - 0.3q$$
$$0.3q = 7 - p$$
$$q = \frac{7-p}{0.3} = \frac{70}{3} - \frac{10}{3}p$$
$$\pi = p\left(\frac{70}{3} - \frac{10}{3}p\right) - 1.1\left(\frac{70}{3} - \frac{10}{3}p\right) - 9$$
$$\pi = (p - 1.1)\left(\frac{70}{3} - \frac{10}{3}p\right) - 9$$

$$\frac{d\pi}{dp} = \frac{70}{3} - \frac{10}{3}p - \frac{10}{3}(p - 1.1) = 0$$
$$\frac{70}{3} + \frac{11}{3} = \frac{20}{3}p$$
$$p = 81/20 = 4.05$$

Use whichever more convenient. Usually inverse demand.

The optimization condition could also be written as follows:

$MR = p\left(1+\frac{1}{\eta}\right) = MC$ or

$$p\left(\frac{\eta+1}{\eta}\right) = MC$$

$$p = \left(\frac{\eta}{1+\eta}\right)MC$$

So, the profit maximizing markup over MC is $\frac{\eta}{1+\eta}$.

Example: Suppose demand is $q = 10000p^{-3}$ and unit cost is constant at \$4 per unit.

$\pi = (10000p^{-3})p - 4(10000p^{-3}) - F$.

Could take derivative, find p. Instead:

$$p = \left(\frac{\eta}{1+\eta}\right)MC = \left(\frac{-3}{1-3}\right)4 = (1.5)4 = 6$$

$$q = 10000(6^{-3}) \approx 46.3$$
$$\pi \approx (6-4)(46.3) - F$$

Lower elasticity, less responsive demand, means higher markup.

IF elasticity AND MC are constant, this gives the optimal price.

Compare markup for η=-4 and η=-2.

3: Applications and Extensions of Optimal Production and Pricing

3rd Degree (Simple) Price Discrimination

Suppose identifiable groups of customers have different demands

Idea – if resale is not possible and you can identify who is who, charge higher price to group with less elastic demand.

$$\pi = p_1(q_1)q_1 + p_2(q_2)q_2 - C(q_1+q_2)$$
$$MR_1 = MR_2 = MC$$
$$p_1 = \frac{\eta_1}{1+\eta_1}MC \quad p_2 = \frac{\eta_2}{1+\eta_2}MC$$

Examples: Student and senior discounts

Example: 3rd Degree Price Discrimination

2 types of customers, inverse demands: $p_1 = 20 - q_1$ and $p_2 = 30 - q_2$.

Cost is $C(Q) = 0.5(q_1+q_2)^2$

$$\pi = (20-q_1)q_1 + (30-q_2)q_2 - 0.5(q_1+q_2)^2$$
$$MR_1 = MC \to 20 - 2q_1 = q_1 + q_2$$
$$MR_2 = MC \to 30 - 2q_2 = q_1 + q_2$$
$$20 - 2q_1 = 30 - 2q_2$$
$$2q_2 = 10 + 2q_1$$
$$q_2 = 5 + q_1$$
$$20 - 2q_1 = q_1 + 5 + q_1$$
$$15 = 4q_1$$
$$q_1 = 3.75$$
$$q_2 = 5 + q_1 = 8.75$$
$$p_1 = 20 - 3.75 = 16.25$$
$$p_2 = 30 - 8.75 = 21.25$$
$$\pi = 16.25 \bullet 3.75 + 21.25 \bullet 8.75 - 0.5(3.75+8.75)^2$$
$$\pi = 168.75$$

What if only one price is charged?

To find total demand invert the inverse demand curves:

$q_1 = 20 - p_1$ and $q_2 = 30 - p_2$, then adding q1 and q2: $q = 50 - 2p$.

Inverse market demand is $p = 25 - 0.5q$. From there:

$$\pi = (25 - 0.5q)q - 0.5q^2$$
$$MR = MC \to 25 - q = q$$
$$q = 12.5$$
$$p = 25 - 0.5 \bullet 12.5 = 18.75$$
$$\pi = 18.75 \bullet 12.5 - 0.5 \bullet 12.5^2 = 156.25$$

Demand and Profit Maximization when q_D/N does not depend on N

Suppose purchases per customer do not depend on the size of the market. Let $f(p;M,p_s,p_c,x)$ be the fraction that purchase.

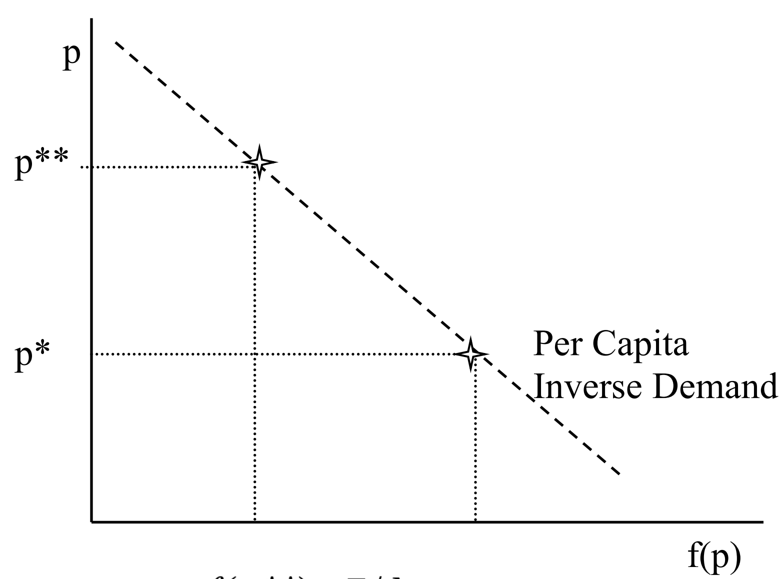
Demand is $q_D=Nf(p;M,p_s,p_c,x)$.

One of the natural applications of this is where basically each person buys either 1 or 0 units and the fraction of the population that purchases depends on the price. The proportion that have a reservation value higher than the price is f(p,**x**) where x just can't include N.

Elasticity is $\frac{dq}{dp}\frac{p}{q}=N\frac{df}{dp}\frac{p}{Nf}=\frac{df}{dp}\frac{p}{f}$. Elasticity does not depend on N, so, markup does not depend on N.

The optimal price will not vary with city size if marginal MC is approximately constant. If two cities are identical except for their size, the optimal price is the same (if there are no capacity limits).

Now, suppose capacity is limited to $\bar{q}$ units. If there is not enough capacity to sell all that is demanded at the otherwise "optimum" price, what is the profit maximizing price? $Nf(p)=\bar{q}$ or $f(p)=\bar{q}/N$



Example:

Of 1000 potential customers, the fraction purchasing is approximated by

$f(p) = 1 - 0.1p$ and constant unit cost is \$2.

$$q = 1000(1 - 0.1p)$$

$$\pi = 1000(1 - 0.1p)(p - 2) - F$$

$$\frac{d\pi}{dp} = 1000[(1 - 0.1p) - .1(p - 2)] = 0$$

$$1 - 0.1p - .01p + .2 = 0$$

$$1.2 = .2p$$

$$p = 6$$

$$q = 1000(1 - 0.1(6)) = 400$$

$$\pi = 400(6 - 2) - F = 1600 - F$$

Now suppose the maximum quantity possible is $\bar{q} = 200$. Can't sell 400. All else equal, want the highest price that will sell out.

$$1000(1 - 0.1p) = 200$$

$$1 - 0.1p = .2$$

$$.8 = 0.1p$$

$$p = 8$$

But, how do you determine how large the capacity should be for a venue when sometimes demand is high and sometimes it is low?

Peak Load Pricing

What if demand differs by time of day, day of week, or season of year?

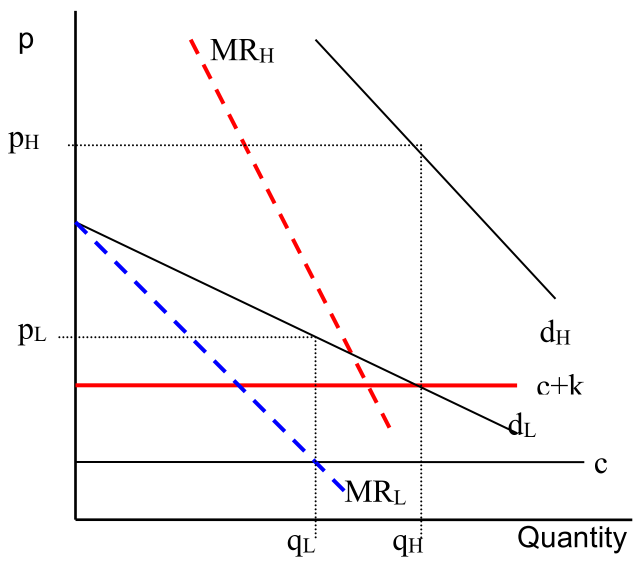
Charge different prices at different times, higher at peak demand.

If insufficient capacity for peak demand at MR=MC, price to sell out.

BUT, how do you determine capacity?

Let c be marginal operating cost and k be marginal capacity cost:

MR off peak = c, and MR at peak = c+k, peak determines capacity



But, what happens as difference between peak and off peak demand gets "small"?

"Shifting Peak" (even more pronounced when demands are interdependent)

It would never make sense to sell more when demand is lower!

Instead, you share capacity costs across both periods, pricing so that fully utilize capacity in both periods.

Correct maximization problem is:

$$Max_{q_L,q_H}\ \pi = p_L(q_L)q_L + p_H(q_H)q_H - cq_L - cq_H - kq_H$$
$$s.t.\ q_H \geq q_L$$

To solve, first just assume it will hold, ignore constraint. Then check.

If it is not violated, all is fine.

If it is violated, now assume $q_H=q_L=q$. Problem becomes:

$$Max_q\ \pi = p_L(q)q + p_H(q)q - cq - cq - kq$$

FOC becomes $MR_L + MR_H = 2c + k$

Example

Inverse demand at high demand is $p_H = 14 - 0.5q_H$ and at low demand is $p_L = 12 - 0.5q_L$. Operating cost is \$2 per unit and capacity cost is \$4 per unit.

1) Assume $q_H>q_L$ and solve

$$\max_{q_H,q_L}\quad (14-0.5q_H)q_H + (12-0.5q_L)q_L - 2q_L - 2q_H - 4q_H$$
$$s.t.\quad q_H \geq q_L$$

$14 - q_H = 6$

$q_H = 8$

$12 - q_L = 2$ (Note, this is just MR=MC)

$q_L = 10$

Breaks assumption!

2) Assume $q_H=q_L$ and solve

$$\max_q\quad (14-0.5q)q + (12-0.5q)q - 8q$$

$14 - q + 12 - q = 8$

$18 = q$

$q = 9$

$p_H = 14 - 0.5(9) = 9.5$

$p_L = 12 - 0.5(9) = 7.5$

Maximizing Profit with Uncertainty

Assume you must choose & produce before you know demand for sure.

ex. weather, fuel prices, income, etc....

Also, could study cost uncertainty

Example: $p_H = 20 - 0.25q_H \quad p_L = 10 - 0.25q_L \quad \Pr(H) = 0.5 \quad MC = 2$

If demand is known before production decision:

High:

$$20 - 0.5q_H = 2$$
$$q_H = 36$$
$$p_H = 20 - 36/4 = 11$$
$$\pi_H = (11-2)36 = 324$$

Low:

$$10 - 0.5q_L = 2$$
$$q_L = 16$$
$$p_L = 10 - 16/4 = 6$$
$$\pi_L = (6-2)16 = 64$$

$$E(\pi) = 0.5(324) + 0.5(64) = 194$$

If demand is not known first:

First, suppose just choose some q and sell whether demand is high or low:

$$E(\pi) = 0.5(20 - 0.25q)q + 0.5(10 - 0.25q)q - 2q$$
$$\frac{\partial E(\pi)}{\partial q} = 0.5(20 - 0.5q) + 0.5(10 - 0.5q) - 2 = 0$$
$$13 = 0.5q$$
$$q = 26$$

If demand turns out high:

$$p_H = 20 - 0.25(26) = 13.50$$
$$\pi_H = (13.50 - 2)26 = 299$$

If demand turns out low:

$$p_L = 10 - 0.25(26) = 3.50$$
$$\pi_L = (3.50 - 2)26 = 39$$

$$E(\pi) = 0.5(299) + 0.5(39) = 169$$

Problem: too much if demand low, too little if demand high

Don't have more to sell if demand is high.

No one makes you sell 26 if demand is low!

NEVER sell more than maximizes Revenue (IF there is free disposal)!

If don't have to sell all when demand is low, may want to have more on hand if demand is high

Plan to sell different amounts depending on demand.

What determines cost?

Build problem assuming sell more at high demand – remember to check!

<u>Generally</u>

$$Max_{q_H, q_L} \quad E(\pi) = \Pr(H) p_H(q_H) q_H + \Pr(L) p_L(q_L) q_L - C(q_H)$$
$$s.t. \quad q_H \ge q_L$$

$$\frac{\partial E(\pi)}{\partial q_H} = \Pr(H) MR_H(q_H) - MC(q_H) = 0 \qquad \frac{\partial E(\pi)}{\partial q_L} = \Pr(L) MR_L(q_L) = 0 \text{ or } MR_L(q_L) = 0$$

IF solving those gives you $q_L > q_H$, need to account for constraint and re solve

$$Max_{q_H, q_L} \quad E(\pi) = \Pr(H) p_H(q) q + \Pr(L) p_L(q) q - C(q)$$

$$\frac{\partial E(\pi)}{\partial q} = \Pr(H) MR_H(q) + \Pr(L) MR_L(q) - MC(q) = 0$$

<u>Example</u>

$$0.5(20 - 0.5q_H) = 2 \qquad 10 - 0.5q_L = 0$$
$$8 = 0.25q_H \qquad 10 = 0.5q_L$$
$$q_H = 32 \qquad q_L = 20$$
$$p_H = 20 - 0.25(32) = 12 \qquad p_L = 10 - 0.25(20) = 5$$

***Remember to check that you satisfied the constraint $q_H \ge q_L$.

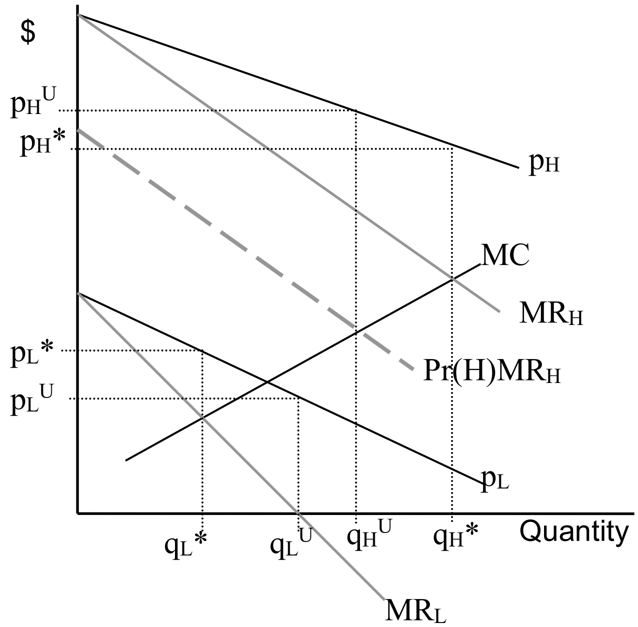
$$E(\pi) = 0.5 \times 12 \times 32 + 0.5 \times 5 \times 20 - 2 \times 32 = 192 + 50 - 64 = 178$$

How much would you pay to know in advance? (100% accurate?)

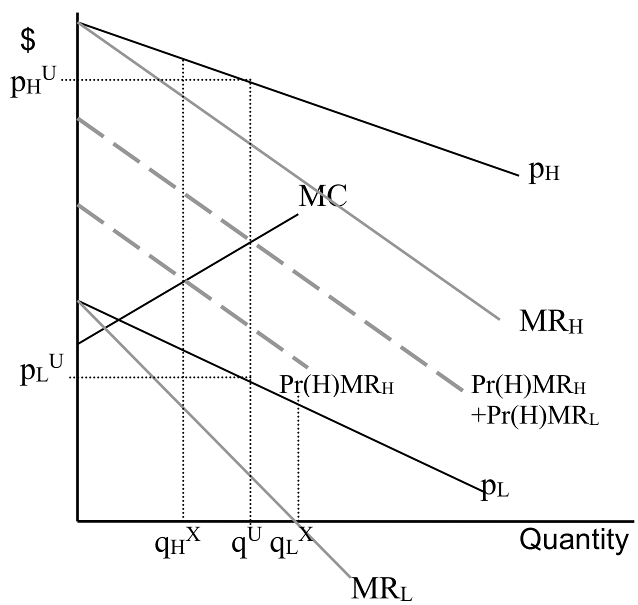
$194 - $178= $16

How much would you pay for info that is not 100% accurate?

Show this solution graphically



From the graph, when would the constraint that $q_H > q_L$ be violated by the solution above? If Pr(H) low or not much difference in low and high demand.



Value of Better but Imperfect Information

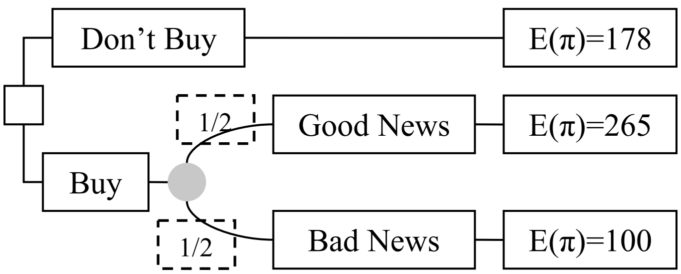
You can buy a forecast, or, build a forecast department, but, it is imperfect

Example: Value of a Demand Forecast (Demand and Cost as Above)

Information Structure			
	OUTCOME		
REPORT	**High**	**Low**	**Total**
Good News	0.4	0.1	0.5
Bad News	0.1	0.4	0.5
Total	0.5	0.5	1

$\Pr(H \mid G) = 4/5 = 0.8 \quad \textit{and} \quad \Pr(H \mid B) = 1/5 = 0.2$

Work thru decision tree:



Already did the "no info" version.

With "good" news:

$$E(\pi) = 0.8(20 - 0.25q_H)q_H + 0.2(10 - 0.25q_L)q_L - 2q_H$$

$$\frac{\partial E(\pi)}{\partial q_H} = 0.8(20 - 0.5q_H) - 2 = 0$$

$14 = 0.4q_H$

$q_H = 35$

$p_H = 20 - 0.25(35) = 11.25$

$$\frac{\partial E(\pi)}{\partial q_L} = 0.5(10 - 0.5q_L) = 0$$

$10 = 0.5q_L$

$q_L = 20$

$p_L = 10 - 0.25(20) = 5$

***Remember to check that you satisfied the constraint $q_H \geq q_L$.

$$E(\pi) = 0.8 \times 11.25 \times 35 + 0.2 \times 5 \times 20 - 2 \times 35 = 315 + 20 - 70 = 265$$

With "bad" news:

$$E(\pi) = 0.2(20 - 0.25q_H)q_H + 0.8(10 - 0.25q_L)q_L - 2q_H$$

$$\frac{\partial E(\pi)}{\partial q_H} = 0.2(20 - 0.5q_H) - 2 = 0$$
$$2 = 0.1q_H$$
$$q_H = 20$$
$$p_H = 20 - 0.25(20) = 15$$

$$\frac{\partial E(\pi)}{\partial q_L} = 0.8(10 - 0.5q_L) = 0$$
$$10 = 0.5q_L$$
$$q_L = 20$$
$$p_L = 10 - 0.25(20) = 5$$

***Remember to check that you satisfied the constraint $q_H \geq q_L$.

$$E(\pi) = 0.2 \times 15 \times 20 + 0.8 \times 5 \times 20 - 2 \times 20 = 60 + 80 - 40 = 100$$

Expected profit with the forecast: $E(\pi \mid Info) = 0.5(265) + 0.5(100) = 182.5$

Value of info = 182.5 – 178 = 4.5

General: $E(\pi \mid Info) = \Pr(G)E(\pi \mid G) + \Pr(B)E(\pi \mid B)$,

$Value\ of\ \ Info = E(\pi \mid Info) - E(\pi \mid NoInfo)$

Violating the constraint

Suppose instead $\Pr(H \mid B) = 0.15$, all else is the same.

$$E(\pi) = 0.15(20 - 0.25q_H)q_H + 0.85(10 - 0.25q_L)q_L - 2q_H$$

$$\frac{\partial E(\pi)}{\partial q_H} = 0.15(20 - 0.5q_H) - 2 = 0$$
$$1 = 0.075q_H$$
$$q_H = 13.33$$

$$\frac{\partial E(\pi)}{\partial q_L} = 0.5(10 - 0.5q_L) = 0$$
$$10 = 0.5q_L$$
$$q_L = 20 > 13.33 = q_H$$

$$E(\pi) = 0.15(20 - 0.25\overline{q})\overline{q} + 0.85(10 - 0.25\overline{q})\overline{q} - 2\overline{q}$$

$$\frac{\partial E(\pi)}{\partial q} = 0.15(20 - 0.5\overline{q}) + 0.85(10 - 0.5\overline{q}) - 2 = 0$$
$$9.5 = 0.5\overline{q}$$
$$\overline{q} = 19$$

$$p_H = 20 - 0.25 \times 19 = 15.25$$
$$p_L = 10 - 0.25 \times 19 = 5.25$$

1) Could combine storage and forecast

2) What if you need to build/book storage capacity in advance?

4. ESTIMATING APPROXIMATIONS

Econometrics: estimation of empirical relationships among economic data (Ex: estimating demand functions) testing predictions of economic models (Ex: Do more dangerous jobs pay more?)

<u>Fitting Approximations with 2 Data Points</u>

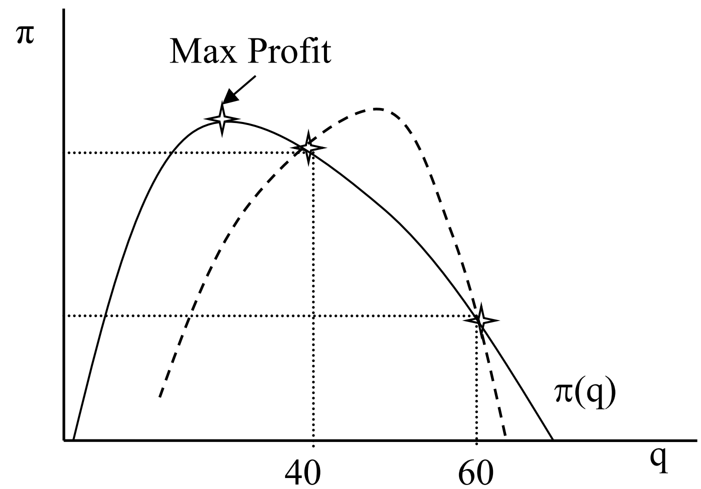
Example: Little and Small Inc. sells 60 units when price is \$4. They lower price to \$3 and sell 70 units. Cost is C=5+2q.

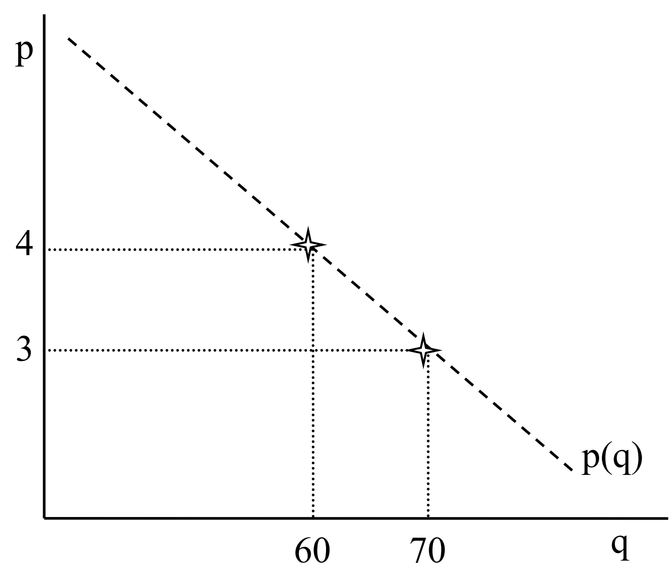
Profit @ 4: 4×60-2×60-5=114.

@ 3: 3×70-2×70-5=65.

Is the optimal price between 3 and 4 or above 4?

Need a demand curve approximation.





Linear Approximation

Find inverse demand:

$$Slope = \frac{\Delta p}{\Delta q} = \frac{4-6}{60-40} = -\frac{2}{20} = -0.1$$

To sell another unit, price must fall by 0.1.

Around a price of 4, as a *linear approximation*, the difference between price and 4 is -0.1 times the difference between quantity and 60, so:

$$\begin{aligned} p-4 &= -0.1(q-60) \\ p &= 4-0.1q+6 \\ p &= 10-0.1q \end{aligned}$$

Could solve for demand instead: $q = 100 - 10p$.

These are just approximations in the range of prices around [3,4].

Profit is:

$$\pi = \left(10-0.1q\right)q - 2q - 5$$

Maximizing:

$$\begin{aligned} \frac{d\pi}{dq} &= (10-0.1q) - 0.1q - 2 = 0 \\ 8 - 0.2q &= 0 \\ 8 &= 0.2q \\ q &= 8/0.2 = 40 \end{aligned}$$

Substituting gives:

$$\begin{aligned} p &= 10 - 0.1(40) = 6 \\ \pi &= 6 \bullet 40 - 2 \bullet 40 - 5 = 155 \end{aligned}$$

**** BEWARE: NO DATA IN THE RANGE OF p=6. CAN YOU BELIEVE THE RESULT? ***

<u>Log Linear or Constant Elasticity Approximation</u>

Find the constant elasticity appropriation for the above example:

q=60 when p=4,

q=70 when p=3.

$60 = a4^{\eta}$ and $70 = a3^{\eta}$. Dividing one by the other:

$$\frac{60}{70} = \frac{a4^{\eta}}{a3^{\eta}}$$

$$\frac{6}{7} = \left(\frac{4}{3}\right)^{\eta}$$

Taking the log of both sides:

$$\ln(6/7) = \eta \ln(4/3)$$

$$\eta = \frac{\ln(6/7)}{\ln(4/3)} = -0.536.$$

Since C(q)=5+2q, the optimal price is p=-0.536(2)=-1.072<0. THIS IS NONSENSE!!! Fails SOC!!!

If demand is inelastic (elasticity less than 1 in absolute value), raising price always increases profit, because revenue goes up and cost goes down!

This approximation is just no good at low prices, low elasticity.

At higher price, elasticity is likely larger, the approximation is likely more useful.

Regression Analysis

How to fit the "best" approximation to more than 2 points

Excel example: data and plot

Draw a line or curve as close to the points as possible: minimize total errors.

Problem: -100 and 100 cancel if sum errors, but, is 200 total error.

More want to minimize total absolute errors

Ordinary Least Squares Regression

Assume a good approximation is:

$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + \varepsilon_i$, where:

Y: dependent variable

Xs: k independent variables

βs: Parameters to estimate

i: indexes n observations

ε: random error, mean 0.

How to estimate the parameters? Predicted value is:

$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \cdots + \hat{\beta}_k X_{ki}$, where $\hat{\beta}s$ are parameters to estimate.

Then, sum of squared errors, SSE, is:

$$SSE = \sum_{i=1}^{n} (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{1i} - \hat{\beta}_2 X_{2i} - \cdots - \hat{\beta}_k X_{ki})^2$$

Take "parital" derivatives of SSE wrt $\hat{\beta}s$, set = 0, solve. Computers do it fast.

Nothing mysterious, or even complicated, to it.

One independent variable example (in excel), electric bill on size of residence:

$Bill = b_0 + b_1 Size$, $dBill/dSize = b_1$

Question: Does size reflect literally only size? What about age?

Two independent variable example, (in excel), size and temperature

$Bill = b_0 + b_1 Size + b_2 Temp$, $\partial Bill/\partial Temp = b_2$

Note change in estimated effect of Size

What if the relationship is not a straight line?

Flexibility of functional form. Only require linearity in parameters. Treat transformed versions of Y and X as dependent and independent variables to get curves of various shapes.

Example: log linear, or, constant elasticity formulation is given by taking logs of both sides of: $Y_i = e^{\beta_0 + \mu_i} X_{1i}^{\beta_1} X_{2i}^{\beta_2} \cdots X_{ki}^{\beta_k}$ to get

$$\ln Y_i = \beta_0 + \beta_1 \ln X_{1i} + \beta_2 \ln X_{2i} + \cdots + \beta_k \ln X_{ki} + \mu_i$$

Coefficients are elasticities

Builds in dependence of the effect of one X on the others…

Could also use X^2, 1/X, etc…

Example: What if think effect of temperature varies with size? Interact Size and Temp by entering their product as a new variable

$Bill = b_0 + b_1 Size + b_2 Temp + b_3 Size \times Temp$, $\partial Bill/\partial Temp = b_2 + b_3 Size$

Note changes in first 2 coefficients

Or could use log linear: Note that it builds in some interaction, as well as some non linearity, since the base is higher for higher size, and, the percentage effect of Temp is constant, the absolute effect of temp is larger at larger sizes…

Categorical, Indicator, or "Dummy" Variables

What about effect of gender, race, an advanced degree on earnings? Effect of being retired on golf games played? Etc…

EX: Effect of pool on electric bill?

$Bill = b_0 + b_1Size + b_2Temp + b_3Size \times Temp + b_4Pool$

Coefficient it total effect on bill, average change in bill when there is a pool, all else equal

Question: Does pool reflect only the direct effect of a pool? Might it reflect other things?

Note changes in coefficients

Example: Log Linear Model. Calculate partial derivative for Temp

5. EVALUATING REGRESSION ANALYSES

Briefly, bias v precision.

Note in excel example that estimates changed as added more variables.

For Excel Example, KNOWN TRUE MODEL is:

$$\text{Bill} = 100 + 6\text{Size} + 0.9\text{Temp} + 0.4\text{Size}\times\text{Temp} + 80\text{Pool}$$

Best Estimate was:

$$\text{Bill} = 59.64 + 20.12\text{Size} + 1.62\text{Temp} + 0.15\text{Size}\times\text{Temp} + 74.93\text{Pool}$$

Both imprecision (noise) and correlations with unobserved errors resulting in less than perfect estimates

Assume for now bias is not a concern… Will come back to this after we know more economics. Focus on precision of the estimates

<u>General (non quantitative) aspects</u>

First, must think hard about what variables to include.

More variables → better if they really belong, but, less data to identify the effects of each variable, so worse estimates of all parameters if they do not

Second, must think hard about what data to use:

More data → better estimates, if it is good data, not if it is low quality data or not available for all important variables

Third, must think hard about form of approximation

Is linear reasonable?

Should it increase then decrease, or, vice versa?

Should there be interactions?

Log linear specific builds in some interaction, as changes proportional

Once you have estimated a model that makes sense based on 1-3, are the signs of individual coefficients “right”. If not, worry something is fishy or ***unstable***, thus unreliable.

Quantitative/Statistical Evaluation of Precision

1) Group of variables explains more than k randomly picked (unrelated) variables.

The p-value of the F-Statistic is roughly the chance that k randomly selected random variables unrelated to y in the population would be as highly correlated with y as this group of k independent variables

For a good model, this probability is small.

Beyond that, quantitative evaluation of precision depends on purpose:

i) Prediction – ex orders to prepare for production

ii) Estimation of parameters – ex elasticity of demand curve to determine profit maximizing price

Prediction

If goal is prediction of Y, bias is not as big a deal (absorbed in intercept). Care about precision of overall estimate.

Root Mean Square Error (RMSE)

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-k-1}}$$ (comment on n–k-1)

RMSE is an estimate of the standard deviation of the error term. The margin of error, or, the "typical" amount your estimate will be off (sort of)

Approximately 95% of the time:

$$\hat{Y}_i - 2\sigma_i < Y_i < \hat{Y}_i + 2\sigma_i$$

Relationship between R^2 and $\hat{\sigma}$.

Does R^2 mean much? Example: Regress City Total Income on City Total Population, High R^2. So what?

Uncertainty, as measured by $\hat{\sigma}$, matters *relative to prediction of Y for your X's of interest*. If $\hat{\sigma}$ is 100, and predicted q of interest is 1,000,000, probably good. But, if predicted q is 350, 95% CI is 50 to 550 at q of interest!

Go back to excel example to illustrate.

<u>Estimation of Parameters</u>

Potential bias is a huge concern here, but, we are not yet ready to address it

Estimated standard error of the coefficient, $s_{\hat{b}}$

Confidence interval: Approximately 95% of the time $\hat{b} - 2s_{\hat{b}} < b < \hat{b} + 2s_{\hat{b}}$

t-stat: $\hat{b}/s_{\hat{b}}$

Reported p-value of t-stat: letting J denote this sample, $P\left(\left|\hat{b}\right| > \left|\hat{b}_J\right| \mid b = 0\right)$

|t-stat| > 2*ish* taken to indicate "statistical" significance, or, p-value about 0.05 or smaller. Same as 0 not in CI.

Just means strong evidence of a relationship, reject Ho of no relationship

Economic significance: Is the coefficient big enough to be important?

Economic precision: Is the SE small enough so the estimate is useful?

EX: Suppose estimated elasticity is -6 with a SE of 2.

95% CI for elasticity indicates profit max markup factor between:

-2/(1-2)=2 and -10/(1-10)=1.11

Go back to example to illustrate

Warnings

1) Do not extrapolate beyond range of data!!!!!!!!!!!!!!!

2) Do not over-fit – gives unstable results!!!!!

3) Best if possible to validate out of sample

6. OVB

Bias is an important issue for parameter estimates AND hypotheses tests!

Ideal: randomized double blind trial (RDBT). Participants randomly assigned to treatment group and a control group where:1) subjects do not know which they are in, 2) neither do researchers.

<u>Example:</u>

At first, all face same price.

"control" $\Delta p=\$1$

"treatment" $\Delta p=\$3$.

So, slope of demand is -2.

	Average Quantity		
Group	Before	After	Δ
Treatment	26	16	-10
Control	25	21	-4
Difference in Difference			**-6**

Hard to get RDBTs with enough subjects and large enough stakes to get meaningful results.

Regression analysis lets us use non-experimental, or, observational, data, which is more readily available

Problem: never sure the explanatory variable of interest is not correlated with something you did not measure that is also correlated with dependent variable. If so, you have not "identified" the effect of interest. "**Omitted Variables Bias**"

In fact, to some degree, OVB is almost always present in empirical economics, because: 1) many (most) variables are endogenous so they are correlated with many other variables, and, 2) no model is perfect, so there are always omitted variables.

“**Endogeneity Bias**” or “**Simultaneous Equations Bias**”

A major cause of omitted variable bias. If variables on the RHS depend on the LHS variable too, they are affected by the error term for the LHS variable. Any endogenous RHS variables ARE correlated with the error term for the LHS variable. Regression is BIASED in an unknown way!

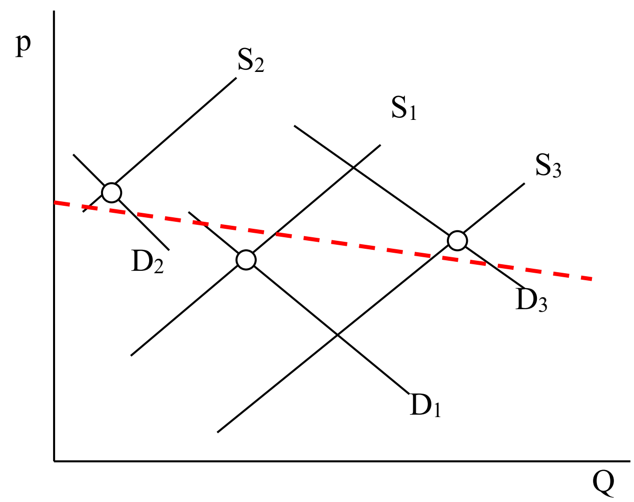
<u>Example</u>: demand estimation where S & D determine p & Q simultaneously.

$Q_D=a-bp$, $Q_S=c+dp$

$$a-bp+\varepsilon_D=c+dp+\varepsilon_S$$

$$(d+b)p=a-c+\varepsilon_D-\varepsilon_S$$

$$p=\frac{a-c+\varepsilon_D-\varepsilon_S}{b+d}$$



Price is correlated with anything that affects supply or demand, hence, it is correlated with the demand error! The true relationship can not be identified (by OLS)!

Measurement Error

Suppose $y=b_0+b_1x+\varepsilon$, but, all you have is a measurement of x that is made with error, $x_M=x+e_M$. Rearranging, $x=x_M-e_M$.

Substitution: $y=b_0+b_1(x_M-e_M)+\varepsilon = b_0+b_1x_M-b_1e_M+\varepsilon$

So, if you regress y on the measured x, the error term is $-b_1e_M+\varepsilon$, which is correlated with x_M since $x_M=x+e_M$

Suppose x_M is higher. Is it because of measurement error, or, because x is higher? This obscures the relationship between y and x in the data. The true relationship can't be identified!

Possible Solutions

1) Panel data

Observe each unit (individual, firm, city, state, etc…) multiple times

Omitted but "fixed effects" captured by dummies for individuals

Example: 4 individuals, i=0-3, Indicators, I_i, 0 omitted:

$y_{it}=a+bx_{it}+ct+d_1I_1+d_2I_2+d_3I_3+\varepsilon_{it}$.

Interpret ds

Then, $\Delta y_{it}=y_{it}-y_{i,t-1}$ is

$\Delta y_{it}=(a-a)+b\Delta x_{it}+c\Delta t+d_1\Delta I_1+d_2\Delta I_2+d_3\Delta I_3+\Delta\varepsilon_{it}$ or

$\Delta y_{it}=c+b\Delta x_{it}+\Delta\varepsilon_{it}$

Regressing differences in y on differences in x, OR, just including dummies, controls for anything about i that does not change over time.

Ex: Collect data on sales, price, advertising, etc… for 40 locations over many periods. Can't include/measure everything. If everything omitted is constant for each location, OK with dummies or differencing.

Problem: demand shocks induce managers to change prices!

Ex: If unmeasured income trending up in some locations, so will price! Looks like price does not cut demand much! WRONG!

May not be reasonable to think unmeasured characteristics are not trending systematically over time for each individual.

2) "Natural" experiments

Look for places where changes totally outside the control of any participants force exogenous changes in endogenous RHS variables

Ex: Want to know impact of minimum wage on employment

Problem: States with higher (real) minimum wage may have it because a) unemployment lower for other reasons, so, not a concern, b) have different unemployment benefits, so, different concerns about created unemployment, that also cause different employment directly.

Potential Natural Experiment: Feds increase minimum minimum wage. Some states not affected. Others are. Potential exogenous change in minimum wage.

Problem: The ones above both the old and new Fed min wage are "different". So, the things that changed in each type before and after the Fed change may well differ in ways that affect unemployment! Social assistance may be trending up in those states, increasing unemployment anyway. Cause D-I-D to underestimate impact on employment

Generally: Groups affected differently by a "natural" experiment are likely to be different in ways that affect both how the "natural" experiment affects them and also the dependent variable. (May have been the cause of the event being taken as a "natural experiment".)

General: Mobile residents face same choice set! Location is not exogenous!

3) “Market Trial”

Create an experiment. Ex: test markets, randomly vary price across outlets

Problems:

1) Participants know which group they are in. May delay purchase, or, cross locations to purchase, etc…
2) Expensive – giving up profit at best guess about price to collect data!

4) Instrumental Variables

Find variables correlated with RHS endogenous variable that have no direct impact on LHS variable.

Use these to generate “predicted” values for the RHS endogenous variables that are not correlated with the error term

Example: use input prices to purge product price of error component correlated with demand. Input prices not direct determinants of demand.

Problem: Wages correlated with income! So are taxes! The price of capital will not vary much from city to city!

Generally: Good instruments are hard to find, partly because all mobile residents face the same array of choices!

Discussion

None of these four solutions are perfect – some are better than others in particular instances.

Arguments can be made that the problem is just completely intractable – can’t get identification from non experimental data where everything is endogenous, measurement error is rampant, data is always incomplete!

Conclusion:

DO NOT GIVE MUCH CREDENCE TO RESULTS OF A NON-EXPERIMENTAL STUDY (in economics, medicine, education, or any other discipline) UNLESS ALL OF THE FOLLOWING APPLY:

1) It is consistent with a simple theory that has a concise formal mathematical expression
2) It is consistent with a believable, intuitive, story
3) Many empirical studies using multiple identification strategies find similar results

<u>All 3 are important!</u>

A persuasive speaker can make almost anything sound good! math provides a check to make sure the "story" is logical and consistent. Repeated independent studies mean the data are at least consistent with the story and the math.

With the right "assumptions" you can get any result you want with a model. It has to be "comprehensible" to make sure it is not doing something silly, and, it needs repeated empirical confirmation to make sure it has real content.

Torturing data can get almost any statistical result! It needs repeated confirmation by independent sources to make sure it was not "forced". Even then, the problems with empirical results are vast and deep. The results must be sensible too give them context and be consistent with formal logic (math) to be sure they are not gibberish!

7. INDIVIDUAL CHOICE

Theory of individual behavior with full information based on three main assumptions (other technical ones we will ignore):

1) Completeness: Individuals can look at any 2 bundles of stuff and say either: a) the first is better (>), b) the second is better (<), or c) they are indifferent (~)

2) More is Better: Any bundle with more of one good and at least as much of all others is better

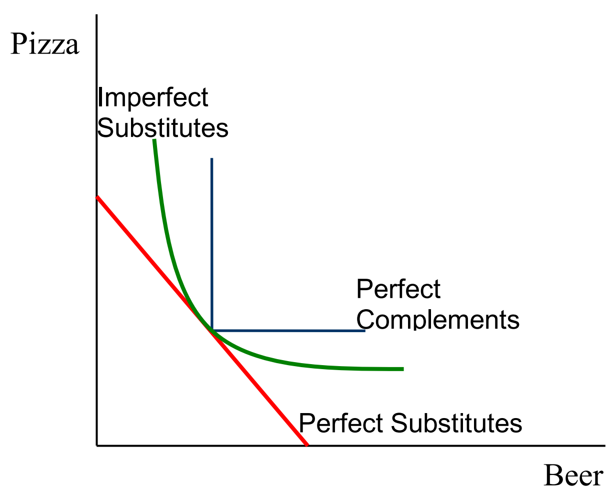
3) Transitivity: If $A \succ B$ and $B \succ C$ then $A \succ C$

Marginal Rate of Substitution: the rate the consumer is willing to substitute x for y. If had one more x, how much less y could you take and remain indifferent?

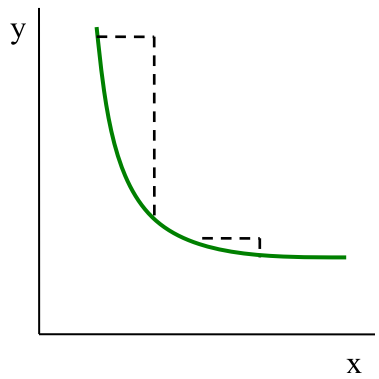
4) Diminishing MRS: As a consumer has more and more of one good, they are willing to trade less and less of the others for still more of the first.

Indifference Curves

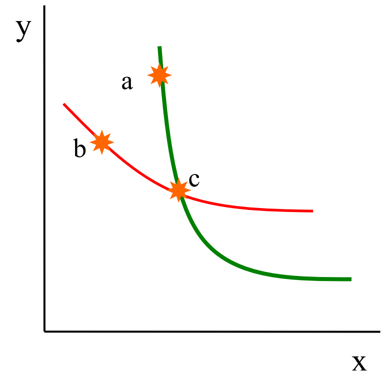
Combinations of goods that the consumer views as "just as good" as each other



Diminishing MRS



Indifference Curves Don't Cross



MRS is the rate substitute x for y, and, the slope of the indifference curve. With more x and less y along an IC, another x is worth less y, so, the indifference curve gets flatter as x increases and y decreases.

The is not possible. Since they are on the same ICs, a~c and b~c, so a~b. But, since more is better, a≻b. CONTRADICTION. Thus, IC's can't cross

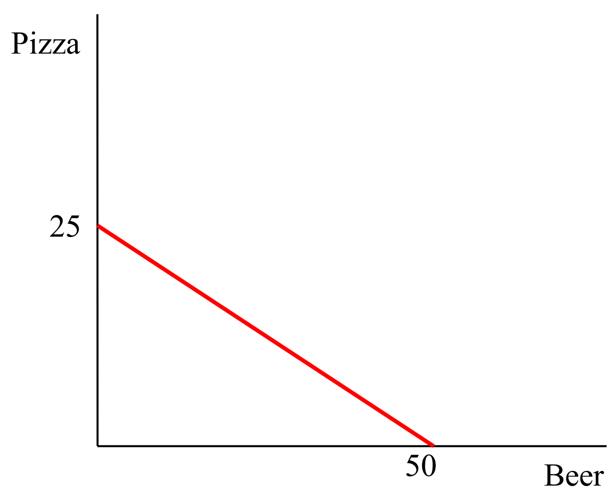
<u>Budget Lines</u>

With 2 goods: $M = p_x x + p_y y$

Or: $y = \dfrac{M}{p_y} - \dfrac{p_x}{p_y} x$

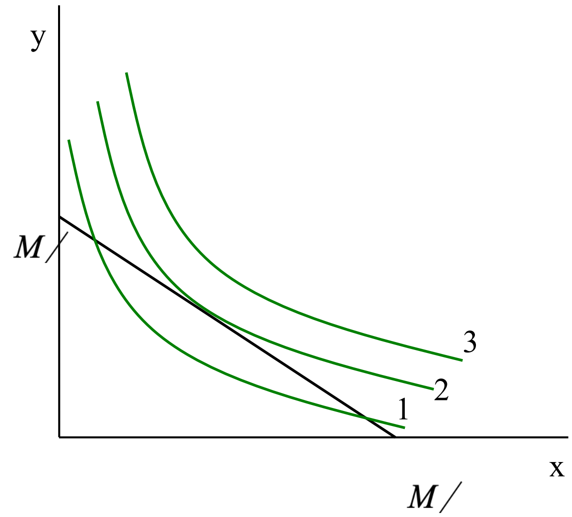
Slope is price ratio

Example: Say price of beer is \$1 per bottle, pizza is \$2 per slice, \$50 to spend



Individual's Problem: Find the best bundle on their budget line

Can do better than IC 1, can't reach 3.



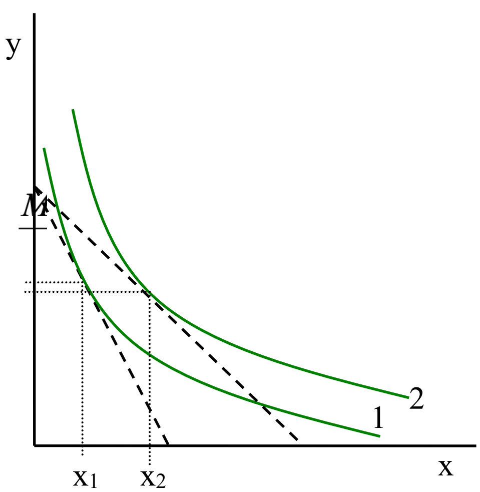
At highest IC:

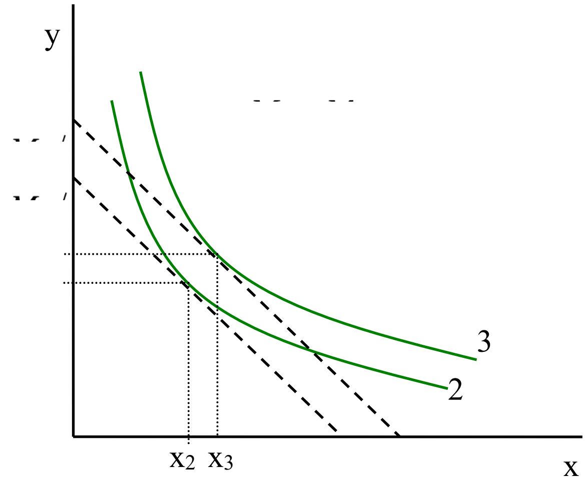
Slope of IC = Slope of Budget Line

MRS=Price Ratio

Interpretation: rate willing to trade goods equals rate market lets you trade them, or, should buy more of the one more valued relative to market prices.

Effect of Price and Income Shifts





Calculus of Individual Behavior

Graphical problem is not always convenient

Can preferences be represented mathematically?

When the assumptions above are reasonable approximations, yes!

Demonstration:

1) Number each indifference curve by x value where it crosses $45°$ line (y=x).

2) Assign that value of "utility", U, to every bundle on that indifference curve.

3) We have mapped every combination of x and y to a unique utility number, U. If a given bundle is preferred, it has a higher U.

4) Thus, there is a utility function, U(x,y), that represents consumer preferences that satisfy assumptions 1-4!

Uniqueness:

Scale does not matter. U(x,y), and .1U(x,y)+100, U(x,y)^2, and ln(U(x,y) all give same preferences. Example, suppose U=x+y+xy. Compare x=2 y=4 to x=3, y=3. 14<15, hence (3,3) is "preferred". Now, say U=2(x+y+xy) and compare the same 2 possibilities,

So, individual's problem is:

$$\max_{x,y} \quad U(x,y)$$
$$subject \quad to \quad p_x x + p_y y \le M$$

OR:

$$\mathcal{L} = U(x,y) + \lambda(M - px - py)$$

FOC:

$$\frac{\partial U}{\partial x} - \lambda p_x = 0$$
$$\frac{\partial U}{\partial y} - \lambda p_y = 0$$
$$M - p_x x - p_y y = 0$$

So, 2 conditions that mirror cost minimization apply to utility maximization:

1) $\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$ Interpretation?

2) $M = p_x x + p_y y$

Solution gives individual demand functions.

Reasonable to *model* preferences with mathematical functions.

8. APPLICATIONS

Application to Compensation Indexing

Suppose want to get a manager to move from Gainesville to Atlanta.

First, assume only care about consumption

Amenities are the same, also, ignore moving costs

What prices vary? Housing and services (Law of one Price / Arbitrage)

Assume for our purposes only the price of housing, R, varies.

Utility depends on consumption of housing, H, and, everything else, E. u(H,E)

Budget line in Gainesville, $M_G=R_GH+E$. Graph shows solution, H_G,E_G

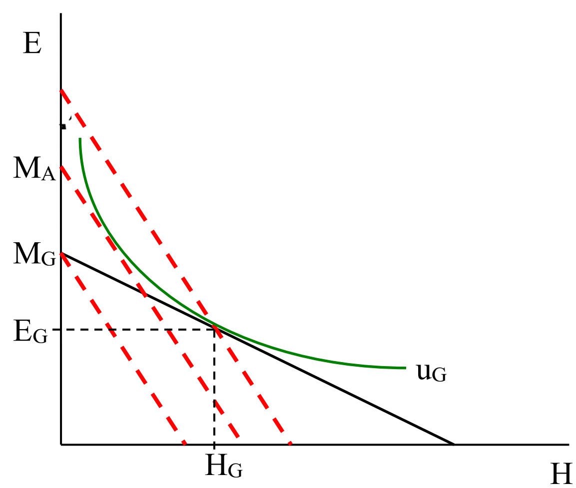
In Atlanta, housing prices higher, $R_A>R_G$

If pay the same as in Gainesville,

impossible to reach the same IC. The manager will not accept the offer.

What to do? Could pay enough to buy the same bundle: $\hat{M}_A = R_A H_G + E_G$.

As shown in graph, this is MORE than needed. Solution is M_A.



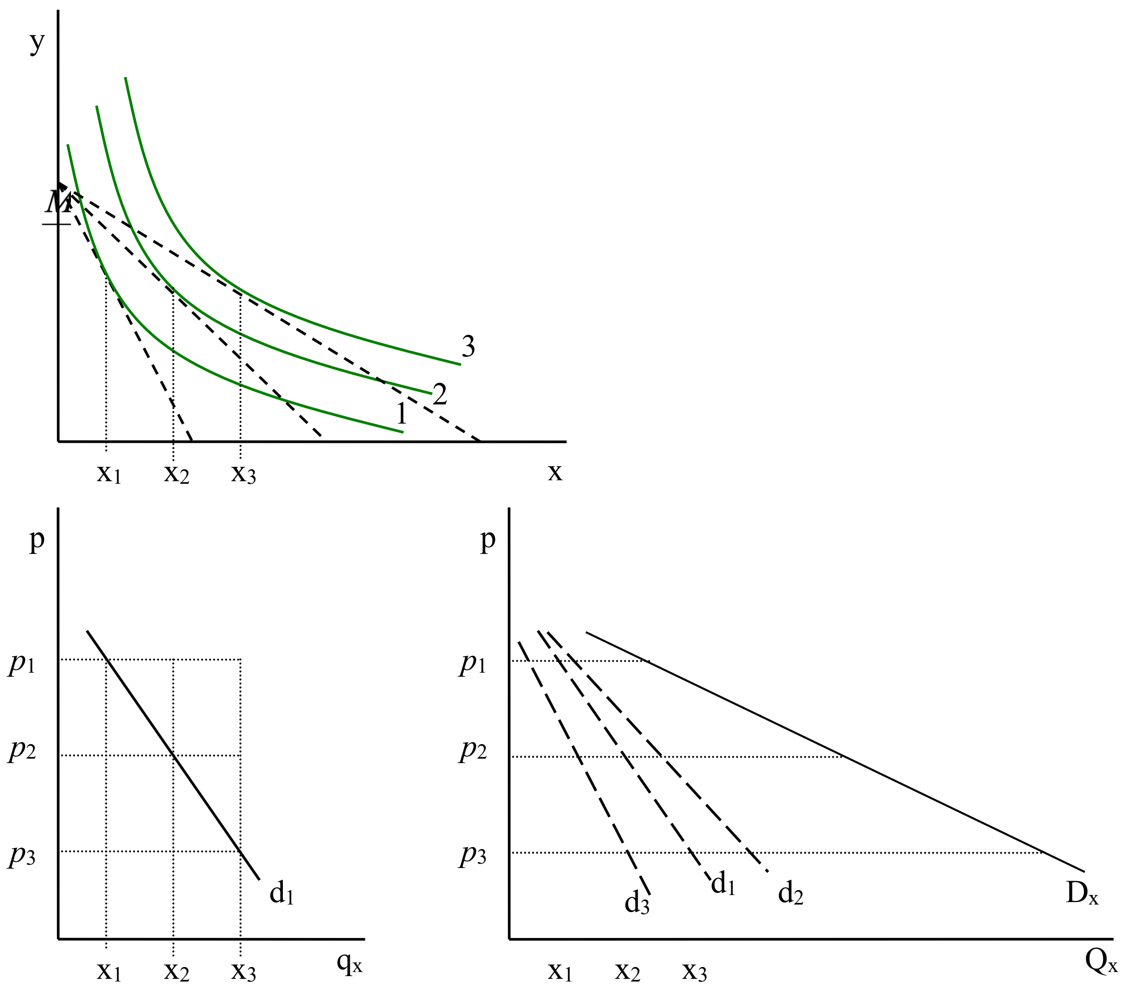
What if Gainesville offers higher "amenities"? Relevant IC is shifted up in Atlanta.

Individual and Market Demand

Derive individual demand, $q_i(p)$ from an indifference map by varying price

Market demand is sum of individual demands: $D(p) = Q_D = \sum_i q_i(p)$

HORIZONTAL sum of inverse demands



Show effect of income shifts

Measuring Consumer Welfare - Consumer Surplus

Consumer Surplus - The difference between the most consumers would willing pay for q and what they actually pay.

A (very) simplified measure of consumer welfare, used extensively. Warning: to be exact, should relate to utility theory!!!!! Relationship between demand functions and preferences is crucial in all applied quantitative policy analysis.

Formally

Let v(q) be the most a consumer would pay for q units.

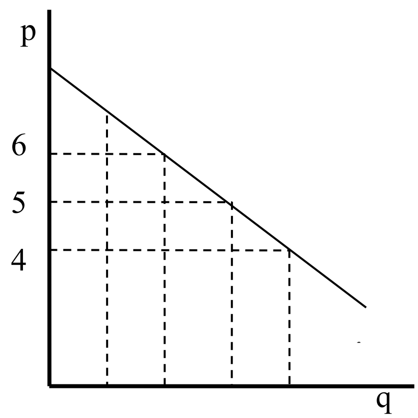
Their "surplus" if price is p is s=v(q)-pq. If they choose q to maximize this:

$s_i = v_i(q_i) - pq_i$

$\frac{ds_i}{dq_i} = \frac{dv_i}{dq_i} - p_i = 0$

$p = \frac{dv_i}{dq_i} \rightarrow i's$ inverse demand

inverting it gives i's demand, $q_i(p)$



p	q	V(q)
8	0	0
7	1	7
6	2	13
5	3	18
4	4	22
3	5	25

Inverse demand is marginal willingness to pay.

Add them all up, get total willingness to pay. Ignoring indivisibilities:

$$v(q) = \int_0^q p(x)dx$$

Subtract the amount paid from v(q), get cs(q)

$$cs(q) = \int_0^q \left(v'(x) - p\right)dx \text{ or } s(q) = \int_0^q v'(x)dx - pq$$

Sum of individual consumers' surpluses is area below demand and above price

$$CS(Q) = \sum_i cs_i(q_i) = \int_0^Q p(x)dx - px$$

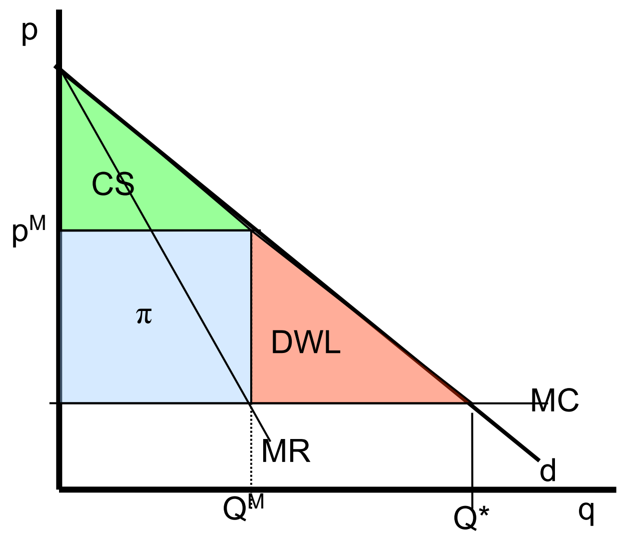
9. NON LINEAR PRICING

Two Part and Block Pricing

Until now we have assume a simple, constant, per unit price.

BUT, firms with market we can generate much more profit with more sophisticated pricing.

Why? See standard graph:



Of total possible surplus value, if sell Q^M @ p^M, giving up CS to consumers and just losing DWL

If sell for p=MC, get max surplus, but, it all goes to CS!

<u>Block pricing:</u>

Figure out how each consumer would buy at p=MC, q_i^*

Package that many together for a total block price, P, equal to the customer's total willingness to pay, $v(q_i^*)$

<u>Two part pricing:</u>

Set per unit price p=MC

Charge each customer a fixed fee, f, for the right to purchase, like Sam's, Fee equals customer's surplus, $f=cs(q_i^*)=v(q_i^*)-cq_i^*$.

Does not work if resale is possible.

<u>Identical Customers</u>

Let C(Q) be total cost, where Q=nq, v(q) be the willingness to pay, and p(q) be the inverse demand function. Remember $p(q)=v'(q)$.

Block Pricing

Profit is the number of consumers times P less Cost.

There is one constraint: each consumer's surplus can not be negative

The problem can be written as:

$$Max_{P,q}\ nP - C(nq)$$
$$s.t.\ \ v(q) \geq P$$

Substituting the constraint into the problem gives

$$\pi = nv(q) - C(nq)$$
$$\frac{d\pi}{dq} = n\left(p(q) - MC\right) = 0$$

Max and solve for q. Use the constraint to find P.

Two-part pricing

Profit is the number of consumers times f+pq less Cost.

There are 2 constraints

1) price is dictated by the demand curve

2) Each consumer's surplus can not be negative

The problem can be written as:

$$\begin{array}{ll} \max_{p,q,f} & \pi = n(f + pq) - C(nq) \\ s.t. & \begin{array}{c} v(q) - pq - f \geq 0 \\ p(q) = p \end{array} \end{array}.$$

Substituting the first constraint into the problem gives:

$$\max_q \pi = nv(q) - C(nq)$$
$$\frac{d\pi}{dq} = n\frac{dv}{dq} - n\frac{dC}{dq} = 0$$
$$\frac{d\pi}{dq} = n\left(p(q) - MC\right) = 0$$
$$MB = v'(q) = p(q) = MC$$

Solve for q. Then, use the constraints to find p and then f.

Example: Block and Two Part Pricing

Identical customers with $v(q) = 10q - 0.25q^2$, $C(nq) = 2nq$.

With simple linear pricing:

$$p = 10 - 0.5q$$
$$\pi = n(10 - 0.5q)q - 2nq$$
$$\frac{d\pi}{dq} = n(10 - q - 2) = 0$$
$$q = 8$$
$$p = 6$$
$$\pi = n(6-2)8 = 32$$

To find the q to max TS(Q)

$$TS = n(10q - 0.25q^2) - 2qn$$
$$\frac{dTS}{dq} = n(10 - 0.5q - 2) = 0$$
$$q = 16$$

With block pricing:

$$P = V(16) = 10(16) - 0.25(16^2)$$
$$P = 96$$
$$\pi = n\left(96 - 2(16)\right) = n64$$

With 2 part pricing:

$$p = 2$$
$$f = cs = v(q) - pq = 96 - 2(16) = 64$$
$$\pi = n64$$

Customers not Identical

If you can segment them, charge each a different “membership” fee. For example, discount membership fees for seniors, teachers, etc…

If you can’t separate them:

a) balance two tradeoffs: i) increasing f above MC of lowest demand type and losing customers but getting more fee revenue from those that remain, and ii) increasing p to increase revenue, losing sales per customer but not losing customers. (Two part pricing more is flexible than block pricing when customers are not identical)

b) use “menu pricing” to get customers to segment themselves

Menu Pricing

When customers differ but can't be explicitly segmented, firms with market power can increase profit by offering choices, customers' choice tells you their type!

A menu of block prices and quantities (P,q) or a menu of two part tariffs (f,p) and customers choose which option they want

Idea: a lower charge for a small package for low demand customers and a higher total charge for a large package for high demand customers

Must ensure each customer actually prefers to choose the menu price and quantity intended for them

<u>Formally</u>

n_H high demand consumer's with willingness to pay $v_H(q_H)$

n_L low demand consumer's with willingness to pay $v_L(q_L)$

$C(n_Hq_H+n_Lq_L)$: Total Cost

(P_H,q_H) block price and q intended for "high" type

(P_L,q_L) block price ad q intended for low type

$$
\begin{aligned}
&Max_{P_H,q_H,P_L,q_L} \quad \pi = n_H P_H + n_L P_L - C(n_H q_H + n_L q_L) \\
&s.t \qquad 1)\ v_L(q_L) - P_L \geq 0 \\
&\qquad\qquad 2)\ v_H(q_H) - P_H \geq 0 \\
&\qquad\qquad 3)\ v_L(q_L) - P_L \geq v_L(q_H) - P_H \\
&\qquad\qquad 4)\ v(q_H) - P_H \geq v_H(q_L) - P_L
\end{aligned}
$$

Which of the 4 constraints will "bind"?

First, you can never make "high" demand customer "break even". Why?

Second, would low demand customer ever claim to be "high"? Why?

Third, would you let "low" type keep any surplus? Why?

So:

$$P_L=v_L(q_L)$$

Fourth, would you give "high" type more than needed to keep them honest? Why?

So:

$$v_H(q_H)-P_H=v_H(q_L)-P_L$$

$$P_H=P_L+v_H(q_H)-v_H(q_L)$$

$$P_H= v_L(q_L)+v_H(q_H)-v_H(q_L)$$

Substituting

$$\pi = n_H\left(v_H(q_H)-v_H(q_L)+v_L(q_L)\right)+n_L v_L(q_L)-C(n_H q_H+n_L q_L)$$

$$\frac{d\pi}{dq_H}=n_H v'_H(q_H)-n_H MC=0$$

$$p_H(q_H)=MC$$

Marginal benefit of another unit to the high type equals MC, high type's q is "efficient".

$$\frac{d\pi}{dq_L}=n_H\left(-v'_H(q_L)+v'_L(q_L)\right)+n_L v'_L(q_L)-n_L MC=0$$

$$p_L(q_L)-MC=\frac{n_H}{n_L}\left(p_H(q_L)-p_L(q_L)\right)>0$$

Marginal benefit of another unit to the low type is higher than MC – that is, low type's q is "too small"!

Never distort "high" type's q. Why?

This is a surprisingly general result with asymmetric info.

Example: Menu Pricing

$V_1(q_1) = 10q_1 - 0.25q_1^2$, $V_2(q_2) = 15q_2 - 0.25q_2^2$, n1=10 and n2=5, and $C(q) = 2q$.

$$\max_{q_1,P_1,q_2,P_2} \quad 10(P_1 - 2q_1) + 5(P_2 - 2q_2)$$

$$\text{subject\_to} \quad \begin{aligned} &10q_1 - 0.25q_1^2 - P_1 \geq 0 \\ &15q_2 - 0.25q_2^2 - P_2 \geq 0 \\ &10q_1 - 0.25q_1^2 - P_1 \geq 10q_2 - 0.25q_2^2 - P_2 \\ &15q_2 - 0.25q_2^2 - P_2 \geq 15q_1 - 0.25q_1^2 - P_1 \end{aligned}$$

The binding constraints are:

$$10q_1 - 0.25q_1^2 = P_1$$
$$15q_2 - 0.25q_2^2 - P_2 = 15q_1 - 0.25q_1^2 - P_1.$$

Substituting the first into the second:

$$15q_2 - 0.25q_2^2 - P_2 = 15q_1 - 0.25q_1^2 - 10q_1 + 0.25q_1^2$$
$$P_2 = 15q_2 - 5q_1 - 0.25q_2^2$$

So:

$$\pi = 10(10q_1 - 0.25q_1^2 - 2q_1) + 5(15q_2 - 0.25q_2^2 - 5q_1 - 2q_2)$$
$$\frac{d\pi}{dq_1} = 10(10 - 0.5q_1 - 2) - 5\bullet 5 = 0$$
$$80 - 5q_1 - 25 = 0$$
$$q_1 = 11$$
$$\frac{d\pi}{dq_2} = 5(15 - 0.5q_2 - 2) = 0$$
$$q_2 = 26$$
$$P_1 = 10(11) - 0.25(11^2) = 79.5$$
$$P_2 = 166$$
$$\pi = 10(79.5 - 2\bullet 11) + 5(166 - 2\bullet 26) = 1145$$

Compare to "Efficient" quantities:

$$10 - 0.5q_1 = 2 \qquad 15 - 0.5q_1 = 2$$
$$q_1 = 16 \qquad\qquad q_1 = 26$$

Graph

Figure shows p(q) or v'(q) for one consumer of each type.

Assumes constant MC, equal numbers of each type.

In left panel, consider the "efficient" quantities for each type, where v'=MC.

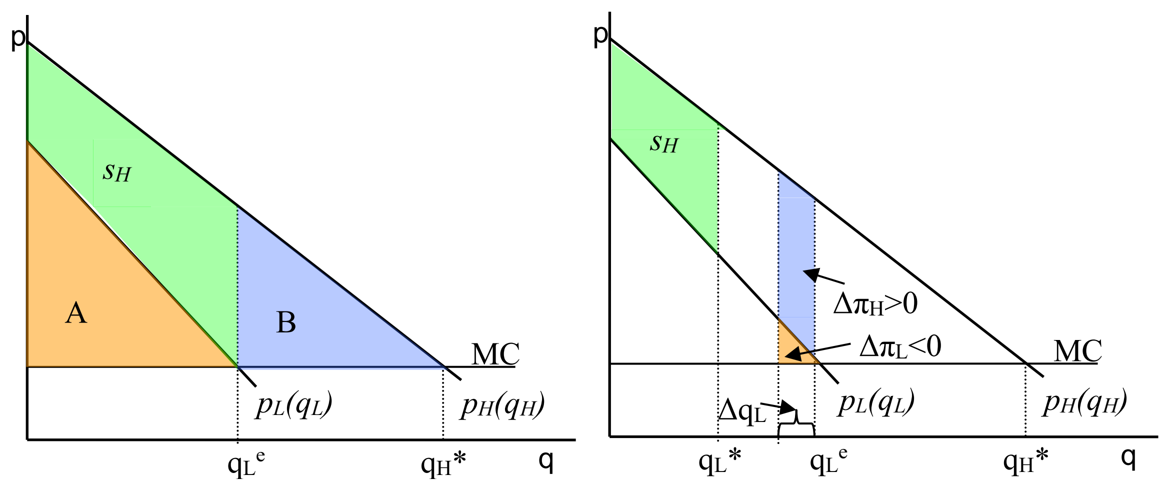
Charge $P_L = v_L(q_L^e)$ to low type.

Buying the "small" bundle, high type could get surplus equal to area labled s_H.

Must let them keep that much surplus if they buy bundle with q_H^e.

Profit from the low type is the area between p_L and MC up to q_L^e, area 'A'.

High type profit is the area between p_H and MC up to q_H^e, less s_H, A+B.



Suppose equal numbers of each type.

Consider shrinking the "small" bundle by a little, Δq_L in the right panel. Profit from the "low" type shrinks. But, you need to let the high type keep less surplus, so, profit from the high type increases by more!

Lower q_L until the marginal loss of π_L equals the marginal gain in π_H.

With equal numbers of each type, the FOC:

$p_L(q_L) - MC = \frac{n_H}{n_L}\left(p_H(q_L) - p_L(q_L)\right)$ becomes $p_L(q_L) - MC = p_H(q_L) - p_L(q_L)$.

Balance the DWL from reducing q_L with the decrease in profit surrendered to the high type to keep them honest.

This can also be implemented as a choice of two part tariffs – higher membership fee gets per unit price at MC (discount over posted price)

Lower membership fee gets higher per unit price.

In the figure, D is the DWL from shrinking the low type's bundle.

p_L* is the per unit price charged to the low type

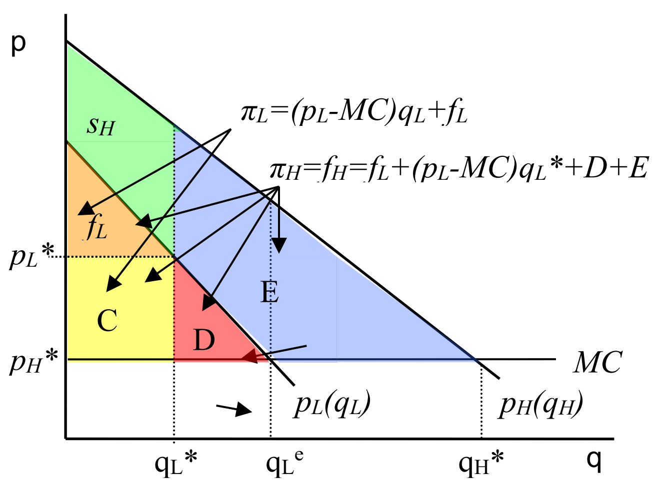
f_L is the fee charged to the low type

s_H is surplus kept by the high type

$p_H^*=MC$ is the price charged to the high type per unit

The high type fee is $v(q_H*)-s_H-p_H*q_H*$ or $f_L+(p_L-MC)q_L*+D+E$

Profits are $\pi_L=(p_L*-MC)q_L*+f_L$ and $\pi_H=f_H=(p_L*-MC)q_L*+f_L+D+E$



10. Uncertainty with Risk Aversion

Basic idea: most people dislike uncertainty with respect to large amounts of wealth. We need a convenient model so we can apply this in economic analysis.

Expected Utility Theorem:

Let w_i index possible wealth outcomes and f_i denote the probability of each outcome. Under our assumptions, there is an expected utility function, $u(w_i)$, such that the gamble with the higher *expected* utility, $EU = E(u) = \sum_i f_i u(w_i)$, is preferred.

We will use this as an "as-if" model.

But, to derive it, apply assumptions 1-4 to preferences over lotteries. Then, one more important assumption:

5) Independence: Suppose $A \succ B$. Then, a "compound" lottery yielding A with prob f and C with prob 1-f is preferred to a "compound" lottery yielding B with prob f and C with prob 1-f.

Interpretation: elements that are the same in 2 lotteries do not affect the choice

Examples: Suppose $u = 10\sqrt{w}$.

Option A is a 50% chance of $0 and a 50% chance of $100.

Option B is a 100% chance of $36.

Which does this consumer prefer?

EU of A is $.5 \times 10\sqrt{0} + .5 \times 10\sqrt{100} = 50$ EV was 50

EU of B is $= 10\sqrt{36} = 60$ EV was 36<50

Option C is an 80% chance of $36 and a 20% chance of $0.

Option D is a 50% chance of 64 and a 50% chance of $0.

Which is preferred?

EU of C is $.8 \times 10\sqrt{36} = 48$ EV = 28.8

EU of D is $= .5 \times 10\sqrt{64} = 40$ EV = 32>28.8 but, C preferred

<u>Certainty Equivalent</u>: certain payoff viewed as equivalent to a gamble.

Risk Neutral: CE=EV

Risk Averse: CE<EV

CE's for lotteries with various probabilities of wealth w1 and w2 are defined by the following relationship:

$u(CE) = fu(w_1) + (1 - f)u(w_2)$.

Generalizes for more probabilities: $u(CE) = \sum_i f_i u(w_i)$

Example: 50% chance at 100, 50% chance at 0. EV=$50. CE if $u = 10\sqrt{w}$?

$10\sqrt{CE} = .5(10\sqrt{0}) + .510\sqrt{100}$, $\sqrt{CE} = .5\sqrt{100}$, $\sqrt{CE} = 5$

But, $25 for certain is just as good.

Constructing a Utility Function for Uncertain Outcomes

Let 100 be the best possible outcome and 0 be the worst possible outcome of a series of gambles.

Compare various certain wealth levels to a gamble between \$100 with probability f and \$0 with probability 1-f.

Ask what probability of \$100 would make an individual indifferent between each certain wealth level and the gamble: $u(w) = fu(100)$, find f. See table.

These probabilities are points on a utility function.

More convenient to scale things so U(100)=100 and U(0)=0, by multiplying by the dollar value of the best possible outcome. Last column: $u(w) = fu(100) = 100f$

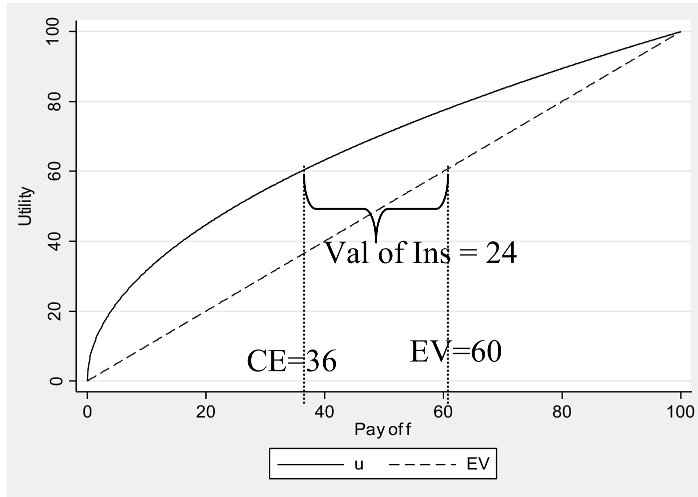
Plot it:

w	f	u(w)
0	0.0	0
16	0.4	40
25	0.5	50
49	0.7	70
81	0.9	90
100	1.0	100

Curve: utility function, $u = 10\sqrt{w}$

The dashed line shows all possible expected values of gambles between \$0 and \$100 for all probabilities of the high payoff between 0 and 1.

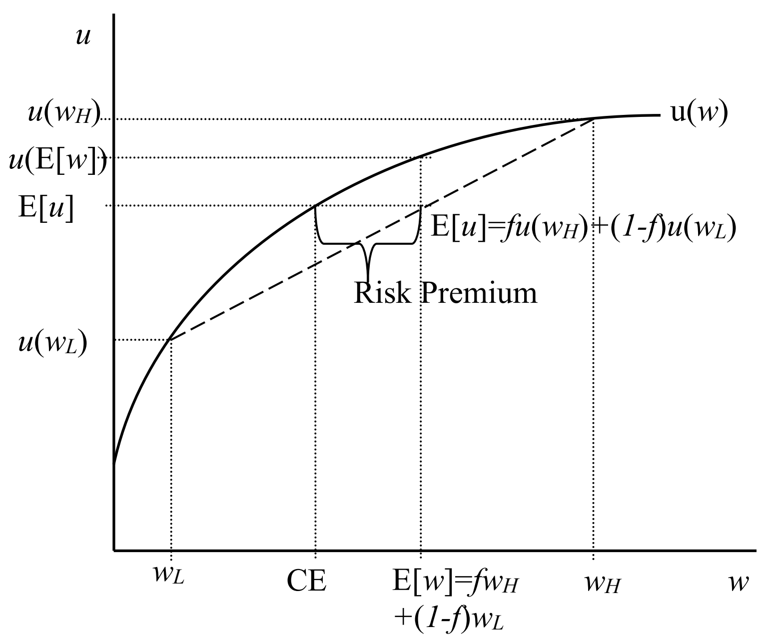
For example, EV of a 60% chance at \$100 is \$60.



Since we scaled the utility function to have a maximum of 100 and a minimum of 0, dashed line is the expected utility of gambles between 0 and 100 for different probabilities.

For example, the expected utility of a 60% shot at \$100 is 60. The utility of \$60 for sure is considerably higher – about 77.

Generalizes to any chord thru any utility function.



Important: For any expected payoff, the utility of that amount for certain is higher. The *CURVATURE* of the expected utility function is crucial!

Expected utility is not affected by scale, but, can only make transformations that are linear in nature, called "affine". Otherwise, would change the curvature

$u = a\sqrt{w} + b$ gives exactly the same preferences and CEs as $u = \sqrt{w}$.

$u = \left(\sqrt{w}\right)^2 = w$ does not give the same CE's as $u = \sqrt{w}$. It is risk neutral.

Risk aversion has to do with the curvature of the utility function. This is apparent from the graph – the curvature determines the difference between CEs and EVs. Thus, any utility function with the same shape, but any scale, represents the same preferences under uncertainty.

<u>Value of Insurance</u>

a) Pool together many people taking the same, independent, risks. Individuals will sell their gamble to the insurance company.

b) If enough sell, law of large numbers says risk becomes very small (not demonstrate this).

c) Insurance company will pay up to expected value, less administrative costs.

d) Individuals will accept certainty equivalent, less transactions costs.

e) Gains from trade between individuals and insurance company are E[w]-CE per insured. This is a gain in well being just as good as making N(E[w]-CE) more output – less administrative and transactions costs, off course.

If insurance industry is perfectly competitive, price of a policy is expected loss plus administrative costs – all gains to consumers

Problem on value of insurance industry!

Limitations of the Expected Utility Model

Allais Paradox

4 Possible Gambles: A, B, C, D

	Probabilities			
Prize	A	B	C	D
\$0mil	0	0.01	0.89	0.9
\$1mil	1	0.89	0.11	0
\$5mil	0	0.1	0	0.1

Expected Utility of Each

$$EU_A = U(1)$$
$$EU_B = 0.01U(0) + 0.89U(1) + 0.1U(5)$$
$$EU_C = 0.89U(0) + 0.11U(1)$$
$$EU_D = 0.9U(0) + 0.1U(5)$$

Comparing A and B, A usual choice:

$$A \succ B$$
$$EU_A > EU_B$$
$$U(1) > 0.01U(0) + 0.89U(1) + 0.1U(5)$$
$$0.11U(1) > 0.01U(0) + 0.1U(5)$$

Comparing C and D, D usual choice:

$$D \succ C$$
$$EU_D > EU_C$$
$$0.9U(0) + 0.1U(5) > 0.89U(0) + 0.11U(1)$$
$$0.01U(0) + 0.1U(5) > 0.11U(1)$$

Both can't be true. This is taken to be a violation of the independence axiom.

In both A and B, 89% chance of \$1. That 89% should not affect choice!

In both C and D, 89% chance of is \$0. That 89% should not affect choice!

Table removes shared probabilities

Other 11% is identical between A and C and B and D.

If A is chosen, C should be too.

	Shared Probabilities Removed			
Prize	A	B	C	D
\$0mil	0	0.01	0	0.01
\$1mil	0.11	0	0.11	0
\$5mil	0	0.1	0	0.1

Ellsberg Paradox

Urn with 60 chips. 1/3 Green (G), 2/3 mixed Orange (O) and Blue (B).

Choose Gamble I or Gamble II

I: \$1 if draw Green VS II: \$1 if draw Blue

Now, Choose Gamble III or Gamble IV

III: \$1 if draw Green or Orange vs IV: \$1 if draw Orange or Blue

Most choose I over II and IV over III. Contradicts subjective expected utility.

Pr(O): subjective prob of orange, Pr(B): subjective prob of Blue.

$EU_I=(1/3)u(1)$ $EU_{II}=Pr(B)u(1)$

$I \succ II$ iff $(1/3)>Pr(B)$

$EU_{III}=(1/3)u(1)+Ou(1)$ $EV_{IV}=Pr(B)u(1)+Pr(O)u(1)$

$IV \succ III$ iff $Pr(B)+Pr(O)>(1/3)+O$, or, $Pr(B)>(1/3)$

Thus, Pr(B)>1/3 and Pr(B)<1/3, contradiction! → Ambiguity Aversion?

Cornell Experiment and Endowment Effect

Cornell Experiment

Candy bars and mugs of equal value distributed randomly to large class after checking that preferences were equally divided

Followed by chance to trade - found too little trading

Endowment Effect

Different than just sentiment – occurs immediately

Sentiment is not “irrational” – it bears utility!

Coase, endowments, and, Transactions Costs

John List

Studied trades in collectible card markets

Little endowment effect with experienced/efficient traders

More present with less experienced traders

Perceived transactions costs can explain some novice behavior

Reasonableness and Uses of Expected Utility

There are 3 ways to think about this.

1) A guide to how decisions ought to be made under uncertainty. Idea: for big decisions, if you could work out your utility function, could help you make more rational calculations. Not of much interest for our purposes.

2) A description of how individuals make every decision in the face of uncertainty. Empirical and experimental evidence indicates this is false, but, the more decisions made in the face of uncertainty, and, the larger the stakes, the more exactly this model approximates behavior.

3) A **model** of how decisions are made under uncertainty that offers a close enough description to major market place decisions to be useful in modeling business decisions, not as a description of the thought process of every individual consumer and worker. This is how we will use it.

Alternatives. None yield tools as easy to use as expected utility.

I) Predictions better match behavior of experienced agents.

II) Takes only a few efficient traders for market prices to come close to model's predictions

11. Production and Cost

Half of profit is revenue, pq. Other half is –Cost, C(q)

Cost is the amount spent on inputs.

If x denotes inputs, p denotes prices, and, i indexes inputs,

$$Cost = p_1x_1 + p_2x_2 + ... + p_nx_n = \sum_{i=1}^{n} p_ix_i$$

The cost function, C(q), denotes the minimum possible cost of producing q units

- No waste
- Choose the lowest cost production process (characterized by the relative intensity of the use of capital, labor, and, other inputs)
- Economic Cost versus Accounting Cost
    - Economic cost includes all opportunity costs
    - Normal return on capital, value of owners time, etc…

Cost depends on prices of inputs and production relationships

- Detailed engineering relationships involved, but…
- We need a convenient framework for *thinking* about and *modeling* cost
- In reality, may need a team of engineers, accountants & economists to really get a handle on the cost

Production, Production Functions, and Input Substitution

General notation: $q = f(x_1, x_2, ..., x_n)$

Marginal Product: $\frac{dq}{dx_i}$

Input Substitution

Red and yellow pencils are prefect substitutes for filling out reports

Recipe calls for X ounces of sweetener per gallon of water

- Red and yellow pencils are *prefect substitutes* for filling out reports
- Water and sweetener may be *perfect complements* in recipes (fixed proportions)
- Generally, production relationships are somewhere in between, imperfect substitutes.
    - Example: workers and capital in digging a hole

Example: $q = 4L^{0.5}K^{0.5}$

Marginal products are:

$\frac{\partial q}{\partial L} = 2\left(\frac{K}{L}\right)^{0.5}$ and $\frac{\partial q}{\partial K} = 2\left(\frac{L}{K}\right)^{0.5}$

MRTS$_{\mathbf{LK}}$: The amount capital for which one unit of labor could be substituted while keeping the output level constant

Example q(L,K)

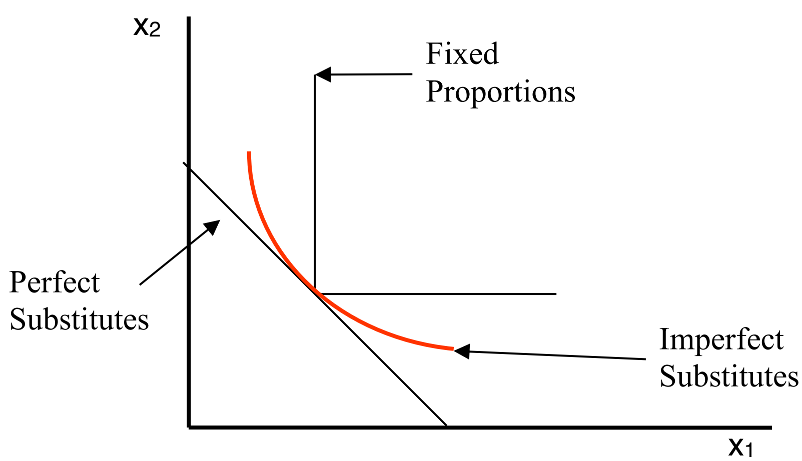
- Say MP_L=10 & MP_K=5. If $\Delta L = 1$, need $\Delta K = -2$ to keep q unchanged
- Can substitute one unit of labor for MP_L/MP_K units of capital,

$$MRTS_{ij} = \frac{MPi}{MPj}$$

- As L↑, MP_L↓, MP_K↑, hence, MRTS diminishes
- Generally: units of x_i required to replace a unit of x_j increases as x_i increases, keeping q constant

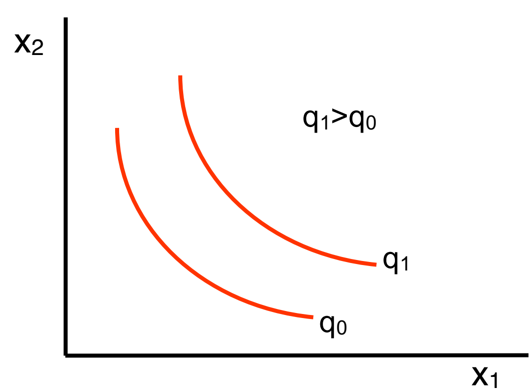
Isoquant – a curve showing combinations of inputs that can produce a given level of output (a level curve)

- If one x increases and all others do not decrease, what happens?
  - x increases → q increases
  - Isoquants slope down
  - Isoquants do not cross
- As more of one input is used, it takes more of it to replace a unit of the other input – diminishing marginal rate of technical substitution.



MRTS is slope of isoquant:

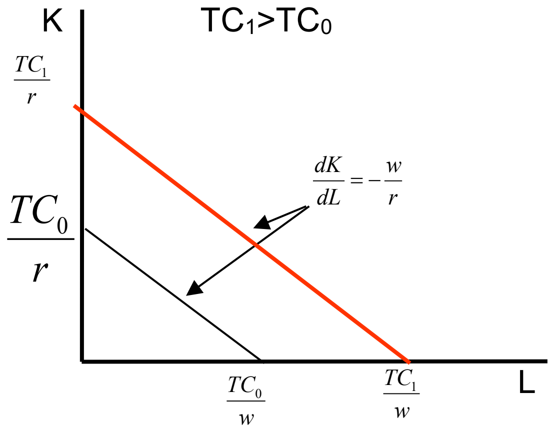
$$MRTS_{LK} = -\frac{dK}{dL} = \frac{MP_L}{MP_K}$$

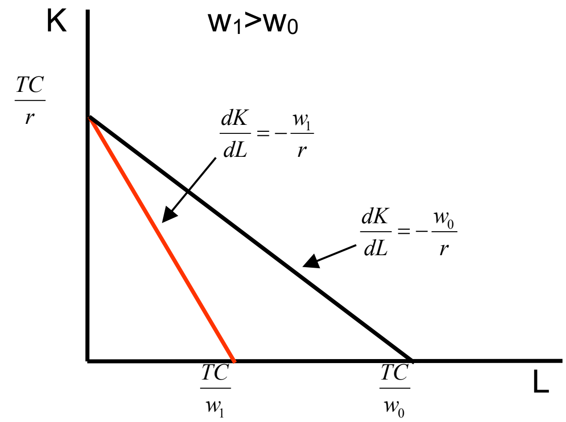


Minimizing Cost

- Isocost – line showing combinations of inputs that cost the same amount

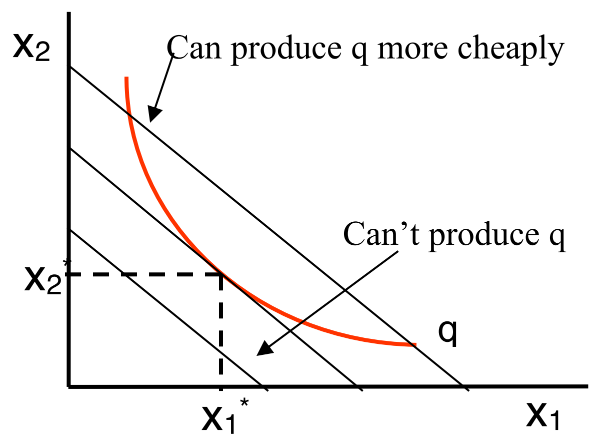
$TC = wL = rK$ or $K = \frac{TC}{r} - \frac{w}{r}K$





- The minimum possible cost of producing output q, is C(q)
- Graphically, find the lowest isocost that can reach the required isoquant
- Isoquant tangent to isocost, or,

$$MRTS_{ij} = \frac{P_i}{P_j} \qquad \frac{MP_i}{MP_j} = \frac{P_i}{P_j}$$



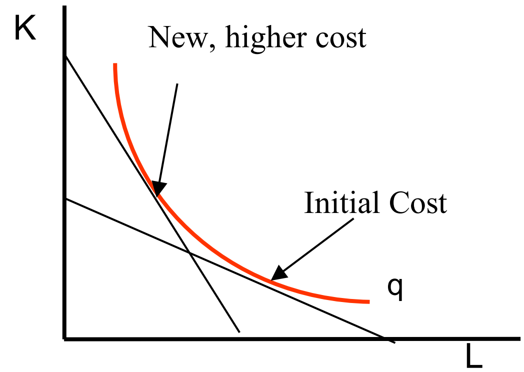
- Other interpretations:

$$\frac{MP_i}{P_i} = \frac{MP_j}{P_j} \qquad \frac{P_i}{MP_i} = \frac{P_j}{MP_j}$$

$$MC_i = MC_j$$

Comparative Statics

- What if w↑?
- MPL/w ↓ → K↑, L↓
- As shown
- What if MPK↑?



<u>Calculus of Cost Minimization</u>

2 Input Example: Minimize the cost of producing any number of units if w=20, r=5, and $q = 4L^{0.5}K^{0.5}$

Marginal products are:

$$\frac{\partial q}{\partial L} = 2\left(\frac{K}{L}\right)^{0.5} \text{ and } \frac{\partial q_A}{\partial K} = 2\left(\frac{L}{K}\right)^{0.5}$$

The optimization condition, $\frac{MP_L}{MP_K} = \frac{w}{r}$, gives:

$$\frac{K}{L} = 4 \text{ or } K = 4L$$

Substituting into the production function:

$$q = 4L^{0.5}(4L)^{0.5} = 8L \text{ so } L = q/8 \text{ and } K = q/2$$

NOTE: If asked for the cost of a specific q, say 40, you could just plug that number in for q here.

Then, substituting:

$$C(q) = wL + rK = 20\frac{q}{8} + 5\frac{q}{2} = 5q$$

General Approach (How programs handle optimization problems): Lagrangian

$\mathcal{L} = wL + rK + \lambda\left[q - f(K,L)\right]$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda MP_L = 0 \Rightarrow w = \lambda MP_L$$
$$\frac{\partial \mathcal{L}}{\partial K} = r - \lambda MP_K = 0 \Rightarrow r = \lambda MP_K$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = q - f(K,L) = 0$$

From the first 2:

Optimization Condition 1: $\frac{w}{r} = \frac{MP_L}{MP_K}$. Generally…

From the second:

Optimization Condition 2: $q = f(K,L)$. Generally…

If x* represents the levels of inputs that minimize cost, then

$$C(q; p_1, p_2, ..., p_n) = \sum_i p_i x_i^*$$

Example: Excel Solver

- Let's use excel solver to find the cost minimizing K & L for an example

  Assume: $q = L_s^{0.2} L_u^{0.25} K^{0.3} M^{0.2}$ w_s=30, w_u=10, r=5, P_M=5

Summary of Cost Minimization

- Choose input combination where "bang per buck" is equal for all inputs
- And, where production target is met
- Result is cost function, depends on q and input prices, $C(q, p_1, p_2, ..., p_n)$

Short Run Cost

- In SR, at least one input is fixed, e.g. plant size
- So, there are fixed costs
- Difference between Fixed Costs and Sunk Costs?
- Substitute fixed value for fixed inputs into production function
- Repeat other steps → SR cost minimizing input demands and SR cost
- Gives SR cost curves

Short Run Curves

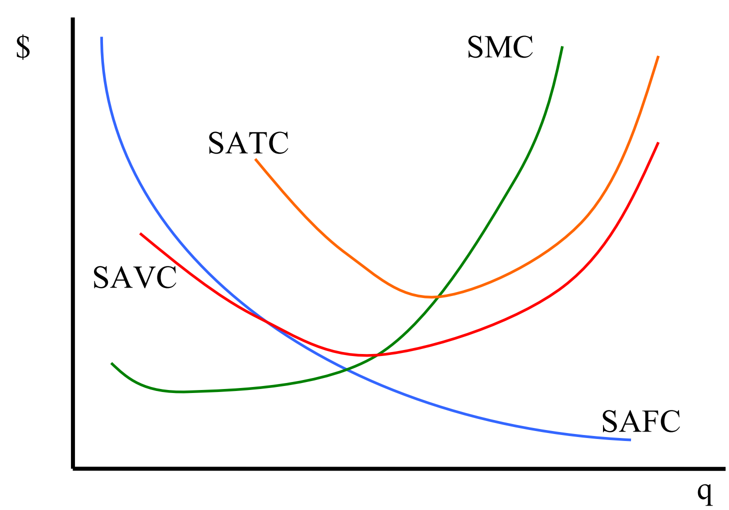
$SMC(q) = \frac{\partial STC(q)}{\partial q}$: Eventually rising with q due to diminishing returns to fixed factors, may (or may not) fall at low q

$SAVC(q) = \frac{SVC}{q}$: Eventually rising with q due to diminishing returns to fixed factors, may (or may not) fall at low q

$AFC_s(q) = \frac{F}{q}$: Falling with q as "overhead" is spread out

$SAC(q) = \frac{STC}{q}$ Tends to fall first with AFC, then, pulled up by rising MC

Where does MC cross AVC, ATC? Why?



2 Input Example in SR:

- K fixed at 4, r = 5, w=20, $q = 4L^{0.5}K^{0.5}$
- With only one variable input, algebra is trivial

$$q = 4L^{0.5}K^{0.5}$$
$$q = 4\left(4^{0.5}\right)L^{0.5}$$
$$q = 8L^{0.5}$$
$$L^{0.5} = \frac{q}{8}$$
$$L = \frac{q^2}{64}$$

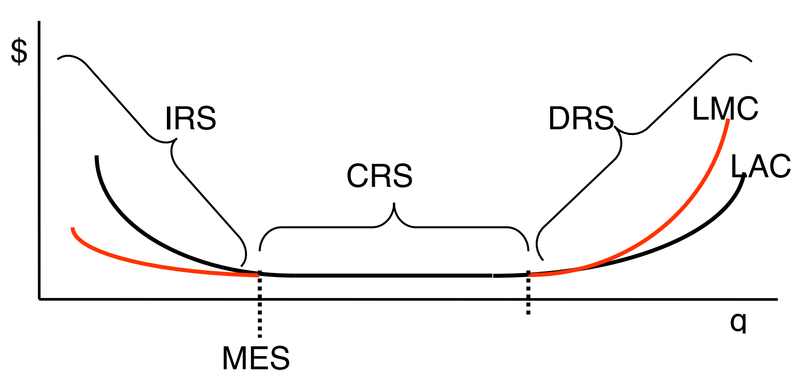
$$STC = 20\frac{q^2}{64} + 5(4) = \frac{5}{16}q^2 + 20$$
$$SMC = \frac{5}{8}q$$
$$SATC = \frac{5}{16}q + \frac{20}{q}$$

$$SAVC == \frac{5}{16}q$$
$$SAFC = \frac{20}{q}$$

Long Run Cost Curves

- No fixed fixed factors or fixed costs - choose optimal plant for each q
- Economies and Diseconomies of Scale (IRS, CRS, DRS)
- Replication argument?
- ATC may decline throughout relevant range (depends on market demand)
- Ability to manage huge organizations may be limited
- Difficult to transmit information in large organizations
- AC may decline then rise
- Where does MC cross AC?

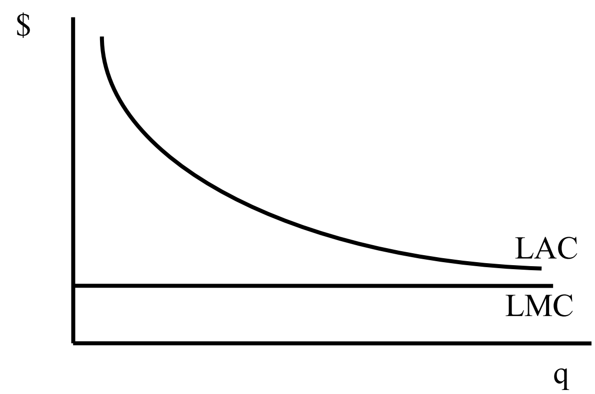
“Typical” LR Cost Curves: (IRS, Range of CRS, DRS)



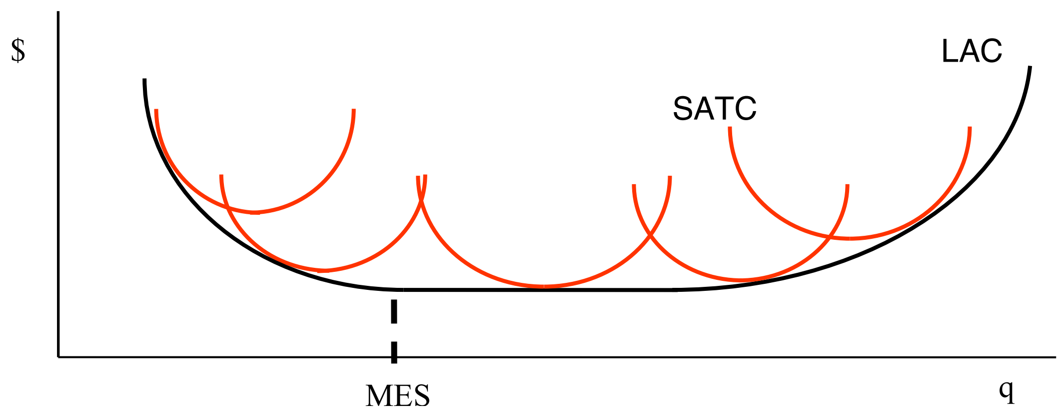
Relevant portion depends on scale of market demand!!!

Economies of Scale Throughout Range of Demand is Possible:

EX: C=F+cq where F is simply not yet “sunk”



<u>Relation of SR to LR Cost Curves</u>



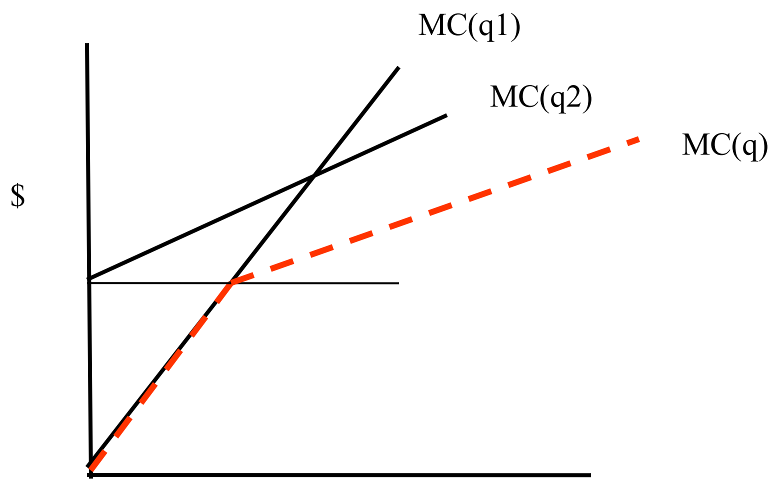
LAC is lower envelope of SATC. WHY is SATC higher except at one quantity???

Economies of Scope: When cost of producing 2 outputs jointly is less than the cost of producing them separately

Cost Complimentarity: When producing more of one good lowers the marginal cost of the other good, more than just sharing of fixed costs

Multi-plant Cost Minimization

- Veronica produces thingamabobs in 2 plants
- One has a high fixed cost, but, a low marginal cost
- One has little fixed cost, but, high marginal cost
- How to minimize total cost?
- If MC in first plant is above MC in the second, how could cost be reduced for the same output?
- Overall MC is the horizontal sum of the individual plant MCs
- Add up quantity in each plant at each MC



- Must be worth bringing both plants online – fixed costs versus sunk costs
- If q is small enough, want to produce in plant with lowest start up cost only
- If q is large enough, want to use both
- In the middle, may use only plant with lowest MC if both have start up costs

Example 3: Managing Cost with Multiple Plants

Suppose $C(q_1) = 20 + .25q_1^2$
$C(q_2) = 10 + q_2^2$

Should production occur in plant 1, 2, or, both?
Must just compare the possibilities.

If both plants are used: $MC_1(q_1) = .5q_1$
$MC_2(q_2) = 2q_2$

Setting MCs equal:

$$q_1 = 4q_2$$
$$q_1 + q_2 = 5q_2$$
$$q_2 = .2q$$
$$q_1 = .8q$$
$$C(q) = 30 + .25(.8q)^2 + (.2q)^2$$
$$C(q) = 30 + .2q^2$$

If all fixed costs are already sunk, use both as long as MC is not always lower in one or the other for the total quantity.

NOTE: If marginal cost is constant in one plant, this may not be the case!

Suppose instead fixed cost is incurred only if q>0
Plant 2 cheapest at low q, because start up cost is smallest.
For example, suppose q=2.
Only plant 1, C=20+0.25(4)=21,
only 2, C=10+4=14.
Both, C=30+.2(4)=30.8

If a lot produced, use both. Example, q=30
Only 1: C=20+0.25(900)=245
Only 2: C=10+900=910
both: C=30+0.2(900)=210

It is possible (but not certain) that in between, it may make sense to use just plant 1 for the lower marginal cost while still avoiding the fixed cost of plant 2.
Example, q=10
Only 1: C=20+0.25(100)=45
Only 2: C=10+100=110
Both: C=30+0.2(100)=50

Beware, cost may be sunk in one plant, but, not the other.

12. ONE SHOT GAMES WITH DISCRETE STRATEGIES

• One player, or many small players, no strategic interdependence

• With a small number of players, each realizes their actions have non-negligible impacts on others, strategic interdependence is important

• Examples – airline price wars, where to locate relative to competitors, whether to slack or not

Game Theory: What are "reasonable ways to play" in situations with strategic interaction

• Types of Games
  - – Simultaneous or Sequential Move
    - • Are the other players' actions known when making decisions?
  - – One Shot or Repeated
    - • Will the same interaction take place more than once?

Representing Games

• Building Blocks
  - – List of players
  - – List of decision nodes and choices available at each node
  - – Payoffs for every outcome (what they are playing for)

• Strategy
  - –Rule giving the decision to be made at every decision node at which a player gets to make a decision
  - – Even very simple games can have very many strategies

One Shot Simultaneous Move Games

Prisoners Dilemma

- 2 partners under interrogation, David and Mike. Enough evidence to convict of a minor charge. Only convict of major charge if one testifies against the other. Drop minor charge in exchange for testimony.
- Each can rat or not.
- Payoffs:
  - – If both rat, both get 24 months, serious charge, lesser dropped for coop.
  - – If neither rats, both get 6 months for lesser charge
  - – If one rats and the other does not, the rat gets 0 months and the other gets 30.
- Strategies: – Rat or Don’t

Interpretation as hard or soft competition – price war

Prisoner's Dilemma

		David	
		Rat	Don't
Mike	Rat	M: 24 | D: 24	M:0 | D:30
	Don't	M:30 | D:0	M:6 | D:6

Solution Concepts

- How would players play, if they max profit, or, utility?

Dominant Strategies

- – “Rat” is Strong Dom Strategy for both players in prisoner’s dilemma
- – Weak Dominance
- – All games don’t have a solution with only dominant strategies

Solution Concepts – Iterated Dominance

Look at Simultaneous Entry Game

- Entrant has no dominant strategy
- Incumbent does – Don't
- What does entrant do if he can count on incumbent not expanding?
- General application of iterated dominance

Simultaneous Entry Game

		Entrant: In	Entrant: Out
Incumbent	Expand	I: 20 E: -20	I:50 E:0
	Don't	I: 30 E: 20	I: 80 E:0

Solution Concepts – Secure Strategy

- Secure, Guarantee, or, Maxi-Min Strategies
- Example – Battle of the sexes
- A "Coordination Game"
- Play "Safe" for boy
- Game "Safe" for girl
- What would girl do if she thinks boy will play safe?

"Safe" play may not be consistent with rational play

Battle of the Sexes

		Boyfriend: Play	Boyfriend: Game
Girlfriend	Play	G:2 B:3	G:0 B:0
	Game	G:1 B:1	G:3 B:2

Solution Concepts – Nash Equilibrium

• Best Response – strategy with the highest payoff for a given strategy of opponent

• Reaction function

  – Rule giving best response to every possible strategy of opponent

It is reasonable to expect everyone to play a best response to what they expect their opponents to play

It is reasonable to expect opponents to only play best responses

So, any reasonable solution involves BOTH players playing best responses to the other's strategy

NE: Intersection of reaction functions – strategies that are best responses to each other

Look at 3 games already on board and mark best responses for each player

– Prisoner's Dilemma: NE, both Rat.

– Entry game: 2 NE. Expand, Don't and Don't Exp, Enter.

– Battle of the Sexes – 2 equilibria. Both to game, both to play.

If all players have a strongly dominant strategy, there is only 1 NE, it is also the "secure" strategy. EX: Prisoners Dilemma

Otherwise, there MAY be more than 1 NE. What is the "reasonable" way to play?

Focal points and Social Norms

Solution concepts may not agree.

Entry game & battle of sexes, intersection of safe strategies not equilibrium.

Mixed Strategy Equilibria

- This game has no pure strategy equilibrium.
- Nash proved there is always at least one equilibrium.
- Look for a “mixed strategy” equilibrium.
- Alex chooses a probability of checking, f_C.
- Sarah chooses a probability of hard, f_H.

Monitoring Game			
		Sarah	
		Hard	Shirk
Alex	Check	A:30 S:20	A:30 S:0
	Don't	A:40 S:20	A:10 S:30

If one player is too predictable, the other will exploit that.

Each is unpredictable only if the other is unpredictable.

The only “solution” is when each is unpredictable enough.

A player is willing to be unpredictable ONLY if payoffs are equal:

$$E(\pi_A \mid Check) = E(\pi_A \mid Don't)$$
$$f_H 30 + (1 - f_H)30 = f_H 40 + (1 - f_H)10$$
$$30 = 10 + f_H 30$$
$$f_H = \frac{2}{3}$$

$$E(\pi_S \mid Hard) = E(\pi_A \mid Shirk)$$
$$f_C 20 + (1 - f_C)20 = f_C 0 + (1 - f_C)30$$
$$20 = 30 - f_C 30$$
$$f_C = \frac{1}{3}$$

(This kind of game is the only type where you may be worse off for moving first.)

Sequential Move Games

Normal Form Reasoning

Entry Game - assume incumbent moves first.

The earlier representation is NOT CORRECT for the sequential game.

The entrant has ***4*** possible strategies.

Imagine entrant writes down strategy, conditional on what incumbent does, for an employee to follow then leaves town.

	Incumbent's Decision	
	expand	don't
Entrant's decision	in	in
	in	out
	out	in
	out	out

A Correct Representation

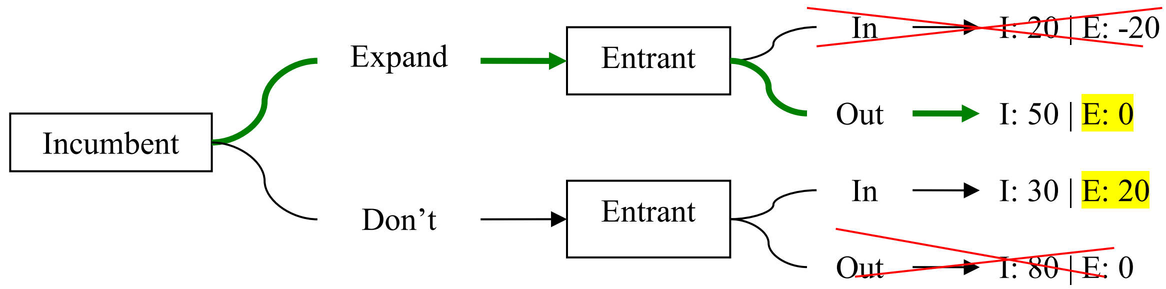
Sequential Entry Game					
		Entrant			
		I,I	I,O	O,I	O,O
Incumbent	Expand	I:20 E:-20	I:20 E:-20	I:50 E:0	I:50 E:0
	Don't	I:30 E:20	I:80 E:0	I:30 E:20	I:80 E:0

2 Nash Equilibria: Expand; O,I and Don't; I,I.

Entrant plays a weakly dominant strategy in the first

Is the second reasonable? It involves ***non-credible threat***

Extensive Form Reasoning – Sequential Move Games



Use Backward Induction or find "rollback" equilibria

One such equilibrium: Expand; O,I

This is also the subgame perfect Nash Equilibrium

Definition of spne

Ex: Suppose entrant plays I,I.

The best response to that is for the incumbent to not expand.

So, Don't | I,I is a NE.

BUT, involves entrant making a non-optimal choice at the sub-game where the incumbent does expand. Not rational at that point, so, not credible.

But, I,I is only weakly dominated, if the incumbent does not expand, the entrant never has to decide whether or not to deliver on threat.

Timing Advantages

There is a first mover advantage for the incumbent.

Incumbent can preempt entry if moves first.

- What if entrant moves first?
- COMMITMENT IS HUGE!
  - – Tying own hands. Sometimes higher payoff if perceived as "crazy". This theme will recur!

Sometimes there is a second mover advantage.

Only time first MIGHT be WORSE off is if no PSNE, only mixed.

Just depends on the details.

Many Strategies

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Hannah</td></tr>
<tr><td>Left</td><td>Middle</td><td>Right</td></tr>
<tr><td rowspan="3">Ray</td><td>Top</td><td>0 , 10</td><td>10 , 0</td><td>30 , 20</td></tr>
<tr><td>Middle</td><td>10 , 20</td><td>20 , 20</td><td>0 , 10</td></tr>
<tr><td>Bottom</td><td>30 , 10</td><td>10 , 20</td><td>10 , 0</td></tr>
</table>

2 equilibria. Is one more likely?

More than 2 players possible also. With three players, third chooses table

Review of One Shot Games

Representations – Normal best for simultaneous – Extensive best for sequential

Solution concepts – Dominant strategies, iterated dominance, “safe”, NE

Multiple Equilibria

Mixed Strategies

Subgame perfection, non credible threats, commitment

13. ONE SHOT GAMES WITH CONTINUOUS STRATEGIES

Price, Quantity, Advertising Expenditure, all examples

Idea remains the same: Given guess of opponent's choice, each player chooses own best response.

Example – Advertising – Simultaneous Move

Firms: Kevin and Morgan.

Prices predetermined (MSRP), p-MC=10

Quantity sold depends on advertising, a_K and a_M

$$q_K = 10 + 0.5a_K - 0.25a_M - 0.005\left(a_K + \alpha a_M\right)^2$$
$$q_M = 10 + 0.5a_M - 0.25a_K - 0.005\left(a_M + \alpha a_K\right)^2$$

Interpret, especially "alpha", why positive? why negative?

Kevin's profit net of advertising is:

$$\pi_K = 10\left(10 + 0.5a_K - 0.25a_M - 0.005\left(a_K + \alpha a_M\right)^2\right) - a_K$$

Maximizing

$$\frac{d\pi_K}{da_K} = 10\left(0.5 - 0.01\left(a_K + \alpha a_M\right)\right) - 1 = 0$$

That is, the ***Net Marginal Benefit***, NMB, is 0

OR, MB=MC

$$5 - 0.1a_K - 0.1\alpha a_M = 1.$$

Solving NMB=0 gives Kevin's choice as a function of what he expects Morgan to choose. That is, Kevin's reaction function:

$$a_K = R_K(a_M) = 40 - \alpha a_M$$

Slope is

$$\frac{da_K}{da_M} = \frac{dR_K}{da_M}(a_M) = -\alpha$$

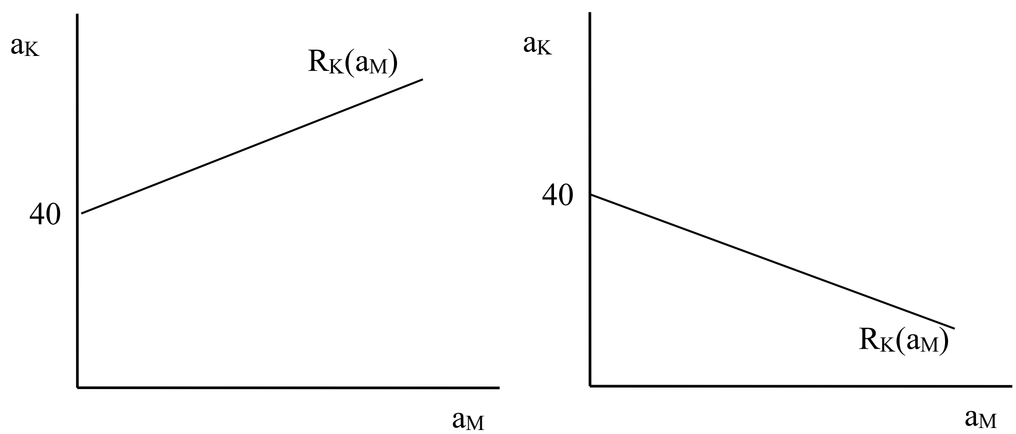
The slope of a reaction function has important implications for strategic interaction

If a_M increases NMB_K, $a_K\uparrow$ as $a_M\uparrow\downarrow$.

If a_M decreases NMB_K, $a_K\downarrow$ as $a_M\uparrow$.

Impact of Morgan's advertising on the NMB of Kevin's advertising:

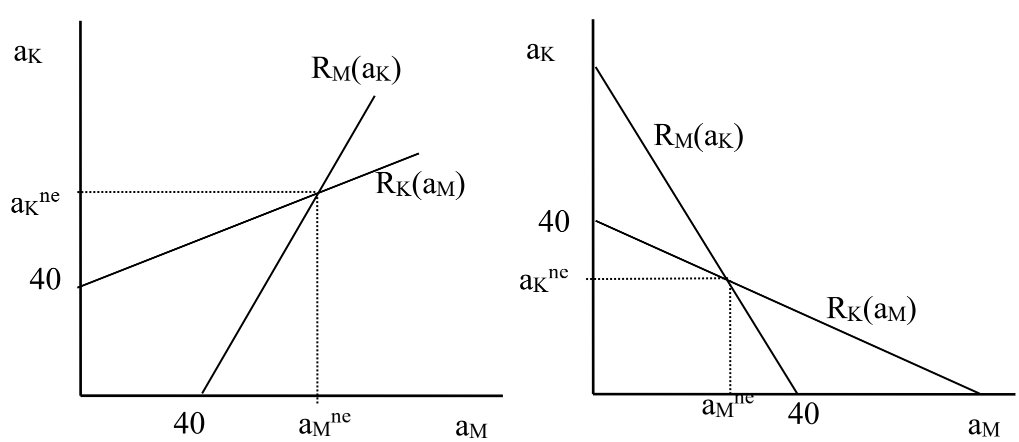
$$\frac{dNMB_K}{da_M} = -10\left(0.01\alpha\right) = -0.1\alpha$$



Morgan's reaction function is found the same way

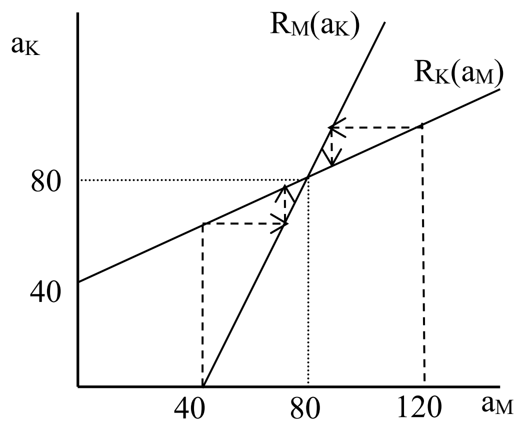
$$a_M = R_M(a_K) = 40 - \alpha a_K$$

Nash Equilibrium is where they cross:



Assume α=-0.5 and solve for NE

$$R_K(a_M) = 40 + 0.5a_M$$
$$R_M(a_K) = 40 + 0.5a_K$$
$$a_K = 40 + 0.5(40 + 0.5a_K)$$
$$= 60 + 0.25a_K$$
$$0.75a_K = 60$$
$$a_K = 80$$
$$a_M = 40 + 0.5(80) = 80$$

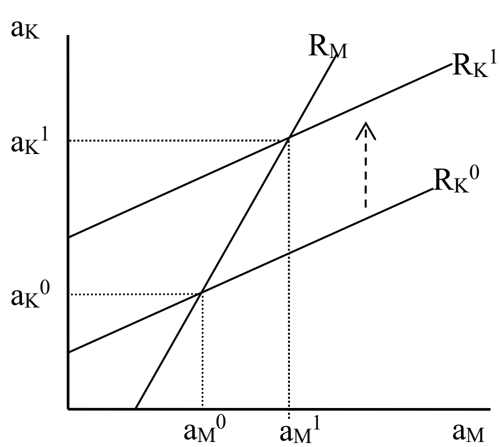


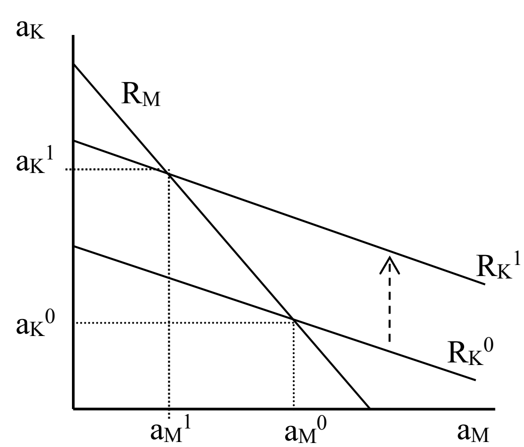
Solving using symmetry instead:

$$10\big(0.5 - 0.01(a + 0.5a)\big) - 1 = 0$$
$$5 - 0.05a - 1 = 0$$
$$0.05a = 4$$
$$a = 80$$

Advertising is higher and profit lower than if the two cooperated. Why?

Comparative Statics – Change in Marginal Production Cost





Example – Advertising – Sequential Move

Suppose Morgan moves first. Would she advertise more or less compared to simultaneous move NE?

Direct effect of a small change in her advertising is 0, since NMB=0.

But, if she can get Kevin to advertise less, it would boost her profit

If reaction fns slope up, Kevin advertises less of she advertises less

If reaction fns slope down, Kevin advertises less of she advertises more

Solve example, again assume α=-0.5.

Morgan knows Kevin will maximize his profit given her advertising

So, she knows he will be on his reaction function: $a_K = R_K(a_M) = 40 - \alpha a_M$

Substituting that into her profit function:

$$\pi_M = 10\left(10 + 0.5a_M - 0.25\left(40 + 0.5a_M\right) - 0.005\left(a_M - 0.5\left(40 + 0.5a_M\right)\right)^2\right) - a_M$$

Simplifying

$$\pi_M = 10\left(0.375a_M - 0.005\left(0.75a_M - 20\right)^2\right) - a_M$$

Maximizing

$$\frac{d\pi_M}{da_M} = 10\left(0.375 - 0.01\left(0.75a_M - 20\right)0.75\right) - 1 = 0$$

$$3.75 - 0.1\left(0.75a_M - 20\right)0.75 - 1 = 0$$

$$a_M = 75.56$$

Continuous Strategies – General

In general $\pi_A(x_A, x_B)$ and $\pi_B(x_B, x_A)$

Simultaneous

Each player optimizes, given the other's strategy:

$$\frac{\partial \pi_A}{\partial x_A} = NMB_A(x_A, x_B) = 0 \text{ and } \frac{\partial \pi_B}{\partial x_B} = NMB_B(x_B, x_A) = 0$$

Solving these gives the reaction functions:

$x_A^R(x_B)$and $x_B^R(x_A)$ or $R_A(x_B)$ and $R_B(x_A)$

Works the same for N players

Determining the slope pf the reaction function:

Depends on the effect of B's strategy on NMB_A. If B's strategy increases (decreases) NMB_A, A's reaction function slopes up (down)

Show graphically

Uses for comparative statics

Sequential

Assume B moves first

$$\pi_B(x_B, x_A^R(x_B))$$

$$\frac{d\pi_B}{dx_B} = \frac{\partial \pi_B}{\partial x_B} + \frac{\partial \pi_B}{\partial x_A}\frac{dx_A}{dx_B} = 0$$

So, B takes account of direct effect and the effect of their choice on A

NOTE: $\frac{\partial \pi_B}{\partial x_B} \neq 0$

B's strategy will be above/below the level where $\frac{\partial \pi_B}{\partial x_B} = 0$ depending both on the slope of A's reaction function and whether B wants A to do more or less x_A.

14. REPEATED GAMES

Meet more than once

Is it possible to get cooperation, or, sustain other outcomes that are not rational in a one shot game?

Cooperation in Repeated Games

Prisoner's Dilemma (hard or soft price competition)

- Known final period, T
- No reputation effects
    - is cooperation possible?
- Players play their best responses in the last period – no reason not to!
- So, no reason to cooperate at T-1
- So, no reason to cooperate at T-2
- ….. cooperation not possible

Price Competition			
		David	
		Hard	Soft
Mike	Hard	M: 0 D: 0	M: 15 H: -10
	Soft	M: -10 D: 15	M: 10 D: 10

• Is it possible to guarantee giving a surprise quiz?

Infinitely Repeated Games

• Two Interpretations
  – Really Never Ends
  – Never sure when it will end

• f: probability the game ends after any period

• r: discount rate

• With an uncertain end period, "trigger strategies" may sustain cooperation
  – Tit for tat
  – Grim Trigger

Trigger Strategies and Cooperation

- Suppose f=0.12, r=0.1
- Consider David's best response to "Grim Trigger" by Mike (Mike cooperates as long as David cooperates, then never again)
- If David plays "Grim Trigger"

$$E[\pi \mid coop] = 10 + \frac{(1-0.12)}{(1+0.1)}10 + \frac{(1-0.12)^2}{(1+0.1)^2}10 + \ldots$$

$$E[\pi \mid coop] = 10\sum_{t=0}^{\infty}\left(\frac{0.88}{1.1}\right)^t = 10\sum_{t=0}^{\infty}0.8^t$$

Some Very useful Math Facts:

$$\sum_{t=0}^{\infty} a^t = \frac{1}{1-a} \qquad \text{So: } \sum_{t=1}^{\infty} a^t = \sum_{t=0}^{\infty} a^t - a^0 = \frac{1}{1-a} - 1 = \frac{1-1+a}{1-a} = \frac{a}{1-a}$$

Aside: perpetuity formula: a=1/(1+r)

$$\sum_{1}^{\infty}\left(\frac{1}{1+r}\right)^t = \frac{\frac{1}{1+r}}{1-\frac{1}{1+r}} = \left(\frac{1}{1+r}\right)\left(\frac{1}{\frac{1+r-1}{1+r}}\right) = \frac{1}{r}$$

So: $E[\pi \mid coop] = 10\left(\frac{1}{1-0.8}\right) = 10(5) = 50$

If David "cheats"

$$E[\pi \mid cheat] = 15 + \frac{(.88)}{(1.1)}0 + \frac{(.88)^2}{(1.1)^2}0 + \ldots = 15$$

50>15

Same holds for Mike - symmetric

So, cooperation possible!!!!

General Version

• Players A and B

• π_{coop}: per period payoff if play "agreed upon" cooperative strategy – was 10

• π_{cheat}: one time payoff if one "cheats" by playing best one period response to other's cooperative play – was 18

• π_{ne}: per period payoff in the Nash Equilibrium to the one shot game – was 0

• Consider A's best response to "Grim Trigger" by B

(B cooperates as long as A cooperates, then never again)

• If A plays "Grim Trigger"

$$E[\pi \mid coop] = \pi_{coop} + \frac{(1-f)}{(1+r)}\pi_{coop} + \frac{(1-f)^2}{(1+r)^2}\pi_{coop} + ... = \pi_{coop}\sum_{t=0}^{\infty}\left(\frac{(1-f)}{(1+r)}\right)^t$$

$$\sum_{t=0}^{\infty} a^t = \frac{1}{1-a} \Rightarrow \sum_{t=0}^{\infty}\left(\frac{1-f}{1+r}\right)^t = \frac{(1+r)}{(r+f)}$$

• So, $E[\pi \mid coop] = \frac{(1+r)}{(r+f)}\pi_{coop}$

• If A "cheats"

$$E[\pi \mid cheat] = \pi_{cheat} + \frac{(1-f)}{(1+r)}\pi_{ne} + \frac{(1-f)^2}{(1+r)^2}\pi_{ne} + ...$$

$$E[\pi \mid cheat] = \pi_{cheat} + \frac{(1+r)}{(r+f)}\pi_{ne} - \pi_{ne}$$

• So, cooperation possible if

$$\frac{(1+r)}{(r+f)}(\pi_{coop} - \pi_{ne}) > \pi_{cheat} - \pi_{ne}$$

Warning: one shot equilibrium play always an equilibrium to the repeated game!

Monitoring, Enforcement, and, Punishment

- In reality, may have to expend resources to monitor opponents strategy
- May observe opponent's play with noise. Opponent may make a mistake, or, random events may intervene.
- So, likely want some scope for forgiveness, or, trapped in non-cooperation
- But, if too forgiving, opponent will cheat often, knowing you will forgive them. Must demand repayment.
- If demand too much repayment, opponent will not be willing to bear it though.
- If you designed strategy right, know partner would not cheat!
- But, if trigger for punishment is tripped by random event, must still punish! That hurts the punisher too!
- Must deliver punishment, anyway, or, everyone will cheat!

Factors Impacting Likelihood of Cooperation

r: higher interest, coop less likely

f: less future, coop less likely

noise: more likely to get punished, less gains from coop

players: more players, monitoring is more expensive, gains to coop split more
    ways, coop less likely
    Ex: splitting monopoly profit over n players

Reputation Games

• *"Recompense injury with justice, and recompense kindness with kindness."* (Confucious)

• *"Life shall go for life, eye for eye, tooth for tooth"* (Deuteronomy 19:21)

• *"An honest man's word is as good as his bond"* (John Ray's English Proverbs, 1670)

• Some players may be "crazy", or, "honest"

• Rational players may play "crazy" to keep other players from discovering for certain that they are not "crazy", "tough", or, "honest"

<u>Cooperation in Reparation Games – Making Promises Credible</u>

Consider this expanded prisoner's dilemma:

Expanded Prisoner's Dilemma				
		Player B		
		Soft	Medium	Hard
Player A	Soft	10 ; 10	-5 ; 15	-10 ; 0
	Medium	15 ; -5	0 ; 0	-5 ; -5
	Hard	0 ; -10	-5 ; -5	-10 ; -10

Suppose the game is played repeatedly with a known end time

If all rational, cooperation not possible, - unraveling

Medium is dominant strategy NE

f: probability matched with a "crazy" opponent

"crazy" play soft until opponent does not play soft, then, plays Hard (at least one time, could play medium thereafter)

In the final period, we know "sane" players will play medium.

Is it a N.E. for "sane" players to play "Soft" before the last period? Partially mimicking "crazy" players?

Compare "Soft, Medium" in last 2 periods to "Medium, Medium" for one same player, *ASSUMING* other sane players play S,M

$$E(\pi \mid S,M) = 10 + \frac{f}{1+r}15 + \frac{1-f}{1+r}0 \qquad E(\pi \mid M,M) = 15 + \frac{f}{1+r}(-5) + \frac{1-f}{1+r}0$$

Sane playing S,M is an equilibrium, *IF*:

$$10 + \frac{f}{1+r}15 \geq 15 - 5\frac{f}{1+r}$$

$$\frac{f}{1+r}20 \geq 5$$

$$\frac{f}{1+r} \geq \frac{1}{4}$$

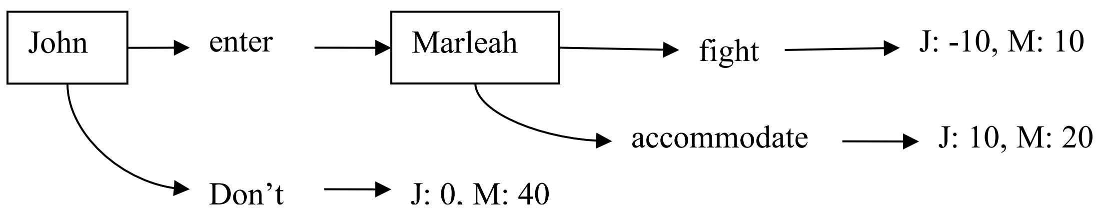
For low r, in this example, f need only be a bit larger than 0.25.

The higher the "punishment", the lower f need be

So, may cooperate early to get a reputation as a "cooperator", but, will "milk" that reputation later on and take advantage of any "crazy" players

Reputation Games - Making threats credible

Fight or Accommodate (entry game, entrant goes first)



• First, no “crazy” players, so, no reputation effects

• Sub-game Perfect Nash Equilibrium is: Enter ; Accommodate

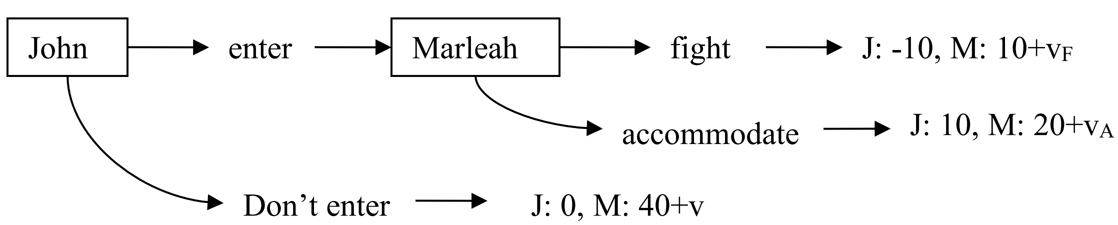
• Now, assume Marleah will play many times against many different opponents

• f: probability Marleah is “crazy” - always fights

• v_F: Value of reputation if sane Marleah fights: may converge to (40)/r here

• v_A: Value of reputation if sane Marleah accommodates: may converge to 20/r

• v: Value of reputation if sane Marleah doesn’t fight or accommodate



• Sane Marleah fights if:

$$10 + v_F > 20 + v_A$$
$$v_F - v_A > 10$$
$$20 / r > 10?$$
$$2 > r?$$

• g: John's assessment of the probability Marleah is sane and fights anyway

• g, v, v_A, and v_F depend on f, other payoffs, r, and how long Marleah will play

• John does not enter if:

$$(1 - f - g)10 + (f + g)(-10) < 0$$
$$1 - (f + g) + (f + g) < 0$$
$$f + g > \frac{1}{2}$$

If Marleah plays a long time and the discount rate is moderate, g and vF may be large even if f is small (We will not prove this.)

Rep may work exactly the same way if there are no crazy players but other player's don't know when she will "retire"

She will "milk" her rep near retirement though!

Value of a Reputation

Rep worth less as the end draws near

Rep must be expensive to acquire in order to be valuable

Otherwise, no one will believe you will sacrifice to keep it.

Part 5: Market Structure

- Environment a firm operates in governs usefulness of particular strategies
- To predict effects of changes in another industry on your firm, must understand the nature of that industry

Pure Monopoly

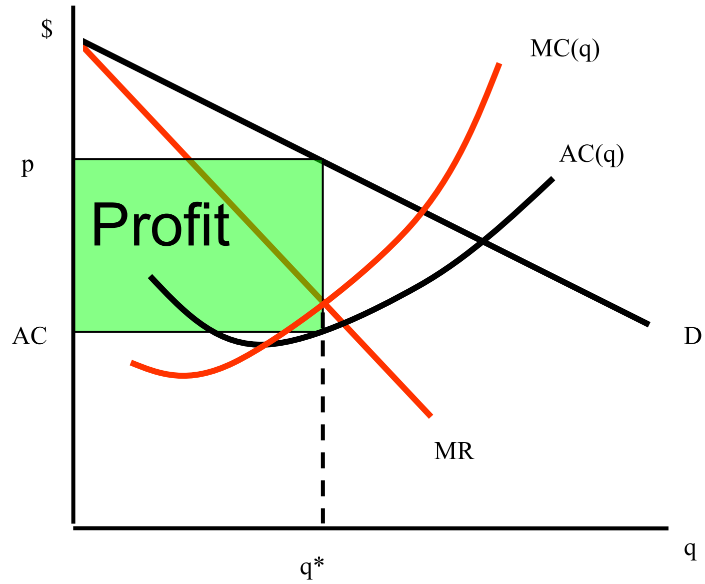
- Simplest market structure, just maximize profit

Remember:

$$MR = p + \frac{dp}{dQ}Q = p\left(1 + \frac{1}{\eta}\right)$$

(basis for running example)

$p = 20 - 0.25Q$, $k = 3$, $c = 2$



Example 1: (basis for running example) $p = 20 - 0.25Q$, $k = 3$, $c = 2$

$$\pi = (20 - 0.25Q)Q - 5Q$$
$$20 - 0.5Q - 5 = 0$$
$$Q = 30$$
$$p = 20 - 30/4 = 12.5$$
$$\pi = 30(12.5 - 5) = 225$$

Having a monopoly does not guarantee profit

- Market can dry up, cost may be high
- Entry may occur – eroding profits
- Barriers to entry are necessary to maintain monopoly
- extreme scale economies, other cost advantages, legal restrictions, patents

15. HOMOGENOUS PRODUCT MARKETS

Homogenous Product Price Competition - Oligopoly

- Marianna & Lindsay are the only two manufacturers of widgets.
- Inverse demand $p(Q)=p(q_M+q_L)$
- They each announce price & meet all demand
- Constant marginal cost, \$c
- Since price is the only difference, all customers buy from whoever is cheapest
- Marianna's best response is to undercut Lindsay slightly
  - (ε is the smallest noticeable amount)

$$R_M(p_L)=\begin{cases} p_L-\varepsilon & \text{if} \quad p_L>c+\varepsilon \\ c+\varepsilon & \text{if} \quad p_L=c+\varepsilon \\ c & \text{if} \quad p_L=c \end{cases}$$

- $p=c+\varepsilon$ or $p=c$ for both firms in the Nash Equilibrium

Price is driven to unit cost in the equilibrium, with only 2 firms!

Often called "Bertrand"

Is this reasonable? Can it be softened?

- Collusion, implicit or explicit, may soften this
- How else?

In NE, each has half the market.

To undercut price and meet all demand, must have capacity to serve entire market

Would anyone build that much capacity when NE is to serve half the market?

Homogenous Product Oligopoly with Capacity Limits

- Suppose both firms M and L must first invest in capacity (q-bar) in period 1, play pricing game in period 2
- Cost of capacity per unit: *k*
- In second period, k times q-bar is fixed cost, so, total cost is:

$$C(q) = \begin{cases} k\bar{q} + cq & if \quad q \leq \bar{q} \\ \infty & if \quad q > \bar{q} \end{cases}$$

- Will not build capacity do not intend to use (unless there are other concerns)
- So, in second period, q=q-bar and $p = p(\bar{q}_M + \bar{q}_L)$
- Looking forward from the first period, Marianna's profit function is:

$$\pi_M = p(\bar{q}_M + \bar{q}_L)\bar{q}_M - k\bar{q}_M - c\bar{q}_M$$

- NOTE: Profit now depends only on capacities chosen in the first period!
- As if the firms compete on the capacities, or quantities they plan to sell!
- If capacity limits are important (significant lead time), model homogenous product oligopoly as if compete on QUANTITY (capacity or plant size) NOT price directly.
- Called "Cournot" competition.

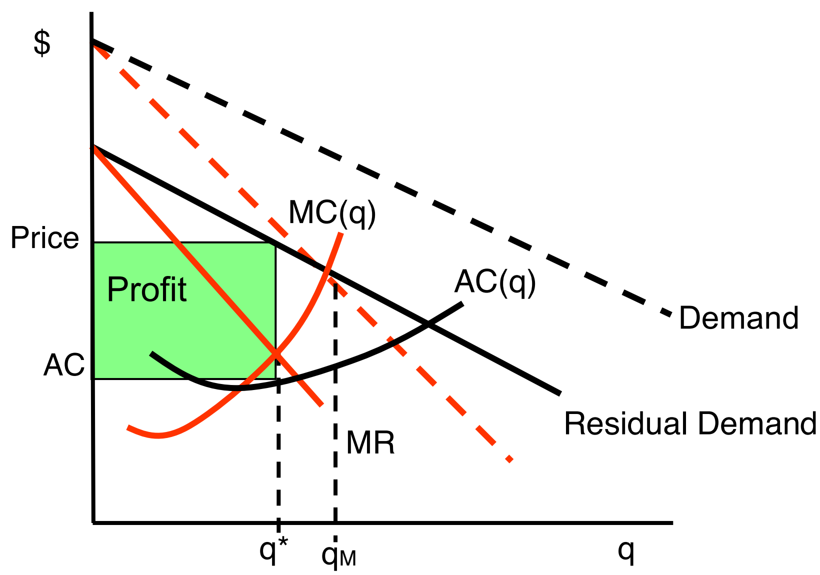
Cournot - Quantity Competition

- Just choose quantity, cost includes k and c, let market determine price

  $$\pi_M = p(\bar{q}_M + \bar{q}_L)\bar{q}_M - C(\bar{q}_M)$$

- May have increasing MC, or constant MC

- SHOW MR+MC in graph with Residual demand
- Subtract opponent's output from Demand to get residual demand



- Maximizing Marianna's profit:

$$\frac{d\pi_M}{dq_M} = p(q_M + q_L) + \frac{dp}{dQ}\frac{dQ}{dq_M}q_M - \frac{dC}{dq_M} = 0$$

$$MR_M = p + \frac{dp}{dQ}q_M$$

$$MR_M = p + \frac{dp}{dQ}Qs_M$$

Under monopoly, MR was $MR = p + \dfrac{dp}{dQ}Q$

- Revenue lost from lowering price is spread over 2 firms.
- MR is closer to p than if there is only one firm.
- Solution gives reaction function: $q_M = R_M(q_L)$

Example: Cournot Competition

2 Firms, Lindsay & Mari

$$\pi_M = (20_M - 0.25(q_L + q_M))q_M - 5q_M$$
$$20 - 0.25q_L - 0.5q_M - 5 = 0$$
$$0.5q_M = 15 - 0.25q_L$$
$$q_M = 30 - 0.5q_L$$

Generally, do the same thing for L's reaction function. Will yield:

$$q_L = 30 - 0.5q_M$$

Graph Reaction Functions

Substitute & solve.

$$q_M = 30 - 0.5(30 - 0.5q_M)$$
$$q_M = 15 + 0.25q_M$$
$$0.75q_M = 15$$
$$q_M = 60/3 = 20$$
$$q_L = 20$$

BUT, you can just use symmetry for this example once you have Mari's reaction function. Since the only difference between the players is their names, their q's must equal one another:

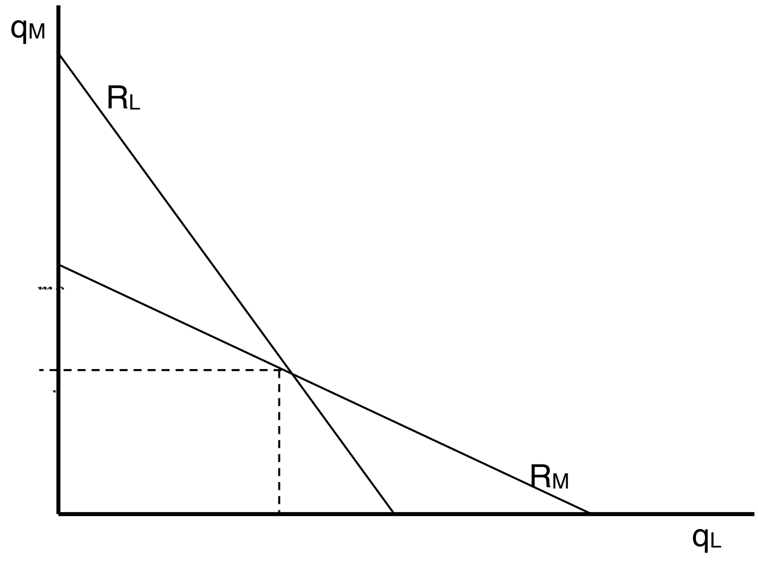
$$q = 30 - 0.5q$$
$$1.5q = 30$$
$$q = 20 = q_L = q_M$$

From there:

$$Q = 40$$
$$p = 20 - 40/4 = 10$$
$$\pi_M = 20(10 - 5) = 100$$
$$\pi_M + \pi_L = \Pi_{COURNOT} = 200$$

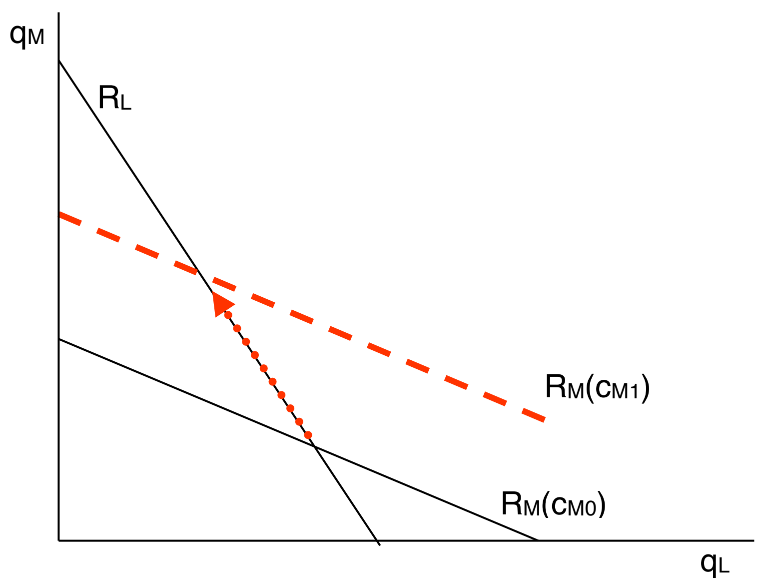
Quantity Competition – General Reaction Functions

- Lindsay's q↑ → Marianna's q↓, Quantities are strategic substitutes
- If opponent produces 0, $q=q^{mon}$
- If opponent's q very high, own q=0
- If opponent $q=q^{mon}$, p>MC, own q>0



Incentives for Cost Management

- If c↓, q↑ more than the direct effect
- Firms may "over-invest" in cost reductions to gain competitive advantage, compared to monopoly facing same initial demand



Show this in MR=MC graph, too

Advertising in Homogenous Product Competition?

Cost to one firm, benefits spread over all firms

Role of trade associations

Quantity Competition with a First Mover – Stackelberg

- Quantity Competition, but, Lindsay moves first.
- Takes Marianna's reaction function as a constraint
- Put yourself in your opponent's place and ask what would they do?
- Generally:

$$\pi_L = p(q_L + R_M(q_L))q_L - C(q_L)$$
$$\frac{d\pi_L}{dq_L} = p + \frac{dp}{dQ}\left(\frac{dQ}{dq_L} + \frac{dR_M}{dq_L}\right)q_L - \frac{dC}{dq_L} = 0$$
$$MR_L = p + \frac{dp}{dQ}\left(1 + \frac{dR_M}{dq_L}\right)q_L = MC_L$$

- Lindsay takes into account the impact her choice will have on Marianna's choice
- <u>First mover advantage</u>: Stackelberg leader gains profit and market share at expense of follower

Moving first is a ***credible*** way to commit to a higher quantity than otherwise rational

Example 3: Stackelberg Competition

Lindsay moves first. She can figure out how Mari will react.

$$\pi_L = (20 - 0.25q_L - 0.25(30 - 0.5q_L))q_L - 5q_L$$
$$\pi_L = (12.5 - 0.125q_L)q_L - 5q_L$$
$$12.5 - 0.25q_L - 5 = 0$$
$$0.25q_L = 7.5$$
$$q_L = 30$$

So,

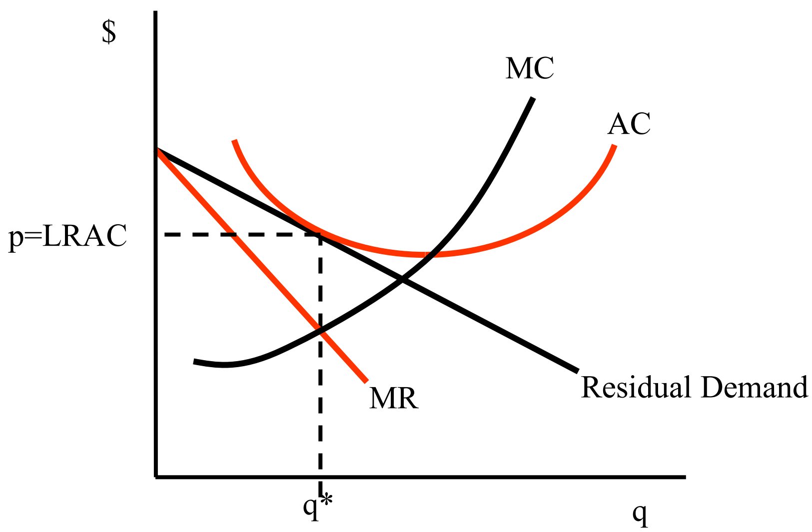
$$q_M = 30 - 0.5(30)$$
$$q_M = 15$$
$$Q = 45$$
$$p = 20 - 0.25(45) = 8.75$$
$$\pi_L = (8.75 - 5)30 = 112.50$$
$$\pi_M = (8.75 - 5)15 = 56.25$$
$$\Pi_{STACKELBERG} = 168.75$$

Comparison of Structures in Example

	Monopoly	2-Firms	
		Cournot	**Stackelberg**
Price	12.5	10	8.75
Total Quantity	30	40	45
Total Profit	225	200	168.75

Homogenous Product Industry With Free Entry

- In LR, as long as no entry barriers are present, if profit is possible, entry occurs
- In LR, for the marginal firm that is just willing or unwilling to enter, price at optimal quantity is less than or equal to LRAC



- If firms are not identical, the lowest cost firms constitute the industry
- Most efficient firm that does not enter would make a loss
- Least efficient firm that enters makes minimal profit

Array of firms that enter from 1 to n with 1 most efficient and n least efficient

Profit is: $\pi_j = p\left(\sum_{i=1}^{n} q_i\right)q_j - C_j(q_j)$

Two conditions define the Nash equilibrium for all potential firms:

Equilibrium Condition 1: all firms in the market maximize profits

$$\frac{d\pi_j}{dq_j} = p\left(\sum_{i=1}^{n} q_i\right) + \frac{dp}{dQ}q_j - \frac{dC_j}{dq_j} = 0$$

$$p + \frac{dp}{dQ}Qs_j = MC_j$$

$$MR_j = MC_j$$

Equilibrium Condition 2: the marginal firm breaks even

$$\pi_j = p\left(\sum_{i=1}^{n} q_i\right)q_n - C_n(q_n) \approx 0$$

or $p \approx LRAC_n$

Example

Demand: $p = 20 - 0.25Q$. Current n=4. Firm 4's cost is $C(q) = 5q + F$.

In the current Nash equilibrium, firms 1-3 produce 40 total units.

How does F effect equilibrium?

$$\pi_4 = (20 - 0.25(40) - 0.25q_4)q_4 - 5q_4 - F$$

$$\pi_4 = (10 - 0.25q_4)q_4 - 5q_4 - F$$

$$\frac{d\pi_4}{dq_4} = 10 - 0.5q_4 - 5 = 0$$

$$q_4 = 10$$

$$Q = 50$$

$$p = 20 - 0.25(50)$$

$$p = 20 - 12.5 = 7.5$$

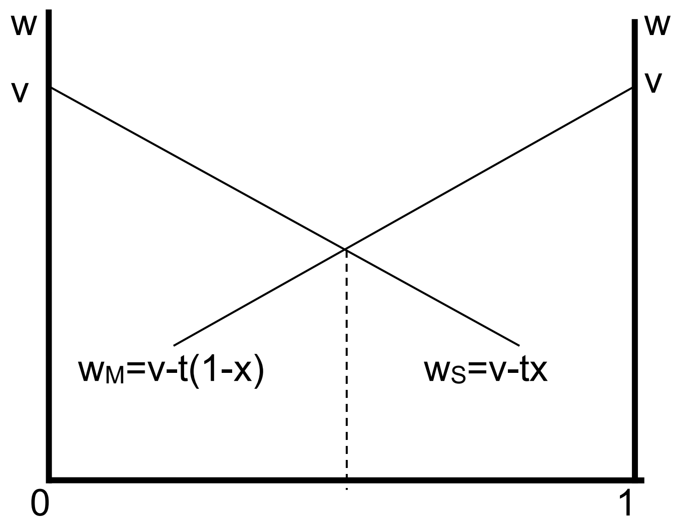
$$\pi_4 = (7.5 - 5)10 - F = 25 - F$$

So, if F>25, exit will occur. If F<25, entry will occur.

16. DIFFERENTIATED PRODUCT MARKETS

Oligopoly - Differentiated Product Price Competition

- Basic spatial differentiation to begin with
- Seitz and Morgan own restaurants at different ends of town.
- Distance between them is 1 unit. Seitz is at 0, Morgan is at 1.
- Let t measure travel cost per unit of distance (x).
- Everyone willing to pay (w) up to \$v less transportation cost for a meal.
- N customers uniformly distributed across the town.



- Purchase from whichever gives most net surplus, S=w-p.
- Indifferent at location $\tilde{x}$ where:
- $v-p_S-t\tilde{x}=v-p_T-t(1-\tilde{x})$, or,
- $t+p_M-p_S=2t\tilde{x}$
- $\tilde{x}=(t+p_M-p_S)/2t$
- $\tilde{x}=1/2+(p_M-p_S)/2t$
- Demand for Seitz is
- $q_S=N/2+N(p_M-p_S)/2t$

Maximize profits, find reaction functions and NE

$$\pi_S = (p_S - c) N\tilde{x}$$

$$\pi_S = (p_S - c) N\left(\frac{1}{2} + \frac{p_M - p_S}{2t}\right)$$

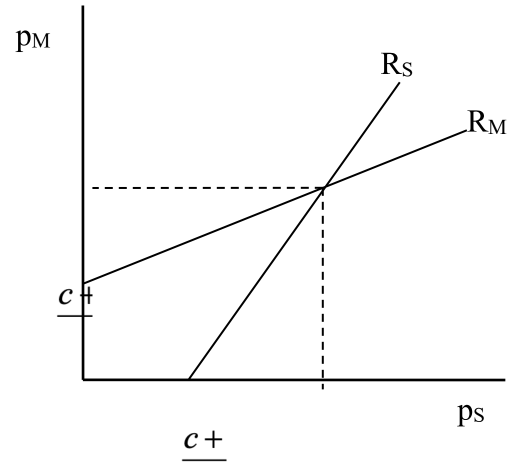
$$\frac{d\pi_S}{dp_S} = N\left(\frac{1}{2} + \frac{p_M - p_S}{2t} - \frac{(p_S - c)}{2t}\right) = 0$$

$$2p_S = t + p_M + c$$

$$p_S = R_S(p_M) = \frac{t+c}{2} + \frac{p_M}{2}$$

By symmetry, $p_M = p_S$

$$p = t + c$$



- Prices are strategic complements.
- If there is no differentiation, t=0, homogenous product Bertrand result, p=c.
- With differentiation, differentiated product Bertrand result, p is increasing with the importance of the product differentiation, t

General Differentiated Product Price Competition

- Seitz and Morgan sell slightly differentiated gadgets, produced at constant MC
- Morgan's demand is $q_M(p_M, p_S)$, unit cost c_M.
- Find reaction function

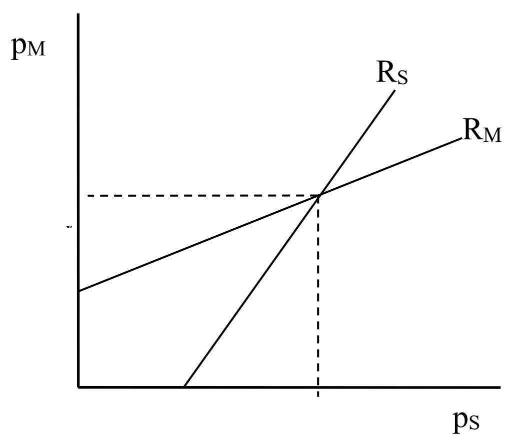
  $$\pi_M = q_M(p_M, p_S)(p_M - c_M)$$

  $$\frac{d\pi_M}{dp_M} = q_M + \frac{dq_M}{dp_M}(p_M - c_M) = 0$$

  Both q and dq/dp are functions of both prices. Solving gives reaction functions:

  $$p_M = R_M(p_S)$$

- Same for Seitz, Graph reaction functions, show NE



Example: Price Competition

Morgan and Seitz. Demand and cost functions are:

$q_M = 10 - p_M + 0.5p_S$, $q_S = 18 - 2p_S + 0.5p_M$, $C(q_M) = 2q_M$ and $C(q_S) = q_S$.

Find Morgan's reaction function:

$$\pi_M = (10 - p_M + 0.5p_S)(p_M - 2)$$

$$\frac{d\pi_M}{dp_M} = 10 - p_M + 0.5p_S - p_M + 2 = 0$$

$$2p_M = 12 + 0.5p_S$$

$$p_M = 6 + 0.25p_S$$

Find Seitz's reaction function:

$$\pi_S = (18 - 2p_S + 0.5p_M)(p_S - 1)$$

$$\frac{d\pi_S}{dp_S} = 18 - 2p_S + 0.5p_M - 2p_S + 2 = 0$$

$$4p_S = 20 + 0.5p_M$$

$$p_S = 5 + 0.125p_M$$

Solve the reaction functions for the equilibrium prices.

$$p_M = 6 + \frac{1}{4}(5 + \frac{1}{8}p_M)$$

$$p_M = 6 + \frac{5}{4} + \frac{1}{32}p_M$$

$$\frac{31}{32}p_M = \frac{29}{4}$$

$$p_M = \frac{32 \bullet 29}{31 \bullet 4} = \frac{232}{31} = 7.48$$

$$p_S = 5 + \frac{1}{8}\frac{232}{31} = 5.94$$

Differentiated Product Price Competition with a First Mover

- Seitz moves first, demand is $q_S(p_S, p_M)$, unit cost c_S.
- Morg's reaction function was $p_M = R_M(p_S)$

$$\pi_S = q_S\left(p_S, R_M(p_S)\right)\left(p_S - c_S\right)$$
$$\frac{d\pi_S}{dp_S} = q_M + \left(\frac{\partial q_s}{\partial p_s} + \frac{\partial q_s}{\partial p_M}\frac{dR_M}{dp_M}\right)\left(p_s - c_s\right) = 0$$

- Seitz takes account of the effect on his price on Morgan's price
- At Seitz's p in the simultaneous play NE, MB of increasing p just equaled MC
- Now, at that p, there is a new effect, on Morgan's price, so, there is now a net benefit to raising price above that level, Seitz's price higher
- Since Seitz's p higher, Morgan's demand is higher, so, her price is higher, and, her profit is higher
- But, since Morg gets to "undercut" (relatively), she gains MORE profit moving second than she would if she moved first

Example: Price Competition with a First Mover

Assume Seitz moves first

$$\pi_S = (18 - 2p_S + 0.5(6 + 0.25p_S))(p_S - 1)$$
$$\pi_S = (18 - 2p_S + 3 + 0.125p_S)(p_S - 1)$$
$$\pi_S = (21 - 1.875p_S)(p_S - 1)$$

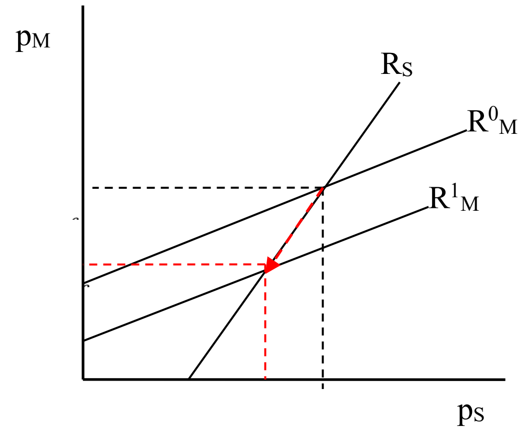
$$\frac{d\pi_S}{dp_S} = 21 - 3.75p_S + 1.875 = 0$$
$$p_S = 6.10 > 5.94$$
$$p_M = 6 + 0.25(6.10) = 7.53 > 7.48$$

- Seitz could have chosen 5.94. Morg would have then chosen 7.48.
- Seitz' profit higher. Morg prefers Seitz to have higher price, her profit higher.
- If you plug in for profits, both are better off than with simultaneous moves.

    Second mover gains most! But, first is better than simultaneous.

Incentives For Cost Management in Differentiated Product Price Competition

- Suppose Morgan adopts a new process that lowers her MC
- Her reaction fn shifts down (why?)
- Because prices are strategic complements, Seitzs' price is lower, too
- This offsets some of the gain to the cost reduction
- Since products are differentiated, even if Seitz did not lower his price, her market share increase would be limited.



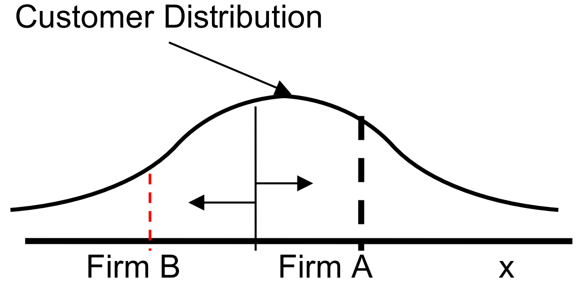
Show effect in MR=MC graph for Morgan

Incentives for Advertising in Differentiated Product Price Competition

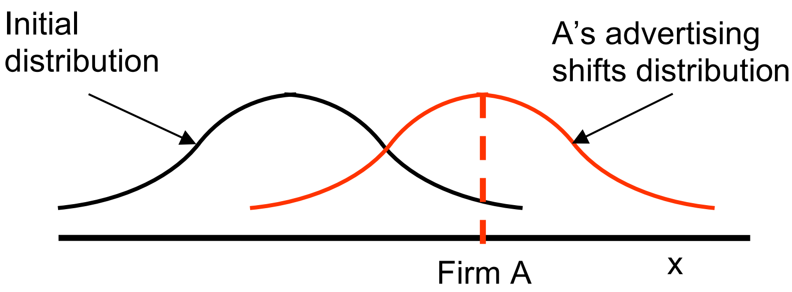
- Look for ways to shift demand to your product
- Two types of advertising:

Informative – let customers know your products characteristics (including price) so those better suited to it can switch

In figure, just letting customers know "B" exists gets them near half the market (at similar prices)



Persuasive – change preferences of consumers



- The line between the two is sometimes blurry
- For "social" reasons, may care about the difference - for our purposes, difference matters only in how to most effectively shift demand in your favor

Optimal Advertising with Differentiated Products

Equates MB to MC - how much does it cost to spend $1 more on advertising???

$$\pi = q(p, A)p - C(q(p, A)) - A$$

$$\frac{d\pi}{dA} = p\frac{\partial q}{\partial A} - \frac{dC}{dq}\frac{\partial q}{\partial A} - 1$$

$$(p - MC)\frac{\partial q}{\partial A} = 1$$

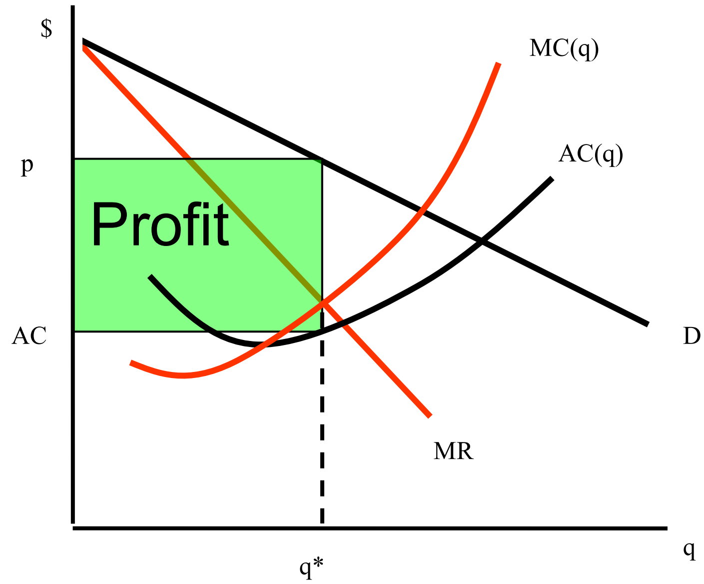
If your advertising increases your price, part of the strategic effect is a resulting increase in your opponent's price.

Full strategic effects depend on how your advertising impacts your opponent's demand, thus, their reaction function. May shift your opponents reaction function down, lowering their price.

Draw examples of each.

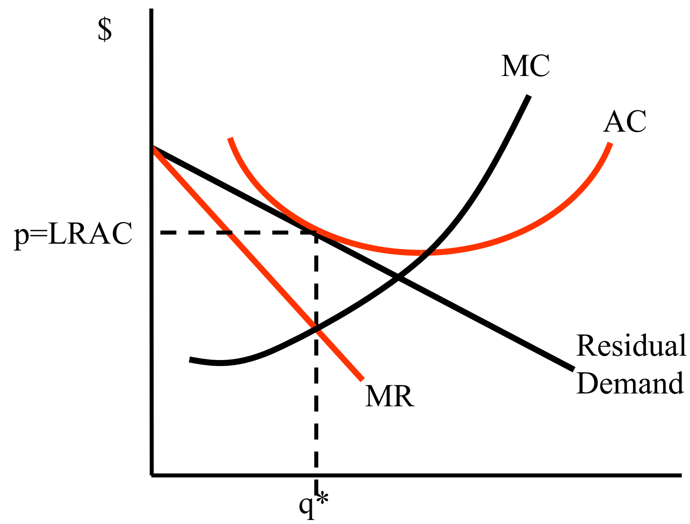
Long Run Equilibrium - Differentiated Product Oligopoly

- Prices of substitutes effect location of the demand curve
- In LR, firms enter if profits are possible absent entry barriers



If economies of scale not too substantial and no other barriers to entry, profit is 0 in the LR

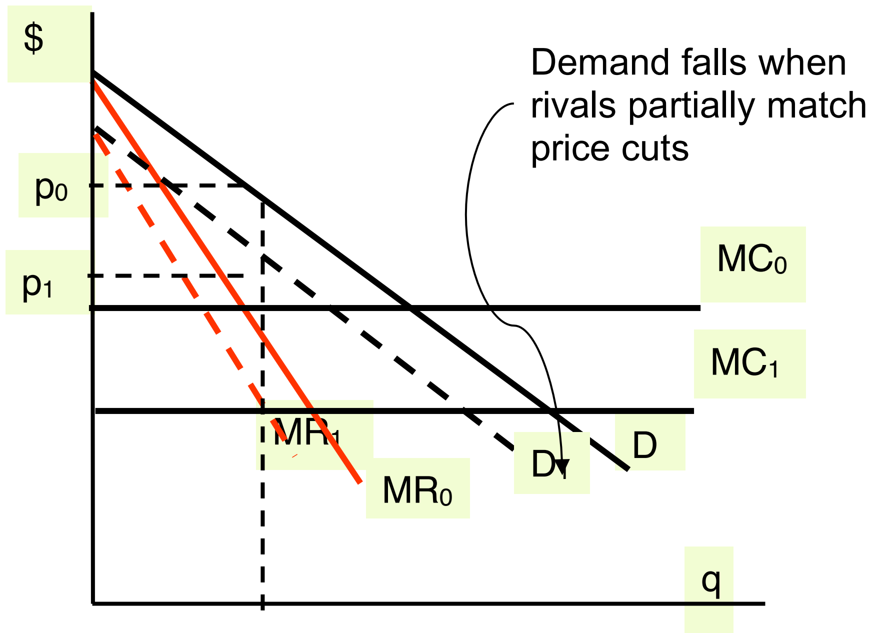
In LR, no two firms producing the same product



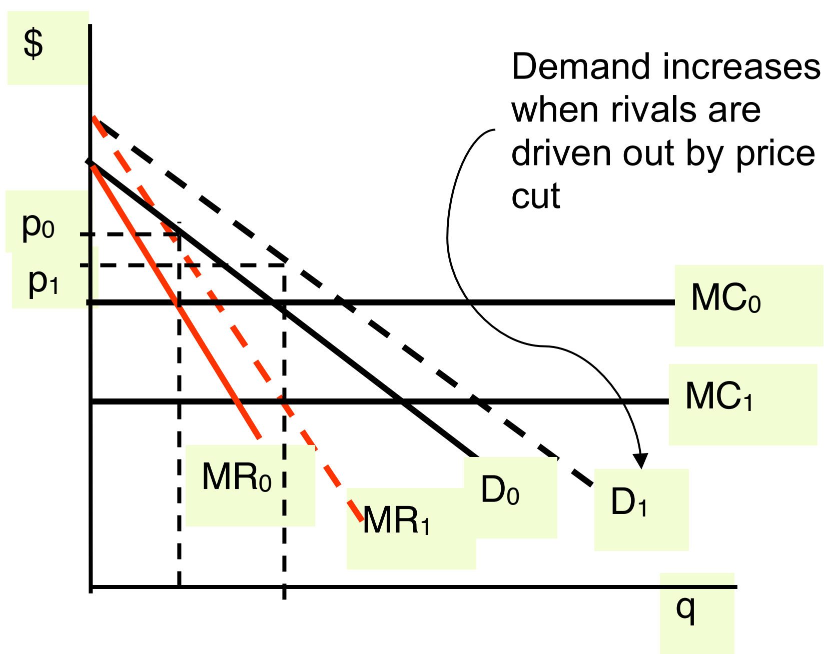
- If the number of firms producing similar products is not large, strategic concerns are more important
- Gains from cost reduction are muted if rivals have some room to match price cuts
- Gains from cost reduction are amplified if rivals were making little or no economic profit, because they may be driven out

- If the number of firms producing similar products is relatively large, strategic interdependence is less important "*Monopolistic Competition*"
- Advertising, product differentiation, cost reduction all still matter
- Less concern over strategic effects on individual competitors, but, closest competitors may still matter strategically

Marginal cost reduction when competitors have some room to match price cuts



Marginal cost reduction when competitors have no room to match price cuts



17. PERFECT COMPETITION

If economies of scale are relatively small (and differentiation is not too large, which is sort of the same thing) free entry means many firms, each with essentially no control over price.

Homogenous Product Industry with Small Firms and Free Entry

$$R_{Monopoly} = p(Q)Q$$

$$MR_{Mon} = p + \frac{dp}{dQ}Q$$

With more firms: $Q = \sum_i q_i$, q/Q is market share, s

$$R_i = p(Q)q_i$$

$$MR_i = p + \frac{dp}{dq_i}q_i = p + \frac{dp}{dQ}q_i = p + \frac{dp}{dQ}q_i\frac{Q}{Q} = p + \frac{dp}{dQ}Qs_i$$

- Small market share gives a price near marginal revenue
- Maximizing profit gives MR=MC
- If marginal firm is least efficient, smallest, s_n is small, $p \approx MC_n$
- In LR, marginal firm breaks even, $p \approx LRAC_n$
- Put them together, $p \approx MC_n \approx LRAC_n$, or $p \approx MinLRAC_n$

Differentiated Product Industry with Small Firms and Free Entry

To Max profits: $p_i = \frac{\eta_i}{1+\eta_i}MC_i$

As there are more and better substitutes, $|\eta_n| \to \infty$, so $p_n \to MC_n$

In LR, marginal firm breaks even, $p_n \approx LRAC_n$

Put them together, $p_n \approx MC_n \approx LRAC_n$, or $p_n \approx MinLRAC_n$

What is the best way to model such markets? Treat all firms as price takers!

Short Run

Start in SR with n firms

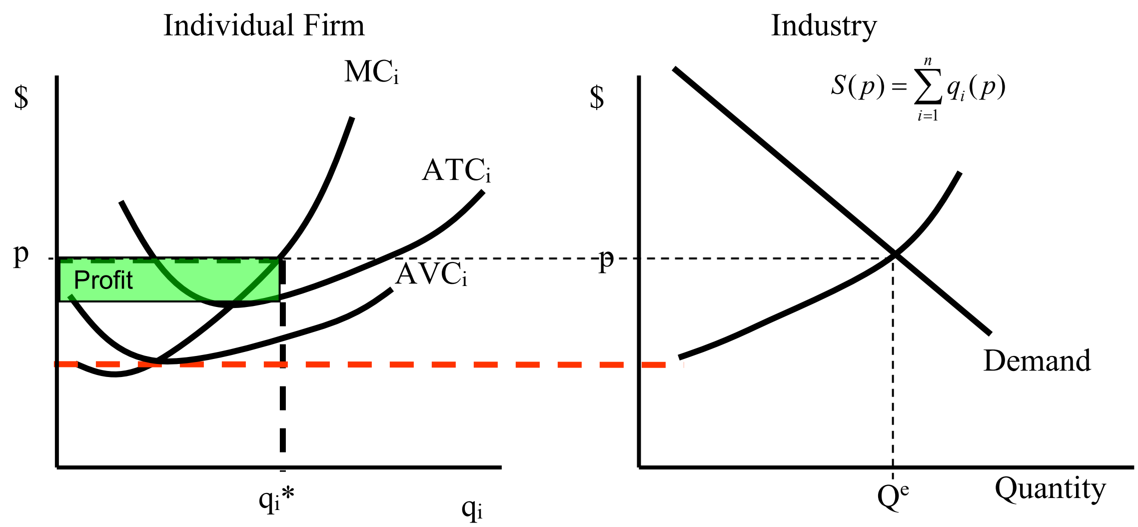
- Choose q to maximize profit

  $\pi = pq_i - C(q_i)$

- $\frac{d\pi}{dq} = p - MC(q_i)$

- Produce where *p=MC*, (in SR produce as long as *p>Min AVC*)
- Solving $p=MC_i$ gives individual's i's supply as a function of p, $q_i(p)$.
- Market supply is $S(p) = Q_S(p) = \sum_i q_i(p)$, the horizontal sum of the individual

  MC curves
- Market equilibrium occurs where supply and demand meet: *D(p)=S(p)*



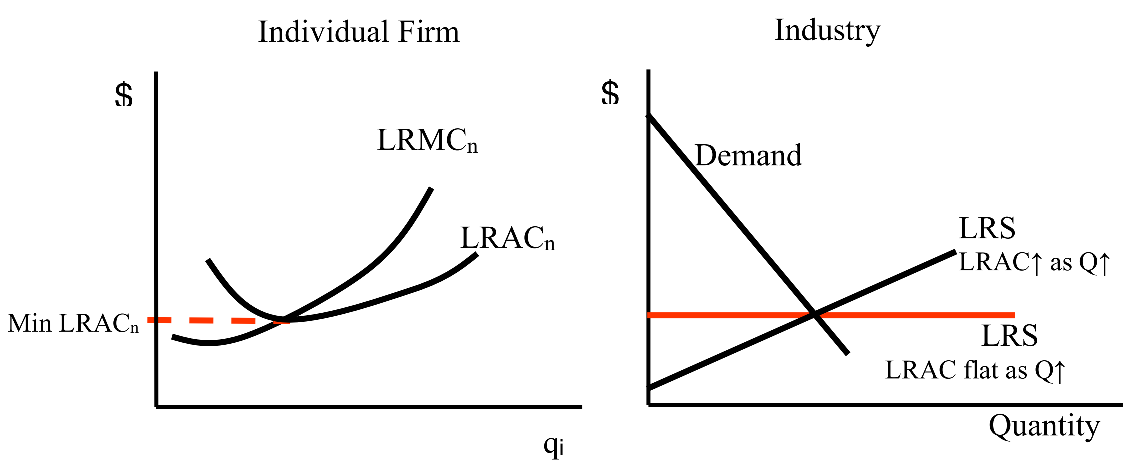
Example: Perfect Competition in the SR

25 identical firms, $C(q) = 10 + 2q + 0.25q^2$ $Q_D = 200 - 10p$

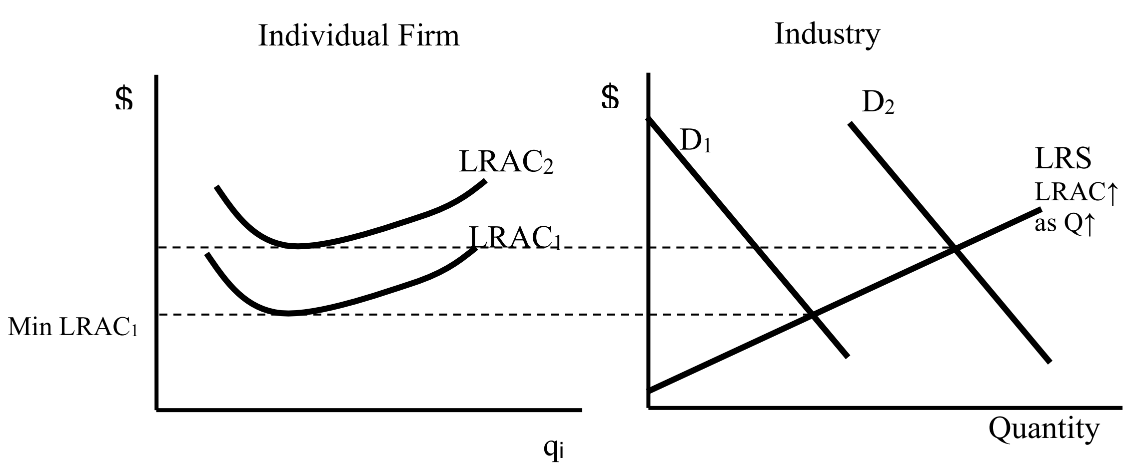
$MC = 2 + 0.5q = p$	$Q_S = Q_D$
$0.5q = p - 2$	$50p - 100 = 200 - 10p$
$q = 2p - 4$	$60p = 300$
$Q_S = 25(2p - 4)$	$p = 5$
$Q_S = 50p - 100$	$Q_D = 200 - 10(5) = 150$
	$q = 150/25 = 6$

Long Run

- Array firms that enter from 1 to n with 1 most efficient and n least efficient
- In LR, must be the case that no firm that has not entered could make a profit



- In LR, p=Min LRAC=LRMC for marginal firm
- If firms are identical and input prices are not driven up by industry expansion, LRS is horizontal at Min LRAC
- If not, it slopes up as less efficient firms enter or input prices are driven up
- What would make a firm "more efficient"? Really an economic "rent" likely to be captured by a scarce factor.



Example: Perfect Competition in the LR

Identical firms, $C(q) = 2q - 0.2q^2 + 0.01q^3, Q_D = 450 - 100p$

$MC = 2 - 0.4q + 0.03q^2$

$AC = 2 - 0.2q + 0.01q^2$

$MC = AC$

$2 - 0.4q + 0.03q^2 = 2 - 0.2q + 0.01q^2$

$0.02q^2 = 0.2q$

$q = 10$

$p = MinAC$

$p = 2 - 2 + 1 = 1$

$Q_D = 450 - 100p = 350$

$n = 350/10 = 35$

Measuring "Value Added" or Welfare in Competitive Markets

Already covered consumer surplus. Quick Review. What about producers?

Producer Surplus

Analogous to consumer surplus - area below price and above supply

In the SR: the same as profit, but for fixed costs, since supply reflects MC

In the LR, LRS reflects min LRAC for various quantities

IF a constant cost industry, PS=Profit=0

If an increasing cost industry, much more complicated!

- Suppose Min LRAC of firm 100 with 100 firms producing 100 each is 10 and with 125 firms producing 100 each is 12 for firm 125. What happened to the Min LRAC of firm 100?
- If costs are increasing due to increased payments to various factors, the infra-marginal units earn economic "rents" – returns above opportunity costs
- Ex: suppose there were little market for pro football. Brett Favre would have played for not much! All the rest he is paid because of high demand is rent.
- Examples: best managers, best farmland, etc…
- Economic rents are "surplus" paid to factors, not share holders. So part of costs in supply curve are really producer side surplus in increasing cost industries

So, like CS, PS is a very simplified measure. Still USEFUL!!

If $p_S(Q)$ is the market inverse supply, $PS = pQ - \int_0^Q p_S(x)dx$

Total Surplus: PS+CS, usually. Draw it!

Evaluated at the market equilibrium:

$$TS = CS + PS = \int_0^Q p_D(x)dx - pQ + pQ - \int_0^Q p_S(x)dx = \int_0^Q \left(p_D(x) - p_S(x)\right)dx$$

SHOW GRAPHICALLY!

Perfect Competition - Summary

- p=MC gives supply in SR, S=D gives equilibrium price
- p=Min LRAC in LR for "highest cost" firm, gives LR supply
- In LR, "efficient" firms may make economic profit, or, factors that make them low cost may earn "rents".
- Running the firm efficiently means simply producing as efficiently as possible
- No role for advertising – homogenous product
    - this is an oversimplification since we apply the model to dense differentiated product markets to
    - In the SR an entrepreneur may make a profit by differentiating, opening a new niche, etc…
- Every unit for which maximum willingness to pay exceeds the minimum LRAC, which equals MC, is produced
- Free trade in perfectly competitive markets is thus socially efficient, maximizes Total Surplus
    - Assumes a lot – particularly no externalities
- Thus, there are laws against restraint of trade
- Easy to do "comparative statics"

18. APPLICATIONS OF SUPPLY AND DEMAND

Demand depends on:

m Income (normal/inferior)

p_{RC} Price of related goods (consumption substitutes and compliments)

n_C Number of consumers

z_C Other stuff

$Q_D = D(p, m, p_{RC}, n_C, z_C)$

Beware of the impact of tastes.

Supply depends on:

n_F Number of firms

p_I Input prices

p_{RP} Prices of related goods, (production substitutes and complements)

z_s Other stuff

$Q_S = S(p, p_I, p_{RP}, n_F, z_S)$

Equilibrium: S(p)=D(p)

CS, PS, TS

Graphical Examples:

1) Your firm uses a lot of gas & a storm knocks out a refinery?
2) Impact of crackdown on illegal immigration on housing prices
3) Impact of construction boom in China on construction in US

Effects of Shocks, Taxes, Price Controls on Consumers & Producers

Tax incidence and deadweight cost of taxation

per unit taxes and ad valorem taxes

SR vs LR

Hurricanes:

Effect on prices

Effect of price controls on CS, PS, Q

Rationing and black markets

Agricultural Subsidies

Direct subsidy and price supports

Applicability of S&D Analysis

1) No precise definition of "supply curve" unless firms are pure price takers!
2) But, for modeling entire industries, if there is a reasonably large number of firms, treating as perfectly competitive firms with an industry supply curve is a very good approximation
3) Even with fewer firms (still several), very useful for organizing thoughts as long as there are several firms and products which are relatively homogenous
4) Market definition matters
    a. Gainesville and Ocala? Gainesville and Miami?
    b. McDonalds and Wendy's? McDonalds and Applebees?

Generally, Why use "as-if" models

19. MARKET STRUCTURE WRAP UP

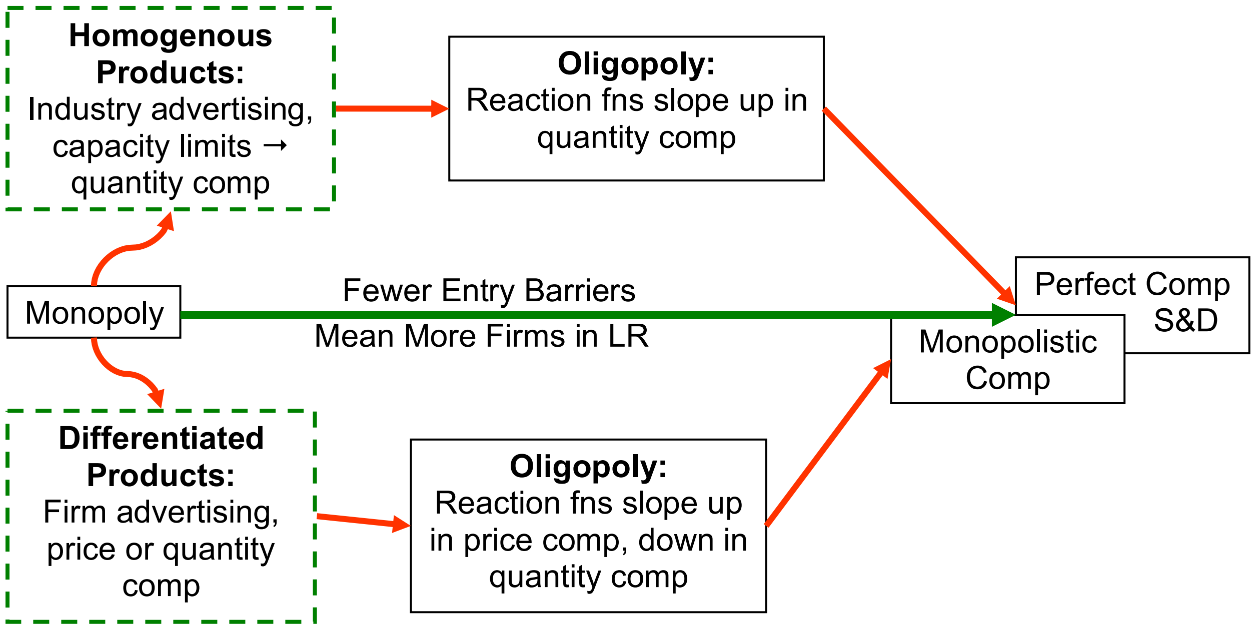
Homogenous Product Industry with Free Entry

- Price competition with p=MC in NE is inconsistent with capacity constraints
- Quantity competition most sensible model
- Least efficient firm that enters makes minimal profit, MR=MC all firms
- Advertising benefits all firms in SR, and, may induce more entry in LR
- Cost reduction benefits your firm and has positive strategic effect
- Early firms may gain first mover advantage - invest in larger capacity - discourages entry and induces others to choose smaller capacity
- Converges to perfect comp as number of firms gets large

Differentiated Product Industry with Free Entry

- Both price and quantity competition are possible (Q comp would work like in homogenous product industries)
- Least efficient firm that enters makes minimal profit, MR=MC all firms
- Advertising benefits more concentrated on firm undertaking expense
    - Strategic effects can reinforce or counter direct effect, depending on price or quantity competition and impact on opponent's demand
- Cost reduction benefits your firm and harms opponent
    - BUT, strategic effect counters direct effect in price competition
    - Strategic effect augments direct effect in quantity competition
- In price competition, there may be SR gains from moving second if the first mover is "committed" to their price for some time
- Converges to "monopolistic" competition as economies of scale become small, similar to perfect competition as become very small

Summary of Models



All else equal, more competitive industries have lower prices and profit, but higher output and total surplus.

Firms would like to pursue strategies that limit competition

The government, Department of Justice, seeks to promote competition, limit monopoly, and limit anti competitive practices.

Measuring Market Structure

- How do particular industries fit?
- Number of firms in industry, n, is indicator
- Relative size of firms, not just n, is important
- How much of industry is concentrated in a few firms?
- 2 common measures

Concentration Ratios

- m firm concentration ratio, C_m, is % of total sales to top m firms
- C_4 most common
- Array of firms from largest, 1, to smallest, n.
- Let s_i be share of firm i's sales (in dollars) in industry total. If R_i is the dollar value of revenue to firm i and R_T is the sum of all sales revenue to all firms in the industry, $s_i = R_i/R_t$
- Then $C_4 = s_1+s_2+s_3+s_4$
- Problem, what if firms 5 and 6 are large as well?

Firm	100s	$10000s^2$
1	20	400
2	20	400
3	10	100
4	10	100
5	10	100
6	10	100
7	10	100
8	4	16
9	4	16
10	2	4
Total	100	1336

Herfindahl Hirschman Index

- HHI considers all firms, placing more weight on larger sales volume

$$HHI = 10{,}000\sum_{1}^{n} s_i^2$$

- Pure monopoly → HHI=10,000
- Many small firms → HHI very small, 0 is limit

Concentration Measures for Selected US Industries - 2007					
Industry	C_4	HHI	Industry	C_4	HHI
Fluid milk	46	1013	Men's neckwear	65	1407
Soft drink	46	710	Printing	11	48
Breakfast cereal	82	3000	Pen & mechanical pencil	74	1957
Bread & bakery products	41	581	Ready-mix concrete	11	57
Sugar	53	856	Dental equip & supply	38	437
Distilleries	79	2090	Basic chemical	18	160
Furniture & related	11	57	Battery	55	958
Wood container & pallet	7	26	Petroleum refineries	47	809
Paper mills	56	883	Automobile	87	2754
Corrugated & solid fiber	33	392	Boat building	39	573
US Census Bureau, http://www.census.gov/epcd/www/concentration.html					

<u>Problems with Concentration Measures</u>

- Definition of relevant market
- Geographic concerns
    - Distilleries & import spirits
    - Ready mix concrete is a purely local market
- Product Concerns
    - Are foil, plastic wrap, and wax paper the same product?
    - Ideally, the products are close substitutes
    - Define by high cross price elasticity of demand
- "Geographic" and "product" concerns are pretty much the same thing
    - concrete in different city is a different product

More Direct Measures of Market Power

- Could measure markup over marginal cost
- Idea, clear but problematic in practice!
- Estimating cost curve of typical firm for specific product is HARD!
- Often just assume MC=AC, measure AC with accounting data
- Most firms produce multiple products, too
- But, especially at any given time in SR, may not be reasonable!
- Are all opportunity costs & a fair return on investment are included, including rents to resource owners that are commingled with the firm?

Market Power Measures for Broad US Industries 1949-1985

Industry	Mark Up Factor	Demand Elasticity Firm	Market	Firm/ Market
Agriculture	1.01	-96.2	-1.8	0.02
Construction	1.24	-5.2	-1	0.19
Durable Manufacturing	1.40	-3.5	-1.4	0.40
Nondurable Manufacturing	1.42	-3.4	-1.3	0.38
Transportation	2.11	-1.9	-1	0.53
Communication and Utilities	2.25	-1.8	-1.2	0.67
Wholesale Trade	2.67	-1.6	-1.5	0.94
Retail Trade	2.25	-1.8	-1.2	0.67
Finance	1.22	-5.5	-0.1	0.02
Services	1.04	-26.4	-1.2	0.05

Shapiro, Matthew D., "Measuring Market Power in U.S. Industry, NBER Working Paper No.2212, 1987

Measures of Market Power

- Mark Up factor is 1 for perfect competition.
- No benchmark for monopoly. Depends on elasticity of market
- Compare demand elasticity of entire market to demand elasticity of a particular firm, take ratio, Market/Firm
- Perfect Comp = 0. Monopoly = 1.
- Idea clear but problematic in practice!
- What price do you use for the industry demand curve?
- How do you combine prices of foil and plastic wrap?

Problems Measuring Market Power

- Often inferring markup and elasticity from observed price and assumed constant marginal cost
- In reality, observations are made with error, costs are imperfectly observed, more than just immediate profit maximization

Market Structure Data - Summing Up

- Industries vary from highly concentrated to very diffuse
- Some firms face price sensitivity far above the market average, some appear to face essentially the market demand curve
- Warning 1 – measure may not always be precise
- Warning 2 – ***nature of competition*** matters as well! Fierce price competition, actual or potential, forces prices near cost even if only a few large firms.

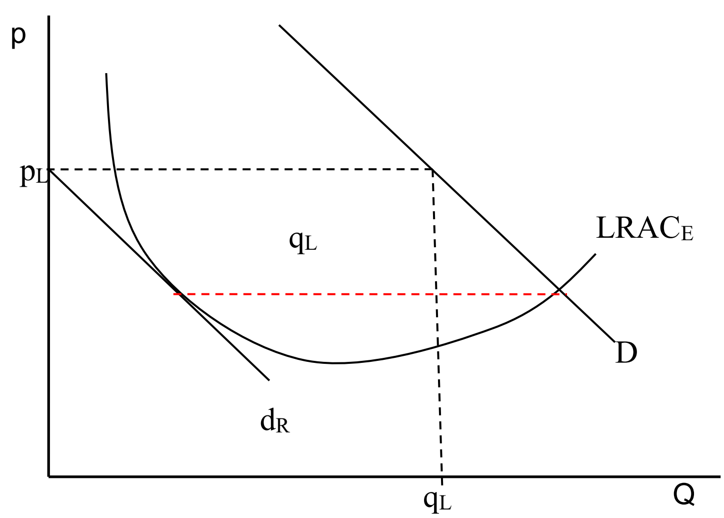
Anti-Trust Policy

- US DOJ considers HHI above 1800 "highly concentrated"
- Close scrutiny of mergers that increase HHI by more than 100 or in industries with HHI over 1800
- May allow for efficiency reasons, or if evidence of foreign competition
- Rule of Reason – only *unreasonable* restraints on trade are violations
- May consider arguments about economies of scale or scope that benefit consumers

Firm Strategies to Reduce Competition

Entry Limit Pricing

Idea: Produce a sufficiently high output at a sufficiently low price to convince potential entrants that if they enter, they would make a loss.



Problem: Once entrant enters, you have no more incentive to produce high output. Entrant can anticipate this!

Solution: Must "commit" to high output. How?

1) Invest in lowering your marginal cost more than otherwise optimal so an entrant can never make a profit – if they invested enough to get a MC low enough to compete, fixed/sunk costs (the costs of the investment in lowering MC) would be so high you both make a loss.
2) Move so far along the learning curve so fast no entrant wants to catch up because they would make too large a loss until they did.
3) Invest in acquiring a reputation as a “tough” or “crazy” competitor.

Considerations:
1) It may not be possible to reduce MC enough to keep competitor out.
2) If it is, must compare profits from limiting entry to profits from accommodating entry.

π_L: profit if limit entry

π_A: profit if accommodate entry

π_M: monopoly profit

$$\pi_M + \frac{\pi_A}{i} < \pi_L + \frac{\pi_L}{i}?$$

$$\pi_M - \pi_L < \frac{\pi_L - \pi_A}{i}?$$

Predatory Pricing

Temporarily charge very low price to drive rival out.

May involve temporarily charging below unit/marginal cost –raises legal issues

Other Considerations:

1) Cost structure must be such that another entrant will not want in
2) Check that present value of profits is actually higher
3) Talk about contestable markets here

Vertical Foreclosure

Use upstream monopoly power to raise costs of downstream rivals. May even be possible to drive downstream competitors out of market.

Increases downstream profits at expense of upstream profits.

Need to optimize, balance downstream gains with upstream losses.

Price Matching Guarantee

1. Is it a good deal for consumers?
2. What happens to incentive to undercut rivals prices?
3. A way to lessen price competition.
4. Conditions:
    a. Have a way to monitor (require advertisement)
    b. Make sure rival's cost not too low

Limiting Entry By Raising Costs

1) Raising Fixed Costs

Suppose you are making $100K profit per year.

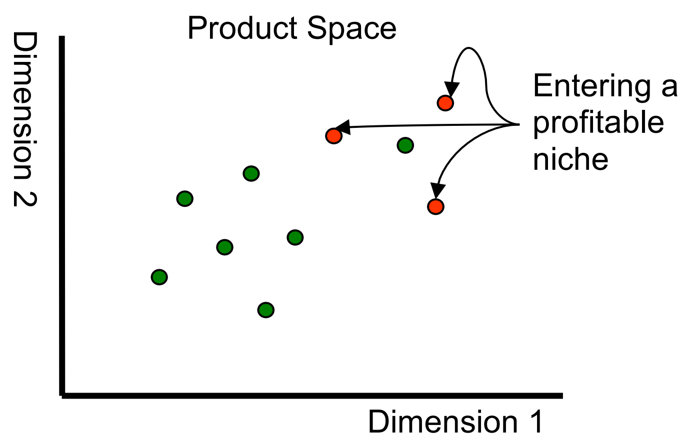
If an entrant enters, you both make $20K per year.

If you convince government to impose a regulation that increases fixed costs by $21K per year, the entrant will not enter and you still make $79K

2) Raising marginal cost can have the same effect. Less efficient socially.

Pre Emptive Entry and Niche Markets

- Look for profitable areas in product space
- Beware of entry near a profitable niche
- Consider preemptive product introductions



Penetration Pricing

Idea: Entrant temporarily charges very low price to break into market, perhaps giving the product away or paying customers to try it

Especially useful or likely where:

Entrant has a superior product

Some factor has created customer lock in

Lock in likely where product value depends on how many customers use it or related "compatible" products – network situations

No consumer wants to be the first to adopt a new software package or file format that is incompatible with what all their colleagues, friends, etc…

20. INPUT PROCUREMENT AND CONTRACTS

To make products, must get inputs.

3 ways to do so: Spot Market, make it yourself, contract with supplier

Spot Market

Works well when near perfect info, standardized products.

Maximizes specialization.

When search/negotiations costs or other transactions costs get high, inefficient, or, may break down entirely.

Vertical Integration

Cuts out transactions costs. Most loss of specialization.

Contracting and Optimal Contract Length

Can minimize transactions costs, keep specialization.

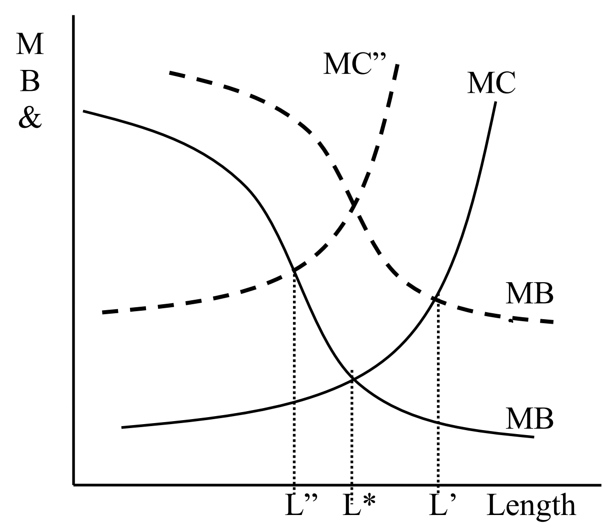
Benefits of contracting, contract length

Reduce transactions cost. The higher TC, the higher the MB of contract length

Costs of contracting and contracting length

If many contingencies, costly to write. More complex the contracting environment, higher MC of contract length.

Unknown Unknowns. Tied down, can't adjust to unforeseen contingencies arises. More uncertainty, MC of length higher.



Input Contracts with Asymmetric Information

Money Box example.

Asymmetric information: when one person knows more pertaining to a market transaction than the others.

Can cause a market to be less efficient, or, to break down entirely.

Stocks and insider trading.

Too many lemons in a used car market.

2 General types

Moral Hazard – when an action is hidden. How hard is the manager or team member really working when the boss is away? Are driver's insured by a company really exercising appropriate caution when they are fully covered?

Adverse Selection – when a characteristic is hidden. The least healthy are more likely to buy full health insurance, all else equal. Making it more expensive than healthy will pay. Leading companies to be unwilling to insure anyone without screening.

Both play an important role in input contracts, in particular, firm structure.

Principal Agent Relationships with Adverse Selection

Idea: other firm or agent the principal is contracting with has, or will have, better information than the principal. For example, about the cost of providing inputs.

Must budget for higher cost agents, or, agent will not accept your terms.

But, if allow higher cost, low cost agents will pretend to be high cost agents!

Solution: Offer a menu of contracts. Buy less from those who say cost is high, but, at a higher per unit price.

This sort of problem is sometimes called "mechanism design".

Model

Need to procure units of an input, q. Gross value to principal is V(q). Cost to supplier is low, c_L, with probability f, and high c_H, with probability (1-f).

Assume can't force a supplier to take a loss. (Even if agreed to it up front because info not yet revealed). Also, assume always want to purchase.

Principal's Problem:

$$
\begin{array}{lll}
\max_{P_L, q_L, P_H, q_H} & f(V(q_L) - P_L) + (1-f)(V(q_H) - P_H) & \\
 & P_L - C_L(q_L) \geq 0 & \text{Participation} \\
\textit{subject to} & P_H - C_H(q_H) \geq 0 & \\
 & P_L - C_L(q_L) \geq P_H - C_L(q_H) & \text{Incentive Compatibility or Truth Telling} \\
 & P_H - C_H(q_H) \geq P_L - C_H(q_L) &
\end{array}
$$

BE CAREFUL. L & H are cost, not demand or price, here.

In general, participation constraint of high cost supplier and selection constraint of low cost supplier bind. **Why?**

Also, generally buy efficient output from lowest cost supplier! **WHY?**

Thus: $P_H = C_H(q_H)$, and $P_L = P_H + C_L(q_L) - C_L(q_H)$. OR: $P_L = C_L(q_L) + \left(C_H(q_H) - C_L(q_H)\right)$

Pay a bonus for higher production! High enough to make lying unattractive.

Substituting from the constraints:

$$E(\pi) = f\left(V(q_L) - C_L(q_L) - C_H(q_H) + C_L(q_H)\right) + (1-f)\left(V(q_H) - C_H(q_H)\right)$$

$$\frac{\partial E(\pi)}{\partial q_L} = f\left(MB(q_L) - MC(q_L)\right) = 0$$

$$MB(q_L) = MC_L(q_L)$$

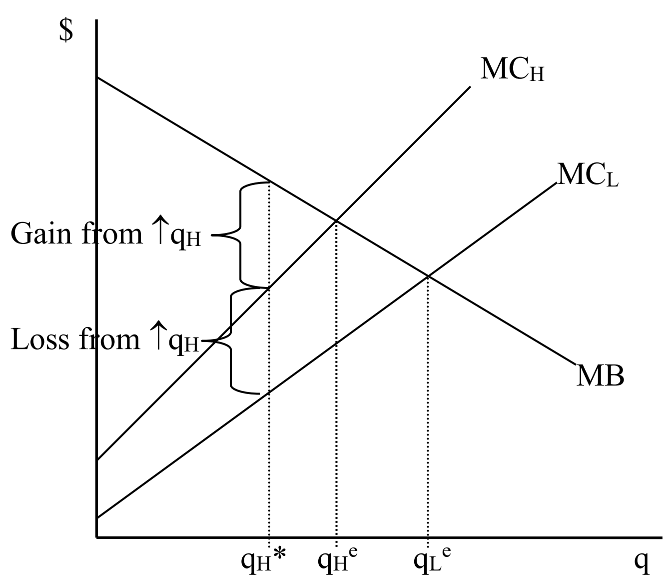
MB=MC if the agent says cost is low.

$$\frac{\partial E(\pi)}{\partial q_H} = f\left(-MC_H(q_H) + MC_L(q_H)\right) + (1-f)\left(MB(q_H) - MC_H(q_H)\right) = 0$$

$$MB(q_H) - MC_H(q_H) = \frac{f}{1-f}\left(MC_H(q_H) - MC_L(q_H)\right)$$

Analog of menu pricing FOC!

If agent says cost is high, restrict q to where MB still > MC. Why? To reduce bonus paid to "low" cost guy to keep him honest!



Example: $V(q) = 10q - .25q^2 \quad c_L = .5 \quad c_H = 2 \quad f = .5$

Binding Constraints: $P_H = 2q_H$, $P_L = P_H + 0.5q_L - 0.5q_H = 1.5q_H + 0.5q_L$

$$E(\pi) = .5(10q_L - .25q_L^2 - P_L) + .5(10q_H - .25q_H^2 - P_H)$$

$$E(\pi) = .5[10q_L - .25q_L^2 - 2q_H - .5(q_L - q_H)] + .5[10q_H - .25q_H^2 - 2q_H]$$

$$\frac{\partial E(\pi)}{\partial q_L} = .5[10 - .5q_L - .5] = 0$$

$$9.5 = .5q_L$$

$$q_L = 19$$

$$\frac{\partial E(\pi)}{\partial q_H} = .5[-2 + .5] + .5[10 - .5q_H - 2] = 0$$

$$-1.5 + 10 - .5q_H - 2 = 0$$

$$6.5 = .5q_H$$

$$q_H = 13$$

$$P_H = 26 \quad P_L = 26 + .5(6) = 29$$

$$E(\pi) = .5(10(19) - .25(19)^2 - 29) + .5(10(13) - .25(13)^2 - 26)$$

$$E(\pi) = .5(70.75) + .5(61.75) = 66.25$$

Compare to certainty.

$$10 - .5q_L = .5$$
$$q_L = 19$$
$$P_L = 9.5$$

$$10 - .5q_H = 2$$
$$q_H = 16$$
$$P_L = 32$$

$$E(\pi) = .5(10(19) - .25(19)^2 - 9.5) + .5(10(16) - .25(16)^2 - 32)$$

$$E(\pi) = .5(90.25) + .5(64) = 77.125$$

MUST MAKE SURE WANT TO ACTUALLY BUY FROM HIGH COST

Contracting in Principal Agent Relationships With Moral Hazard

How can a principal motivate an agent when effort is not directly observed?

Make payment contingent on observed outcomes of job performance. If no uncertainty, no problem.

But if there is uncertainty coupled with unobservable effort – there is an important moral hazard problem. If work hard, and, have bad luck, low pay.

So, must pay more on average to get agent to agree to contract to overcome risk aversion, and, pay more when the firm does well to motivate the unobservable effort.

Important idea**: Tradeoff between risk and incentives in contracting.

Possible signals on which to base pay:

a) Sales
b) Cost
c) Profit
d) Complaints
e) Reports of monitors
f) other firms – "Yardstick Competition"

The better the signal is correlated with effort – the stronger the incentives tied to it should be. The noisier the signal, the weaker the incentives tied to it should be.

Model

Value (before cost of contract) is either high, V_H, or, low, V_L.

Effort, e, is either high, e_H, or low, e_L.

f: $Pr(V_H|e_H)$, g: $Pr(V_H|e_L)$

Agent's utility function is u(w)-c(e).

u(w) is a standard expected utility function

c(e) is the disutilty of effort, or, the utility cost of effort.

For simplicity, c(H)=c, c(L)=0.

u_R: agent's reservation utility, $u(w_R)$. w_R: reservation wage, wage must earn for certain with low effort to make willing to accept offer.

w_H wage if value is high, w_L wage if value is low.

Principal's problem:

$$\min_{w_H, w_L} \quad fw_H + (1-f)w_L$$
$$subject \quad to \quad fu(w_H) + (1-f)u(w_L) - c \geq u(w_R) \qquad PC$$
$$fu(w_H) + (1-f)u(w_L) - c \geq gu(w_H) + (1-g)u(w_L) \qquad IC$$

In general, both the constraints "bind". ***Why?***

Solving, first work from the IC

$$u(w_L) + f\left(u(w_H) - u(w_L)\right) - c = u(w_L) + g\left(u(w_H) - (w_L)\right)$$
$$\left(f-g\right)\left(u(w_H) - u(w_L)\right) = c \qquad Interpretation?$$
$$u(w_H) - u(w_L) = \frac{c}{f-g}$$

Now go to the PC

$$u(w_L) + f\left(u(w_H) - u(w_L)\right) - c = u(w_R)$$
$$u(w_L) + \frac{f}{f-g}c - c = u(w_R)$$
$$u(w_L) = u(w_R) + c - \frac{f}{f-g}c$$
$$u(w_L) = u(w_R) + c\frac{f-g-f}{f-g}$$
$$u(w_L) = u(w_R) - c\frac{g}{f-g}$$

Substitute into previous result:

$$u(w_H) = u(w_L) + \frac{c}{f-g}$$
$$u(w_H) = u(w_R) - c\frac{g}{f-g} + \frac{c}{f-g}$$
$$u(w_H) = u(w_R) + c\frac{1-g}{f-g}$$

Now, how do f, g, c, and w_R change the incentive contract, and, WHY???

Comparative Statics		
	Change in Endogenous Variables	
Exogenous Variable	w_H	w_L
w_R or u_R	+	+
C	+	-
f	-	+
g	+	-

Compare to Certainty

From above:

$$u(w_H) = u(w_R) + c\frac{1-g}{f-g}. \text{ As f}\rightarrow 1,\ u(w_H) \rightarrow u(w_R) + c$$

w_H decreases

And:

$$u(w_L) = u(w_R) - c\frac{g}{f-g} \text{ As g}\rightarrow 0,\ u(w_L) \rightarrow u(w_R)$$

w_L increases

Uncertainty and imperfect info means lower "base" pay and larger "bonus".

THAT means more introducing uncertainty into risk averse manager's contract.

That increases expected compensation costs!

Trade off between risk and incentives!!!

Example: $f = 0.8 \quad g = 0.4 \quad u = w^{0.5} \quad u_R = 10 \quad c = 8$

With certainty:

$w^{0.5} - 8 = 10$

$w^{0.5} = 18$

$w = 324$

With uncertainty:

$$\begin{aligned} &Min && 0.8w_H + 0.2w_L \\ &s.t. && 0.8w_H^{0.5} + 0.2w_L^{0.5} - 8 \geq 10 \\ & && 0.8w_H^{0.5} + 0.2w_L^{0.5} - 8 \geq 0.4w_H^{0.5} + 0.6w_L^{0.5} \end{aligned}$$

Both constraints bind. Working from the incentive compatibility constraint:

$0.4w_H^{0.5} - 0.4w_L^{0.5} = 8$

$w_H^{0.5} - w_L^{0.5} = 20$

$w_H^{0.5} = w_L^{0.5} + 20$

Substituting into the participation constraint:

$0.8(w_L^{0.5} + 20) + 0.2w_L^{0.5} - 8 = 10$

$w_L^{0.5} + 16 = 18$

$w_L^{0.5} = 2$

$w_L = 4$

$w_H^{0.5} = 20 + w_L^{0.5} = 22$

$w_H = 484$

$E(w) = 0.8 \times 484 + 0.2 \times 4 = 388$

Extra cost due to uncertainty:

$388-$324=$64

May have both Adverse Selection and Moral Hazard in the same transaction.

Pitfalls of Contracting with Incentives

Commitment is crucial – must actually deliver incentive pay in the way expected. Also – can kill morale if there is a misunderstanding

“Ratchet” effect. Do not revise incentives rapidly based on past performance under incentive system.

Be extremely careful how these incentives are structured. If not just right, can have bad consequences. Encourage wrong behavior.

Powerful incentives may be much more appropriate for sophisticated decision makers (top management and CEOs) than for others.

With bad corporate governance, can get bad contracts, excessive CEO pay, gaming the system, etc…

Upshot, labor and procurement contracts should build in incentives for effort provision and for telling the truth about hidden information. But, do so with great care.

Remember, recruiting individuals with a “reputation” as honest and fostering repeated interactions can have similar effects – is worth paying for. (As long as they are not too near retirement!)

21. THE FIRM

Why is economic activity organized in firms? How should firms be organized? What is the relationship between the manager and stockholders? What is the role of the manager?

To produce goods, need inputs, production processes. Ways to get them:

1) Spot Market
2) Contract with a supplier, worker, or partner
3) Integrate the activities into a firm

Spot Markets

Anonymous buyers and sellers, near perfect information, standardized products.

Allows high degree of specialization, low unit cost

Problems arise if:

- high transactions costs (search, negotiation)
- highly specific product / input / labor needs
- imperfect or asymmetric information (wrt quality, for example)

When spot market breaks down due to any of these, can lead to role for firms.

Consider 3 related problems in some detail: Team Production, Relationship Specific Investments, and, Double Marginalization

Team Production & the Free Rider Problem

Two people participate in a "team" to produce something of value $V(e_A,e_B)$.

e_A and e_B denote effort, not just hours.

$C_A(e_A)$, $C_B(e_B)$: $ must be paid to willingly put forth that much effort.

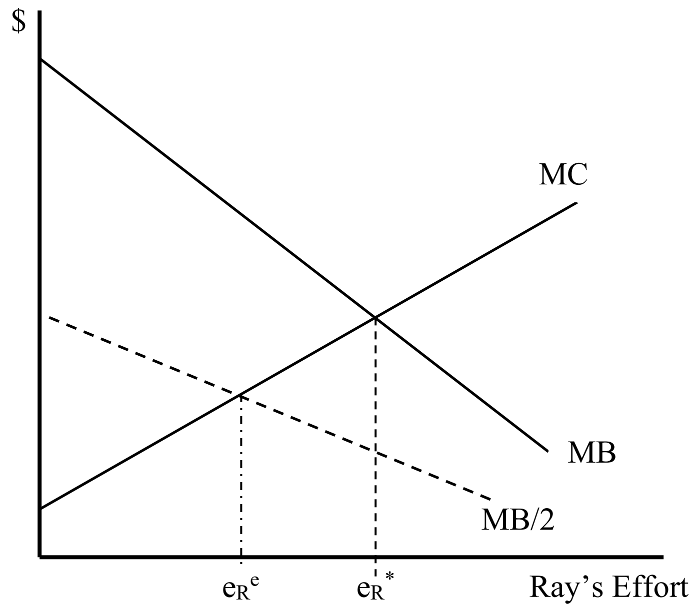
S: payment less cost of effort

From joint view, should set MB=MC for both. Gives "efficient" efforts, e*.

Taking B's effort as given at the efficient level, graph shows MB_A and MC_A.

Suppose they "split" value. Each gets half the MB of their effort, bears the full cost.

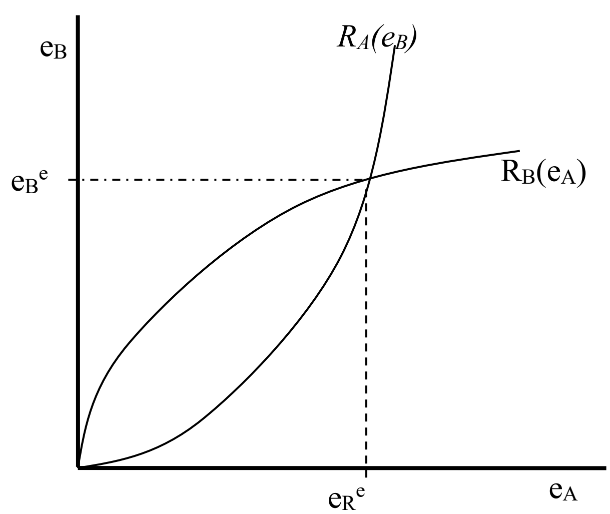
Figure shows for A, holding B's effort at the most efficient level. But, true for B as well.



Fact that B puts in less effort reduces the MB of A's effort, so A puts in even less. And, so on. So, both will put in too little effort.

This is known as "free riding"

Solution of MB_A=MC_A yields A's reaction function, $R_A(e_B)$. Same for B. NE is where the two cross.



Formally

Maximizing total surplus (cooperative):

$$S_{TOT} = V(e_A, e_B) - C_A(e_A) - C_B(e_B)$$

$$\frac{\partial S_{TOT}}{\partial e_A} = MB_A - MC_A = 0$$

$$\frac{\partial S_{TOT}}{\partial e_B} = MB_B - MC_B = 0$$

Solution yields efficient effort levels for each, e*.

Now, suppose they share gross value.

Maximizing individual surplus:

$$S_A = V(e_A, e_B)/2 - C_A(e_A)$$

$$\frac{dS_A}{de_A} = \frac{1}{2}\frac{dV}{de_A} - \frac{dC_A}{de_A} = 0$$

$$MB_A/2 = MC_A$$

This gives $R_A(e_B)$. Same for B!

NE efforts below efficient levels.

Example: $V = 8e_R^{.5}e_W^{.5},\ C_R(e_R) = .25e_R^2,\ C_W(e_W) = .25e_R^2$

Treat as non-cooperative game

$$S_R = 4e_R^{.5}e_W^{.5} - .25e_R^2$$

$$\frac{dS_R}{de_R} = 2\frac{e_W^{.5}}{e_R^{.5}} - .5e_R = 0$$

Using symmetry:

$$2 = .5e_R$$

$$e_R = e_W = 4$$

$$S = 8(2)(2) - .25(16) - .25(16)$$
$$S = 32 - 4 - 4 = 24$$

If work together efficiently:

$$S = 8e_R^{.5}e_W^{.5} - .25e_R^2 - .25e_W^2$$

$$\frac{dS_R}{de_R} = 4\frac{e_W^{.5}}{e_R^{.5}} - .5e_R = 0$$

Using symmetry:

$$4 = .5e_R$$

$$e_R = e_W = 8$$

$$S = 8\sqrt{8}\sqrt{8} - .25(64) - .25(64)$$
$$S = 64 - 16 - 16 = 32$$

In non-cooperative Nash equilibrium, each **FREE RIDES** on the other's effort – producing where half the MB to the firm equals their own MC of effort → inefficiently low effort, both worse off than if cooperated.

Team incentive problems get worse as # on team increases, with n members, each gets 1/n of the MB of their effort.

Solutions:

1) Work with "crazy" (honest, etc…) partners, or, those that have invested in good reputation & value keeping it. Harder as team size increases.
2) Trigger strategies to sustain cooperation if the interaction is repeated. Harder as team size increases.
3) Binding Contracts. Paid only if put forth efficient effort. May be difficult or not be feasible.

A Further Complication: Unobservable Effort and Noise

If effort not observable, not directly contactable.

Solution: write the contract on output. Only pay if value corresponds to value at efficient effort levels. May not eliminate free riding as a nash equilibrium

Problem: What if one member puts forth efficient effort other shirks? Neither gets paid. Is this really enforceable? Would you agree to such a contract?

Problem: what if just a bit of uncertainty in relationship between effort and output? Need incentive contracts. Reduces efficiency – trade off between incentives and insurance.

4) Form a firm. Stockholders are "residual claimants" – have the incentive to monitor and enforce contract for efficient effort. They pay workers only if they put forth efficient effort and they keep all surplus. So, they have incentive to monitor and to enforce contract. Must be able to commit to actually paying, and, labor contract must be enforceable.

Relate all this to the principal's reputation for being "honest". Example: coaches.

Relationship Specific Investments

Specialized investments may be required to facilitate exchange.

Investment has little or no value if the 2 parties do not trade.

Three Common Types:

1) Location, or, site specificity – coal mine or rail line next to power plant.

2) Physical Asset specificity – particular machinery that matches facilities of trading partner

3) Specific Human Capital (May relate to team production)

Immediate Consequence: only a few parties. No market, no prices, everything hinges on negotiations, bargaining, specific to that particular situation.

Repeated bargaining can be ***very*** costly.

<u>Hold-Up Problem:</u>

Suppose the supplier sinks $1 million into a specific investment to develop a product just for you. After that, cost will be $100 per unit.

Once sunk, you have the incentive to renegotiate price closer to $100.

Anticipating this opportunism, the supplier will under invest in development.

Also possible on buyer side.

Generally, the possibility of such opportunism with specific investments leads to inefficiently low investment.

Generally: net value if supplier and buyer trade efficient amounts after up front sunk investment of I_S by supplier and I_B by buyer is $V(I_S,I_B)$.

Assume bargaining power is such that they just evenly split Gains from trade (This is the equilibrium with equal bargaining power under Nash bargaining).

Efficient solution:

$$S = V(I_S, I_B) - I_S - I_B$$

$$\frac{\partial V}{\partial I_S} = 1 \quad \frac{\partial V}{\partial I_B} = 1$$

Non-Cooperative Solution:

Renegotiation means each will get half of V after investment is sunk

$$S_S = \frac{V(I_S + I_B)}{2} - I_S$$

$$\frac{1}{2}\frac{\partial V}{\partial I_S} = 1 \quad \frac{1}{2}\frac{\partial V}{\partial I_B} = 1$$

In general, both under invest in product development in the NE.

Example: $V = 100 I_S^{0.5}$.

Efficient Solution:

$$S = 100 I^{.5} - I$$

$$\frac{\partial S}{\partial I} = \frac{50}{I^{.5}} - 1 = 0$$

$$I = 2500$$

$$S = 100(50) - 2500 = 2500$$

$$S/2 = 1250$$

Non-Cooperative Solution

Renegotiation means supplier will get half of $100I^{.5}$.

$$\pi = \frac{100 I^{.5}}{2} - I$$

$$\frac{\partial \pi}{\partial I} = \frac{25}{I^{.5}} - 1 = 0$$

$$I = 625$$

$$\pi = 50(25) - 625 = 625$$

$$S = 100(25) - 625 = 1875$$

Show graphically

Solutions:

1) <u>Dual sourcing</u> or developing at least 2 customers. Can be expensive!

<u>2) Contracting</u>

If all terms, requirements, contingencies, easily specified up front, contracting can

a) get around hold-up problem

b) reduce costs of repeated bargaining

But, there are contracting costs as well

a) costs of negotiating contract

b) costs of being tied down – may change your mind about what you want, how you want it, demand may dry up, etc….

Optimal contract length balances these costs

More specialized investments → longer contract

More bargaining costs → longer contract length

More complex contracting environment → Shorter Contract Length

<u>3) Vertical Integration – Firm Formation</u>

If contracting environment costly, or infeasible, due to complexity, uncertainty, or, unenforceability, may produce input internally.

That has costs as well

a) loss of specialization – not as efficient

b) still have to deal with internal contracts, even if "implicit". Monitoring, incentives, etc…. Just puts final say on disputes in the hands of a CEO – saves negotiating costs, cuts though complexity, etc… *as long as CEO trusted*

Good CEO reputation (never not doing what is expected/stated) becomes ***EXTREMELY*** important! Observed failing that once → implicit contracts become much less effective.

Double Marginalization and Vertical Integration

Two firms with market power, Upstream, U, and Downstream, D.

Upstream produces a component used by the downstream firm to serve final demand.

For simplicity, 1 unit of upstream output needed per unit of downstream output

Example: Engines to be used in Aircraft, etc….

Total cost is: $C_{TOT}(q) = C_U(q) + C_D(q)$

Inverse dem`nd for finished product faced downstream is p(q)

To maximize joint (total) profits: $MR_D = MC_U + MC_D$

If profits are maximized independently, upstream firm charges price above their MC to downstream firm

Downstream firm's marginal cost is price charged by upstream firm plus other costs

Downstream Firm's Problem

$$\pi_D = p(q)q - p_U q - C_D(q)$$
$$MR_D - p_U - MC_D = 0$$
$$MR_D = p_U + MC_D > MC_U + MC_D$$
$$p_U = MR_D - MC_D$$

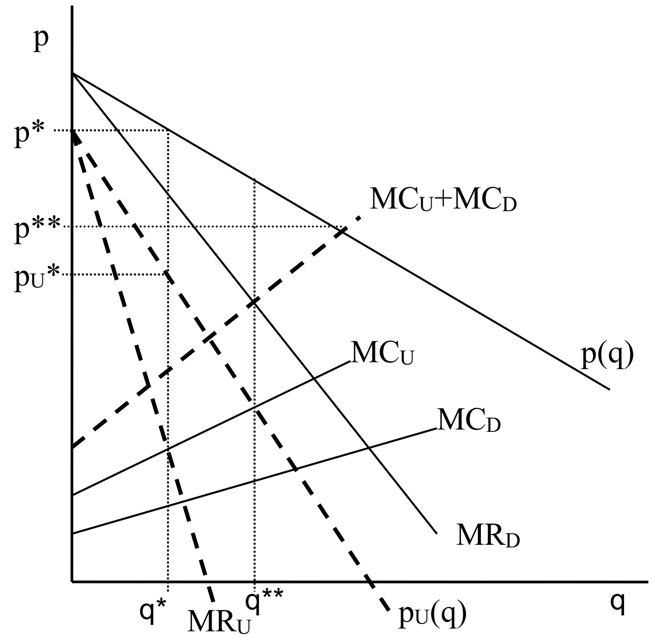
So: 1) since downstream charges a price above that marginal cost, final price is too high, 2) yields residual demand for upstream firm (for graph).

Problem: There are two markups over marginal cost – "Double Marginalization". Final price is too high, final output is too low. Result – lower total profits.

Graphical Analysis

Maximizing downstream profit gave: $MR_D = p_U + MC_D$

Derived inverse demand faced by upstream firm is: $p_U = MR_D - MC_D$



Ex: $p_1 = 11 - .25q_1$, $c_U = .2$, $c_D = .8$

Joint:

$$11 - 0.5q = 1$$
$$q = 20$$
$$p = 11 - 20/4 = 6$$
$$\pi = (6-1)20 = 100$$

Independently:

Downstream:

$$11 - 0.5q = p_U + 0.8$$
$$p_U = 10.2 - 0.5q$$

Upstream:

$$10.2 - q = .2$$
$$q = 10$$
$$p_U = 10.2 - 10/2 = 5.2$$
$$p = 11 - 10/4 = 7.5$$
$$\pi_U = (5.2 - 0.2)10 = 50$$
$$\pi_D = (7.5 - 6)10 = 15$$
$$\pi_D + \pi_U = 65$$

Solutions:

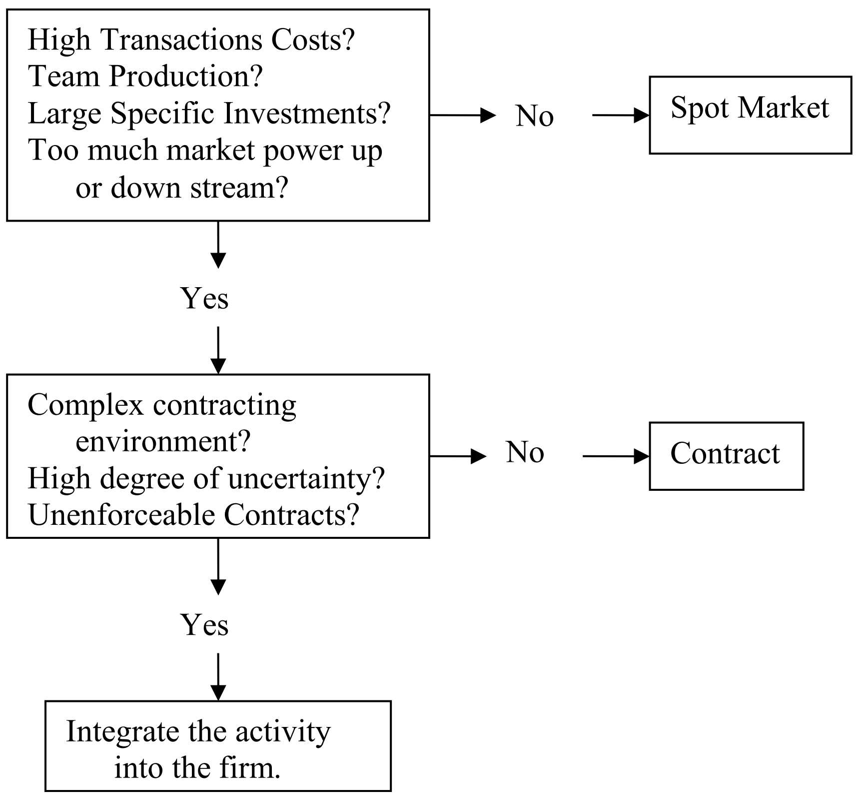
1) Repeated interaction, reputations
2) Explicit Contracts
3) Vertical Integration – Not perfect, still may have problem between divisions!

Transfer Pricing

In firms divided into upstream and downstream divisions, must ensure that upstream division charges $p_U = MC_U$ to downstream division to avoid double marginalization

What is a firm?

A firm is a collection of implicit and explicit agreements between agents (management and workers) and principals (stockholders and management), spelling out responsibilities and who has final say in disputes, constructed to deal with productive relationships and activities too complex to handle through the spot market or contracting.



Free Enterprise: Firms represent collectivized economic activity, but we let the market (partially) decide the distribution, extent, and boundaries of collectivization. Just call the collectivized groupings “firms”.