Gus Lipkin - Midterm
Monday, March 1, 2021 5:29 PM

Question 1

Demand may be high, q=100-2p, with probability 0.6, or low, q=100-4p. Cost is 10 per unit.

The timeline differs from the ones we worked before. The order is:

i) All production is completed and a price is posted.
ii) Nature determines the level of demand, but the firm cannot observe it.
iii) Consumers purchase all they want at the posted price.
iv) The firm only learns if demand was high or low based on how much consumers purchase.
v) Unsold items are disposed of for free.

a) Set up the profit maximization problem, find the optimal price and quantity and the maximum expected profit. Some hints:
- ~~set up profit as a function of q_H and q_L~~ since we are stuck with a single price no matter if demand is high or low. This means you will take derivatives with respect to p, not q.
- Production costs depend only on the amount produced
- For this problem, you can safely assume the firm would not choose production below quantity demanded at the fixed price when demand is high.
- If you see how to use those three things, this problem is simpler and less tedious than the version of optimal production and pricing we worked before.

$q_H = 100 - 2p_H$ $\quad q_L = 100 - 4p_L$

$2p_H = 100 - q_H$

$p_H = (100 - q_H)/2$ $\quad p_L = (100 - q_L)/4$

$\pi = .6[(100-q_H)/2]q_H + .4[(100-q_L)/4]q_L - 10(q_H + q_L)$

$\mathrm{EC}(\pi) = .6(q_H)(p_H) + .4(q_L)(p_L) - 10(q_H + q_L)$ $\quad \mathrm{EC}(\pi) = .6(100-2p_H) + .4(100-4p_L)p - 10(100-4p_L)$

derivative-calculator.net ... $\frac{d\pi}{dp} = 60 - 2.4p + 40 - 3.2p + 20 = 0$

$\frac{d\pi}{dq_H} = \frac{3q_H - 100}{5}$ $\quad \frac{d\pi}{dq_L} = -\frac{q_L}{5}$ $\quad p^* = 21.43$

$\hookrightarrow q_H = \frac{100}{3}$ $\quad q_H = 57.14$ $\quad q_L = 14.29$

$\hookrightarrow q_L = 0$

$p_H = (100 - \frac{100}{3})/2$ $\quad p_L = (100 - 0)/4$ $\quad \mathrm{EC}(\pi) = .6 \cdot 57.14(3 \cdot 4 \cdot 14.29)(21.43)$

$p_H = 100/3$ $\quad p_L = 25$ $\quad = 20.57.14$

$= 23.71$

$\pi = .6 \cdot \frac{100}{3} \cdot \frac{100}{3} + .4 \cdot 0 \cdot 25 - 10(\frac{100}{3} + 0)$

$\pi = 666.66 + 0 - 333.33$

$\pi = 333.33$

b) Suppose unsold product can be stored for future use at a cost of 2 per unit stored, saving $10 per unit in production costs next period. Next period's savings should be discounted. Assume the discount rate is 5%. Set up the profit maximization problem, find the optimal price and quantity and the maximum expected profit.

Without discounting... $\quad +10 - 2$

$\pi = .6[(100-q_H)/2]q_H + .4[(100-q_L)/4]q_L - 10(q_H + q_L) + (.4 \cdot .8)(q_H - q_L)$

$\mathrm{EC}(\pi) = .6(100-2p) + .4(100-4p)p - 10(100-2p) + .4(.8)(100-2p - (100-4p))$

derivative... $\frac{d\pi}{dp} = 60 - 2.4p + 40 - 3.2p + 40 + 6.02 = 0$

$\pi =$ $\quad \hookrightarrow p^* = 22.50$

$q_H = 54.99$ $\quad q_L = 7.99$ $\quad + \frac{(.4 \cdot .8)(q_H - q_L)}{1.05}$

derivative-calculator.net ...

$\frac{d\pi}{dq_H} = -\frac{6q_H - 320}{105}$ $\quad \frac{d\pi}{dq_L} = -\frac{4q_L - 70}{105}$

$\hookrightarrow q_H = 40.333$ $\quad \hookrightarrow q_L = 15.239$

$p_H = (100 - 40.333)/2$ $\quad p_L = (100 - 15.239)/4$

$p_H = 29.833$ $\quad p_L = 21.19$

$\pi = 333.33 + \frac{(.4 \cdot .8)(40.333 - 15.239)}{1.05} = 469.814$ $\quad \mathrm{EC}(\pi) = 411.46$

c) **Explain intuitively why and how the solution in b changes from a.**

In part A, the profit function is an all or nothing scenario where anything produced and not sold is considered waste and discarded. In part B, anything extra that is produced and not sold can be resold at a later date, but for less profit. This means that it is safer to take a risk and produce more. In part A, you under-produce for both high and low demand so you don't go over and have to discard product. In part B, you can overestimate demand a little and still make money back. However, if you over-produce too much, it will begin to eat away at your profits as either the quantity above demand increases, time passes, or both.

d) It costs 40 per period to maintain the storage facility of part b, it will last 5 years, and the interest rate is 5%. What is the value of obtaining such a facility? That is, what is the most the firm should be willing to pay for it? Assume payment for it is made one period before its first use.

Shed value $= \frac{40}{.05} \cdot (1 - \frac{1}{1.05^5}) = 38.095 \cdot .2165 = 8.247$

$\pi =$ $\quad + \frac{(.4 \cdot .8)(q_H - q_L)}{1.05^5}$

$\frac{d\pi}{dq_H} = -\frac{432233 \cdot 03911 - 4576610100}{2042205 \cdot 05}$ $\quad \frac{d\pi}{dq_L} = \frac{4034101 + 8130000}{2042205 \cdot 05}$

$\hookrightarrow q_H = 37.512$ $\quad \hookrightarrow q_L = -12.536$

$\mathrm{EMP}_H = (411.46 - 28.71)\sum_{t=1}^{5}(\frac{1}{1.05})^t = 327.96$ $\quad$ ↳ This doesn't make sense. I'm going to call it 0

$p_H = (100 - 37.512)/2$ $\quad p_L = (100 - -12.536)/4$

$p_H = 31.2439$ $\quad p_L = 26.6282$

$\pi = .6 \cdot 37.512 \cdot 31.2439 + .4 \cdot -12.536 \cdot 26.6282 - 10(37.512 + -12.536)$

↳ $\pi = 171.298$ $\quad \rightarrow 469.814 - 171.298 = 298.516$

The firm should only buy the shed if they are given $298.516

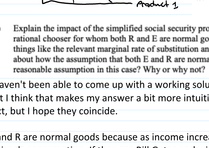
Question 2

Suppose someone is 60 years old and will live 25 more years. Currently they have saved $250 (thousand). They earn a wage, w, of 50 (thousand) per year they work, W. They care only about years they do not work, R (retirement) and how much they spend over the rest of their life, E (for expenditure, in thousands). Ignore discounting and such for this problem.

a) Write the equation for their budget constraint, showing the possibilities for years retired and expenditures. Draw it, placing R on the horizontal axis and E on the vertical axis.

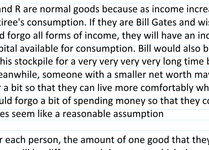
Budget $= 250 + 50(25 - R) - E$

↳ $E = 250 + 50(25 - R)$



b) Suppose now that the government announces a social security program in which they will pay everyone a stipend, s, of $20 (thousand) per year in retirement, provided they are over 60. This is an oversimplified version of social security, but simply ignore other aspects. Write the new budget constraint. Draw it in the figure.

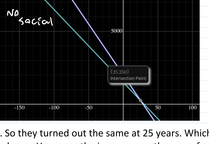
$E = 250 + 50(25 - R) + 20(25 - R)$



c) Draw indifference curves tangent to each budget line, assuming both R and E are normal goods. Any indifference curves satisfying the axioms of rational choice, and for which both E and R are normal goods, are fine. Label the individual's solution for years worked and expenditure with each budget constraint (a and b). Make sure the (R,E) pairs in the solutions you label respect the budget constraints.

I just noticed that there's definitely something wrong with my functions. I should be able to maximize it when I'm not on an axis but I just can't figure out what I'm doing wrong. I'm not able to find a good way to balance the years of retirement and the yearly expenditure. Any ratio of any value existing or derived always gives me the axis values.

Anyways, here's what I know about indifference curves: The indifference curve is where the marginal rate of substitution is equal to the utility of one good over the utility of another. The indifference curve should be tangent to the budget line at that point. I'm definitely having a dumb moment (more than one) and can't find the MRS even though I'm describing it to you. I tried finding the derivatives of the equations I gave above but because it's all constants, the derivative is also a constant. Example curve:



d) Explain the impact of the simplified social security program on years worked for any rational chooser for whom both R and E are normal goods. Be precise, meaning talk about things like the relevant marginal rate of substitution and market rate of trade. Also, talk about how the assumption that both E and R are normal goods matters. Does that seem a reasonable assumption in this case? Why or why not?

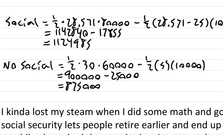
I haven't been able to come up with a working solution for this problem yet but I think that makes my answer a bit more intuitive, rather than based in fact, but I hope they coincide.

E and R are normal goods because an income increases, so too will the retiree's consumption. If they are Bill Gates and wish to retire without working and forgo all forms of income, they will have an incredibly high amount of capital available for consumption. Bill would also be able to retire and live off of his stockpile for a very very very very long time because it is so large. Meanwhile, someone with a smaller net worth may choose to keep working for a bit so that they can live more comfortably while they are retired. Or they could forgo a bit of spending money so that they could be retired longer. This does seem like a reasonable assumption

For each person, the amount of one good that they're willing to give up for the other will be different and the rate at which these curves change will also be different. This curve is the indifference curve from part c.

All that said, let's take my equations from part a and b, even if they're wrong. We can set them equal to each other to find out how long someone would have to work for an equal amount of spending money per year after 60 both with and without the social security program.

$\frac{250 + 50(25 - R)}{25} = \frac{250 + 50(25 - R) + 20(25 - R)}{25}$ $\quad \rightarrow R = 25$



Shoot. Have I been doing it right the whole time?

Ok. So they turned out the same at 25 years. Which is great because you're dead now. However, the income over the years from social security was much greater than that of the person who did not take part in the social security program.

$\text{Social} = \frac{1}{2} \cdot 28.571 \cdot 70000 - \frac{1}{2}(28.571 - 25)(10000)$
$= 1142857 - 17855$
$= 1124985$

$\text{No Social} = \frac{1}{2} \cdot 30 \cdot 60000 - \frac{1}{2}(5)(10000)$
$= 900000 - 25000$
$= 875000$

I kinda lost my steam when I did some math and got confused. Basically, social security lets people retire earlier and end up in the same spot as they would've been had they not had assistance and worked longer.