Take Home Final

Sunday, April 25, 2021 6:46 PM

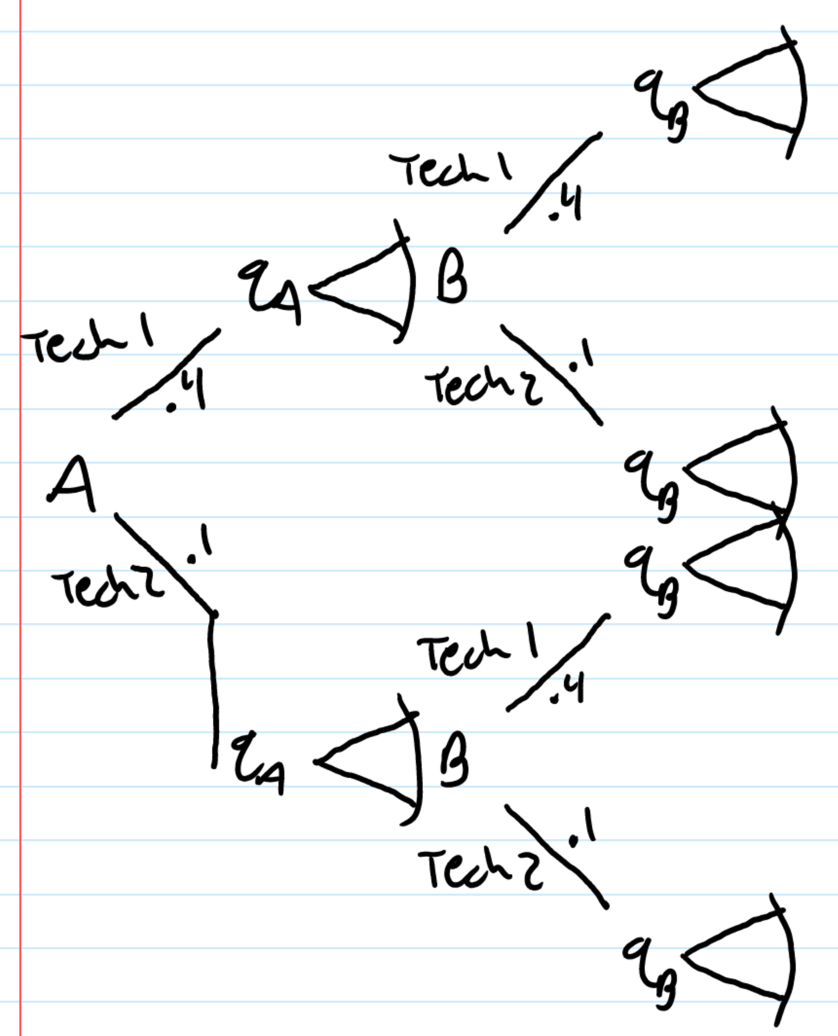
Question 1

We worked through much of this one in class. Here you will finish the analysis.

Amy and Bob compete in homogeneous product markets. Units are chosen for price and quantity so that the linear approximation of demand is p=1-q. This looks quite special and simplistic. But, by choosing units for price and quantity appropriately, *any* demand curve has a local approximation that can be written this way. So really we have to assume we are conducting this exercise as an approximation in the local area of the solution.

Each must choose technology and quantity. The cost functions have the form C=F+cq. For technology 1, c=0.4 and F=0. For technology 2, c=0.1 and F>0. Amy moves first.

A) Diagram the extensive form of the game.



B) Work out when Bob would or would not choose c=0.1 as a function of F and Amy's quantity.

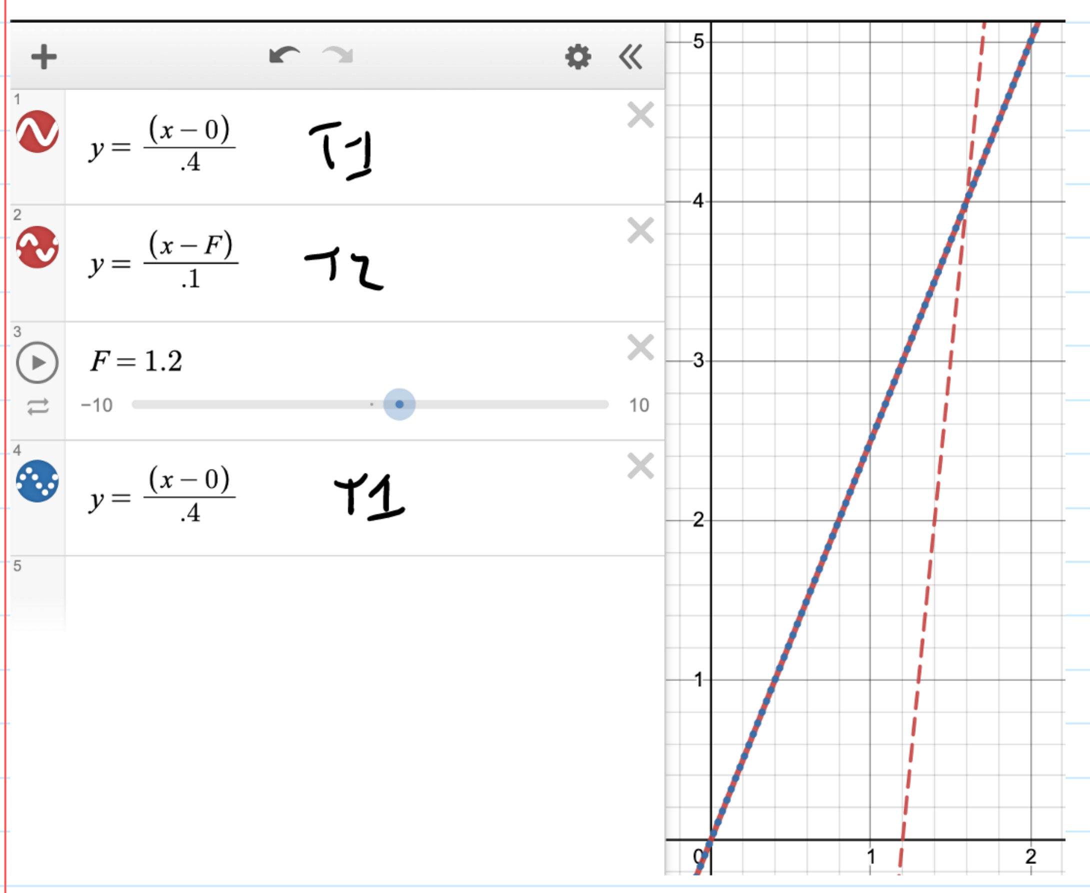
$C_A = f_A + c_a q_A \rightarrow q_A = \frac{C_A - f_A}{c_a}$

$x = C_A$

$\rightarrow \frac{x}{.4} = \frac{x-f}{.1} \rightarrow \frac{.1x}{.4} = x - f \rightarrow .25x = x - f \rightarrow f = .75$

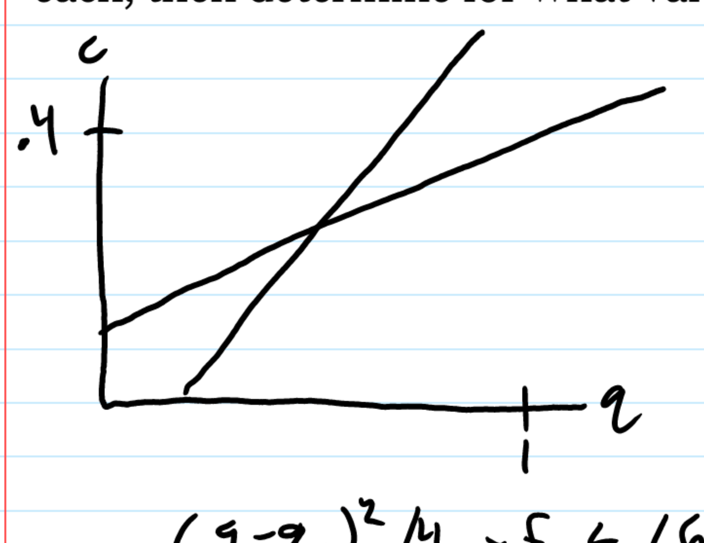
If $0 \le f \le .75$, bob will choose Tech 1 with c=.1 otherwise, he chooses tech 2

C) Assume F is so high a monopolist would choose technology 1 with c=0.4. In that case Bob, moving second, definitely chooses technology 1. Why?



If F is so high a monopolist would choose tech 1, then Bob chooses tech 1 also because if the monopolist doesn't believe they can cross the threshold to where tech 2 is better, then there is no way that Bob, not a monopolist, could make it to that point either.

D) What is the highest value of F for which Amy chooses technology 2, the low MC technology? At this point, there are two Stackelberg games to analyze, one for each of Amy's choices of technology. To answer this question, you must find Amy's profit in the solution to each, then determine for what values of F she would prefer the second technology.



$f + .1 = .4 \rightarrow f = .3$

$P = 1 - Q$

$P_A = (1 - q_A - q_B)q_A - .4q_A$

$(.9 - q_A)^2/4 - f < (.6 - q_A)^2/4 \rightarrow f > -\frac{.6q_A - .45}{4}$

$C = f + cq$

$\pi = (1 - q_A)q_A - Cq_A$

$\pi = (1 - q_A)q_A - q_A\left(\frac{f + cq_A}{}\right)$

$= q_A - q_A^2 - q_A\left(\frac{-.6q_A + .45}{4}\right) - cq_A^2$

$= q_A - q_A^2 - .15q_A^2 + .1125q_A - cq_A^2$

$= -cq_A^2 - 1.15q_A^2 + 1.115q$

$c = .1$

$= -.1q^2 - 1.15q^2 + 1.115q$

$\pi = -1.25q^2 + 1.115q$

$\frac{d\pi}{dq} = \frac{-500q - 223}{200} \rightarrow q = -223/500$

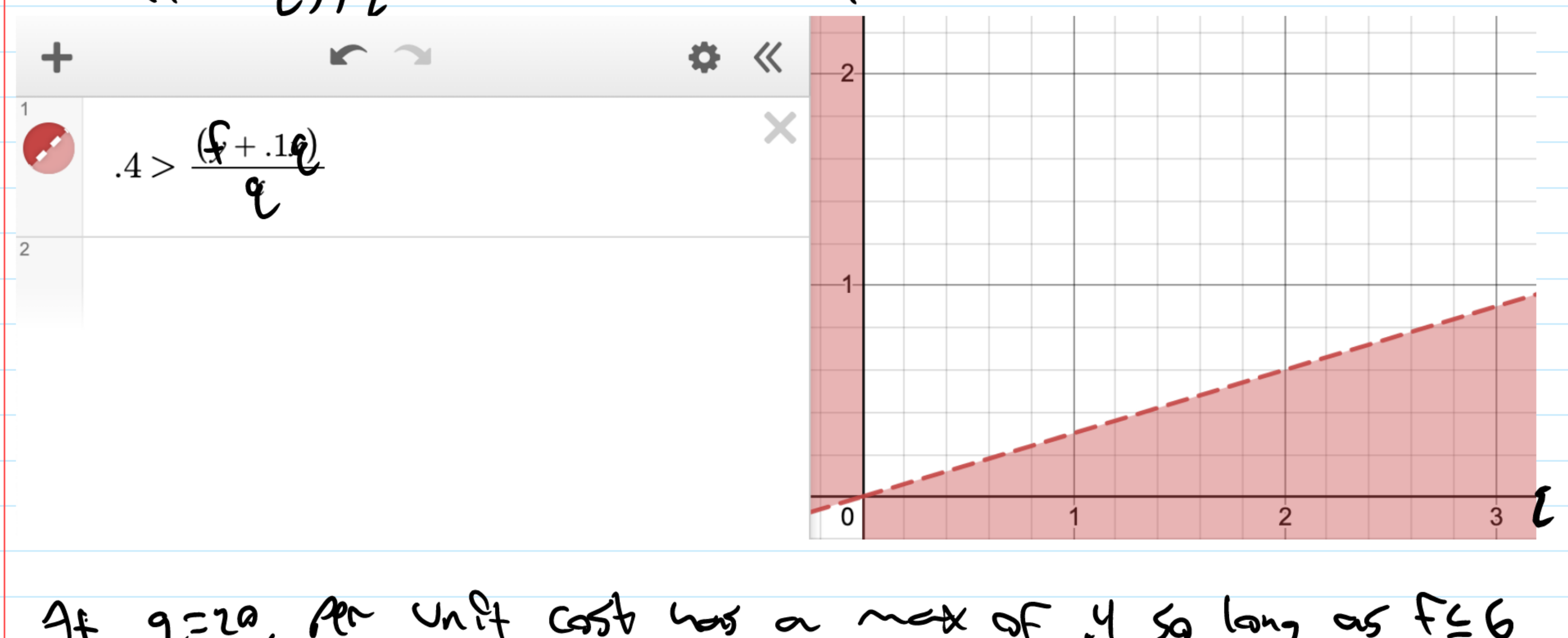
$f > -.6(-223/500) - .45/4 \rightarrow f = \frac{.2676 + .45}{4} = .1794$

If F > .1794, then Amy prefers technology 2

E) Are there any cases where Amy would produce so much that Bob stays out (his quantity is 0)? To think about this question, since Bob's marginal cost is 0.4 and his fixed cost is 0, what would price have to be if Bob produced 0 to keep him from producing 0? What would Amy have to produce to drive price to that level? What would Amy's profit be if she produced that much? How does that compare with your work from (D)?

If Amy's total per unit cost is less than .4, Bob won't produce because then he can't compete. Amy would have to choose tech 2 for this to happen.

$.4 > (f + .1q)/q$



At q=20, per unit cost has a max of .4 so long as $f \le 6$

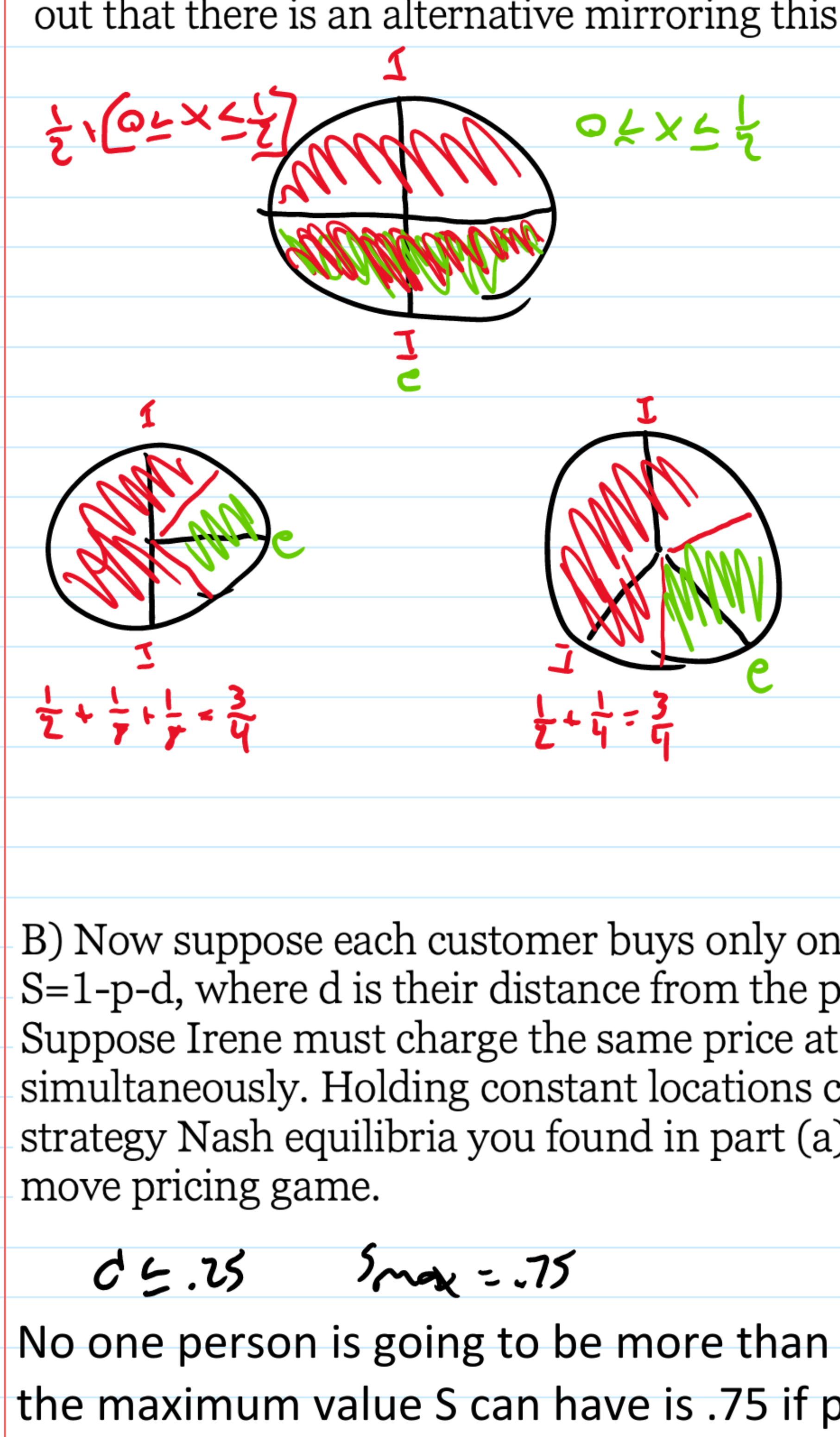
F) In the case where Amy chooses technology 2, might society be better off is she was prevented from doing so? Might society be better off if Amy was a monopolist and did not face potential competition from Bob? Why?

If Amy chose technology 2, she could flood the market until her cost per unit is incredibly low and block out any other competitors. If she was prevented from doing this, there would be a bit more competition and prices would be lower. Or she could be regulated like a utility so that people could still afford her product. Alternatively, if she was a monopolist and did not face competition from Bob, there would be no reason to flood the market but then we'd be in the same spot as before where she could charge whatever she wanted and would need to be regulated.

Question 2

Irene and Eddie compete in differentiated products where the differentiation can be modeled as location on a circle of circumference 1. 100 customers are uniformly distributed around the circle. Irene, the incumbent, already has a product at location 0. She moves first and introduces one more product at a location of her choosing. Then Eddie introduces a product at a location of his choosing, having observed Irene's choice.

A) Assume all customers purchase one unit of the nearest product. Where will Irene and Eddie locate the two new products in the pure strategy Nash equilibria? Hint: there are two such equilibria, but they mirror one another. I suggest just drawing a circle and playing with locations to get some intuition. Then posit an intuitive equilibrium and prove it is one, rather than constructing the equilibrium from some maximization process. You can then just point out that there is an alternative mirroring this solution with the same properties.



Irene will open her second store directly across the circle from her first store. Eddie will then open his store at the same place as one of Irene's existing stores. Irene is guaranteed to get 50% of the total market (the half of the population closest to the store that doesn't have Eddie next door). The remaining half of the market will be split between Irene and Eddie where 50 = Iq + Eq (Irene's quantity and Eddie's quantity). They each have potential for between 0 and 50 customers and their competitor will get the remainder.

B) Now suppose each customer buys only one product to maximize their consumer surplus, S=1-p-d, where d is their distance from the product they choose and p is the price they pay. Suppose Irene must charge the same price at both of her locations. Prices are announced simultaneously. Holding constant locations corresponding to either of the (mirror) pure strategy Nash equilibria you found in part (a), find the Nash equilibria of the simultaneous move pricing game.

$d \le .25 \qquad S_{max} = .75$

No one person is going to be more than .25 away from a store. This means that the maximum value S can have is .75 if p is 0. If Irene's price is higher than Eddie's she'll lose all of the sales on that side. This means that their equilibrium prices have to be equal. How that specific value is decided, I don't know.

C) Consider a game where locations are chosen as in (a) and then prices are chosen as in (b). Are the locations from (a) an equilibrium in this game? If not, why?

If prices are chosen separately and simultaneously, then the location game in part a is not an equilibrium in this game. In part a, Irene has to offer the same price in both locations so if Eddie offers a lower price in accordance with part b, he could conceivably pull customers from over the 50% mark if his prices were low enough. Irene would want to share as little customer space as possible so long as she puts her second location anywhere that's not equidistant and Eddie chooses the side with a larger customer base, Eddie will end up with 25% of the customers and Irene the other 75%.