Problem 12

Tuesday, February 9, 2021 7:34 PM

Inverse demand will be p_H=12-0.005q_H with probability 0.4, and otherwise p_L=12-0.01q_L. Any product not sold must be disposed of at a cost of $1 per unit. For parts a and b, production must take place *before* demand uncertainty is resolved and cost per unit is constant at $3.

$E(\pi) = .4(12-.0005q_H)q_H + .6(12-.01q_L)q_L - 3q_H - .6\cdot 1(q_H - q_L)$

a. Find the profit maximizing prices and quantities for each state of demand.

$\cancel{\pi = (12-.005q_H)q_H + (12-.01q_L)q_L - 3(q_H+q_L)}$

$\cancel{\frac{d\pi}{q_H} = 9 - \frac{q_H}{100} \to q_H = 900}$ $\frac{d\pi}{dq_H} = .4(12-.01q_H) - 3 - .6 = 0 \to q_H = 700$

$\cancel{\frac{d\pi}{dL} = 9 - \frac{q_L}{50} \to q_L = 450}$ $\frac{d\pi}{dq_L} = .6(12-.02q_L) + .6 = 0 \to q_L = 650$

$\cancel{P_H = 12 - .005(900) = 7.5}$
$\cancel{P_L^H = 12 - .01(450) = 7.5}$ ~~→ that's no good... or is it?~~

$\cancel{\pi_H = 900 \cdot 7.5 = 4050}$
$\cancel{\pi_L = 450 \cdot 7.5 = 2025}$ $\cancel{\to E(\pi) = (.4 \cdot 4050) + (.6 \cdot 2025) = 2855}$

$P_H = 12 - .0005 \cdot 300 = 10.5$
$P_L^H = 12 - .01 \cdot 650 = 5.5$ $q_L \,!< q_H$ so $q_L = q_H$

$\cancel{\pi = (12-.005q)q + (12-.01q)q - 3q}$
$\cancel{\frac{d\pi}{q} = 21 - \frac{3q}{100} \to \frac{3q}{100} = 21 \to 3q = 2100 \to q}$

~~wrong it's -3(2q)~~
$E(\pi) = .4(12-.0005q)q + .6(12-.02q)q - 3q$
$\frac{d\pi}{dq} = .4(12-.01q) + .6(12-.02q) - 3 = 0$
$\to q = 562.5$

$\cancel{\pi = (12-.005q)q + (12-.01q)q - 6q}$

$\cancel{\frac{d\pi}{q} = 18 - \frac{3q}{100} \to q = 600}$

$\cancel{P_H = 12 - .005(600) = 9 = P_H}$
$\cancel{P_L^H = 12 - .01(600) = 6 = P_L}$

$P_H = 12 - .0005 \cdot 562.5 = 9.19$
$P_L = 12 - .01 \cdot 562.5 = 6.38$
~~→ high. forgot to account for probability~~
$\to E(\pi) = (.4 \cdot 9.19 + .6 \cdot 6.38 - 3) 562.5$
$= 2531.25$

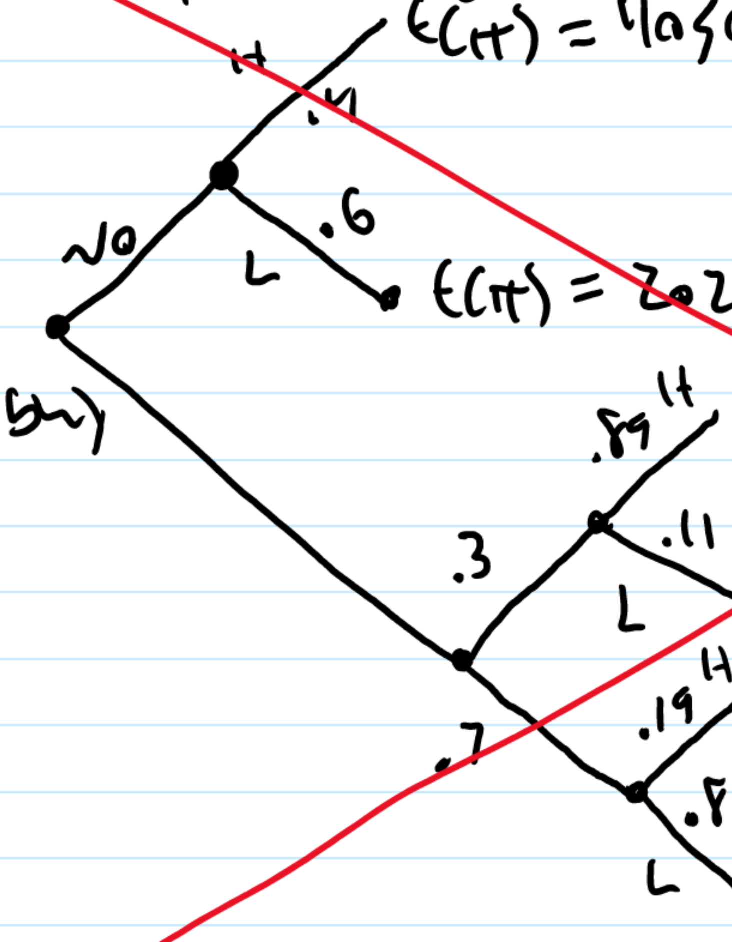
$\cancel{\pi_H = 600 \cdot 9 = 5400}$
$\cancel{\pi_L = 600 \cdot 6 = 3600}$

$\cancel{E(\pi) = (.4 \cdot 5400) + (.6 \cdot 3600) = 4320}$

b. Suppose you can set up an analytics program to obtain additional information on the probability of high demand. Your best guess is that with probability 0.3 they will tell you the probability of high demand is 0.89, and that otherwise they will tell you the probability of high demand is 0.19. What is the analytics program worth per period?

$\cancel{\pi_L = (12-.01q)q - 3q}$

$\cancel{\pi_H = (12-.005q_H)q_H + (12-.01q_L)q_L - 3(q_H+q_L) - (q_H - q_L)}$



$\cancel{E(\pi) = 4050 - (3 \cdot 900) = 1350 \cdot .4 = 540}$

$\cancel{E(\pi) = 2025 - (3 \cdot 450) = 675 \cdot .6 = 405}$

$\cancel{E(\pi_H) = [2855 - (4050 - 2025)] \cdot .3 \cdot .89 = 221.61}$

$\cancel{E(\pi_L) = (2025 \cdot .3 \cdot .11) = 66.825}$

$\cancel{E(\pi_H) = [2855 - (4050 - 2025)] \cdot .7 \cdot .19 = 269.325}$

$\cancel{E(\pi_L) = (2025 \cdot .7 \cdot .81) = 1148.175}$

$\cancel{E(\text{info}) = 1148.175 - 540 = 608.175}$

$E(H) = .89(12-.0005q_H)q_H + .11(12-.01q_L)q_L - 3q_H - .11 \cdot 1(q_H - q_L)$

$\frac{d\pi}{dq_H} = .89(12-.01q_H) - 3 - .11 = 0$ $\frac{d\pi}{dq_L} = .11(12-.02q_L) + .11 = 0$

$\to q_H = 850.6$ $\to q_L = 650$

$P_H = 12 - .0005 \cdot 850.6 = 7.75$ $P_L = 12 - .01 \cdot 650 = 5.50$

$q_H > q_L$ is good

$E(\pi) = .89 \cdot 7.75 \cdot 850.6 + .11 \cdot 5.5 \cdot 650 - 3 \cdot 850.6 - .11 \cdot 1(850.6 - 650) = 3686.4$

π $P(H) = .19$
$E(\pi) = .19(12-.0005q)q + .81(12-.01q)q - 3q$

$\frac{d\pi}{dq} = .19(12-.01q) + .81(12-.02q) - 3 = 0 \to q = 497.24$

$P_H = 12 - .0005 \cdot 497.24 = 9.51$
$P_L^H = 12 - .01 \cdot 497.24 = 7.03$

$E(\pi) = (.19 \cdot 9.51 + .81 \cdot 7.03 - 3) 497.24 = 2237.57$

$E(\pi \mid \text{info}) = .3 \cdot 3686.4 + .7 \cdot 2237.57 = 2672.22$
$E(\pi \mid \text{no}) = 2531.25$

$\to \text{Info} = 140.97$

c. Assume you do not have recourse to additional information as in (b). Instead, suppose that in addition to your current production line (that costs $3 per unit) you could add a just in time production line with a cost of $5 per unit. Find the maximum expected profit if you add this line, and therefore its value per production period. *Hint*: Since your base line cost is only $3 per unit, you would always use it to produce any units you are certain to sell (low demand sales). The question is whether or not it saves money to use the just in time line for additional production when demand is high. The answer determines how you add the unit cost of units q_H through q_L, and the potential disposal cost, to the problem setup.

$\cancel{\pi = (12-.005q_H)q_H + (12-.01q_L)q_L - 3(q_H+q_L) - 5(q_H - q_L)}$

$\cancel{\text{or } 50/(.04/12) = 15000}$

$\cancel{\frac{d\pi}{dq_H} = 4 - \frac{q_H}{100} \to q_H = 400}$ $E(\pi) = .4(12-.0005q_H)q_H + .6(12-.01q_L)q_L - 3q_L - .4 \cdot 5(q_H - q_L)$

$\cancel{\frac{d\pi}{dq_L} = 14 - \frac{q_L}{50} \to q_L = 700}$ $\frac{d\pi}{dq_H} = .4(12-.01q_H) - .4 \cdot 5 = 0 \to q_H = 700$

$\cancel{P_H = 12 - .005(400) = 10}$ $\frac{d\pi}{dq_L} = .6(12-.02q_L) - 3 + .4 \cdot 5 = 0 \to q_L = 516.67$

$\cancel{\pi = 400 \cdot 10 = 4000}$ $P_H = 12 - .0005 \cdot 700 = 8.50$
$P^H = 12 - .01 \cdot 516.67 = 6.83$

$E(\pi) = .4 \cdot 8.5 \cdot 700 + .6 \cdot 6.83 \cdot 516.67 - .4 \cdot 5(700 - 516.67) = 2581.67$

Value JIT $= 2581.67 - 2531.25 = 50.42$

~~If demand is high, the JIT line has a profit of 4000 while the normal line has a profit of 4050. Thus, the JIT line is not a good idea.~~

d. Continuing from (c), assume the safe rate of interest is 4% annually (so 0.04/12 monthly), and that you make one production run per month. Using the fact that the present value of a perpetual payment of $V starting one period from the current period is $V/r, calculate an upper bound of the expected present value of adding a just in time production line.

$\cancel{\frac{.04}{12} = .003}$

$\cancel{4050 - 4000 = 50}$

$\cancel{\sum_{i=1}^{\infty} \frac{50}{(1+.003)^i} = 15000}$ $\sum_{i=1}^{\infty} \frac{50.42}{(1+.04/12)^i} = 15125$