1) Consider an economy with three people. All have Von-Neuman Morgenstern utility functions $u = \sqrt{c}$ where c is the amount of income left for consumption after taxes. The three individuals' incomes, m, are \$50, \$45, and \$5. The government must raise \$20 in tax revenue to fund necessary services. The government may levy a tax rate of t on income to raise money. The government can also make a lump sum transfer of s to each individual. Each individual's after tax income, for consumption, is then $c=(1-t)m+$s. Consider the following two tax systems. I) A flat tax rate of 20% with a lump sum transfer of 0. II) A tax rate of 50% together with a lump sum transfer of 10.

a) Show that if the tax plan is chosen by simple majority rule, plan I wins 2-1. Assume individuals vote for the plan that gives them the highest utility and are not altruistic.

Individual	Utility under	
	Plan I	Plan II
1	$u=(0.8\times50)^{0.5}=6.32$	$u=(0.5\times50+10)^{0.5}=5.92$
2	$u=(0.8\times45)^{0.5}=6$	$u=(0.5\times45+10)^{0.5}=5.70$
3	$u=(0.8\times5)^{0.5}=2$	$u=(0.5\times5+10)^{0.5}=3.53$
E(u)	4.77	5.05

As shown in the table, individuals 1 and 2 both get a higher utility form plan I, so would vote for it if they are not altruistic.

b) Show that if the individuals had to choose the tax plan from behind a veil of ignorance, plan II would win unanimously.

As shown in the table, if each person had an equal chance of having any of these income levels, that is the choice were made from behind a veil of ignorance, Plan II has the higher expected utility, due to the risk of ending up with an income of only \$10.

c) What, if anything, does this suggest about majority rule voting on public policy regarding taxation and redistribution?

If there is a relatively small chance of ending up very poor, majority rule it likely to end up with too little redistribution to the poor compared to what the majority rule decision would be if everyone had an equal chance of being the ones that end up poor at the time the system of taxation and redistribution is voted on.

2) Find the set of rationalizable strategies for the game in the table to the right. Explain your reasoning.

		Cathy				
		C1	C2	C3	C4	C5
Robert	R1	5, 0	**4**, 3	3, 5	3, 6	1, **7**
	R2	**6**, 1	2, **5**	1, 4	3, 2	2, 1
	R3	2, 4	2, 4	1, 5	0, 5	0, **6**
	R4	3, 1	**4**, 2	**4**, **3**	**4**, 1	**3**, 2

Best replies are marked in bold green.
Round 1) R3, C1, and C4 are never a best response and so are eliminated.
Round 2) R2 is never a best response, and so is eliminated.
Round 3) C2 is never a best response, and may be eliminated.
Round 4) R1 may be eliminated.
Round 5) C5 is eliminated.
That leaves the solution as R4, C3.

3) A firm with a constant marginal cost of \$2.4 faces inverse demand of p=10-0.1q with probability 0.4 (low demand) and otherwise faces inverse demand of p=10-0.05q (high demand). The firm must produce output before demand uncertainty is resolved. Any output not sold must be disposed of. Disposal is free.

a) Show that the quantities and prices in each state of the world are q_L=50, p_L=5, q_H=60, and p_H=7. Also, what is expected profit?

$$E(\pi) = 0.4(10-0.1q_L)q_L + 0.6(10-0.05q_H)q_H - 2.4q_H$$

$$0.4(10-0.2q_L) = 0 \quad 0.6(10-0.1q_L) - 2.4 = 0$$

$$q_L = 50 \qquad q_H = 60$$

$$p_L = 5 \qquad p_H = 7$$

$$E(\pi) = 0.4 \bullet 5 \bullet 50 + 0.6 \bullet 7 \bullet 60 - 2.4 \bullet 60 = 208$$

b) Suppose the firm can build a facility to store any unused output from one period to the next. What is the most the firm would be willing to pay per period for such a facility? Ignore discounting from one period to the next.

The key is to realize that saving unused output saves \$2.4 the next period So:

$$E(\pi) = 0.4(10-0.1q_L)q_L + 0.6(10-0.05q_H)q_H - 2.4q_H + 0.4 \bullet 2.4(q_H - q_L)$$

$$E(\pi) = 0.4(10-0.1q_L)q_L + 0.6(10-0.05q_H)q_H - 0.6 \bullet 2.4q_H - 0.4 \bullet q_L$$

$$0.4(10-0.2q_L-2.4) = 0 \quad 0.6(10-0.1q_H-2.4) = 0$$

$$q_L = 38 \qquad q_H = 76$$

$$p_L = 6.2 \qquad p_H = 6.2$$

$$E(\pi) = 0.4 \bullet 5.8 \bullet 42 + 0.6 \bullet 6.2 \bullet 76 - 4 \bullet 76 + 0.4 \bullet 4 \bullet 34 = 231.04$$

They would pay up to 231.04-208=23.04.

4) Two firms, A and B, engage in quantity competition. Both firm's constant marginal cost is \$2. Both firms have fixed costs of \$5. Inverse demand is p=5-0.1(q_a+q_b).

a) Show that if A moves first the equilibrium quantities and price are q_a=15, q_b=7.5, and p=2.75.

First, start at the end to find firm B's reaction function.

$$\pi_B = (5 - 0.1q_A - 0.1q_B)q_B - 2q_B - 5$$
$$3 - 0.1q_A - 0.2q_B = 0$$
$$q_B = 15 - 0.5q_A$$

Then use that to find A's output.

$$\pi_A = (5 - 0.1(15 - 0.5q_A) - 0.1q_A)q_A - 2q_A - 5$$
$$1.5 - 0.2q_A + 0.1q_A = 0$$
$$q_A = 15$$

From there we can get firm B's quantity, price, and profits.

$$q_B = 15 - 0.5(15) = 7.5$$
$$p = 5 - 0.1 \cdot 22.5 = 2.75$$
$$\pi_A = (2.75 - 2)15 - 5 = 6.25$$
$$\pi_B = (2.75 - 2)7.5 - 5 = 0.625$$

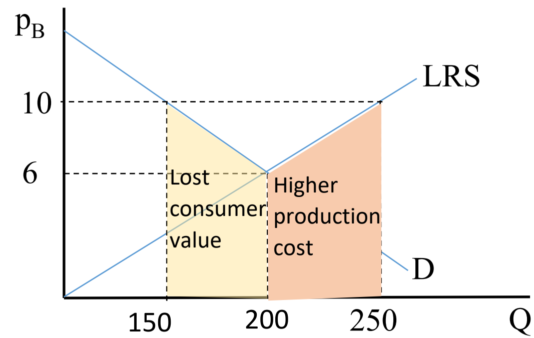
b) Suppose fixed costs are not sunk. That is, they only must be paid if a firm decides to produce more than 0. Describe how you would determine whether firm A could profitably produce enough to prevent form B from producing anything at all. You don't need to do the math, just explain what you would do.

We would need to solve these two equations for q_A and q_B:

$$\pi_B = (5 - 0.1q_A - 0.1q_B)q_B - 2q_B - 5 = 0$$
$$q_B = 15 - 0.5q_A$$

This represents firm B making its best response (its reaction function) and still making no profit. If q_A were a tiny bit larger than the solution to these equations, firm B would make a loss if they produced, and so would choose to stay out. We would then need to calculate A's profit at that quantity. If it is higher than the answer to part (a), a would produce this entry limiting quantity. Otherwise they would stay with the solution from part (a).

5) In this question, you will use supply and demand to analyze the impact of an agricultural price support program. Consider an agricultural market where the equilibrium price is $6 and the equilibrium quantity is 200. The government implements a price support program to keep the price at $10. At that price, the quantity demanded by consumers is only 150 while the quantity supplied is 250. Assume the supply price for unit 150 would have been $3. The government keeps the price at $10 by purchasing and destroying the surplus product. How much does the program cost taxpayers? How much value added is destroyed in the product market? Show an explain your calculations. Illustrate with a supply and demand diagram. Hint: Sketch a diagram first to guide your analysis. Then redraw it neatly when you have the details worked out.



It costs taxpayers $10*100=1000 to purchase the surplus. By itself, this is just a transfer from tax payers to producers. The program does have significant real net costs though.

The full value of units 150 to 200 to customers is lost since before the price floor consumers consumed those units and now they do not. This is equal to 0.5*4*50+6*50=400.

In addition, producers cost go up by the area under the supply curve between units 200 and 250. This is equal to 6*50+0.5*4*50=400.

This was not required, but to sum up, the net cost is $800.