8 Extra Problems

Monday, February 22, 2021 5:42 PM

Utility is $U=H^{0.5}E$ where H is housing in square feet and E is expenditure on everything else. R is monthly rent per square foot. Monthly disposable income is M.

1) Solve for housing demand as a function of M and R. Also solve for the demand for E as a function of M and R.

$E = M - RH$

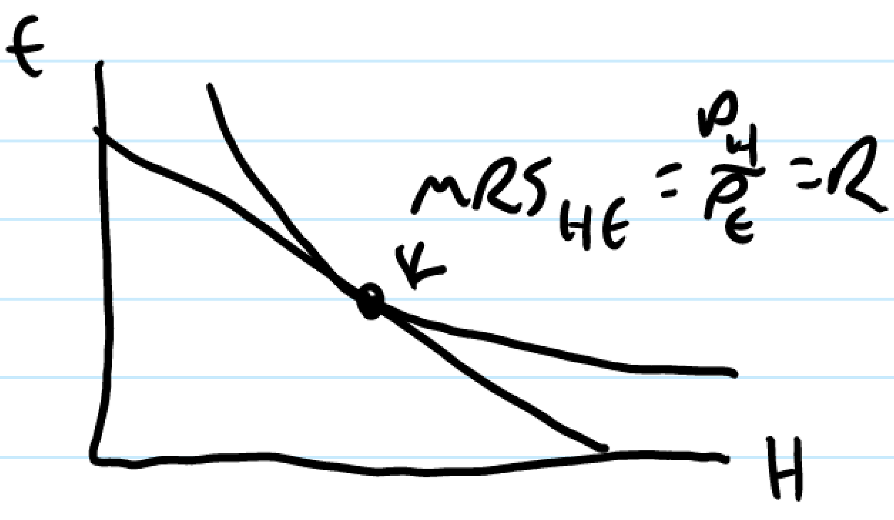
$\frac{du}{dx} = E/2\sqrt{H}$ $\frac{du}{dy} = \sqrt{x}$

$MRS_{HE} = \frac{E}{2H^{.5}H^{.5}} = \frac{E}{2H} = R$

$M = E + RH = 2RH + RH = 3RH$

$H = M/3R$

$E = 2RH = 2R\frac{M}{3R} = \frac{2}{3}M = E$



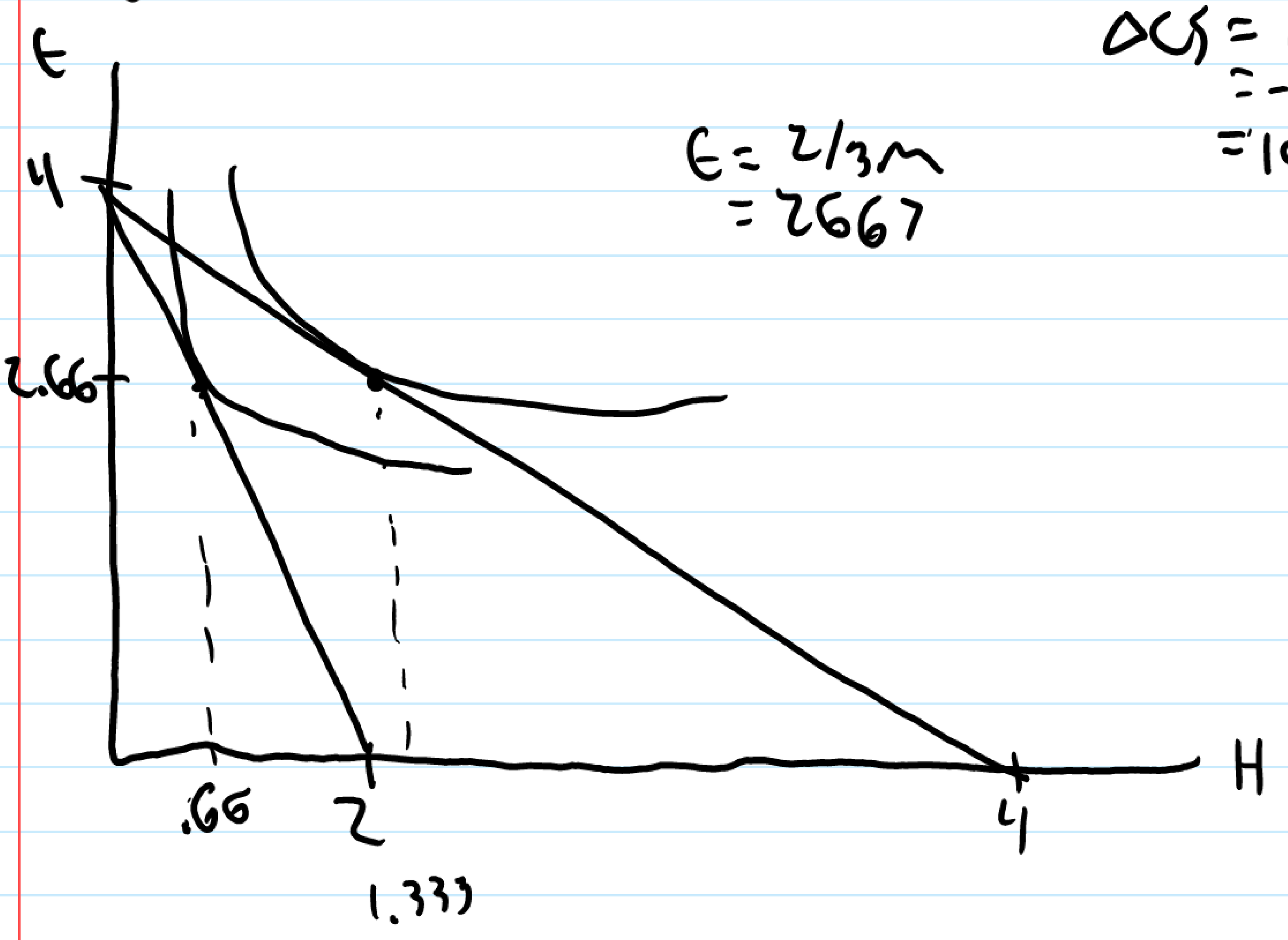
2) The indirect utility function, V, gives maximum utility as a function of income and prices. Substitute your solutions from #1 into the utility function to find the indirect utility function.

$V = \left(\frac{M}{3R}\right)^{.5} \cdot \left(\frac{2M}{3}\right)$

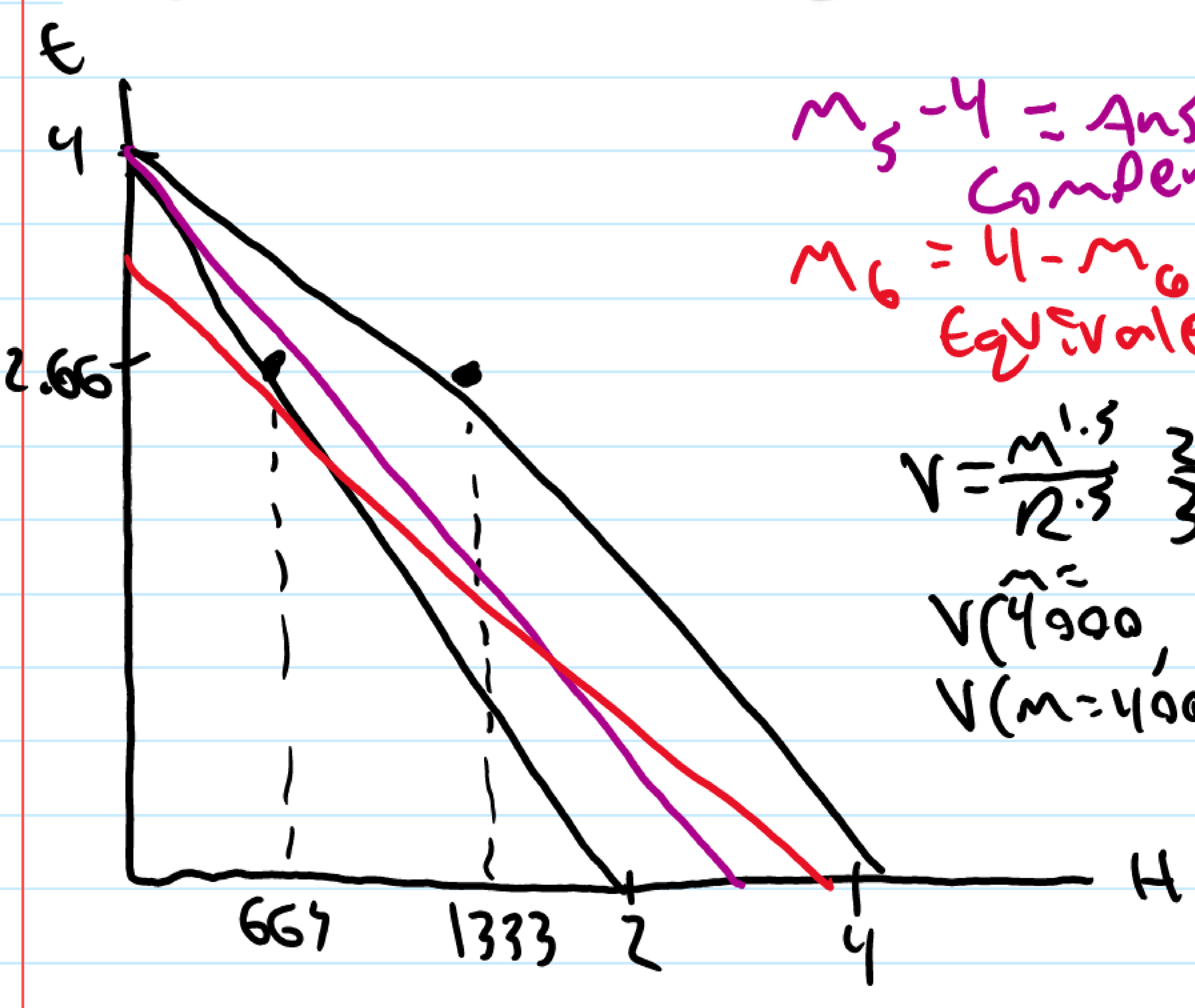
3) Suppose income is 4000 and rent increases from 1 to 2. How does housing consumption change? Be specific. Sketch both the consumer's optimization problem and the demand curve.

$\frac{4000}{3 \cdot 1} - \frac{4000}{3 \cdot 2} = 1333\,^1/_3 - 666\,^2/_3$

$\Delta CS = (2-1) \cdot 667 - \frac{1}{2}(2-1) \cdot (1333-667)$
$= -3/2 - 667$
$= 1000$



4) What is utility at each level of rent?



$M_5 - 4$ = Answer to 5 Compensating Variation

$M_6 = 4 - M_6$ = Answer to 6 Equivalent Variation

$V = \frac{M^{1.5}}{R^{.5}} \frac{2}{3}^{1.5}$

$V(M=4000, R=1) = 2 \cdot (4000/3)^{1.5} = 97373$

$V(M=4000, R=2) = \sqrt{2}\,(4000/2)^{1.5} = 68853$

No unit because it is a measure of utility

5) How much additional income is needed to keep the individual just as happy at R=2 as they were at R=1? Formally, this is called the compensating variation, CV. Show this in the depiction of the consumer optimization problem.

$\left(\frac{M_5}{3}\right)^{1.5} \frac{2}{\sqrt{2}} = 97373 \rightarrow M_5 = 5039.69$

6) At R=1, what is the most the consumer would pay to avoid rent increasing to 2? This is called the equivalent variation, EV. Show this in the depiction of the consumer optimization problem.

$\left(\frac{M_6}{3}\right)^{1.5} \frac{2}{\sqrt{1}} = 68853 \rightarrow M_6 = 3174.8$

$CV = 5040 - 4000 = 1040$
$EV = 4000 - 3175 = 825$
$\Delta CS = 1000$

7) Looking at the demand curve, using a linear approximation between the two rent levels, what is the change in consumer surplus when price goes from 1 to 2?

8) How does the change in CS compare to the EV and CV? Can you intuitively explain the differences?