1. Alice is a monopolist.

a) Suppose Alice does not know what the state of demand will be when production takes place and output is delivered to the market. Set up expected profit and find the quantity to maximize it (Q). Then, find price, profit, and consumer surplus in each state of the world (p_H, p_L, π_{AH}, π_{AL}, CS_H, and CS_L). Finally, calculate expected profit and expected consumer surplus.

$$\pi_A = 0.5Q(1.2 - 0.002Q) + 0.5Q(0.8 - 0.002Q) - 0.2Q$$
$$\pi_A = Q(1 - 0.002Q) - 0.2Q$$
$$\frac{d\pi_A}{dQ} = 1 - 0.004Q - 0.2 = 0$$
$$0.004Q = 0.8$$
$$Q = 200$$
$$p_H = 1.2 - 0.002 \times 200 = 0.8$$
$$p_L = 0.8 - 0.002 \times 200 = 0.4$$

$$\pi_H = (0.8 - 0.2)200 = 120$$
$$\pi_L = (0.4 - 0.2)200 = 40$$
$$E(\pi) = 0.5 \times 120 + 0.5 \times 40 = 80$$
$$CS_H = 0.5(1.2 - 0.8)200 = 40$$
$$CS_L = 0.5(0.8 - 0.4)200 = 40$$
$$E(CS) = 0.5 \times 40 + 0.5 \times 40 = 40$$

b) Suppose Alice knows the state of demand when production takes place and output is delivered. Find the profit maximizing quantity in each state (Q_H and Q_L), and the resulting prices, profits, and consumer surplus levels (p_H, p_L, π_{AH}, π_{AL}, CS_H, and CS_L). Calculate expected profit and expected consumer surplus viewed before the state of the world is determined.

$$\pi_{AH} = Q_{AH}(1.2 - 0.002Q_{AH}) - 0.2Q_{AH}$$
$$\frac{d\pi_{AH}}{dQ_{AH}} = 1.2 - 0.004Q_{AH} - 0.2 = 0$$
$$0.004Q_{AH} = 1$$
$$Q_{AH} = 250$$
$$p_H = 1.2 - 0.002 \times 250 = 0.7$$
$$\pi_{AH} = (0.7 - 0.2)250 = 125$$
$$CS_H = 0.5(1.2 - 0.7)250 = 62.5$$
$$E(\pi_A) = 0.5 \times 125 + 0.5 \times 45 = 85$$

$$\pi_{AL} = Q_{AH}(0.8 - 0.002Q_{AL}) - 0.2Q_{AL}$$
$$\frac{d\pi_{AL}}{dQ_{AL}} = 0.8 - 0.004Q_{AL} - 0.2 = 0$$
$$0.004Q_{AL} = 0.6$$
$$Q_{AL} = 150$$
$$p_L = 0.8 - 0.002 \times 150 = 0.5$$
$$\pi_{AL} = (0.5 - 0.2)150 = 45$$
$$CS_L = 0.5(0.8 - 0.5)150 = 22.5$$
$$E(CS) = 0.5 \times 62.5 + 0.5 \times 22.5 = 42.5$$

c) What is the value of information to the firm? How much better or worse off are consumers if the firm has the information on the state of the world? What does this suggest about the social and private value of information?

The information increases the firm's expected profit by $5, by allowing the firm to match quantity to conditions. Consumers also gain 2.5, so the social value of information exceeds the private value in this case.

2. Alice and Bob are homogenous product duopolists competing in quantities.

a) Suppose neither knows the state of the world before choosing quantity. Set up expected profits, find their reaction functions and the equilibrium quantity. From there, find the price, individual profit, total profit, and consumer surplus in each state. Find the expected value of individual and total profit and consumer surplus.

$$\pi_A = 0.5q_A\left(1.2-0.002\left(q_A+q_B\right)\right)$$
$$+0.5q_A\left(0.8-0.002\left(q_A+q_B\right)\right)-0.2q_A$$
$$\pi_A = q_A\left(1-0.002\left(q_A+q_B\right)\right)-0.2q_A$$
$$\frac{d\pi_A}{dq_A}=1-0.002q_B-0.004q_A-0.2=0$$
$$q_A=200-0.5q_B$$

Bob's reaction function would be found the same way. But, the game is symmetric with a unique NE, the quantities are the same in equilibrium, so:

$$1.5q_A=200$$
$$q_A=q_B=400/3=133.33$$
$$Q=266.67$$

$$p_H=1.2-0.002\times 266.67=0.67$$
$$p_L=0.8-0.002\times 266.67=0.27$$
$$\pi_{iH}=\left(0.67-0.2\right)133.33=62.22$$
$$\pi_{iL}=\left(0.27-0.2\right)133.33=8.89$$
$$E\left(\pi_i\right)=0.5\times 62.22+0.5\times 8.89=35.56$$
$$E\left(\pi_A+\pi_B\right)=71.11$$
$$CS_H=0.5\left(1.2-0.67\right)800/3=71.11$$
$$CS_L=0.5\left(0.8-0.27\right)800/3=71.11$$
$$E\left(CS\right)=0.5\times 71.11+0.5\times 71.11=71.11$$

b) Suppose both know the state of the world before choosing quantity. Find their reaction functions and the equilibrium quantity, price, individual and total profit, and consumer surplus in each state. Find the expected value of individual and total profit and consumer surplus.

$$\pi_{AH}=q_{AH}\left(1.2-0.002\left(q_{AH}+q_{BH}\right)\right)-0.2q_{AH}$$
$$\frac{d\pi_{AH}}{dq_{AH}}=1-0.002q_B-0.004q_A=0$$
$$q_{AH}=250-0.5q_{BH}$$
$$1.5q_{AH}=250$$
$$q_{AH}=q_{BH}=500/3=166.67$$
$$Q=333.33$$
$$p_H=1.2-0.002\times 1000/3=0.53$$
$$\pi_A=\pi_B=\left(0.53-0.2\right)166.67=55.56$$
$$\pi_A+\pi_B=111.11$$
$$CS_H=0.5\left(1.2-0.53\right)333.33=111.11$$

$$\pi_{AL}=q_{AL}\left(0.8-0.002\left(q_{AL}+q_{BL}\right)\right)-0.2q_{AL}$$
$$\frac{d\pi_{AL}}{dq_{AL}}=0.6-0.002q_{BL}-0.004q_{AL}=0$$
$$q_{AL}=150-0.5q_{BL}$$
$$1.5q_{AL}=150$$
$$q_{AL}=q_{BL}=100$$
$$Q=200$$
$$p_L=0.8-0.002\times 200=0.4$$
$$\pi_{AL}=\pi_{BL}=\left(0.4-0.2\right)100=20$$
$$\pi_{AL}+\pi_{BL}=40$$
$$CS_L=0.5\left(0.8-0.4\right)200=40$$

$$E\left(\pi_i\right)=0.5\times 111.11+0.5\times 20=37.78$$
$$E\left(CS\right)=0.5\times 111.11+0.5\times 40=75.56$$
$$E\left(\pi_A+\pi_B\right)=75.56$$

c) Suppose Alice knows the state of the world before choosing quantity and Bob does not. Bob will choose one quantity, q_B. Based on that quantity, Alice will have two reaction functions, one for each state of the world, and so two different quantities for the high and low states of demand, q_{AH} and q_{AL}. Bob will have one reaction function arising from maximizing expected profit over states of the world, in which in which both the demand intercept and Alice's quantity are discrete random variables. Find q_{AH}, q_{AL}, q_B, p_H, p_L, π_{AH}, π_{AL}, π_{BH}, π_{BL}, CS_H, and CS_L. Finally, find the expected value of individual and total profit and consumer surplus.

Set up & maximize Bob's expected profit:

$$\pi_B = 0.5q_B\left(1.2-0.002\left(q_{AH}+q_B\right)-0.002q\right)$$
$$+0.5q_B\left(0.8-0.002\left(q_{AL}+q_B\right)\right)-0.2q_B$$
$$\pi_A = q_B\left(1-0.001\left(q_{AH}+q_{AL}\right)-0.002q_B\right)-0.2q_B$$
$$\frac{d\pi_B}{dq_B}=1-0.001\left(q_{AH}+q_{AL}\right)-0.004q_B-0.2=0$$
$$q_B = 200-0.25\left(q_{AH}+q_{AL}\right)$$

From 2b, we know $q_{AH}=250-0.5q_B$ and $q_{AH}=150-0.5q_B$. So:

$$q_B = 200-0.25\left(400-q_B\right)$$
$$0.75q_B = 100$$
$$q_B = 400/3 = 133.33$$
$$q_{AH} = 183.33$$
$$q_{AL} = 83.33$$
$$Q_H = 316.67$$
$$Q_L = 216.67$$

$$p_H = 1.2-0.002\times 316.67 = 0.57$$
$$p_L = 0.8-0.002\times 216.67 = 0.37$$
$$\pi_{AH} = \left(0.57-0.2\right)183.33 = 67.22$$
$$\pi_{BH} = \left(0.57-0.2\right)133.33 = 48.89$$
$$CS_H = 0.5\left(1.2-0.57\right)316.67 = 100.28$$
$$\pi_{AL} = \left(0.37-0.2\right)83.33 = 13.89$$
$$\pi_{BL} = \left(0.37-0.2\right)133.33 = 22.22$$
$$CS_L = 0.5\left(0.8-0.37\right)216.67 = 46.94$$
$$E\left(\pi_A\right) = 40.56$$
$$E\left(\pi_B\right) = 35.56$$
$$E\left(\pi_A+\pi_B\right) = 76.11$$
$$E\left(CS\right) = 73.61$$

d) Charlie owns the information on the state of the world and is considering selling it to Alice only, Bob only, or both. If Charlie sells to only one party, everyone is sure the information will remain proprietary and disseminate no further. Will Charlie sell to one or both, and at what price? Hint: You must first determine how much one of the competitors would pay for the information if they were sure the other would not have it AND how much they would pay if they were sure the other WOULD have it.

From above, if one firm has the information and the other does not, it increases their profit by 40.56-35.56=5, so that is the most one would pay knowing the other firm would not get the information. If one has the information, the most the one without it would pay for it is 37.78-35.56-2.22. So, the most Charlie could receive selling to both is 2×2.22=4.44. Thus, Charlie will sell only to one firm.

e) What is consumer surplus if Charlie sells to only one and what is it if he sells to both?

If one firm has the information, expected CS is 73.61, while if both have it, it is 75.56.

f) What does all this suggest about the social and private value of information?

In this problem, unlike the #1, expected consumer surplus is higher if one firm has the information than if neither does, and higher yet if both have it. Contrasting the two problems, competition between the firms means the information is used in a way that ultimately is better for consumers, and the more broadly it is spread, the better for them.