Midterm Exam – March 1, 2021 - Solution

Question 1

Demand may be high, q=100-2p, with probability 0.6, or low, q=100-4p. Cost is 10 per unit.

The timeline differs from the ones we worked before. The order is:

i) All production is completed and a price is posted.
ii) Nature determines the level of demand, but the firm cannot observe it.
iii) Consumers purchase all they want at the posted price.
iv) The firm only learns if demand was high or low based on how much consumers purchase.
v) Unsold items are disposed of for free.

a) Set up the profit maximization problem, find the optimal price and quantity and the maximum expected profit. Some hints:
   - Set up profit as a function of p, not q, since we are stuck with a single price no matter if demand is high or low. This means you will take derivatives with respect to p, not q.
   - Production costs depend only on the amount produced
   - For this problem, you can safely assume the firm would not choose production below quantity demanded at the fixed price when demand is high.
   - If you see how to use those three things, this problem is simpler and less tedious than the version of optimal production and pricing we worked before.

First, expected profit is

$$E(\pi)=0.6pq_H(p)+0.4pq_L(p)-10q_H(p)$$

Since we are choosing p, substitution gives:

$$E(\pi)=0.6(100-2p)p+0.4(100-4p)p$$
$$-10(100-2p)$$

$$\frac{d\pi}{dp}=60-2.4p+40-3.2p+20=0$$

p^*=21.43

If demand is high, q is 57.14

If demand is low, q is 14.29

$$E(\pi)=(0.6\times 57.14+0.4\times 14.29)21.43$$
$$-10\times 57.14$$
$$=285.71$$

As an aside, for the educational value, note that the derivative generally is

$$\frac{dE(\pi)}{dq}=\Pr(H)\left(q_H+p\frac{dq_H}{dp}\right)+\Pr(L)\left(q_L+p\frac{dq_L}{dp}\right)-MC\frac{dq_H}{dp}=0$$

$$=\Pr(H)(MR_H-MC)\frac{dq_H}{dp}+\Pr(L)MR_L\frac{dq_L}{dp}=0$$

So, this still related to MR and MC as we understand them, just translated to price and reflecting that the price must be the same.

b) Suppose unsold product can be stored for future use at a cost of 2 per unit stored, saving \$10 per unit in production costs next period. Next period's savings should be discounted. Assume the discount rate is 5%. Set up the profit maximization problem, find the optimal price and quantity and the maximum expected profit.

First, expected profit is

$$E(\pi) = 0.6pq_H(p) + 0.4pq_L(p) - 10q_H(p)$$
$$+0.4\left(\frac{10}{1.05} - 2\right)\left(q_H(p) - q_L(p)\right)$$

Since we are choosing p, substitution gives:

$$E(\pi) = 0.6(100-2p)p + 0.4(100-4p)p$$
$$-10(100-2p)$$
$$+0.4(7.524)\left((100-2p)-(100-4p)\right)$$

$$E(\pi) = 0.6(100-2p)p + 0.4(100-4p)p$$
$$-10(100-2p) + 0.4\times 7.524\times 2p$$

$$\frac{d\pi}{dp} = 60 - 2.4p + 40 - 3.2p + 20 + 6.02 = 0$$

$$p^* = 22.50$$

If demand is high, q is 54.99
If demand is low, q is 9.99

$$E(\pi) = (0.6\times 54.99 + 0.4\times 9.99)22.50 - 10\times 54.99 + 0.4(7.524)(54.99 - 9.99) = 411.46$$

c) Explain intuitively why and how the solution in b changes from a.

With storage, there is an incentive to sell less when demand is low to save money on production later. This results in setting a higher price to make more profit when demand is high. The higher price in turn results in less sales, but more profit, when demand is high, but lower profit when demand is low. The reduction in profit when demand is low is more than made up for by the reduction in future costs from saving output for later when demand is low.

d) It costs 40 per period to maintain the storage facility of part b, it will last 5 years, and the interest rate is 5%. What is the value of obtaining such a facility? That is, what is the most the firm should be willing to pay for it? Assume payment for it is made one period before its first use.

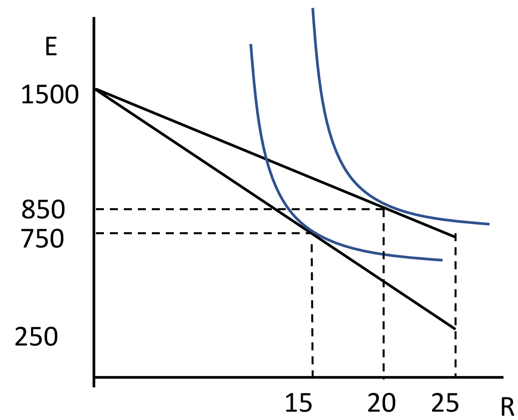
First note the increase in expected profit per period is 411.46-285.71=125.75. Net of maintenance costs this is 75.75. Also note 1/1.05=0.9523. Then:

$$ENPV = 75.75\sum_{t=1}^{5} 0.9523^t = 327.96$$

Question 2

Suppose someone is 60 years old and will live 25 more years. Currently they have saved \$500 (thousand). They earn a wage, w, of 50 (thousand) per year they work, W. They care only about years they do not work, R (retirement) and how much they spend over the rest of their life, E (for expenditure, in thousands). Ignore discounting and such for this problem.

a) Write the equation for their budget constraint, showing the possibilities for years retired and expenditures. Draw it, placing R on the horizontal axis and E on the vertical axis.



Note E is 250+50W and W=25-R. Thus E=250+50(25-R) or E=1500-50R.

b) Suppose now that the government announces a social security program in which they will pay everyone a stipend, *s*, of \$20 (thousand) per year in retirement, provided they are over 60. This is an oversimplified version of social security, but simply ignore other aspects. Write the new budget constraint. Draw it in the figure.

Note E is 250+50W+20R, and W=25-R. Thus E=1500-50R+20R or E=1500-30R.

c) Draw indifference curves tangent to each budget line, assuming both R and E are normal goods. Any indifference curves satisfying the axioms of rational choice, and for which both E and R are normal goods, are fine. Label the individual's solution for years worked and expenditure with each budget constraint (a and b). Make sure the (R,E) pairs in the solutions you label respect the budget constraints.

d) Explain the impact of the simplified social security program on years worked for any rational chooser for whom both R and E are normal goods. Be precise, meaning talk about things like the relevant marginal rate of substitution and market rate of trade. Also, talk about how the assumption that both E and R are normal goods matters. Does that seem a reasonable assumption in this case? Why or why not?

The first crucial point is that *w-s* is the price of a year of retirement and it is now lower! The rational choice occurs where the willingness to sacrifice consumption for another year of retirement just equals the relative price of retirement in terms of sacrificed consumption. At the old solution $MRS_{ER}=w=50>30=w-s$. Thus, if we imagine staying on the same indifference curve, R should increase and E decrease until MRS_{ER} diminishes to 30. This is the substitution effect.

The second point is that the stipend makes the individual richer in real—it expands their consumption possibilities set. If both goods are normal, the income effect of the decline in the price of retirement will tend to increase consumption of both goods. The substitution effect will tend to increase R but decrease E. This, assuming the goods are both normal means that unambiguously R increases—it is not a Giffen good. If one thinks most people would like to spend less time working if they were to receive an inheritance, or win the lottery, or something of that sort, then the assumption that R is normal makes sense.