# Problem 15

✓

Ben's preferences are represented by $U=0.3\ln H+0.7\ln E+0.1S$, where $H$ is square feet of housing consumed monthly, $E$ is the amount spent monthly on everything else, and $S=1$ if he lives somewhere sunny like Florida (no snow or sleet and little freezing weather) and 0 otherwise. He is considering 2 jobs, one in Tampa and one in Boston. The job in Boston pays $7,000 per month. Housing costs $4 per square foot monthly in Boston and $1.5 per square foot monthly in Tampa. Calculate the salary in Tampa that would make Ben indifferent between the job in Tampa and the job in Boston. Illustrate with a figure.
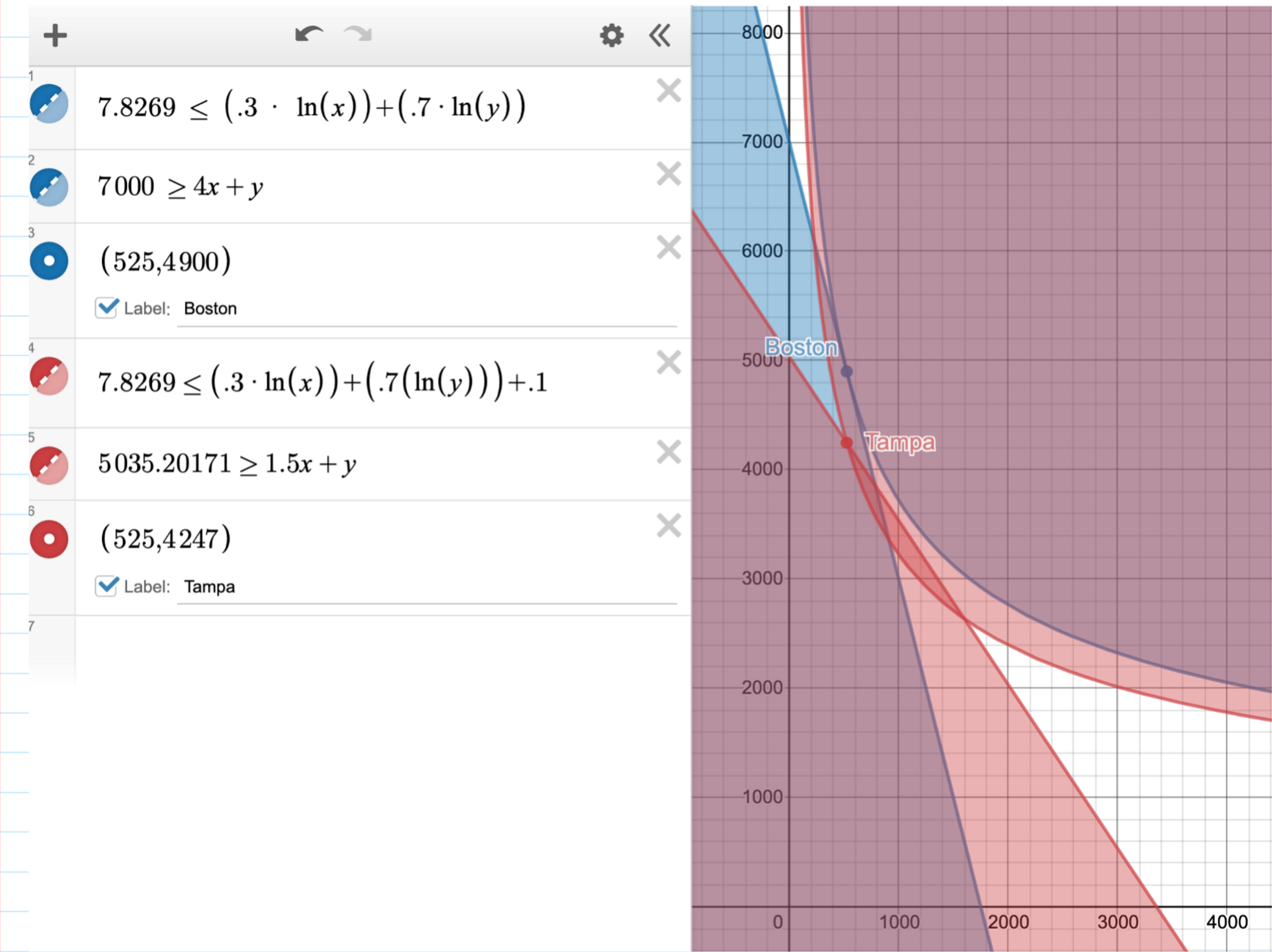
$E_{boston} = 7000 - 4H$          $E_{Tampa} = X - 1.5H$

| | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | Square ft | Rent | | Income | | Everything | | Utility | | Diff |
| 2 | Boston | 525.000042 | 2100.00017 | | 7000 | | 4899.99983 | | 7.82691282 | | 2.7001E-13 |
| 3 | Tampa | 525 | 787.5 | | 5035.20171 | | 4247.70171 | | 7.82691282 | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |

I know we didn't learn solver in this class and I could do it by hand but I was too uncertain of what to do to do that to myself.

I wasn't having any luck with solver by trying to set the income as the same and realized it was because I wasn't using the utility. I added the utility in but solver still wouldn't do anything. That's because there were two unknowns which "cancelled" out, so to speak, and made the solution not exist because ln(0) is not a good time. Then I realized that I could maximize the utility of Boston by itself to find the square footage in Boston and use the same number in Tampa because otherwise the comparison doesn't make sense and isn't fair. With the new square footage, I set diff equal to the absolute value of the utility of Tampa minus the utility of Boston. I then asked solver to try and minimize the difference while changing the income in Tampa. Thankfully, this worked and I got an income of 4247.70.



Desmos graph panel:

1. $7.8269 \le (.3 \cdot \ln(x))+(.7 \cdot \ln(y))$
2. $7000 \ge 4x + y$
3. $(525, 4900)$ — Label: Boston
4. $7.8269 \le (.3 \cdot \ln(x))+(.7(\ln(y)))+.1$
5. $5035.20171 \ge 1.5x + y$
6. $(525, 4247)$ — Label: Tampa

## Math this time

Ben's preferences are represented by $U=0.3\ln H+0.7\ln E+0.1S$, where $H$ is square feet of housing consumed monthly, $E$ is the amount spent monthly on everything else, and $S=1$ if he lives somewhere sunny like Florida (no snow or sleet and little freezing weather) and 0 otherwise. He is considering 2 jobs, one in Tampa and one in Boston. The job in Boston pays $7,000 per month. Housing costs $4 per square foot monthly in Boston and $1.5 per square foot monthly in Tampa. Calculate the salary in Tampa that would make Ben indifferent between the job in Tampa and the job in Boston. Illustrate with a figure.

**max Boston**

$$\frac{MU_H}{MU_E} = \frac{.3}{H} \cdot \frac{E}{.7} = \frac{4}{1} = \frac{P_H}{P_E} \quad \rightarrow \quad E = \frac{28}{3}H$$

$$7000 = 4H + \frac{28}{3}H = \frac{40}{3}H \quad \rightarrow H = 525$$

$$E = 7000 - 4 \cdot 525 = 4900 \quad \rightarrow U = .3\ln 525 + .7\ln 4900 = 7.827$$

**Tampa must have same utility**

$$\frac{MU_H}{MU_E} = \frac{.3}{H} \cdot \frac{E}{.7} = 1.5 = \frac{P_H}{P_E}$$

$$E = 3.5H$$

$$y = 1.5H + 3.5H = 5H \quad \rightarrow \quad H = \frac{y}{5} \quad \rightarrow \quad E = W - \frac{3W}{10} = .7W$$

$$U = .3\ln .2 + .3\ln W$$
$$+ .7\ln .7 + .7\ln W + .1$$
$$= 7.827$$

$$\ln W = 7.727 - .3\ln .2 - .7\ln .7 = 8.46 \quad \rightarrow \text{don't forget } -.1$$

$$W = e^{8.46} = 4722$$
$$H = 944$$
$$E = 3306$$