Problem 7

✓

A firm sells q_B mugs of beer at price p_B, and q_P slices of pizza at price p_P. The inverse demand for mugs of beer is $p_B = 5 - 0.25q_B + 0.1q_P$ and the inverse demand for pizza slices is $p_P = 4 - 0.5q_P + 0.1q_B$. It costs \$1/mug to serve beer and \$2/slice to serve pizza. Find the prices and quantities that maximize profit and the maximum profit.

	A	B	C	D	E	F	G
1	item	c	p	q	unit cost	profit	
2	beer		2.9	10	1	19	
3	pizza		3	4	2	4	
4						23	<- Solver

Hailey, Troy, and myself can't seem to organically find a right answer without solver

↳ we thought q_B and q_P had to be integers

$$\pi = (5 - .25q_B + .1q_P)(q_B) + (4 - .5q_P + .1q_B)(q_P) - (1 \cdot q_B) - (2 \cdot q_P)$$

$$= -.25q_B^2 + .2q_B q_P + 4q_B - .5q_P^2 + 2q_P$$

~~$$\frac{d\pi}{dq_B} = -\frac{q_B}{2} + \frac{q_P}{5} + 4$$~~

~~$$\frac{q_B}{2} = \frac{q_P}{5} + 4 \rightarrow q_B = \frac{2q_P}{5} + 8$$~~

~~$$a = -\frac{(2q_P/5 + 8)}{2} + \frac{q_P}{5} + 4 \rightarrow q_P = 0$$~~

~~$$a = -\frac{q_B}{2} + \frac{a}{5} + 4$$~~

~~$$-4 = -q_B/2$$~~

~~$$-8 = -q_B$$~~

~~$$q_B = 8$$~~

$$\frac{d\pi}{dq_B} = (5 - .5q_B + .1q_P) + .1q_P - 1 = 0$$
$$.5q_B = 4 + .2q_P$$
$$q_B = 8 + .4q_P$$

$$\frac{d\pi}{dq_P} = (4 - q_P + .1q_B) + .1q_B - 2 = 0$$
$$q_P = 2 + .2q_B$$

$$q_B = 8 + .4(2 + .2q_B)$$
$$q_B = 8 + .8 + .08q_B$$
$$.92q_B = 8.8$$
$$q_B = 9.57$$

$$q_P = 2 + .2(9.57) = 3.91$$

$$p_B = 5 - (.25 \cdot 9.57) + (.1 \cdot 3.91) = 3$$
$$p_P = 4 - (.5 \cdot 3.91) + (.1 \cdot 9.57) = 3$$

$$\pi = (3-1)(9.57) + (3-2)(3.91) = 23.04$$