Problem 25

Tuesday, April 6, 2021 8:34 PM

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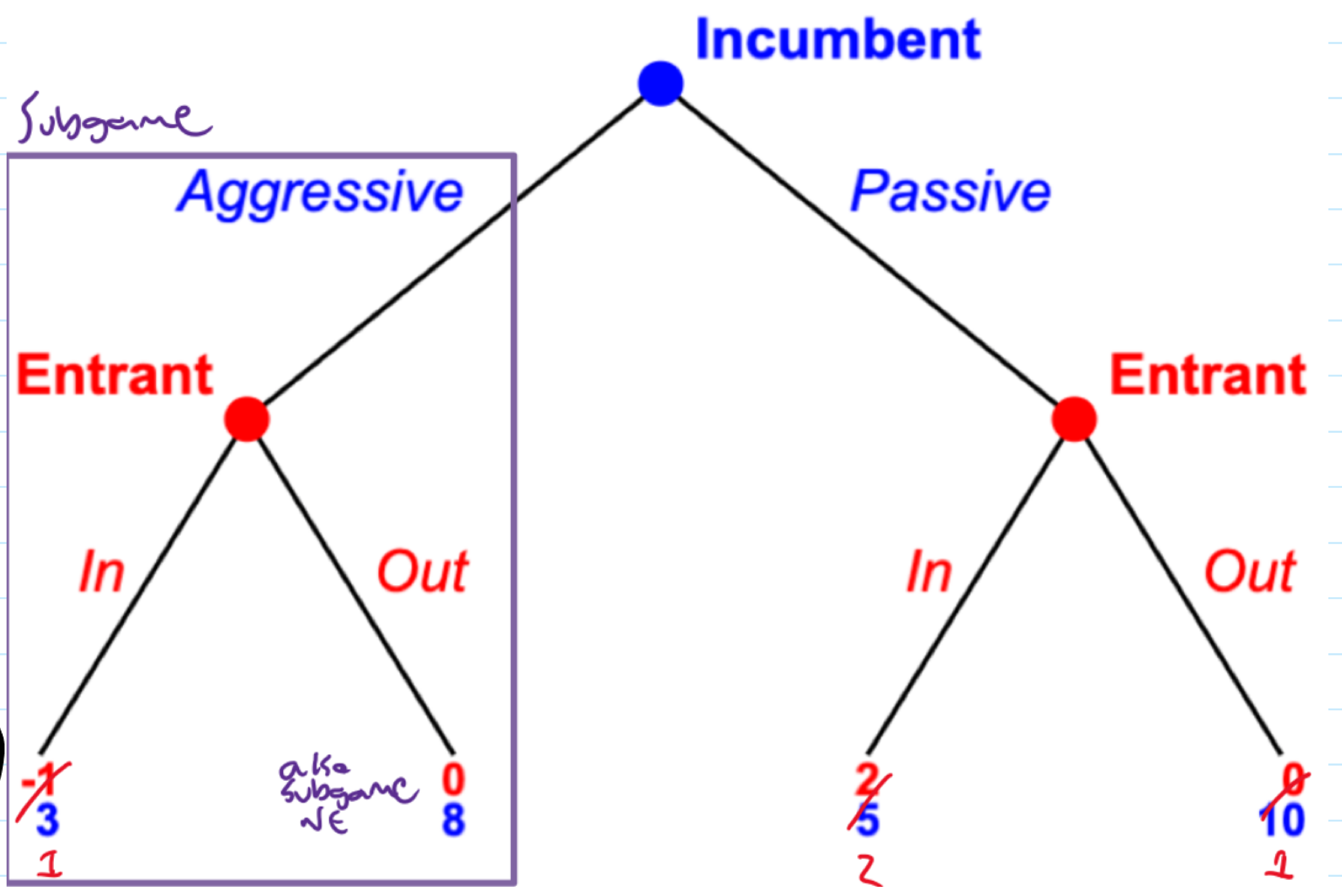
Continuation of problem 25. The incumbent is indefinitely lived. The probability of continuing after any round is *f* and the discount rate is *r*. Each entrant plays only one round and knows how the incumbent played in the past. We want to determine the value to the incumbent of maintaining the ability to introduce and withdraw new product varieties on short notice.

a) Suppose the incumbent has to move first each round because it takes time to introduce and position a new product. Calculate the expected present value of the incumbent's profit.

	Aggro	Passive
In	-1,3	2,5
Out	0,8	0,10



$3q+8(1-q)=5k+10(1-k)$

$3q+8-8q=5k+10-10k$

$-5q+8=-5k+10$

$-5q=-5k+2$

$q=-k-2/5$

$3(k-2/5)+8(1-(k-2/5))=-5k+10$

$q=0 \quad k=.4$

$E(\pi)=3(0)+8(1-0)+5(.4)+10(.6)$

$=0+8+2+6$

$=16$

$\frac{1}{1-\delta} \to \delta=\frac{f}{1+r}$ something fraction

$\left[\delta / \left(\frac{1}{1-\delta}\right)(1-\delta)\right](1-\delta^T)$ ← adjust for $t=0$

↳ +1

$E(\pi)=8$

$\sum_{t=0}\left(\frac{f}{1+r}\right)^t \to$ $\boxed{8\cdot\frac{1}{1-(f/1+r)} - E(\pi)}$

$E(\pi)=8\frac{1+r}{1+r-f}$

b) Suppose the incumbent can maintain the capacity to introduce a new product on short notice, and similarly to withdraw it from the market whenever it chooses. Determine the values of *r* and *f* for which the following strategies constitute a sequentially rational Nash Equilibrium. (Sequential rational is closely akin to subgame perfection. It means the strategy is rational at every point, even those that will not actually be reached in the equilibrium.) **Incumbent:** In each round, if the entrant enters, introduce the new product and withdraw it at the end of the round, and if the entrant does not enter do not introduce the new product. **Entrant:** Enter if the incumbent has ever been passive, and do not enter otherwise.

r = discount rate

f = prob of continue

$V=10\frac{1+r}{1+r-f}$

$V=3+10\sum_{t=1}^{\infty}\left(\frac{f}{1+r}\right)^t=10\frac{1+r}{1+r-f}-7$

$8\cdot\frac{1}{1-f/1+r}+3\cdot\frac{1}{1-(1-f)/1+r}=-1\cdot\frac{1}{1-\frac{f}{1+r}}+0$

$V_{deviate}=5\frac{1+r}{1+r-f}$

$r=\frac{-2f+1}{4}$

$V\geq V_{deviate} \to f\geq\frac{2}{7}(1+r)$

Plug in... $f=\frac{5}{2}$?

$r=-1$?

I'm definitely doing something wrong, but I keep getting $r=-1$

c) How much might it be worth to the incumbent to be able to introduce and withdraw new products on short notice, as a function of *r* and *f*?

I would calculate expected profit for the incumbent if the other player enters the market

$\cancel{3\cdot\frac{1}{1-f/1+r}+5\cdot\frac{1}{1-\frac{1-f}{1+r}}=\frac{8r^2-5fr+13r-2f+5}{(f+r)(r+1-f)}}$

$(10-8)\frac{1+r}{1+r-f}$