

# Applications and Extensions of Optimal Production and Pricing

## Simple (3<sup>rd</sup> degree) Price Discrimination

charge different prices to different groups  
↳ senior discount

groups must:

- ↳ be identifiable
- ↳ have specific willingness to pay
- ↳ resale should be impossible

market w/ less elastic demand

$$P_1 = \frac{3}{1+3} MC \quad P_2 = \frac{3}{1+3} MC$$

Example: Two groups. Max Profit w/ discrimination  
 $P_1 = 20 - q_1$     $P_2 = 30 - q_2$     $C(Q) = .5(q_1 + q_2)^2$

Two Prices:  $\pi = (20 - q_1)q_1 + (30 - q_2)q_2 - .5(q_1 + q_2)^2$

$$MR_1 = MR_2 = MC$$

$$MR_1 = MC$$

$$20 - 2q_1 = q_1 + q_2$$

$$MR_2 = MC$$

$$30 - 2q_2 = q_1 + q_2$$

$$\downarrow$$

$$20 - 2q_1 = 30 - 2q_2$$

$$2q_2 = 10 + 2q_1$$

$$q_2 = 5 + q_1$$

$$\downarrow$$

$$20 - 2q_1 = q_1 + 5 + q_1$$

$$15 = 4q_1$$

$$q_1 = 3.75$$

$$\rightarrow q_2 = 5 + q_1 = 8.75$$

$$P_1 = 20 - 3.75 = 16.25$$

$$P_2 = 30 - 8.75 = 21.25$$

$$\downarrow$$

$$\pi = 16.25(3.75) + 21.25(8.75) - .5(3.75 + 8.75)^2 = 168.75$$

one price?

$$P_1 = 20 - q_1 \rightarrow q_1 = 20 - P_1$$

$$P_2 = 30 - q_2 \rightarrow q_2 = 30 - P_2$$

$$P_1 = P_2 = P \rightarrow q = 50 - 2P \rightarrow P = 25 - .5q$$

$$\downarrow$$

$$\pi = (25 - .5q)q - .5q^2$$

$$MR = MC$$

$$25 - q = q$$

$$q = 12.5$$

$$P = 25 - .5(12.5) = 18.75$$

$$\pi = 18.75(12.5) - .5(12.5)^2 = 156.25$$

$$\text{Extra Profit} \rightarrow 168.75 - 156.25 = 12.5$$

## Profit Maximization when Purchases per Capita don't depend on market size

$$q/N = f(P, P_s, P_c, M, Z) \rightarrow \text{Purchases per Capita}$$

$N$  = city size

## Maximizing Profit with ~ pre-determined capacity constraint

Theaters, stadiums, etc

## Profit Maximization with Uncertainty

Expected Profit

## Value of Information with Continuous Decisions

## Peak Load Pricing - Determining Capacity when demand varies

### Chapter 16 in Game Theory

$$\text{To max } \pi, MR_L = MC_L = C \text{ and } MR_H = MC_H = C + k$$

$$\pi = P_H(q_H)q_H + P_L(q_L)q_L - Cq_L - (C + k)q_H \rightarrow q_H \geq q_L$$

↳ if  $q_H < q_L$  then  $q_H = q_L$

$$MR_H + MR_L = 2C + k$$