Problem 19

Saturday, March 20, 2021 1:37 PM

Individual preferences for risky monetary outcomes (x) are well approximated by the von Neumann-Morgenstern utility function $u = \sqrt{x}$. Income is $40,000. There is a 0.05 probability of a $30,000 loss.

a) Calculate the certainty equivalent of an individual's lottery and the risk premium, and show them graphically.

$E(w) = (40000 \cdot .95) + (10000 \cdot .05) = 38500$

$\sqrt{u(CE)} = \frac{19}{20} u(40000) + \frac{1}{20} u(10000)$

$CE = 38025$

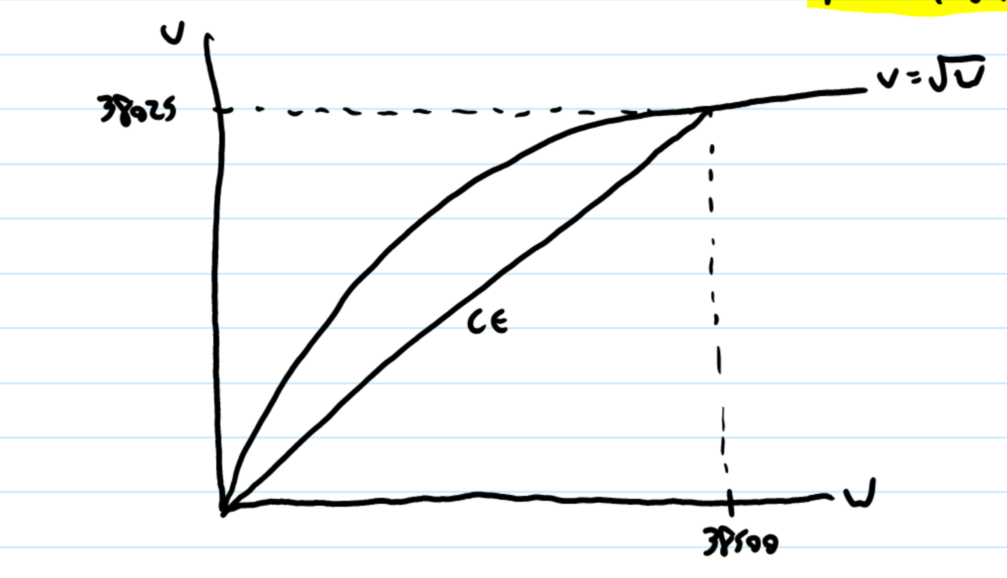
$\sqrt{40000 - P} = \sqrt{38025} \rightarrow 40000 - P = 38025$

$P = 1975$

$\sqrt{CE} = .95\sqrt{40k} + .05\sqrt{10k}$
$= 190 + 5 = 195$
$CE = 195^2 = 38025$

did math backwards

$RP = E(x) - CE$
$= 38500 - 38025$
$= 475$



b) Suppose there are 2 identical individuals and losses are independent. Neither, one, or both may suffer a loss. If the two pool their risk, and so share any losses equally, what is the certainty equivalent of an individual's lottery? In terms of the certainty equivalent, how much value did they gain from pooling their risk?

F	W
$1/20^2$	10000
$38/400$	25000
$19^2/20^2$	40000

No loss: $P(x=40k) = .95^2 = .9025$
1 loss: $P(x = 40k - 30k/2 = 25k) = 2 \cdot .95 \cdot .05 = .095$
2 loss: $P(x = 10k) = .05^2 = .0025$

$\sqrt{CE} = \frac{1}{400}\sqrt{10000} + \frac{38}{400}\sqrt{25000} + \frac{361}{400}\sqrt{40000}$ ✓

$\hookrightarrow CE = 38326.213$ ✓

Value gained $= 38326.213 - 38025 = 301.213$ ← real value /2 ?

there are 2 people so ↑ ·2

c) Suppose there are 5 identical individuals facing independent losses, with the total number of losses therefore following the binomial distribution. That is, the probability of exactly k losses in n trials if the chance of an individual loss is p is $P(k \mid n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$. If all 5 pool their risk, what is the certainty equivalent of each individual's lottery? How much value is created (in total) by pooling risk rather than each individual bearing their own risk?

$\sum_{i=0}^{5} P(k|n)$

$\frac{5!}{1!(4)!} .05^1 (1-.05)^4 + \frac{5!}{2!\,3!} .05^2 (.95)^3 + \frac{5!}{3!\,2!} .05^3 (.95)^2 +$

$\frac{5!}{4!\,1!} .05^4 .95^1 + \frac{5!}{5!\,0!} .05^5 .95^0 =$

	A	B	C	D	E	F	G	H
1	n	k	n-k	P(k|n)	Good W	Bad W	Total W	CE
2	5	0	5	0.773780938	40000	0	40000	154.7561875
3	5	1	4	0.203626563	32000	2000	34000	37.54688291
4	5	2	3	0.021434375	24000	4000	28000	3.586656951
5	5	3	2	0.001128125	16000	6000	22000	0.167327978
6	5	4	1	2.96875E-05	8000	8000	16000	0.003755205
7	5	5	0	3.125E-07	0	10000	10000	0.00003125
8							Total CE	38439.85369
9							Value Gained	414.8536863
10							Total Value	2074.268432

d) Suppose there are 100,000 identical individuals and risks are independent. A risk neutral insurance company insures everyone at the actuarially fair rate. What is that rate? What is each individual's now certain income? How much value is added by the insurance company? each individual gains 475 for a total of 47.5 M

3/20/2021 R Notebook

R Notebook

```
getValue <- function(n) {
df <- as.data.frame(matrix(data = NA, nrow = n + 1, ncol = 8))
colnames(df) <- c("N", "K", "NK", "PKN", "GoodW", "BadW", "TotalW", "CE")
df$N <- n
df$K <- 0:(nrow(df) - 1)
df$NK <- df$N - df$K
df$PKN <- (factorial(df$N) / (factorial(df$K) * factorial(df$NK))) *
  (.05^df$K) * (.95^df$NK)
df$GoodW <- 40000 * df$NK / n
df$BadW <- 10000 * df$K / n
df$TotalW <- df$GoodW + df$BadW
df$CE <- df$PKN * sqrt(df$TotalW)
return(((sum(df$CE)^2) - 38025)*5)
}

df2 <- as.data.frame(matrix(data = NA, nrow = 71, ncol = 2))
colnames(df2) <- c("N", "Value")
df2$N <- 100:170
df2$Value <- sapply(df2$N, getValue)
df2
```

N <int>	Value <dbl>
100	2361.068
101	2361.206
102	2361.342
103	2361.475
104	2361.606
105	2361.734
106	2361.859
107	2361.982
108	2362.103
109	2362.222

1-10 of 71 rows Previous 1 2 3 4 5 6 ... 8 Next

```
library(tidyverse)
```

```
## — Attaching packages ——————————————————— tidyverse 1.3.0 —
```

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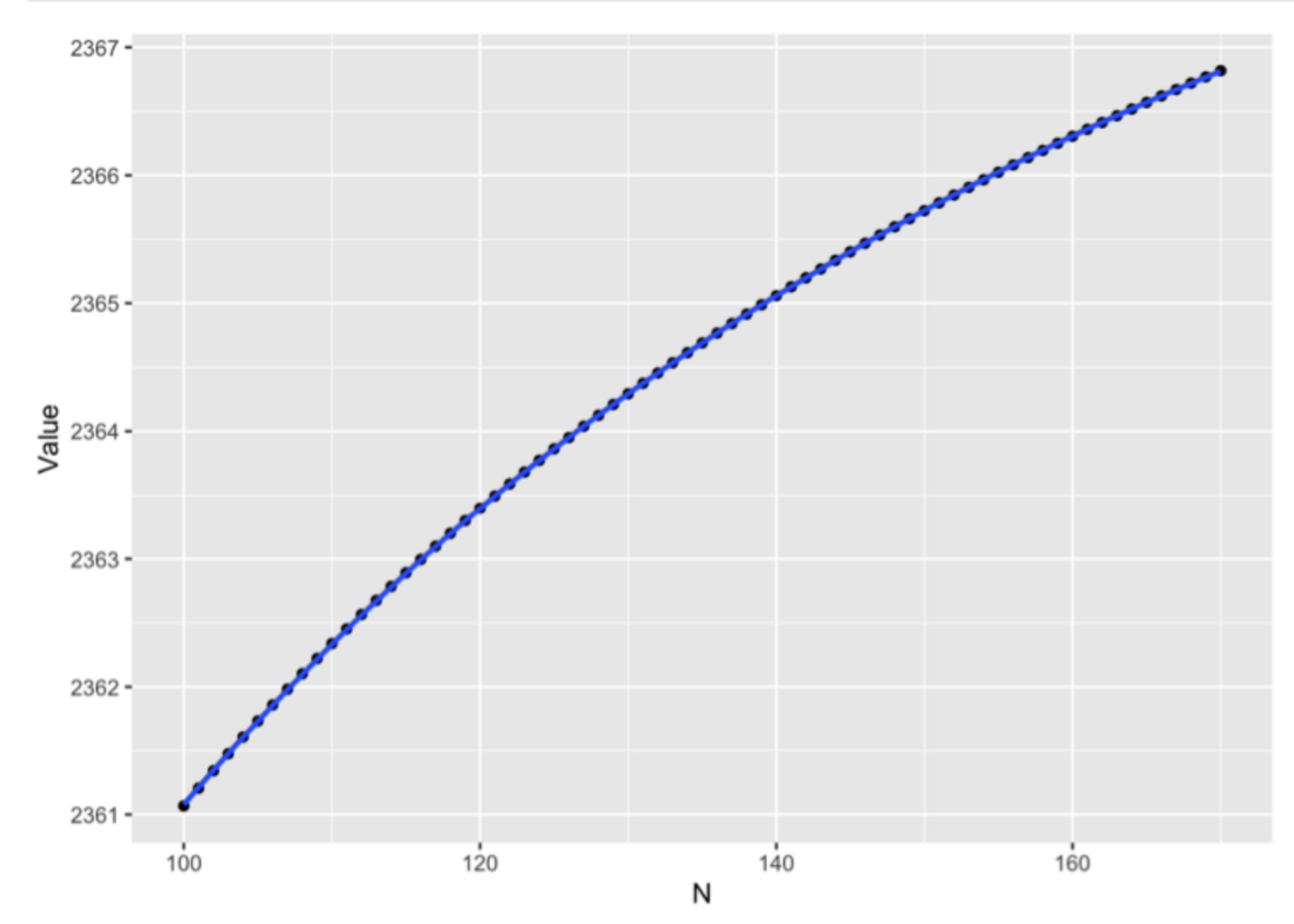
3/20/2021 R Notebook

```
## ✓ ggplot2 3.3.2     ✓ purrr   0.3.4
## ✓ tibble  3.0.4     ✓ dplyr   1.0.2
## ✓ tidyr   1.1.2     ✓ stringr 1.4.0
## ✓ readr   1.4.0     ✓ forcats 0.5.0
```

```
## — Conflicts ——————————————————— tidyverse_conflicts() —
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
df2 %>%
  ggplot(aes(x = N, y = Value)) +
  geom_point() +
  geom_smooth(formula = y ~ sqrt(x))
```

```
## `geom_smooth()` using method = 'loess'
```



I'm not really sure how I'm supposed to do this and 100000! Is a really really big number. I did think that the number would eventually converge so I decided to do my best to check. It turns out that it does converge, to what I don't know, ~~but it does follow y=sqrt(x), just like the utility function.~~

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e) For the binomial distribution, the mean number of occurrences is *np* and the standard deviation of the total number of occurrences is $\sqrt{np(1-p)}$. Suppose regulations require the insurance company in (d) to hold reserves so that there is only a 0.000001 probability they will experience losses over annual revenue plus reserves, which they could not cover. How much is needed in reserves? With large numbers of observations, the binomial distribution is very closely approximated by the normal distribution, with the appropriate mean and standard deviation. So, you can use the normal approximation with the implied mean and standard deviation, or, you can use any program that can calculate the inverse of the cumulative binomial distribution (excel will do it using the binom.inv() function). company must be able to cover 5371 losses revenue = 150m. Thus reserves must be 9.93m

binom.inv(100000, .05, .000001) = 4676

f) Reinsurance is insurance purchased by insurance companies to protect the company against a very rare circumstance regarding an exceptionally high level of claims. Interpret the role of reinsurance companies in light of (e).

Reinsurance companies exist because regular insurance companies pool risk among their policy holders so if each insurance company has 100,000 people and a reinsurance company has 10 insurance companies, then the risk is spread over 1,000,000 people. It allows for aggregated risk pooling.