Problem 17

Saturday, March 6, 2021 3:07 PM

✓

There are two types of consumers for a particular beer, with inverse demands given by p_1=3.5-0.25q_1 and p_2=3.3-0.4q_2, respectively, where q is the number of bottles per period. Marginal cost is $0.5 per bottle.

a) Assuming 40% of customers are type 1 and the remainder are type 2, set up the optimization problem with all 4 constraints, though only 2 will bind.

$P_1 = 3.5 - .25q_1 \qquad P_2 = 3.3 - .4q_2$

$\pi = (3.5 - .25q_1)q_1 + (3.3 - .4)q_2 - .5(q_1 + q_2)$

$q_1 = .4q \qquad q_2 = .6q \qquad q_1 + q_2 = q \qquad q \geq 0$

$\pi = [3.5 - (.25 \cdot .4 \cdot q)](.4q) + [3.3 - (.4 \cdot .6 \cdot q)](.6q) - .5q$

b) Solve the problem to find the optimal bundle for each consumer type, their prices, the firm's profits, and each consumer type's surplus. Constraints 2+3 bind

$P_2 = 3.3q - .2q^2$

$\pi = (3.5 - .1q)(.4q) + (3.3 - .24q)(.6q) - .5q$

$= -.184q^2 + 2.88q$

$P_1 = P_2 + (3.5q_1 - .125q_1^2) - (3.5q_2 - .125q_2^2)$

$P_1 = 3.5q_1 - .125q_1^2 - 3.5q_2 - .125q_2^2$

$\frac{d\upsilon}{d\pi} = -.368q + 2.88 \rightarrow q = 7.826$

$\pi = .4(3.5q_1 - .125q_1^2 - .2q_2 - .045q_2^2 - .5q_1) +$

$.6(3.3q_2 - .2q_2^2 - .5q_2)$

$\frac{d\pi}{dq_1} = .4(3 - .25q_1) = 0$

$q_1 = 12$

$\frac{d\pi}{dq_2} = .6(2.5 - .4q_2) - .4(.2 + .15q_2) = 1.6$

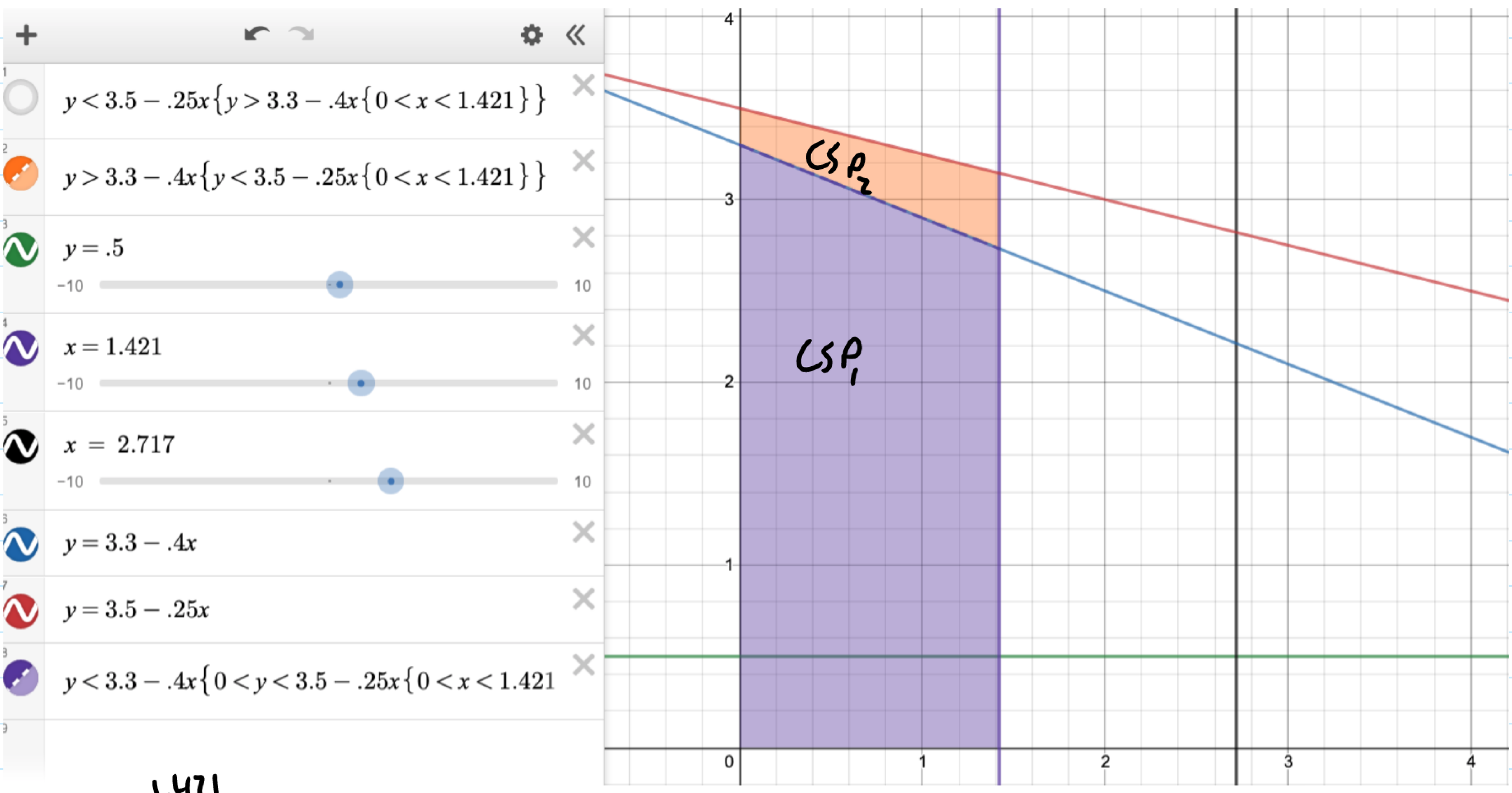
$q_1 = .4q = .4 \cdot 7.826 = 3.13$

$q_2 = .6q = .6 \cdot 7.826 = 4.69$

$P_1 = 3.5 - .25(3.13) = 2.717$

$P_2 = 3.3 - .4(4.69) = 1.421$

$\pi = (3.13 \cdot 2.717) + (4.69 \cdot 1.421) - .5(7.826) = 11.269$



$CSP_2 = \int_0^{1.421} (3.5 - .25x) - (3.3 - .4x) = .4356$

$CSP_1 = \int_0^{1.421} 3.3 - .4x = 4.2854$

c) How much additional profit could be made if the company could engage in perfect 1st degree price discrimination?

$P_2 = MC \Rightarrow 3.3 - .4q_2 = .5 \rightarrow q = 7$

$P_2 = 3.3 \cdot 7 - .2 \cdot 7^2 = 13.3$

$\pi = .4(24 - .5 \cdot 12) + .6(13.3 - .5 \cdot 7)$

$= 13.08$

$\pi = 11.269 + .4356 + 4.2854 = 15.99$

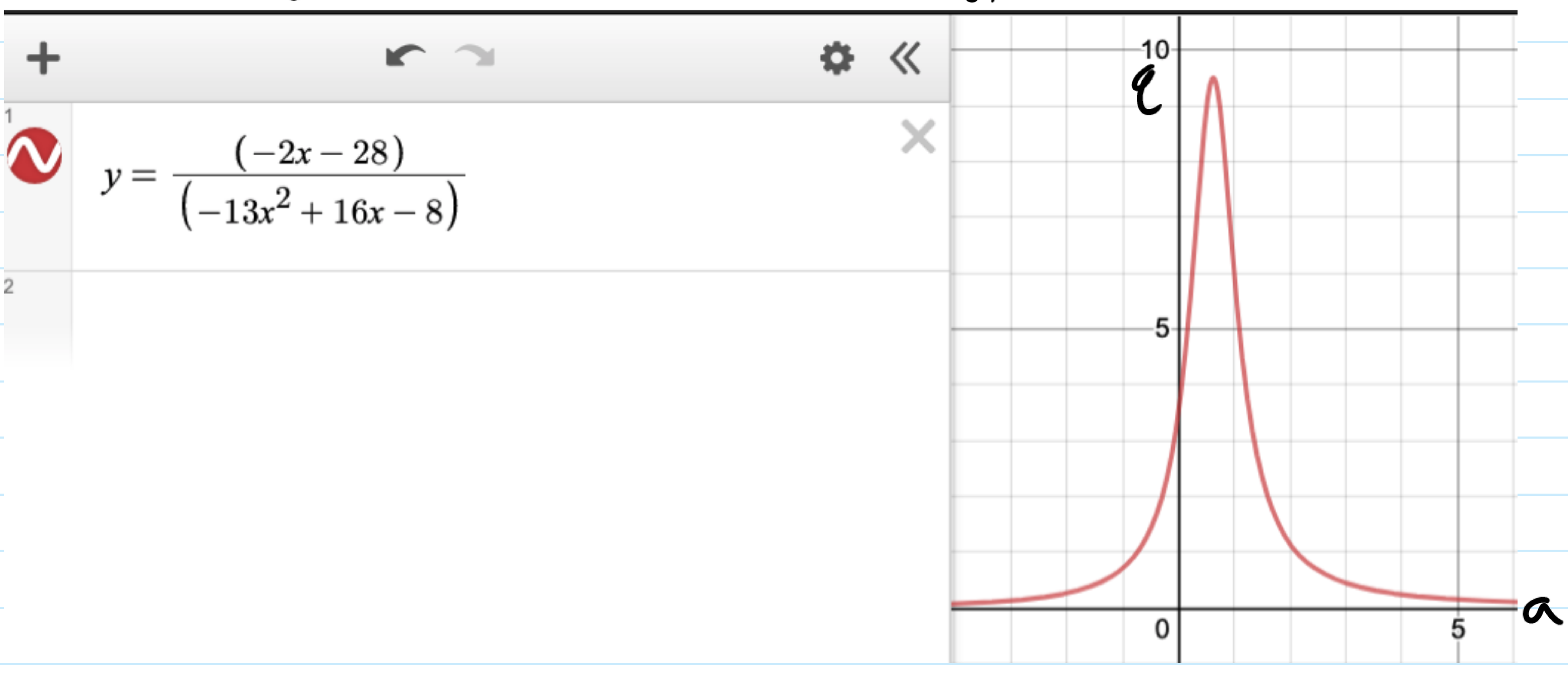
d) Let α represent the fraction of consumers that are type 1, while all remaining customers are type 2. Set up the optimization problem and solve for the optimal bundles and prices as a function of α. What happens to the bundles and prices as α increases? Explain in intuitive terms.

$q_1 = \alpha q \qquad q_2 = (1 - \alpha)q$

$P_1 = 3.5 - .25\alpha q \qquad P_2 = 3.3 - .4(1 - \alpha)q$

$\pi = (\alpha q)(3.5 - .25\alpha q) + ((1 - \alpha)q)(3.3 - .4(1 - \alpha)q) - .5(q)$

$\frac{d\upsilon}{d\pi} =$



I think I messed up somewhere. This is not a nice looking graph. Nevertheless, I'll explain what I think is happening. You want as many high type customers as possible but after a certain amount of them, you'll begin to lose money on the missing low types.