

Problem 26

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Alice and Bob repeatedly play the one shot game to the right, in which they sell differentiated products. Each can choose price high as they would with no competition, lower, or at cost. Alice and Bob may each be normal or crazy, but do not know the type of their opponent. A crazy player prices high until their competitor does not, and charges at cost forever after their opponent charges a price below high. The probability of a crazy player is g . The discount rate is r . The number of repetitions is known to be T . Think of $1/2-h$ as reflecting how much pricing at cost hurts an opponent charging lower prices, where $(1/2) > h > (1/10)$. Please use the table included below.

		Bob		
		High	Lower	At Cost
Alice	High	3/4, 3/4	1/3, 1	1/10, 0
	Lower	1, 1/3	1/2, 1/2	h, 0
	At Cost	0, 1/10	0, h	0, 0

a) What is the one shot Nash equilibrium if all players are rational?

NE is $1/2, 1/2 \rightarrow$ lower, lower $eq =$ above cost and $<$ monopoly

b) What price will rational players charge in the last round, and why?

They will both choose lower and lower because they have the opportunity to maximize their profits. Profits with lower are always higher on average than high or at cost.

c) For what values of r , g , and h is it a Nash equilibrium for sane players to charge high prices in all rounds before the last? Hint: work out if it is equilibrium play in the second to last round. Interpret your result.

$$\frac{x}{n} \cdot \left(1 - \frac{1}{(1+r)^T}\right) \text{ In } T-1: \frac{2g + 2(1-g)}{(1+r)^T} + \frac{\dots}{(1+r)^T}$$

$$t_{-1} = \text{high} \quad t = \text{low} \rightarrow \frac{3}{4} + \frac{[(1-g)/2 + g]}{(1+r)}$$

$$t_{-1} = \text{low} \quad t = \text{high} \rightarrow 1 + \frac{[(1-g)/2 + g]}{(1+r)}$$

$$(1+r)/3$$