11.6 Autoregressive Processes - 1

Monday, February 8, 2021 6:57 PM

$Y_t = \rho_1 Y_{t-1} + r_t + $

- bMa
- $\hookrightarrow E(r_t) = 0$
- $\hookrightarrow Var(r_t) = \sigma^2$

} AR(1)

$\rho_1 = 1 \Rightarrow$ random walk

Y_0 Fixed Start

$E(Y_t) = Y_0$

$Y_t = \rho(\rho Y_{t-2} + r_{t-1}) + r_t$

$Y_t = \rho^2 Y_{t-2} + \rho r_{t-1} + r_t$

$Y_t = \rho^2(\rho Y_{t-3} + r_{t-2}) + \rho r_{t-1} + r_t$

$= \rho^t Y_0 + \sum_{h=0}^{t} \rho^h r_{t-h}$ $\Rightarrow \rho^0 = 1$ h = lag order

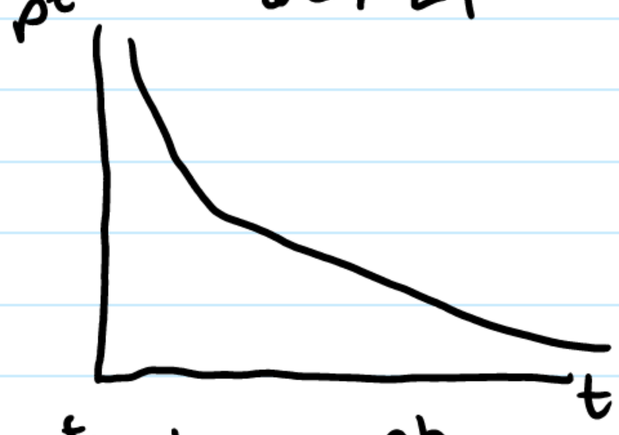
$E(Y_t) = \rho^t Y_0 + \sum_{h=0}^{t} \rho^h E(r_{t-h})$

$\hookrightarrow = 0$

$|\rho| < 1$

$E(Y_t) \to 0$ as $t \to \infty$ if $|\rho_1| < 1$

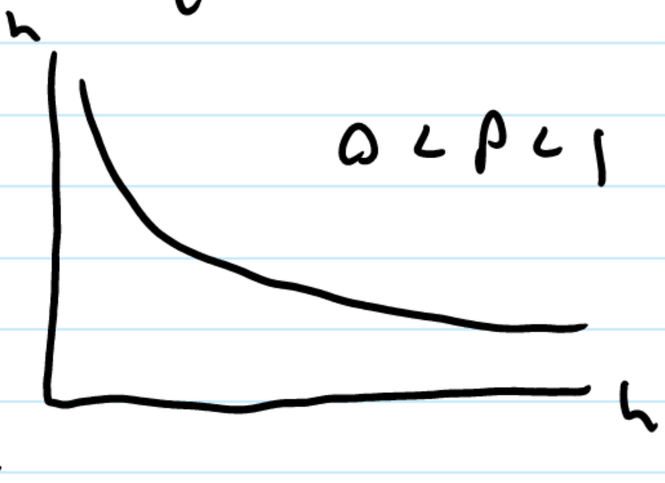
$0 < \rho < 1$



$Y_t = \rho^t Y_0 + \sum_{h=0}^{t} \rho^h r_{t-h}$

$|\rho| < 1$

$Y_t + Y_{t-h} \to$ almost independent if h is large



$Y_t = \rho_1 Y_{t-1} + r_t$

$\Delta Y_t = Y_t - Y_{t-1} = \rho_1 Y_{t-1} - Y_{t-1} + r_t = (\rho_1 - 1) Y_{t-1} + r_t$

$\Delta Y_{t-1} =$

$Y_t = \rho_1 Y_{t-1} + r$

$\hookrightarrow 0 < \rho < 1$

$\Delta Y_t = Y_t - Y_{t-1} = (\rho_1 - 1) Y_t + r_t$

$\rho = .9$	$\rho = .8$
$\rho - 1 = .1$	$(\rho - 1) = .2$
$\rho = .5$	$(\rho - 1) = -.5$