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ECO 4044 ~ Economic Analysis for Technologists

Midterm Portfolio

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1 March 2021

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1 Book Notes

Sunday, January 10, 2021 8:09 PM

Aim

Apply microeconomic tools "systematically to problems faced by firms and their managers"

"applied microeconomic analysis underpins much of the information upon which business students will base professional decisions"

Must use good sources well

The Goal of the firm (and its management)

Firms procure inputs and produce outputs

$\pi = R - C$
- C → cost
- R → revenue
- π → profit

Who claims profit (residual revenue)?
→ The firms shareholders are the **residual claimants**

Σ(all shares) = **expected present value** of all future profits

Shareholders want money and will seek to provide their **agents** (firm management) with incentives to maximize share value

Present Value

$FV = PV(1+r)^t$
- t = time periods
- r = interest rate
- PV = present value
- FV = future value

$PV = FV/(1+r)^t$ → **discounts** FV backward

Example:
$r = 10\%$ $PV = \$1$ $t = 3$ $FV = ?$
$FV = PV(1+r)^t$
$FV = 1(1+.1)^3$
$FV = 1(1.1)^3$
$FV = 1.331$

$r = 10\%$ $FV = 3$ $t = 2$ $PV = ?$
$PV = FV/(1+r)^t$
$= 3/(1+.1)^2$
$= 2.48$

π_t = value of π after period t

$\Sigma_t = \pi_t/(1+r)^t$

Example:
$\pi_0 = -124$ $\pi_1 = 21$ $\pi_2 = 72$ $\pi_3 = 45$ $r = 8\%$ $NPV = ?$
→ $NPV = -124 + \frac{21}{1.08} + 72/1.08^2 + 45/1.08^3 = -7.10$

Information Structure

Who knows what

3 types:

Complete and perfect information
Everyone knows everything

Simple risk and uncertainty

Risk: everyone shares estimates of probabilities of each outcome
Uncertainty: when there is not enough information to determine the amount of risk
Subjective probability: estimate risk based on personal knowledge

Asymmetric Information

People have different amounts of information

This can lead to two forms of market failure...

Adverse selection: when someone has information about inherent market characteristics that others don't have

Moral hazard: someone has information about an action they've taken that others can't do and are therefore shielded from consequences

Expected value and attitudes towards risk

Expected Value (EV)
Certainty equivalent (CE)

EV = average outcome over many iterations

i = outcome index
x_i = value of outcome i
f_i = probability of i

Expected value of x $E(x) = \Sigma_i f_i x_i$

$Pr(x_i)$ = probability that x takes on the specific value of x_i

Example:
$i_1 = .8$ $x_{i_1} = 40$ $i_2 = .2$ $x_{i_2} = -100$

$Pr(\pi = 40) = .8$ and $Pr(\pi = -100) = .2$
$E(x) = \sum_{i=1}^{n} Pr(x_i)\, x_i = .8 \cdot 40 + .2 \cdot -100 = 12$

CE of a lottery is the sum of money for certain that is viewed as exactly equivalent to the lottery

Risk neutral
Risk averse
Risk neutral

Expected Present Value and the value of the firm

Assume firms are risk neutral

Diversify stocks to balance out risk

Expected Present Value (EPV)

$EPV = \Sigma_t [\Sigma_i f_{it} X_{it}]/(1+r)^t$

Example:
Year one: .4 for 100, -200
Year two: .7 for 400, -200
$r = 7\%$

$PEV = [.4 \cdot 100 + .6 \cdot -200]/1.07 + (.7 \cdot 400 + .3 \cdot -200)/1.07^2$
$PEV = -74.76 + 192.15 = 117.39$

Value of Information - Part 1 - yes/no decisions

Value of better information vs perfect information

s = success f = failure π = payoff

Decision trees...

Example:
$Pr(s) = .5$ $\pi_s = 100$ $\pi_f = -60$ $\pi = ?$

$E(\pi \mid \text{No Info}) = \max((Pr(s)\pi_s + Pr(f)\pi_f), 0)$
$E(\pi \mid \text{No Info}) = \max(.5(100) + .5(-60), 0)$
$E(\pi \mid \text{No Info}) = \max(20, 0) = 20$

Dont proceed —1— 0
Proceed —.5— 100, —.5— -60 } .5(100) - .5(60) = 20 > 0

Significant chance of failure

What's to come

More math!

Marginal revenue and marginal cost
→ at maximum π, $MR = MC$

Perfectly competitive markets

Uses and Limits of Models

Models don't tell the whole story

1.1 Profit and Present Value

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Analytical approach

Profit = revenue - cost $\rightarrow \pi = P \cdot q - C(q)$

Present value

\$1 = PV $\rightarrow$ FV = 1.05 · 1 = 1.05 if interest = 5%

$FV_1 = (1+r)PV$

$V_1 = (1+r)V_0$

$V_2 = (1+r)(1+r)V_0 = (1+r)^2 V_0$

$V_t = (1+r)^t V_0$

PV of some FV?

$PV = FV/(1+r)^t$

$V_0 = \pi_0 + \pi_1/(1+r) + \pi_2/(1+r)^2 + \ldots$

$V_0 = \Sigma_t \pi_t/(1+r)^t \quad (1/1+r)^t < 1 \quad \delta = 1/1+r \quad \delta^t =$

-future is uncertain!

1.2 Expected Value

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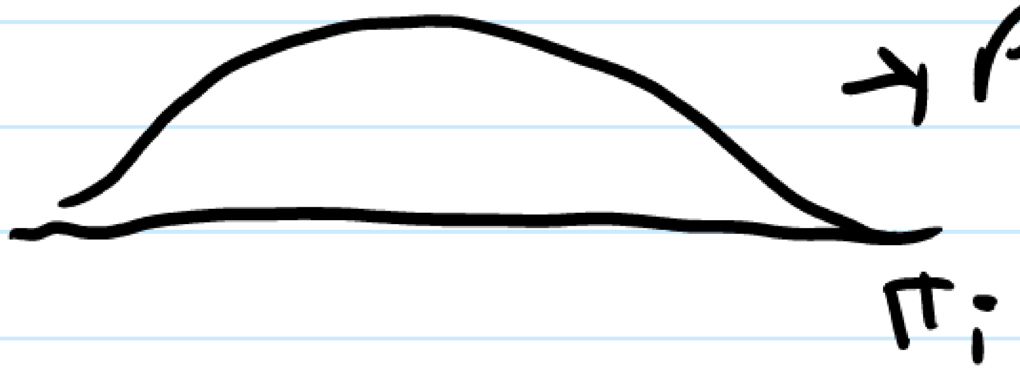
$\sum_i Pr(X=x_i)\cdot x_i$ = Expected Value

F = Probability

i	x_i (π_i)	F_i	
1	-1	.4	$=(.4\cdot -1)$
2	10	.5	$=(.5\cdot 10)$
3	30	.1	$=(.1\cdot 30)$

$= -.4 + 5 + 3 = 7.6 = E(\pi)$

Continuous outcomes...



→ Probability density function (PDF)

$E(\pi) = \int_{-\infty}^{\infty} x f(x)\,dx$

Attitudes towards Risk

- future is a lottery
- Preferences over Lotteries

Certainty Equivalent

- certain outcome viewed as equivalent to the lottery

Risk averse → $CE < EV$

Diversification

$EPV = \left[\sum_i F_{it}\cdot \pi_{it}\right]/(1+r)^t = \sum_t E(\pi_t)/(1+r)^t$

$CE > EV$ = Risk Loving

1.3 Information Structures

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- complete + perfect info
- Incomplete information
  - Risk vs uncertainty

Risk	uncertainty
↓	↓
objective probability	subjective probability

- Asymmetric information

1 Extra Problems

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t	π
1	-10
2	5
3	10
4	5
5	3

$r = 4\%$

$V_0 = -10/1.04 + 5/1.04^2 + 10/1.04^3 + 5/1.04^4 + 3/1.04^5 = 10.637$

i	π_i	f_i
1	-10	.4
2	4	.4
3	50	.2

$E(\pi) = (.4 \cdot -10) + (.4 \cdot 4) + (.2 \cdot 50) = 7.6$

t=1

i	f	π
1	.4	-110
2	.6	10

t=2

i	f	π
1	.7	-10
2	.3	50

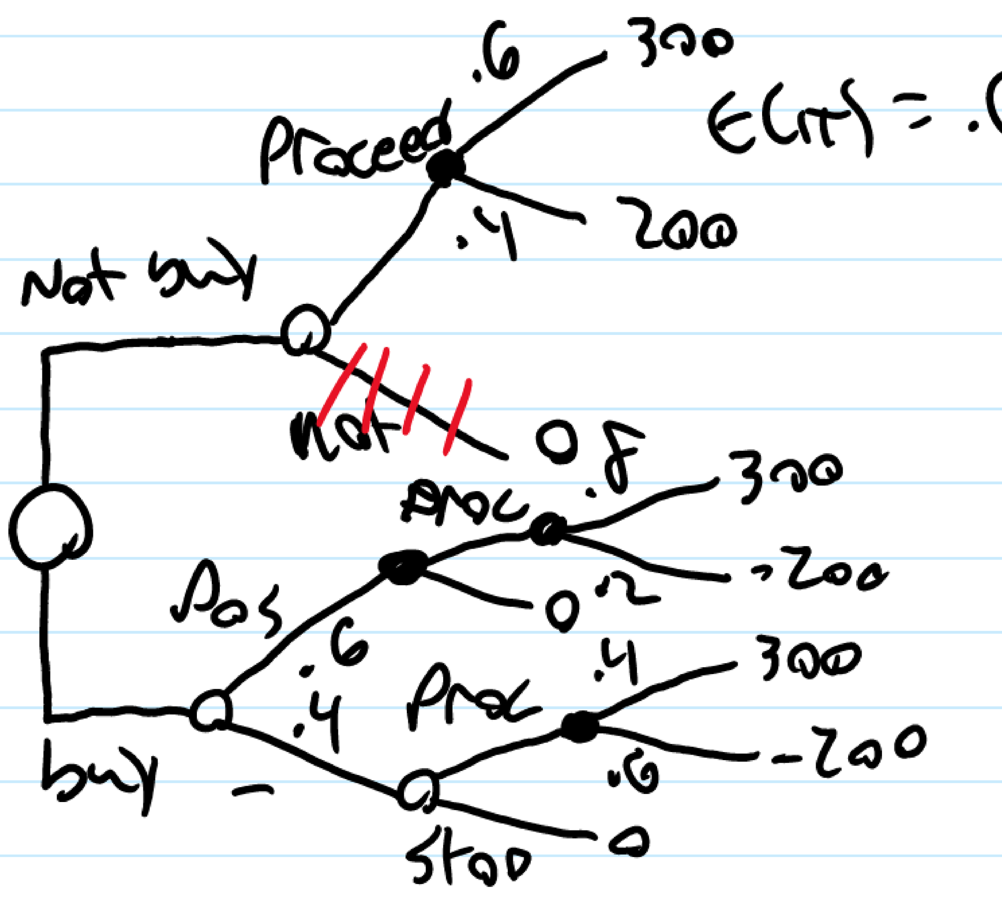
$r = 4\%$

$EPV = (.4 \cdot -40)/(1.04) + (.6 \cdot 10)/1.04 + (.7 \cdot -10)/1.04^2 + (.3 \cdot 50)/1.04^2 = -2.2189$

Suppose a project returns 300 with probability 0.6 and loses 200 otherwise.

A test can be conducted to provide additional information about the probability of success. You think the probability of a positive result is 0.6. You think the probability of success with a positive test result is 0.8, and you think the probability of success with a negative test result is 0.4.

What, if anything, is the value of the information provided by the test?



$E(\pi) = .6 \cdot 300 - .4 \cdot 200 = 100$

$E() = .4 \cdot 300 - .6 \cdot 200 = 0$

Info =

A. Suppose inverse demand is p=1000-2q. Find the quantity that maximizes revenue, the corresponding price, and the maximum revenue.

B. Suppose cost is 1000+100q+0.5q². Find the profit maximizing quantity, price, and the maximum profit.

C. Illustrate.

D. Explain why the price that maximizes revenue is too low. What is profit at that price?

a) Inverse demand: $0 = P - 1000 - 2q$
$2q = 1000 - P$
$q = 500 - 1/2 P$

Revenue $= P \cdot q$
$= P \cdot (500 - 1/2 P)$
$= 500P - 1/2 P^2$
$P = 500 \quad q = 250$

$\rightarrow R = 1000q - 2q^2$
$dR = 1000 - 4q$

B) $\pi = p \times q - c \qquad C = 1000 + 100q + 1/2 q^2$

$(1000q - 2q^2) - (1000 + 100q + 1/2 q^2) = -2.5q^2 + 1100q - 1000$

$\frac{d\pi}{dq} = -5q + 1100 \rightarrow q = 220 \rightarrow P = 560$

c)

1/27 Problems

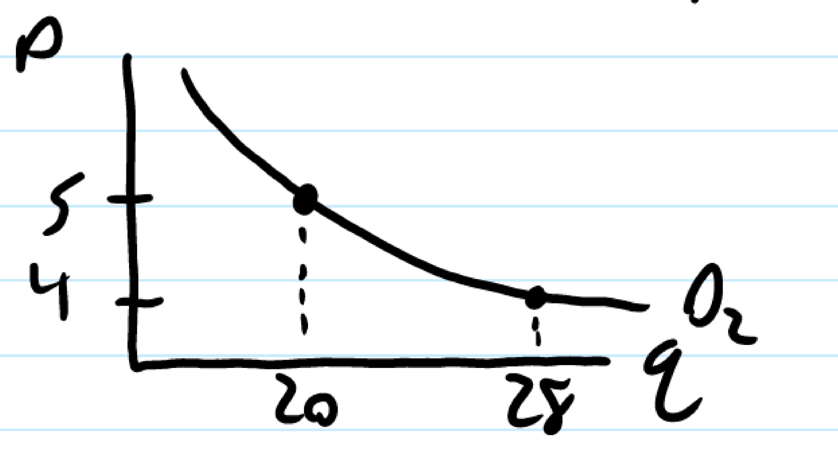
$q_1 = 100 P_1$
$C = F + 2q$

$\varepsilon_2^d = -2$
If $P_2 = 5,\ q_2 = 20$

1) $P_1^* = ?$ 2) $P_2^* = ?$
3) $q_1^* = ?$ 4) $q_2^* = ?$
5) Constant ε approx for market 2

$MR = P(1 + 1/\varepsilon)$
$MR = P(1 + 1/3) = 2 \rightarrow P^* = \varepsilon/(1+\varepsilon) \cdot MC \rightarrow P_1 = -4/(-1-4) \cdot 2 = 2.67$

$P_2^* = -2/(1-2) \cdot 2 = 4$
$q_1^* = 100/2.67^4 = 1.97$



$P_2 = 5 \rightarrow q_2 = 20$

$\% \Delta P = -20\%$
$\% \Delta q = 40\%$

$q_2 = A P_2^{-2} \qquad 20 = A/5^2 \rightarrow A = 20 \cdot 25 = 500 \qquad q_2 = 500/P^2$

$\frac{\Delta q}{\Delta P} \frac{P}{q}$

2 Book Notes

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Cost, Demand, and Profit Maximization

Cost, Its Determinants, and Marginal Cost

$C(q)$ = firm cost

$C = C(q; w, r, z)$ → cost of q units depends on q, wage rate (w), capital costs (r), and other factors (z)

Marginal cost = cost of making one more unit

$MC = dC/dq$ d = change (derivative)

4 ways to approximate cost:

$C(q) = f + cq \rightarrow c > 0,\ f \geq 0,\ MC = c$
$C(q) = f + cq^{d} \rightarrow c > 0,\ d > 0,\ f \geq 0,\ MC = cdq^{d-1}$
$C(q) = f + aq + bq^2 \rightarrow a > 0,\ b > 0\ f \geq 0,\ MC = a + 2bq$
$C(q) = f + aq + bq^2 + cq^3 \rightarrow a > 0,\ b < 0,\ c > 0,\ f \geq 0,\ MC = a + 2bq + 3cq^2$

Demand and its Determinants, Inverse Demand

$q = q(P)$

m = income

Price of substitutes (P_s) and complements (P_c)

n = market size

Z = other variables

$q = q(P, m, P_s, P_c, n, Z)$

↳ implies that q depends on P

$P = P(q, m, P_s, P_c, n, Z)$

$P = P(q)$

Measuring the Sensitivity of Quantity Demanded to Price

Elasticity is a percentage

Elasticity of demand = η (eta) $= \%\Delta q / \%\Delta p = \Delta q/q \,/\, \Delta p/p$

η not constant over demand

Demand Approximations

Linear Demand Approximations

ε = random error

$q_0 = A - Bp$

$p = \frac{A}{B} - \frac{1}{B}q$

Log Linear (Constant Elasticity) Demand Approximations

$q = Ap^{-B}$

$\ln(q) = \ln(A) - B\ln(p)$

$dq/dp = -BAp^{-B-1}$

η w/ respect to independents are constant and equal to that variable's coefficient

Revenue and Marginal Revenue

Revenue = price • quantity

Firms have **market power** if they have a non-negligible effect on market price

Marginal revenue = gain in revenue from one unit sold

$MR = P + \frac{dP}{dq}q$

Marginal Revenue and Elasticity

$dR/dq = P\left(1 + \frac{1}{\eta}\right)$

Profit Maximization

$q^{\#}$ = profit maximizing q

↳ $P^{\#}$

$P = (\eta/(1+\eta))\,MC$

2.1 Value of Information

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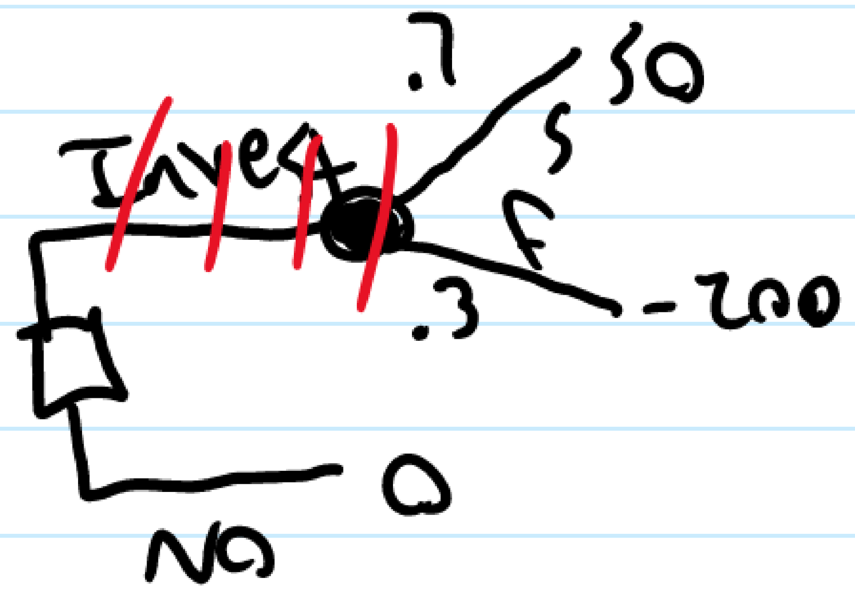
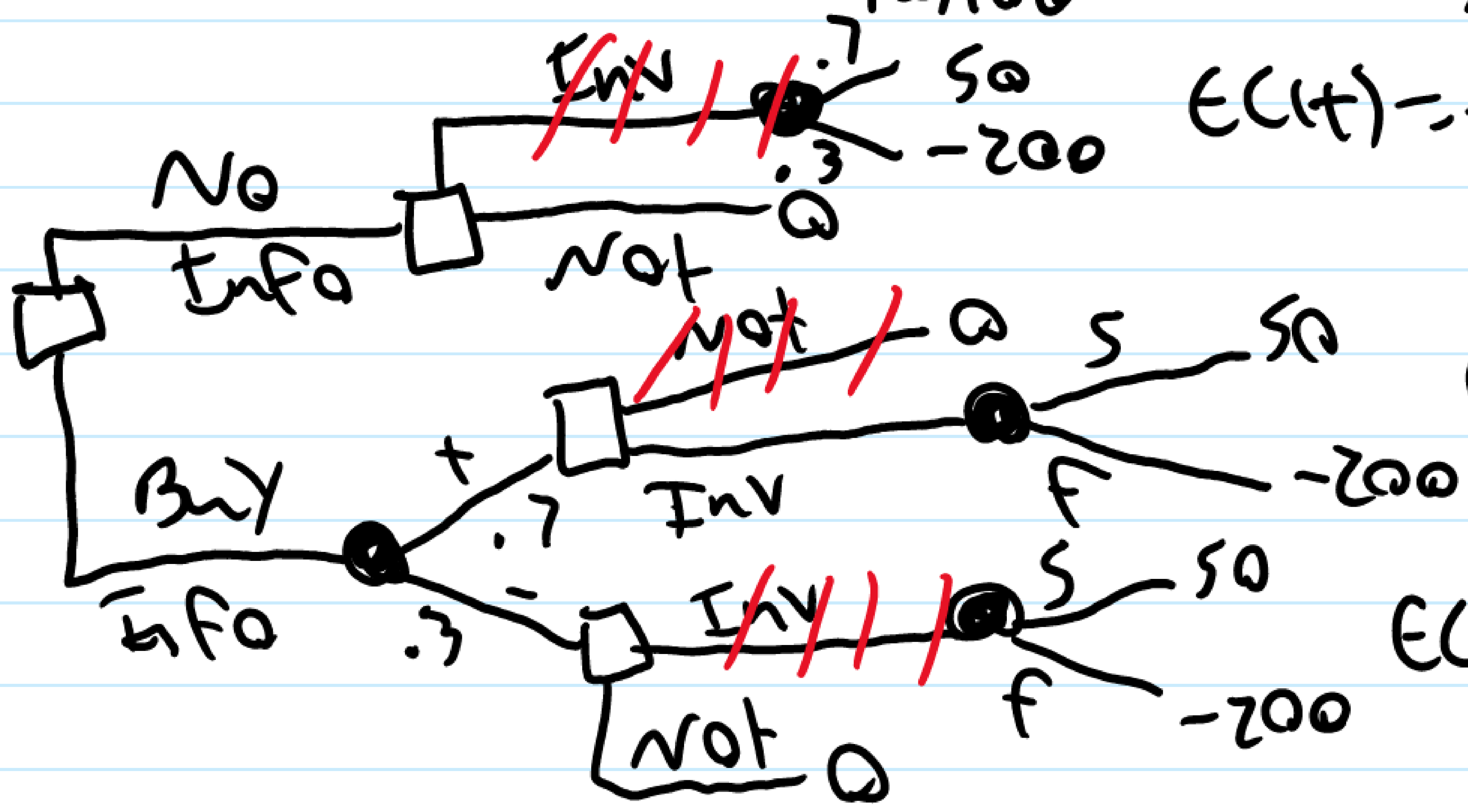
$Pr(S) = .7$
$\pi_S = 50$
$\pi_F = -200$



$E(\pi | inv) = .7 \cdot 50 + .3 \cdot 200 = 35 - 60 = -25$

	Test		
	Pos	Neg	
Succ	63	5	70/100
Fail	7	25	30/100
	70/100	30/100	



$E(+) = -25$

$E(\pi) = \frac{63}{70} \cdot 50 - \frac{7}{70} \cdot 200 > 0$
$= 25$

$E(\pi) = \frac{7}{30} \cdot 50 - \frac{23}{30} \cdot 200 < 0$
$= -16/2$

$E(\pi | info) = .7 \cdot 25 + .3 \cdot 0 = 17.5 > 0$

Value of info: $17.5 - 0 = 17.5$

$P(S|+) > P(S)$
$P(S|-) < P(S)$

Demand and profit maximization

$\pi = R - C$

$R = p \cdot q - C(q)$

Approximate cost?

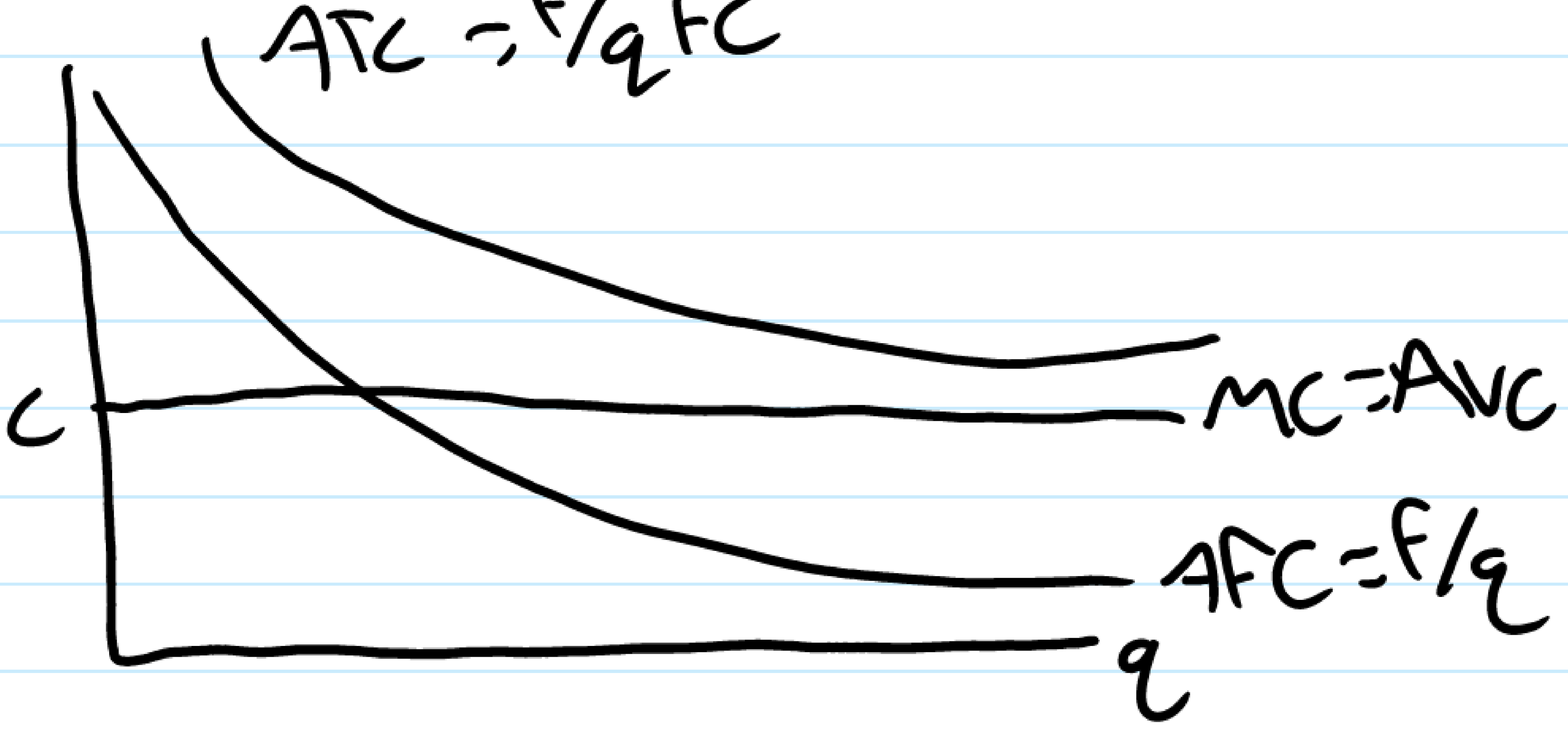
$C = f + cq$

$MC = \frac{dC(q)}{dq}$

$ATC = C(q)/q$

$AVC = TVC/q$

$AFC = TFC/q$



Quadratic

$C = f + aq + bq^2$

$\frac{dc}{dq} = a + 2bq$

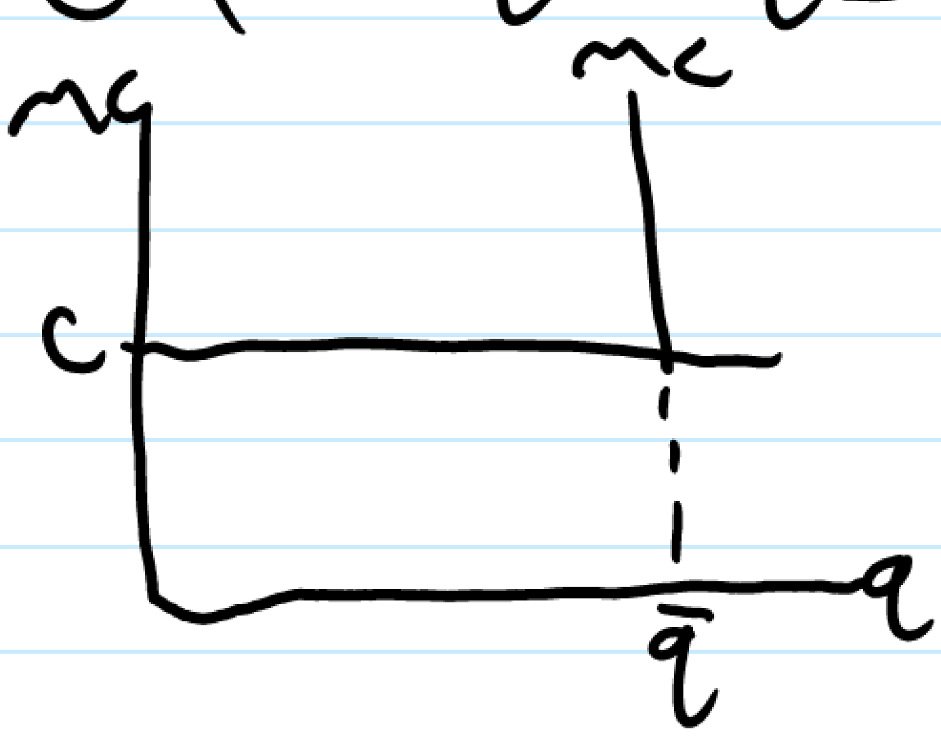
Cubic

$C = f + aq + bq^2 + cq^3$

$\frac{dc}{dq} = a + 2bq + 3cq^2 \Rightarrow b < 0, c > 0$, mc → U shape

Constant mc w/ fixed capacity

$C = f + cq \quad q \le \bar{q} \qquad q > \bar{q}$



2.3 Cost Concepts

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opportunity cost

fixed costs

sunk costs

Quasi-fixed costs

$C = F + cq = 0 \quad q > 0 \quad q = 0$

Cost shifters

w = wage
r = price of capital

Demand and revenue

q(p) demand
p(q) inverse demand

linear: $q = a - bp$

Log-linear: $q = ap^3$

Log-linear demand = constant elasticity

$q = ap^3$

$\ln q = \ln a - 3\ln p$

2.5 Elasticity of Demand

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dq/dp

$\eta = \%\Delta q / \%\Delta p < 0$

$\eta = \Delta q/q \,/\, \Delta p/p \rightarrow$ point elasticity

$q = ap^3$

$\frac{dq}{dp} = 3ap^{3-1}$

2.6 Demand Shifters

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$\Delta P \rightarrow$ quantity demanded changes

Income Normal $m\uparrow$, $\vec{D}$

Inf. $m\uparrow$ $\overleftarrow{D}$

Prices of substitutes $\rightarrow P_s\uparrow \rightarrow \vec{D}$

Prices of complements $\rightarrow P_c\uparrow \rightarrow \overleftarrow{D}$

$q = b_0 + b_p P + b_s P_s + b_c P_c + b_N N + b_Z Z$

Dumb!

$\ln q = b_0 + b_p \ln P + b_s \ln P_s + \ldots$

elasticities

2.7 Demand and Revenue

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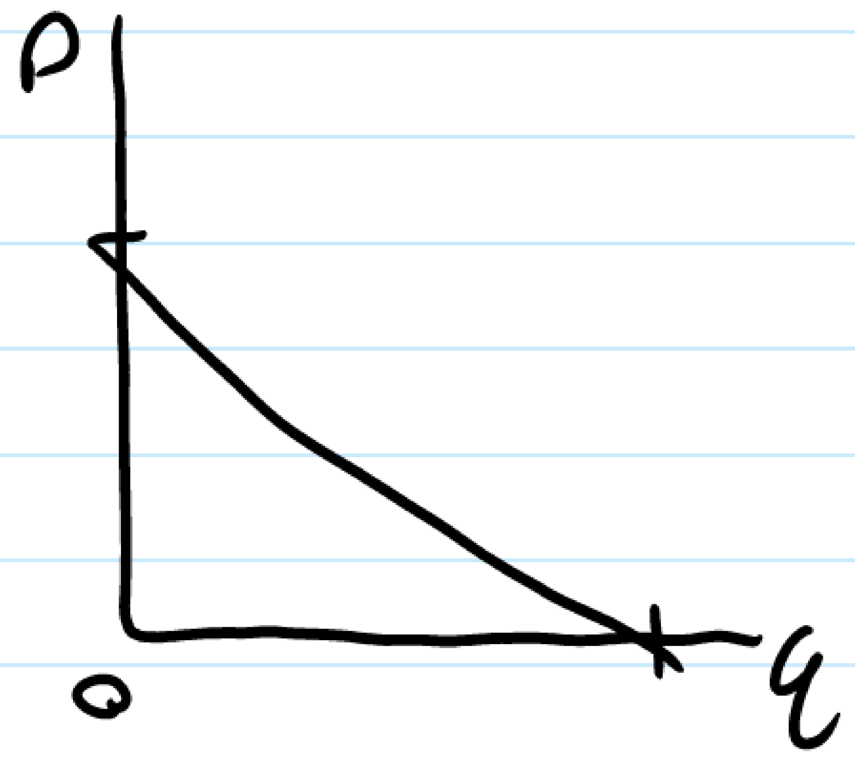
p	q	R
10	100	1000
9	120	1080
8	140	1120
7	160	1120
6	180	1080
5	200	1000
4	220	880



$\frac{\Delta q}{\Delta p} = -20$

$q = 100 - 20(p - 10)$
$= 100 - 20p + 200$
$= 300 - 20p \quad \rightarrow p = 15 - .05q$

$R(q) = p \cdot q$

$MR = \frac{dR}{dq} = \frac{dp}{dq} \cdot q + 1 \cdot p \quad \rightarrow \frac{dp}{dq} < 0$

$\hookrightarrow MR = p + \frac{dp}{dq} \cdot \frac{q}{p} p = p(1 + 1/3) = p(1+3/3)$

2.8 Profit Maximization

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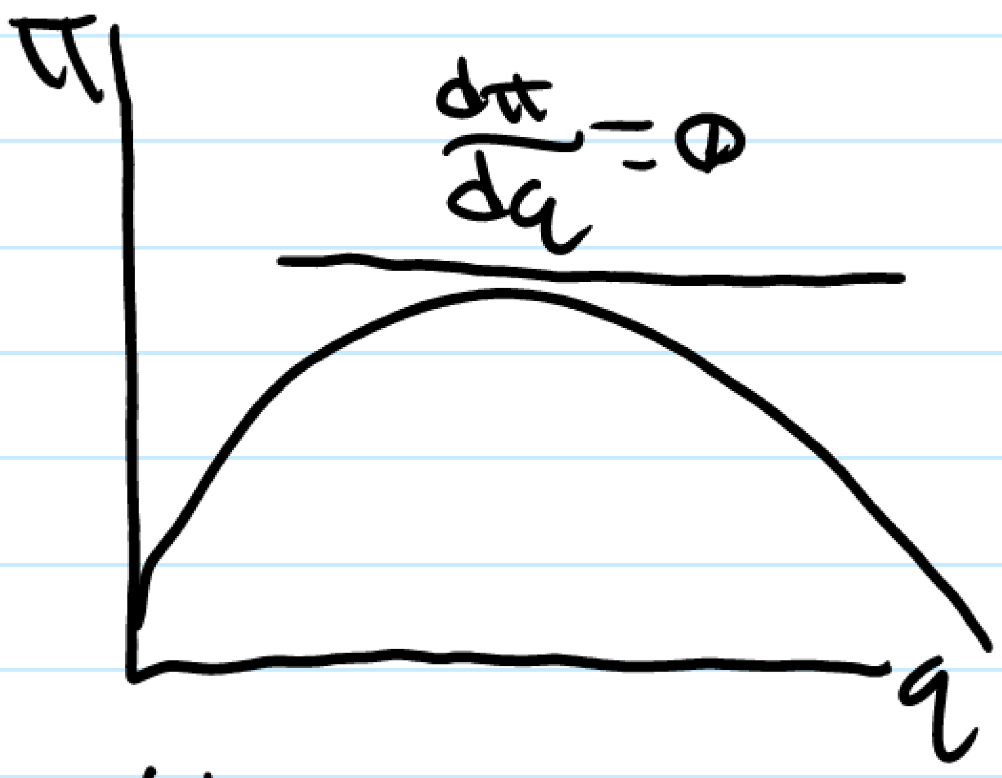
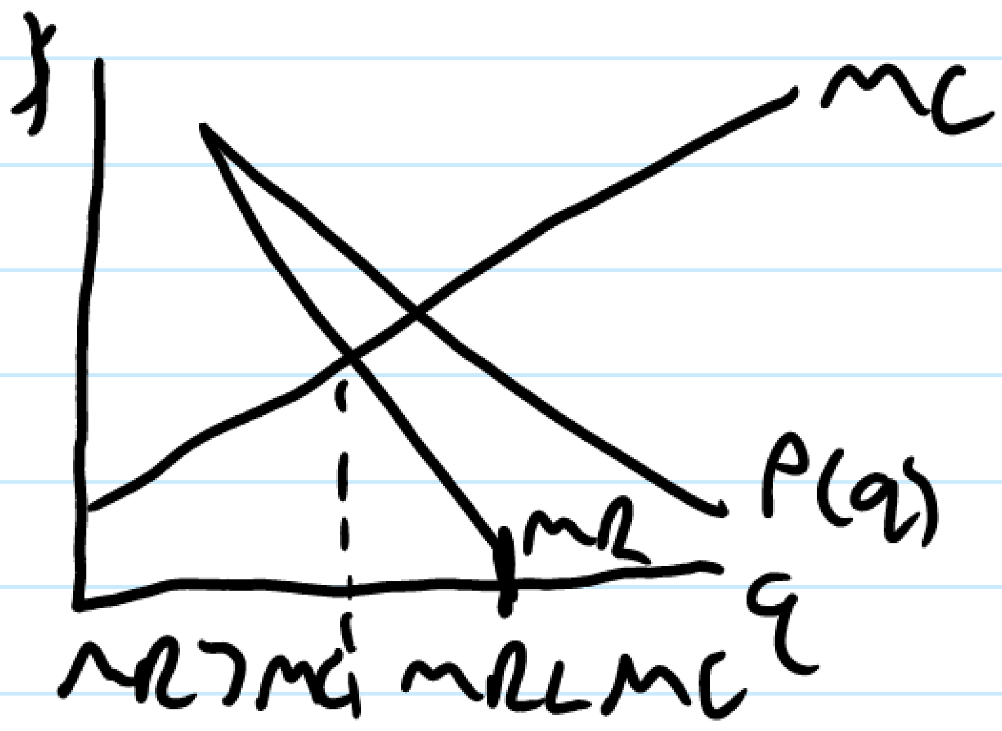
$$\pi(q) = P(q) \cdot q - C(q)$$



$$\frac{d\pi}{dq} = MR - MC = 0$$



2.9 Profit Maximization Example 1

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$P = 15 - q/20 \qquad C = 100 - 5q$

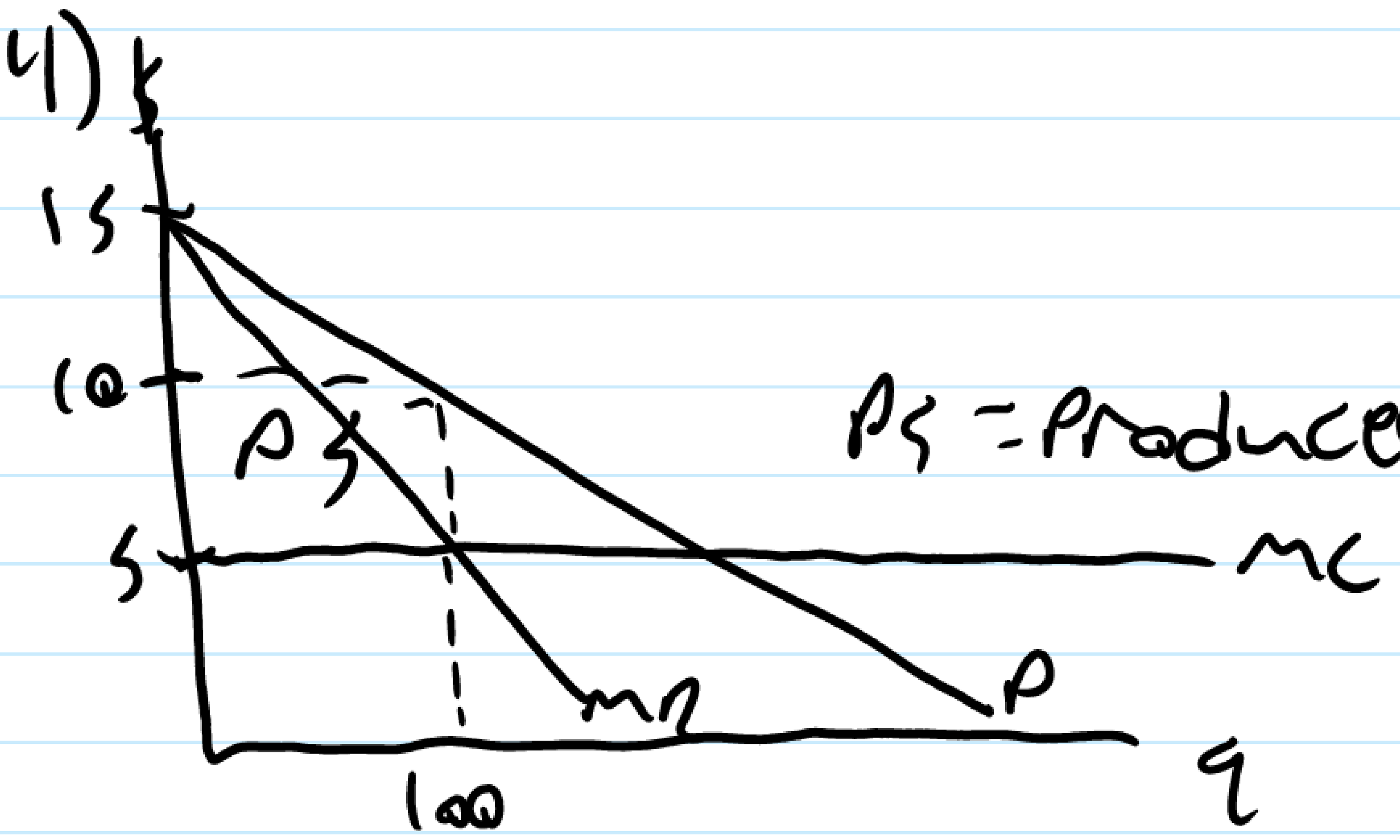
1) write $\pi(q)$
2) find MR, MC
3) find P^*, q^*, π^*
4) illustrate

1) $\pi = (15 - q/20)q - 5q - 100$

2) $MR = 15 - q/10$
$MC = \text{derivative } 100 - 5q = 5$

3) $15 - q/10 = 5 \rightarrow q = 100$
$P = 15 - 100/20 \Rightarrow P = 10$
$R = 10 \cdot 100 = 1000$
$C = 100 + 5(100) = 600$
$\pi = 1000 - 600 = 400$

4)



PS = Producer Surplus

2.10 Profit Maximization - More Examples

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$MC = 5 \qquad \varepsilon = -3$

$MR = P(1 + 1/\varepsilon) = MC$

$P = \left(\frac{\varepsilon}{1+\varepsilon}\right) MC = \frac{-3}{-2} \cdot 5 = 7.5$

$\varepsilon = -1/2 \Rightarrow P = \frac{-1/2}{1 - 1/2} \cdot 5 = -5$

3 Book Notes

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Applications and Extensions of optimal production and pricing

Simple (3rd degree) Price Discrimination

charge different prices to different groups
↳ senior discount

groups must:
↳ be identifiable
↳ have specific willingness to pay
↳ resale should be impossible

Market w/ less elastic demand

$P_1 = \frac{3_1}{1+3_1} MC$ $\quad P_2 = \frac{3_2}{1+3_2} MC$

Example: Two groups. Max profit w/ discrimination

$P_1 = 20 - q_1 \quad P_2 = 30 - q_2 \quad C(Q) = .5(q_1+q_2)^2$

Two Prices: $\pi = (20-q_1)q_1 + (30-q_2)q_2 - .5(q_1+q_2)^2$

$MR_1 = MR_2 = MC$

$MR_1 = MC \qquad MR_2 = MC$
$20 - 2q_1 = q_1 + q_2 \qquad 30 - 2q_2 = q_1 + q_2$

$20 - 2q_1 = 30 - 2q_2$
$2q_2 = 10 + 2q_1$
$q_2 = 5 + q_1$

$20 - 2q_1 = q_1 + 5 + q_1$
$15 = 4q_1$
$q_1 = 3.75 \rightarrow q_2 = 5 + q_1 = 8.75$
$P_1 = 20 - 3.75 = 16.25 \qquad P_2 = 30 - 8.75 = 21.25$

$\pi = 16.25(3.75) + 21.25(8.75) - .5(3.75 + 8.75)^2 = 168.75$

one price?

$P_1 = 20 - q_1 \rightarrow q_1 = 20 - P_1 \qquad P_2 = 30 - q_2 \rightarrow q_2 = 30 - P_2$

$P_1 = P_2 = P \rightarrow q = 50 - 2P \rightarrow P = 25 - .5q$

$\pi = (25 - .5q)q - .5q^2$
$MR = MC$
$25 - q = q$
$q = 12.5$
$P = 25 - .5(12.5) = 18.75$
$\pi = 18.75(12.5) - .5(12.5)^2 = 156.25$

Extra profit $\rightarrow 168.75 - 156.25 = 12.5$

Profit maximization when purchases per capita don't depend on market size

$q_D/N = f(P, P_3, P_c, M, Z) \rightarrow$ Purchases per capita

N = city size

Maximizing Profit with a pre-determined capacity constraint

Theaters, stadiums, etc

Profit maximization with Uncertainty

Expected Profit

Value of information with continuous Decisions

Peak Load Pricing - Determining capacity when demand varies

Chapter 16 in Game Theory

To max π, $MR_L = MC_L = c$ and $MR_H = MC_H = c + k$

$\pi = P_H(q_H)q_H + P_L(q_L)q_L - cq_L - (c+k)q_H \rightarrow q_H \geq q_L$

↳ if $q_H < q_L$ then $q_H = q_L$

$MR_H + MR_L = 2c + k$

$P^* = \left(\frac{\varepsilon}{1+\varepsilon}\right) MC$

$\pi = P_1(q_1)q_1 + P_2(q_2)q_2 - C(q_1+q_2)$

$\frac{d\pi}{dq_1} = MR_1 - \frac{dc}{dq} = 0 \qquad MR_1 = MC$

$\frac{d\pi}{dq_2} = MR_2 - MC = 0 \qquad MR_2 = MC$

$MR_1 = MR_2$

$P_1 = \frac{\varepsilon_1}{1+\varepsilon_1} MC \qquad P_2 = \frac{\varepsilon_2}{1+\varepsilon_2} MC$

1) Must be able to differentiate
2) No resale

Example:

$P_1 = 20 - 1/2\, q_1 \qquad P_2 = 15 - 1/2\, q_2 \qquad C = (q_1+q_2)^2$

$\pi = (20 - 1/2\, q_1)q_1 + (15 - 1/2\, q_2)q_2 - (q_1+q_2)^2$

$20 - q_1 = 2(q_1+q_2)$
$15 - q_2 = 2(q_1+q_2)$
$20 - q_1 = 15 - q_2$
$5 + q_2 = q_1$

$15 - q_2 = 2(5 + q_2 + q_2)$
$15 - q_2 = 10 + 4q_2$
$5 = 5q_2$
$q_2 = 1 \qquad P_2 = 14.5$
$q_1 = 6 \qquad P_1 = 17$

$\pi = 6 \cdot 17 + 1 \cdot 14.5 - 7^2 = 67.5$

$1/2\, q_1 = 20 - P_1$
$q_1 = 40 - 2P_1$
$1/2\, q_2 = 15 - P_2$
$q_2 = 30 - 2P_2$
$q = 70 - 4P$

$\frac{d\pi}{dq} = \frac{70}{4} - 1/2\, q - 2q = 0$
$q = 7$
$P = \frac{70}{4} - 1/4(7)$
$= 63/4 = 15.75$
$\pi = 7 \cdot 63/4 - 49 = 61.25$

3.3-4 Pricing When Per

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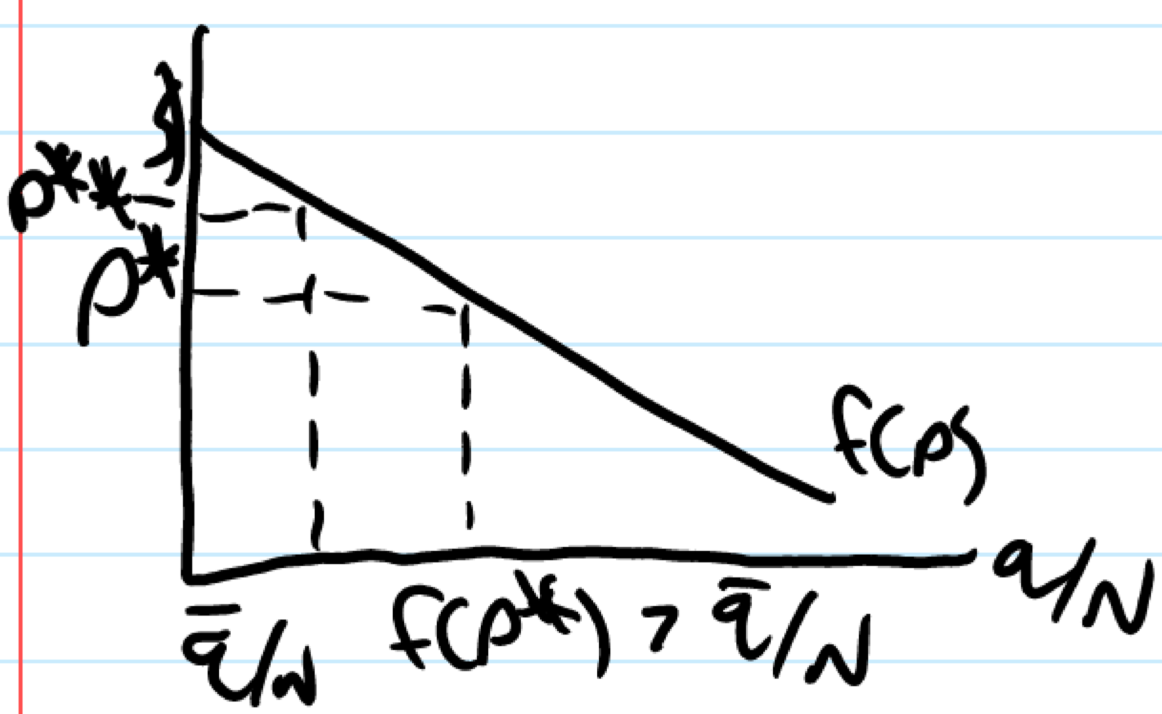
$q = N(\beta_0 + \beta_p p) \rightarrow q/N = a - bp$

all else in a

$q = Nf(p)$ $\quad f = q^d/N$

$\frac{dq}{dp} \cdot \frac{p}{q} = N \frac{df}{dp} \cdot \frac{p}{Nf(p)} \rightarrow \eta = \frac{df}{dp} \cdot p/f \quad p^* = (\eta/_{1+\eta}) mc$

"Law of one price"



Example:

$N = 1000, \ f(p) = 1 - .1p \quad mc = 2$

$\pi = 1000[p(1-.1p) - 2(1-.1p)]$
$= 1000(p-2)(1-.1p)$
$\frac{d\pi}{dp} = 1000(-.1(p-2) + 1(1-.1p)) = 0$
$.2 + 1 - .2p = 0$
$.2p = 1.2$
$p = 6$ $\quad f = 1 - .1 \cdot 6$
$q = 400$ $\quad = .4$
$\pi = (6-2) \cdot 400 - F$
$= 1600 - F$

$\bar{q} = 200$
$f = 200/1000 = .2$ $\quad .2 = 1 - .1p$
$.1p = .8$
$p = 8$

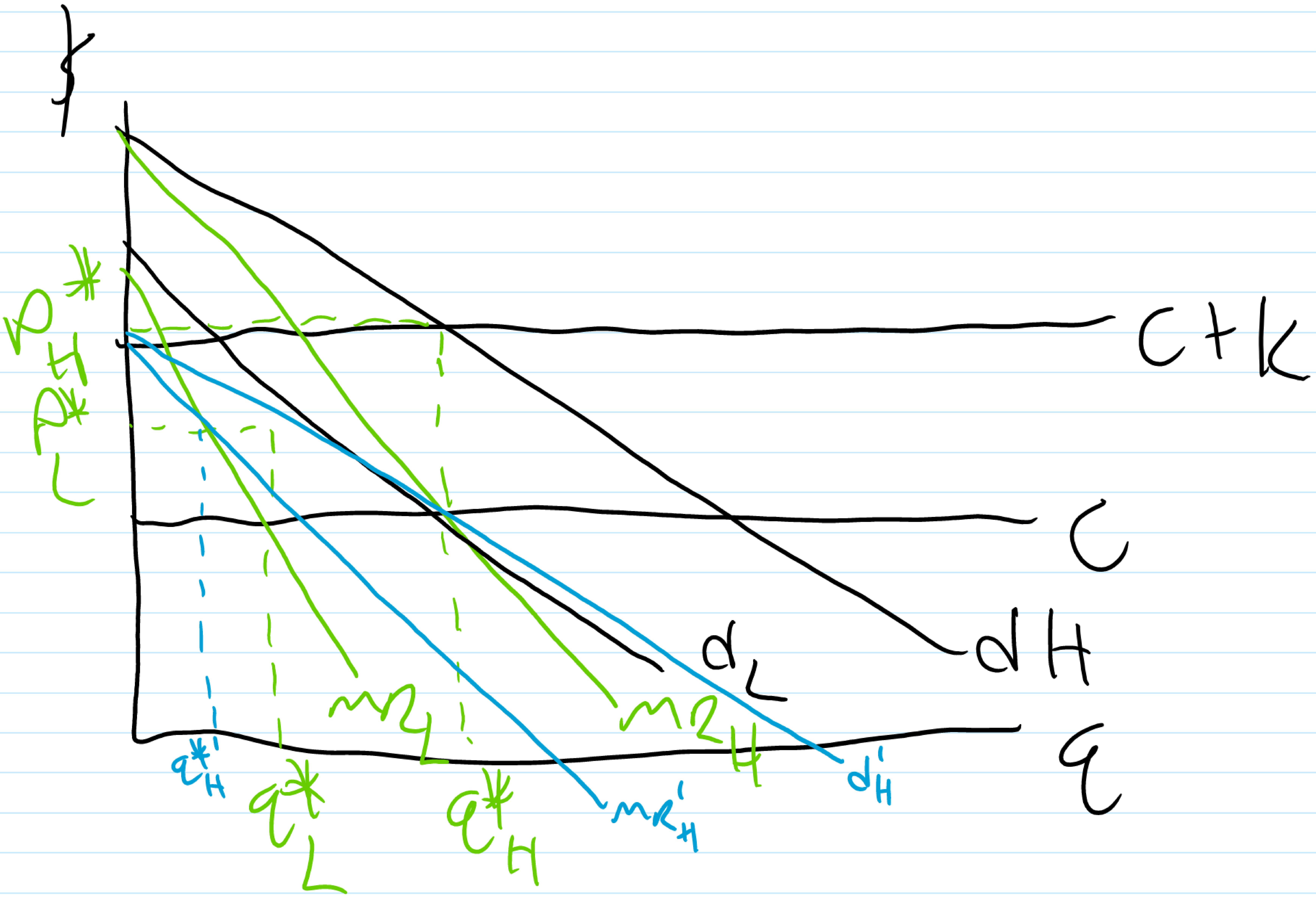
$\pi = (8-2) \cdot 200 - F$
$= 1200 - F$

3.5 Peak Load Pricing - Informal

Sunday, January 24, 2021 3:18 PM

$C(q) = (c+k)q$
- k = cap cost
- c = op cost

$C(q) = cq \Rightarrow q <$ capacity



$MR_H = c + k$

$MR_L = c$

"shifting peak" to adjust demand

3.6 Peak Load Pricing - Formal

Sunday, January 24, 2021 3:30 PM

$\pi = P_L(q_L)\cdot q_L + P_H(q_H)\cdot q_H - cq_L - cq_H - k\bar{q}$

q_L, q_H s.t. $q_H \geq q_L$

1) Ignore Constraint

$MR_L = c$
$MR_H = c+k$ $\Rightarrow q_L, q_H, P_L, P_H$

Check if $q_H \geq q_L$
↳ yes = done
↳ no = step 2

2) Sub $q_H = q_L = q$

$\max_q = P_L(q)\cdot q + P_H(q)\cdot q - (2c+k)q$
↳ q_L, P_H, P_L

3.7 Peak Load Pricing Example

Sunday, January 24, 2021 3:37 PM

$P_H = 14 - \tfrac{1}{2} q_H \qquad P_L = 12 - \tfrac{1}{2} q_L \qquad c = 2 \qquad k = 4$

1) Assume $q_H > q_L$

$MR = 14 - q_H = 4 + 2 \qquad\qquad = 12 - q_L = 2$

$q_H = 8 \qquad\qquad q_L = 10$

$\cancel{q_L > q_H}$

2) Assume $q_H = q_L$

$\pi = (14 - \tfrac{1}{2} q)q + (12 - \tfrac{1}{2} q)q - (2 \cdot 2 + 4)q$

$= 14 - q + 12 - q - 8 = 0$

$2q = 18$

$q = 9 \Rightarrow P_H = 14 - \tfrac{1}{2} 9 = 9.5 \qquad P_L = 12 - \tfrac{1}{2} 9 = 7.5$

3.8-9 Production and Pricing

Saturday, January 30, 2021 11:48 AM

Timeline

Produce — learn demand — Post Price — Sell

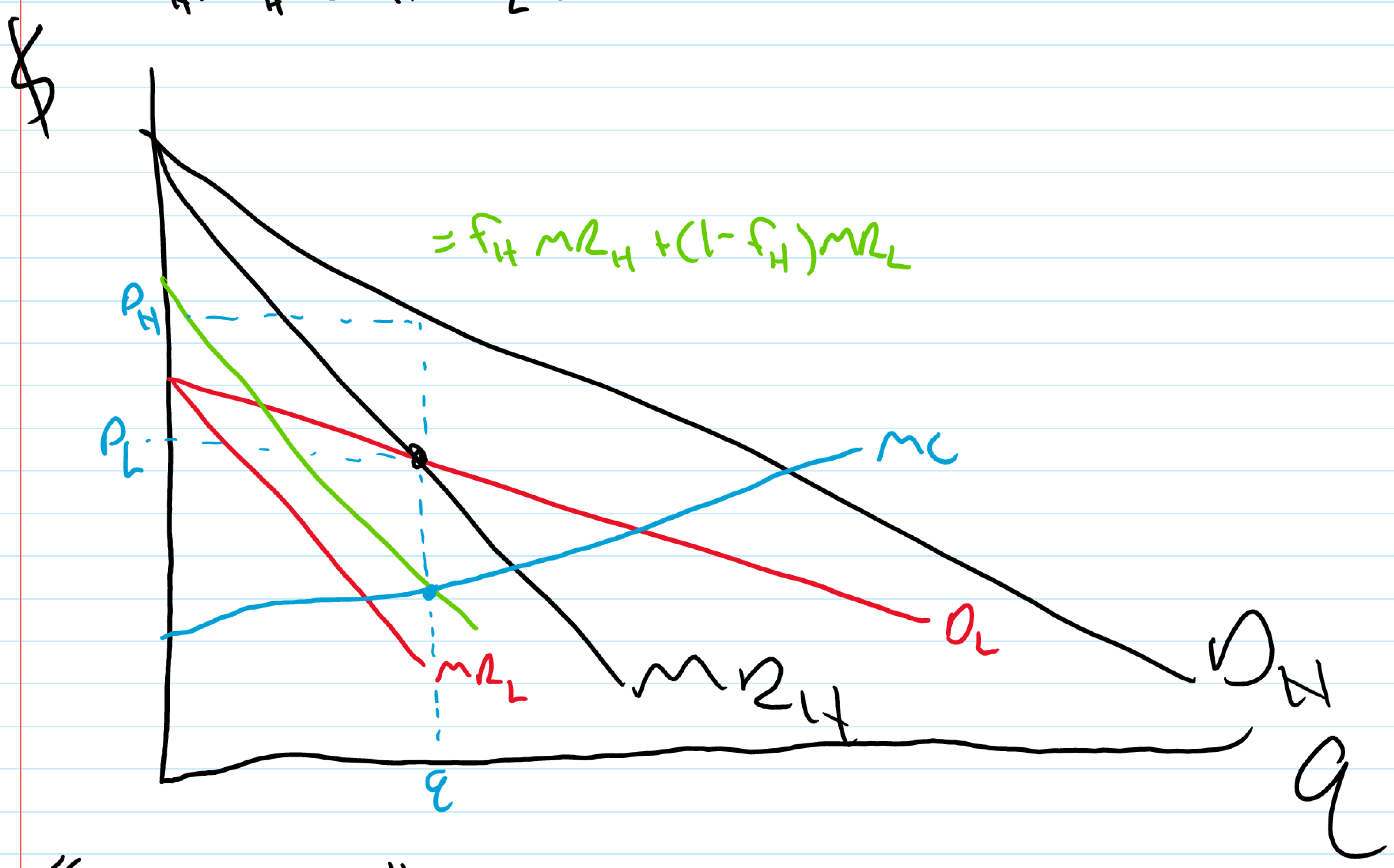
$Pr(H)$ = Probability of high demand
$Pr(L)$ = Prob of Low demand

q_H p_H q_L P_L

$$E(\pi) = Pr(H)\, p_H(q)\, q + (1 - Pr(H))\, P_L(q)\, q - C(q)$$

$$\frac{d\pi}{dq} = f_H MR_H + (1 - f_H) MR_L - MC = 0$$

$$f_H MR_H + (1 - f) MR_L = MC$$



"Free disposal"

$$E(\pi) = Pr(H) P_H(q_H) q_H + Pr(L) P_L(q_L) q_L$$

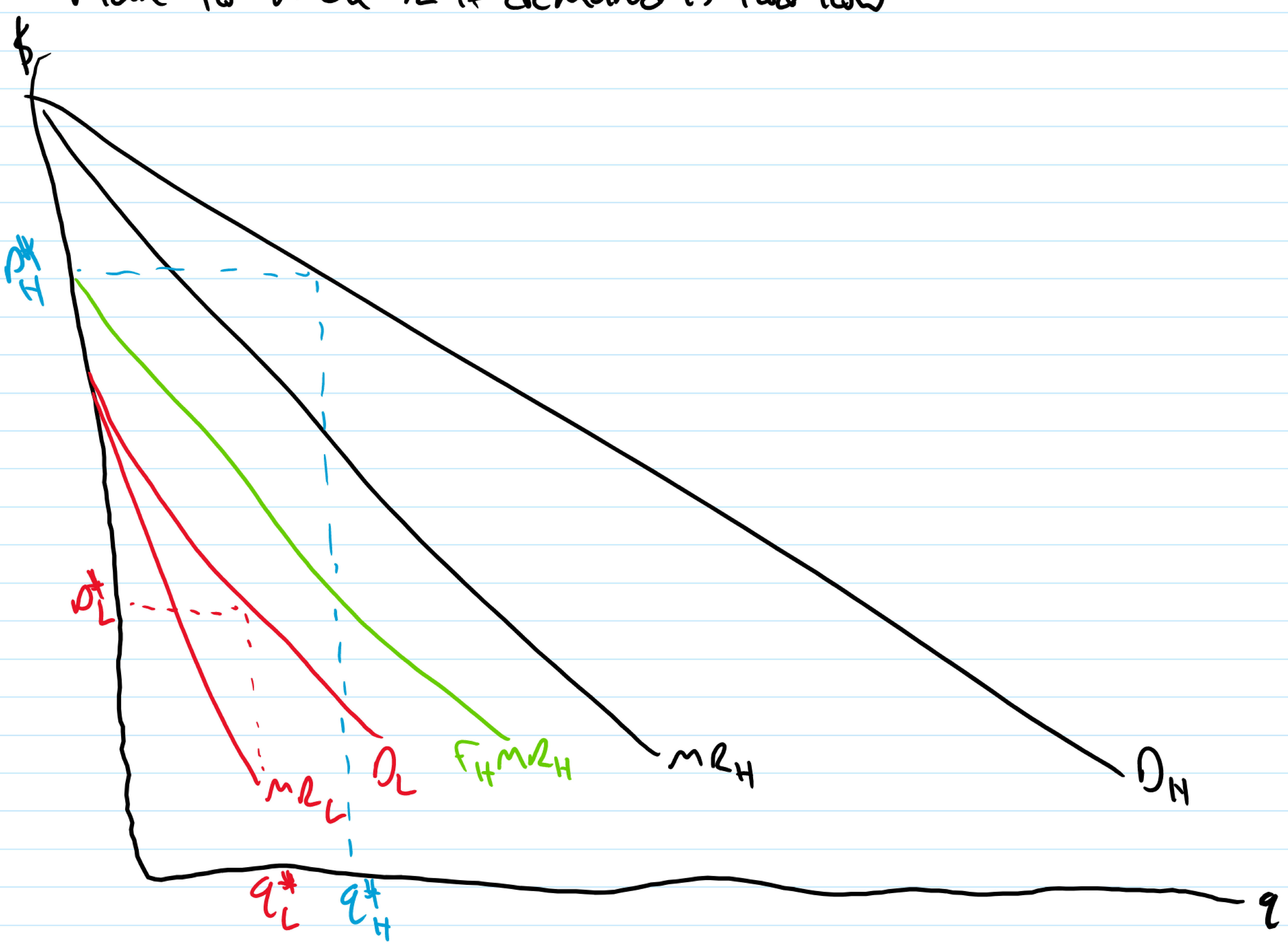
s.t. $q_H \le q$ $\quad q_H = q$
$q_L \le q$

$$\max E(\pi) = Pr(H) P_H(q_H) + Pr(L) P_L(q_L) - C(q_H)$$
s.t. $q_H \ge q_L$

$$\frac{dE(\pi)}{dq_H} = Pr(H) \cdot MR_H - MC = 0$$

$$\frac{dE(\pi)}{dq_L} = MR_L = 0$$

Plan to max R if demand is too low



$q^*_H > q^*_L$

If $q^*_H < q^*_L$, nonsense constraint violated!
Go back to $q_H = q_L = q$

3.10-12 Production and Pricing

Saturday, January 30, 2021 12:15 PM

$P_H = 20 - 1/4\, q_H$ $\quad P_L = 10 - 1/4\, q_L$ $\quad f_H = .5$ $\quad MC = 2$

$E(\pi) = \frac{1}{2}(20 - 1/4 q)q + \frac{1}{2}(10 - 1/4\, q)q - 2q$

$\hookrightarrow \frac{1}{2}(20 - 1/2 q) + \frac{1}{2}(10 - 1/2)q = 2$

$\hookrightarrow 10 - 1/4 q + 5 - 1/4 q = 2$

$13 = 1/2 q$

$q = 26$

$P_H = 20 - 26/4 = 13.50$, $\quad \pi_H = (13.50 - 2)26 = 299$

$P_L = 10 - 26/4 = 3.50$; $\quad \pi_L = (3.5 - 2)\,26 = 39$

$\hookrightarrow E(\pi) = \frac{1}{2}299 + \frac{1}{2}39 = 169$

$R_L = (10 - 1/4 q_L)q_L \to 10 - 1/2 q_L = 0 \to q_L = 20$ $\boxed{\text{Max Revenue}}$

"Free disposal"

$E(\pi) = \frac{1}{2}(\underbrace{20 - 1/4\, q_H}_{\hookrightarrow Pr(H) \hookrightarrow P_H})q_H + \frac{1}{2}(\underbrace{10 - 1/4\, q_L}_{\hookrightarrow Pr(L) \hookrightarrow P_L})q_L - \underbrace{2q_H}_{\hookrightarrow C}$

$\frac{dE(\pi)}{dq_H} = \frac{1}{2}(20 - 1/2 q_H) - 2 = 0$

$20 - 1/2 q_H = 4$

$1/2\, q_H = 16$

$q_H = 32 \to P_H = 20 - 32/4 = 12$

$E(\pi) = 1/2 \cdot 12 \cdot 32 + \frac{1}{2} \cdot 5 \cdot 20 - 2 \cdot 32$

$= 1/2(10 \cdot 32) + \frac{1}{2}(5 \cdot 20)$

$= 160 + 50$

$= 210$

Example with perfect information

High

$MR_H = 20 - 1/2 q_H = 2 \to q_H = 36 \to P_H = 20 - \frac{36}{4} = 11 \to \pi_H = 324$

Low

$MR_L = 10 - 1/2 q_L = 2 \to q_L = 16 \to P_L = 10 - \frac{1}{4} \cdot 16 = 6 \to \pi_L = 64$

$E(\pi) = \frac{1}{2}324 + \frac{1}{2}64 = 194$

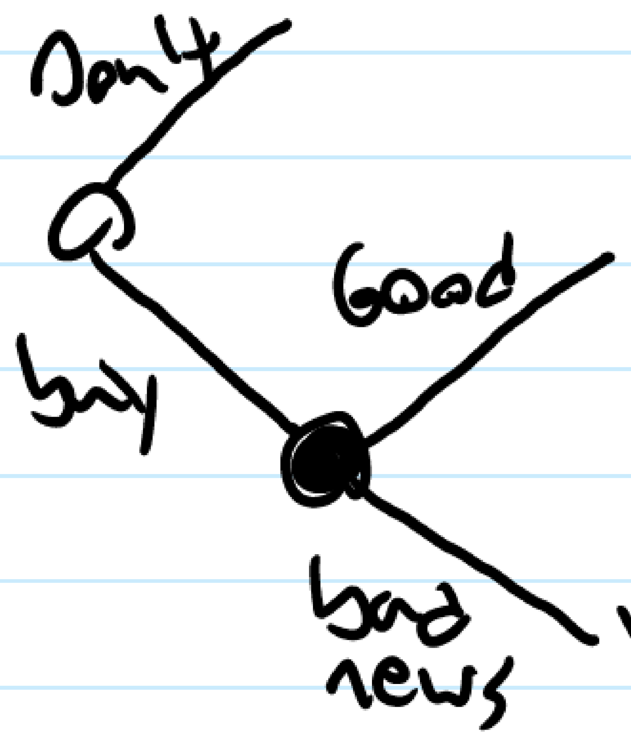
Value of perfect info $= 194 - 178 = 16$

3.13 Value of Information 2

Saturday, February 6, 2021 4:14 PM

Produce → Learn demand → announce price → sell

↑ add buy forecast/test → get results →



work → $E(\pi | GN)$

$E(\pi / \text{do buy}) = Pr(G) E(\pi | GN) + Pr(B) E(\pi | BN)$

work → $E(\pi | BN)$

Value = best π w/ info − best π w/out info

3.14-15 Value of Information 2 - Example

Saturday, February 6, 2021 4:23 PM

$Pr(H) = .7$ Buy info, $Pr(H|+) =$ and $Pr(H|-) =$
$Pr(+) =$

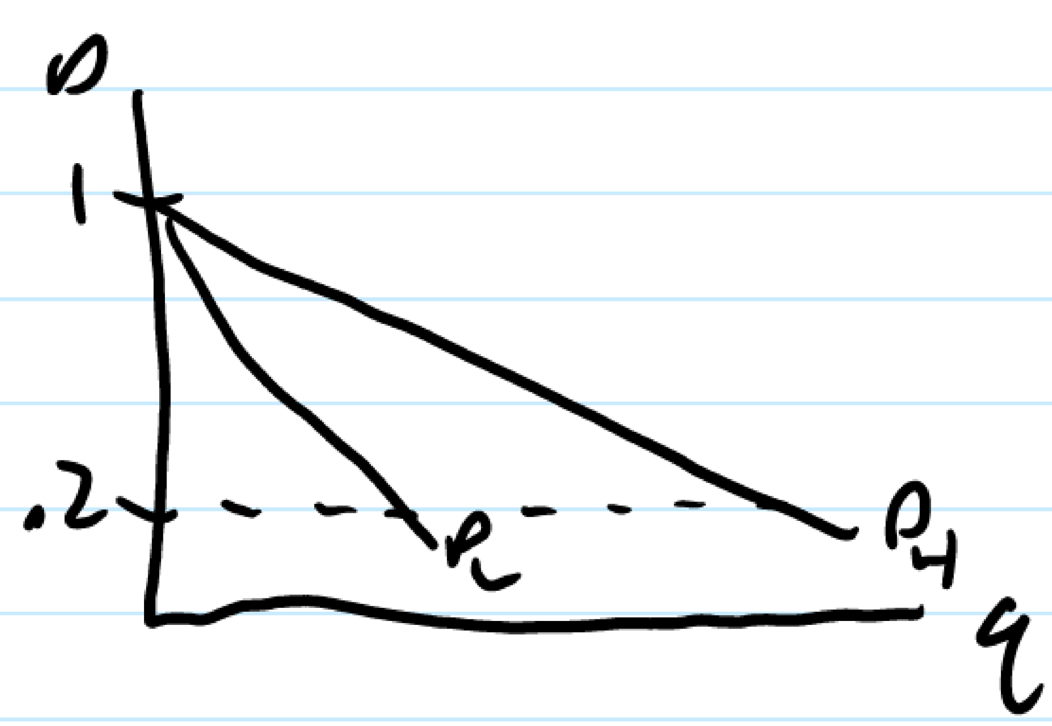
	Pos	Neg	Total
High	13	1	14
Low	1	5	6
Total	14	6	20

$P(H|+) = Pr(H \text{ and } +) / Pr(+)$
$P(H|-) = Pr(H \text{ and } -) / [1 - Pr(+)] = [P(H) - P(H \text{ and } +)] / [1 - P(+)]$

$P(H)$ $P(+)$ $P(H|+)$ $P(H|-)$ → only 3/4 are independent

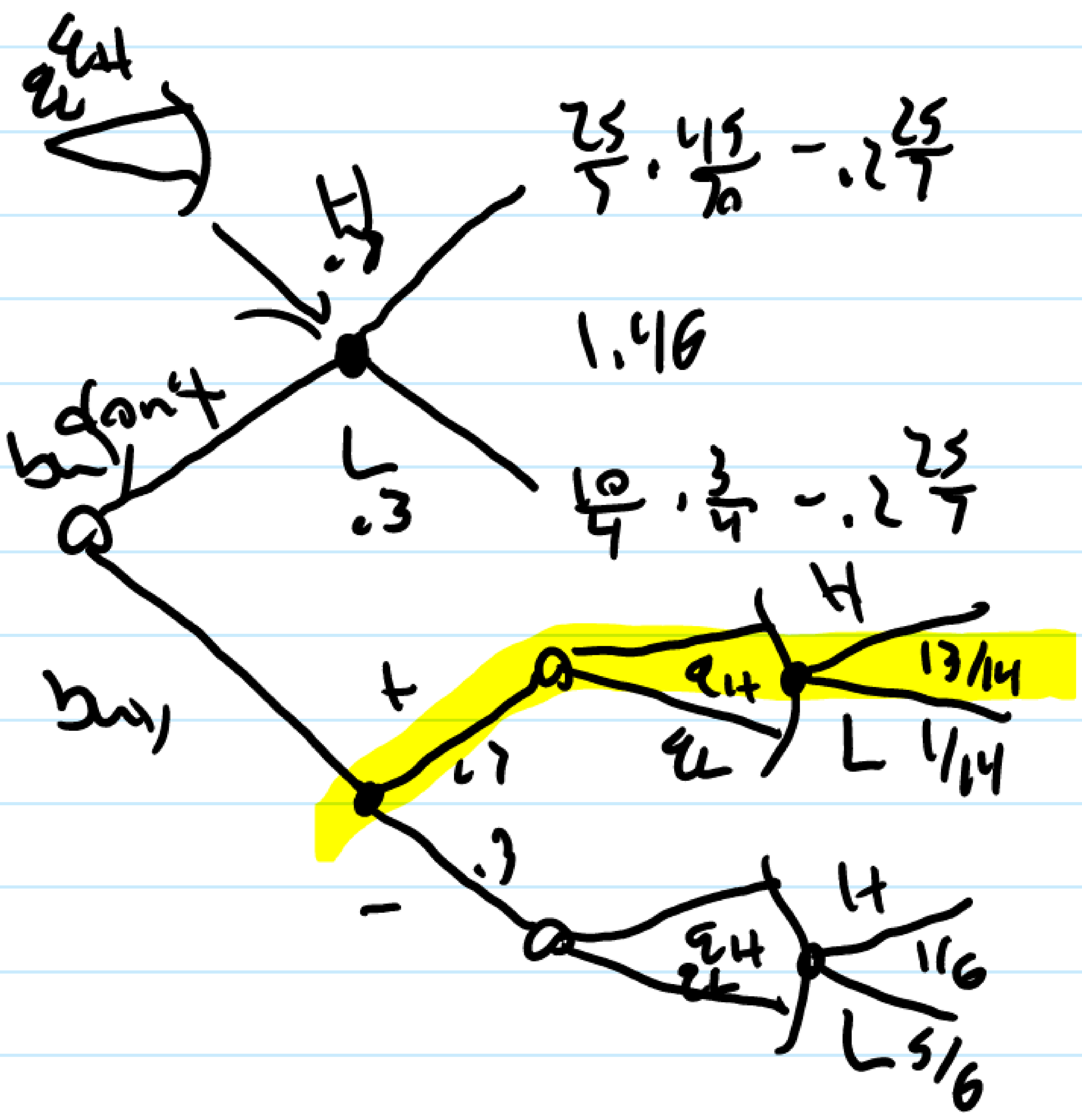
$Pr(H) = .7$ $Pr(+) = .7$
$Pr(L) = .3$ $Pr(-) = .3$

$mc = .2$



$Pr(H|+) = 13/14$ $Pr(H|-) = 1/6$

$P_H = 1 - .1q_H$ $P_L = 1 - .2q_L$



Solve before class
$E(\pi) = \frac{13}{14}(1 - q_H/10)q_H + \frac{1}{14}(1 - q_L/5)q_L - .2q_H$

$E(\pi) = 1.52$ $P_H = .61$ $q_L = 2.5$ $P_L = .5$

$.7(1 - .2q_H) = .2$
$1 - .2q_H = 2/7$
$.2q_H = 5/7$
$q_H = 25/7$

$1 - .4q_L = 0$
$q_L = 10/4$ → $P_L = 1 - \frac{10}{40} = .75$

$E(\pi) = .7\,\frac{45}{70}\,\frac{25}{7} + .3\,\frac{3}{4}\,\frac{10}{4} - \frac{1}{3}\,\frac{25}{7} = 1.46$

$\frac{1}{6}(1 - .1q)q + \frac{5}{6}(1 - .2q)q - .2q = E(\pi)$ for $q = q$

↳ $q = 2.18$ → $P_H = 1 - .1(2.18) = .782$
$P_L = 1 - .2(2.18) = .564$

↳

3 Extra Problems

Monday, February 1, 2021 6:13 PM

$P_1 = 20 - 1/4 q_1$ $\quad P_2 = 20 - 1/2 q_2$ $\quad Cost = \frac{1}{2}(q_1+q_2)^2 + 2\bar{q}$

$\bar{q}$ = capacity

$\pi = (20 - 1/2 q_1)q_1 + (20 - 1/2 q_2)q_2 - 1/2(q_1+q_2)^2 - 2q_1$

$\frac{d\pi}{dq_1} = 20 - q_1 - 2 \cdot 1/2(q_1+q_2) - 2 = 0$

$= 20 - 1/2 q_1 - q_1 - q_2 - 2 = 0$

$\frac{d\pi}{dq_2} = 20 - q_2 - (q_1 + q_1) = 0$

$20 - 2q_2 = q_1$

Inverse demand is p=100-2q with probability 0.6 and otherwise it is p=80-2q. Cost is 20 per unit. Output not sold can be repurposed at a value of 5 per unit.

Determine quantities and prices for each state of demand and expected profit.

$\pi = .6(100 - 2q_1)q_1 + .4(80 - 2q_2)q_2 - 20q_1 + (.4 \cdot 5)(q_1 - q_2)$

$\frac{d\pi}{q_1} = \frac{-12x - 210}{5} \rightarrow = \frac{-32}{5} = 17.5$

$\frac{d\pi}{q_2} = \frac{-8x - 150}{5} \rightarrow -\frac{75}{4} = 18.75$

$\mapsto \; !(q_H > q_L)$

try again

$\pi = .6(100 - 2q)q + .4(80 - 2q)q - 20q$

4 Book Notes

Friday, February 5, 2021 9:04 AM

Estimating and Interpreting Approximations

It can be difficult to maximize profit w/ mult. curves

Fitting a Linear Demand Approximation w/ 2 points

Example:

$p = \$4$ and $q = 60$ or $p = \$3$ and $q = 70$

Find $\frac{\Delta p}{\Delta q} = \frac{4-3}{60-70} = -.1 \rightarrow \Delta p = -.1\Delta q$

$(p-4) = -.1(q-60)$

$p = 4 - .1(q-60)$

$p = 10 - .1q$

$C = 5 + 2q \rightarrow \pi = (10 - .1q)q - 5 - 2q$

$\frac{d\pi}{dq} = 8 - .2q = 0 \rightarrow q = 40$

$p = 10 - .1(40) = 6 \rightarrow \pi = 6(40) - 5 - 2(40) = 155$

p may not be \$6 because we only know data from \$3 - \$4

Fitting a Log-linear demand approximation w/ 2 points

$q_D = ap^b \rightarrow 3^d$

↳ scale coefficient of demand

Example

$p = \$3$ + $q = 60$ or $p = \$4$ + $q = 70$

$60 = a4^n$ $70 = a3^n$

$\frac{60}{70} = \frac{a4^n}{a3^n} \rightarrow \frac{6}{7} = (\frac{4}{3})^n \rightarrow \ln(\frac{6}{7}) = n\ln(\frac{4}{3}) \rightarrow n = -.5358$

$60 = a4^{-.5358} \rightarrow a = 126.11$

$q = 126.11p^{-.5358}$

$p^* = (\frac{n}{1+n})mc \rightarrow p^* = (\frac{-.536}{1-.536})2 = -2.31$

$|-.5358| < 1$, raise price

Regression - Fitting the best approximation w/ many data points

^ = estimates

Lots of stuff about regression

Interactions

Interaction term is a new independent var that is the product of two or more other ind. vars.

Categorical or Dummy Variables

remove data contamination

Flexibility of Functional Form - Log and Other Transformations

Evaluating Regression Analyses

Bias and Imprecision

Evaluating the model and specification data

don't over-fit data

Evaluating the signs and magnitudes of the coefficient estimates

make sure estimates are reasonable

Evaluating the statistical significance of the results

ANOVA

$MSE = SSE / n-k-1$

↳ $n-k-1$ = degrees of freedom

mean square $MSM = \frac{SSM}{k}$

F statistic $= \frac{MSM}{MSE} = \frac{SSM/k}{SSE/(n-k-1)}$

Evaluating the Accuracy of Results - Dependent Variable Prediction

$R^2 = \frac{SSM}{SST}$

root mean square error $(RMSE) = \sqrt{MSE}$

Evaluating the Accuracy of Results - coefficient estimates

point estimates

Improving precision

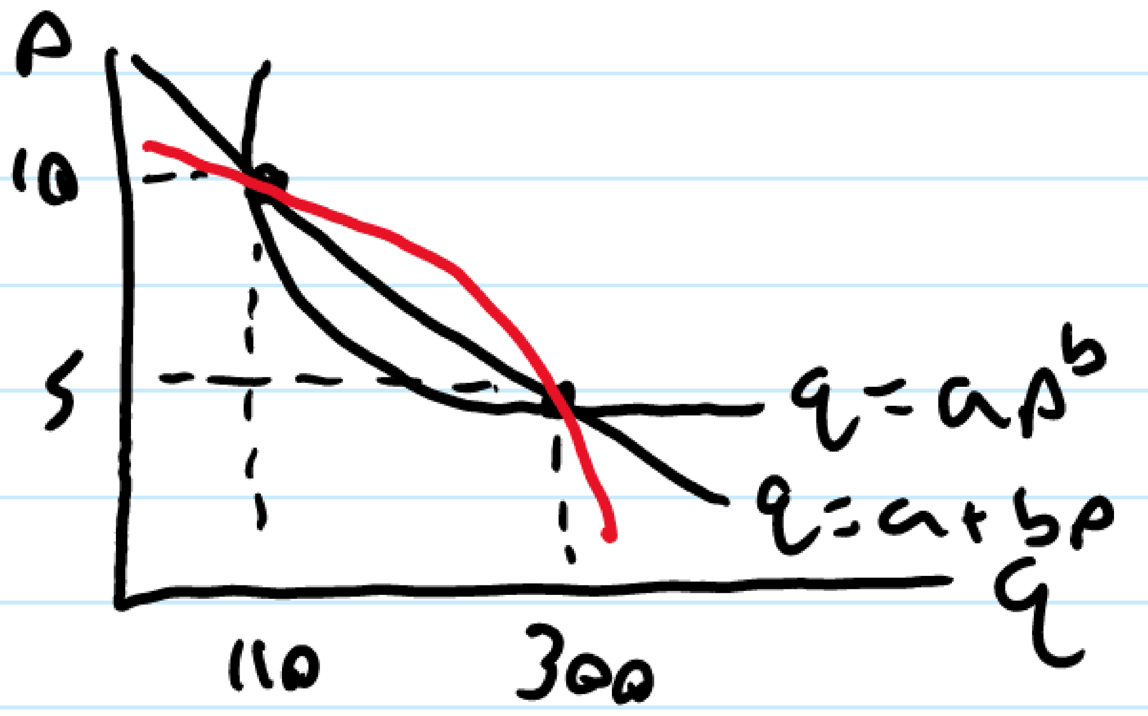
better data + better models

Evaluating Results - An Extended Example

likely that any empirical model suffers from omitted variable bias

Limits of Approximations

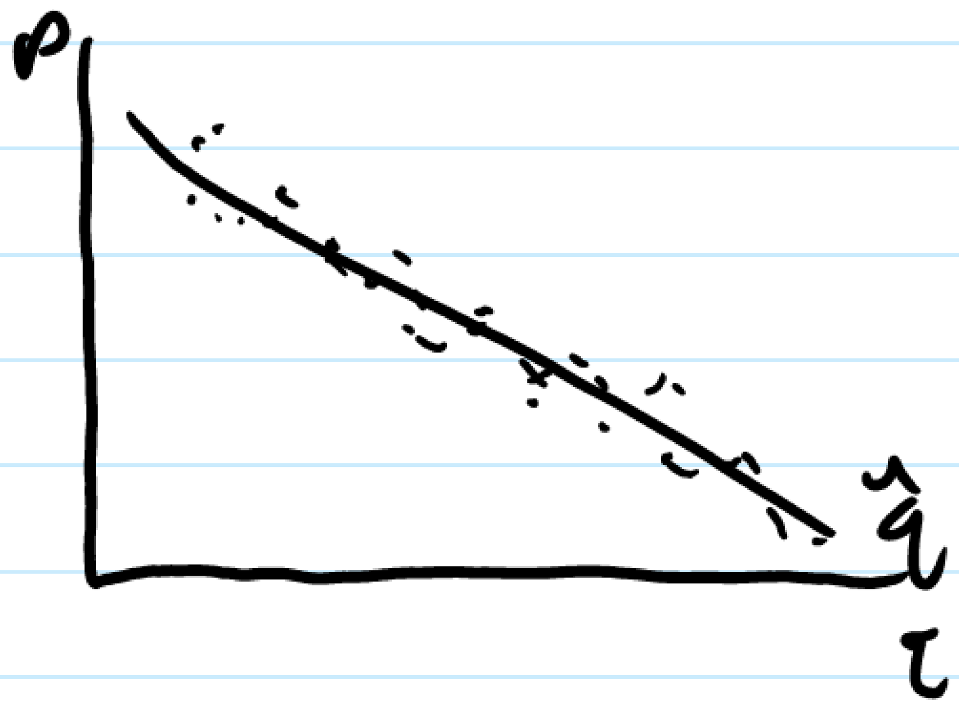
don't extrapolate beyond the data range



→ b = elasticity

est demand → est $p^* = 15$

simple w/ 1 predictor, complex w/ many



Regression

$\sum_i (y_i - \hat{y}_i)^2 = SSR$ choose $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_k$

Evaluating a Regression

1) To predict Y only

$RMSE_Y = s$

$95\% = \hat{Y} + 2s$ ← not 5

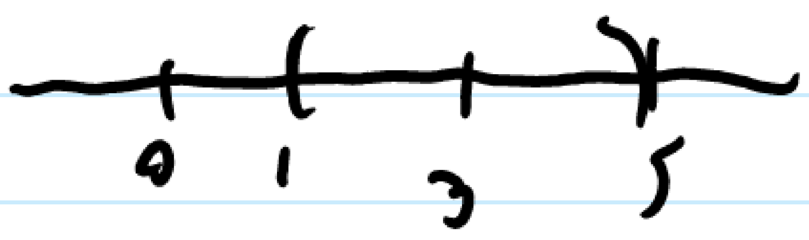
$RMSE = \sqrt{SSR/(N-k-1)}$

Evaluating coefficients

$\hat{Y} = 1 + 3x$

$B_0 \quad B_1 \qquad s_{\hat{\beta}_1} = .5$

$95\ CI = \hat{\beta}_1 \pm 2 s_{\hat{\beta}_1}$



6 Book Notes

Saturday, February 6, 2021 12:46 PM

Omitted Variable Bias

Reducing Bias

Market Trials

Natural Experiments

Instrumental Variables

find another variable that correlates w/ one and has no relationship w/ any other

Panel Data Techniques

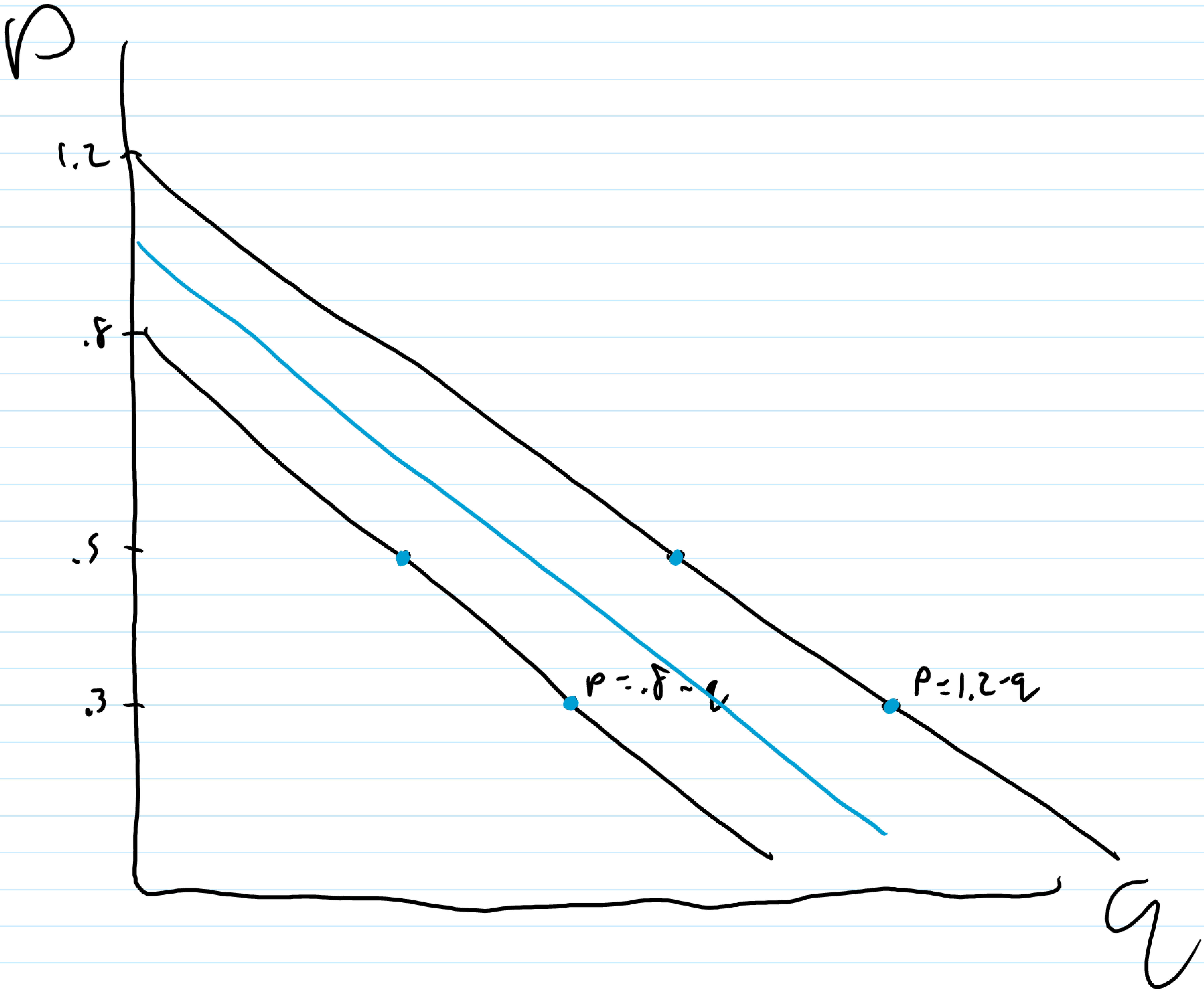
Regression Discontinuity designs

6.1 Omitted Variables

Tuesday, February 9, 2021 5:02 PM

$q = 1 + \varepsilon - p$ $\quad \varepsilon$ = random noise

$q = .8 - p$ $\quad q = 1.2 - p$



6.2-3 Omitted

Tuesday, February 9, 2021 5:31 PM

Prediction vs Causation

$E(q|p)$

Need $q^D = a - bP$ Need cause → effect

$$\left.\frac{\Delta q}{\Delta p}\right|_D = -1$$

Need random assignment of "predictors" independent Vars + exogenous

1) randomize
2) Natural Experiment
3) Instrumental Variable
   ↳ driver of price but not demand
4) Dif in Dif
   ↳ dif individual datasets then find those differences

7 Book Notes

Friday, February 12, 2021 7:31 AM

Individual Choice

Underlying Assumptions about Preferences

Completeness → all decisions available
more is better
Transitivity → A > B and B > C thus A > C
MRS_{xy} = marginal rate of substitution of good x for good y
↳ This can diminish

Indifference Curves and Preferences

different combinations w/ equal utility
Perfect complements vs perfect substitutes
Imperfect substitutes have marginal returns
Curves cannot cross

Utility functions, marginal utility, and the MRS

Marginal utility = derivative
$MRS_{xy} = MU_x / MU_y$

Budget Constraints

budget line $= m P_x X + P_y Y$ → $P_x + P_y$ = cost of $X + Y$
$X + Y$ = Qty. of $X + Y$
m = budget constraint

↳ $Y = \frac{m}{P_y} - \frac{P_x}{P_y} X$

Individual choice

Interpretation of the solution of the Individual's choice problem

$MRS_{xy} = \frac{P_x}{P_y}$ → optimality condition
↳ $\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$

Price Changes and Income Shifts

Substitution effect = induced by price change
income effect = change in purchasing power
Giffen Good = substitution ↓ income ↑ = income wins

Individual choice - The Calculus Version

$\max_{x,y} U(X,Y)$ subject to $P_x X + P_y Y \leq m$

$U(X, \frac{m}{P_y} - \frac{P_x}{P_y} X)$

Lagrange multiplier $L = U(X,Y) + \lambda[m - P_x X - P_y Y]$

use system of equations to solve ↑

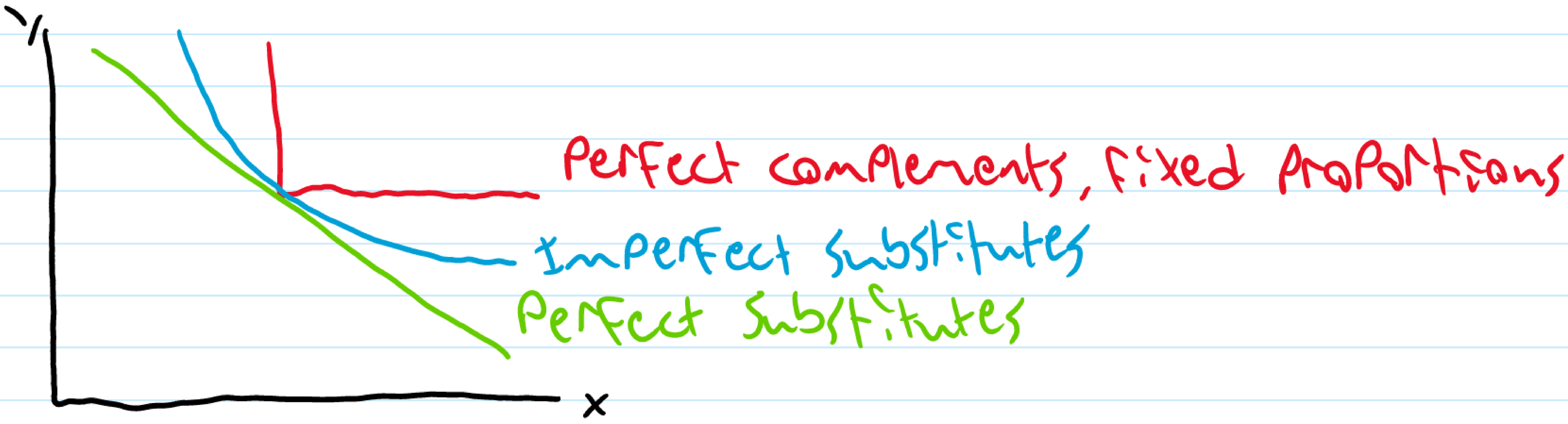
7.1 Axioms of Rational Choice

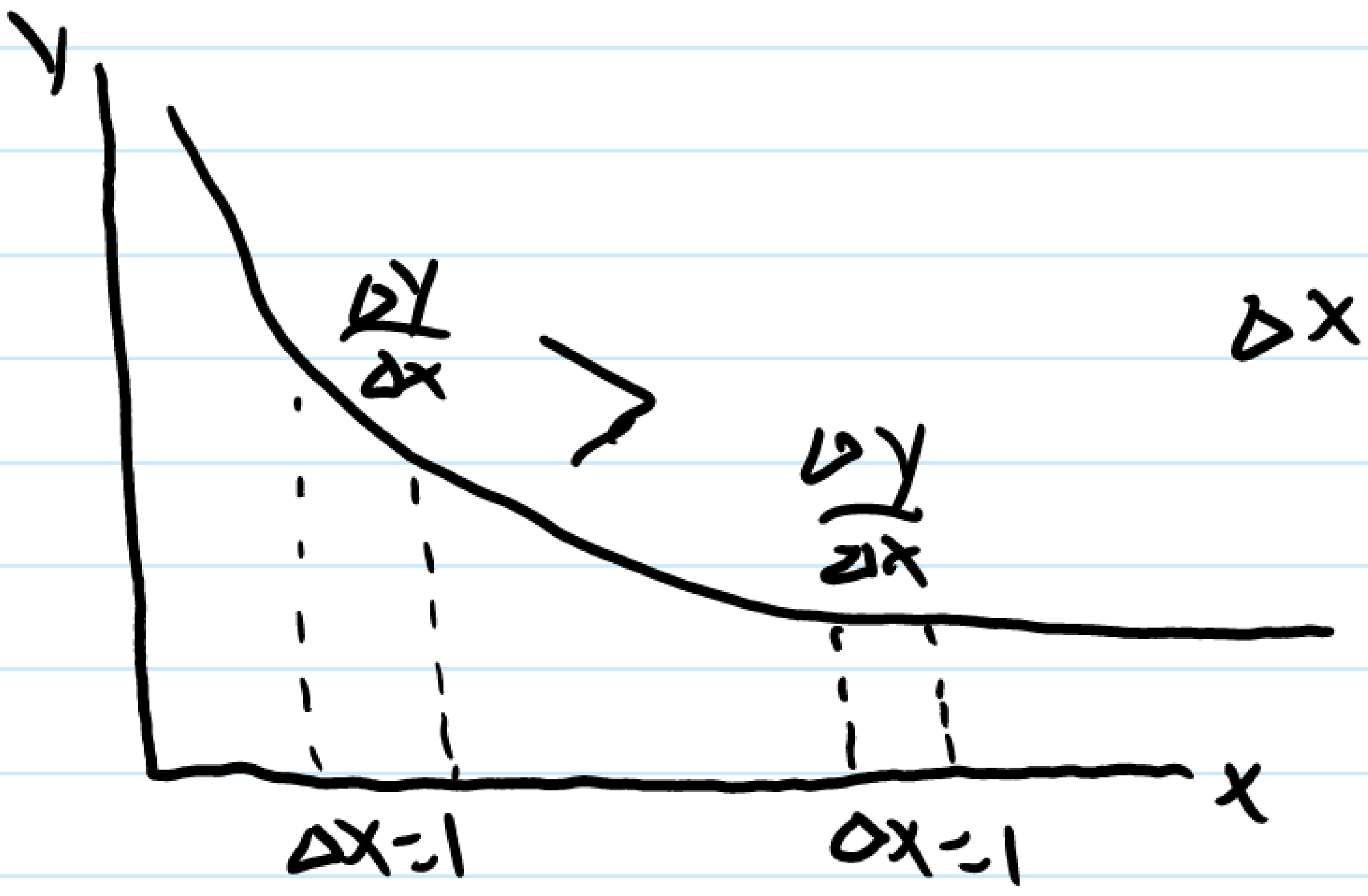
Friday, February 12, 2021 11:36 AM

1) Completeness
   "bundle" → things you can choose
   ↳ A > B or A < B or A = B
2) More is better
3) Transitivity
   ↳ if A > B and B > C, A > C
   ↳ Independence of irrelevant options
4) Diminishing marginal rate of substitution

7.2 Indifference Curves

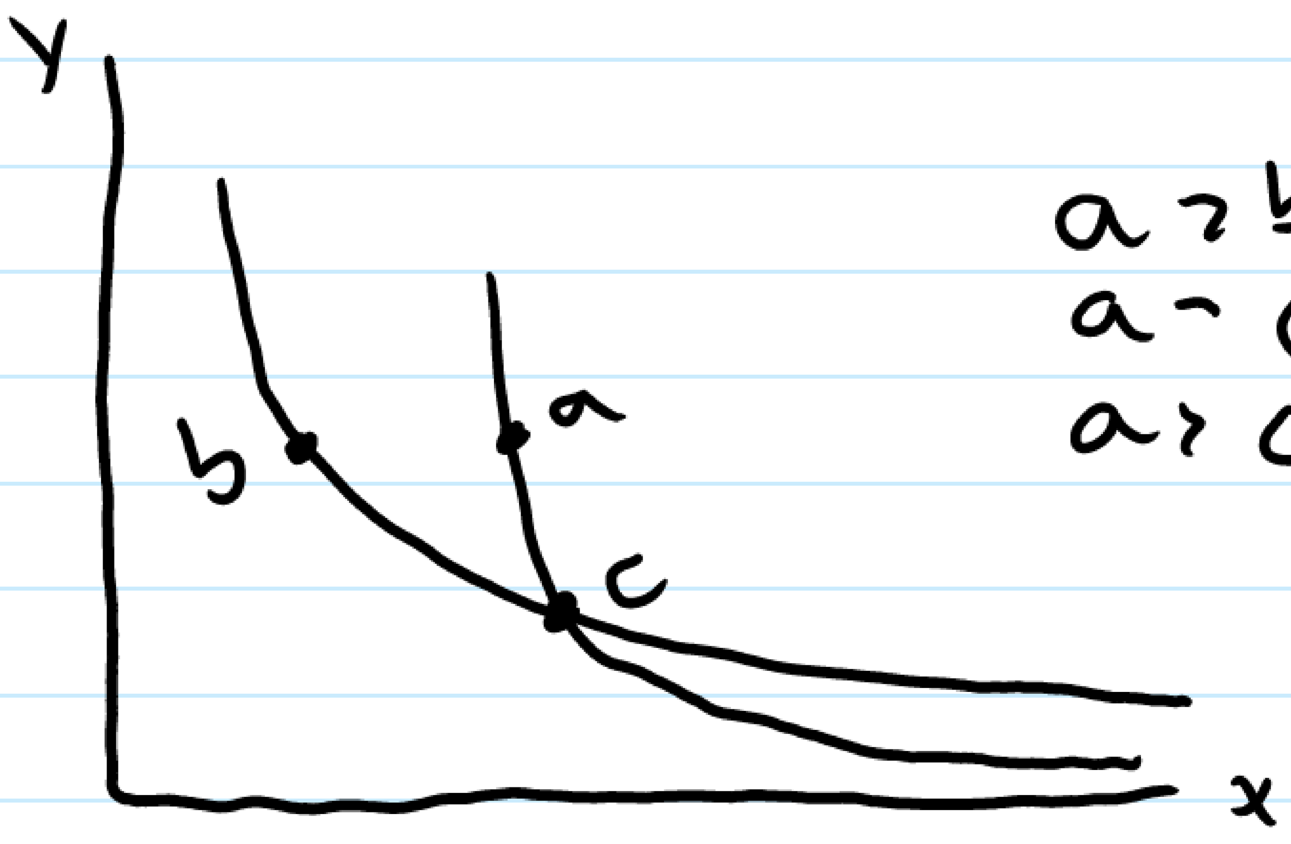
Friday, February 12, 2021 11:47 AM





$\Delta x \to 0$

Indifference curves cannot cross



$a > b$

$a \sim c$ $b \sim c$ → contradiction

$a > c$

7.3 Budget Constraint

Friday, February 12, 2021 11:57 AM

$m \quad P_x \quad P_y$

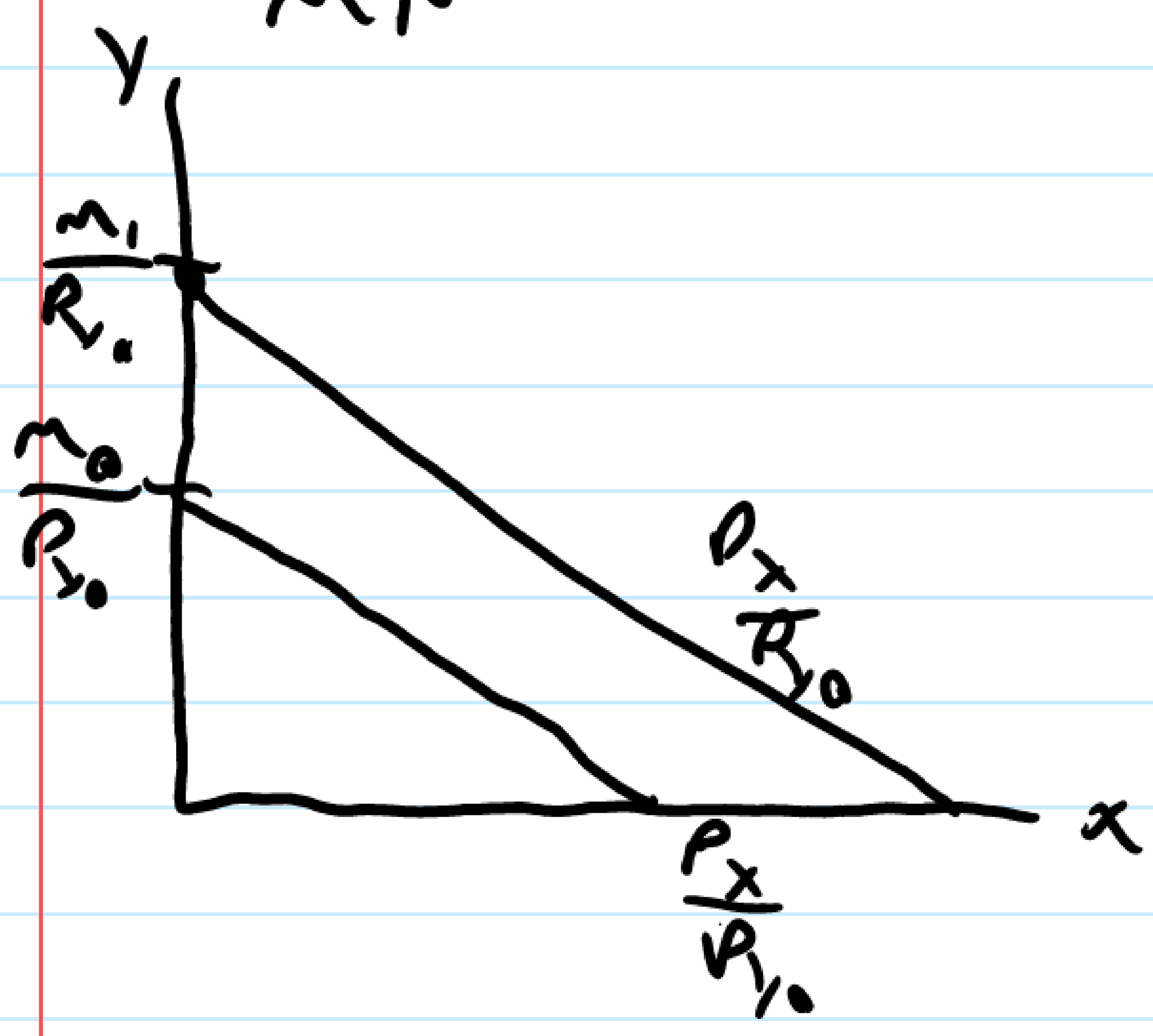
$m \geq P_x X + P_y Y$

$m = P_x X + P_y Y \Rightarrow Y = \frac{m}{P_y} - \frac{P_x}{P_y} X \rightarrow$ budget line

↳ price ratio → rate market dictates

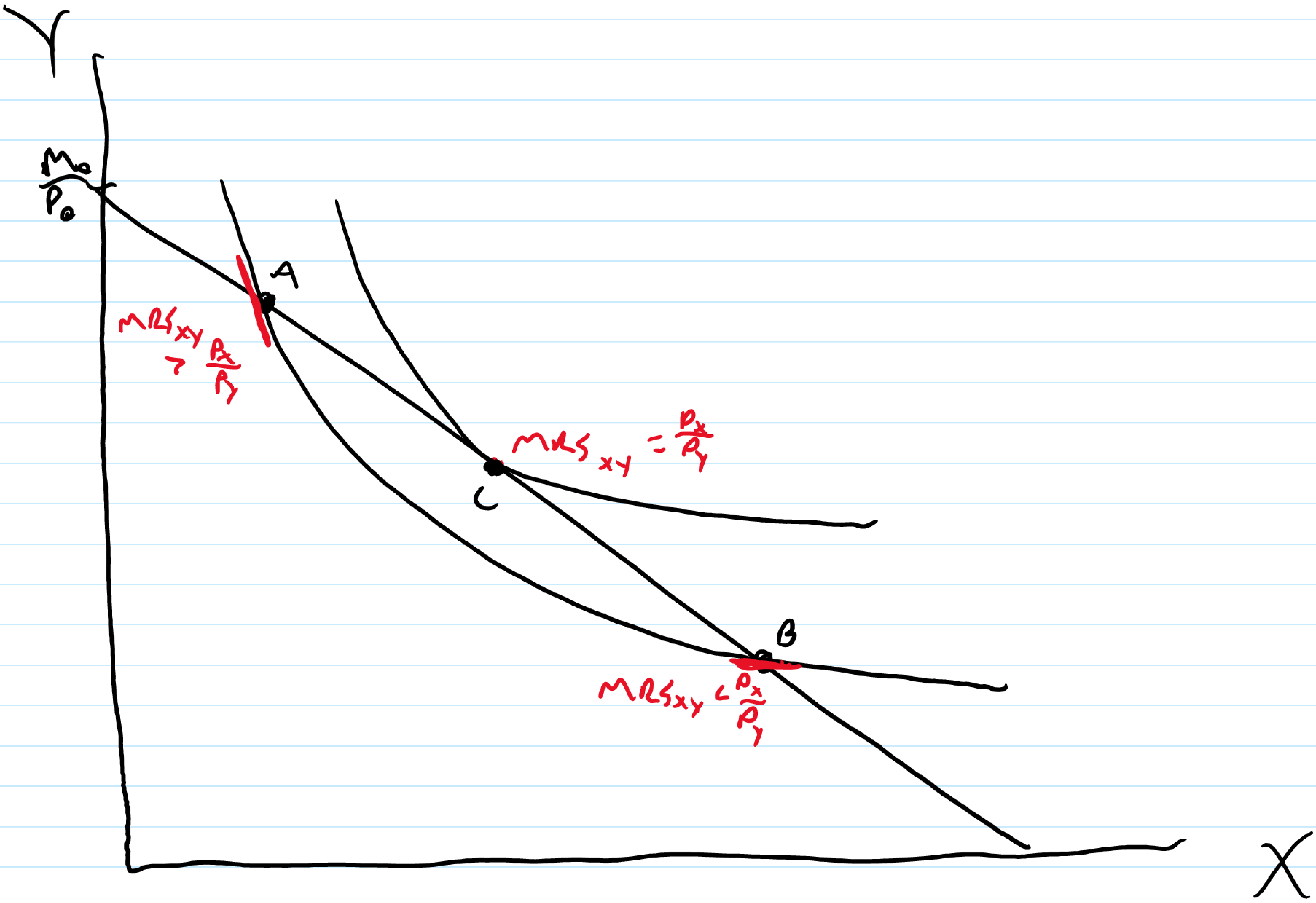
↳ y intercept

$m \uparrow$



7.4 Rational Choice and

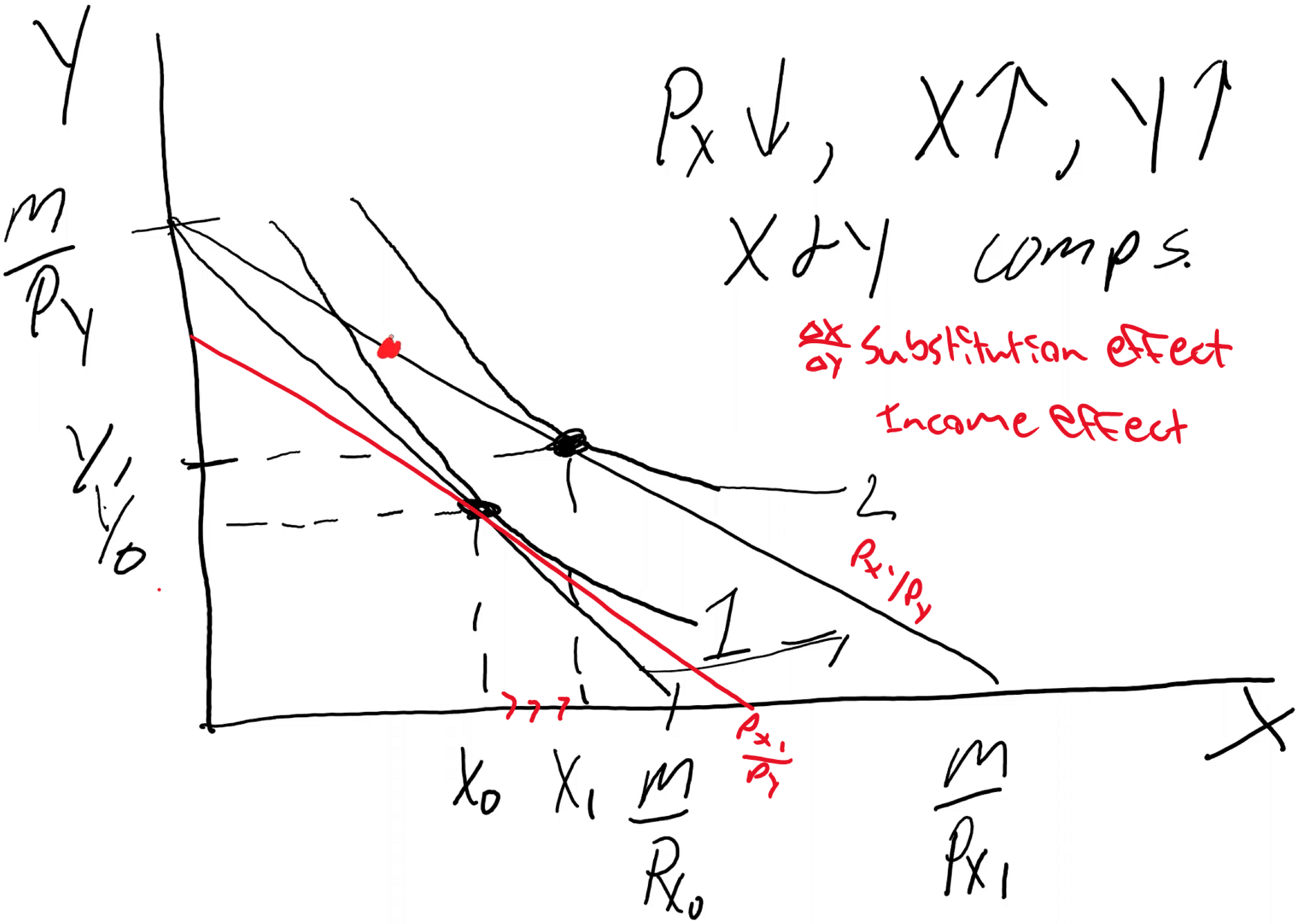
Friday, February 12, 2021 12:02 PM



Line moves as price shifts

7.5 Substitution and

Friday, February 12, 2021 2:15 PM



SE Hold Utility constant but ΔPx $\hat{x} - x_0$: SE

Higher real income $x_1 - \hat{x}$: IE

$$\Delta x = x_1 - x_0 = \underbrace{(x_1 - \hat{x})}_{IE} + \underbrace{(\hat{x} - x_0)}_{SE}$$

IF $P_x \uparrow \rightarrow x \uparrow$

What if x is a normal good

IF x inferior & |IE| > |SE| → Giffen goods

7.6 Giffen Goods

Friday, February 12, 2021 2:21 PM

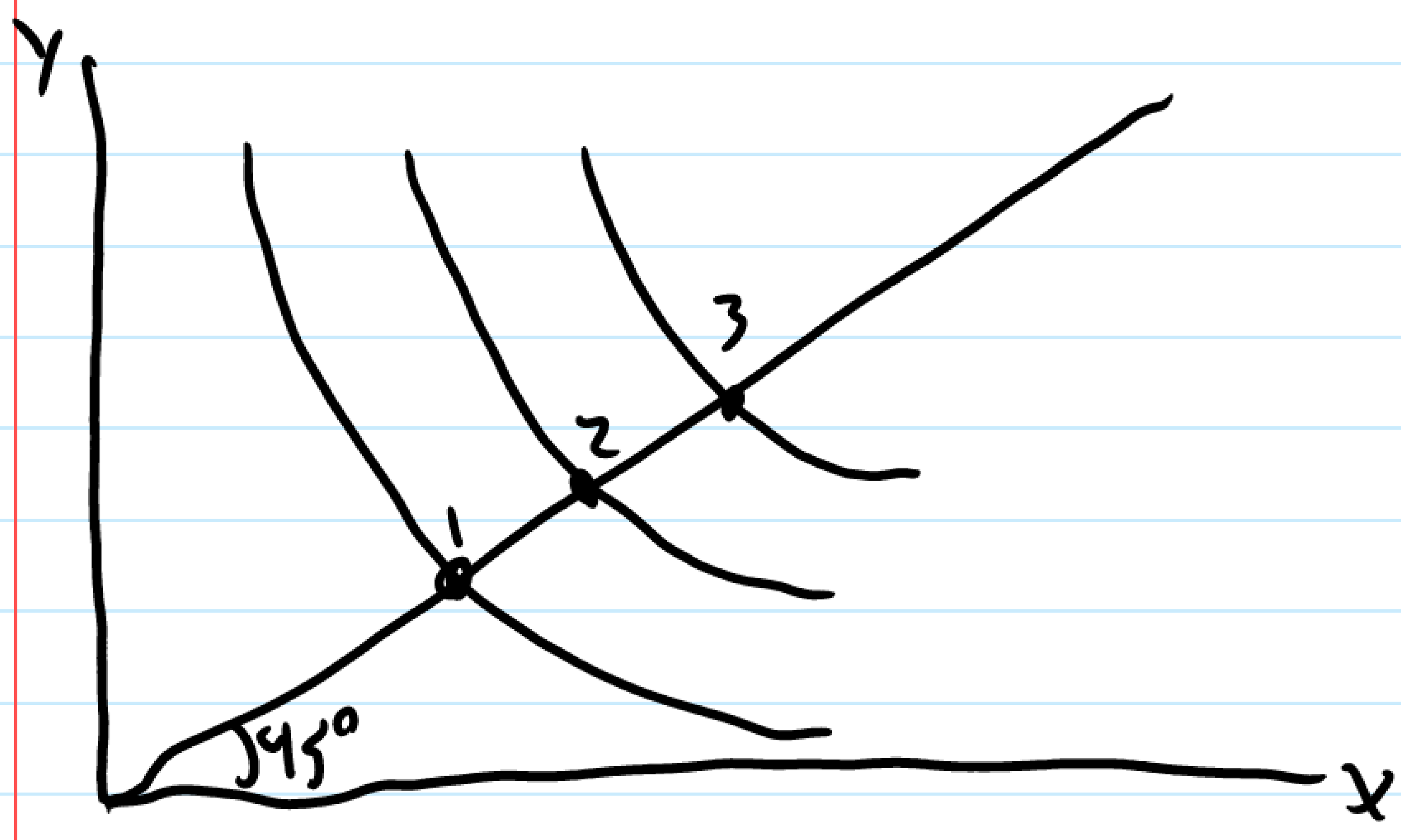
|IE| big if Huge % of income spent on it

Inf good, huge budget share? Poor

7.7 Utility Functions

Friday, February 12, 2021 2:32 PM

Preferences can be represented w/ a utility function



$U(x, y)$ incr in x, y

$|U(x,y)| + 2001$

$\ln(U(x, y))$

↳ only care about ranking

Utility is ordinal, not cardinal

$\max_{x,y} U(xy)$ subject $m = P_x X + P_y Y$

7.8 Calculus of Choice

Friday, February 12, 2021 2:37 PM

Lagrangian

$$L = U(x,y) + \lambda(m - P_x X - P_y Y)$$

$$\frac{dL}{dx} = \frac{dU}{dx} - \lambda P_x = 0$$

$$\frac{dL}{d\lambda} = m - P_x X - P_y Y = 0 \rightarrow m = P_x X + P_y Y$$

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y} \quad \text{and} \quad \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

$$MU_x = 2 \quad \text{and} \quad MU_y = 1 \rightarrow 2y = 1x$$

7 Extra Problems

Monday, February 15, 2021 5:54 PM

Income = 2000 (E)verything (F)ood $P_e = P_f = 1$

a) original budget line

b) 500 income assistance

c) $500 food gift card

d) Can anyone be better off w/ b than c? Illustrate

Anyone that spends < $500 on food

e) Can anyone be better off with c than b? If so, Illustrate

No one is better off because they are now limited.

f) Why would a Public Policy be like c than b?

If the gov't wants that to be spent on food

Applications and Extensions of Consumer Theory

Compensation Indexing and Compensating Differentials

H = housing costs

$V(H, E)$

$M = RH + E$

Individual Choice, Individual Demand, and Market Demand

market demand is the sum of individual demand

Willingness to Pay and Consumer Surplus

$V = WTP$

WTP - market cost = consumer surplus

↳ $CS(q) = V(q) - Pq$

↳ $\frac{dcs}{dq} = \frac{dv}{dq} - P = 0$

$V'(q)$ = marginal WTP

↳ $P = V'(q)$

8.1 Compensating Differentials

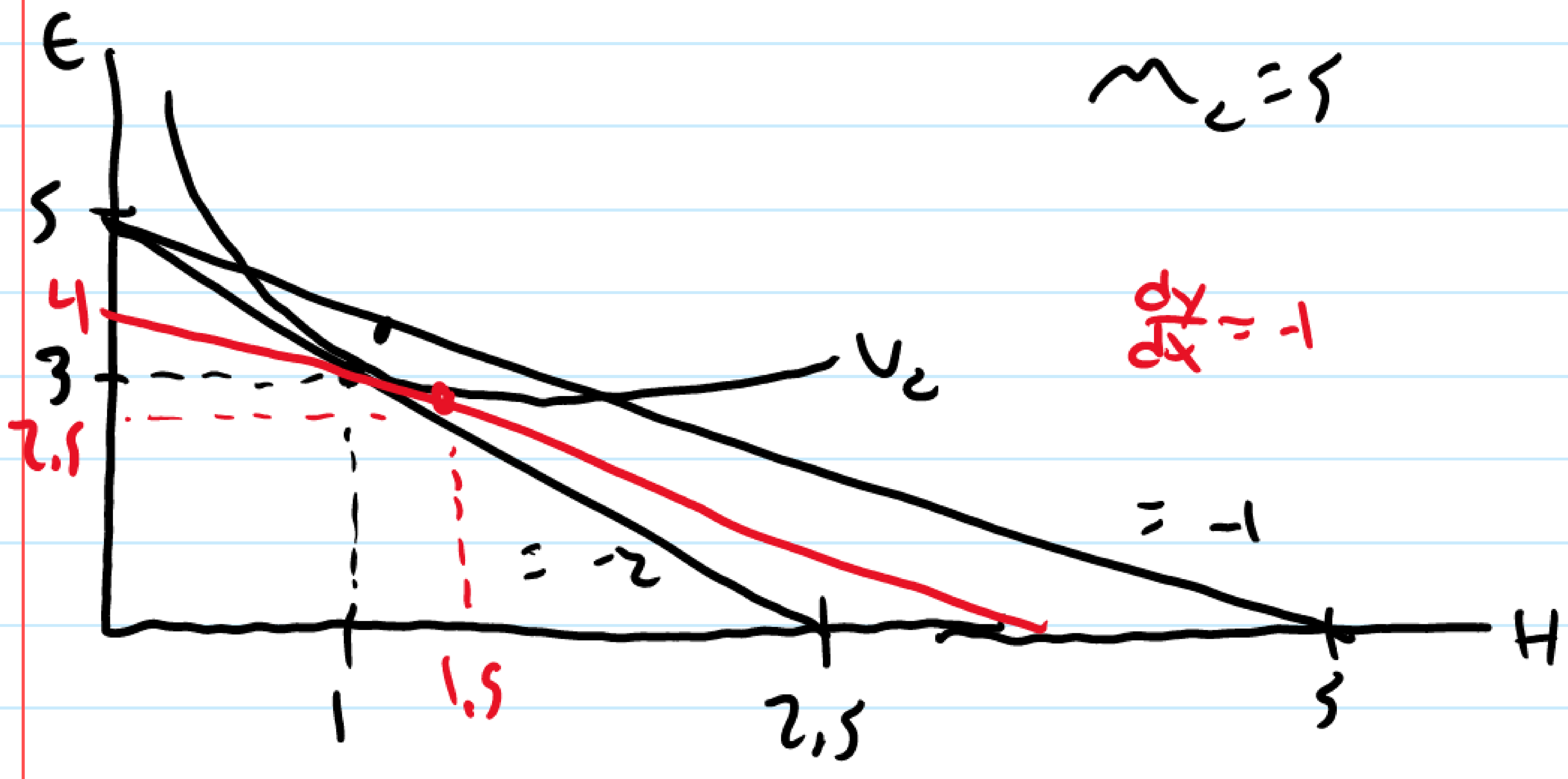
Saturday, February 20, 2021 3:59 PM

$R = \text{rent}/\text{sq ft}$

Lakeland vs Cleveland

\$1 \$2

$m = E + RH$



8.2 Individual

Saturday, February 20, 2021 4:12 PM

$q_i(p)$ = individual demand

Market demand = sum individual demand

8.3 Demand

Saturday, February 20, 2021 4:19 PM

$v(x, y)$ subject $m = P_x x + P_y y$

money metric: $v(x,y) = v(x) + $ expenditure only

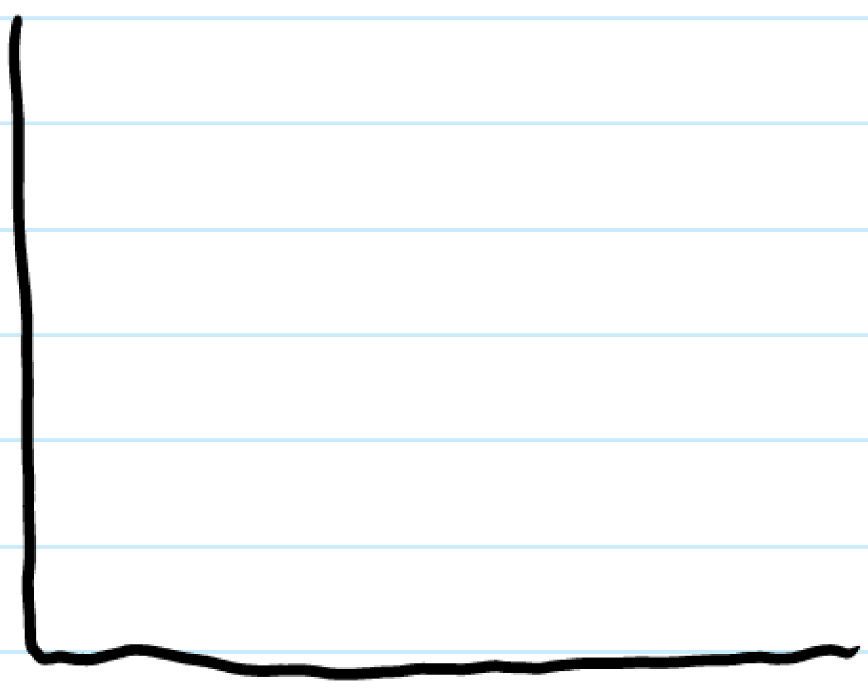
$v(x, \epsilon) = v(x) + \epsilon \quad P_\epsilon = 1$

$v = v(x) + m - P_x x \quad m = \epsilon + P_x x$

max $v(x) + m - P_x x$

$\frac{dv}{dx} = \frac{dv}{dx} - P_x = 0$

↳ inverse demand for x



$\frac{dv}{dx}$ = marginal value of x

change in m don't affect

$v(x)$ total value to consumer = WTP

$\frac{dv}{dx}$ = marginal WTP

$v = 10q - \frac{1}{2}q^2 + \epsilon \quad m = 100$

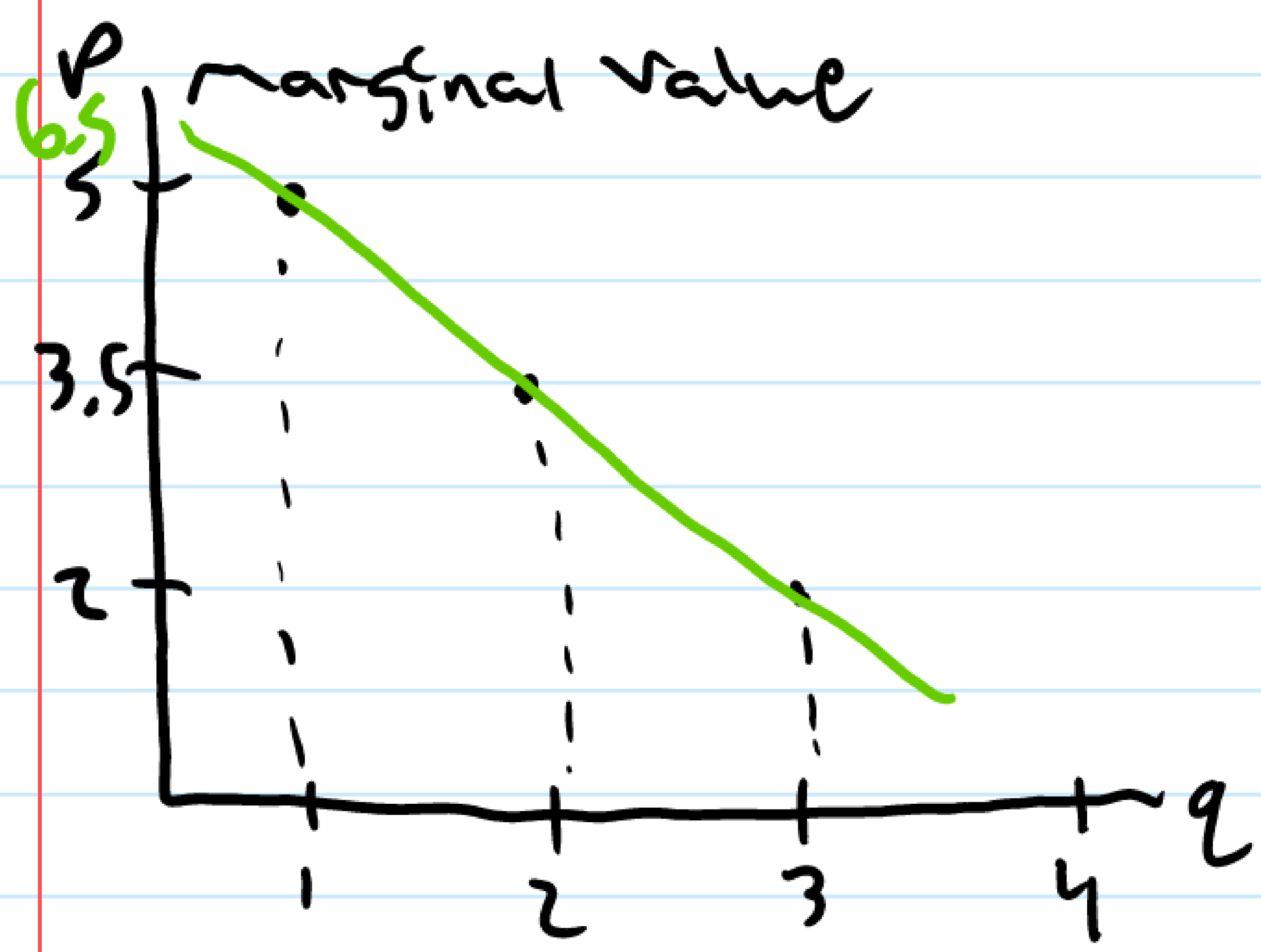
$= 100 + 10q - \frac{1}{2}q^2 - Pq$

$\frac{du}{dx} = 10 - q = P \quad 10 - q$ = inverse demand

$Q = \sum_{i=1} q_i(P)$

8.4 Consumer Surplus

Saturday, February 20, 2021 4:26 PM



$CS = \frac{1}{2}(6.5 - 1.99) =$

$CS = v(q) - Pq$

$CS_i = \int_0^{q_i} P(x)\,dx - Pq_i$

$CS_{total} = \int_0^{Q} P(x)\,dx - PQ$

Utility is $U=H^{0.5}E$ where H is housing in square feet and E is expenditure on everything else. R is monthly rent per square foot. Monthly disposable income is M.

1) Solve for housing demand as a function of M and R. Also solve for the demand for E as a function of M and R.

$E = M - RH$

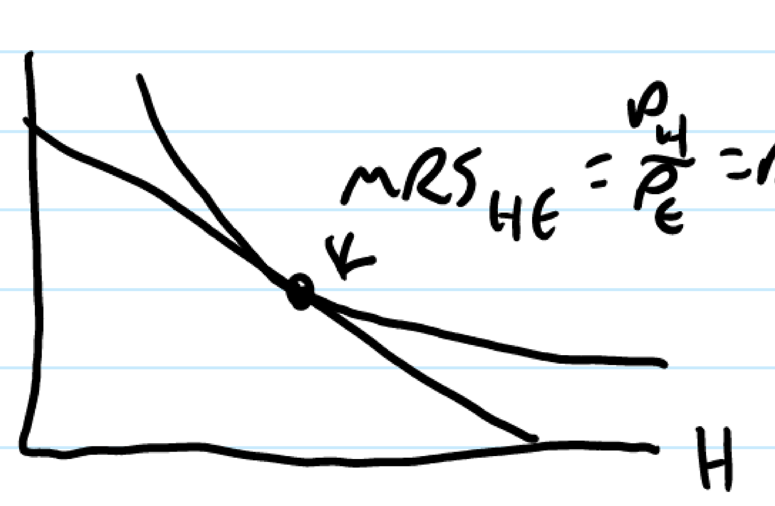
$\frac{du}{dx} = E/2\sqrt{H}$ $\qquad \frac{du}{dy} = \sqrt{x}$

$MRS_{HE} = \frac{E}{2H^{.5}H^{.5}} = \frac{E}{2H} = R$

$M = E + RH = 2RH + RH = 3RH$

$H = M/3R$

$E = 2RH = 2R\frac{M}{3R} = \frac{2}{3}M = E$



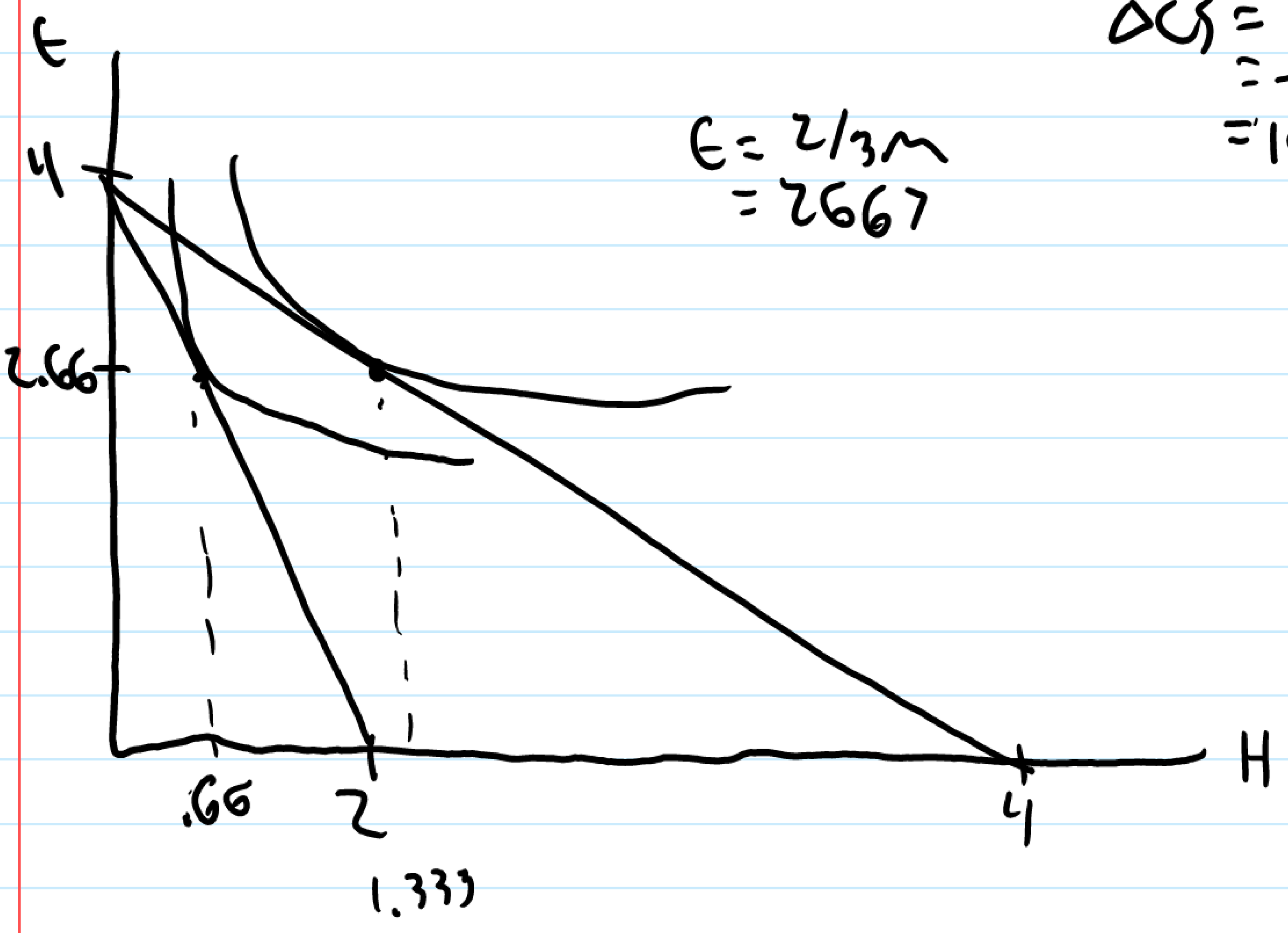
2) The indirect utility function, V, gives maximum utility as a function of income and prices. Substitute your solutions from #1 into the utility function to find the indirect utility function.

$V = \left(\frac{M}{3R}\right)^{.5} \cdot \left(\frac{2M}{3}\right)$

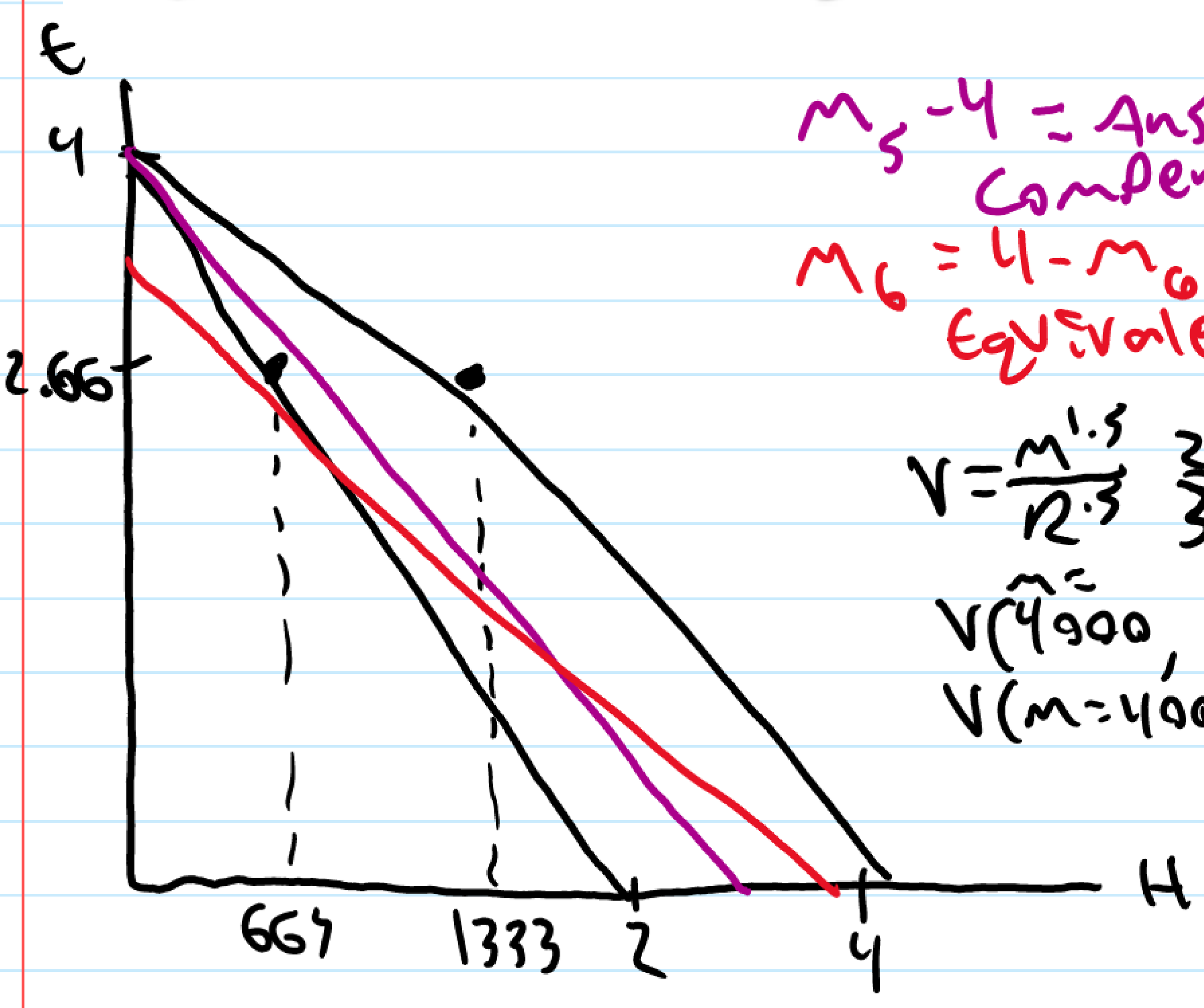
3) Suppose income is 4000 and rent increases from 1 to 2. How does housing consumption change? Be specific. Sketch both the consumer's optimization problem and the demand curve.

$\frac{4000}{3\cdot 1} - \frac{4000}{3\cdot 2} = 1333\,1/3 - 666\,2/3$

$\Delta CS = (2-1)\cdot 667 - \frac{1}{2}(2-1)\cdot(1333-667)$
$= -3/2 - 667$
$= 1000$



4) What is utility at each level of rent?



$M_5 - 4$ = Answer to 5 Compensating Variation

$M_6 = 4 - M_6$ = Answer to 6 Equivalent Variation

$V = \frac{M^{1.5}}{R^{.5}} \frac{2}{3}^{1.5}$

$V(M=4000, R=1) = 2\cdot(4000/3)^{1.5} = 97373$

$V(M=4000, R=2) = \sqrt{2}\,(4000/3)^{1.5} = 68853$

No unit because it is a measure of utility

5) How much additional income is needed to keep the individual just as happy at R=2 as they were at R=1? Formally, this is called the compensating variation, CV. Show this in the depiction of the consumer optimization problem.

$\left(\frac{M_5}{3}\right)^{1.5} \frac{2}{\sqrt{2}} = 97373 \rightarrow M_5 = 5039.69$

6) At R=1, what is the most the consumer would pay to avoid rent increasing to 2? This is called the equivalent variation, EV. Show this in the depiction of the consumer optimization problem.

$\left(\frac{M_6}{3}\right)^{1.5} \frac{2}{\sqrt{1}} = 68853 \rightarrow M_6 = 3174.8$

$CV = 5040 - 4000 = 1040$

$EV = 4000 - 3175 = 825$

$\Delta CS = 1000$

7) Looking at the demand curve, using a linear approximation between the two rent levels, what is the change in consumer surplus when price goes from 1 to 2?

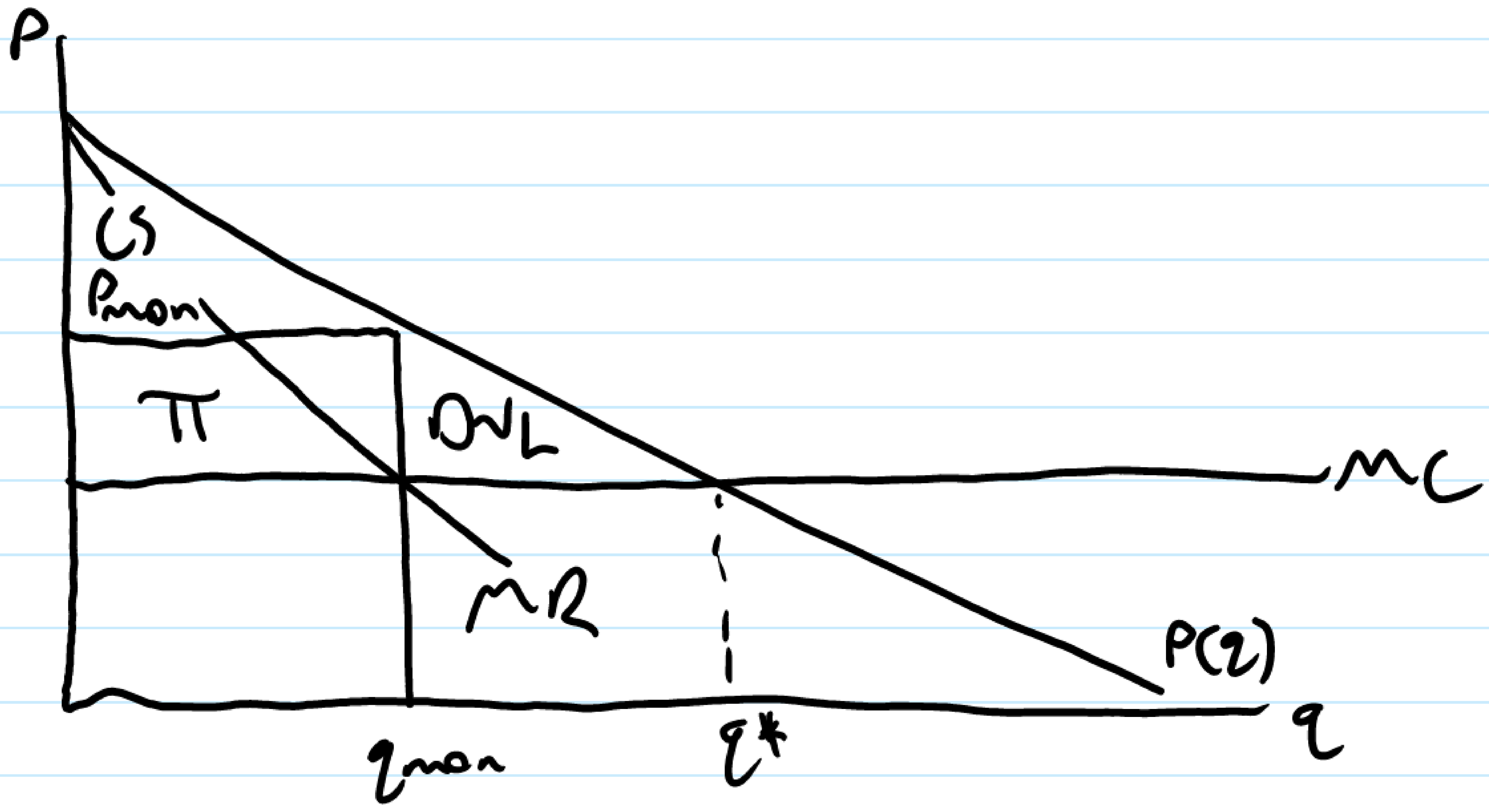
8) How does the change in CS compare to the EV and CV? Can you intuitively explain the differences?

9 Book Notes

Sunday, February 21, 2021 10:26 AM

Non Linear Pricing

Block Pricing and Two-Part Pricing



To get rid of DWL, MWTP = MC

Block Pricing: sell a bundle at one price (24-Pack of soda)

↳ $\pi = nP - C(nq^*)$

↳ $\pi = nV(q) - C(nq)$

↳ $\frac{d\pi}{dq} = n\frac{dv}{dq} - \frac{dc}{d(nq)} \cdot \frac{d(nq)}{dq}$

$\frac{dv}{dq} = MWTP$

Two-Part Pricing: Costco membership + goods

↳ $\pi = n(f + Pq) - C(nq)$

Menu Pricing: different prices for different bundles

Participation vs selection constraints

Problem 1

Saturday, January 23, 2021 12:08 PM

√

What is the present value of the payments in the table? Time is measured with the present being 0. Assume the riskless annual rate of return is 2.5%.

Time	1	2	3	4	5
Value	-50	-50	-100	-200	500

$V = -50/(1.025) + -50/1.025^2 + -100/1.025^3 + -200/1.025^4 + 500/1.025^5$

$= -48.78 - 47.59 - 92.85 - 181.19 + 441.92$

$= 71.505$

Problem 2

Saturday, January 23, 2021 12:35 PM

Use the fact that the present value of a perpetuity paying $X per period starting in one year is X/r, where r is the riskless rate of return, to determine the present value of annual payments of $X accruing for 20 years, starting one year from now. Hint: Think of it as a perpetuity less the appropriately discounted value of a perpetuity starting 20 years from now.

$$X/r - \sum_{t=1}^{20} X/(1+r)^t$$

$$V_0 = \frac{X}{r} - \left(\frac{X}{r}\right)\left(\frac{1}{(1+r)^{20}}\right) = \frac{X}{r} \cdot \left(1 - \frac{1}{(1+r)^{20}}\right)$$

Problem 3

Saturday, January 23, 2021 12:38 PM

What is the present value of the uncertain payment stream in the table? Time is measured with the present being 0. P(End) is the probability the payment stream is permanently terminated before that period's payment is made, conditional on the previous period's payment having been made. So, you have to work out the probability the venture survives long enough for each payment to be made. The riskless annual rate of return is 4%.

Time	1	3	6	10
P(End)	0.1	0.1	0.4	0.7
Value	-10	-5	60	1000

$$V = \left(\frac{-10}{1.04^1}\right)(.9) - \left(\frac{5}{1.04^3}\right)(.9\cdot.9) + \left(\frac{60}{1.04^6}\right)(.9\cdot.9\cdot.6) + \left(\frac{1000}{1.04^{10}}\right)(.9\cdot.9\cdot.6\cdot.3)$$

$$= -8.56 - 3.60 + 23.045 + 98.497$$

$$= 109.2885$$

Multiplying probabilities because each one is conditional on the ones before it being true

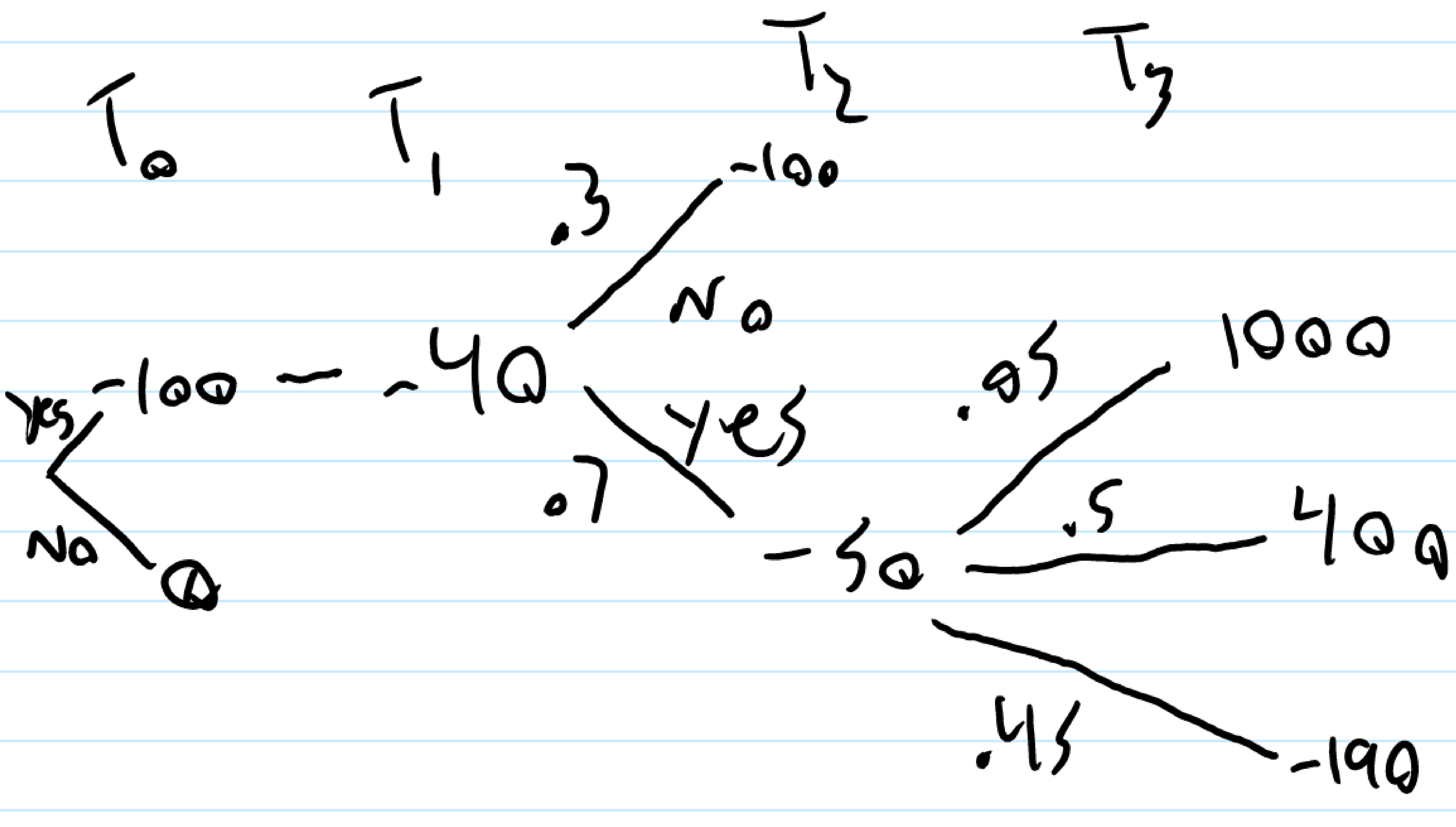
Problem 4

Saturday, January 23, 2021 12:46 PM

√

Jason is considering developing a process innovation. It requires an initial investment of $100, then another investment of $40 after one year. Jason thinks the probability it will turn out to be feasible after two years is 0.7. If it is feasible, it will then take another expenditure of $50 (2 years from the initial investment) to complete. It will then be ready to demonstrate 3 years from the initial investment. Jason thinks there is a 0.05 probability that with a successful demonstration he will sell his innovation for $1,000 and a 0.5 probability he will sell it for $400, and that otherwise there will be no interest. The annual discount rate (riskless rate of return) is 5%. There are no other costs and Jason is risk neutral.

a. Illustrate the decision(s) to be made with a decision tree.



b. What is the present expected value of the project?

$$\left(\frac{-100}{1.05^0}\right)(1) - \left(\frac{40}{1.05^1}\cdot 1\right) + \left(\frac{0}{1.05^2}\right)(.3) - \left(\frac{-50}{1.05^2}\right)(.7)$$
$$+\left[\left(\frac{1000}{1.05^3}\right)(.05) + \left(\frac{400}{1.05^3}\right)(.5) + \left(\frac{0}{1.05^3}\right)(0)\right]\left[.7\right] = -18.67$$

c. What probability of selling the project for $400 would make Jason indifferent between pursuing it or not, assuming P(1000) stays the same?

$0 = -100 - \frac{40}{1.05} + .7\left(-\frac{50}{1.05^2} + \left(P\frac{400}{1.05^3} + .05\frac{1000}{1.05^3}\right)\right)$

Using Solver... = .577875

$P = .577$

	A	B	C	D	E	F	G	H	I
1	time	0	1	2	2	3	3	3	
2	value	-100	-40	0	-50	1000	400	0	
3	prob	1	1	0.3	0.7	0.05	0.5772	0.3728	
4	1.05								sum
5		-100	-38.1	0	-31.75	43.192	199.44	0	0

d. Jason may obtain an expert's opinion of the feasibility of his idea for a fee. Suppose the consultant's studied opinion is completely accurate and Jason thinks there is a 70% chance they will find the innovation feasible. How much is the opinion worth?

$(.05\cdot 1000) + (.5\cdot 400) + (.45\cdot -190) = 164.5$

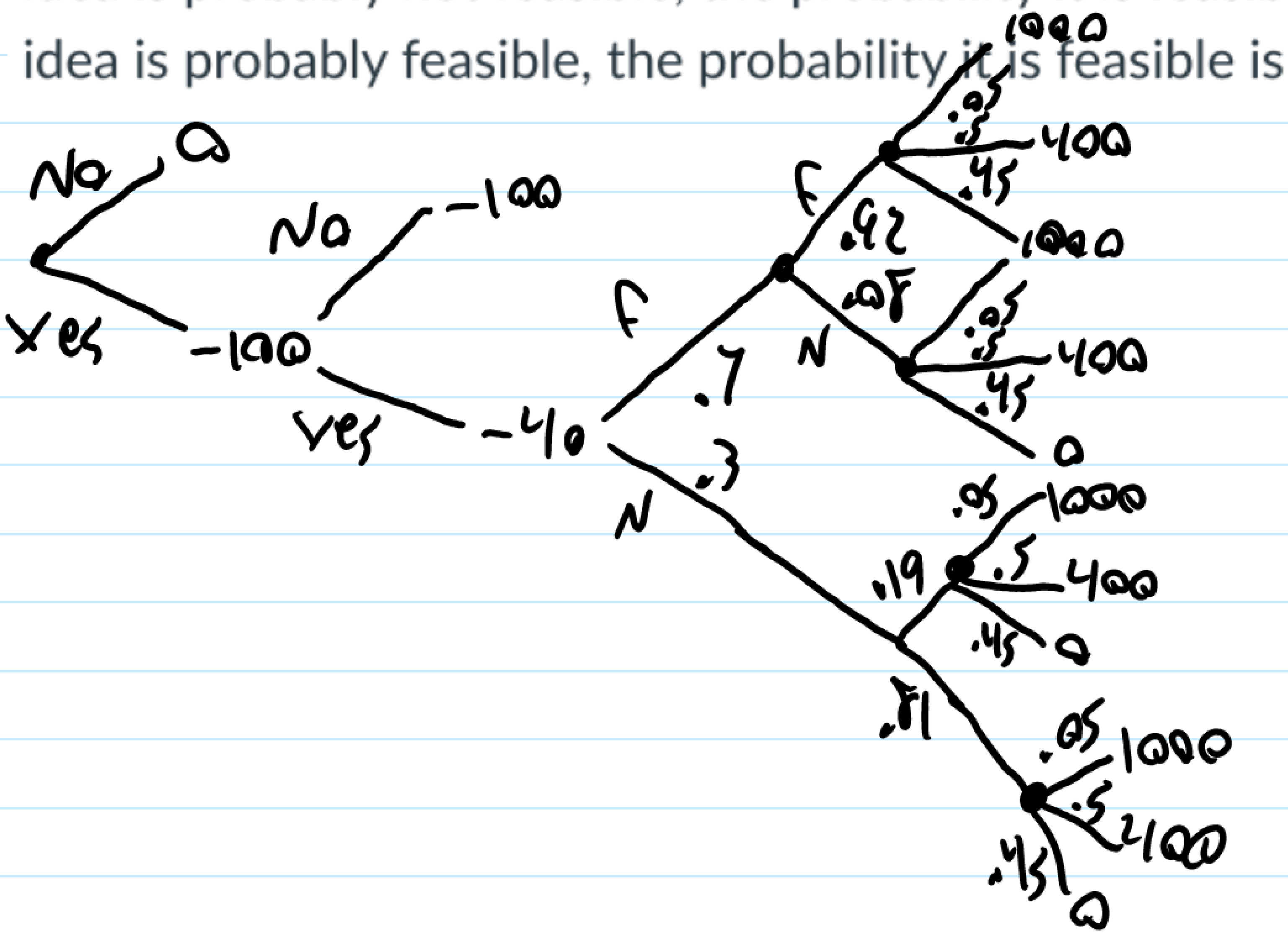
$164.5 - (-100) = 264.5$

$EPV = -100 - \frac{40}{1.05} + .7\left(-\frac{50}{1.05^2} + \left(.5\frac{400}{1.05^3} + .05\frac{1000}{1.05^3}\right)\right)$

If info, $EPV = .7\cdot 32.51 + .3\cdot 0 = 22.76$

Info − no $= 22.76 - 0 = 22.76$

e. Suppose, having dealt with consultants on similar projects, Jason guestimates there is a 0.7 probability the consultant will report the innovation is probably feasible and otherwise the consultant will report the idea is probably not feasible. Jason thinks that if the consultant says the idea is probably not feasible, the probability it is feasible is 0.19 and that if the consultant says the idea is probably feasible, the probability it is feasible is 0.92. How much is the opinion worth?



This is definitely wrong :(

$PV = -100 - \frac{40}{1.05^1} + \left(\frac{50}{1.05^2}\cdot .7\cdot .92\cdot 164.5\right)$

$EPV = -100 - \frac{40}{1.05} + .92\left(-\frac{50}{1.05^2} + \left(.5\frac{400}{1.05^3} + .05\frac{1000}{1.05^3}\right)\right) = 18.86$

Thus, with info $EPV = .7\cdot 18.86 + .3\cdot 0 = \boxed{13.2}$

Problem 5

Saturday, January 30, 2021 3:02 PM

After incurring a cost of \$300 to set up a cafeteria for a day, each meal costs \$4 to prepare and serve. If the inverse demand for meals on Sunday is $p=9-0.025q$, what price and quantity maximize profit, and what is maximum profit? Illustrate with a figure.

$\pi = (9 - .025q)q - 4q - 300$

$q = (9-P)/.025$

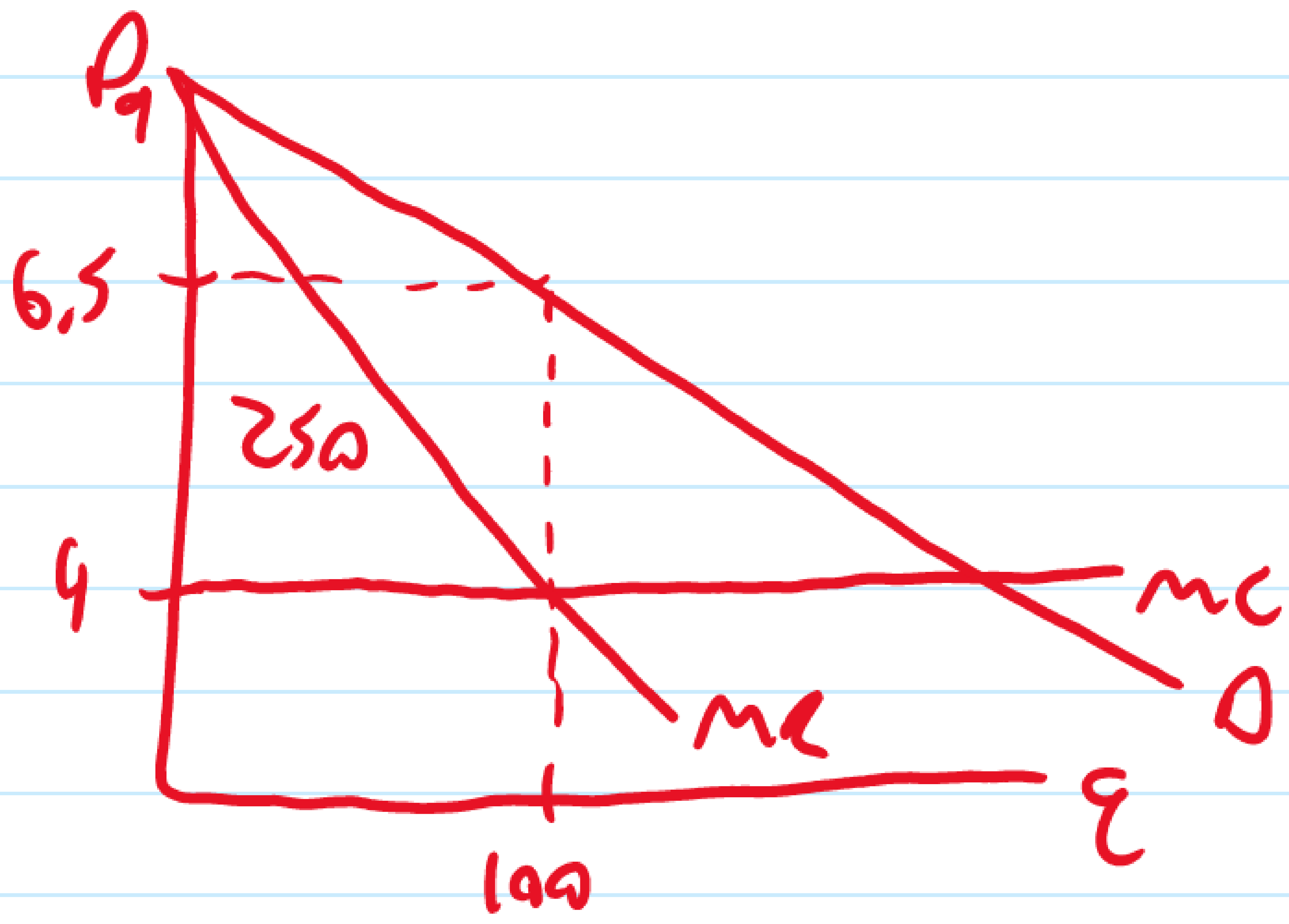
$\frac{d\pi}{dq} = (9 - .025q) - .25q - 4$

$\frac{d\pi}{dq} = 0 = 9 - .025q - .025q - 4$

$0 = 5 - .05q$

$5 = .05q$

$q = 100$



$P = 9 - .025(100)$

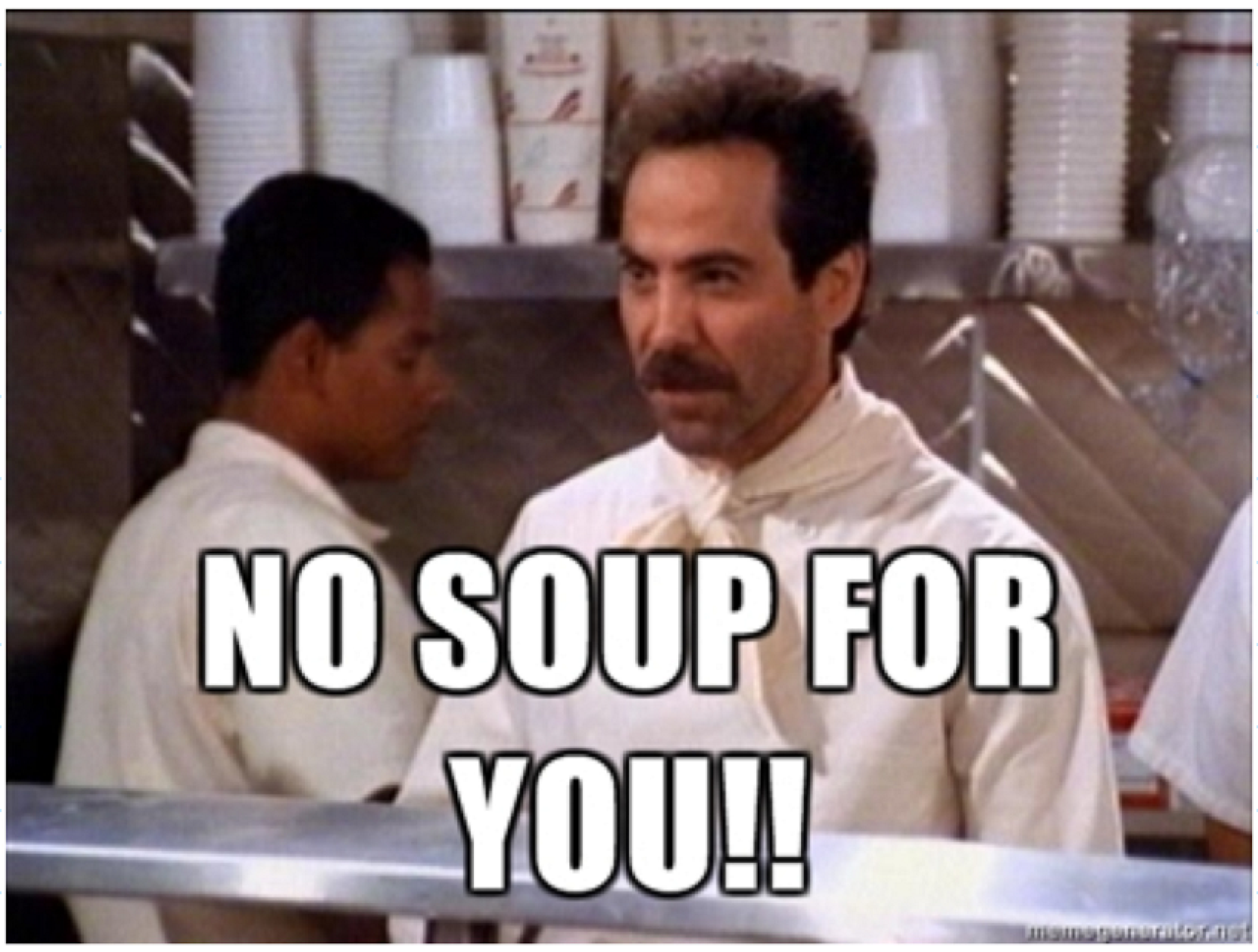
$= 9 - 2.5$

$= 6.5$

$\pi = (6.5 \cdot 100) - (4 \cdot 100) - 300$

$\pi = -50$

↳ profit is negative because it's a soup kitchen and they operate at a loss.



→ because they have no money

← Illustrate with a figure.

Problem 6

Saturday, January 30, 2021 3:31 PM

Inverse demand is, p=-5+0.5m-0.75q, where m is per capita income. If the cost per unit is constant at \$5, calculate the profit maximizing price as a function of per capita income. How much does the profit maximizing price increase per \$1 increase in per capita income?

$\pi = (-5 + .5m - .75q)q - 5q$

$= -5q + .5mq - .75q^2 - 5q$

$\hookrightarrow -10q + .5mq - .75q^2$

$\frac{d\pi}{dq} = -10 + .5m - 1.5q \quad \rightarrow q^* = m - 20/3$

$0 = -10 + .5m - 1.5q$

$10 = .5m - 1.5q$

$10 - .5m = -1.5q$

$q = -6.\overline{66} + .\overline{33}m$

$P = -5 + .5m - .75(-6.66 + 1/3m)$

$P = -5 + .5m + 5 - .25m$

$P = .25m$

~~IF it's 1.5m...~~

~~$\ldots 10 - 1.5m = -1.5q$~~

~~$q = -\frac{20}{3} + m$~~

~~$P = -5 + 1.5m - .75(-\frac{20}{3} + m)$~~

~~$P = -5 + 1.5m + 5 - .75m$~~

~~$P = .75m$~~

~~$P = .75m / .25m$~~

~~$= 3$~~

The Price increases by 300% per \$1 increase in per capita income

Problem 7

Saturday, January 30, 2021 3:50 PM

A firm sells q_B mugs of beer at price p_B, and q_P slices of pizza at price p_P. The inverse demand for mugs of beer is $p_B=5-0.25q_B+0.1q_P$ and the inverse demand for pizza slices is $p_P=4-0.5q_P+0.1q_B$. It costs $1/mug to serve beer and $2/slice to serve pizza. Find the prices and quantities that maximize profit and the maximum profit.

	A	B	C	D	E	F	G
1	item	c	p	q	unit cost	profit	
2	beer		2.9	10	1	19	
3	pizza		3	4	2	4	
4						23	<- Solver

Hailey, Troy, and myself can't seem to organically find a right answer without solver

We thought q_B and q_P had to be integers

$$\pi = (5-.25q_B+.1q_P)(q_B) + (4-.5q_P+.1q_B)(q_P) - (1\cdot q_B) - (2\cdot q_P)$$

$$= -.25q_B^2 + .2q_Bq_P + 4q_B - .5q_P^2 + 2q_P$$

$$\frac{d\pi}{dq_B} = -\frac{q_B}{2} + \frac{q_P}{5} + 4$$

$$\frac{q_B}{2} = \frac{q_P}{5} + 4 \rightarrow q_B = \frac{2q_P}{5} + 8$$

$$0 = \frac{-(2q_P/5+8)}{2} + \frac{q_P}{5} + 4 \rightarrow q_P = 0$$

$$0 = \frac{-q_B}{2} + \frac{0}{5} + 4$$

$$-4 = -q_B/2$$

$$-8 = -q_B$$

$$q_B = 8$$

$$\frac{d\pi}{dq_B} = (5-.5q_B+.1q_P) + .1q_P - 1 = 0$$

$$.5q_B = 4 + .2q_P$$

$$q_B = 8 + .4q_P$$

$$\frac{d\pi}{dq_P} = (4 - q_P + .1q_P) + .1q_P - 2 = 0$$

$$q_P = 2 + .2q_B$$

$$q_B = 8 + .4(2 + .2q_B)$$

$$q_B = 8 + .8 + .08q_B$$

$$.92q_B = 8.8$$

$$q_B = 9.57$$

$$q_P = 2 + .2(9.57) = 3.91$$

$$P_B = 5 - (.25\cdot 9.57) + (.1\cdot 3.91) = 3$$

$$P_P = 4 - (.5\cdot 3.91) + (.1\cdot 9.57) = 3$$

$$\pi = (3-1)(9.57) + (3-2)(3.91) \approx 23.04$$

Problem 8

Saturday, January 30, 2021 4:37 PM

✓

Demand is given by $q=400p^{-2}$. Cost per unit is \$10. What are the profit maximizing price and quantity and what is the maximum profit?

$q = 400p^{-2}$ $MC = 10$ $\eta^d = -2$

$p = 10(^{-2}/_{1-2}) = {}^{-20}/_{-1} = 20$

$q = 400p^{-2} = 400(20)^{-2} = 400 \cdot {}^{1}/_{400} = \underline{1}$

$\cancel{\pi = 20 \cdot 1 - 10}$ $\pi = (20-10)(1) = 10$

Problem 9

Saturday, January 30, 2021 4:43 PM

At the current price of \$8, you sell 24 units. Cost is \$4/unit. Based on publicly available estimates, you think the elasticity of demand is approximately -3. Estimate the profit maximizing price and the quantity sold and profit at that price.

Current $\pi = 24 \cdot 4 = 96$

$mc = 4 \qquad \eta^d = -3$

$P = mc(\eta^d / (1+\eta^d))$

$P = 4(-3/(1+-3))$

$P = 4(-3/-2)$

$P = 4/1.5$

$P = 6$

$-3 = \frac{\Delta q}{\Delta p} \cdot \frac{p}{q} = \frac{\Delta q}{-2} \cdot 8/24 \Rightarrow \Delta q = 18$

$q = 24 + 18 = 42$

$\pi = (6-4)(42) = 2 \cdot 42 = 84$

Problem 10

Tuesday, February 9, 2021 6:09 PM

√

Inverse demand for movie tickets is:

p_s=16-0.01q_s for senior citizens and $.01q_s + P_s = 16$ $\quad .01q_s = 16 - P_s$ $\quad q_s = \frac{16-P_s}{.01}$

p_A=24-0.01q_A for others. $q_A = \frac{24-P_A}{.01}$

The marginal cost of serving one more movie goer is $4.

Determine the profit maximizing prices for each type of customer.

$$\pi = (16 - .01q_s)q_s + (24 - .01q_A)q_A - 4(q_s + q_A)$$

$$\frac{d\pi}{q_s} = 12 - \frac{q_s}{50} \Rightarrow \frac{q_s}{50} = 12 \Rightarrow q_s = 12 \cdot 50 = 600 = q_s$$

~~I'm definitely missing the $q_H > q_L$ step~~

✓

	A	B	C	D	E
1		Quantity	Price	Cost	Total
2	Senior	600.000016	9.99999984	2400.00007	3600
3	Other	1000.00001	13.9999999	4000.00002	10000
4	A bit of solver to make sure I'm on the right track				13600

$$\frac{d\pi}{q_A} = 20 - \frac{q_A}{50} \Rightarrow \frac{q_A}{50} = 20 \qquad q_A = 1000$$

I'm not sure this is right.

$$P_s = 16 - .01(600) = 10 = P_s$$
$$P_A = 24 - .01(1000) = 14 = P_A$$

$\cancel{\pi_{max} = 1300}$

$$\pi = (10 \cdot 600) + (14 \cdot 1000) - (4 \cdot \underset{q_A + q_s}{1600}) = 13600$$

Problem 11

Tuesday, February 9, 2021 6:43 PM

✓

Inverse demand for evening movie tickets is p_E=25-0.01q_E and inverse demand for matinee tickets is p_M=15-0.01q_M.

$25 - P_E = .01q_E$

a. Assuming the marginal cost of serving one more customer is $2 holding capacity constant, and that cost of adding capacity is $2 per unit, determine profit maximizing capacity, prices and quantities.

$Cost = 2(q_E+q_m) + 2(q_E+q_m)$

k = capacity

	A	B	C	D	E	F
1		Quantity	Price	Capacity	Cost	Total
2	Evening	1050	14.5	1050	4200	11025
3	Matinee	550	9.5	550	2200	3025
4	A bit of solver to make sure I'm on the right track					14050

$\pi = (25-.01q_E)q_E + (15-.01q_m)q_m - 2(q_E+q_m) - \cancel{2(q_H - q_l)} - 2q_E$

↳ assume $q_E = q_h$

↓

fix solver

	A	B	C	D	E	F
1		Quantity	Price	Capacity	Cost	Total
2	Evening	1050	14.5	1050	2900	12325
3	Matinee	650	8.5	650	2600	2925
4	A bit of solver to make sure I'm on the right track					15250

$\frac{d\pi}{q_E} = 21 - \frac{q_E}{50} \rightarrow q_E = 1050$ $\frac{d\pi}{dq_m} = 15 - .02q_m - 2 \rightarrow q_m = 650$

~~$\frac{d\pi}{q_m} = 15 - \frac{q_m}{50} \rightarrow q_m = 750$ → hmm solver disagrees~~

Something's wrong with my profit function

	A	B	C	D	E	F
1		Quantity	Price	Capacity	Cost	Total
2	Evening	1050	14.5		2700.00002	12525
3	Matinee	750	7.5	750	1500	4124.999999
4	A bit of solver to make sure I'm on the right track					16650

That's better. B3 must equal D3 and E2 = 2·(B2 − D3) + (2·B2)

$\pi = (25-.01q_E)q_E + (15-.01q_m)q_m - (2\cdot q_m) - [2(q_E - q_m) + (2\cdot q_E)]$

$P_E = 25 - .01(1050) = 14.5 = P_E$

$P_m = 15 - .01(750) = 7.5 = P_m$

~~If Cost = $2q_m + 2(q_E - q_m) - 2q_E$ then capacity is the lower q which is $q_m = 750$~~

b. Find the value of per unit capacity cost, k, at which the constraint that matinee quantity is less than or equal to capacity is just binding. That is, at all lower values of k, matinee ticket sales will be less than capacity, and at k or higher, matinee and evening sales both equal capacity.

$Cost = 2q_m + k(q_E - q_m) - 2q_E$

Easiest to find where they equal

$q_E = q_m$ so Cost = 0?

	A	B	C	D	E	F
1		Quantity	Price	Capacity	Cost	Total
2	Evening	900	16	900	1800	12600
3	Matinee	900	6	900	1800	3600
4	A bit of solver to make sure I'm on the right track					16200
5			Capacity Cost	0		

= k

double checking w/ solver gives the same answer

To triple check, remove D2 === D3 and set k = 2

	A	B	C	D	E	F
1		Quantity	Price	Capacity	Cost	Total
2	Evening	1050	14	1050	2700	12525
3	Matinee	750	7	750	1500	4125
4	A bit of solver to make sure I'm on the right track					16650
5			Capacity Cost	2		

There's the same answer as Part A. Either both parts are very wrong or I did it right both places!

$\pi = (25-.01q_E)q_E + (15-.01q_m)q_m - 2q_E - 2q_m - kq_E$

Assume $q_m = 650$ from Part A

$\frac{d\pi}{dq_E} = 25 - .02q_E - 2 - k = 0$

$.02q_E = 23 - k$

$q_E = 1150 - 50k = 650$

$50k = 500$

$k = 10$

Problem 12

Tuesday, February 9, 2021 7:34 PM

√

Inverse demand will be p_H=12-0.005q_H with probability 0.4, and otherwise p_L=12-0.01q_L. Any product not sold must be disposed of at a cost of $1 per unit. For parts a and b, production must take place *before* demand uncertainty is resolved and cost per unit is constant at $3.

$E(\pi) = .4(12-.0005q_H)q_H + .6(12-.01q_L)q_L - 3q_H - .6 \cdot 1(q_H - q_L)$

a. Find the profit maximizing prices and quantities for each state of demand.

$\cancel{\pi = (12-.005q_H)q_H + (12-.01q_L)q_L - 3(q_H+q_L)}$

$\cancel{\frac{d\pi}{q_H} = 9 - \frac{q_H}{100} \to q_H = 900}$ $\frac{d\pi}{dq_H} = .4(12-.01q_H) - 3 - .6 = 0 \to q_H = 300$

$\cancel{\frac{d\pi}{dL} = 9 - \frac{q_L}{50} \to q_L = 450}$ $\frac{d\pi}{dq_L} = .6(12-.02q_L) + .6 = 0 \to q_L = 650$

$\cancel{P_H = 12 - .005(900) = 7.5}$
$\cancel{P_L^H = 12 - .01(450) = 7.5}$ ~~→ that's no good... or is it?~~

$\cancel{\pi_H = 900 \cdot 7.5 = 4050}$
$\cancel{\pi_L = 450 \cdot 7.5 = 2025}$ $\cancel{\to E(\pi) = (.4 \cdot 4050) + (.6 \cdot 2025) = 2855}$

$P_H = 12 - .0005 \cdot 300 = 10.5$
$P_L^H = 12 - .01 \cdot 650 = 5.5$

q_L !< q_H so $q_L = q_H$

$\cancel{\pi = (12-.005q)q + (12-.01q)q - 3q}$
$\cancel{\frac{d\pi}{q} = 21 - \frac{3q}{100} \to \frac{3q}{100} = 21 \to 3q = 2100 \to q}$

~~wrong it's~~ $\cancel{-3(2q)}$
$E(\pi) = .4(12-.0005q)q + .6(12-.02q)q - 3q$
$\frac{d\pi}{dq} = .4(12-.01q) + .6(12-.02q) - 3 = 0$
$\to q = 562.5$

$\cancel{\pi = (12-.005q)q + (12-.01q)q - 6q}$

$\cancel{\frac{d\pi}{q} = 18 - \frac{3q}{100} \to q = 600}$

$P_H = 12 - .0005 \cdot 562.5 = 9.19$
$P_L = 12 - .01 \cdot 562.5 = 6.38$

$\cancel{P_H = 12 - .005(600) = 9 = P_H}$
$\cancel{P_L^H = 12 - .01(600) = 6 = P_L}$

~~→ ugh. forgot to account for probability~~

$\to E(\pi) = (.4 \cdot 9.19 + .6 \cdot 6.38 - 3)562.5$
$= 2531.25$

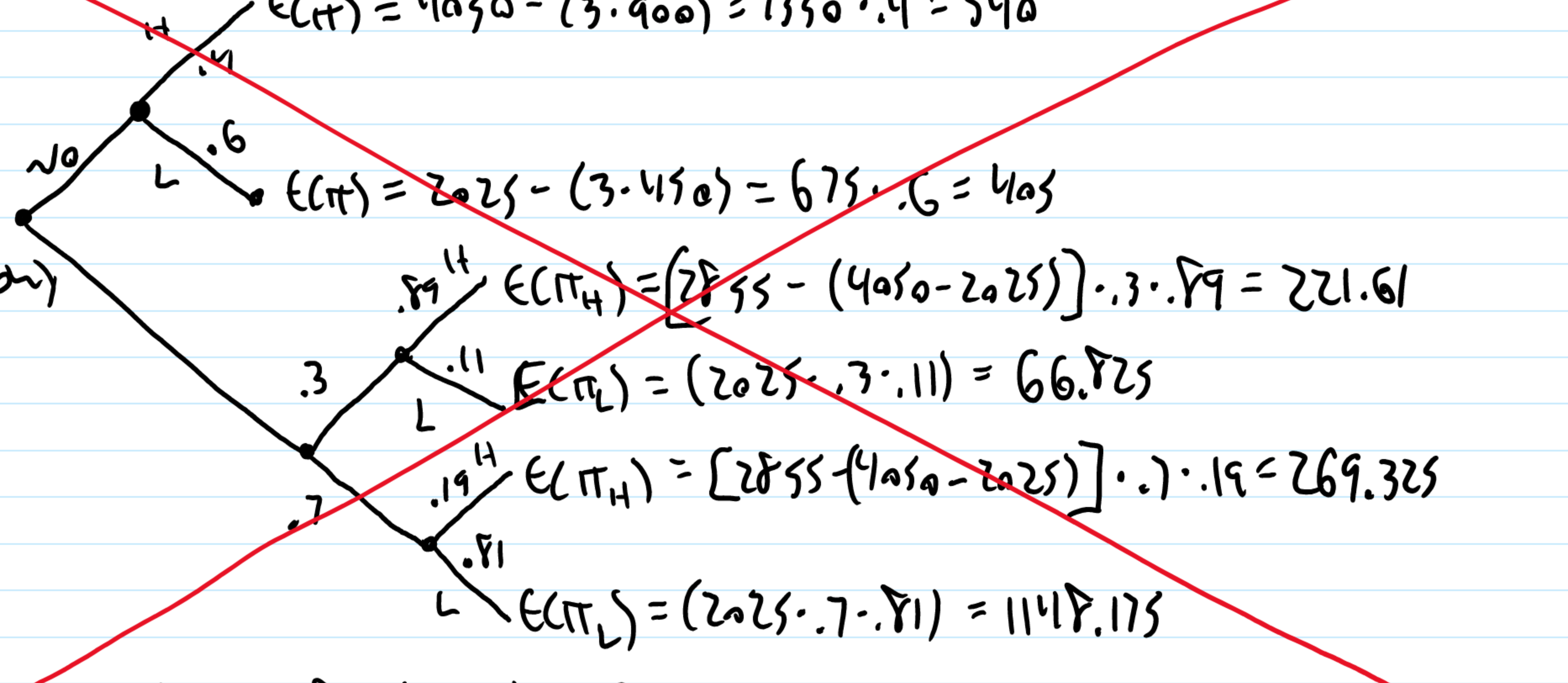
$\cancel{\pi_H = 600 \cdot 9 = 5400}$
$\cancel{\pi_L = 600 \cdot 6 = 3600}$

$\cancel{E(\pi) = (.4 \cdot 5400) + (.6 \cdot 3600) = 4320}$

b. Suppose you can set up an analytics program to obtain additional information on the probability of high demand. Your best guess is that with probability 0.3 they will tell you the probability of high demand is 0.89, and that otherwise they will tell you the probability of high demand is 0.19. What is the analytics program worth per period?

$\cancel{\pi_L = (12-.01q)q - 3q}$

$\cancel{\pi_H = (12-.005q_H)q_H + (12-.01q_L)q_L - 3(q_H+q_L) - (q_H - q_L)}$



$\cancel{E(\text{info}) = 1148.175 - 540 = 608.175}$

$E(\pi) = .89(12-.0005q_H)q_H + .11(12-.01q_L)q_L - 3q_H - .11 \cdot 1(q_H - q_L)$

$\frac{d\pi}{dq_H} = .89(12-.01q_H) - 3 - .11 = 0$ $\frac{d\pi}{dq_L} = .11(12-.02q_L) + .11 = 0$

$\to q_H = 850.6$ $\to q_L = 650$

$P_H = 12 - .0005 \cdot 850.6 = 7.75$ $P_L = 12 - .01 \cdot 650 = 5.50$

$q_H > q_L$ is good

$E(\pi) = .89 \cdot 7.75 \cdot 850.6 + .11 \cdot 5.5 \cdot 650 - 3 \cdot 850.6 - .11 \cdot 1(850.6 - 650) = 3686.4$

If $P(H) = .19$
$E(\pi) = .19(12-.0005q)q + .81(12-.01q)q - 3q$

$\frac{d\pi}{dq} = .19(12-.01q) + .81(12-.02q) - 3 = 0 \to q = 497.24$

$P_H = 12 - .0005 \cdot 497.24 = 9.51$
$P_L^H = 12 - .01 \cdot 497.24 = 7.03$

$E(\pi) = (.19 \cdot 9.51 + .81 \cdot 7.03 - 3)497.24 = 2237.57$

$E(\pi | \text{info}) = .3 \cdot 3686.4 + .7 \cdot 2237.57 = 2672.22$
$E(\pi | \text{no}) = 2531.25$

$\to$ Info $= 140.97$

c. Assume you do not have recourse to additional information as in (b). Instead, suppose that in addition to your current production line (that costs $3 per unit) you could add a just in time production line with a cost of $5 per unit. Find the maximum expected profit if you add this line, and therefore its value per production period. *Hint*: Since your base line cost is only $3 per unit, you would always use it to produce any units you are certain to sell (low demand sales). The question is whether or not it saves money to use the just in time line for additional production when demand is high. The answer determines how you add the unit cost of units q_H through q_L, and the potential disposal cost, to the problem setup.

$\cancel{\pi = (12-.005q_H)q_H + (12-.01q_L)q_L - 3(q_H+q_L) - 5(q_H - q_L)}$
$\cancel{\text{or } 50/(.04/12) = 15000}$

$\cancel{\frac{d\pi}{dq_H} = 4 - \frac{q_H}{100} \to q_H = 400}$ $E(\pi) = .4(12-.0005q_H)q_H + .6(12-.01q_L)q_L - 3q_L - .4 \cdot 5(q_H - q_L)$

$\cancel{\frac{d\pi}{dq_L} = 14 - \frac{q_L}{50} \to q_L = 700}$ $\frac{d\pi}{dq_H} = .4(12-.01q_H) - .4 \cdot 5 = 0 \to q_H = 700$

$\cancel{P_H = 12 - .005(400) = 10}$ $\frac{d\pi}{dq_L} = .6(12-.02q_L) - 3 + .4 \cdot 5 = 0 \to q_L = 516.67$

$\cancel{\pi = 400 \cdot 10 = 4000}$ $P_H = 12 - .0005 \cdot 700 = 8.50$
$P^H = 12 - .01 \cdot 516.67 = 6.83$

$E(\pi) = .4 \cdot 8.5 \cdot 700 + .6 \cdot 6.83 \cdot 516.67 - .4 \cdot 5(700 - 516.67) = 2581.67$

Value JIT = 2581.67 - 2531.25 = 50.42

~~If demand is high, the JIT line has a profit of 4000 while the normal line has a profit of 4050. Thus, the JIT line is not a good idea.~~

d. Continuing from (c), assume the safe rate of interest is 4% annually (so 0.04/12 monthly), and that you make one production run per month. Using the fact that the present value of a perpetual payment of $V starting one period from the current period is $V/r, calculate an upper bound of the expected present value of adding a just in time production line.

$\cancel{\frac{.04}{12} = .003}$

$\cancel{4050 - 4000 = 50}$

$\cancel{\sum_{i=1}^{\infty} \frac{50}{(1+.003)^i} = 15000}$ $\sum_{i=1}^{\infty} \frac{50.42}{(1+.04/12)^i} = 15125$

Problem 13

Monday, February 22, 2021 8:34 PM

✓

Chris' preferences are represented by , where *S* is the number of pizza slices he eats and *B* is the number of mugs of beer he drinks. Pizza costs $2 per slice, beer costs $3 per mug, and Chris has $36 to spend on beer and pizza.

a) Find the beer and pizza consumption bundle that maximizes his utility.

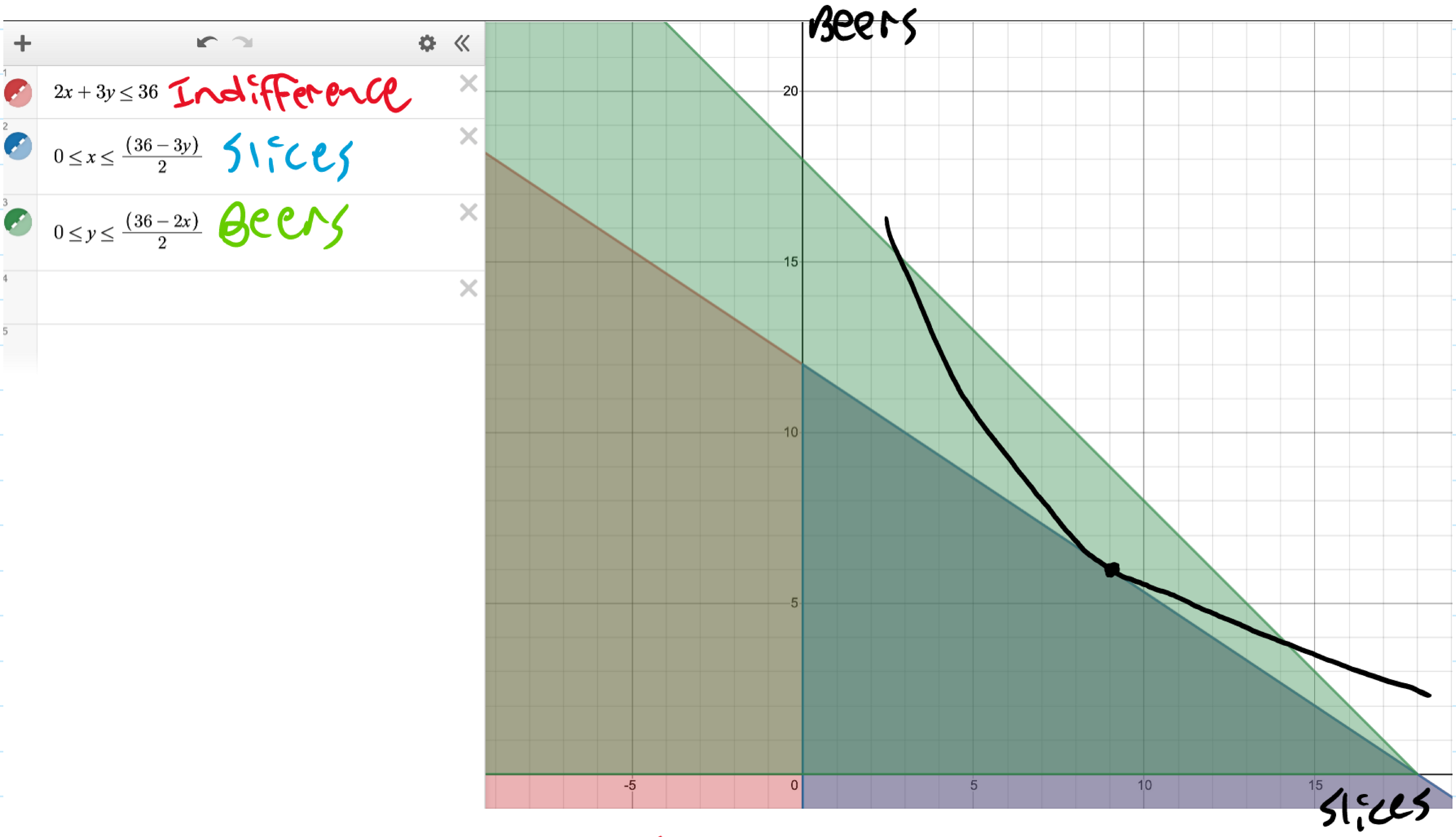
$36 = 2S + 3B$

There are lots of combinations that maximize $36

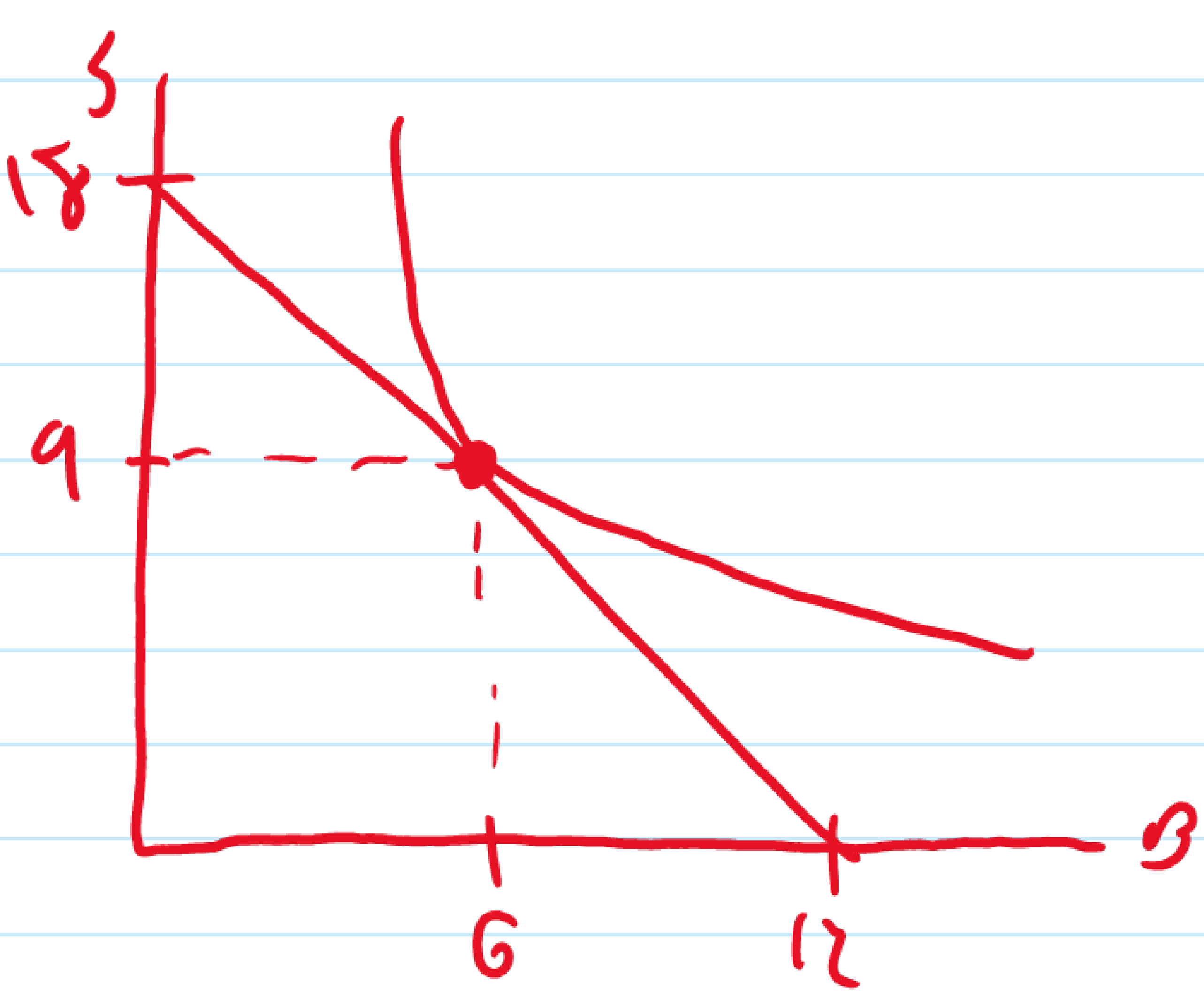
S	B	S·B
0	12	0
3	10	30
6	8	42
9	**6**	**54**
12	4	48
15	2	30
18	0	0

→ I would choose this one because they're closest

b) Sketch the budget line and the indifference curve corresponding to Chris' choice.



↑
different axes



Problem 14

Monday, February 22, 2021 8:56 PM

✓

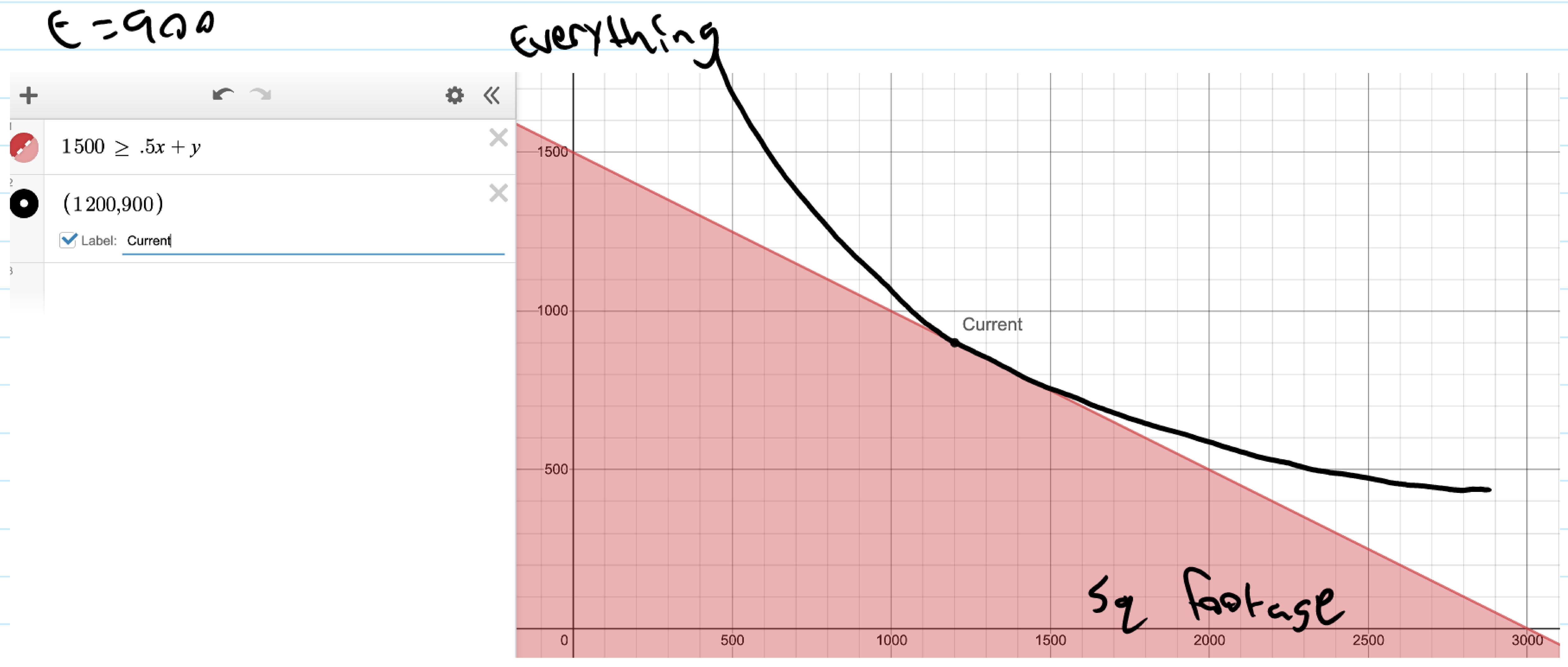
For purposes of this question, divide all things into housing, *H*, and money spent on everything else, *E*. Consider a household with a monthly income of $1,500 facing a cost of $1 per square foot to rent housing who chooses to live in a 600 square foot residence.

a) Draw the household's budget line and an indifference curve appropriate to their choice.

$1500 = H + E$
$1500 = 600 + E$
$E = 900$

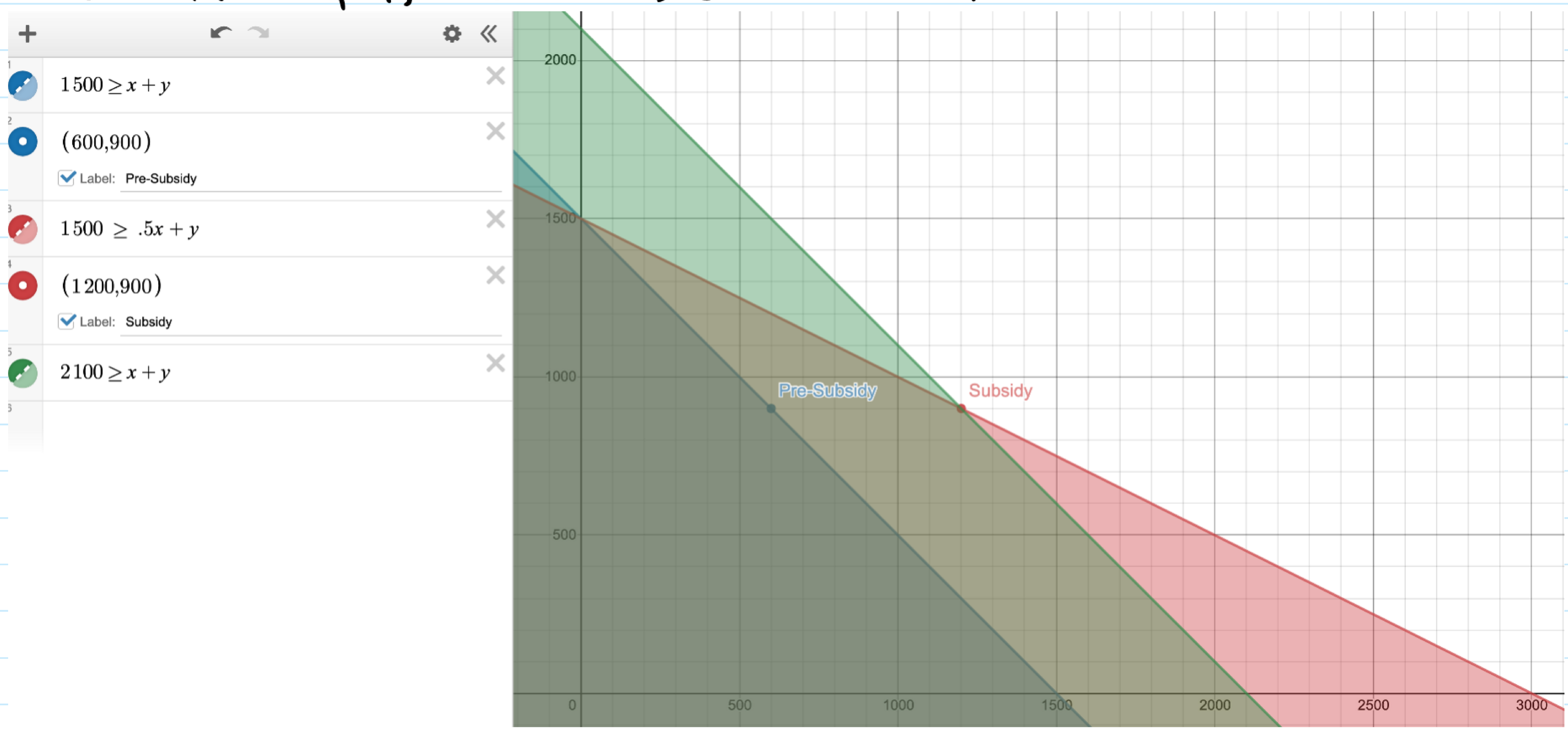
b) Consider a proposal to subsidize housing under which this household would face a price of $0.50 per square foot. In that circumstance, suppose the household chooses to rent a 1,200 square foot residence. Show the new budget line and indifference curve.

$1500 = .5H + E$
$1500 = .5(1200) + E$
$1500 = 600 + E$
$E = 900$



c) Now, for the analytical part... Use your figure to show that if the money spent subsidizing housing had been given to the household to spend as they saw fit (shifting their original budget line out but leaving the housing price at $1 per square foot), the household would rent a smaller residence, spend more on other things, and reach a higher indifference curve, and so be better off. Explain why this is so. As part of your explanation, consider the rate housing can actually be changed into other things (given by market prices) compared to the rate the household is willing to trade housing for other things at their optimum choice given the subsidized price.

New total $ is $.5 \cdot 1200 \leq 600 + 1500 = \$2100$



We start with the blue line where we have an unsubsidized budget of $1500. The point chosen by the household is $600 for rent which leaves $900 for everything else. Then, if we subsidize half of housing, the subsidized point is still $600 for housing but is converted to $1200 of buying power with the subsidy. The amount left for everything else remains the same at $900. Once the limitations on the subsidy are removed, the total buying power becomes $2100. At any point past a housing cost of $1200, the marginal rate of utility for buying anything else decreases faster than the housing-limited subsidy. Meanwhile, the marginal rate of utility for any housing less than $1200 increases much faster than the subsidized housing. Thus, someone not bound by spending limitations would spend, at most, the same on housing as someone bound by those limitations and has incentive to spend less on housing so they have more for everything else.

Problem 15

Monday, February 22, 2021 9:24 PM

Ben's preferences are represented by $U=0.3\ln H+0.7\ln E+0.1S$, where H is square feet of housing consumed monthly, E is the amount spent monthly on everything else, and $S=1$ if he lives somewhere sunny like Florida (no snow or sleet and little freezing weather) and 0 otherwise. He is considering 2 jobs, one in Tampa and one in Boston. The job in Boston pays \$7,000 per month. Housing costs \$4 per square foot monthly in Boston and \$1.5 per square foot monthly in Tampa. Calculate the salary in Tampa that would make Ben indifferent between the job in Tampa and the job in Boston. Illustrate with a figure.

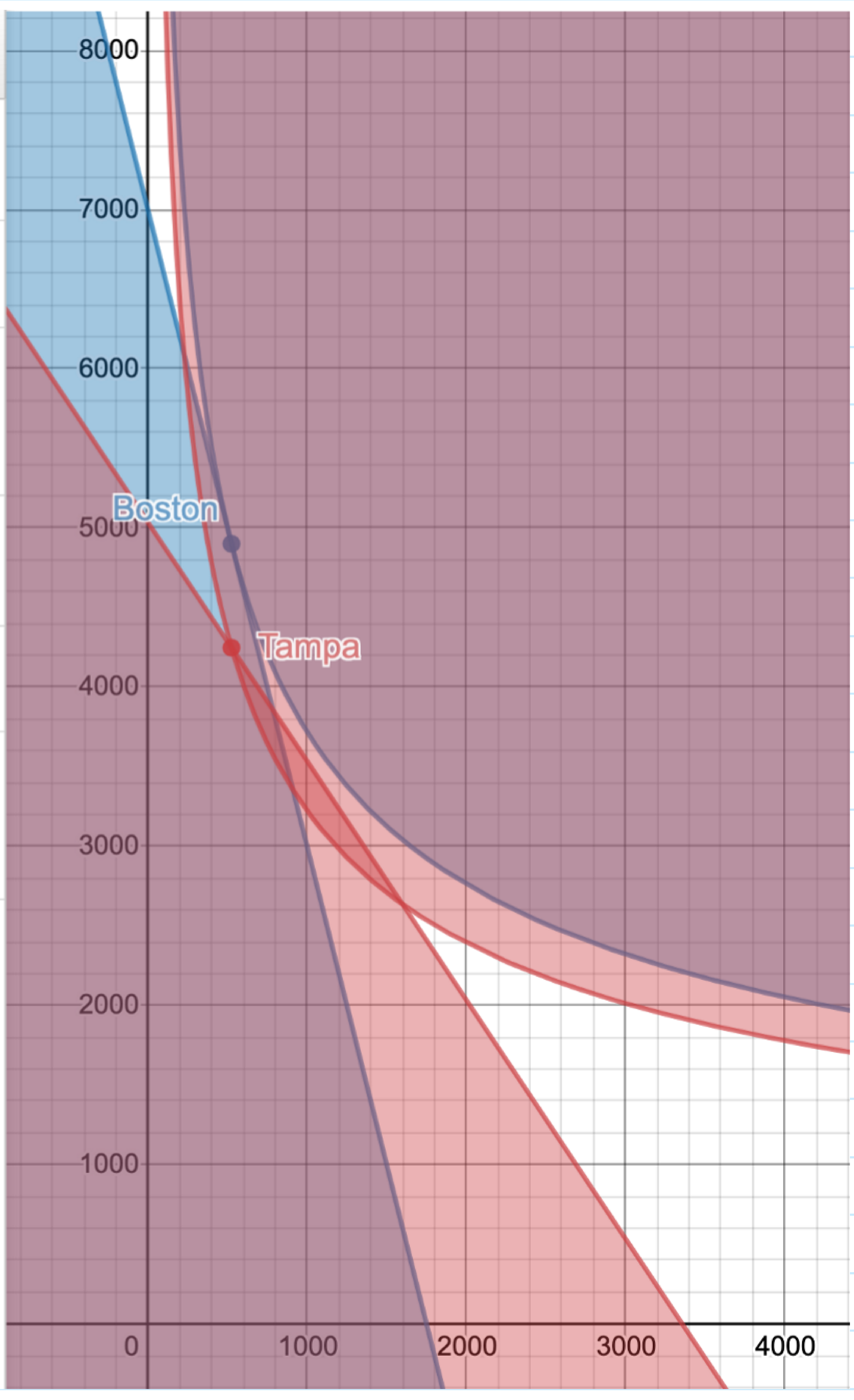
$E_{boston} = 7000 - 4H$ $\qquad E_{Tampa} = X - 1.5H$

	A	B	C	D	E	F	G	H	I	J	K
1		Square ft	Rent		Income		Everything		Utility		Diff
2	Boston	525.000042	2100.00017		7000		4899.99983		7.82691282		2.7001E-13
3	Tampa	525	787.5		5035.20171		4247.70171		7.82691282		
4											
5											

I know we didn't learn solver in this class and I could do it by hand but I was too uncertain of what to do to do that to myself.

I wasn't having any luck with solver by trying to set the income as the same and realized it was because I wasn't using the utility. I added the utility in but solver still wouldn't do anything. That's because there were two unknowns which "cancelled" out, so to speak, and made the solution not exist because ln(0) is not a good time. Then I realized that I could maximize the utility of Boston by itself to find the square footage in Boston and use the same number in Tampa because otherwise the comparison doesn't make sense and isn't fair. With the new square footage, I set diff equal to the absolute value of the utility of Tampa minus the utility of Boston. I then asked solver to try and minimize the difference while changing the income in Tampa. Thankfully, this worked and I got an income of 4247.70.

1. $7.8269 \le (.3 \cdot \ln(x)) + (.7 \cdot \ln(y))$
2. $7000 \ge 4x + y$
3. $(525, 4900)$ Label: Boston
4. $7.8269 \le (.3 \cdot \ln(x)) + (.7(\ln(y))) + .1$
5. $5035.20171 \ge 1.5x + y$
6. $(525, 4247)$ Label: Tampa



Math this time

Ben's preferences are represented by $U=0.3\ln H+0.7\ln E+0.1S$, where H is square feet of housing consumed monthly, E is the amount spent monthly on everything else, and $S=1$ if he lives somewhere sunny like Florida (no snow or sleet and little freezing weather) and 0 otherwise. He is considering 2 jobs, one in Tampa and one in Boston. The job in Boston pays \$7,000 per month. Housing costs \$4 per square foot monthly in Boston and \$1.5 per square foot monthly in Tampa. Calculate the salary in Tampa that would make Ben indifferent between the job in Tampa and the job in Boston. Illustrate with a figure.

Max Boston

$\frac{MU_H}{MU_E} = \frac{.3}{H} \cdot \frac{E}{.7} = \frac{4}{1} = \frac{P_H}{P_E} \rightarrow E = \frac{28}{3} H$

$7000 = 4H + \frac{28}{3}H = \frac{40}{3} H \rightarrow H = 525$

$E = 7000 - 4 \cdot 525 = 4900 \Rightarrow U = .3\ln 525 + .7\ln 4900 = 7.827$

Tampa must have same utility

$\frac{MU_H}{MU_E} = \frac{.3}{H} \cdot \frac{E}{.7} = 1.5 = \frac{P_H}{P_E}$

$E = 3.5H$

$Y = 1.5H + 3.5H = 5H \rightarrow H = W/5 \rightarrow E = W - \frac{3W}{10} = .7W$

$U = .3\ln .2 + .3\ln W$
$+ .7\ln .7 + .7\ln W + .1$
$= 7.827$

$\ln W = 7.727 - .3\ln .2 - .7\ln .7 = 8.46 \rightarrow$ don't forget $-.1$

$W = e^{8.56} = 4722$

$H = 944$

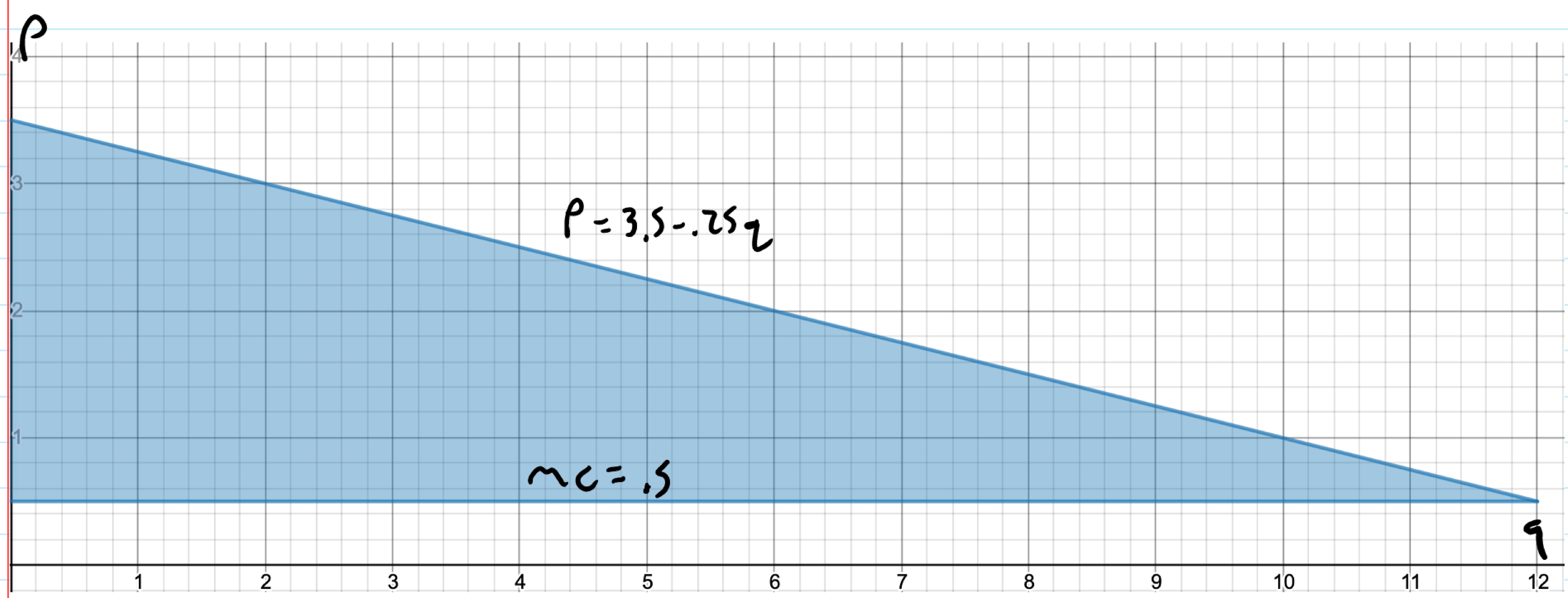
$E = 3306$

Problem 16

Monday, February 22, 2021 10:00 PM

An individual's inverse demand for a particular beer is $p = 3.5 - 0.25q$, where q is the number of bottles per period. The marginal cost is \$0.5 per bottle.

a) If bottles are sold at marginal cost, what is consumer surplus per consumer?



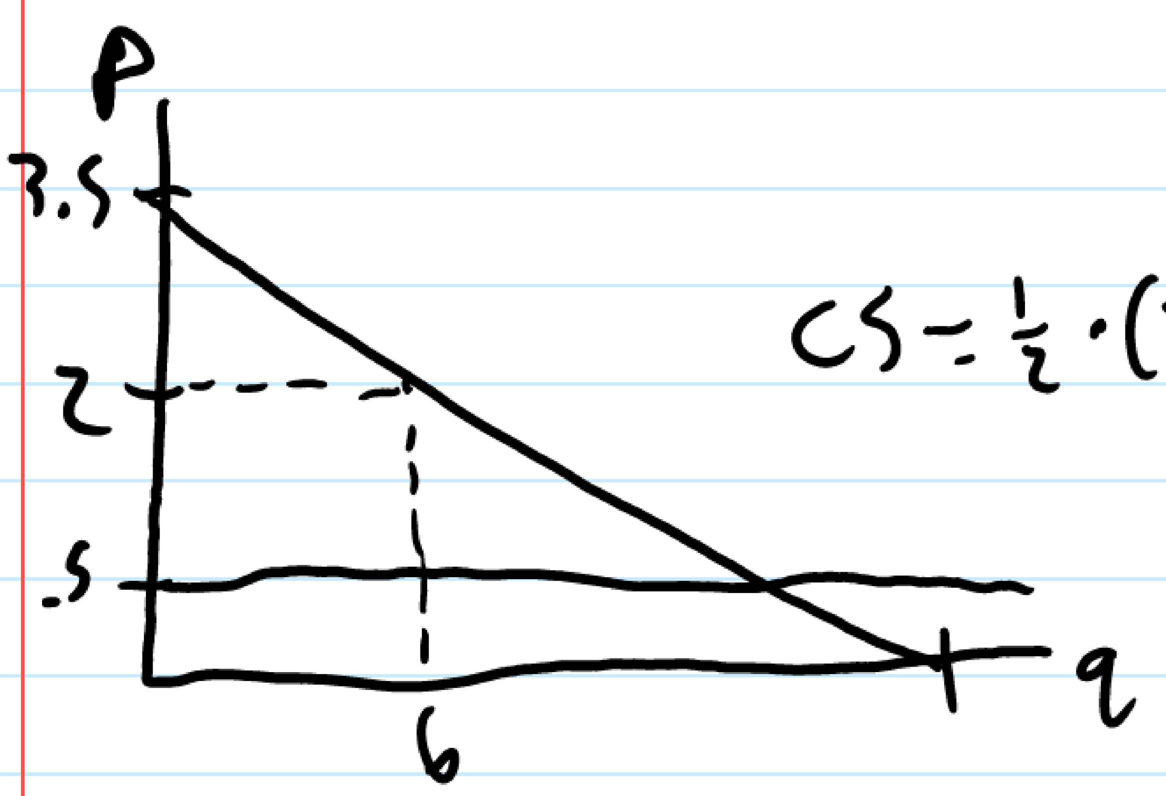
$\text{Area} = (3.5 - .5)\cdot(12)\cdot\frac{1}{2} = 3\cdot 12\cdot\frac{1}{2} = 3\cdot 6 = 18 \rightarrow$ Consumer surplus

b) If bottles must be sold one at a time at a posted price and the firm maximizes profit, what are profit and consumer surplus per customer?

$\pi = (3.5 - .25q)q - .5q \rightarrow \frac{d\pi}{dq} = 3 - .5q \rightarrow .5q = 3 \rightarrow q = 6$

$P = 3.5 - (.25\cdot 6) = 2$

$P - MC = 2 - .5 = 1.5$ per cust



$CS = \frac{1}{2}\cdot(3.5 - 2)\cdot 6 = \frac{1}{2}\cdot 1.5\cdot 6 = 4.5$

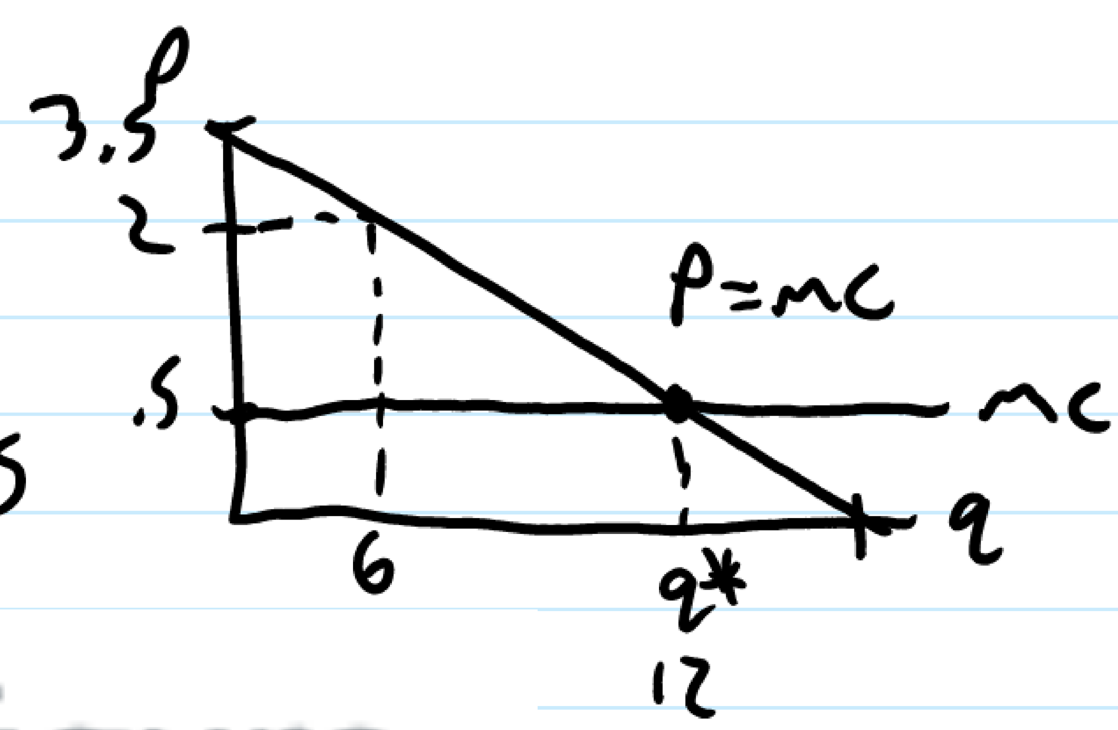
$CS = 4.5$
$\pi = 9$

c) Suppose the firm can sell packages of any number of bottles it chooses and resale is not possible. What number of bottles should be bundled together, and what price should the bundle be sold at, to maximize profit?

$.5\cdot 12 + 18 = \$24$

$3.5 - .25q = \underset{MC}{.5}$
$3 = .25q$
$q = 12$

$\rightarrow P = 3.5 - .25(12) = 3.5 - 3 = .5$



d) Illustrate a-c with a figure.

Grocery Gus or: How I Learned to Stop Worrying and Love the Log Card

Author: Gus Lipkin

Date: February 22, 2021

Table of Contents

Introduction and Background

Author's note

Defense contractors generally don't like when you go into too much detail about what you do or how their company works, even if it is for a paper for school. As such, I will be describing where I work and my job as a supermarket.

My Origin Story

I was originally hired as an intern with MiGrocery with the express purpose of collecting information on items that were delivered to be stocked in our canned goods department. Each canned good is supposed to have a log card with it. Each log card describes where the product was made and who handled it along with any climate conditions so we know if the product is safe for the sales floor. I was given a list of SKUs and a login for a piece of software we will call GrocerDocs which allows for realtime tracking of each SKU and a serial number for each unit of that SKU.

Unfortunately, the people who stock the warehouse and are not very organized and items of the same SKU do not always end up stocked together. In addition, just because GrocerDocs says an item is in a location does not mean it will actually be there when I look for it and each item may not have its required log card with it. Someone else could have taken the item without updatingGrocerDocs or taken the log card and not uploaded it to GrocerDocs, or both.

GrocerDocs will tell me where each item is, but there's no way to check and see all of the locations I will need to visit without checking the location atribute for each SKU which could have several hundred serial numbers scattered about the warehouse where I work and on the sales floor that I do not have access to. I quickly realized that going SKU by SKU and searching all the different locations that for each specific serial number was inefficient. I found a way to export GrocerDocs' complete inventory and used R to extract a list of every serial number for every SKU I needed to find and order the list by location and export this information to a spreadsheet. I also added some columns to help me better keep track of product as it moved around the warehouse and so I knew which items I had already looked for and which ones I had uploaded. With my new program, I could now sync with GrocerDocs every morning to get the newest locations. Once I showed this to my supervisors, they thought I was a "tech wizard". Thus began the onslaught of spreadsheet generator requests.

The Spinoff Series

My supervisor, the canned food department head wanted a spreadsheet that had the item names and SKUs down the left and the item serial numbers listed horizontally with each serial number cell highlighted based on its status. She had recently spent several days creating this spreadsheet but since item status can change, it was no longer up to date and she needed a new one. Creating it by hand would take me several days each time, but automating it would only take several days once. Once completed, she showed the other department managers and the bakery manager decided he wanted one too. Then, the district manager decided he wanted me to make a spreadsheet that was very similar but only listed items on the sales floor. The catch was that he wanted a single file for each department with a worksheet for every store. While there is no list of what each store should have on the sales floor, I could make a good effort at figuring it out by comparing across stores and making sure every store has the same number. Unfortunately for him, it was my day off and I was not going to accept my regular hourly rate. Ultimately, we agreed on a bonus of five hours of pay for coming in and creating the document.

At this point, my supervisor, the canned goods department manager, realized that I had not done any work for her in several weeks because I was working on generating spreadsheets of all sorts for other department heads. She wants me back on the log card project. By creating the location sorted spreadsheet of every item I need to find, I have already optimized the process as much as I can.

The Dilemma

Is it more economical to keep me on the log card project full time, keep me on the log card project and loan me out occassionally, or switch me to special projects full time?

Analysis

Profit Maximization

At first glance, this presents itself as a constrained optimization problem where the total payoff is a function of my time spent on the log card project, my time spent on special projects, the time it would take someone else to complete the same work, and the urgency of each project. Traditionally, we would need to account for differences in pay, but the problem is much simpler if everyone is paid the same. It is also very easy to add that element back in later as a function of time spent on the project.

We can use the standard π as profit. We can also define my time on log cards and special projects as M_L and M_S, respectively. The time someone else would take is O_L and O_S. The urgency of each project should be a ratio that sums to one which we can call W_L and W_S as we are weighing our options. The constant value of each finished project is defined as V_L and V_S. However, each project is not yet finished so D_L and D_S denote the percent progress of the project on a zero to one scale. Thus, expected profit is written as

$$\pi = W_L V_L D_L + W_S V_S D_S - 13(M_L + O_L + M_S + O_S)$$

Because the weight adds to one, the equation can be quickly rewritten as

$$\pi = W_L V_L D_L + (1 - W_L) V_S D_S - 13(M_L + O_L + M_S + O_S)$$

The portion completed of each project, D, can be re-written as a function of the total time taken to complete the project for each person and the total time it will take to complete the project. We can call this total time for each project T_L and T_S. I have already optimized the log card project so D_L becomes $\frac{M_L+O_L}{T_L}$. For any special projects, we can say that it will take me one-tenth the time it will take someone else. D_S can become $\frac{M_S}{.1T_S} + \frac{O_S}{T_S}$ which is simplified to $\frac{O_S+10M_S}{T_S}$. The initial equation is then rewritten as

$$\pi = W_L V_L \frac{M_L + O_L}{T_L} + (1 - W_L) V_S \frac{O_S + 10M_S}{T_S} - 13(M_L + O_L + M_S + O_S)$$

Finally, in order to make this giant mess usable, T_L and T_S must be able to be said in terms of each other. Each special project takes about twenty-five hours to complete. Based on current progress, we can estimate that the log card project would take about ten twenty-five hour weeks to complete. Thus, $T_L = 10T_S$. In order to make sure we are comparing the same amout of time, we must multiply any special project values by ten as well. This means that my and anyone else's time is limited to 250 hours as well. O_S and $10M_S$ become $250 - O_L$ and $10(250 - M_L)$, respectively. The profit equation becomes

$$\pi = W_L V_L \frac{M_L + O_L}{250} + 10(1 - W_L) V_S \frac{(250 - O_L) + 10(250 - M_L)}{250} - 13(M_L + O_L + 10(10(250 - M_L)) + 10(250 - O_L))$$

This cannot be simplified much more unless some of the variables are assigned numeric values. While I do not know the true values of W_L, V_L, or V_S, I can most certainly make something up. Let us say that the log cards are four times as important as any given special project and so $W_L = .8$. However, special projects are considerably more profitable and so $V_S = 5V_L$ and $V_S = \$10,000,000$.

$$\pi = (.8 * 2,000,000) \frac{M_L + O_L}{250} + 10(1 - .8)10,000,000 \frac{(250 - O_L) + 10(250 - M_L)}{250} - 13(M_L + O_L + 10(10(250 - M_L)) + 10(250 - O_L))$$

Simplifying,

$$\pi = 1,600,000 \frac{M_L + O_L}{250} + 20,000,000 \frac{(250 - O_L) + 10(250 - M_L)}{250} - 13(M_L + O_L + 10(10(250 - M_L)) + 10(250 - O_L))$$

$$\pi = 6400M_L + 6400O_L + 220000000 - 80000O_L - 800000M_L - 357500 - 1287M_L - 117O_L$$

$$\pi = 219642500 - 794887M_L - 73717O_L$$

The important part here is not the final profit equation because, as expected, the profit would be higher if MiGrocery did not have to pay employees, but rather that one hour of my time has a value of 794887 while the value of someone else's time is only 73717. My time is 10.87 times more valuable than someone else's for the same task if the constants given above retain their value.

Proper profit maximization requires taking the first derivative of the profit function. With all of the guesses about the values of constants I made and the ratios that allowed me to put one variable in terms of another, this quickly fell by the wayside. With proper numbers, the equation could be put into solver to give the amount of hours per project that myself and someone else should work on to maximize profit.

Expected Marginal Utility

As we established in *Profit Maximization*, I provide no added value to the log card project over any other person performing the same task. I will concede, of course, that I already know which buttons to press to update my spreadsheets and no one else does, but those tasks are easy to teach in under fifteen minutes and can therefore be ignored. Because I provide no added value to the project, when my supervisor asks me to work on log cards, they have a marginal utility of zero while I am doing that work.

What is the company's expected marginal utility for each unit of time I am on a special project? This is relatively easy to answer because of the nature of the special projects I am assigned to. Each special project has progress that can be measured by tasks completed or percent completion when done by hand such as transcribing part numbers into a spreadsheet or trying to find duplicate part numbers. However, when writing a program that does these tasks it is exceptionally rare that a computer running the program would take as long as a human would to do the same task.

`Assumption 1: A computer program will always take less time to complete a given task than a human will`

Of course, that program does take time to write. It takes me about a week to write a fully-fledged solution to any special project. If I am able to re-purpose existing code for a new project, that time is cut down to just one or two days.

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Assumption 2: I will take a week to finish writing a program to complete any given task
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It should now be clear that if any special project is expected to take more than a week to complete, I should be tasked with writing a program to do the task. If the task is expected to be repeated, I should be asked to write a program if the total time spent on the task is more than a week. It may take me more time in the beginning, but at some point the programming time and task time will reach a breakeven point at which point the program will be more efficient.

Conclusion

I may be tooting my own horn, so to speak, but I do not think that MiGrocery could have hired a better and more productive intern. I have shown time and time again that I am able to use skills in my toolset to expertly optimize current workloads and to automate tasks that could take weeks to complete by hand. Through profit maximization techniques with estimated numbers, I have shown that I can be more than ten times as productive as an employee doing the same task by hand. In discussing expected marginal utility, we learned that it makes sense for me to take on any special project that is expected to take more than a week to complete or that is an ongoing project. Because I am so much more productive on special projects, it makes sense for me to be switched to those full time rather than kept on log cards and loaned out to special projects on occasion.