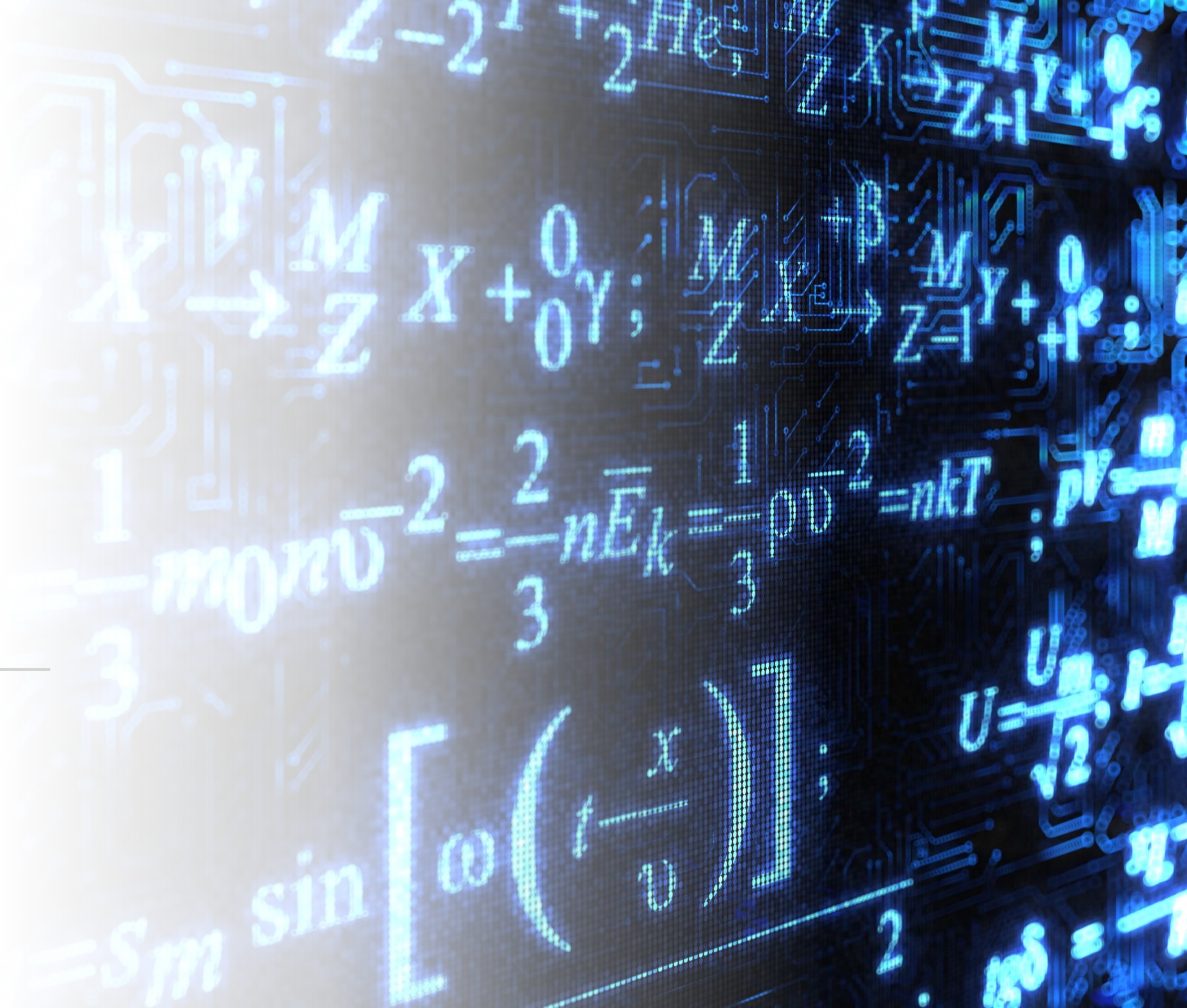
Linear Algebra Review



Sravani Vadlamani

Statistics I

Matrix

- A matrix is a rectangular array of elements arranged in rows and columns.

$$A_{m\,X\,n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & . & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & . & . & . & a_{2n} \\ a_{31} & a_{32} & a_{33} & a_{34} & . & . & . & a_{3n} \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & . & . & . & a_{mn} \end{bmatrix}$$

All elements can be identified by a typical element a_{ij} where i = 1,2, ….m denotes rows and j = 1, 2, ….n denotes columns

Row and Column Vector

- A matrix that has a single column is called a column vector.
- A matrix that has a single row is row vector

Transpose of a Matrix

- Formed by interchanging the rows and columns
- A matrix of order m X n becomes n X m when transposed

$$A_{2\,X\,3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$A'_{3X2} = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

(A')' = A; (kA)' = kA' where k is a scalar

Symmetric Matrix

- If A’ = A, the matrix is called symmetric
- A square matrix
  - same number of rows and columns i.e., m = n
  - Off-diagonal elements are symmetric i.e., $a_{ij} = a_{ji}$ *for all i and j*

$$A = \begin{bmatrix} 4 & 5 & -3 \\ 5 & 7 & 2 \\ -3 & 2 & 10 \end{bmatrix}$$

Diagonal Matrix

$$I = \begin{bmatrix} 1 & 0 & . & . & . & 0 \\ 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & 1 \end{bmatrix} = \mathrm{diag}(\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \ldots..\mathrm{a}_\mathrm{n})$$

I is the identity matrix which has 1’s on the diagonal and 0’s on the off diagonal

Matrix Addition and Subtraction

- Possible if the matrices are of the same dimension
- Addition/Subtraction of matrix A and B is the addition/subtraction of the corresponding elements of A and B

$C = A + B \;\Rightarrow\; c_{ij} = a_{ij} + b_{ij}$ for all i and j

$A = \begin{bmatrix} 2 & 5 \\ 6 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 6 \\ 5 & 8 \end{bmatrix}$ then $C = \begin{bmatrix} 2 & 11 \\ 11 & 3 \end{bmatrix}$

- $A \pm B = B \pm A$ (commutative)
- $(C \pm B) \pm C = A \pm (B \pm C)$ - Associative
- $(A \pm B)' = A' \pm B'$

Scalar Multiplication

- If k is a scalar and A is a matrix, the product k times A is called scalar multiplication.
- Each element of A is multiplied by the scalar k
- B= kA $\Rightarrow$ b_{ij} = k a_{ij} for all i and j

Matrix Multiplication

- To multiply two matrices A and B and store the result in a matrix called C
- C = AB
- number of columns of A = number of rows of B

$$C_{m\ X\ n} = A_{m\ X\ p} * B_{p\ X\ n}$$

Matrix Multiplication

$$F = AD = \begin{bmatrix} 6 & 8 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 3 & -8 & 1 \\ 9 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6*3+8*9 & 6*(-8)+8*2 & 6*1+8*5 \\ (-2)*3+4*9 & (-2)*(-8)+4*2 & (-2)*1+4*5 \end{bmatrix}$$

$$= \begin{bmatrix} 90 & -32 & 46 \\ 30 & 24 & 18 \end{bmatrix}$$

Matrix Multiplication

- A (m x 1) column vector multiplied by a (1 x n) row vector becomes an (m x n) matrix.
- A (1 x m) row vector multiplied by a (m x 1) column vector becomes a scalar.
- In general, $AB \neq BA$.
- But, $kA = Ak$ if k is a scalar and A is a matrix.
- And, $AI = IA$ if A is a matrix and I is the identity matrix and conformable for multiplication.

Trace of a Square Matrix

- Sum of diagonal elements
- $tr(A) = diag(a_{11} + a_{22} + a_{33} + \ldots\ldots a_{nn})$
- $tr(A)$ = A, if A is scalar
- $tr(A')$ = $tr(A)$, as A is square
- $tr(kA)$ = k $tr(A)$, where k is a scalar
- $tr(I_n) = n, trace\ of\ an\ identity\ matrix\ is\ its\ dimension$
- $tr(A \pm B)$ = $tr(A) \pm tr(B)$
- $tr(AB)$ = $tr(BA)$

Hilbert Matrix

- A square matrix with entries being the unit fractions

$$H_{ij} = \frac{1}{i+j-1}$$

- 3 x 3 Hilbert matrix

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

SVD of a Matrix

http://timbaumann.info/svd-image-compression-demo/