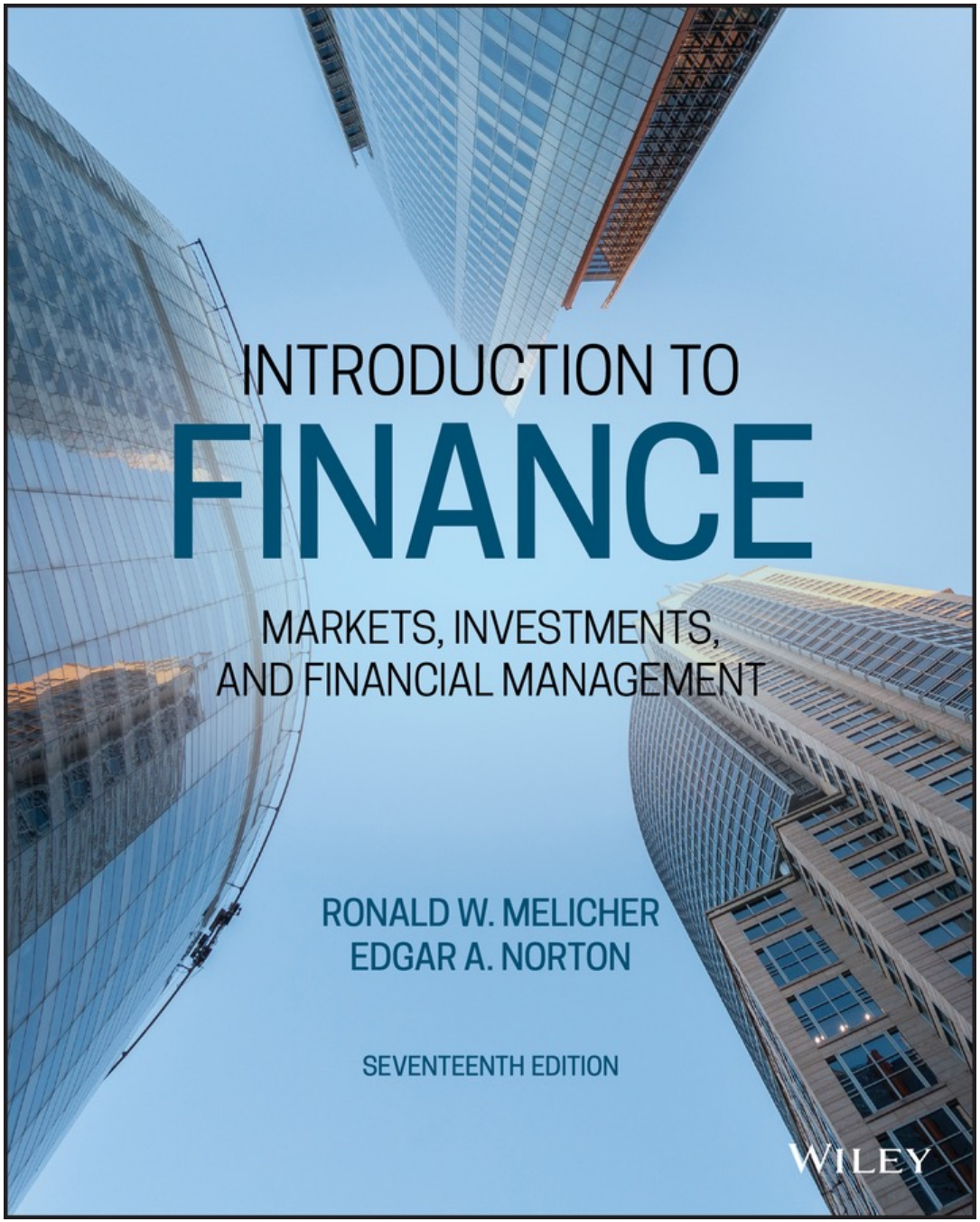
Chapter 9

Time Value of Money



Learning Objectives

- LO 9.1 Explain what is meant by the "time value of money" and the concept of simple interest.
- LO 9.2 Describe the process of compounding to determine future values.
- LO 9.3 Describe the process of discounting to determine present values.

Learning Objectives

- LO 9.4 Explain how to find interest rates and time requirements for problems involving compounding or discounting.
- LO 9.5 Define an *annuity* and describe how to find the future value of an ordinary annuity.
- LO 9.6 Explain how to calculate the present value of an ordinary annuity.
- LO 9.7 Describe how to find interest rates and time requirements for problems involving ordinary annuities.

Learning Objectives

- LO 9.8 Explain how to determine periodic ordinary annuity payments.
- LO 9.9 Explain how to calculate future and present values when time intervals are less than one year, and describe how to estimate the cost of consumer credit.
- LO 9.10 (Learning Extension) Understand and calculate annuity due problems involving future and present values.

Section 9.1 Basic Time Value Concepts

- LO 9.1 Explain what is meant by the "time value of money" and the concept of simple interest.

Finance: Six Principles

1. Money has a time value
2. Higher returns are expected for taking on more risk
3. Diversification of investments can reduce risk
4. Financial markets are efficient in pricing securities
5. Manager and stockholder objectives may differ
6. Reputation matters

[Note: Principles were initially discussed in Chapter 1]

Basic Concepts

- Chapter 9 Concentrates on the Time Value of Money Principle

  Money can grow or increase over time if it can be saved (or invested) and a return earned on the savings (or investment)

- Time Value of Money Definition

  Math of finance whereby a financial return is earned over time by saving or investing money

Basic Concepts

- Present Value

  Value of an investment or savings amount today or at the present time

- Future Value

  Value of an investment or savings amount at a specified future time

- Simple Interest

  Interest earned only on the principal of the initial investment

Time Value of Money Example: Simple Interest

- Basic Information

  You have $1,000 to save or invest for one year and a bank will pay you 8% for use of your money. What will be the value of your savings after one year?

- Basic Equation

  Future value = Present value + (Present value x Interest rate)

  Future value = $1,000 + ($1,000 x .08) = $1,000 + $80 = $1,080

Time Value of Money Example: Simple Interest (continued)

- Alternative Basic Equation

  Future value = Present value x (1 + Interest rate)

  Future value = $1,000 x (1 + .08) = $1,000 x 1.08 = $1,080

Section 9.2 Compounding to Determine Future Values

- LO 9.2 Describe the process of compounding to determine future values.

Compounding Basics

- Compounding

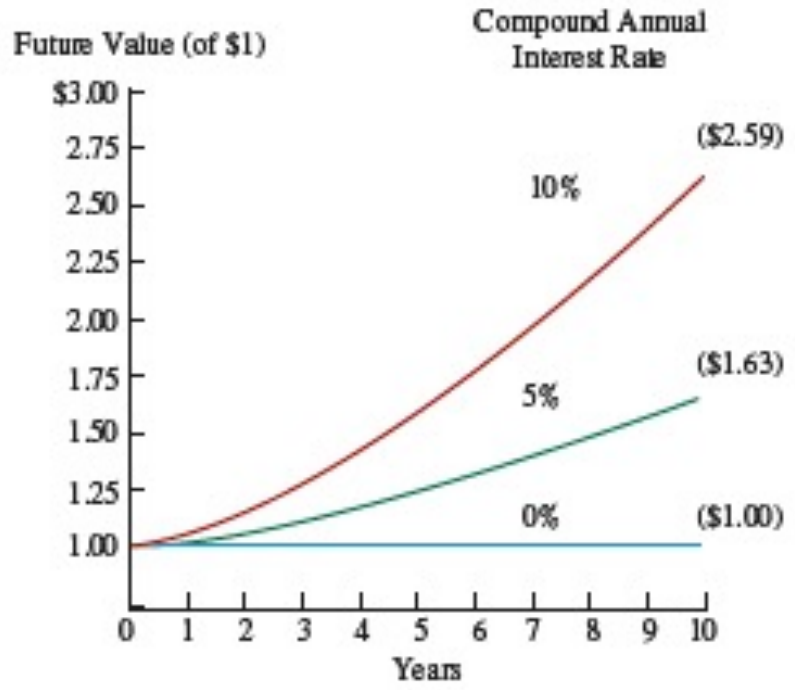
  Arithmetic process whereby an initial value increases or grows at a compound interest rate over time to reach a value in the future

- Compound Interest

  Involves earning interest on interest in addition to interest on the principal or initial investment

- Figure 9.1 shows graphically the impact of compounding on values

Figure 9.1 Future Value, Interest Rate, and Time Period Relationships



Compounding to Determine Future Values: Example

- Basic Information

  You plan to invest $1,000 now for two years and a bank will pay you compound interest of 8% per year. What will be the value after two years?

- Basic Equation

  Future value = Present value x [(1 + Interest rate) x (1 + Interest rate)]
  Future value = $1,000 x (1.08 x 1.08) = $1,000 x 1.1664 = $1,166.40

Four Ways to Solve Time Value of Money Problems

- "By Hand" Using Equations
- Financial Calculators
- Spreadsheet Programs
- Tables-Based Methods

Using Interest Factor Tables to Solve Future Value Problems

- Future Value: $(FV_n) = PV(FVIF_{r,n})$
  Where: PV = present value amount
  $FVIF_{r,n}$ = pre-calculated future value interest factor for a specific interest rate (r) and specified time period (n)
- Example: What is the future value of $1,000 invested now at 8% interest for 10 years?

  $FV_{10} = \$1,000(2.1589) = \$2,158.90$

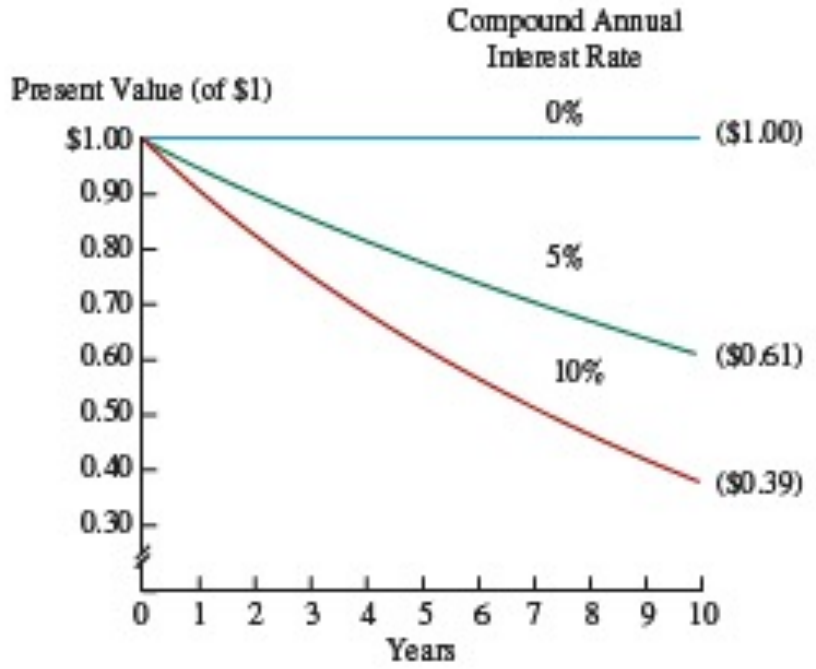
Section 9.3 Discounting to Determine Present Values

- LO 9.3 Describe the process of discounting to determine present values.

Basic Discounting Concepts

- Discounting
  Arithmetic process whereby a future value decreases at a compound interest rate over time to reach a present value
- Basic Equation
  Present value = Future value x {[1/(1 + Interest rate)] x [1/(1 + Interest rate)]}
- Figure 9.2 shows graphically the impact of discounting on values

Figure 9.2 Present Value, Interest Rate, and Time Period Relationships



Discounting to Determine Present Values: Example

- Basic Information

  A bank agrees to pay you $1,000 after two years when interest rates are compounding at 8% per year. What is the present value of this payment?

- Basic Equation

  Present value = Future value x {[1/(1+ Interest rate)] x [1/(1 + Interest rate)]}
  Present value = $1,000 x (1/1.08 x 1/1.08) = $1,000 x 0.8573 = $857.30

Using Interest Factor Tables to Solve Present Value Problems

- Present Value: $(PV) = FV_n(PVIF_{r,n})$
  Where: FV_n = future value amount
  $PVIF_{r,n}$ = pre-calculated present value interest factor for a specific interest rate (r) and specified time period (n)
- Example: What is the present value of $1,000 to be received 10 years from now if the interest rate is 8%?

  PV = $1,000(0.4632) = $463.20

Section 9.4 Finding Interest Rates and Time Requirements

- LO 9.4 Explain how to find interest rates and time requirements for problems involving compounding or discounting.

Interest Factor Tables: Finding Interest Rates or Time Periods

- Four Basic Variables

  FV = future value

  PV = present value

  r = interest rate

  n = number of periods

- Key Concept

  Knowing the values for any three of these variables allows solving for the fourth or "unknown" variable

Finding Interest Rates: Example

- Basic Problem

  Assume that the present value of an investment is $1,000, the future value is $1,403 and the time period is five years. What compound interest rate would be earned on this investment?

- Financial Calculator Solution

  Enter PV (present value) = 1000, FV (future value) = 1403, N (number of years) = 5, and press CPT (compute) followed by the %i key to find an r (interest rate) of 7.01

  [Note: some calculators may require either the PV entry or FV entry to be negative.]

Finding Time Periods: Example

- Basic Problem

  Assume that the present value of an investment is $1,000, the future value is $1,403 and the interest rate is 7 percent. What length of time does this investment involve?

- Financial Calculator Solution

  Enter PV (present value) = 1000, FV (future value) = 1403, %i = 7, and press CPT (compute) followed by the N key to find the n number of years to be 5.01

  [Note: some calculators may require either the PV entry or FV entry to be negative.]

Time Required for an Investment to Double in Value

- Typical Question

  How long will it take, at a 6 percent interest rate (or rate of return), for an investment to double in value?

- Rule of 72

  A shortcut method used to approximate the time required for an investment to double in value

- Application of Rule of 72

  72 divided by 6 percent indicates the investment will double in about 12 years

Section 9.5 Future Value of an Annuity

- LO 9.5 Define an *annuity* and describe how to find the future value of an ordinary annuity.

Annuity Terms and Concepts

- Annuity

  A series of equal payments (or receipts) that occur over a number of time periods
- Ordinary Annuity

  Exists when equal payments (or receipts) occur at the end of each time period
- Annuity Due

  Exists when equal payments (or receipts) occur at the beginning of each time period

Finding the Future Value of an Annuity

- Basic Equation:
  $FVA_n = PMT\{[(1 + r)^n - 1]/r\}$
- Where:

  FVA = future value of ordinary annuity

  PMT = periodic equal payment

  r = compound interest rate, and

  n = total number of periods

Future Value of an Annuity: Example

- Basic Information

  You plan to invest $1,000 each year beginning next year for three years at an 8% compound interest rate. What will be the future value of the investment?

- Basic Equation:

  $FVA_n = PMT\{[(1 + r)^n - 1]/r\}$

  $FVA_3 = \$1{,}000\{[(1 + 0.08)^3 - 1]/0.08\} =$
  $\$1{,}000[(1.2597 - 1)/0.08] =$
  $\$1{,}000(3.246) = \$3{,}246$

Section 9.6 Present Value of an Annuity

- LO 9.6 Explain how to calculate the present value of an ordinary annuity.

Finding the Present Value of an Annuity

- Basic Equation:

  $PVA_n = PMT\{[1 - (1/(1 + r)^n)]/r\}$

- Where:

  PVA = present value of ordinary annuity

  PMT = periodic equal payment

  r = compound interest rate, and

  n = total number of periods

Present Value of an Annuity: Example

- Basic Information

  You will receive $1,000 each year beginning next year for three years at an 8% compound interest rate. What will be the present value of the investment?

- Basic Equation

$$PVA_n = PMT\{[1 - (1/(1 + r)^n)]/r\}$$

$$PVA_3 = \$1{,}000\{[1 - (1/(1.08)^3)]/0.08\}$$

$$= \$1{,}000\{[1 - 0.7938]/0.08\}$$

$$= \$1{,}000(0.2062/0.08)$$

$$= \$2{,}577$$

Section 9.7 Interest Rates and Time Requirements for Annuities

- LO 9.7 Describe how to find interest rates and time requirements for problems involving ordinary annuities.

Finding Interest Rates or Time Periods for Annuities

- Five Basic Variables for Annuity Problems

  FV = future value

  PV = present value

  r = interest rate

  n = number of periods

  PMT = payment amount (ordinary annuities)

- Using Financial Calculators

  Knowing the values for either the FV or PV and two of the other variables allows solving for the "unknown" variable

Finding Interest Rates for Annuities

- Example: Assume that the future value of an ordinary annuity is $5,751, the annual payment is $1,000, and the time period is five years. What is the interest rate for this problem?
- Using a Financial Calculator: Enter FV = 5751, PMT = –1000 (for both TI and HP calculators), and N = 5. Press the CPT key followed by the %i key to find an r of 7 percent.

Finding Time Periods for Annuities

- Example: Assume the future value of an ordinary annuity is $5,751, the annual payment is $1,000, and the interest rate is 7 percent. How long would it take for the $1,000 annual investments to grow to $5,751?
- Using a Financial Calculator: Enter FV = 5751, PMT = –1000, %i = 7, and press the CPT key followed by the N key to find an n of five years.

Section 9.8 Determining Periodic Annuity Payments

- LO 9.8 Explain how to determine periodic ordinary annuity payments.

Periodic Annuity Loan Payments

- Amortized Loan

  A loan repaid in equal payments over a specified time period

- Process

  Solve for the periodic payment amount

- Loan Amortization Schedule:

  A schedule of the breakdown of each payment between interest and principal, as well as the remaining balance after each payment

Section 9.9 More Frequent Time Intervals and The Cost of Consumer Credit

- LO 9.9 Explain how to calculate future and present values when time intervals are less than one year, and describe how to estimate the cost of consumer credit.

Compounding or Discounting More Often than Once a Year

- Basic Equation

  $FV_n = PV(1 + r/m)^{nxm}$

  Where m = number of compounding periods per year and the other variables are as previously defined

- Example

  What is the future value of a two-year, $1,000, 8% interest loan with semiannual compounding?

  $FV_n = \$1,000(1 + 0.08/2)^{2x2}$

  $= \$1,000(1.04)^4 = \$1,000(1.1699)$

  $= \$1,169.90$

Comparing APR and EAR

- Annual Percentage Rate (APR)

  Determined by multiplying the interest rate charged (r) per period by the number of periods in a year (m)
- APR Equation

  APR = r x m
- Example

  What is the APR on a car loan that charges 1% per month?
  APR = 1% x 12 months = 12%

Comparing APR and EAR

- Effective Annual Rate (EAR)

  True interest rate when compounding occurs more frequently than annually
- EAR Equation: $EAR = (1 + r)^m - 1$
- Example: What is the EAR on a credit card loan with an 18% APR and with monthly payments?

  Rate per month = 18%/12 = 1.5%

  $EAR = (1 + 0.015)^{12} - 1$

  $= 1.1956 - 1 = 19.56\%$

Learning Extension 9.10 Annuity Due Problems

- LO 9.10 Understand and calculate annuity due problems involving future and present values.

Annuity Due Problems

- Annuity Due
  Exists when the equal periodic payments occur at the beginning of each period
- Future Value of an Annuity Due ($FVAD_n$) Equation:
  $FVAD_n = FVA_n \times (1 + r)$
  $FVAD_n = PMT\{[((1 + r)^n - 1)/r] \times (1 + r)\}$

Annuity Due Problems: Example

- Basic Information

  You plan to invest $1,000 each year beginning now for three years at an 8% compound interest rate. What will be the future value of this investment?

- Equation: $FVAD_n = FVA_n \times (1 + r)$

  $FVAD_n = PMT\{[((1 + r)^n - 1)/r] \times (1 + r)\}$

  $FVAD_3 = \$1,000\{[((1.08)^3 - 1)/0.08] \times (1.08)\} = \$1,000[(3.246) \times (1.08)] = \$1,000(3.506) = \$3,506$

Web Links

- www.chase.com
- www.citibank.com
- www.federalreserve.gov