Problem 5. [7 marks]

Prove, by induction on n, that a slithy Boolean expression in CNF with at most n variables has at most n clauses and is satisfiable.

So:

base: let $n=1$

(x_1) so it is proven for $n=1$ that a slithy booleen expression in CNF only contains a single clause

Inductive Hypothesis: As the base case holds we can now assume the expression is true for n.

hence: $\underbrace{(x_1 \vee x_2) \wedge \ldots \wedge (x_{n-1} \vee x_n)}_{\text{at most } n \text{ clauses.}}$

Inductive Step: We assume for n a now must prove for the $n+1$ case.

now: we know for n^{th} case our expression has form such that:

$(C_1) \wedge \ldots \wedge (C_n)$ # where C_n is short-form for a clause

At it's most maximal form by the inductive hypothesis.
so: by introducing a new variable x_{n+1} we can create a new slithy expression such that.

$(C_1) \wedge \ldots \wedge (C_n) \wedge (x_{n+1})$

now containing $n+1$ clauses,

BUT: there exists no way in which to create more clauses.

hence a slithy boolean expression containing $n+1$ variables contains a maximum of $n+1$ clauses.

So: By induction it is proven that a slithy boolean expression with n variables can contain at most n clauses

$\blacksquare$