Context-Free Languages

Now refer to the description of **Walks** on page 22.

Let SAW be the language of strings over the alphabet $\{\mathtt{N}, \mathtt{S}, \mathtt{E}, \mathtt{W}\}$ that represent self-avoiding walks.

Problem 8. [8 marks]

Prove or disprove: The language SAW is context-free.

Your submission can be typed or hand-written, but it must be in PDF format and saved as a file `prob8.pdf`.

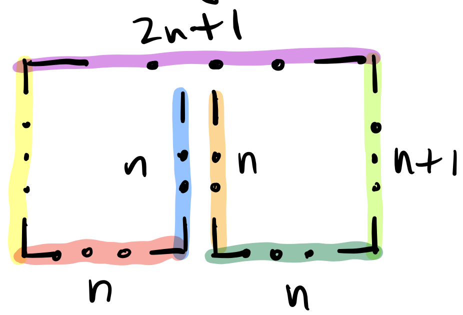
Suppose the language SAW is context-free and let K be the number of non-terminal symbols in a CFG for SAW.

now let $N = 2^k$ for simplicity.

Then by the pumping lemma for context free languages every $w \in SAW$ such that $|w| > 2^k - 1$ can be written in the form $w = uvwxyz$ where $|vy| \geq 1$, $|vxy| \leq 2^k$ and all $i \geq 0$, such that $uv^ixy^iz \in SAW$.

It's important to note that the rules on SAW dictate that every substring of $w \in SAW$ must follow that $|N| \neq |S| \vee |E| \neq |W|$

now: we choose a string $w = S^n W^n N^{n+1} E^{2n+1} S^{n+1} W^n N^n$
which by map diagram we can represent as:

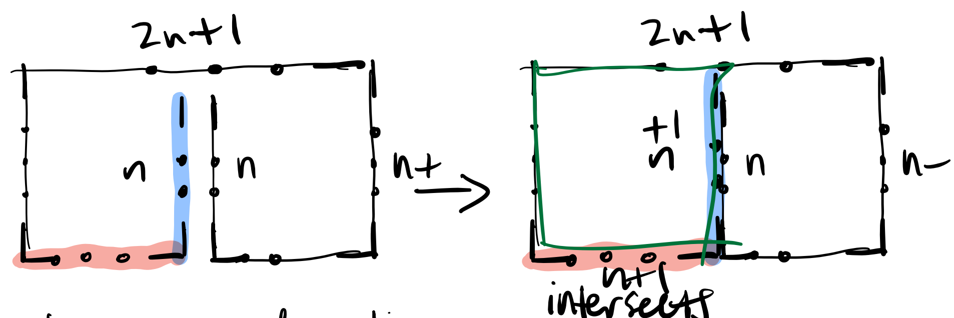


Case 1: $S^n W^n \ldots$

pumping S - if $i=1 \Rightarrow S^{n+1} W^n N^{n+1} E^n$,
$|S| = |W|, |W| = |E| \Rightarrow \notin$ SAW

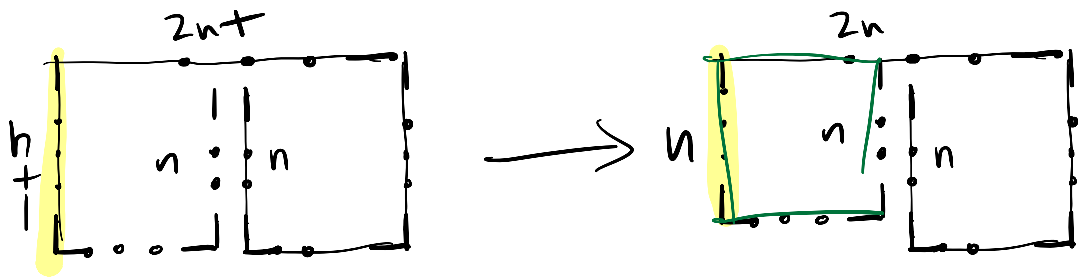
Case 2:
pumping W - $i=1 \Rightarrow W^{n+1} N \; E^{2n+1} S^{n+1} W^n$
equal parts $\Rightarrow \in$ SAW.

Case 3:
pumping W & S -

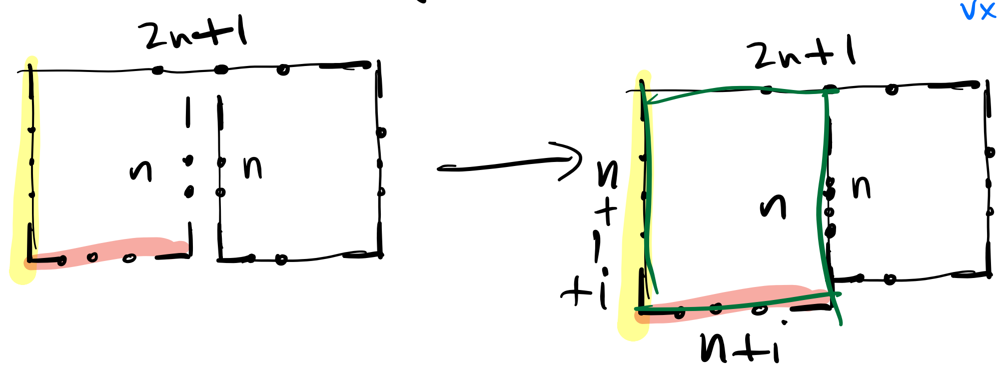


when pumped lines will intersect
$\Rightarrow \notin$ SAW.

Case 4: pumping N^{n+1}, if we let $i=0$ then we will reach $S^n W^n N^n E^n$ as a substring for which does not satisfy SAW.

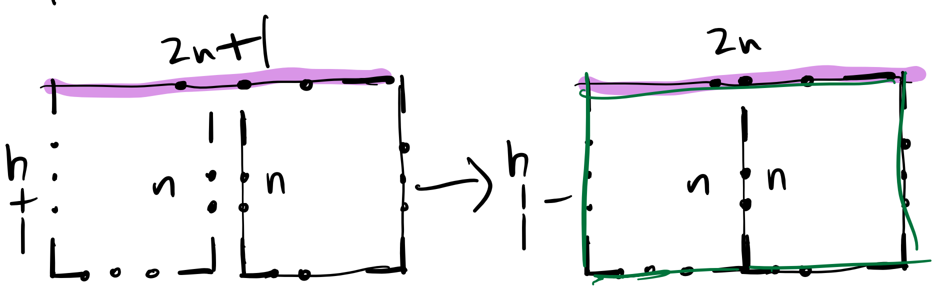


Case 5: pumping combination of $\underbrace{W^n N}_{vxy}{}^{n+1}$



Any combination of W's and N's will result in an intersection occurring such that some substring of w

Case 6: pumping E, if we let $i=0$ then $W^n N^{n+1} E^{2n} S^{n+1} W^n$, which forms a a substring that does not exist in a word that is an element of SAW



*I ran out of time, the rest of the cases are mostly similar to the ones provided except on the opposite side.
Apologies for the hand waviness, only came up with this late friday night.