UNIVERSITY
OF WARSAW

Faculty of Mathematics, Informatics and Mechanics
Institute of Informatics

Warsaw, 10th July 2019

PhD Committee
Jagiellonian University, Kraków

Report on the PhD thesis of Grzegorz Guśpiel
New combinatorial structures for several algorithmic problems

Overall evaluation. The presented thesis is a very good dissertation.

It includes a number of interesting results concerning polynomial-time algorithms (or respective NP-hardness proofs) of a few combinatorial problems. It shows wide interests of the author and his ability to contribute to various subareas of the theory of algorithms.

Notably, a number of results appeared as single-authored (or co-authored with the co-advisor) publications, while in the main multi-author publication there is a clear attribution of part of the work done by the author of the thesis. In all cases, the contribution of the author of the thesis are strong and of top technical quality.

Finally, the presented results are well-written and the thesis is well-organized.

Consequently, the presented thesis satisfies all the requirements to award a PhD degree to the candidate. Furthermore, in my opinion the dissertation crosses the bar for a distinction and thus I support awarding a PhD degree with a distinction.

Overview of the thesis. The thesis consists of four independent chapters.

Banacha 2, 02-097 Warsaw, Poland
+48 22 55 44 419
e-mail: m.pilipczuk@mimuw.edu.pl

- In Chapter 1, the author presents the first deterministic in-place subquadratic algorithm for permutation inversion.
- In Chapter 2, bar visibility representations of graphs are studied; a near-linear time algorithm is presented for the extension problem for st-planar graphs, while an NP-hardness lower bound is presented for the extension problem for planar undirected graphs.
- In Chapter 3, an NP-hardness proof is shown for finding a perfect matching with minimum number of crossings.
- In Chapter 4, the author continues a study from his master's thesis on universal targets of homomorphisms, tightening the previously obtained bounds.

Permutation inversion. With extra $\mathcal{O}(n)$ space, the task of computing an inversion of a permutation of the set $\{1, 2, \ldots, n\}$ is trivial. However, with only $\mathcal{O}(\log n)$ space (i.e., an *in-place* algorithm), the task becomes much more interesting. The direct approach (reverse every cycle of the permutation independently) leads to $\Theta(n^2)$-time algorithm, with the main bottleneck being multiple visits of long cycles of the permutation: the algorithm visits a cycle of length ℓ exactly ℓ times, spending time $\mathcal{O}(\ell)$ on each visit. Previously, the only known subquadratic algorithms were randomized.

In his single-authored manuscript, the author of the thesis develops an $\mathcal{O}(n^{3/2})$-time algorithm via an intricate marking scheme of visited long cycles in the permutation that allow the algorithm to realize that it is revisiting a long cycle of the permutation with only $\mathcal{O}(\sqrt{n})$ steps. I found the solution and its main ideas very cute and interesting on the technical level.

Visibility representations. Various variants of visibility representations of graphs is a wide and deeply studied area in graph drawing and computational geometry. In Chapter 2, a study of one of the more classical variants, bar visibility representations, is presented. The main algorithmic contribution is a polynomial-time algorithm for the extension problem on planar st-graphs via dynamic programming on SPQR trees. The dynamic programming algorithm, while quite technical, is also a rather natural approach for the problem and follows well-paved paths.

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However, a direct implementation of the approach leads to $\mathcal{O}(n^2)$-time algorithm. The main contribution of the author of the thesis is speeding it up to $\mathcal{O}(n \log^2 n)$. The speed-up is achieved as follows. The $\mathcal{O}(n^2)$-time algorithm reduces the most complicated case in the dynamic programming algorithm to a resolution of a 2-SAT formula, but the number of constraints can grow quadratically with the size of the graph. The author observes that the added constraints have some particular structure, related to the structure exploited for orthogonal range queries algorithms for low dimensions, and uses this structure to encode the same logic with only $\widetilde{\mathcal{O}}(n)$ constraints.

In my opinion, this speed-up is the main technical contribution of Chapter 2 and required a very insightful observation into the studied problem. According to the statement of the author, the speed-up is his contribution to the content of Chapter 2 (which appeared as multi-authored publication in *GD'16*, an internation conference on graph drawing, and *Algorithmica*, a very good international journal in Theoretical Computer Science), making it a strong point of the thesis.

Furthermore, the chapter contains an NP-hardness proof of the extension problem in (arbitrary) planar undirected graphs. The reduction is nontrivial, but somewhat standard in the area; according to the statement of the author, he took part in developing the reduction.

Bipartite matchings with minimum number of crossings. Given a bipartite graph $G = (V_1, V_2, E)$ with $|V_1| = |V_2| = n$ and fixed orderings π_1, π_2 of V_1, V_2, respectively, we ask for a perfect matching in G that minimizes the number of pairs of edges from the matching that *cross* (i.e., their endpoints in π_1 are in different order than in π_2). Interestingly, the problem appears in the literature as an alternative formulation of some token-swapping problem and its complexity has been open for a while.

The author develops a technically involved NP-hardness proof of the problem, answering an open problem from the literature. The contribution of the author, together with complementary contributions of other authors,[1] appeared at *SoCG'19* (a top international conference in computational ge-

[1]The contribution of the author of the thesis has been merged with the contributions of the other authors into a single paper after the author of the thesis published his NP-

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ometry).

Universal targets for homomorphisms of edge-colored graphs. Finally, in the last chapter of the thesis, the author continues and tightens the results obtained in his master's thesis.[2] The topic here is a universal target of graph homomorphisms for edge-colored graphs: for fixed integer k and a graph class $\mathcal{F}$, we ask for a minimum-sized k-edge-colored graph H such that for every k-edge-colored graph $G \in \mathcal{F}$ there exists a (color-respecting) homomorphism from G to H. It turns out that the crucial parameters of $\mathcal{F}$ here are bounds on the acyclic chromatic number and maximum average degree of elements of $\mathcal{F}$. The author tightens the previously obtained bounds via an improved construction of the universal target.

Summary. While the overall evaluation of the thesis has been presented at the begining of the report, let me conclude by pointing out the main strength of the thesis. By providing a number of insightful, technically involved, and full of ideas contributions (with clear authorship attribution), the author undoubtfully proved his great problem-solving skills. Excellently presented, the thesis satisfies the requirements for a PhD degree with big margin and in my opinion deserves a distinction.

Yours faithfully,

Marcin Pilipczuk

hardness proof as a preprint on the arXiv preprint server.

[2]Notably, the results from master's thesis also appeared as a publication with his advisor in Journal of Combinatorial Theory series B, a top international journal in combinatorics.

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