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HEAT TRANSFER IN MHD VISCOELASTIC FLUID FLOW ON STRETCHING SHEET WITH HEAT SOURCE/SINK, VISCOUS DISSIPATION, STRESS WORK, AND RADIATION FOR THE CASE OF LARGE PRANDTL NUMBER

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Heat Transfer in MHD Viscoelastic Fluid Flow on Stretching Sheet with Heat Source/Sink, Viscous Dissipation, Stress Work, and Radiation for the Case of Large Prandtl Number

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An analysis is carried out to study the magnetohydrodynamic boundary layer flow behavior and heat transfer characteristics of a viscoelastic fluid flow over a stretching sheet with radiation and for the case of large Prandtl numbers. The basic boundary layer equations of momentum and heat transfer, which are nonlinear partial differential equations, are converted into nonlinear ordinary differential equations by means of similarity transformation. The resulting nonlinear ordinary differential equations of momentum are solved exactly. Similarly, the energy equation is transformed to a confluent hypergeometric differential equation using a new variable and Rosseland approximation for radiation. The analytic solutions for temperature profile and heat transfer characteristics are obtained in terms of Kummer's function, and their asymptotic limits for large Prandtl numbers are also obtained. The effects of magnetic field, viscoelastic parameter, viscous dissipation, heat generation/absorption, work done due to deformation, and radiation on flow and heat transfer characteristics are discussed through several graphs. To assess the validity and accuracy of the present work, heat transfer results were compared to those of previously published work of Nataraja et al. (1977). This comparison shows excellent agreement between the results.

Keywords Heat generation/absorption; MHD; Radiation; Stretching sheet; Viscoelastic fluid

Introduction

Due to the increasing importance of non-Newtonian fluids in modern technology and industries, the investigation of such fluids is desirable. Studies of laminar boundary layer flows of non-Newtonian fluids have received much attention because the power needed in stretching a sheet and the heat transfer rate in non-Newtonian fluids are quite different from those of Newtonian fluids. The boundary layer behavior on a moving continuous solid surface is an important type of flow arising in many engineering processes like polymer processing, particularly in manufacturing processes of artificial film and artificial fibers. Some applications of dilute polymer solution, such as 5.4% solution of polyisobutylene in cetane, materials manufactured by extrusion processes, and heat-treated materials traveling between a feed roll and a windup roll or conveyor belts, possess the characteristics of a moving continuous

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surface. Thus the study of boundary layer flow of a viscoelastic fluid has been the main subject of a large number of researchers in the past several decades (Sakiadis, 1961; Beard and Walters, 1964; Crane, 1970; Chen and Char, 1966; Garge and Rajagopal, 1991). None of these works take into account the heat transfer phenomenon. Although Lawrence and Rao (1992) presented work on heat transfer in the flow of viscoelastic fluid over a stretching sheet, they did not consider viscous dissipation; viscoelastic fluid flow generates heat by means of viscous dissipation. However, in the recent past, the study of non-Newtonian fluid flow had received immense interest because of the ever-increasing application of plastic films and artificial fibers in industry. Hence Chang (1989) derived another closed form solution of the non-Newtonian flow problem of Rajagopal et al. (1984). Chang et al. (1991) provided the uniform one-parameter solutions and obtained the solutions of Troy et al. (1987) and Chang (1989). Lawrence and Rao (1995) discussed the uniqueness of the solution of Rajagopal et al. (1984). Bujurke et al. (1987) have presented a work on momentum and heat transfer in the second-order viscoelastic fluid flow over a stretching sheet with internal heat generation and viscous dissipation. Anderson (1992) has obtained an exact analytical solution of magnetohydrodynamic (MHD) flow of a viscoelastic Walter's liquid B past a stretching sheet.

Thermal radiation effect might play a significant role in heat transfer analysis. Raptis and Perdikis (1998) analyzed viscoelastic fluid flow and heat transfer past a semi-infinite porous plate having constant suction in the presence of thermal radiation. Viscous dissipation, which is an important parameter in the heat transfer analysis of non-Newtonian fluid flow, is also excluded from their study. Raptis (1998) studied boundary layer flow and heat transfer of micropolar fluid past a continuously moving plate with viscous dissipation in the presence of radiation. Raptis (1999) has also investigated viscoelastic fluid flow past a semi-infinite plate taking into consideration radiation using the Rosseland approximation (Brewster, 1992) when the free stream velocity and temperature of the plate are not constant. However, his work does not consider the effects due to heat source/sink, viscous dissipation, and suction/blowing. Kumari and Nath (2004) studied radiation effect on viscous fluid flow over a solid surface immersed in a saturated porous medium.

Cortel (2005) studied flow and heat transfer through porous medium over a stretching sheet with internal heat generation/absorption and suction/blowing. Later, Cortel (2006) studied flow and heat transfers of a viscoelastic fluid over stretching sheet with viscous dissipation and work done due to deformation. Siddheshwar and Mahabaleswar (2005) studied MHD flow and heat transfer in a viscoelastic fluid over a stretching sheet with viscous dissipation, internal heat generation/absorption, and radiation, but they did not take into account the permeable stretching boundary condition. Khan (2006) studied flow and heat transfer in a viscoelastic fluid over a stretching sheet with viscous dissipation, internal heat generation/absorption, and radiation to semi-infinite porous stretching sheet, but excluded the work done due to deformation in the analysis.

Motivated by the work done by various authors, the present work concentrates on the study of MHD boundary layer viscoelastic fluid flow over a stretching sheet taking into consideration the effects due to viscous dissipation, temperature-dependent heat source/sink, work done due to deformation, and thermal radiation (which has been excluded in Nataraja et al., 1977) using the Rosseland approximation (Brewster, 1992) and also some results on asymptotic limits for larger Prandtl numbers. A series solution to the energy equation in terms of Kummer's

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and parabolic cylindrical functions are obtained, and some asymptotic cases are studied. Several closed-form analytical solutions are presented for special conditions.

Mathematical Formulation and Solution

The Coleman-Noll constitutive equation based on the postulate of gradually fading memory for an incompressible second-order fluid is

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2, \tag{1}$$

where T is the stress tensor, p is the pressure, μ, α_1, α_2 are material constants with $\alpha_1 < 0$, and A_1 and A_2 are defined as

$$A_1 = (grad\ \vec{V}) + (grad\ \vec{V})^T, \tag{2}$$

$$A_2 = \frac{dA_1}{dt} + A_1.(grad\ \vec{V}) + (grad\ \vec{V})^T.A_1, \tag{3}$$

where $\vec{V}$ denotes velocity.

The model (1) displays normal-stress differences in shear flow and is an approximation to simple fluid in the sense of retardation. This model is applicable to some dilute polymer solutions and is valid at low rates of shear. Coleman and Markovitz (Coleman and Markovitz, 1964; Markovitz and Coleman, 1964) have presented relations for α_1 and α_2 and suggested several practical methods for measuring the material constants μ, α_1, α_2.

Momentum Transfer

Two-dimensional flow of an incompressible electrically conducting viscoelastic fluid of Walter's liquid B type past a stretching sheet is considered. The flow is generated due to the stretching sheet along the x-axis by application of two equal and opposite forces. This flow obeys the rheological equation of state derived by Beard and Walters (1964); further, the flow field is exposed under the influence of a uniform transverse magnetic field. Hence, in view of Equations (1)–(3) the MHD viscoelastic boundary layer flow equation takes the form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial x \partial y}\frac{\partial u}{\partial y}\right\} - \frac{\sigma B_0^2}{\rho}u \tag{5}$$

Here the fluid is at rest and the motion is created by the stretching of the sheet with velocity u_w; u and v are the velocity components along the x and y directions, respectively, where x and y are distances along and normal to the sheet. B_0 is applied magnetic field, σ is electrical conductivity of the fluid, $k_o = -\alpha_1/\rho$ is first moment of the distribution function of relaxation times, and υ is the kinematic viscosity. Uniform magnetic field of strength B_0 is applied in the transverse direction of the sheet, and induced magnetic field is assumed to be negligible.

We assume the boundary sheet with velocity u_w and stretching sheet with stretching rate c.

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The boundary conditions for the above flow situation are

$$\begin{aligned} &u = u_w = cx \quad v = 0 \quad \text{at } y = 0 \\ &u \to 0 \quad u_y = 0 \quad \text{as } y \to \infty \end{aligned} \tag{6}$$

Introducing the stream function ψ

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

and defining

$$\bar{\psi} = \frac{\psi}{u_0 L} = \frac{\bar{x}}{\sqrt{\text{Re}}} f(\eta) \tag{8}$$

where

$$\bar{x} = \frac{x}{L}; \quad \bar{y} = \frac{y}{L}, \quad u_0 = cL, \quad \eta = \sqrt{\text{Re}}\,\bar{y}$$

L and u_0 are characteristic length and velocity respectively. The transformation coordinate η normal to the surface indicates the order of the momentum boundary layer thickness.

Substituting the values of (7) and (8) in (4)–(6), we obtain the following equation

$$f'^2 - ff'' = f''' - M_n f' - k_1\{2f'f''' - f''^2 - ff^{iv}\} \tag{9}$$

where $k_1 = k_0 c/\nu$ is the viscoelastic parameter, $Mn = \sigma B_0^2/\rho c$ is the magnetic parameter, and the corresponding boundary conditions are

$$\begin{aligned} &f'(0) = 1, \quad f(0) = 0, \quad \text{at } \eta = 0 \\ &f'(\eta) = 0, \quad \text{as } \eta \to \infty \end{aligned} \tag{10}$$

Following Rajagopal et al. (1984), the solution of Equation (9) is obtained in the form (see Troy et al., 1987; Nataraja et al., 1977; Rao, 1996)

$$f = \frac{(1 - e^{-m\eta})}{m} \qquad \text{for } 0 \le k_1 < 1 \tag{11}$$

where $m = \sqrt{\frac{1+Mn}{1-k_1}}$.

Hence, the resultant solutions of velocity components are

$$u = cxf'(\eta) \qquad v = \frac{-cL}{\sqrt{\text{Re}}} f(\eta) \tag{12}$$

Heat Transfer Analysis

The governing thermal boundary layer equation of energy with viscous dissipation, internal heat generation/absorption, and work done due to deformation and thermal radiation for temperature T is given by

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 - \rho k_0 \frac{\partial u}{\partial y}.\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + Q(T - T_\infty) - \frac{\partial q_r}{\partial y} \tag{13}$$

where K is the thermal conductivity of the fluid, μ is the coefficient of viscosity of the fluid, T_∞ is the fluid temperature far away from the sheet, and q_r is the radiative heat flux. The term Q represents the heat source when $Q > 0$ and the sink when $Q < 0$. The fifth term on the right-hand side of Equation (13) represents the radiation effect, which has been excluded in Khan (2006), ρ is density, $\upsilon = \mu/\rho$ kinematic viscosity, and c_p is specific heat at constant pressure.

Using the Rosseland approximation for radiation

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{14}$$

Here, σ is the Stefan-Boltzmann constant and k^* is the absorption coefficient. Further, we assume that the temperature difference within the flow is such that T^4 is expressed in a Taylor series form. Hence, expanding T^4 about T_∞ and neglecting higher order terms, we obtain

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4 \tag{15}$$

Therefore, Equation (13) is simplified to the form

$$\begin{aligned} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial u}\right)^2 - \frac{k_0}{c_p}\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \\ + \frac{Q}{\rho c_p}(T - T_\infty) + \frac{1}{3\rho c_p}\frac{16\sigma T_\infty^3}{k^*}\frac{\partial^2 T}{\partial y^2} \end{aligned} \tag{16}$$

The relevant boundary conditions are

$$\begin{aligned} &T = T_w \equiv T_\infty + A\left(\frac{x}{L}\right)^2 \quad \text{at } y = 0 \\ &T \to T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{17}$$

We define nondimensional temperature variable as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{18}$$

Substituting Equation (18) in (16) and also considering u and v from Equation (12), Equation (16) takes the form

$$\frac{(1 + Nr)}{\Pr}\theta'' + f\theta' - (2f' - \beta)\theta + Ec\{(f'')^2 - k_1 f''(f'f'' - ff''''')\} = 0 \tag{19}$$

where

$Ec = \frac{u_0^2}{c_p A}$ is the Eckert number
$\beta = \frac{Q}{\rho c_p}$ is the heat source/sink parameter
$\Pr = \frac{\mu c_p}{k}$ is the Prandtl number
$\mathrm{Re} = \frac{L u_0}{\nu}$ is the Reynolds number
$k_1 = \frac{k_0 c}{\nu}$ is the viscoelastic parameter
$Nr = \frac{16\sigma\, T_\infty^3}{3k^* k_\infty}$ is the radiation parameter

Here we make a remark that the equation for the stretching sheet problem involving a second-order liquid can be obtained from Equations (9) and (19) by replacing k_1 with $-k_1$.

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Similarly, boundary conditions (17) take the form

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{20}$$

Introducing a new independent variable

$$\xi = -re^{-m\eta}$$

where

$$r = \frac{\Pr}{m^2} \tag{21}$$

Substituting (22) in Equation (23) and considering the value of f, we obtain

$$(1+Nr)\xi\theta''(\xi) + [(1+Nr) - r - \xi]\theta'(\xi) + \left(2 + \frac{r}{\xi}\right)\theta(\xi) = -\frac{Ec}{r}(1+Nr)\xi \tag{22}$$

The corresponding boundary conditions are

$$\theta(-r) = 1 \quad \theta(0) = 0 \tag{23}$$

Equations (22) and (23) constitute a nonhomogeneous boundary value problem. Denoting the solution of the homogeneous part of Equation (22) by θ_c and the particular integral by θ_p the solution of Equation (22), subject to the boundary conditions (23) is

$$\theta(\xi) = \theta_c(\xi) + \theta_p(\xi) \tag{24}$$

where

$$\theta_c(\xi) = M\left[p-2, b+1; \frac{\xi}{(1+Nr)}\right]$$

where

$$p = \frac{a+b}{2}, \quad a = \frac{r}{(1+Nr)}, \quad b = \left\{a^2 - \frac{4\beta r}{(1+Nr)}\right\}^{1/2}, \quad r = \frac{\Pr}{m^2} \tag{25}$$

The relation $\frac{4\beta r}{(1+Nr)} \leq a_0^2$ must be satisfied in order to have real values of b where M is Kummer's function (Abramowitz and Stegun, 1972) and is defined by

$$M[a_0, b_0, z] = \sum_{n=1}^{\infty} \frac{(a_0)_n}{(b_0)_n} \frac{z^n}{n!}$$

which satisfies the differential equation $z\frac{d^2M}{dz^2} + (b_0 - z)\frac{dM}{dz} - a_0 M = 0$ and the Pochhammer symbol:

$$(a_0)_n = a_0 + (a_0+1)(a_0+2)\ldots\ldots\ldots\ldots(a_0+n-1)$$
$$(b_0)_n = b_0 + (b_0+1)(b_0+2)\ldots\ldots\ldots\ldots(b_0+n-1)$$

For large values of z, $M(a_0, b_0, z) = \frac{\Gamma(b_0)}{\Gamma(a_0)} e^z z^{a_0 - b_0}\left\{1 - o\left(\frac{1}{z}\right)\right\}$

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The particular integral of Equation (22) is

$$\theta_p(\xi) = \left\{ \frac{Ec(1+Mn)}{2-\beta-\frac{4(1+Nr)}{r}} \right\} \left(\frac{\xi}{r} \right)^2 \tag{26}$$

Now making use of the boundary conditions of Equation (23) and changing the variable ξ to η we obtain the solution in the following form of Kummer's function:

$$\theta(\eta) = \frac{(1-c_1)}{M\left[\frac{a+b}{2}-2, b+1; \frac{-r}{1+Nr}\right]} \; e^{-m\frac{(a+b)}{2}\eta} M\left[\frac{a+b}{2}-2, b+1; \frac{-re^{-m\eta}}{(1+Nr)}\right] + c_2 e^{-2m\eta}$$

where

$$c_1 = \frac{-rEc(1+Mn)}{4(1+Nr)-2r+\beta r} \tag{27}$$

Careful examination of the solution for θ given in Equation (26) of Khan (2006) and Equation (21) reveals a change in the form of the parameter Ec(1-k$_1$), which turns out to be $\frac{Ec(1+Mn)}{(1+Nr)}$ due to the presence of the magnetic term in the momentum equation and radiation term in the thermal boundary layer equation.

The nondimensional surface velocity and temperature gradient obtained from Equations (12) and (27) are

$$f'(0) = -m \tag{28}$$

$$\theta'(0) = \left\{ 1 + \frac{r\ Ec(1+Mn)}{4(1+Nr)-2r+\beta r} \right\} \left\{ \frac{a+b-4}{2(b+1)} \frac{mr}{(1+Nr)} \frac{M\left[\frac{(a+b)}{2}-1, b+2; \frac{-r}{1+Nr}\right]}{M\left[\frac{(a+b)}{2}-2, b+1; \frac{-r}{1+Nr}\right]} - mp \right\} + \frac{2r\ m\ Ec(1+Nr)}{4(1+Nr)-2r+\beta r} \tag{29}$$

Asymptotic Results for Large Prandtl Numbers

The asymptotic results for the temperature variable $\theta(\eta)$ for large Prandtl numbers are obtained by Nataraja et al. (1977) as

$$\theta = \frac{Ec}{\left(2-\frac{4}{r}-\alpha\right)} e^{-2m\eta} + \Theta$$

In our case we define

$$\theta = \frac{Ec(1+Mn)}{2-\frac{4(1+Nr)}{r}-\beta} e^{-2m\eta} + \Theta \tag{30}$$

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The boundary layer equation for energy (20) and the corresponding boundary layer conditions (21) are transformed to the form

$$\frac{(1+Nr)}{\Pr}\Theta'' + f\Theta' - (2f' - \beta)\Theta = 0 \tag{31}$$

$$\begin{aligned} &\Theta = 1 - \frac{Ec(1+Mn)}{\left(2 - \frac{4(1+Nr)}{r} - \beta\right)} \quad \text{at } \eta = 0 \\ &\Theta \to 0 \quad \text{as } \eta \to \infty \end{aligned} \tag{32}$$

Since the thermal boundary layer thickness is of the order of $1/\sqrt{\mathrm{PrRe}}$, the transformation coordinate η in equation (31) is further modified to $\tau = \sqrt{\Pr}\eta$.

Using this relation, the stream function $f(\eta)$, as well as its derivatives, and the thermal boundary layer Equation (31) takes the form

$$\sqrt{\frac{\Pr}{(1+Nr)}}f(\eta) \simeq \tau \tag{33}$$

$$f'(\eta) \simeq 1 \tag{34}$$

$$\ddot{\Theta} + \tau\dot{\Theta} + (\beta - 2) = 0 \tag{35}$$

The boundary conditions (32) take the form

$$\begin{aligned} &\Theta = 1 - \frac{Ec(1+Mn)}{\left(2 - \frac{4(1+Nr)}{r} - \beta\right)} \quad \text{at } \tau = 0 \\ &\Theta \to 0 \quad \text{as } \tau \to \infty \end{aligned} \tag{36}$$

Here dots denote differentiation with respect to τ. The approximations (33) and (34) for the velocity field are valid only inside the thermal boundary layer. The solution of Equations (35) and (36) is

$$\begin{aligned} \Theta(\tau) = &\left\{1 - \frac{Ec(1+Mn)}{2 - \frac{4(1+Nr)}{r}}\right\} \\ &\times \left[M\left(\frac{3-\beta}{2}, \frac{1}{2}, \frac{\tau^2}{2}\right) - \sqrt{2}\frac{\Gamma\left(\frac{4-\beta}{2}\right)}{\Gamma\left(\frac{3-\beta}{2}\right)}\tau M\left(\frac{4-\beta}{2}, \frac{3}{2}, \frac{\tau^2}{2}\right)\right]e^{-\frac{\tau^2}{2}} \end{aligned} \tag{37}$$

The nondimensional surface temperature gradient obtained from Equations (30) and (37) is

$$\Theta'(0) = \frac{-2\,m\,Ec\,(1+Mn)}{2 - \frac{4(1+Nr)}{r} - \beta} - \sqrt{\frac{2\,\Pr}{(1+Nr)}}\frac{\Gamma\left(\frac{4-\beta}{2}\right)}{\Gamma\left(\frac{3-\beta}{2}\right)}\left\{1 - \frac{Ec(1+Mn)}{2 - \frac{4(1+Nr)}{r} - \beta}\right\} \tag{38}$$

The viscoelasticity has no effect on heat transfer rate for Ec $=0$ according to the asymptotic formula (38).

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Results and Discussion

We have considered the boundary layer flow and heat transfer in viscoelastic non-Newtonian fluid flow over a stretching sheet in the presence of a transverse magnetic field and thermal radiation. Linear stretching of the boundary with viscoelasticity, magnetic parameter, Prandtl number, Eckert number, and radiation parameter are taken into consideration in this study. The basic boundary layer partial differential equations, which are highly nonlinear, have been converted into a set of nonlinear ordinary differential equations by applying similarity transformations, and their analytical solutions are obtained. Also, some asymptotic results for large Prandtl numbers are obtained in terms of the special case of Kummer's function. Since the present problem is an extension of Nataraja et al. (1977) to the case of a flow region with magnetic effect and emitting thermal radiation, we study the analysis of effects of thermal radiation and asymptotic results for large Prandtl numbers and various heat transfer characteristics in different physical situations of viscous dissipation, work done due to deformation, viscoelasticity, and magnetic parameter.

Figures 1 and 2 are plotted for dimensionless velocity gradient $f'(\eta)$ with similarity variable η for different values of viscoelastic parameter k_1 and in the absence and presence of magnetic parameter respectively. These graphs reveal that as we increase viscoelastic parameter k_1 there is a decrease in velocity. As the velocity, both transverse as well as horizontal, is a decreasing function of η, where it is an exponential function with negative argument. As in our case m is a function of viscoelastic parameter k_1 and magnetic parameter Mn that gives the slope of the exponentially decreasing velocity profiles. Thus, the effect of viscoelasticity is to increase the momentum boundary layer thickness.

The effects of magnetic parameter on velocity are shown in Figures 3 and 4 in the absence and presence of viscoelastic parameter k_1 respectively. These graphs reveal that the combined effect of magnetic parameter and viscoelastic parameter is to decrease horizontal velocity in the boundary layer. Further, it is observed that

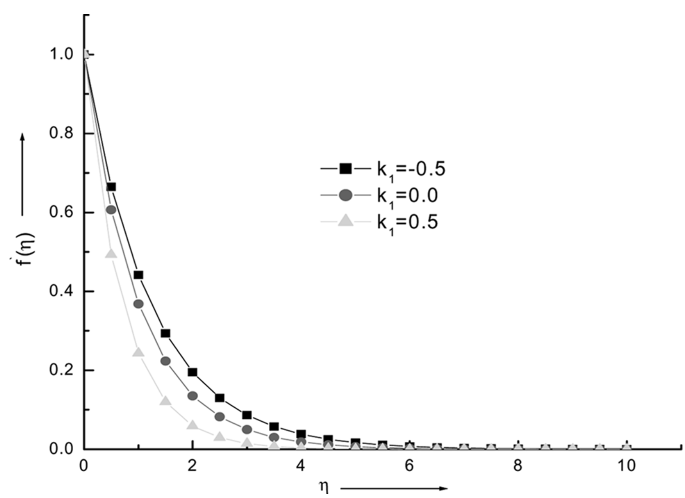

Figure 1. Plot of horizontal velocity component $f'(\eta)$ *vs* η with $\mathrm{Mn} = 0.0$ for different values of k_1.

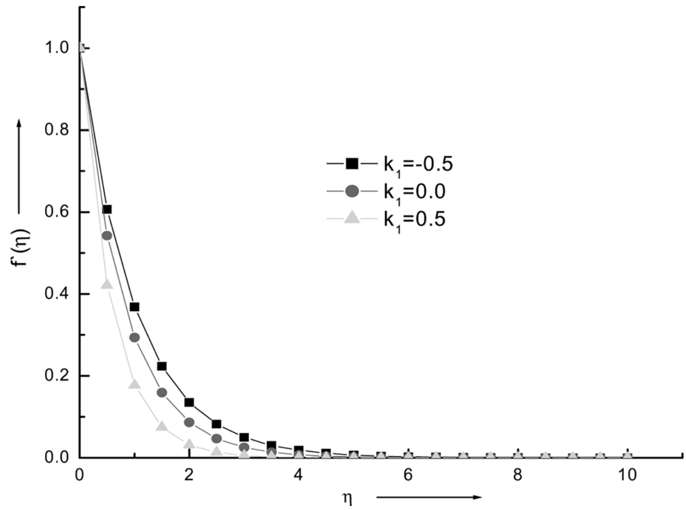

Figure 2. Plot of horizontal velocity component $f'(\eta)$ *vs* η with $\mathrm{Mn} = 0.5$ for different values of k_1.

the magnitude of dimensionless surface velocity gradient $f''(0) = m$ increases with the magnetic parameter Mn but decreases with the viscoelastic parameter k_1. This implies that the role played by the viscoelastic parameter k_1 is to reduce the skin friction at the sheet, and the influence of magnetic field is to increase power needed to stretch the sheet. The introduction of a transverse magnetic field in a direction normal to the flow has a tendency to create a drag due to Lorentz force, which tends to resist the flow, and hence the horizontal velocity in the boundary layer decreases. This result is true even in the presence of heat source/sink parameter β.

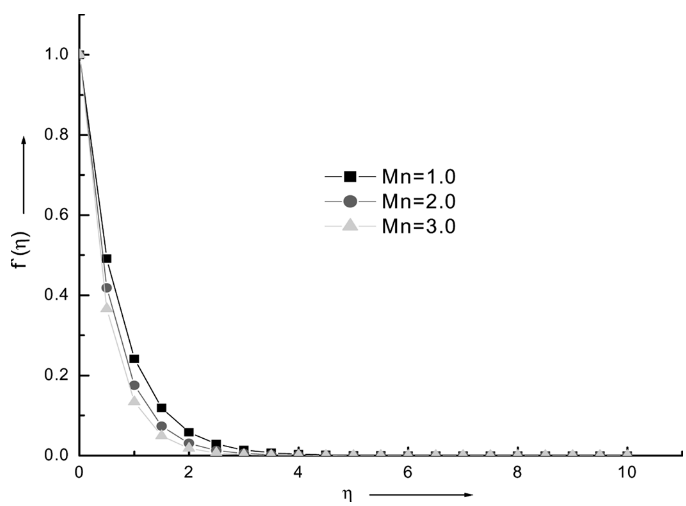

Figure 3. Plot of horizontal velocity component $f'(\eta)$ *vs* η with $\mathrm{k}_1 = 0.0$ for different values of Mn.

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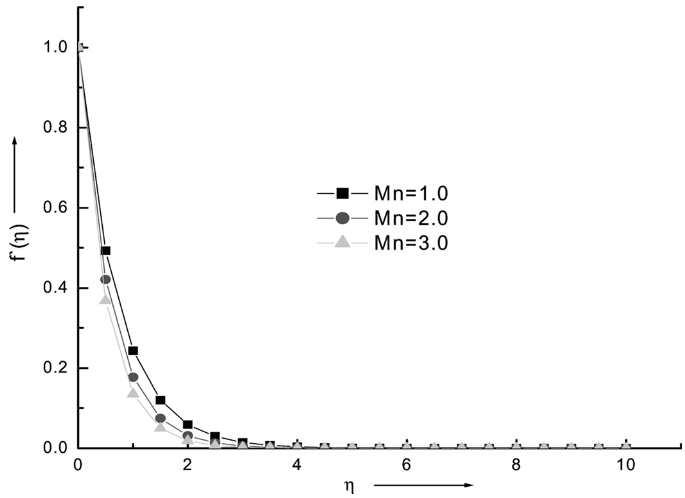

Figure 4. Plot of horizontal velocity component $f'(\eta)$ *vs* η with $k_1 = 0.01$ for different values of Mn.

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Figure 5 is drawn for temperature profiles for various values of magnetic parameter Mn keeping other parameters fixed. We notice that the effect of magnetic parameter is to increase the temperature in the boundary layer; it is obvious that the Lorentz force has the tendency to increase the temperature profile, and the effect on the flow and thermal fields becomes more so as the strength of the magnetic field increases. The effect of magnetic parameter is to increase the wall temperature gradient; this is quite consistent with the physical situation as the application of magnetic field introduces additional skin frictional heating, due to stress work that results in higher temperature on the wall with the increase of thermal boundary layer thickness.

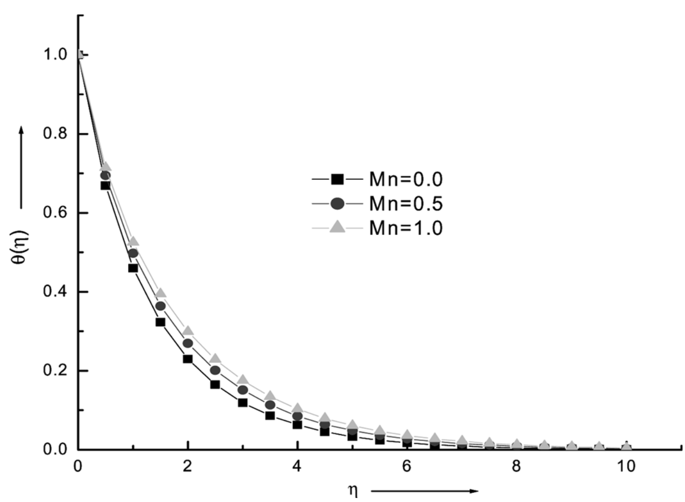

Figure 5. Temperature profiles for various values of magnetic parameter Mn with $k - 1 = 0.1$, $Pr = 1.0$, $Ec = 0.5$, $\beta = -0.2$, $Nr = 1.0$.

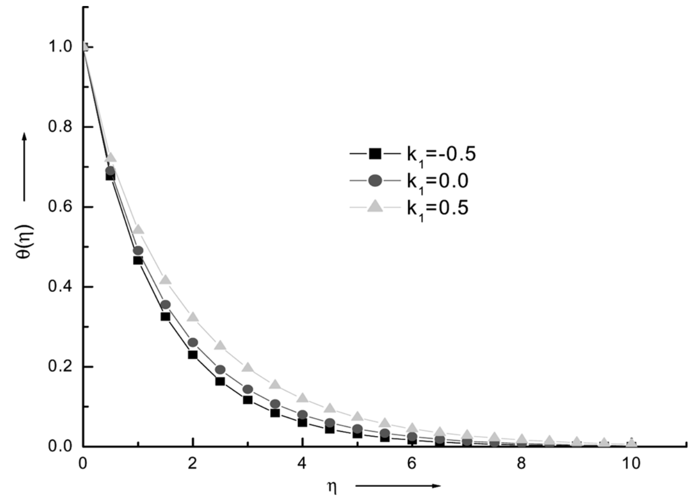

Figure 6. Temperature profiles for various values of viscoelastic parameter k_1 with Mn = 0.5, Pr = 1.0, Ec = 0.5, $\beta = -0.2$, Nr = 1.0.

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Figure 6 is plotted for temperature profiles for different values of viscoelastic parameter k_1. In this case temperature profiles follow patterns similar to the case in Figure 5, however, comparison of Figures 6 and 5 reveals that the effect of viscoelastic parameter k_1 is to increase the temperature throughout the boundary layer except at the wall where it attains unity. This is due to the fact that thermal boundary layer thickness increases in the case of increasing viscoelastic parameter, but the increase in thermal boundary layer thickness is not appreciable since the viscoelastic parameter is usually not large and it can be taken as a small perturbation parameter, as done by Beard and Walters (1964) and Rajeswari and Rathna (1962).

The effect of Prandtl number Pr on heat transfer may be analyzed from Figure 7. This graph reveals that the increase of Prandtl number results in the decrease of temperature distribution at a particular point of the flow region. This is because there would be a decrease of thermal boundary layer thickness with the increase in the values of Prandtl number. The increase of Prandtl number means slow rate of thermal diffusion. It is observed that the wall temperature distribution is unity on the wall for all values of Pr, Ec, and k_1. The effect of increasing values of Prandtl number is to decrease temperature at a point in the flow field, as there would be a thinning of the thermal boundary layer as a result of reduced thermal conductivity.

Figure 8 demonstrates the dimensionless temperature profile $\theta'(\eta)$ for various values of Eckert number Ec, and the effect of increasing values of Eckert number Ec is to increase wall temperature due to the heat addition by means of frictional heating. If the Eckert number is large enough the heat transfer may reverse direction.

The effect of internal heat generation is especially pronounced for high values of β (Figure 9). The fluid temperature is greater when there is heat generation. This is logical because internal heat energy generation results in an increase of heat transfer close to the plate and this will induce more flow along the sheet. The temperature distribution is lower throughout the field for negative values of β, and it is higher

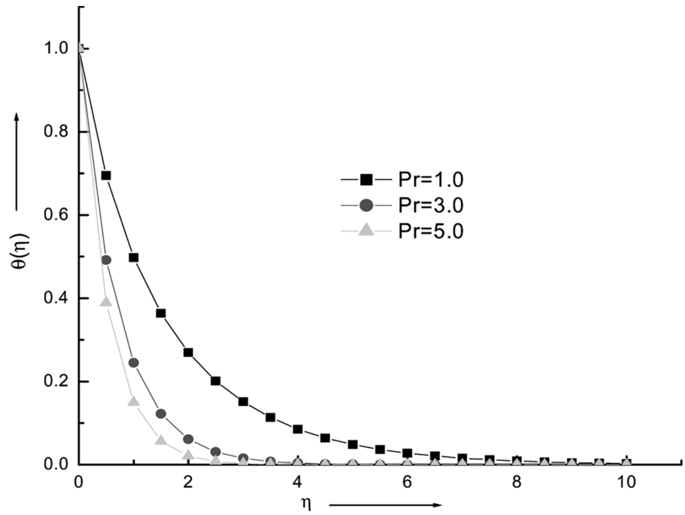

Figure 7. Temperature profiles for various values of Prandtl number Pr with Mn = 0.5, $k_1 = 0.1$, Ec = 0.5, $\beta = -0.2$, Nr = 1.0.

for positive values of β. Physically $\beta > 0$ implies $T_w > T_\infty$, and there will be a supply of heat to the flow region from the wall. Similarly, $\beta < 0$ implies $T_w > T_\infty$, and there will be heat transfer from flow region to the wall.

Figure 10 depicts the influence of radiation parameter Nr on temperature profile. It is observed that increasing radiation parameter Nr is to increase temperature throughout the boundary layer. The increase of radiation parameter Nr implies the release of heat energy from the flow region by means of radiation; this can also be explained by the fact that the effect of radiation is to increase

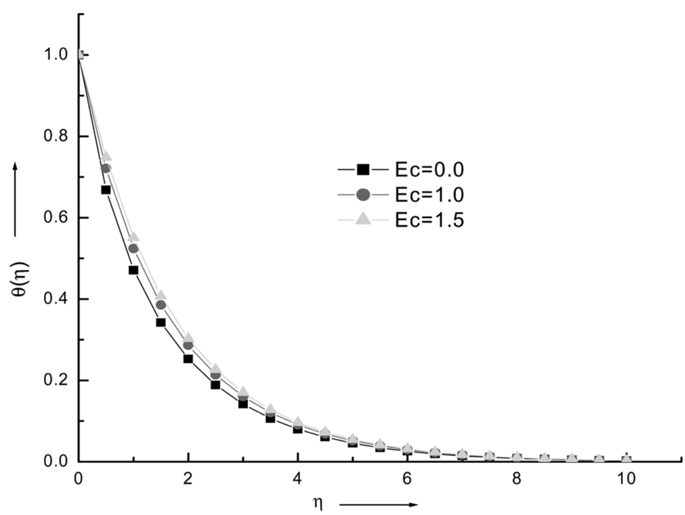

Figure 8. Temperature profiles for various values of Eckert number Ec with Mn = 0.5, $k_1 = 0.1$, Pr = 1.0, $\beta = -0.2$, Nr = 1.0.

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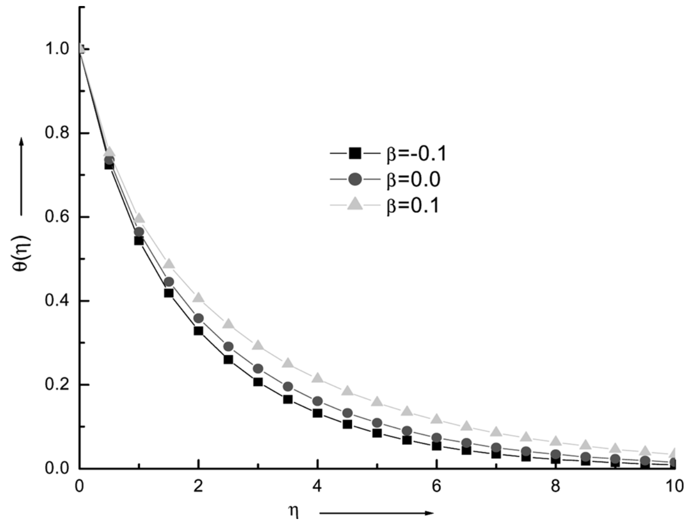

Figure 9. Temperature profiles for various values of heat source/sink parameter β with $Mn = 0.5$, $k_1 = 0.1$, $Pr = 1.0$, $Ec = 0.5$, $Nr = 1.0$.

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the rate of energy transport to the fluid and accordingly increase the fluid temperature.

Numerical values of wall temperature gradient $\theta'(0)$ for different values of Pr with exact solution and asymptotic solution are recorded in Table I for large Prandtl numbers, comparing the results of Nataraja et al. (1977) and the present study. The table reveals that the values of wall temperature gradient with $Nr = 0$ and $Mn = 0$ are approximately similar to the results of Nataraja et al. (1977). We notice that wall temperature gradient $\theta'(0)$ decreases by increasing the values of the Prandtl number.

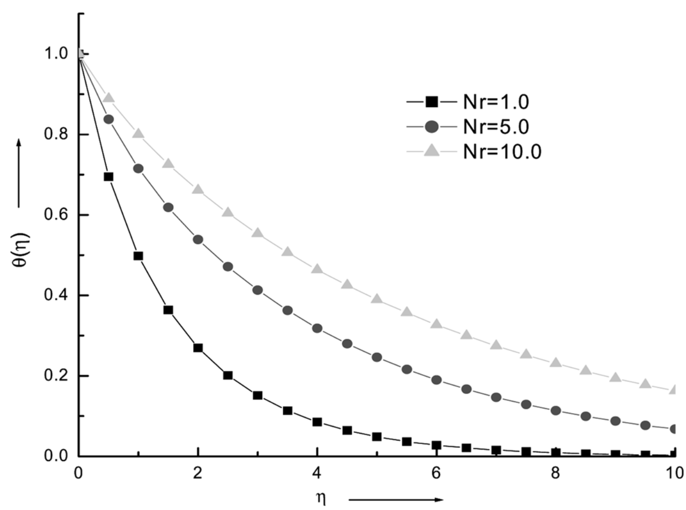

Figure 10. Temperature profiles for various values of radiation parameter with $Mn = 0.5$, $k_1 = 0.1$, $Pr = 1.0$, $Ec = 0.5$, $\beta = -0.2$.

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Table I. Comparison of nondimensional temperature distribution between previously published work and present results with $Mn = 0 = Nr$

	$-\theta'(0)$ Equation (29)				Asymptotic results $-\theta'(0)$ Equation (38)			
	Nataraja et al. (1977)	Present study	Nataraja et al. (1977)	Present study	Nataraja et al. (1977)	Present study	Nataraja et al. (1977)	Present study
Pr	$k_1 = 0.0$	$k_1 = 0.2$	$k_1 = 0.0$	$k_1 = 0.2$	$k_1 = 0.0$	$k_1 = 0.2$	$k_1 = 0.0$	$k_1 = 0.2$
				$Ec = 0.0$				
1	1.3333	1.33333	1.3000	1.30001	1.5958	1.59577	1.5958	1.59577
5	3.3165	3.31648	3.2857	3.28566	3.5682	3.56825	3.5682	3.56825
10	4.7969	4.79687	4.7667	4.76669	5.0463	5.04627	5.0463	5.04627
15	5.9320	5.93201	5.9020	5.90210	6.1804	6.18039	6.1804	6.18039
100	15.712	15.7120	15.683	15.6827	15.958	15.9577	5.9580	5.95770
400	31.669	31.6705	31.641	31.6414	31.915	31.9154	31.915	31.9154
				$Ec = 0.5$				
1	1.1667	1.16667	1.1440	1.1440	1.4947	1.49471	1.4891	1.48905
5	2.7679	2.76795	2.7609	2.76086	2.9148	2.91481	2.9022	2.90216
10	3.9229	3.92285	3.9231	3.92315	4.0943	4.09431	4.1095	4.10953
15	4.7978	4.79778	4.8023	4.80229	4.9745	4.97451	4.9971	4.99709
100	12.214	12.214	12.235	12.2349	12.397	12.3971	12.439	12.4393
400	24.215	24.2156	24.244	24.2439	24.399	24.3990	24.449	24.4489

Table II. Values of heat transfer coefficient for different values of nondimensional physical parameter in the case of large Prandtl numbers

Pr	k_1	Mn	Ec	Nr	β	$-\theta'(0)$ Equation (29)	$-\theta'(0)$ Equation (38)
1	0.0	0.5	0.25	5.0	−0.1	0.37010	0.64419
10						1.63421	2.01208
100						5.50655	5.73641
1000						17.3889	17.6261
1	0.2					0.35228	0.64580
10						1.60757	1.99971
100						5.49925	5.75286
1000						17.3990	17.6641
1	0.5					0.31631	0.64892
10						1.53701	1.99923
100						5.47278	5.77655
1000						17.4191	17.7539
1	0.2	0.0	0.25	5.0	−0.1	0.39298	0.64985
10						1.71013	2.06165
100						5.79782	6.02783
1000						18.4974	18.7300
1		0.25				0.37055	0.64773
10						1.65651	2.02372
100						5.64456	5.88951
1000						17.9422	18.1933
1		0.4				0.35918	0.35918
10						1.62666	1.62666
100						5.55653	5.55653
1000						17.6150	17.6150
1	0.2	0.5	0.0	5.0	−0.1	0.37296	0.66394
10						1.76042	2.09956
100						6.31120	6.63940
1000						20.6714	20.9956
1			0.3			0.34815	0.64218
10						1.57700	1.97974
100						5.33686	5.57555
1000						16.7445	16.9977
1			0.5			0.33161	0.62767
10						1.45473	1.89986
100						4.68730	4.86632
1000						14.1266	14.3325
1	0.2	0.5	0.25	0.0	−0.1	1.18507	1.54907
10						4.23675	4.47501
100						13.4940	13.7586
1000						42.4375	42.7028

(*Continued*)

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Table 2. Continued

Pr	k_1	Mn	Ec	Nr	β	$-\theta'(0)$ Equation (29)	Equation (38)
1				1.0		0.760077	1.1038
10						2.95047	3.07534
100						9.55184	9.81494
1000						30.0522	30.3175
1				10.0		0.22635	0.47987
10						1.11726	1.47819
100						4.03365	4.26679
1000						12.8686	13.1331
1	0.2	0.5	0.25	0.5	−0.01	0.39064	0.63453
10						1.66736	1.96252
100						5.65188	5.61654
1000						17.8804	17.2246
1					0.0	0.29201	0.63327
10						1.54336	1.95836
100						5.34190	5.60113
1000						16.9029	17.1749
1					0.2	0.28092	0.63200
10						1.53663	1.95418
100						5.32589	5.58567
1000						16.8524	17.1252

The effect of viscous dissipation Ec is to reduce the wall temperature gradient $\theta'(0)$. Hence by increasing the values of viscoelastic parameter k_1 and Eckert number Ec the rate of heat transfer can be controlled considerably. Significant increase of Eckert number reverses the direction of heat transfer to the stretching sheet. The heat source/sink parameter β does not affect the general behavior of heat transfer phenomenon, as observed by Vajravelu and Rollins (1999) and Nataraja et al. (1977), hence β is taken as zero in Table I.

The values of wall temperature gradient $\theta'(\eta)$ in the case of analytical solution and asymptotic solution are recorded in Table II for large Prandtl numbers. From this table we notice that the effect of increasing values of Prandtl number is to lower the wall temperature, whereas the effect of increasing values of viscoelastic parameter k_1 and Eckert number is to enhance the wall temperature due to heat addition by means of frictional heating. The asymptotic results are in good agreement when the Prandtl number is greater than 10. The maximum relative errors are no more than 5%, and the least for the parameters we considered. The larger the Prandtl number, the less the relative error.

We observed that when Eckert number is small, the value of $|\theta'(0)|$ is negative and decreases more negatively with Pr, implying that the heat transfer rate from wall to the fluid increases with Pr. This situation is more magnified when the Prandtl number is large because the thermal boundary layer thickness will be compressed $\left(\tau \frac{1}{\sqrt{\mathrm{PrRe}}}\right)$. Comparison of temperature gradient in Table II reveals that

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there would be a higher temperature for the situation when $\mathrm{Ec} = 0.5$ than for the case when $\mathrm{Ec} = 0.0$. This is in conformity with the fact that energy is stored in the fluid region as a consequence of dissipation due to viscosity and elastic deformation.

Increasing the value of magnetic parameter Mn also increases wall temperature in the boundary layer. The simple formula (Equation (38)) can be directly used in practical applications for large Prandtl number. The wall temperature increases with the increase in the thermal radiation parameter, which can be observed in Table II. As the radiation parameter Nr increases, the fluid temperature as well as the dimensionless rate of heat transfer from the plate to the fluid increases.

Conclusion

In the present study an investigation into the effect of magnetic field on the viscoelastic fluid flow over a stretching sheet is considered. Linear stretching of the wall, dissipation due to viscosity and elastic deformation, temperature-dependent heat source/sink, and thermal radiation are taken into consideration. Analytical solutions of the governing boundary layer partial differential equations, which are highly nonlinear and in uncoupled form, have been obtained in terms of confluent hypergeometric function (Kummer function) and its special forms, and asymptotic results on large Prandtl numbers are also examined. Different analytical expressions are obtained for nondimensional velocity and temperature for different physical situations. The specific conclusions derived from this study are mentioned below.

1. The combined effect of magnetic field and viscoelastic parameter is to decrease horizontal velocity in the boundary layer. This result is consistent with the fact that the introduction of tensile stress due to viscoelasticity and magnetic field is to cause transverse contraction of the boundary layer, hence velocity decreases.
2. The combined effect of viscous dissipation energy and deformation energy due to viscoelastic property of the fluid is to increase the wall temperature.
3. There would be a higher temperature at a particular point in the flow region in the presence of magnetic field when the Prandtl number is low.
4. An increase of Prandtl number caused reduction in the thickness of the thermal boundary layer and hence the asymptotic results of nondimensional temperature gradient tend to be closer to the exact values given by Equation (38) as Pr increases.
5. The effect of viscoelastic parameter and the Eckert number is to increase the temperature distribution in the flow region.
6. The effect of heat source in the boundary layer generates energy, which causes the temperature to increase, while the presence of heat absorption effects cause reduction in the fluid temperature.
7. The increase of thermal radiation reduces a significant increase in the thickness of the thermal boundary layer of the fluid, resulting in increasing temperature. As the radiation parameter increases the fluid temperature, the dimensionless rate of heat transfer from the sheet to the fluid, increases (see Datti et al., 2004; Abel et al., 2005; Abo-Eldahab, 2004; Kumari and Nath, 2004; Mbeledogu and Ogulu, 2007).
8. The limiting cases of our results ($\mathrm{Mn} = 0$, $\mathrm{Nr} = 0$) are in excellent agreement with the results Nataraja et al. (1977).

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Nomenclature

B_0	magnetic field strength
c	stretching rate
c_p	specific heat at constant pressure
Ec	Eckert number
K	thermal conductivity
k_1	viscoelastic parameter
k_0	coefficient of viscoelasticity
L	characteristic length
M	confluent hypergeometric function
Mn	magnetic parameter
m	wall heat flux parameter
N_r	radiation parameter
Pr	Prandtl number
Q	heat generation rate
q_r	radioactive heat flux
Re	Reynolds number
T_w	wall temperature
T_∞	temperature far away from the plate
u, v	velocity components along x and y directions
x, y	distance along and perpendicular to the surface

Greek letters

α	real root of equation
β	heat source/sink parameter (volumetric coefficient of thermal expansion)
η	dimensionless space variable
Θ	dimensionless temperature
θ	dimensionless temperature
μ	coefficient of viscosity
ν	kinematic viscosity
ρ	density
σ	electric conductivity
ψ	dimensionless stream function

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