HW4 · 20224314 강현우

5.1.6. let $z = \frac{X-\mu}{\sigma}$

since $\begin{cases} P(Z \le -0.25335) = 0.4 = P(X \le 0) \\ P(Z \le 0.12566) = 0.55 = P(X \le 10) \end{cases}$

$\Rightarrow \frac{0-\mu}{\sigma} \approx -0.25335\ldots, \quad \frac{10-\mu}{\sigma} \approx 0.12566$

$\frac{10}{\sigma} \approx 0.379$, $\begin{cases} \sigma \approx 26.385 \\ \mu \approx 6.684 \end{cases}$

$\frac{\mu}{\sigma} \approx 0.25335$

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5.1.8 find a, b s.t. $P(Z \le a) = 0.25$, $P(Z \le b) = 0.75$

by using normal distribution calculator, we can find

$a = -0.67449$, $b = 0.67449$

$\Rightarrow$ inter quatile range : $[a, b] = [-0.67449, 0.67449]$

In the case of $N(\mu, \sigma^2)$, $\begin{cases} P(X \le \text{lower quartile}) = P(Z \le -0.67449) \\ P(X \le \text{upperquartile}) = P(Z \le 0.67449) \end{cases}$

& since $Z = \frac{X-\mu}{\sigma}$, $\begin{cases} \frac{\text{lower quartile of } X - \mu}{\sigma} = a \\ \frac{\text{upper quartile of } X - \mu}{\sigma} = b \end{cases}$

$\Rightarrow$ inter quatile range of $x$ : $[-0.67449 \times \sigma + \mu, +0.67449 \times \sigma + \mu]$

5.1.16.

let $p_l = p(\text{weight of brick} < 1300)$

$p_m = p(1300 \leq \text{weight of brick} \underline{<} 1330)$

$p_h = p(\text{weight of brick} > 1330)$

then $p$ -"10 bricks case"- $= {}_{10}C_3 \cdot p_l^3 \cdot {}_7C_4 \cdot p_m^4 \cdot {}_3C_3 \cdot p_h^3$

so, we should calculate $p_l$, $p_m$, $p_h$.

by normal distribution calculator.

$p_l = p(\text{weight of brick} < 1300) = p(z < \frac{-20}{15}) = 0.091$

$p_m = p(1300 \leq \text{weight of brick} \underline{<} 1330) = p(-\frac{4}{3} \leq z < \frac{2}{3}) = 0.656$

$p_h = p(\text{weight of brick} > 1330) = p(z > \frac{2}{3}) = 0.252$

$\therefore p \approx 0.0569$.

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5.2.12. (a) let $X_i$ is $i$th student's question time. for $i \in \{1, 2, \cdots, 5\}$

since $X_i$ is i.i.d, $Y_i = \frac{X_i - 8}{2}$ is chi-square distribution.

$Y \sim Gam(\frac{5}{2}, \frac{1}{2})$ (s.t. $Y = \sum_{i=1}^{5} Y_i$ & $X = \sum_{i=1}^{5} X_i$)

then $p(X \geq 45) = p(Y \geq \frac{5}{2}) = 0.07524$

(b) let $x$ = time of finishing talking to third student

$y$ = time of starting headache. of TA.

since $x$ & $y$ is independence, $f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2} \cdot \exp(-\frac{1}{2} \cdot (\frac{(x-\mu_1)^2}{\sigma_1^2} + \frac{(y-\mu_2)^2}{\sigma_2^2}))$

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(b) $X = \sum_{i=1}^{3} X_i$, $E(X) = \sum_{i=1}^{3} E(X_i) = 24 = \mu_1$

$Var(X) = \sum_{i=1}^{3} Var(X_i) = 12$. $\therefore \sigma_1 = 2\sqrt{3}$

since $\mu_2 = 28$, $\sigma_2 = 5$,

$$\therefore f(x,y) = \frac{1}{2\pi \cdot 2\sqrt{3} \cdot 5} \cdot \exp\left(-\frac{1}{2}\left(\frac{(x-24)^2}{12} + \frac{(y-28)^2}{25}\right)\right)$$

since $x, y$ is time, change them as $t$.

$$p = \int_0^{\infty} \frac{1}{20\sqrt{3}\pi} \exp\left(-\frac{1}{2}\left(\frac{(t-24)^2}{12} + \frac{(t-28)^2}{25}\right)\right)$$

$\approx 0.02637$

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5.2.14. let $x$ = finish time of worker 1's fourth task

$y$ = finish time of worker 2's third task.

since $x, y$ is independence $f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2} \cdot \exp\left(-\frac{1}{2} \cdot \left(\frac{(x-\mu_1)^2}{\sigma_1^2} + \frac{(y-\mu_2)^2}{\sigma_2^2}\right)\right)$

Let's find $\sigma_1, \sigma_2, \mu_1, \mu_2$.

since $\begin{cases} \mu_1 = 4 \cdot 13 = 52, \ \sigma_1 = 0.5\sqrt{4} = 1 \\ \mu_2 = 17 \cdot 3 = 51, \ \sigma_2 = 0.6 \cdot \sqrt{3} \approx 1.039 \end{cases}$

$$f(x,y) = \frac{1}{2\pi \cdot 1 \cdot 1.039} \exp\left(-\frac{1}{2}\left(\frac{(x-52)^2}{1^2} + \frac{(y-51)^2}{(1.039)^2}\right)\right) \cdot dt$$

$\therefore p = \int_0^{\infty} f(t,t) \cdot dt$ old.

5.3.2. · 각각의 Binomial로부터 $\mu, \sigma$을 구하여 normal distribution으로 근사하여 계산한다.

이때 $Y \sim N(\mu, \sigma) = N(np, \sqrt{npq})$ 로 여기겠다.

(a) $P(X \geq 7) \approx P(Y \geq 7) \sim N(3, \sqrt{2.1})$

$\approx 0.0029$

(b) $P(9 \leq X \leq 12) \approx P(9 \leq Y < 12) \sim N(10.5, \frac{\sqrt{21}}{2})$

$\approx 0.74368 - 0.25632$

$\approx 0.48736$

(c) $P(X \leq 3) \approx P(Y \leq 3) \sim N(3.5, \frac{\sqrt{7}}{2})$

$\approx 0.35272$

(d) $P(9 \leq X \leq 11) \approx P(9 \leq Y \leq 11) \sim N(7.8, 1.652)$

$\approx 0.97362 - 0.76617$

$\approx 0.20745$

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5.3.6. $X \sim B(15{,}000, \frac{1}{125})$

then $P(X \geq 135) \approx P(Y \geq 135)$ s.t. $Y \sim N(120, 10.910)$

$\therefore P(X \geq 135) \approx 0.08458$.

3.3.14.

let $X$: time to failure of an electrical component

since $X$ has a Weibull distribution,

$$E(X) = \frac{1}{\lambda}\Gamma\left(1+\frac{1}{a}\right) = \frac{1}{0.058}\Gamma(1.4) \approx 15.84$$

$$Var(X) = \frac{1}{\lambda^2}\left\{\Gamma\left(1+\frac{2}{a}\right) - \left[\Gamma\left(1+\frac{1}{a}\right)\right]^2\right\}$$

$$\approx \frac{1}{0.058^2}\left\{(0.931383) - (0.887263)^2\right\} \approx 14.0688$$

$$\Rightarrow P(X \geq 20) = P\left(Z \geq \frac{20-15.84}{3.75}\right) \approx P(Z \geq 1.109)$$

$$\approx 0.13372.$$

let $Y = B(500, 0.13372)$

$P(Y \geq 125) \approx P(Y' \geq 125)$ s.t. $Y' \sim N(66.86, 7.610)$

$$\approx 0$$

∴ 500개중 적어도 125개의 component가 20번 이상의 실패횟수를 가질 확률은 0에 가깝다.