HW2 - GSDS 2022314 강현구

2.5.2. (a)

A. $P(X_i | Y=1) = \dfrac{P(X_i \cap Y=1)}{P(Y=1)}$

$\therefore P(X=1|Y=1) = \dfrac{0.12}{0.32}$, $P(X=2|Y=1) = \dfrac{0.08}{0.32}$, $P(X=3|Y=1) = \dfrac{0.07}{0.32}$, $P(X=4|Y=1) = \dfrac{0.05}{0.32}$

$E(X|Y=1) = \sum_{x=1}^{4} x \cdot P(x|Y=1) = 1 \cdot \dfrac{0.12}{0.32} + 2 \cdot \dfrac{0.08}{0.32} + 3 \cdot \dfrac{0.07}{0.32} + 4 \cdot \dfrac{0.05}{0.32} = \dfrac{0.69}{0.32}$

$\sigma(X|Y=1)^2 = Var(X|Y=1) = E(X^2|Y=1) - E(X|Y=1)^2 = \dfrac{1.87}{0.32} - \left(\dfrac{0.69}{0.32}\right)^2 \approx 5.843 - 4.649$

≈ 1.194

$\left(* E(X^2|Y=1) = \dfrac{0.12+0.32+0.63+0.80}{0.32} = \dfrac{1.87}{0.32} \right)$

$\therefore \sigma(X|Y=1) \approx \sqrt{1.194} = 1.09270 \cdots$

(b)

A. $P(Y|X=2) = \dfrac{P(Y \cap X=2)}{P(X=2)}$

$\therefore P(Y=1|X=2) = \dfrac{0.08}{0.24}$, $P(Y=2|X=2) = \dfrac{0.15}{0.24}$, $P(Y=3|X=2) = \dfrac{0.01}{0.24}$

$E(Y|X=2) = \sum_{y=1}^{3} y \cdot P(y|X=2) = \dfrac{1 \cdot 0.08 + 2 \cdot 0.15 + 3 \cdot 0.01}{0.24} = \dfrac{0.39}{0.24}$

$\sigma^2(Y|X=2) = E(Y^2|X=2) - E(Y|X=2)^2 = \dfrac{0.77}{0.24} - \left(\dfrac{0.39}{0.24}\right)^2 \approx 3.208 - 2.640$

≈ 0.568

$\left(* E(Y^2|X=2) = \dfrac{1 \cdot 0.08 + 4 \cdot 0.15 + 9 \cdot 0.01}{0.24} = \dfrac{0.77}{0.24} \right)$

$\therefore \sigma(Y|X=2) \approx \sqrt{0.568} \approx 0.7536$

2.6.4.

A: $E(A+B) = E(A) + E(B)$.

$Var(A+B) = Var(A) + Var(B) + \cancel{2 \cdot cov(A,B)}$ 0 since A&B is independent

$\therefore E(A+B) = 61 mm \quad Var(A+B) = 1.0\ mm$

2.6.16

(a) $E(X_1+X_2/2) = \frac{1}{2}(E(X_1)+E(X_2)) = \frac{1}{2}(W+W) = W$.

$\sigma^2(X_1+X_2/2) = \frac{1}{4}(Var(X_1) + Var(X_2) + \cancel{2cov(X_1,X_2)}) = \frac{1}{4}(9+16)$

$\therefore \sigma(X_1+X_2/2) = \frac{5}{2}$

(b) $Var(\delta X_1 + (1-\delta)X_2) = \delta^2 \cdot Var(X_1) + (1-\delta)^2 Var(X_2)$ $\cancel{cov}$ 0.

$= 9\delta^2 + 16(1-\delta^2)$

$= 25\delta^2 - 32\delta + 16 = (5\delta - \frac{16}{5})^2 + 16 - (\frac{16}{5})^2$.

$\therefore$ when $\delta = \frac{16}{25}$, $\sigma(B)$ is minimum. & $\min(\sigma(B)) = \sqrt{16 - (\frac{16}{5})^2} = \frac{2\sqrt{5}}{5}$

2.9.6.

(a) $f(x) = \int_1^2 f(x,y)\,dy = \int_1^2 4x(2-y)\,dy = [{}_1^2\, 8xy - 2xy^2]$

$= 16x - 8x - \cancel{8}x + 2x = 2x$.

(b) To prove X, Y is independent, we check $f(x) \cdot f(y) = f(x,y)$.

since $f(y) = \int_0^1 4x(2-y)\,dx = [{}_0^1\, 2x^2(2-y)] = 2(2-y)$,

$\therefore f(x) \cdot f(y) = 2 \cdot (2-y) \cdot 2x = 4x(2-y) = f(x,y)$. X, Y is independen

(c) $cov(X,Y) = 0$. since X, Y is independent.

(d) $p(X \mid Y=1.5) = f(x, 1.5) = 2x$.

2.9.16.

(a). Since $\int_5^6 Ax \cdot dx = 1$, $\frac{36-25}{2} \cdot A = 1$

$\therefore A = \frac{2}{11}$

(b) $F(x) = \int_5^x \frac{2}{11} x\,dx = \left[\frac{1}{11} x^2 \right]_5^x = \frac{x^2-25}{11}$

(c) $E(X) = \int_5^6 x \cdot \frac{2}{11} \cdot x \cdot dx = \left[\frac{2}{33} x^3 \right]_5^6 = \frac{2}{33}(216-125) = \frac{182}{33}$

(d) $\sigma^2(X) = E(X^2) - E(X)^2 = \frac{671}{22} - \left(\frac{182}{33}\right)^2 \approx 30.5 - 30.4168 \approx 0.0832$

$\times\ E(X^2) = \int_5^6 x^2 \cdot \frac{2}{11} x\,dx = \left[\frac{1}{22} x^4 \right]_5^6 = \frac{1}{22}(671) = 30.5$

$\therefore \sigma(X) \approx \sqrt{0.0832} \approx 0.2884$.

2.9.24.

(a) since $\begin{cases} E(Y) = E(c+dX) = c+d \cdot 250 = 100 \\ \sigma^2(Y) = \sigma^2(c+dX) = d^2 \cdot 16 = 1 \end{cases}$

$\therefore d = \frac{1}{4}.\ c = \frac{75}{2}$

(b) $E(10X) = 10 \cdot E(X) = 2500$

$\sigma(10X) = 10 \cdot \sigma(X) = 40$

3.1.12

A. $P(X \geq 10) = P(X=10) + P(X=11) + P(X=12)$

$= {}_{12}C_{10}\,(0.93)^{10} \cdot (0.07)^2 + {}_{12}C_{11} \cdot (0.93)^{11} \cdot 0.07 + {}_{12}C_{12}\,(0.93)^{12}$.

3.2.6. (a) $E(\text{number of container. drops} \ldots) = \frac{1}{p} = \frac{100}{37}$

(b) $\frac{r}{p} = \frac{100}{37} \times 3$

(c) $p = 1 - p(X=0) - p(X=1) - p(X=2)$

$= 1 - {}_{10}C_0 \cdot (0.63)^{10} - {}_{10}C_1 \cdot (0.63)^9 \cdot (0.37) - {}_{10}C_2 \cdot (0.63)^8 \cdot (0.37)^2$

(d) $p = \binom{x-1}{r-1} \cdot p^r \cdot (1-p)^{x-r} \Rightarrow \binom{9}{2} \cdot (0.93)^3 \cdot (0.07)^7$

3.2.10.

A. $E(\text{negative binomial distribution}) = \frac{r}{p} \Rightarrow 6 \cdot 3 = 18$

3.38.

A. let 'chocolate pick' = C, 'strawberry pick' = S

$p = p(C=4) + p(C=3) + p(C=2)$

chocopick straw pick pick selection

$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 6 \cdot 5C_4}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} + \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 5C_2}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} \cdot \frac{9 \cdot 8 \cdot 6 \cdot 5 \cdot 4 \cdot 5C_3}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}$

$= \frac{9 \cdot 8 \cdot 6(210 + 350 + 200)}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} = \frac{\cancel{760}\ 152}{\cancel{35}\ 7 \cdot 13 \cdot 11} \approx 0.1518$