

1.1.6 A bag contains balls that are either red or blue and either dull or shiny. What is the sample space when a ball is chosen from the bag?

1.2.6 Two fair dice are thrown, one red and one blue. What is the probability that the red die has a score that is *strictly greater* than the score of the blue die? Why is this probability less than 0.5? What is the complement of this event?

1.3.6 If $P(A) = 0.4$ and $P(A \cap B) = 0.3$, what are the possible values for $P(B)$?

1.4.10 Consider again Figure 1.25 and the two assembly lines.

Calculate the probabilities:

- (a) Both lines are at full capacity conditional on neither line being shut down
- (b) At least one line is at full capacity conditional on neither line being shut down
- (c) One line is at full capacity conditional on exactly one line being shut down
- (d) Neither line is at full capacity conditional on at least one line operating at partial capacity

1.2.11 **Supplementary problem for 1.4.10**

A factory has two assembly lines, each of which is *shut down* (S), at *partial capacity* (P), or at *full capacity* (F). The sample space is given in Figure 1.25, where, for example, (S, P) denotes that the first assembly line is shut down and the second second one is operating at partial capacity.

(S, S)	(S, P)	(S, F)
0.02	0.06	0.05

(P, S)	(P, P)	(P, F)
0.07	0.14	0.20

(F, S)	(F, P)	(F, F)
0.06	0.21	0.19

Figure 1.25

1.5.14 If a fair die is rolled six times, what is the probability that each score is obtained exactly once? If a fair die is rolled seven times, what is the probability that a 6 is not obtained at all?

1.6.6 The weather on a particular day is classified as either cold, warm, or hot. There is a probability of 0.15 that it is cold and a probability of 0.25 that it is warm. In addition, on each day it may either rain or not rain. On cold days there is a probability of 0.30 that it will rain, on warm days there is a probability of 0.40 that it will rain, and on hot days there is a probability of 0.50 that it will rain. If it is not raining on a particular day, what is the probability that it is cold?

2.1.8 Four cards are labeled \$1, \$2, \$3, and \$6. A player pays \$4, selects two cards at random, and then receives the sum of the winnings indicated on the two cards. Calculate the probability mass function and the cumulative distribution function of the *net* winnings (that is, winnings minus the \$4 payment).

2.2.10 Sometimes a random variable is a mix of discrete and continuous components. For example, suppose that the dial-spinning game is modified in the following way. First a fair coin is tossed and if a head is obtained, the player wins \$500 and the dial is not spun. However, if a tail is obtained, the player spins the dial and receives winnings of

$$\$1000 \times \frac{\theta}{180}$$

as before. In this game there is a probability of 0.5 of winning \$500, with all the other possible winnings between \$0 and \$1000, being equally likely. The coin toss provides a discrete element to the winnings, and the dial spin provides a continuous element. The best way to describe the probabilistic properties of *mixed* random variables such as this is through a cumulative distribution function. The cumulative distribution function of the winning from this game is given in Figure 2.32.

(a) What is the probability of winning less than \$200?

(b) What is the probability of winning between \$400 and \$700?

Interpret your answers.

2.3.14 Consider again the archery problem discussed in Problem 2.2.9. What is the expected deviation from the center of the target? What is the median deviation (median of deviations)?

2.2.9 **Supplementary problem for 2.3.14**

An archer shoots an arrow at a circular target with a radius of 50 cm. If the arrow hits the target, the distance r between the point of impact and the center of the target is measured. Suppose that this distance has a cumulative distribution function

$$F(r) = A + \frac{B}{(r + 5)^3}$$

for $0 \leq r \leq 50$.

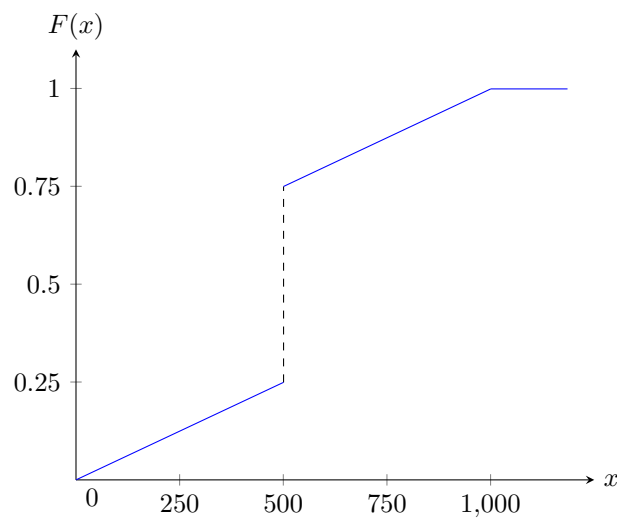


Figure 2.32

2.4.8 Consider again the plastic bending capabilities discussed in Problems 2.2.8 and 2.3.13.

- (a) What is the variance of the deformity angle? (b) What is the standard deviation of the deformity angle? (c) Find the upper and lower quartiles of the deformity angle. (d) What is the interquartile range?

2.2.8 **Supplementary problem for 2.4.8**

The bending capabilities of plastic sheets are investigated by bending sheets at increasingly large angles until a deformity appears in the sheet. The angle θ at which the deformity first appears is then recorded. Suppose that this angle takes values between 0° and 10° with a probability density function

$$f(\theta) = A(e^{10-\theta} - 1)$$

for $0 \leq \theta \leq 10$ and $f(\theta) = 0$ elsewhere.