HW11. 2022314 강현우.

12.1.2.

(a) $E[Y_{20}] = \beta_0 + \beta_1 \cdot 20$

$= 123 - 43.2 = 79.8.$

(b) $E[Y_{30}] - E[Y_{20}] = 10 \cdot \beta_1 = -21.6$

(c) $P(Y_{25} < 60) = P\left(\frac{Y_{25} - E(Y_{25})}{6} < \frac{60 - E(Y_{25})}{6} \right) \because E(Y_{25}) = 69$

$= P(\text{z-value} < -2.19512)$

$\therefore P = 0.01426.$

(d) $P(30 < Y_{40} < 40) = P\left(\frac{30 - \bar{Y}_{40}}{6} < \frac{Y_{40} - \bar{Y}_{40}}{6} < \frac{40 - \bar{Y}_{40}}{6} \right)$

$\because \bar{Y}_{40} = 36.6$

$= P(-1.604 < \text{z-value} < 0.82)$

$= 0.7401$

(e) $P(Y_{27.5} > Y_{30}) = P(Y_{27.5} - Y_{30} > 0) =$

$= P\left(\frac{Y_{27.5} - Y_{30} - (\hat{Y}_{27.5} - \hat{Y}_{30})}{Var(Y_{27.5} - Y_{30})} > \frac{-(\hat{Y}_{27.5} - \hat{Y}_{30})}{Var(Y_{27.5} - Y_{30})} = \frac{-5.4}{\sqrt{2} \cdot 6} \right)$

$\approx P(\text{z-value} > -0.9313)$

$\therefore P : 0.8242$

12.24.

(a). $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum(x_i-\bar{x})\cdot(y_i-\bar{y})}{\sum(x_i-\bar{x})^2} = \frac{6664310}{7040256} \approx 1.063$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x} \approx 6345.2499 - 1.063 \times 8593.625 \approx -2789.7734$

$y = 1.063x - 2789.7734$

(b) $1000 \cdot \hat{\beta}_1 = 1063$

(c) $\hat{y}_{10000} = 1.063 \times 10000 - 2789.7734$

$= 7840.2266$

(d) $\hat{\sigma}^2 = \frac{SSE}{n-2} = \frac{11219019.7}{14} = 801358.55.$

(e) $E[Y_{20000}] = 18472.57.$ $var(\hat{\sigma}) \approx 895.18632$

12.26

(a). $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = -0.32079$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x} \approx 38.55 + 0.32079 \times 46.4 = 53.434656.$

$y = -0.32079x + 53.434655.$

(b) additional five years affect -1.60395 about VO_2-max.

(c) $\hat{y}_{50} = -0.32079 \times 50 + 53.434655 = 37.395155$

(d) $\hat{y}_{15} = 48.622805.$

(e) $\hat{\sigma}^2 = \frac{SSE}{n-2} \approx 57.35373$

#12.3.2.

(a) confidence interval of β_1 : $\hat{\beta}_1 \pm t_{\frac{\alpha}{2},n-2} \dfrac{\hat{\sigma}}{\sqrt{S_{xx}}}$

$\Rightarrow [\hat{\beta}_1 - t_{0.025,20} \cdot se(\hat{\beta}_1), \ \hat{\beta}_1 + t_{0.025,20} \times s.e.(\hat{\beta}_1)]$

$\Rightarrow [56.33 - 2.086 \times 3.78, \ 56.33 + 2.086 \times 3.78]$

$= [48.44492, \ 64.21508]$

(b) Since $\beta_1 = 50.0$ is included in 95% C.I. of β_1,

null hypotesis can't be rejected.

12.3.8.

$y = 0.805074 \cdot x + 12.86392$.

(a) $s.e.(\hat{\beta}_1) = \hat{\sigma} / \sqrt{S_{xx}} \approx 0.3148$

$\because \ \hat{\sigma}^2 = \dfrac{SSE}{n-2} \approx 3.981188$, $S_{xx} = 40.15972$

(b) $\beta_1 \pm t_{\frac{\alpha}{2},n-2} \, se.(\beta_1)$

$\Leftrightarrow [0.805074 - 2.808 \times 0.3148, \ 0.805074 + 2.808 \times 0.3148]$

$= [-0.0788844, \ 1.6890324]$

(c). Since $\beta_1 = 0$. is cluded in C.I. of β_1,

H_0 can't be rejected

12.4.2.

$$\hat{y}_{x_0} \pm t_{\frac{\alpha}{2}, n-2}\, \hat{\sigma} \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$$

$$y = 34.6 \times x + 12.08$$

when $x = 40$, & 95% confidence interval.

$$y \in [\underbrace{1396.08}_{\hat{y}_{40}} - \underbrace{2.132}_{t_{0.025,15}} \times \underbrace{0.4666}_{\sqrt{\frac{1}{n} + \frac{(x_0-\bar{x})^2}{S_{xx}}}} \times 4.2011,\ 1396.08 + 4.1792]$$

$$[1391.9007,\ 1400.2592]$$

12.4.8.

$$y = 0.805074 \cdot x + 12.86392$$

$$\hat{\sigma}^2 = \frac{SSE}{n-2} \approx 3.981188 \qquad S_{xx} = 40.15972$$

$$\hat{y}_{70} = 69.2191 \qquad t_{0.005,22} = 2.819$$

$$\Rightarrow y_{70} \in [69.2191 - \underbrace{2.819}_{t} \times \underbrace{1.995}_{\hat{\sigma}}\ \underbrace{0.2239}_{\sqrt{\frac{1}{n} + \frac{(x_0-\bar{x})^2}{S_{xx}}}},\ 69.2191 + 1.2592]$$

$$([67.9598,\ 70.4783]$$

12.5.4.

$y = -0.32079x + 53.434655.$

$\hat{\sigma}^2 = \frac{SSE}{n-2} \approx 57.35373$, $S_{xx} = 183.5158$.

$\hat{y}_{50} = 37.3951$

$Y_{50} \in [37.3951 - \underbrace{2.101}_{t_{0.025,18}} \cdot \underbrace{7.5732}_{\hat{\sigma}} \cdot \underbrace{0.3473}_{\sqrt{\frac{1}{n} + \frac{(x-50)^2}{S_{xx}}}},\ 37.3951 + 5.5260]$

$\Leftrightarrow [31.8690, 42.9211]$.

$\therefore$ 95% prediction interval of Y_{50} : $[31.8690, 42.9211]$

12.5.8.

$\hat{\sigma}^2 = \frac{SSE}{n-2} = \frac{329.77}{28} = 11.7775$

$\bar{x} = \frac{\sum x_i}{30} = 20.112$,

$S_{xx} = Var(X) = E[X^2] - E[X]^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$

$= 14.78456$

$\hat{y}_{22} = 51.98 + 3.44 \times 22 = 127.66$

In 95% confidence,

$Y_{22} \in [127.66 - \underbrace{2.049}_{t_{0.025,28}} \times \underbrace{3.4318}_{\hat{\sigma}} \times \underbrace{0.5239}_{\sqrt{\frac{1}{n} + \frac{(22-\bar{x})^2}{S_{xx}}}},\ 127.66 + 3.6840]$

$\therefore$ 95% confidence, of $Y_{22} \in [123.9759, 131.3440]$