

**3.4.8** In a scanning process, the number of misrecorded pieces of information has a Poisson distribution with parameter  $\lambda = 9.2$ .

- (a) What is the probability that there are between six and ten misrecorded pieces of information?
- (b) What is the probability that there are no more than four misrecorded pieces of information?

**3.5.4** A researcher plants 22 seedlings. After one month, independent of the other seedlings, each seedling has a probability of 0.08 of being dead, a probability of 0.19 of exhibiting slow growth, a probability of 0.42 of exhibiting medium growth, and a probability of 0.31 of exhibiting strong growth. What is the expected number of seedlings in each of these four categories after one month? Calculate the probability that after one month:

- (a) Exactly three seedlings are dead, exactly four exhibit slow growth, and exactly six exhibit medium growth.
- (b) Exactly five seedlings are dead, exactly five exhibit slow growth, and exactly seven exhibit strong growth.
- (c) No more than two seedlings have died.

**3.8.14** The number of imperfections in an object has a Poisson distribution with a mean  $\lambda = 8.3$ . If the number of imperfections is 4 or less, the object is called “top quality.” If the number of imperfections is between 5 and 8 inclusive, the object is called “good quality.” If the number of imperfections is between 9 and 12 inclusive, the object is called “normal quality.” If the number of imperfections is 13 or more, the object is called “bad quality.” The number of imperfections in different objects are independent of each other.

- (a) A set of seven articles is taken. What is the probability that the set has exactly two top-quality, two good-quality, two normal-quality and one bad-quality objects?
- (b) A set of ten articles is taken. What are the expectation and the standard deviation of the number of normal quality objects in the set.
- (c) A set of eight articles is taken. What is the probability that the sum of the number of top quality and good quality objects is three or less?

**4.2.6** Imperfections in an optical fiber are distributed according to a Poisson process such that the distance between imperfections in meters has an exponential distribution with parameter  $\lambda = 2\text{m}^{-1}$ .

- (a) What is the expected distance between imperfections?
- (b) What is the probability that the distance between two imperfections is longer than 1 meter?
- (c) What is the distribution of the number of imperfections in a 3-meter stretch of fiber?
- (d) What is the probability that a 3-meter stretch of fiber has no more than four imperfections?

**4.2.10** The lengths of telephone calls can be modeled by an exponential distribution with parameter  $\lambda = 0.3$  per minute, with the call lengths being independent. What is the probability that out of ten telephone calls, two will be shorter than 1 minute, four will last between 1 minute and 3 minutes, and the other four will last longer than 3 minutes?

**4.3.6** Recall Problem 4.2.7 concerning the arrivals at a factory first-aid room.

- (a) What is the distribution of the time between the first arrival of the day and the fourth arrival?
- (b) What is the expectation of this time?
- (c) What is the variance of this time?
- (d) By using (i) the gamma distribution and (ii) the Poisson distribution, show how to calculate the probability that this time is longer than 3 hours.

**4.2.7** **Supplementary problem for 4.3.6**

The arrival times of workers at a factory first-aid room satisfy a Poisson process with an average of 1.8 per hours.

- (a) What is the value of the parameter  $\lambda$  of the Poisson process?
- (b) What is the expectation of the time between two arrivals at the first-aid room?
- (c) What is the probability that there is at least 1 hour between two arrivals at the first-aid room?
- (d) What is the distribution of the number of workers visiting the first-aid room during a 4-hour period?
- (e) What is the probability that at least four workers visit the first-aid room during a 4-hour period?

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**4.4.6** The lifetime in minutes of a mechanical component has a Weibull distribution with parameters  $a = 1.5$  and  $\lambda = 0.03$ .

- (a) What are the median, upper quartile, and 99th percentile of the lifetime of a component?
- (b) If 500 independent components are considered, what are the expectation and variance of the number of components still operating after 30 minutes?

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**4.5.6** The proportion of tin in a metal alloy has a beta distribution with parameters  $a = 8.2$  and  $b = 11.7$ .

- (a) What is the expected proportion of tin in the alloy?
- (b) What is the standard deviation of the proportion of tin in the alloy?
- (c) What is the median proportion of tin in the alloy?

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**4.8.12** A hole is drilled into the Antarctic ice shelf and a core is extracted that provides information on the climate when the ice was formed at different times in the past. Suppose that a researcher is interested in high-temperature years, and that the places in the core corresponding to high-temperature years occur according to a Poisson process with parameter  $\lambda = 0.48$  per cm.

- (a) What is the expected distance in cm between adjacent high-temperature years?
- (b) What is the expected distance in cm between one high-temperature year and the tenth high-temperature year that followed after it?
- (c) What is the probability that the distance between two adjacent high-temperature years is less than 0.5 cm?
- (d) Suppose that a 20-cm section of core is analyzed.

What is the probability that the number of high-temperature years in this section of core is between 8 and 12 inclusive?