22.11.13. HW 9 - 2022434 [illegible]

10.1.8.

p: positive result being incorrect.

since binary case, & one-sided test.

we should check 0.1 is included in $\left(0, \hat{p}+\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \cdot z_\alpha\right)$

$\hat{p} = \frac{23}{324}$, $z_{0.01} = 2.327$.

$\Rightarrow$ 99% confidence interval $(0, 0.104187\dots)$

$\therefore$ screening test is hard to acceptable.

10.2.2.

two sided confidence interval $p_A - p_B = \left(p_A - p_B - \sqrt{Var(p_A)+Var(p_B)} \cdot z_{\alpha/2},\ p_A - p_B + \sqrt{Var(p_A)+Var(p_B)} \cdot z_{\alpha/2}\right)$

$\hat{p}_A = \frac{261}{302} \approx 0.8642$, $Var(\hat{p}_A) = \frac{\hat{p}_A \cdot (1-\hat{p}_A)}{302} \approx (0.0197)^2$. $\sqrt{Var(\hat{p}_A)+Var(\hat{p}_B)} \approx 0.02481$

$\hat{p}_B = \frac{401}{454} \approx 0.8833$, $Var(\hat{p}_B) = \frac{\hat{p}_B \cdot (1-\hat{p}_B)}{454} \approx (0.0150)^2$.

(a) $= (-0.0190 - 0.02481 \times 2.576,\ -0.0190 + 0.02481 \times 2.576)$

$(-0.08291,\ 0.04491)$

(b) $= (-0.0190 - 0.02481 \times 1.645,\ -0.0190 + 0.02481 \times 1.645)$

$(-0.05981245,\ 0.02181245)$

(c) $= (-0.190,\ -0.190 + 0.02481 \times 1.645)$

(d) : z-value $= \frac{0.0190}{0.02481} \approx 0.7658$. p-value ≈ 0.222.

10.2.12.

$H_0: p_A \geq p_B$. $H_a: p_A < p_B$.

$$z\text{-value} = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{Var(\hat{p}_A) + Var(\hat{p}_B)}} \approx -4.\cancel{3}36 \qquad \hat{p}_A = \frac{22}{\cancel{52}} \qquad \hat{p}_B = \frac{\cancel{64}}{601}$$

$\Rightarrow$, p-value ≈ 0, then H_0 is rejected. Modification is more attract.

10.3.6.

$n_1 = 225$, $n_2 = 223$, $n_3 = 152$.

then $\hat{p}_2 = \frac{n_2}{n_1+n_2+n_3} = 0.3716 \qquad \hat{p}_3 = \frac{n_3}{n_1+n_2+n_3} = 0.253$.

$H_0: p_2 = p_3$, $H_1 = p_2 \neq p_3$.

$$z\text{-value} = \frac{\hat{p}_2 - \hat{p}_3}{\sqrt{Var(\hat{p}_2) + Var(\hat{p}_3)}} \approx 4.4583.$$

$\therefore$ p-value ≈ 0. three formulation are not equally popular.

10.3.10.

$p_1 = \frac{83}{205}$, $p_2 = \frac{75}{205}$, $p_3 = \frac{47}{205}$

$H_0: p_1 = p_3$, $H_a: p_1 \neq p_3$.

$$z\text{-value} = \frac{\hat{p}_1 - \hat{p}_3}{\sqrt{Var(\hat{p}_1) + Var(\hat{p}_3)}} \approx 3.8905.$$

$\therefore$ p-value ≈ 0. There is sufficient evidence to conclude 3 products do not have equal probabilities ~~of~~

10.3.12.

weibull distribution $f(x;k,\lambda) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \cdot e^{-(\frac{x}{\lambda})^k}$.

$$\chi^2 = \sum_{i=1}^{k} \frac{(x_i - e_i)}{e_i}$$

$$e_i = \frac{0.45}{1.56} \cdot \left(\frac{i}{1.56}\right)^{-0.55} \cdot e^{-(\frac{i}{1.56})^{0.45}}$$

$x_i = (12, 53, 39)$ 1hour 일 때. $k = 0.45$, $\lambda = 0.065$.

$e_i = (16.24,$ 24hour 일 때 $k = 0.45$, $\lambda = 1.56$

Weibull 분포.

10.4.2.

	Dead	slow	Medium	strong	
FX	48	111	186	142	487
F1	71	89	174	181	575
F2	63	95	181	190	529
	182	295	541	513	1531

To check whether FX, F1, F2 is same or not

χ^2-의 p-value check 필요.

$$\chi^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} \frac{(x_{ij} - e_{ij})^2}{e_{ij}}$$

≈ 13.65911

$df = (r-1)(c-1) = 6$

$\therefore p = 0.324$, it is hard to think 3sets are different.

10.4.6. -check

x_{11}	x_{12}	$x_{1\cdot} = x_{11}+x_{12}$
x_{21}	x_{22}	$x_{2\cdot} = x_{21}+x_{22}$
$x_{\cdot 1} = x_{11}+x_{21}$	$x_{\cdot 2} = x_{12}+x_{22}$	

$$\frac{n}{x_{i\cdot} \cdot x_{\cdot j}}$$

$$\chi^2 = \sum_{i=1}^{2}\sum_{j=1}^{2} \frac{(x_{ij}-e_{ij})^2}{e_{ij}}$$

since $e_{ij} = \frac{x_{i\cdot}}{n} \times \frac{x_{\cdot j}}{n} \times n.$

$$= \sum_{i=1}^{2}\sum_{j=1}^{2} \frac{x_{ij}^2}{e_{ij}} - 2x_{ij} + e_{ij}$$

$$\left(\because \sum_{i=1}^{2}\sum_{j=1}^{2} x_{ij} = n, \quad \sum_{i=1}^{2}\sum_{j=1}^{2} e_{ij} = n. \right.$$

$$= \sum_{i=1}^{2}\sum_{j=1}^{2} \frac{n \cdot x_{ij}}{x_{i\cdot} \cdot x_{\cdot j}} - n.$$

$$= \sum_{i=1}^{2} \frac{n}{x_{i\cdot}} \sum_{j=1}^{2} \frac{x_{ij}}{x_{\cdot j}} - n$$

$$= \sum_{i=1}^{2} \frac{n}{x_{i\cdot}} \left(\frac{x_{i1}}{x_{\cdot 1}} + \frac{x_{i2}}{x_{\cdot 2}} \right) - n$$

$$= \frac{n}{x_{1\cdot}} \left(\frac{x_{11}}{x_{\cdot 1}} + \frac{x_{12}}{x_{\cdot 2}} \right) + \frac{n}{x_{2\cdot}} \left(\frac{x_{21}}{x_{\cdot 1}} + \frac{x_{22}}{x_{\cdot 2}} \right) - n.$$

$$= n \left(\frac{x_{2\cdot}(x_{\cdot 1} \cdot x_{12} + x_{\cdot 2} \cdot x_{11}) + x_{1\cdot} \cdot (x_{\cdot 2} \cdot x_{21} + x_{\cdot 1} \cdot x_{22})}{x_{1\cdot} \cdot x_{2\cdot} \cdot x_{\cdot 1} \cdot x_{\cdot 2}} \right) - n.$$

#10.4.10

A	9	17	31	57
B	4	9	36	49
C	15	19	56	90
	28	45	123	196

$\Rightarrow \chi^2 = 5.023351$

p-value $= 0.2848$.

$\therefore$ It is hard to think different

10.7.20. $\hat{p}_A = \frac{56}{94}$, $\hat{p}_B = \frac{64}{153}$

(a). $H_0 : p_A \le \frac{1}{2}$, $H_\alpha : p_A > \frac{1}{2}$

$$Z\text{-value} = \frac{\frac{56}{94} - \frac{1}{2}}{\sqrt{\frac{1}{94}\cdot\left(\frac{56}{94}\cdot\frac{38}{94}\right)}} \approx 1.89156.$$

then p-value ≈ 0.029.

$\therefore$ In 5% significance level, p_A is better than 50%.

(b) 99% confidence interval

$$\left(\hat{p}_A - \hat{p}_B - \sqrt{Var(\hat{p}_A)+Var(\hat{p}_B)}\cdot Z_{0.005},\ \hat{p}_A - \hat{p}_B + \sqrt{Var(\hat{p}_A)+Var(\hat{p}_B)}\cdot Z_{0.005}\right).$$

$\Leftarrow$ $(0.011486,\ 0.\cancel{343}439)$

(c) $\chi^2 \approx 6.646131$, p-value ≈ 0.009937!

$\therefore$ p_A, p_B is different.