2.5.2 Consider Example 19 on air conditioner maintenance.

(a) Suppose that a location has only one air conditioner that needs servicing. What is the conditional probability mass function of the service time required, and the conditional expectation and standard deviation of the service time?

(b) Suppose that a location requires a service time of two hours. What is the conditional probability mass function of the number of air conditioner units serviced, and the conditional expectation and standard deviation of the number of air conditioner units serviced?

Example 19 **Supplementary problem for 2.5.2**

A company that sevices air conditioner units in residences and office blocks is interested in how to schedule its technicians in the most efficient manner. Specifically, the company is interested in how long a technician takes on a visit to a particular location, and the company recognizes that this mainly depends on the number of air conditioner units at the location that need to be serviced.

If the random variable X, taking the values 1, 2, 3, and 4, is the *service time* in hours taken at a particular location, and the random variable Y, taking the values 1, 2, and 3, is the *number of air conditioner units* at the location, then these two random variables can be thought of as jointly distributed.

Suppose that their joint probability mass function p_{ij} is given in Figure 2.58. The figure indicates, for example, that there is a probability of 0.12 that $X = 1$ and $Y = 1$, so that there is a probability of 0.12 that a particular location chosen at random has one air conditioner unit that takes a technician one hour to sevice. Similarly, there is a pprobability of 0.07 that a location has three air conditioner units that take four hours to service. Notice that this is a valid probability mass function since

$$\sum_i \sum_j p_{ij} = 0.12 + 0.08 + \cdots + 0.07 = 1.00$$

X = service time (hrs)

Y = number of air conditioner units	1	2	3	4
1	0.12	0.08	0.07	0.05
2	0.08	0.15	0.21	0.13
3	0.01	0.01	0.02	0.07

Figure 2.58

2.6.4 A machine part is assembled by fastening *two* components of type A and *one* component of type B end to end (all components are independent). Suppose that the lengths of components of type A have an expectation of 37.0 mm and a standard deviation of 0.7 mm, whereas the lengths of components of type B have an expectation of 24.0 mm and a standard deviation of 0.3 mm. What are the expectation and variance of the length of the machine part?

2.6.16 An object has a weight W. When it is weighed with machine 1, a value X_1 is obtained. When it is weighed with machine 2, a value X_2 is obtained. The value X_1 has a mean W and a standard deviation 3. The value X_2 has a mean W and a standard deviation 4. The values X_1 and X_2 are independent.

(a) Suppose that an engineer uses the value $A = (X_1 + X_2)/2$ to estimate the weight. What are the expectation and the standard deviation of A?

(b) Suppose that an engineer uses the value $B = \delta X_1 + (1 - \delta)X_2$ to estimate the weight. What value of δ gives an estimate B with the smallest standard deviation? What is this smallest standard deviation?

2.9.6 Suppose that the random variables X and Y have a joint probability density function

$$f(x, y) = 4x(2 - y)$$

for $0 \leq x \leq 1$ and $1 \leq y \leq 2$.

(a) What is the marginal probability density function of X?

(b) Are the random variables X and Y independent?

(c) What is Cov(X, Y)?

(d) What is the probability density function of X conditional on $Y = 1.5$?

2.9.16 A continuous random variable has a probability density function

$$f(x) = Ax$$

for $5 \leq x \leq 6$.

(a) What is the value of A?

(b) What is the cumulative distribution function of the random variable?

(c) What is the expectation of the random variable?

(d) What is the standard deviation of the random variable?

2.9.24 Bricks have weights that have a mean 250 and a standard deviation 4.
(a) Suppose X is the weight of a randomly chosen brick. Let $Y = c + dX$. What are the values of c and d such that Y has a mean 100 and a standard deviation 1.
(b) Suppose 10 bricks are chosen at random. What are the mean and the standard deviation of the total weight of these 10 bricks?

3.1.12 There is a probability of 0.93 that a visitor to a website will bounce (leave the website without clicking on any links).
What is the probability that at least 10 of the next 12 visitors to the website will bounce?

3.2.6 A supply container dropped from an aircraft by parachute hits a target with a probability of 0.37.
(a) What is the expected number of container drops needed to hit a target?
(b) If hits from three containers are required to provide sufficient supplies, what is the expected number of containers dropped before sufficient supplies have been provided?
(c) What is the probability that sufficient supplies are provided by ten container drops?
(d) What is the probability that it is the tenth container dropped that completes the provision of supplies?

3.2.10 Consider a fair six-sided die. The die is rolled until a 6 is obtained for the third time. What is the expectation of the number of die rolls needed?

3.3.8 A plate has 15 cupcakes on it, of which 9 are chocolate and 6 are strawberry. A child randomly selects 5 of the cupcakes and eats them. What is the probability that the number of chocolate cupcakes remaining on the plate is between 5 and 7 inclusive?