

KSE801 – Recommender System and Machine Learning on Graph

## Lecture 2: Neighborhood-Based Collaborative Filtering

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# RECALL: BASIC MODELS OF RECOMMENDATION

- **Collaborative filtering (CF) – Only ratings**
  - Memory-based approach (Neighborhood-based CF) (Week 2)
  - Model-based approach (Week 3 ~ 4)
- **Side information-based Recommendation (Week 5 ~ 6)**
  - Content-based recommendation – Only contents (Week 1)
  - Content-based CF – Rating + Contents
    - Text, image, social network, etc
- **Advanced topics**
  - Sequential Recommendation & Graph-based Recommendation (Week 7)

# OUTLINE

- Neighborhood-based Collaborative Filtering
- A Regression Modeling View of Neighborhood Methods
- Evaluating Recommender System

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# COLLABORATIVE FILTERING (CF)

- **The most prominent approach to generate recommendations**
  - Used by large, commercial e-commerce sites
  - Well-understood, various algorithms and variations exist
  - Applicable in many domains (book, movies, DVDs, ..)
- **Approach**
  - Use the "wisdom of the crowd" to recommend items
- **Basic assumption and idea**
  - Users give ratings to items (implicitly or explicitly)
  - **Customers who had similar tastes in the past, will have similar tastes in the future**



# NEIGHBORHOOD-BASED CF: INTRODUCTION

- Among the earliest algorithms developed for collaborative filtering

- Main idea**

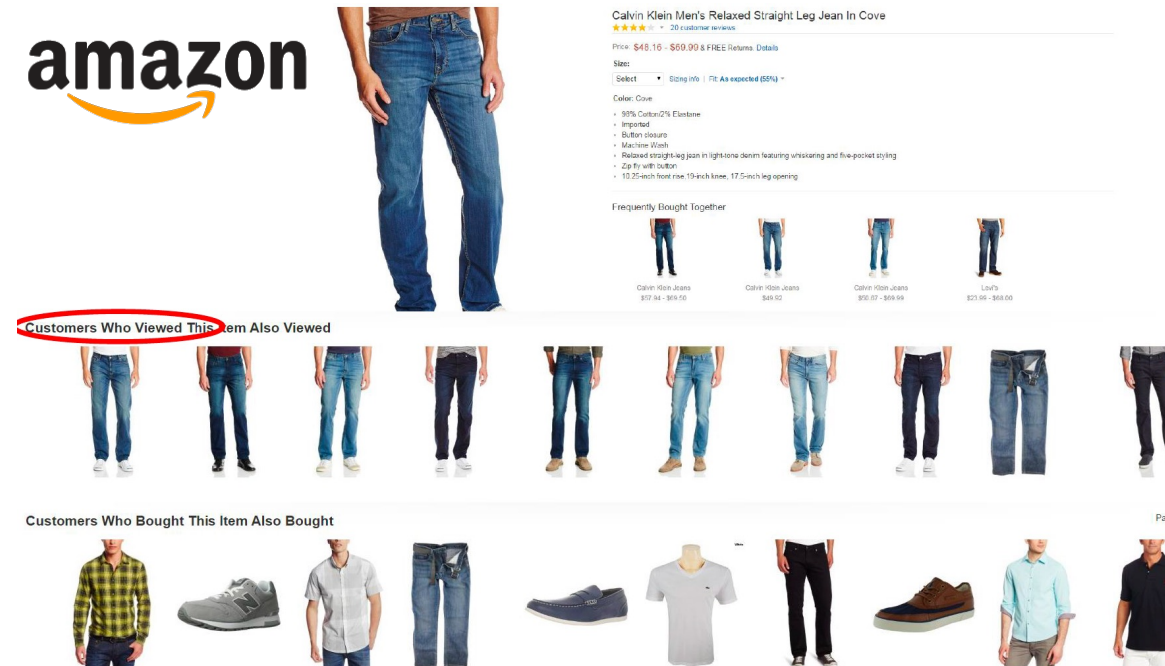
- Similar users display similar patterns of rating behavior**
  - Similar items receive similar ratings**

→ User-based CF

→ Item-based CF

- How do we define similarity between users and items?**

- Q:** How can we measure the **similarity** between two **users**?
    - A:** In terms of the **items** they purchased!
  - Q:** How can we measure the **similarity** between two **items**?
    - A:** In terms of the **users** who purchased them!



# INPUT AND OUTPUT OF CF

- **Input**

- Only a matrix of given user–item ratings

- **Output types**

- A (numerical) prediction indicating to what degree the current user will like or dislike a certain item
  - Explicit feedback: Rating prediction
- A top-N list of recommended items
  - Implicit feedback: Ranking

# USER-BASED COLLABORATIVE FILTERING

(SIMILARLY DEFINED FOR ITEM-BASED CF)

- Given an "active user" (Alice) and an item  $i$  not yet seen by Alice,
  - find a set of users who liked the same items as Alice in the past **and** who have rated item  $i$
  - use, e.g. the average of their ratings to predict, if Alice will like item  $i$
  - do this for all items Alice has not seen and recommend the best-rated
- **Basic assumption and idea**
  - If users had similar tastes in the past they will have similar tastes in the future
  - User preferences remain stable and consistent over time
- **Example:** Determine whether Alice will like or dislike “*Item 5*”, which Alice has not yet rated or seen

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1



# USER-BASED COLLABORATIVE FILTERING

(SIMILARLY DEFINED FOR ITEM-BASED CF)

## ▪ Some first questions

- How do we measure similarity?
- How many neighbors should we consider?
- How do we generate a prediction from the neighbors' ratings?

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Explicit feedback

	Item1	Item2	Item3	Item4	Item5
Alice	1		1		?
User1		1			
User2		1	1		
User3		1		1	
User4	1		1		1

Implicit feedback

# NOTATIONS

- Definitions

- $I_u$  = Set of items purchased by user  $u$
- $U_i$  = Set of users who purchased item  $i$

$$R = \begin{matrix} & \underbrace{\hspace{1.5cm}}_{\text{Items}} & \\ \begin{pmatrix} 1 & 0 & \cdots & 1 \\ 0 & 0 & & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{pmatrix} & \underbrace{\hspace{1cm}}_{\text{Users}} \end{matrix}$$

$R_u$ : Binary representation of items purchased by user  $u$

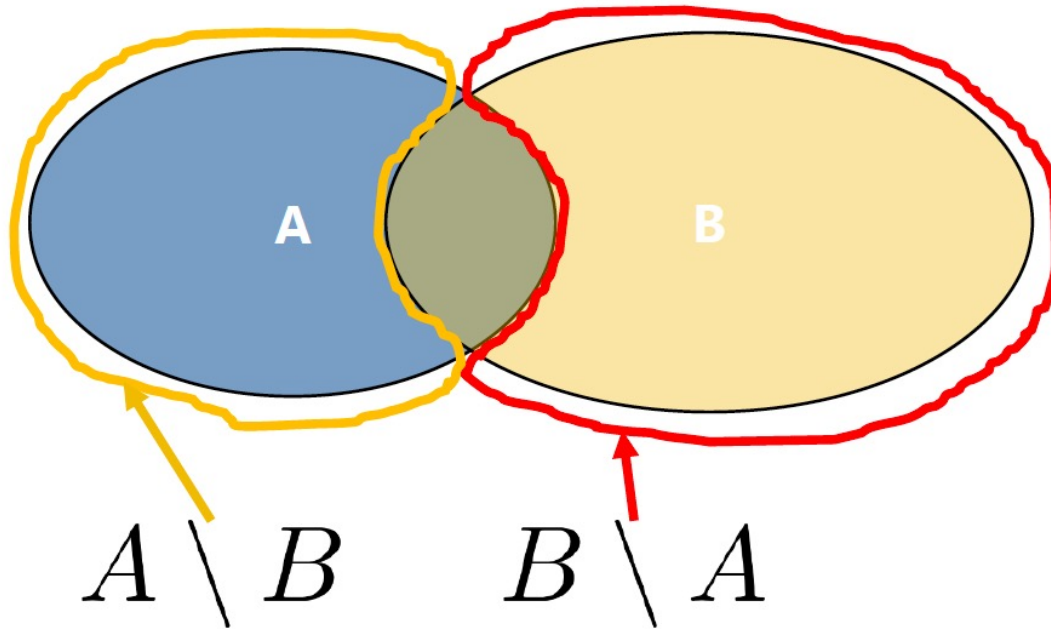
$R_{:,i}$ : Binary representation of users who purchased item  $i$

$$I_u = \{i | R_{u,i} = 1\}, U_i = \{u | R_{u,i} = 1\}$$

# MEASURING USER SIMILARITY: IMPLICIT FEEDBACK

## 1) Euclidean distance

- Between two items  $i, j$  or Between two users  $u, v$



$$\text{Euclidean distance } (A, B) = |A \setminus B| + |B \setminus A|$$

$$|U_i \setminus U_j| + |U_j \setminus U_i|$$

### Example 1

$$U_1 = \{1, 4, 8, 9, 11, 23, 25, 34\}$$

$$U_2 = \{1, 4, 6, 8, 9, 11, 23, 25, 34, 35, 38\}$$

$$|U_1 \setminus U_2| + |U_2 \setminus U_1| = 3$$

### Example 2

$$U_3 = \{4\}$$

$$U_4 = \{5\}$$

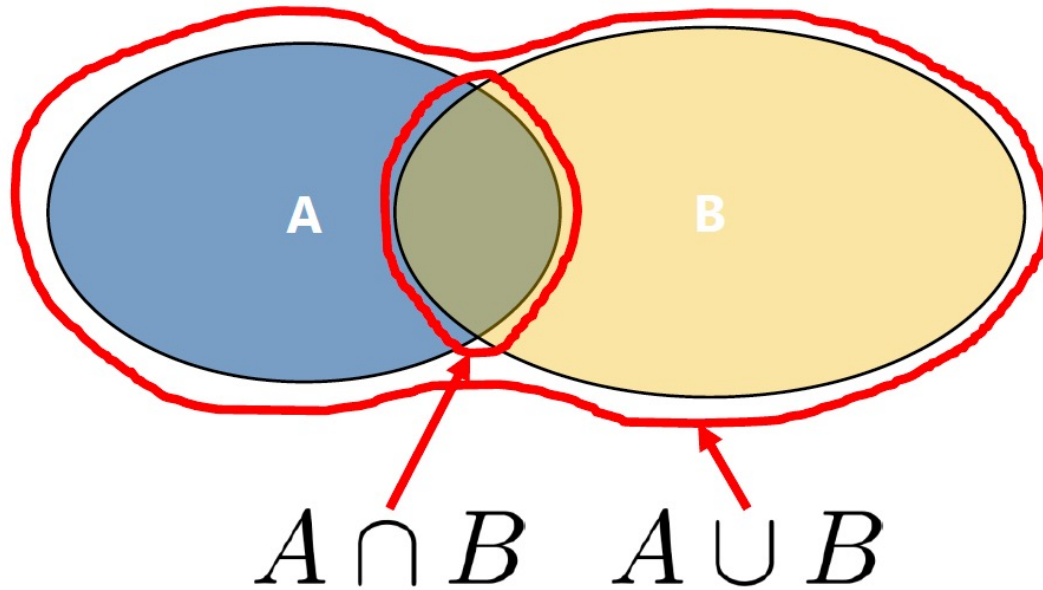
$$|U_3 \setminus U_4| + |U_4 \setminus U_3| = 2$$

**Problem:** Euclidean distance favors small sets, even if they have few elements in common

# MEASURING USER SIMILARITY: IMPLICIT FEEDBACK

## ▪ 2) Jaccard similarity

- Between two items  $i, j$  or Between two users  $u, v$



$$\text{Jaccard}(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

$$\text{Jaccard}(U_i, U_j) = \frac{|U_i \cap U_j|}{|U_i \cup U_j|}$$

$$(0 \leq \text{Jaccard}(U_i, U_j) \leq 1)$$

**Maximum of 1** if the **two users purchased exactly the same set** of items. i.e.,  $U_i = U_j$

**Minimum of 0** if the **two users purchased completely disjoint sets** of items. i.e.,  $U_i \cap U_j = \emptyset$

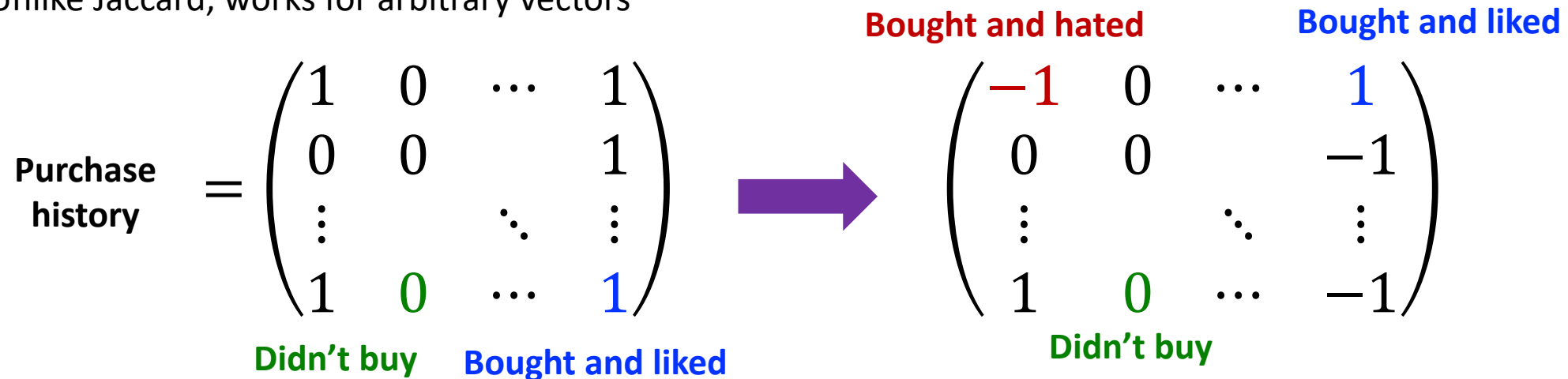
**Problem:** Jaccard similarity can only take care of binary relevance scores

# MEASURING USER SIMILARITY: IMPLICIT FEEDBACK

## 3) Cosine similarity

- Between two items  $i, j$  or Between two users  $u, v$
- Unlike Jaccard, works for arbitrary vectors

$$\text{Cosine Similarity } (A, B) = \frac{A \cdot B}{\|A\| \|B\|}$$



If  $\text{cosine}(I_u, I_v) = 1$ ?

- Angle between  $I_u, I_v = 0$
- Users  $u$  and  $v$  rated the same items and they all agree

If  $\text{cosine}(I_u, I_v) = -1$ ?

- Angle between  $I_u, I_v = 180$
- Users  $u$  and  $v$  rated the same items and they all disagree

If  $\text{cosine}(I_u, I_v) = 0$ ?

- Angle between  $I_u, I_v = 90$
- Users  $u$  and  $v$  rated completely different sets of items

**What if we have numerical ratings?**

# MEASURING USER SIMILARITY: EXPLICIT FEEDBACK

- A popular similarity measure in user-based CF: **Pearson correlation**

- Possible similarity values between  $-1$  and  $1$

- **Different users may provide ratings on different scales**

- **Solution:** Subtract the average

- Values are negative for below-average ratings and positive for above-average ratings

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

sim = 0.85  
sim = 0.00  
sim = 0.70  
sim = -0.79

Items rated by both users  $u$  and  $v$

Average rating by user  $v$

$$Pearson(u, v) = \frac{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R}_u)(R_{v,i} - \bar{R}_v)}{\sqrt{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R}_u)^2 \sum_{i \in I_u \cap I_v} (R_{v,i} - \bar{R}_v)^2}}$$

# MAKING PREDICTIONS

- **A common prediction function**

- **User-based CF:** The most similar items should be the most relevant when predicting future ratings, so the user's past ratings of those items are given the highest weights

$$\hat{R}_{u,i} = \frac{\sum_{v \in U_i} \text{sim}(u, v) * R_{v,i}}{\sum_{v \in U_i} \text{sim}(u, v)} \quad \Rightarrow \quad \hat{R}_{u,i} = \bar{R}_u + \frac{\sum_{v \in U_i} \text{sim}(u, v) * (R_{v,i} - \bar{R}_v)}{\sum_{v \in U_i} \text{sim}(u, v)}$$

- **Item-based CF**

$$\hat{R}_{u,i} = \frac{\sum_{j \in I_u} \text{sim}(i, j) * R_{u,j}}{\sum_{j \in I_u} \text{sim}(i, j)} \quad \Rightarrow \quad \hat{R}_{u,i} = \bar{R}_{:,i} + \frac{\sum_{j \in I_u} \text{sim}(i, j) * (R_{u,j} - \bar{R}_{:,j})}{\sum_{j \in I_u} \text{sim}(i, j)}$$

- **How many neighbors, i.e.,  $|U_i|$ ,  $|I_u|$ ?**

- Only consider positively correlated neighbors (or higher threshold)
- Can be optimized based on data set (e.g., Cross-validation)
- Often, between 50 and 200

# USER-BASED CF vs. ITEM-BASED CF

- **Item-based methods** often provide more relevant recommendations because **a user's own ratings are used** to perform the recommendation
  - i.e., Similar items are those that are consumed by the user, and the user's own ratings on those items are used to extrapolate the ratings of the target item
- Item-based methods can also provide a **reason for the recommendation**

*Because you watched "Secrets of the Wings," [the recommendations are...]*

- For user-based methods, "neighboring users" are **simply a set of anonymous users**
- In social recommendation setting, actual friends can be "neighboring users", which can provide explanations for the recommendation (Later in this course)



# IMPROVING THE METRICS

- **Not all neighbor ratings might be equally "valuable"**
  - Agreement on commonly liked items is not so informative as agreement on controversial items
  - **Possible solution:** Give more weight to items that have a higher variance
- **Value of number of co-rated items**
  - Use “significance weighting”, by e.g., linearly reducing the weight when the number of co-rated items is low
- **Case amplification**
  - Intuition: Give more weight to "very similar" neighbors, i.e., where the similarity value is close to 1.
- **Neighborhood selection**
  - Use similarity threshold or fixed number of neighbors

# NEIGHBORHOOD-BASED CF: PROS/CONS

- Neighborhood-based CF is also known as **memory-based CF** (Why?)
  - vs. Model-based CF
- **Pros**
  - Requires minimal knowledge (only rating/implicit feedback)
  - Produces good-enough results in most cases
- **Challenges**
  - **Sparsity / Cold-start problem**
    - We need enough users in the system
    - New items need to get enough ratings.
    - New users need to provide enough ratings (cold start)
  - **Scalability**
    - Nearest neighbor require computation that grows with both the number of users and the number of items
    - If one user purchases one item, this will change the rankings of **every other item that was purchased by at least one user in common**
  - **Diversity**: Does not encourage diverse results

# SPARSITY / COLD-START PROBLEM

- How to recommend new items? What to recommend to new users?
- **New User Problem:** the system must first learn the user's preferences from the ratings
  - E.g., Watcha asks new users to rate 10 movies when they register
  - Hybrid RS, which combines content-based and collaborative techniques, can help
- **New Item Problem:** Until the new item is rated by a substantial number of users, the RS is not able to recommend it
  - Hundreds/thousands of new items every day
    - Yahoo News: ~100 new articles / day
    - eBay or Amazon: >1000 items / day ???



# OUTLINE

- Neighborhood-based Collaborative Filtering
- A Regression Modeling View of Neighborhood Methods
- Evaluating Recommender System

# A REGRESSION MODELING VIEW OF NEIGHBORHOOD METHODS

- Recall the prediction function of item-based CF

$$\hat{R}_{u,i} = \bar{R}_{:,i} + \frac{\sum_{j \in I_u} \text{sim}(i,j) * (R_{u,j} - \bar{R}_{:,j})}{\sum_{j \in I_u} \text{sim}(i,j)} \quad \begin{array}{l} \forall u \in \{1 \dots m\} \\ \forall i \in \{1 \dots n\} \end{array}$$

- The predicted rating  $\hat{R}_{u,i}$  is a **weighted linear combination** of other ratings of the same user  $u$ , i.e.,  $I_u$
- If we allow  $I_u$  to contain all items, then the function is similar to linear regression
  - Linear regression: The coefficients are determined by optimization model
  - This case: The coefficients are **heuristically** determined by  $\text{sim}(i,j)$
- Can we think about an optimization-based neighborhood methods?

$$W^{item} \in R^{n \times n}$$

$$\hat{R}_{u,i} = \bar{R}_{:,i} + \frac{\sum_{j \in I_u} \text{sim}(i,j) * (R_{u,j} - \bar{R}_{:,j})}{\sum_{j \in I_u} \text{sim}(i,j)} \quad \Rightarrow \quad \hat{R}_{u,i} = \bar{R}_{:,i} + \sum_{j \in I_u} W_{i,j}^{item} * (R_{u,j} - \bar{R}_{:,j})$$

# SPARSE LINEAR MODELS (SLIM)

- SLIM **does not restrict the regression coefficients to only the neighborhood** of the target item

$$\hat{R}_{u,i} = \sum_{j \in I_u} W_{i,j}^{item} * R_{u,j} \quad \longrightarrow \quad \hat{R}_{u,i} = \sum_{j=1}^n W_{i,j}^{item} * R_{u,j} \quad \longrightarrow \quad \hat{R} = RW^{item}$$

- The target item itself is excluded on the right-hand side to prevent overfitting
  - This can be achieved by  $W_{t,t}^{item} = 0$ , for  $t = 1, \dots, n$

Elastic-net regularizer

$$\min_W \frac{1}{2} \|R - RW^{item}\|_F^2 + \frac{\beta}{2} \|W^{item}\|^2 + \lambda |W^{item}|$$

subject to  $W^{item} \geq 0$   
 $diag(W^{item}) = 0$

$$|W| = \sum_{i=1}^n \sum_{j=1}^n |W_{i,j}|$$
$$\|W\|^2 = \sum_{i=1}^n \sum_{j=1}^n w_{ij}^2$$

# RECALL: SHRINKAGE METHODS (IN LINEAR REGRESSION)

- Shrinkage methods *constrain* or *regularize* the coefficient estimates, or equivalently shrink the coefficient estimates to zero
- It turns out that shrinking estimated coefficients towards zero can significantly reduce their variance
- Two best-known techniques are **ridge regression** and **lasso**


- Overview**

- Linear regression

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{X}\beta\|^2$$


- Ridge regression

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} (\|\mathbf{y} - \mathbf{X}\beta\|^2 + \lambda \|\beta\|^2), \text{ where } \|\beta\|^2 = \sum_{j=1}^p \beta_j^2$$

 **L2 norm**

- Lasso

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} (\|\mathbf{y} - \mathbf{X}\beta\|^2 + \lambda |\beta|), \text{ where } |\beta| = \sum_{j=1}^p |\beta_j|$$

 **L1 norm**

# RECALL: RIDGE vs. LASSO

- The Lasso and ridge regression coefficient estimates solve the following problems
- For each value of  $\lambda$ , there exists a value for  $s$  such that
  - Ridge regression

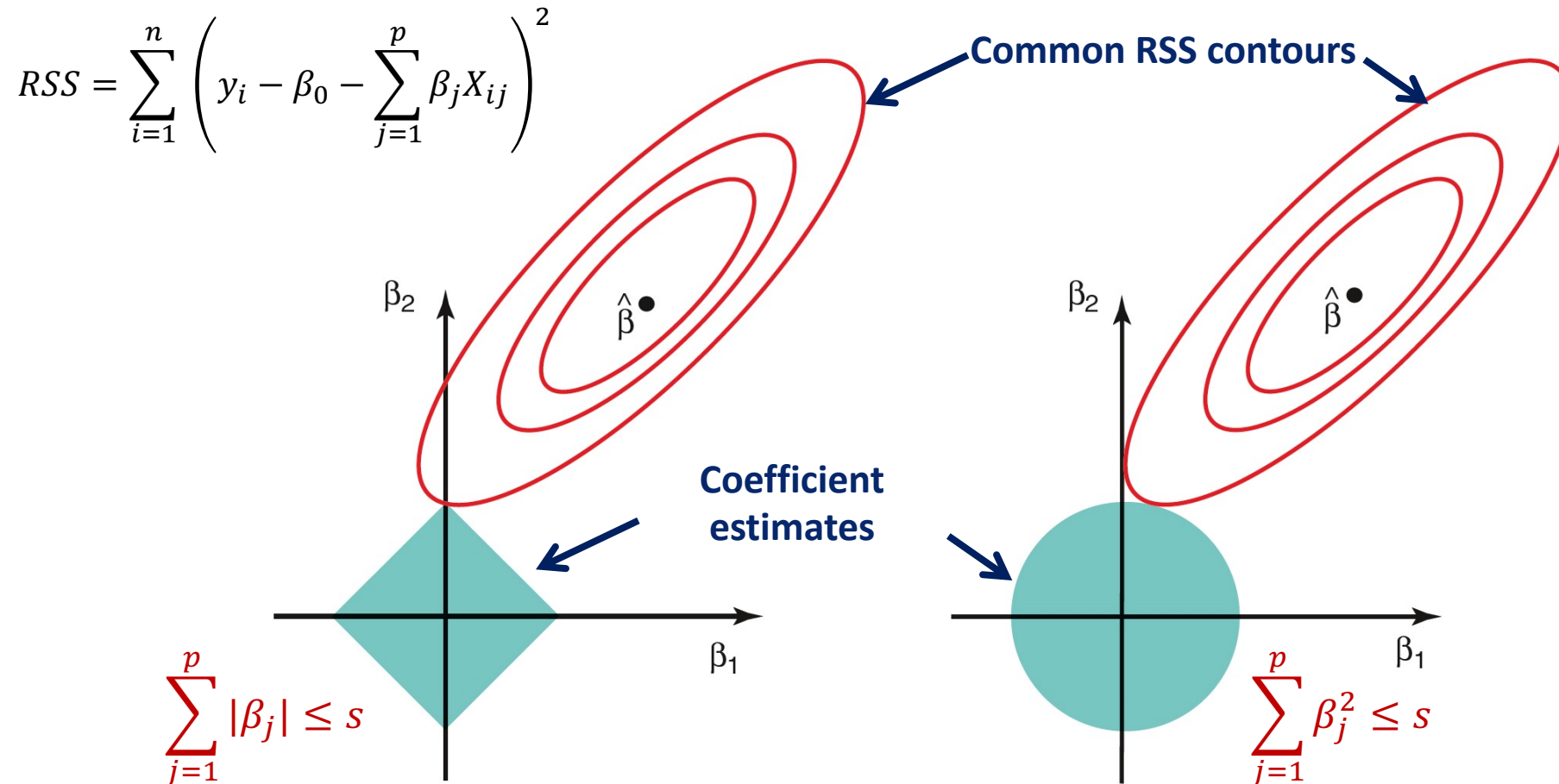
$$\sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^p \beta_j X_{ij} \right)^2 \quad \text{subject to} \quad \sum_{j=1}^p \beta_j^2 \leq s$$

- Lasso

$$\sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^p \beta_j X_{ij} \right)^2 \quad \text{subject to} \quad \sum_{j=1}^p |\beta_j| \leq s$$



# THE LASSO PICTURE: COMPARING CONSTRAINT FUNCTIONS



**FIGURE 6.7.** Contours of the error and constraint functions for the lasso (left) and ridge regression (right). The solid blue areas are the constraint regions,  $|\beta_1| + |\beta_2| \leq s$  and  $\beta_1^2 + \beta_2^2 \leq s$ , while the red ellipses are the contours of the RSS.

# OUTLINE

- Neighborhood-based Collaborative Filtering
- A Regression Modeling View of Neighborhood Methods
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# EVALUATING RECOMMENDER SYSTEM

- Recall, there are broadly two tasks in recommender system
- We need different metrics for different tasks
- **1) Rating prediction → Regression**
  - **Input:** set of ratings for user/item pairs
  - **Output:** map from user/item pair to predicted rating
  - Metrics: Root Mean Square Error (RMSE), Mean Absolute Error (MAE)
- **2) Item ranking (top-k recommendation) → Ranking**
  - **Input:** set of user/item pairs such as shopping data, and an integer  $k$
  - **Output:** a list of  $k$  items for each user which are most likely to be bought by him/her
  - Metrics: Precision, Recall, ROC/AUC, F1, AP, RR, NDCG

	GLADIATOR	GODFATHER	BEN-HUR	GOODFELLAS	SCARFACE	SPARTACUS
U <sub>1</sub>	1			5		2
U <sub>2</sub>		5			4	
U <sub>3</sub>	5	3		1		
U <sub>4</sub>			3			4
U <sub>5</sub>				3	5	
U <sub>6</sub>	5		4			?

(a) Ordered ratings

	GLADIATOR	GODFATHER	BEN-HUR	GOODFELLAS	SCARFACE	SPARTACUS
U <sub>1</sub>	1			1		1
U <sub>2</sub>		1			1	
U <sub>3</sub>	1	1		1		
U <sub>4</sub>			1			1
U <sub>5</sub>				1	1	
U <sub>6</sub>	1		1			?

(b) Unary ratings

# EVALUATION OF RATING PREDICTION

- **Datasets with items rated by users**

- MovieLens datasets 100K-10M ratings
- Netflix 100M ratings

- **Metrics measure error rate**

- **Mean Absolute Error (MAE):** Computes the deviation between predicted ratings and actual ratings

$$MAE = \frac{1}{n} \sum_{i=1}^n |p_i - r_i|$$

- **Root Mean Square Error (RMSE):** Similar to  $MAE$ , but places more emphasis on larger deviation

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (p_i - r_i)^2}$$

# EVALUATION OF ITEM RANKING

		Reality	
		Actually Good	Actually Bad
Prediction	Rated Good	True Positive (tp)	False Positive (fp)
	Rated Bad	False Negative (fn)	True Negative (tn)

Confusion matrix

## ■ Precision

- A measure of exactness, determines the fraction of relevant items retrieved out of all items retrieved
- E.g. the proportion of recommended movies that are actually good

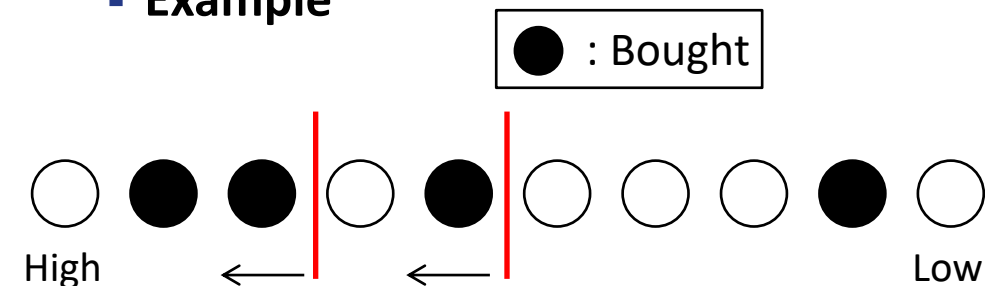
$$\text{Precision} = \frac{tp}{tp + fp} = \frac{|good\ movies\ recommended|}{|all\ recommendations|}$$

## ■ Recall

- A measure of completeness, determines the fraction of relevant items retrieved out of all relevant items
- E.g. the proportion of all good movies recommended

$$\text{Recall} = \frac{tp}{tp + fn} = \frac{|good\ movies\ recommended|}{|all\ good\ movies|}$$

## ■ Example

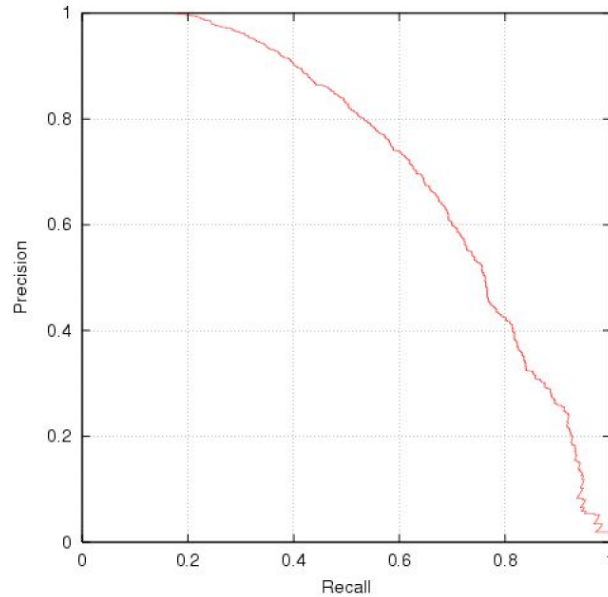


Assuming that 3 items are recommended:

- 2 out of 3 recommended items are actually bought: Precision@3=2/3
- 2 out of 4 bought items are recommended: Recall@3=2/4
- Ex) Precision@5 = 3/5, Recall@5 = 3/4

# TRADEOFF BETWEEN PRECISION AND RECALL

- Typically when a recommender system is tuned to increase precision, recall decreases as a result (or vice versa)

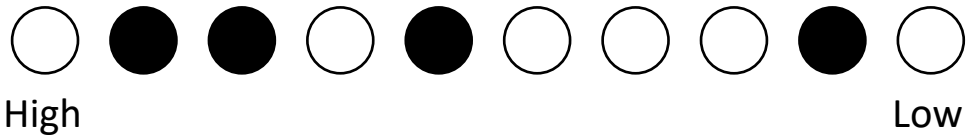


- The **F<sub>1</sub> Metric** attempts to combine Precision and Recall into a single value for comparison purposes.
  - May be used to gain a more balanced view of performance

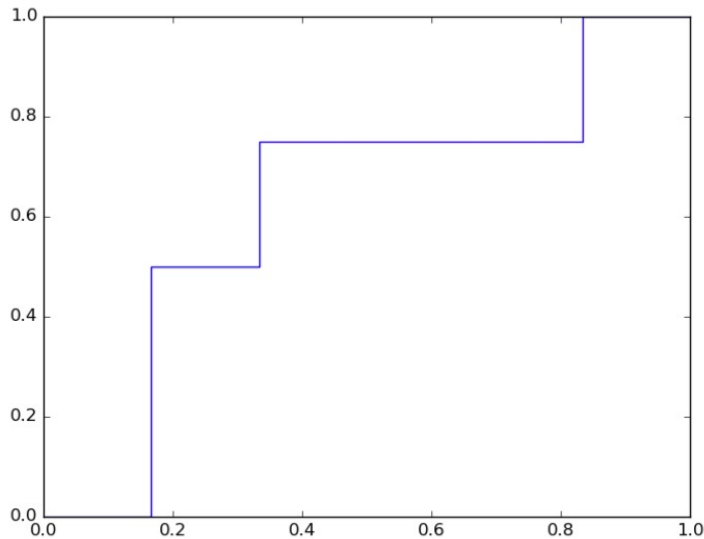
$$F_1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

# ROC & AUC

● : Bought



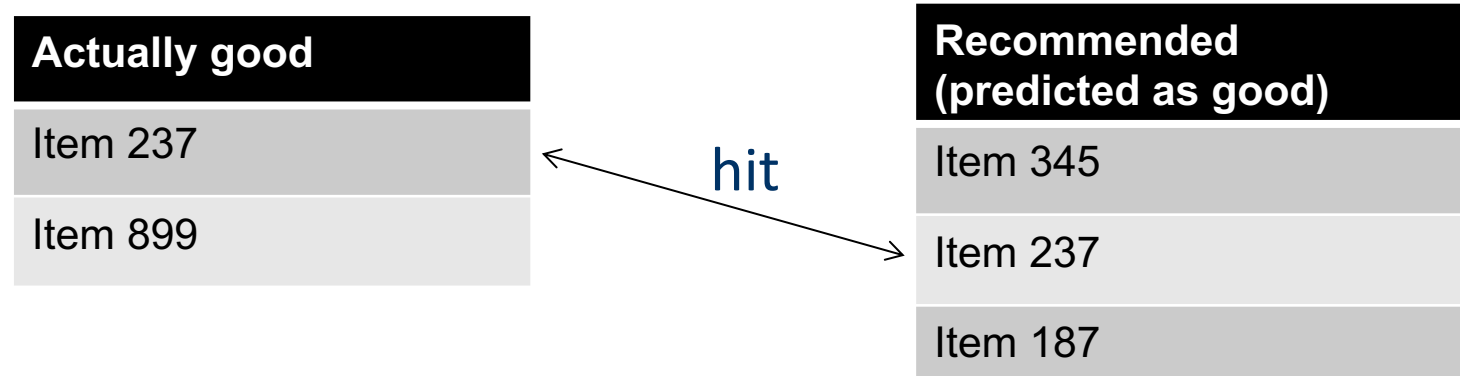
Num. recommendation	1	2	3	4	5	6	7	8	9	10
Num. whites	1	1	1	2	2	3	4	5	5	6
Num. blacks	0	1	2	2	3	3	3	3	4	4



- Divide the first and second row by total number of white and blacks respectively, and plot the values
- This curve is called **ROC curve**
- The area under this curve is called **AUC**
- Higher AUC is better (max =1)

# RANK POSITION MATTERS

- For a user:



- **Rank metrics** extend recall and precision to take the **positions of correct items** in a ranked list into account
  - Relevant items are more useful when they appear earlier in the recommendation list
  - Particularly important in recommender systems as lower ranked items may be overlooked by users
  - Average precision (AP), Reciprocal rank (RR), Normalized discounted cumulative gain (NDCG)



# AVERAGE PRECISION

- **Average Precision (AP)** is a ranked precision metric that places emphasis on highly ranked correct predictions (hits)
- Example

Rank	Hit?
1	
2	✓
3	✓
4	✓
5	

$$AP = \frac{1}{3} \left( \frac{1}{2} + \frac{2}{3} + \frac{3}{4} \right) = \frac{23}{36} \approx 0.639$$

Rank	Hit?
1	✓
2	
3	
4	✓
5	✓

$$AP = \frac{1}{3} \left( \frac{1}{1} + \frac{2}{4} + \frac{3}{5} \right) = \frac{21}{30} = 0.7$$

$$mAP = \frac{1}{|U|} \sum_{i=1}^{|U|} AP(u)$$
$$AP(u) = \frac{1}{|S|} \sum_{k=1}^n prec@k \cdot rel(k)$$

# RECIPROCAL RANK

- **Reciprocal Rank (RR)** is the sum of the inverse of the rank of the relevant items (hit) in a given list.

- Example

Rank	Hit?
1	
2	X
3	X
4	X
5	

$$RR = \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

Rank	Hit?
1	X
2	
3	
4	X
5	X

$$RR = 1 + \frac{1}{4} + \frac{1}{5}$$

$$MRR = \frac{1}{|U|} \sum_{u=1}^{|U|} RR(u)$$

$$RR(u) = \sum_{i=1}^k \frac{relevance_i}{rank_i}$$

# NORMALIZED DISCOUNTED CUMULATIVE GAIN (NDCG)

## ▪ Discounted cumulative gain (DCG)

- Logarithmic reduction factor

$$DCG_{pos} = rel_1 + \sum_{i=2}^{pos} \frac{rel_i}{\log_2 i}$$

- $pos$  denotes the position up to which relevance is accumulated
- $rel_i$  returns the relevance of recommendation at position  $i$

Rank	Hit?
1	
2	X
3	X
4	X
5	

## ▪ Idealized discounted cumulative gain (IDCG)

- Assumption that items are ordered by decreasing relevance

$$IDCG_{pos} = rel_1 + \sum_{i=2}^{|h|-1} \frac{rel_i}{\log_2 i}$$

$$DCG_5 = \frac{1}{\log_2 2} + \frac{1}{\log_2 3} + \frac{1}{\log_2 4} = 2.13$$

$$IDCG_5 = 1 + \frac{1}{\log_2 2} + \frac{1}{\log_2 3} = 2.63$$

## ▪ Normalized discounted cumulative gain (nDCG)

- Normalized to the interval [0..1]

$$nDCG_{pos} = \frac{DCG_{pos}}{IDCG_{pos}}$$

$$nDCG_5 = \frac{DCG_5}{IDCG_5} \approx 0.81$$

# CONCLUSION

- Neighborhood-based Collaborative Filtering
  - User-based CF / Item-based CF
  - Measuring similarity
  - Pros and cons of neighborhood-based CF
  - Sparsity / Cold-start problem
- A Regression Modeling View of Neighborhood Methods
  - Sparse Linear Model (SLIM)
- Evaluating Recommender System
  - Rating prediction and item ranking

**Coming up next:**  
**Model-based CF**