12.1.2

(a) $123.0 - 2.16 \times 20 = 79.8$

(b) $-2.16 \times 10 = -21.6$

(c) The purity has the distribution $X = N(123.0 - 2.16 \times 25, 4.1^2) = N(69.4.1^2)$. $P(X \leq 60) = 0.014$.

(d) $P(30 \leq N(123.0 - 2.16 \times 40, 4.1^2) \leq 40) = 0.743$

(e) Remark that, for any two independent normal distribution $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, we have $N(\mu_1, \sigma_1^2) - N(\mu_2, \sigma_2^2) = N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$. Therefore, $P(N(123.0 - 2.16 \times 30, 4.1^2) \leq N(123.0 - 2.16 \times 27.5, 4.1^2))$$= P(N(-5.4, 5.80^2) \leq 0) = 0.824$.

12.2.4

(a) We remark that

$$\hat{\beta}_1 = \frac{n \sum_{i=1}^n x_i y_i - \left(\sum_{i=1}^n x_i\right)\left(\sum_{i=1}^n y_i\right)}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i\right)^2}, \quad \hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}.$$

Using this, we have $y = \hat{\beta}_0 + \hat{\beta}_1 x = -2277 + 1.003x$

(b) $1.003 \times 1000 = 1003$

(c) $1.003 \times 10000 - 2277 = 7753$

(d) $\hat{\sigma}^2 = \frac{\mathrm{SSE}}{n-2} = 774211$. We remark that $n = 16$ and $\mathrm{SSE} = \sum_{i=1}^n (y_1 - (\hat{\beta}_0 + \hat{\beta}_1 x_i))^2$.

(e) $1.003 \times 20000 - 2277 = 17756$, but it is inaccurate since this is extrapolation.

12.2.6

(a) $y = 54.218 - 0.338x$

(b) No. $\hat{\beta}_1 < 0$ means that aerobic fitness gets worse with age. The predicted change in VO2$-$max for an additional 5 years of age is $5\hat{\beta}_1 = -1.68$.

(c) $54.218 - 0.338 \times 50 = 37.33$

(d) $54.218 - 0.338 \times 15 = 49.153$, but it is inaccurate since this is extrapolation.

(e) $\hat{\sigma}^2 = \frac{\mathrm{SSE}}{20-2} = 57.29$.

12.3.2 (a) A two-sided 95% confidence interval for β_1 is

$$(\hat{\beta}_1 - t_{0.025,20} s.e.(\hat{\beta}_1), \hat{\beta}_1 + t_{0.025,20} s.e.(\hat{\beta}_1)) = (48.44, 64.22) \quad \textbf{(+5 points)}$$

(b) The t-statistic for testing $H_0 : \beta_1 = 50.0$ is

$$t = \frac{\hat{\beta}_1 - 50.0}{s.e.(\hat{\beta}_1)} = 1.675 \quad \textbf{(+2 points)}$$

Then, the p-value is

$$\text{P-value} = 2P(T > 1.675) = 0.11 > 0.05$$

where $T \sim t_{20}$. Thus, H_0 is accepted. **(+3 points)**

12.3.8 (a) The standard error of $\hat{\beta}_1$ is

$$s.e.(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}} = \sqrt{\frac{SSE}{(n-2)S_{xx}}} = 0.064 \quad \textbf{(+3 points)}$$

(b) A two-sided 99% confidence interval for β_1 is

$$(\hat{\beta}_1 - t_{0.005,22} s.e.(\hat{\beta}_1), \hat{\beta}_1 + t_{0.005,22} s.e.(\hat{\beta}_1)) = (0.625, 0.985) \quad \textbf{(+3 points)}$$

(c) The t-statistic for testing $H_0 : \beta_1 = 0$ is

$$t = \frac{\hat{\beta}_1}{s.e.(\hat{\beta}_1)} = 12.527 \quad \textbf{(+2 points)}$$

Then, the p-value is

$$\text{P-value} = 2P(T > 12.527) = 0 < 0.01$$

where $T \sim t_{22}$. Thus, H_0 is rejected. **(+2 points)**

12.4.2 A two-sided 95% confidence interval for the expected response at $x^* = 40.0$ is

$$\left(\hat{\beta}_0 + \hat{\beta}_1 x^* - t_{0.025,15}\hat{\sigma}\sqrt{\frac{1}{17} + \frac{(x^* - \bar{x})^2}{S_{xx}}}, \hat{\beta}_0 + \hat{\beta}_1 x^* + t_{0.025,15}\hat{\sigma}\sqrt{\frac{1}{17} + \frac{(x^* - \bar{x})^2}{S_{xx}}}\right)$$

$= (1392, 1400)$ **(+10 points)**

12.4.8 From 12.3.8, $\hat{\beta}_0 = 12.8639, \hat{\beta}_1 = 0.8051$. The expected resistance at $70°F$ is $\hat{y} = 69.22, \hat{\sigma}^2 = 3.981, \bar{x} = 70.58$, and $S_{xx} = 963.83$. The two-sided 99% confidence interval for average resistance is

$$\left(\hat{y} - t_{0.005,n-2}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{70-\bar{x}}{S_{xx}}}, \hat{y} + t_{0.005,n-2}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{70-\bar{x}}{S_{xx}}}\right) = (68.07, 70.37).$$

(+10 points)

12.5.4 From 12.3.8, $\hat{\beta}_0 = 54.21805, \hat{\beta}_1 = -0.33767$. The expected VO2-max measurement is $\hat{y} = 54.21805 - 0.33767 \cdot 50 = 37.33455, \hat{\sigma}^2 = 57.29, \bar{x} = 46.4$, and $S_{xx} = 3486.8$. The two-sided 95% prediction interval for VO2-max measurement is

$$\left(\hat{y} - t_{0.025,n-2}\hat{\sigma}\sqrt{\frac{n+1}{n} + \frac{50-\bar{x}}{S_{xx}}}, \hat{y} + t_{0.025,n-2}\hat{\sigma}\sqrt{\frac{n+1}{n} + \frac{50-\bar{x}}{S_{xx}}}\right) = (21.01, 53.66).$$

(+10 points)

12.5.8 Expected $y = 51.98 + 3.44 \cdot 22 = 127.66$**(+2 points)**, $t_{0.025,28} = 2.048$, $S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = 443.44$, $\hat{\sigma} = \frac{SSE}{28} = 3.43$**(+3 points)**, $S = \hat{\sigma}\sqrt{\frac{n+1}{n} + \frac{(x_* - \bar{x})}{S_{xx}}} = 3.501$**(+3 points)**. Therefore, the range is $(120.49, 134.83)$**(+2 points)**.