7.2.4 Suppose that $E(X_1) = \mu$, $Var(X_1) = 1$, $E(X_2) = \mu$, and $Var(X_2) = 7$.
(a) What is the variance of
$$\hat{\mu_1} = \frac{X_1}{2} + \frac{X_2}{2}$$
(b) What value of p minimizes the variance of
$$\hat{\mu} = pX_1 + (1-p)X_2?$$
(c) What is the relative efficiency of $\hat{\mu_1}$ to the point estimate with the smallest variance that you have found?

7.2.8 Suppose that $X \sim B(10, p)$ and consider the point estimate
$$\hat{p} = \frac{X}{11}$$
(a) What is the bias of this point estimate?
(b) What is the variance of this point estimate?
(c) Show that this point estimate has a mean square error of
$$\frac{10p - 9p^2}{121}$$
(d) Show that this mean square error is smaller than the mean squre error of $X/10$ when $p \leq 21/31$.

7.3.8 In a consumer survey, 234 people out of a representative sample of 450 people say that they prefer product A to product B. Let p be the proportion of all consumers who prefer product A to product B. Construct a point estimate of p. What is the standard error of your point estimate?

7.3.28 The pH levels of food items prepared in a certain way are normally distributed with a standard deviation of $\sigma = 0.82$. An experimenter estimates the mean pH level by averaging the pH levels of a random sample of n items.
(a) If $n = 5$, what is the probability that the experimenter's estimate is within 0.5 of the true mean value?
(b) If $n = 10$, what is the probability that the experimenter's estimate is within 0.5 of the true mean value?
(c) What sample size n is needed to ensure that there is a probability of at least 99% that the experimenter's estimate is within 0.5 of the true mean value?

7.4.2 Suppose that a set of observations is collected from a beta distribution, with an average of $\overline{x} = 0.782$ and a variance of $s^2 = 0.0083$. Obtain point estimates of the parameters of the beta distribution.

7.4.4 If the random variables $X_1, \ldots, X_k$ have a multinomial distribution with parameters n and $p_1, \ldots, p_k$, the likelyhood function is

$$L(x_1, \ldots, x_k, p_1, \ldots, p_k) = \frac{n!}{x_1! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k} \tag{1}$$

Maximize this likelihood subject to the condition that

$$p_1 + \cdots + p_k = 1$$

in order to find the maximum likelihood estimates $\hat{p}_i$, $1 \leq i \leq k$.

7.7.8 **Programming Errors**

Consider the data set of programming errors given in DS 6.7.3. Construct a point estimate of the average number of errors per month. What is the standard error of your point estimate?

7.7.12 Suppose that among 24,839 customers of a certain company, exactly 11,842 feel "very satisfied" with the service they received. In order to estimate the satisfaction levels of the customers, a manager contacts a random sample of 80 of these customers and finds out how many of them were "very satisfied." What is the probability that the manager's estimate of the proportion of "very satisfied" customers in this group is within 0.10 of the true value?

7.7.23 Components have lengths that are independently distributed as a normal distribution with $\mu = 723$ and $\sigma = 3$. If an experimenter measures the lengths of a random sample of 11 components, what is the probability that the experimenter's estimate of μ will be between 722 and 724?