HW 7 - ~~2022~~4314 GSDS 강현무.

8.1.8. Since $-t_{\alpha/2,n-1} \le \frac{\bar{X}-\mu}{S/\sqrt{n}} \le t_{\alpha/2,n-1}$,

Length of t-interval $= 2 \cdot t_{\alpha/2,n-1} \cdot S/\sqrt{n} \le 0.2$

given $\alpha = 0.01$, $S = 0.15$, $(2 \cdot t_{0.005,n-1} \cdot 0.15 \times 5)^2 \le (\sqrt{n})^2$

8.1.9

$\therefore$ when $n=19$ it satisfy the condition.

$(2 \cdot 2.898 \cdot 0.15 \times 5)^2 \approx 18.8964 \le 19$

8.1.10. $L_0 = 2 \cdot t_{\alpha/2,n-1} \frac{S}{\sqrt{n}} \le 0.05$

given $\alpha = 0.01$, $S = 0.124$, $(2 \cdot t_{0.005,n-1} \cdot 0.124 \times 20)^2 \le n$

$\therefore$ when $n=167$, it satisfy the condition

$t_{0.005,166} \approx 2.604$ $(2 \cdot 2.604 \cdot 0.124 \times 20)^2 \approx 166.8189 \le 167$

8.1.14. 95% one tail confidence interval $= \left[\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le t_{0.05}\right]$

it means $C = \bar{X} - t_{0.05,n-1} \cdot \frac{\sigma}{\sqrt{n}}$

given $\bar{X} = 11.80$, $n=19$, $C = 11.80 - 1.734 \cdot 2.0/\sqrt{19} \approx 11.0044$

$\therefore$ $C \approx 11.0044$

8.1.16. Confidence interval $= (\bar{X} - t_{\alpha/2,n-1} \cdot S/\sqrt{n},\ \bar{X} + t_{\alpha/2,n-1} \cdot S/\sqrt{n})$

then $6.668 = \bar{X} - t_{\alpha/2,n-1} \cdot S/\sqrt{n}$, $7.054 = \bar{X} + t_{\alpha/2,n-1} \cdot S/\sqrt{n}$

$\Rightarrow t_{\alpha/2,15} \approx 1.7545$ $\alpha/2 = 0.05$.

$\therefore \alpha$ (confidence level) $= 0.1$

8.2.4. (a). $H_0 : \mu = 90.0$ V.S $H_A : \mu \neq 90.0$.

two tail confidence interval $= \left(\bar{X} - t_{\alpha/2, n-1} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + t_{\alpha/2, n-1} \cdot \frac{\sigma}{\sqrt{n}}\right)$

$= \left(87.9 - t_{\alpha/2, 43} \cdot \frac{5.90}{\sqrt{44}}, 87.9 + t_{\alpha/2, 43} \cdot \frac{5.90}{\sqrt{44}}\right)$

H_0.이 성립하려면. $\left.\begin{array}{l} \mu = 90 \leq 87.9 + t_{\alpha/2, 43} \cdot \frac{5.90}{\sqrt{44}} \\ 90 \geq 87.9 - t_{\alpha/2, 43} \cdot \frac{5.90}{\sqrt{44}} \end{array}\right]$ 을 충족 해야한다.

이를 충족시키려면. $\left.\begin{array}{l} t_{\alpha/2, 43} \geq 2.36099 \\ t_{\alpha/2, 43} \leq -2.36099 \end{array}\right)$ 이여야한다.

이때 유의확률 α는. 0.023. 이다. 따라서 p-value는. $0.\cancel{023}$ 이다.

(b) $H_0 : \mu < 86.0$, V.S. $H_A : \mu > 86.0$

$$p = P(T < t) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < t\right)$$

Since $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = -2.13613$, p-value $= 0.019$.

8.2.10. $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ $\sigma = 10$, $n = 29$. $H_0 : \mu \geq 420$

(a) test stastic z falls in acceptance interval.

$A = \{z : z \leq z_\alpha\}$

$\therefore$ when. $\alpha = 0.1$, $z < 1.282$.

(b) test static z fall in reject interval. $A\{z : z > z_\alpha\}$

when $\alpha = 0.01$ $z > 2.327$

(c) Since one-tail confidence interval: $\left(\bar{X} - z_\alpha \frac{\sigma}{\sqrt{n}}, \infty\right)$

then '$\bar{X} - z_\alpha \cdot \frac{\sigma}{\sqrt{n}} < 420$'이 확률이 α다.

$\Rightarrow$ $z_\alpha : > 2.3156$ $\therefore$ when $\alpha = 0.1$, reject

$\alpha = 0.01$. accept

(d) $p \approx 0.0103$

8.2.18. $H_0: \mu = 1.1$, $H_A: \mu \neq 1.1$

by DS 6.1.7. $\bar{X} = 1.1105$, $S = 0.0529$, $n = 125$

$$P = P\left(\left|\frac{\bar{X}-\mu}{S/\sqrt{n}}\right| < t_{\alpha/2,124}\right)$$

$$= P(2.21916 < t_{\alpha/2,124}) \approx 0.028.$$

$\therefore$ when $\alpha = 0.05$, H_0 is rejected.

8.2.24 $H_0: \mu \le 70$, $H_A: \mu > 70$.

$$P = P\left(\frac{\bar{X}-\mu}{S/\sqrt{n}} < t_{\alpha,24}\right)$$

$$\approx P(1.3239 < t_{\alpha,24}) \approx 0.099.$$

$\therefore$ when $\alpha = 0.1$, H_0 is rejected. So it is depend on α.

but $\alpha = 0.05$, H_0 is accepted.