9.2.8 **Antibiotic Efficacies (with $\alpha = 0.05$)**
Eight cultures of a bacterium are split in half. One half is tested using a standard antibiotic and the other half is tested using a new antibiotic. The data values in DS 9.2.8 are the times taken to kill the bacterium. Use an appropriate hypothesis test to assess whether there is any evidence that the new antibiotic is quicker than the standard antibiotic.

9.3.2 In an unpaired two-sample problem an experimenter observes $n = 14$, $\bar{x} = 32.45$, $s_x = 4.30$ from population A and $m = 14$, $\bar{y} = 41.45$, $s_y = 5.23$ from population B.
(a) Use the pooled variance method to construct a 99% two-sided confidence interval for $\mu_A - \mu_B$.
(b) Construct a 99% two-sided confidence interval for $\mu_A - \mu_B$ without assuming equal population variances.
(c) Consider a two-sided hypothesis test of $H_0 : \mu_A = \mu_B$ without assuming equal population variances. Does a size $\alpha = 0.01$ test accept or reject the null hypothesis? Write down an expression for the exact p-value.

9.3.6 The thicknesses of $n = 41$ glass sheets made using process A are measured and the statistics $\bar{x} = 3.04$ mm and $s_x = 0.124$ mm are obtained. In addition, the thicknesses of $m = 41$ glass sheets made using process B are measured and the statistics $\bar{y} = 3.12$ mm and $s_y = 0.137$ mm are obtained. Use a pooled variance procedure to answer the following questions.
(a) Does a two-sided hypothesis test with size $\alpha = 0.01$ accept or reject the null hypothesis that the two processes produce glass sheets with equal thicknesses on average?
(b) What is a two-sided 99% confidence interval for the difference in the average thicknesses of sheets produced by the two processes?

9.3.10 In an unpaired two-sample problem, an experimenter observes $n = 38$, $\bar{x} = 5.782$ from population A and $m = 40$, $\bar{y} = 6.443$ from population B. Suppose that the experimenter wishes to use values $\sigma_A = \sigma_B = 2.0$ for the population standard deviations.
(a) What is the exact p-value for the hypothesis testing problem $H_0 : \mu_A \geq \mu_B$ versus $H_A : \mu_A < \mu_B$?
(b) Construct a 99% one-sided confidence interval that provides an *upper bound* for $\mu_A - \mu_B$.

9.3.14 Consider again the data set in Problem 9.3.2 with sample sizes $n = m = 14$. If a two-sided 99% confidence interval for the difference in population means is required with a length no larger than $L_0 = 5.0$, what *additional* sample sizes would you recommend be obtained from the two populations?

9.3.22 The breaking strengths of 14 randomly selected objects produced from a standard procedure had a mean of 56.43 and a standard deviation of 6.30. In addition, the breaking strengths of 20 randomly selected objects produced from a new procedure had a mean of 62.11 and a standard deviation of 7.15. Perform a hypothesis test to investigate whether there is sufficient evidence to conclude that the new procedure has a larger breaking strength on average than the standard procedure. (**with** $\alpha = 0.05$)

9.7.9 **Comparing Two Population Variances**

For use with Problem 9.7.10

Recall that if S_x^2 is the sample variance of a set of n observations from a normal distribution with variance σ_A^2, then

$$S_x^2 \sim \sigma_A^2 \frac{\chi^2_{n-1}}{n-1}$$

and that if S_y^2 is the sample variance of a set of m observations from a normal distribution with variance σ_B^2, then

$$S_y^2 \sim \sigma_B^2 \frac{\chi^2_{m-1}}{m-1}$$

(a) Explain why

$$\frac{\sigma_A^2 S_y^2}{\sigma_B^2 S_x^2} \sim F_{m-1,n-1}$$

(b) Show that part (a) implies that

$$P\Big(F_{1-\alpha/2,m-1,n-1} \le \frac{\sigma_A^2 S_y^2}{\sigma_B^2 S_x^2} \le F_{\alpha/2,m-1,n-1}\Big) = 1-\alpha$$

or alternatively that

$$P\Big(\frac{1}{F_{\alpha/2,n-1,m-1}} \le \frac{\sigma_A^2 S_y^2}{\sigma_B^2 S_x^2} \le F_{\alpha/2,m-1,n-1}\Big) = 1-\alpha$$

(c) Deduce that

$$\frac{\sigma_A^2}{\sigma_B^2} \in \Big(\frac{s_x^2}{s_y^2\ F_{\alpha/2,n-1,m-1}}, \frac{s_x^2\ F_{\alpha/2,m-1,n-1}}{s_y^2}\Big) = 1-\alpha$$

is a 1-α level two-sided confidence interval for the ratio of the population variances.

If such a confidence interval contains the value 1, then this indicates that it is plausible that the population variances are equal. An unfortunate aspect of these confidence intervals, however, is that they depend heavily on the data being normally distributed, and they should be used only when that is a fair assumption. You may be able to obtain these confidence intervals on your computer package.

9.7.10 A sample of $n = 18$ observations from population A has a sample standard deviation of $s_x = 6.48$, and a sample of $m = 21$ observations from population B has a sample standard deviation of $s_y = 9.62$. Obtain a 90% confidence interval for the ratio of the population variances.

9.7.14 **Reinforced Cement Strengths (with $\alpha = 0.05$)**
The strengths of nine reinforced cement samples were tested using two procedures. Each sample was split into two parts, with one part being tested with procedure 1 and the other part being tested with procedure 2. The resulting data set is given in DS 9.6.7. Use an appropriate hypothesis test to assess whether there is any evidence that the two testing procedures provide different results on average.

9.7.22 **Sphygmomanometer and Finger Monitor Systolic Blood Pressure Measurements (compare the two methods with $\alpha = 0.05$)**
A sphygmomanometer is a standard instrument for measuring blood pressure in the arteries consisting of a pressure gauge and a rubber cuff that wraps around the upper arm. DS 9.6.13 compares blood pressure readings for 15 patients using this standard method and a new method based upon a simple finger monitor.