REGRA DA CADEIA

Em cada quadro abaixo, suponha que todas as funções envolvidas sejam diferenciáveis o que, como sabemos, acarreta a existência de todas as derivadas parciais listadas.

Também, apenas com o intuito de tornar as notações menos "carregadas" nos abstraímos de em cada derivada parcial colocarmos os pontos nos quais ela é calculada, por exemplo: (x_0, y_0) ou (x_0, y_0, z_0); ou mesmo, apenas x_0 nos casos de derivadas ordinárias.

01 $u = f(x, y)$;
$x = g(r, s)$; $y = h(r, s)$;

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r};$$
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s};$$

02 $u = f(x, y, z)$;
$x = g(r, s)$; $y = h(r, s)$; $z = p(r, s)$;

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial r};$$
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial s};$$

03 $u = f(x)$;
$x = g(r, s)$;

$$\frac{\partial u}{\partial r} = \frac{du}{dx}\frac{\partial x}{\partial r};$$
$$\frac{\partial u}{\partial s} = \frac{du}{dx}\frac{\partial x}{\partial s};$$

04 $u = f(x, y)$;
$x = g(r, s, t)$; $y = h(r, s, t)$;

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r};$$
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s};$$
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t};$$

05 $u = f(x, y, z)$;
$x = g(r, s, t)$; $y = h(r, s, t)$; $z = p(r, s, t)$;

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial r};$$
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial s};$$
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t};$$

06 $u = f(x)$;
$x = g(r, s, t)$;

$$\frac{\partial u}{\partial r} = \frac{du}{dx}\frac{\partial x}{\partial r};$$
$$\frac{\partial u}{\partial s} = \frac{du}{dx}\frac{\partial x}{\partial s};$$
$$\frac{\partial u}{\partial t} = \frac{du}{dx}\frac{\partial x}{\partial t};$$

07 $u = f(x, y)$;
$x = g(t)$; $y = h(t)$;

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt};$$

08 $u = f(x, y, z)$;
$x = g(t)$; $y = h(t)$; $z = p(t)$;

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt};$$

09 $u = f(x)$;
$x = g(t)$;

$$\frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt};$$

Outras situações podem, facilmente, ser compostas a partir destes. Como, por exemplo: se $u = f(x, y, z, w)$ com $x = g(r, s)$; $y = h(r, s)$; $z = p(r, s)$; $w = q(r, s)$; Temos o quadro **10**; ou, se $u = f(x, y)$ com $x = g(r, s, t, l)$; $y = h(r, s, t, l)$; Temos o quadro **11**:

10
$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial r} + \frac{\partial u}{\partial w}\frac{\partial w}{\partial r};$$
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial s} + \frac{\partial u}{\partial w}\frac{\partial w}{\partial s};$$

11
$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r};$$
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s};$$
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t};$$
$$\frac{\partial u}{\partial l} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial l} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial l};$$