Lista 3

2) verifique se a integral imprópria converge ou diverge, no caso de convergência, ache seu valor.

• a. $\int_1^{\infty} \frac{1}{x^{4/3}} dx = \lim_{b \to \infty} \int_1^b \frac{1}{x^{4/3}} dx = \frac{-3}{\sqrt[3]{b}} + 3 = \frac{-3}{\sqrt[3]{\infty}} + 3 = -0 + 3 = 3$

$\int_1^b \frac{1}{x^{4/3}} dx = -\frac{1}{(\frac{4}{3}-1)x^{(4/3-1)}}\Big|_1^b = -\frac{1}{\frac{1}{3}x^{1/3}}\Big|_1^b = -1 \cdot \frac{3}{x^{1/3}}\Big|_1^b$

$= -\frac{3}{\sqrt[3]{x}}\Big|_1^b = \frac{-3}{\sqrt[3]{b}} - \left(-\frac{3}{\sqrt[3]{1}}\right) = \frac{-3}{\sqrt[3]{b}} + 3$ $\quad * \int \frac{1}{x^n} dx = -\frac{1}{(n-1)x^{n-1}} \quad n \neq 1$

• b. $\int_{-\infty}^{-1} 1/x^3 \, dx = \lim_{a \to \infty} \int_a^{-1} 1/x^3 \, dx = -\frac{1}{2} + \frac{1}{2\infty^2} = -\frac{1}{2} + 0 = -\frac{1}{2}$

$\int_a^{-1} \frac{1}{x^3} dx = -\frac{1}{(3-1)x^{3-1}}\Big|_a^{-1} = -\frac{1}{2x^2}\Big|_a^{-1} = \left(-\frac{1}{2(-1)^2}\right) - \left(-\frac{1}{2a^2}\right)$

$= -\frac{1}{2} + \frac{1}{2a^2}$

• c. $\int_{-\infty}^{+\infty} \frac{1}{1+x^2} dx = \lim_{a \to -\infty} \int_a^0 \frac{1}{1+x^2} dx + \lim_{b \to \infty} \int_0^b \frac{1}{1+x^2} dx =$

$\lim_{a \to -\infty} \arctan(x)\Big|_a^0 + \lim_{b \to \infty} \arctan(x)\Big|_0^b =$

$\lim_{a \to -\infty} [\arctan(0) - \arctan(a)] + \lim_{b \to \infty} [\arctan(b) - \arctan(0)] =$

$\lim_{a \to -\infty} -\arctan(a) + \lim_{b \to \infty} \arctan(b) = \frac{\pi}{2} + \frac{\pi}{2} = \frac{2\pi}{2} = \pi$

$* \lim_{x \to -\infty} \arctan(x) = -\pi/2$

$$\cdot\ d.\ \int_2^{\infty} \frac{1}{(x-1)^2}\,dx = \lim_{b\to\infty}\int_2^{b} \frac{1}{(x-1)^2}\,dx = \lim_{b\to\infty}\int_2^{b} \frac{1}{t^2}\,dt =$$

$$= \lim_{b\to\infty}\left(-\frac{1}{(2-1)t^{2-1}}\right)\Bigg|_2^{b} = \lim_{b\to\infty} -\frac{1}{t}\Bigg|_2^{b} = \lim_{b\to\infty} \frac{-1}{x-1}\Bigg|_2^{b} =$$

$$\lim_{b\to\infty}\left(-\frac{1}{b-1}+\frac{1}{2-1}\right) = \lim_{b\to\infty}\left(-\frac{1}{b-1}+1\right) = -\lim_{b\to\infty}\frac{1}{b-1} + \lim_{b\to\infty} 1$$

$$= 0 + 1 = 1$$

$$*\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\cdot\arctan\left(\frac{x}{a}\right)$$

$$\cdot\ e.\ \int_{-\infty}^{\infty} \frac{1}{x^2+6x+12}\,dx = \int_{-\infty}^{\infty} \frac{1}{x^2+6x+9+3}\,dx = \int_{-\infty}^{\infty} \frac{1}{(x+3)^2+3}\,dx$$

$$= \lim_{a\to-\infty}\int_a^{0} \frac{1}{(x+3)^2+3}\,dx + \lim_{b\to\infty}\int_0^{b} \frac{1}{(x+3)^2+3}\,dx =$$

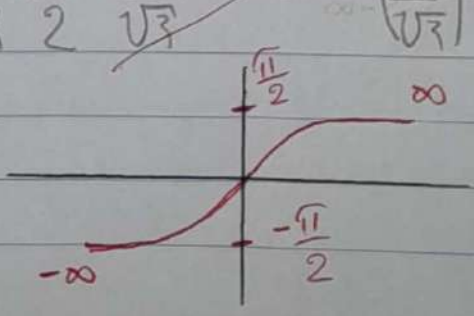
$$= \lim_{a\to-\infty} \frac{1}{\sqrt{3}}\cdot\arctan\left(\frac{x+3}{\sqrt{3}}\right)\Bigg|_a^{0} + \lim_{b\to\infty} \frac{1}{\sqrt{3}}\cdot\arctan\left(\frac{x+3}{\sqrt{3}}\right)\Bigg|_0^{b}$$

$$= \lim_{a\to-\infty}\left[\frac{1}{\sqrt{3}}\cdot\arctan\left(\frac{0+3}{\sqrt{3}}\right) - \frac{1}{\sqrt{3}}\cdot\cancel{\arctan\left(\frac{a+3}{\sqrt{3}}\right)}^{\to -\infty}\right] +$$

$$\lim_{b\to\infty}\left[\frac{1}{\sqrt{3}}\cdot\cancel{\arctan\left(\frac{b+3}{\sqrt{3}}\right)}^{\to \infty} - \frac{1}{\sqrt{3}}\cdot\arctan\left(\frac{0+3}{\sqrt{3}}\right)\right]$$

$$= \left(\cancel{\frac{1}{\sqrt{3}}\cdot\arctan\left(\frac{3}{\sqrt{3}}\right)} - \frac{1}{\sqrt{3}}\cdot -\frac{\pi}{2}\right) + \left(\frac{1}{\sqrt{3}}\cdot\frac{\pi}{2} - \cancel{\frac{1}{\sqrt{3}}\cdot\arctan\left(\frac{3}{\sqrt{3}}\right)}\right)$$

$$= \frac{\pi}{2\sqrt{3}} + \frac{\pi}{2\sqrt{3}} = \frac{2\pi}{2\sqrt{3}} = \boxed{\frac{\pi}{\sqrt{3}}}$$



• f. $\int_0^1 \frac{1}{\sqrt{1-x}}\,dx = \lim_{b\to 1}\int_0^b \frac{1}{\sqrt{1-x}}\,dx = \lim_{b\to 1}\int_0^b -\frac{1}{\sqrt{t}}\,dt$

$= \lim_{b\to 1} -2\sqrt{t}\,\Big|_0^b = \lim_{b\to 1} -2\sqrt{1-x}\,\Big|_0^b = \lim_{b\to 1}\left(-2\sqrt{1-b}+2\sqrt{1-0}\right)$

$= \lim_{b\to 1} -2\sqrt{1-b}+2 = -2\cdot 0+2 = \boxed{2}$ $\quad * \int \frac{1}{\sqrt{x}} = 2\sqrt{x}$

• g. $\int_0^1 x\ln(x)\,dx = \lim_{a\to +0}\int_a^1 x\ln(x)\,dx = \lim_{a\to +0}\left(\frac{x^2}{2}\ln(x)-\frac{x^2}{4}\right)\Big|_a^1$

$= \lim_{a\to +0}\left[\left(\cancelto{0}{\frac{1^2}{2}\ln(1)}-\frac{1^2}{4}\right)-\left(\frac{a^2}{2}\ln(a)-\frac{1^2}{4}\right)\right] = \lim_{a\to +0}\left(-\frac{1}{4}-\frac{a^2}{2}\ln(a)+\frac{a^2}{4}\right)$

$= -\frac{1}{4}+\lim_{a\to +0}\left(\cancelto{0}{-\frac{a^2}{2}\ln(a)}+\cancelto{0}{\frac{a^2}{4}}\right) = -\frac{1}{4}-0+0 = \boxed{-\frac{1}{4}}$

$u = \ln(x) \quad dv = x\,dx \quad du = 1/x\,dx \quad v = \frac{x^2}{2}$

$\int x\ln(x)\,dx = \frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx = \frac{x^2}{2}\ln(x) - \int \frac{x}{2}\,dx$

$= \frac{x^2}{2}\ln(x) - \frac{x^2}{4} + C$

$\lim_{a\to +0} -\frac{a^2}{2}\ln(a) = \lim_{a\to +0}\left(-\frac{\ln(a)}{2/a^2}\right) = \lim_{a\to +0}\left(-\frac{\frac{1}{a}}{-\frac{4}{a^3}}\right) =$

$\lim_{a\to +0} -\frac{1}{a}\cdot\left(-\frac{a^3}{4}\right) = \lim_{a\to +0}\frac{a^2}{4} = 0$

1. Calcule cada limite abaixo, se existir

• a. $\lim_{x\to 0}\frac{x}{tg(x)} = \lim_{x\to 0}\frac{1}{sec^2 x} = \frac{1}{1} = \boxed{1}$

L'Hospital

$t = \frac{1}{x}$ $x = \frac{1}{t}$ Dado $x \to +\infty$ e $t = \frac{1}{x}$, então $t \to 0$

- b. $\lim\limits_{x \to +\infty} x \operatorname{sen}\left(\frac{1}{x}\right) = \lim\limits_{x \to +\infty} \left(\frac{1}{t} \operatorname{sen}(t)\right) = \lim\limits_{t \to 0} \left(\frac{\operatorname{sen}(t)}{t}\right)$

$\overset{!}{=} \lim\limits_{t \to 0} \left(\frac{\cos(t)}{1}\right) = \boxed{1}$

- c. $\lim\limits_{x \to \frac{\pi}{2}} \frac{\ln(\operatorname{sen}(x))}{(\pi - 2x)^2} = \lim\limits_{x \to \frac{\pi}{2}} \frac{-\frac{1}{\operatorname{sen}^2 x}}{8} = -\frac{1}{\operatorname{sen}^2 x} \cdot \frac{1}{8} = \frac{-1}{8 \operatorname{sen}^2 x} = \frac{-1}{8 \cdot 1^2} = \boxed{\frac{-1}{8}}$

$\frac{[\ln(\operatorname{sen}(x))]''}{[(\pi - 2x)^2]''} = \frac{(\cot(x))'}{(4\pi - 8x)'} = \frac{-\operatorname{cossec}^2 x}{8} = \frac{-\left(\frac{1}{\operatorname{sen}(x)}\right)^2}{8}$

- d. $\lim\limits_{x \to 0} \frac{2^x - 3^x}{x} = \lim\limits_{x=0} (\ln(2) 2^0 - \ln(3) \cdot 3^0) = \ln(2) \cdot 1 - \ln(3) \cdot 1 = \boxed{\ln\left(\frac{2}{3}\right)}$

$\frac{(2^x - 3^x)'}{x'} = \frac{\ln(2) \cdot 2^x - \ln(3) \cdot 3^x}{1} = \ln(2) \cdot 2^x - \ln(3) \cdot 3^x$

e. $\lim\limits_{x \to 0} \frac{x - \operatorname{sen}(x)}{tg^3(x)} = \lim\limits_{x \to 0} \left(\frac{\cos^4(x)}{3 + 3\cos x}\right) = \frac{1^4}{3+3} = \boxed{\frac{1}{6}}$

$\frac{(x - \operatorname{sen}(x))'}{(tg^3(x))'} = \frac{1 - \cos(x)}{3 tg^2(x) \cdot \sec^2(x)} = \frac{1 - \cos(x)}{\left(\frac{3 \operatorname{sen}^2(x)}{\cos^4(x)}\right)} = 1 - \cos(x) \cdot \frac{\cos^4(x)}{3 \operatorname{sen}^2(x)}$

$= \frac{(1 - \cos(x)) \cdot (\cos^4(x))}{3(1^2 - \cos^2(x))} = \frac{(1 - \cos(x)) \cdot (\cos^4 x)}{3[(1 - \cos(x)) \cdot (1 + \cos(x))]} = \frac{\cos^4(x)}{3 + 3\cos(x)}$

$3 tg^2(x) \cdot \sec^2(x) = 3\left(\frac{\operatorname{sen}(x)}{\cos(x)}\right)^2 \cdot \left(\frac{1}{\cos(x)}\right)^2 = 3\left[\frac{\operatorname{sen}^2 x}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\right] = \frac{3 \operatorname{sen}^2 x}{\cos^4 x}$

• f. $\lim\limits_{x \to +\infty} \dfrac{1 - e^{1/x}}{-3/x} = \dfrac{\frac{e^{1/x}}{x^2}}{3/x^2} = \dfrac{e^{1/x}}{x^2} \cdot \dfrac{x^2}{3} = \dfrac{e^{1/x}}{3} = \dfrac{e^{1/\infty}}{3} \cong \dfrac{e^0}{3} = \dfrac{1}{3}$

$(1 - e^{1/x})' = 0 - (e^{1/x} \cdot (\frac{1}{x})') = -\left[e^{1/x}\left(\dfrac{1' \cdot x - 1 \cdot x'}{x^2}\right)\right] = -\left[e^{1/x} \cdot \left(-\dfrac{1}{x^2}\right)\right] = \dfrac{e^{1/x}}{x^2}$

$\left(-\dfrac{3}{x}\right)' = \dfrac{-3' \cdot x - (-3) \cdot x'}{x^2} = \dfrac{0 \cdot x - (-3) \cdot 1}{x^2} = \dfrac{3}{x^2}$

• g. $\lim\limits_{x \to 0} \dfrac{\cos x - \cosh x}{x^2} = \dfrac{-\cos x - \frac{e^x + e^{-x}}{2}}{2} = \dfrac{-1 - \frac{1+1}{2}}{2} = \dfrac{-1-1}{2} = \dfrac{-2}{2} = -1$

$(\cos x - \cosh x)'' = (-\operatorname{sen} x - \operatorname{senh} x)' = -\cos x - \cosh x = -\cos x - \dfrac{e^x + e^{-x}}{2}$

$(x^2)'' = (2x)' = 2$