RESOLUÇÃO · LISTA · DERIVADAS PARCIAIS

01 Em cada caso, encontre $f_x(x,y)$ e $f_y(x,y)$ se:

(a) $f(x,y) = 6x + 3y - 7$;

Solução: Temos; $f_x(x,y) = \dfrac{\partial(6x+3y-7)}{\partial x} = 6 + 0 = 6$;

e $f_y(x,y) = \dfrac{\partial(6x+3y-7)}{\partial y} = 0 + 3 - 0 = 3$;

(b) $f(x,y) = xy^2 + x^2y^3 + x^3y^4$;

Solução: Temos; $f_x(x,y) = \dfrac{\partial(xy^2+x^2y^3+x^3y^4)}{\partial x} = y^2\dfrac{\partial(x)}{\partial x} + y^3\dfrac{\partial(x^2)}{\partial x} + y^4\dfrac{\partial(x^3)}{\partial x} =$

$= y^2 + 2xy^3 + 3x^2y^4$;

e $f_y(x,y) = \dfrac{\partial(xy^2+x^2y^3+x^3y^4)}{\partial y} = x\dfrac{\partial(y^2)}{\partial y} + x^2\dfrac{\partial(y^3)}{\partial y} + x^3\dfrac{\partial(y^4)}{\partial y} =$

$= 2xy + 3x^2y^2 + 4x^3y^3$;

(c) $f(x,y) = (x+2y)^{10}.(2x+y)^9$;

Solução: Temos;

$f_x(x,y) = \dfrac{\partial[(x+2y)^{10}.(2x+y)^9]}{\partial x} = \dfrac{\partial(x+2y)^{10}}{\partial x}.(2x+y)^9 + (x+2y)^{10}.\dfrac{\partial(2x+y)^9}{\partial x} =$

$= 10(x+2y)^9.\dfrac{\partial(x+2y)}{\partial x}.(2x+y)^9 + (x+2y)^{10}.9(2x+y)^8.\dfrac{\partial(2x+y)}{\partial x} =$

$= 10(x+2y)^9.(2x+y)^9 + 18(x+2y)^{10}(2x+y)^8 =$

$= 2(x+2y)^9.(2x+y)^8[5(2x+y) + 9(x+2y)] =$

$= 2(x+2y)^9.(2x+y)^8.(19x+23y)$;

e $f_y(x,y) = \dfrac{\partial[(x+2y)^{10}.(2x+y)^9]}{\partial y} = \dfrac{\partial(x+2y)^{10}}{\partial y}.(2x+y)^9 + (x+2y)^{10}.\dfrac{\partial(2x+y)^9}{\partial y} =$

$= 10(x+2y)^9.\dfrac{\partial(x+2y)}{\partial y}.(2x+y)^9 + (x+2y)^{10}.9(2x+y)^8.\dfrac{\partial(2x+y)}{\partial y} =$

$= 20(x+2y)^9.(2x+y)^9 + 9(x+2y)^{10}.(2x+y)^8 =$

$= (x+2y)^9.(2x+y)^8.[20(2x+y) + 9(x+2y)] =$

$= (x+2y)^9.(2x+y)^8.(49x+38y)$;

(d) $F(x,y) = \dfrac{2x-3y}{x^2+y^2}$;

Solução: Temos;

$$F_x(x,y) = \frac{\partial\left[\frac{2x-3y}{x^2+y^2}\right]}{\partial x} = \frac{\frac{\partial(2x-3y)}{\partial x}.(x^2+y^2) - \frac{\partial(x^2+y^2)}{\partial x}.(2x-3y)}{(x^2+y^2)^2} =$$

$$= \frac{2(x^2+y^2)-2x(2x-3y)}{(x^2+y^2)^2} = \frac{2y^2-2x^2+6xy}{(x^2+y^2)^2};$$

e $$F_y(x,y) = \frac{\partial\left[\frac{2x-3y}{x^2+y^2}\right]}{\partial y} = \frac{\frac{\partial(2x-3y)}{\partial y}.(x^2+y^2) - \frac{\partial(x^2+y^2)}{\partial y}.(2x-3y)}{(x^2+y^2)^2} =$$

$$= \frac{-3(x^2+y^2)-2y(2x-3y)}{(x^2+y^2)^2} = \frac{3y^2-3x^2-4xy}{(x^2+y^2)^2};$$

(e) $F(x,y) = x\,\text{sen}\,y + y\,\text{sen}\,x + xy$;

Solução: Temos;

$$F_x(x,y) = \frac{\partial(x\,\text{sen}\,y + y\,\text{sen}\,x + xy)}{\partial x} = \text{sen}\,y\frac{\partial(x)}{\partial x} + y\frac{\partial(\text{sen}\,x)}{\partial x} + y\frac{\partial(x)}{\partial x} =$$

$$= \text{sen}\,y + y\cos x + y;$$

e $$F_y(x,y) = \frac{\partial(x\,\text{sen}\,y + y\,\text{sen}\,x + xy)}{\partial y} = x\frac{\partial(\text{sen}\,y)}{\partial y} + \text{sen}\,x\frac{\partial(y)}{\partial y} + x\frac{\partial(y)}{\partial y} =$$

$$= x\cos y + \text{sen}\,x + x;$$

(f) $F(x,y) = (x^2+y^2)^{-1/2}$;

Solução: Temos;

$$F_x(x,y) = \frac{\partial\left[(x^2+y^2)^{-1/2}\right]}{\partial x} = -\frac{1}{2}(x^2+y^2)^{-3/2}.\frac{\partial(x^2+y^2)}{\partial x} = -\frac{1}{2}(x^2+y^2)^{-3/2}.2x = \frac{-x}{(x^2+y^2)^{3/2}};$$

e

$$F_y(x,y) = \frac{\partial\left[(x^2+y^2)^{-1/2}\right]}{\partial y} = -\frac{1}{2}(x^2+y^2)^{-3/2}.\frac{\partial(x^2+y^2)}{\partial y} = -\frac{1}{2}(x^2+y^2)^{-3/2}.2y = \frac{-y}{(x^2+y^2)^{3/2}};$$

(g) $F(x,y) = e^{y/x}.\ln\left(\frac{x^2}{y}\right);$

Solução: Temos;

$$F_x(x,y) = \frac{\partial\left[e^{y/x}.\ln\left(\frac{x^2}{y}\right)\right]}{\partial x} = \frac{\partial\left(e^{y/x}\right)}{\partial x}.\ln\left(\frac{x^2}{y}\right) + e^{y/x}.\frac{\partial\left[\ln\left(\frac{x^2}{y}\right)\right]}{\partial x} =$$

$$= e^{y/x}.\frac{\partial\left(y/x\right)}{\partial x}.\ln\left(\frac{x^2}{y}\right) + e^{y/x}.\frac{1}{\frac{x^2}{y}}.\frac{\partial\left(\frac{x^2}{y}\right)}{\partial x} =$$

$$= e^{y/x}.\ln\left(\frac{x^2}{y}\right).\left(\frac{-y}{x^2}\right) + e^{y/x}.\frac{y}{x^2}.\frac{2x}{y} = \frac{e^{y/x}}{x}\left[2 - \frac{y}{x}.\ln\left(\frac{x^2}{y}\right)\right];$$

e $$F_y(x,y) = \frac{\partial\left[e^{y/x}.\ln\left(\frac{x^2}{y}\right)\right]}{\partial y} = \frac{\partial\left(e^{y/x}\right)}{\partial y}.\ln\left(\frac{x^2}{y}\right) + e^{y/x}.\frac{\partial\left[\ln\left(\frac{x^2}{y}\right)\right]}{\partial y} =$$

$$= e^{y/x}.\frac{\partial\left(y/x\right)}{\partial y}.\ln\left(\frac{x^2}{y}\right) + e^{y/x}.\frac{1}{\frac{x^2}{y}}.\frac{\partial\left(\frac{x^2}{y}\right)}{\partial y} =$$

$$= e^{y/x}.\ln\left(\frac{x^2}{y}\right).\frac{1}{x} + e^{y/x}.\frac{y}{x^2}.\left(\frac{-x^2}{y^2}\right) = e^{y/x}\left[\frac{1}{x}\ln\left(\frac{x^2}{y}\right) - \frac{1}{y}\right];$$

(h) $F(x,y) = x\cosh y + y\,\mathrm{senh}\,x;$

Solução: Temos;

$$F_x(x,y) = \frac{\partial\left[x\cosh y + y\,\mathrm{senh}\,x\right]}{\partial x} = \cosh y\frac{\partial(x)}{\partial x} + y\frac{\partial(\mathrm{senh}\,x)}{\partial x} = \cosh y + y\cosh x;$$

e $$F_y(x,y) = \frac{\partial\left[x\cosh y + y\,\mathrm{senh}\,x\right]}{\partial y} = x\frac{\partial(\cosh y)}{\partial y} + \mathrm{senh}\,x\frac{\partial(y)}{\partial y} = x\,\mathrm{senh}\,y + \mathrm{senh}\,x;$$

(i) $F(x,y) = x^y;$

Solução: Temos;

$$F_x(x,y) = \frac{\partial(x^y)}{\partial x} = \frac{\partial\left[e^{\ln x^y}\right]}{\partial x} = \frac{\partial\left[e^{y\ln x}\right]}{\partial x} = e^{y\ln x}.\frac{\partial(y\ln x)}{\partial x} = e^{\ln x^y}.\frac{y}{x} = \frac{x^y.y}{x};$$

e

$$F_y(x,y) = \frac{\partial(x^y)}{\partial y} = \frac{\partial\left[e^{\ln x^y}\right]}{\partial y} = \frac{\partial\left[e^{y\ln x}\right]}{\partial y} = e^{y\ln x}.\frac{\partial(y\ln x)}{\partial y} = e^{\ln x^y}.\ln x.\frac{\partial(y)}{\partial y} = x^y.\ln x;$$

(02) Em cada caso, encontre $f_x(x,y,z)$, $f_y(x,y,z)$ e $f_z(x,y,z)$ se:

(a) $f(x,y,z) = 2x^2+3y^2+4z^2$;

Solução: Temos;

$f_x(x,y,z) = \dfrac{\partial(2x^2+3y^2+4z^2)}{\partial x} = 4x$; $f_y(x,y,z) = \dfrac{\partial(2x^2+3y^2+4z^2)}{\partial y} = 6y$;

e $f_z(x,y,z) = \dfrac{\partial(2x^2+3y^2+4z^2)}{\partial z} = 8z$;

(b) $f(x,y,z) = xy^2z + x^2yz + xyz^2$;

Solução: Temos;

$f_x(x,y,z) = \dfrac{\partial(xy^2z+x^2yz+xyz^2)}{\partial x} = y^2z\dfrac{\partial(x)}{\partial x} + yz\dfrac{\partial(x^2)}{\partial x} + yz^2\dfrac{\partial(x)}{\partial x} = y^2z + 2xyz + yz^2$;

$f_y(x,y,z) = \dfrac{\partial(xy^2z+x^2yz+xyz^2)}{\partial y} = xz\dfrac{\partial(y^2)}{\partial y} + x^2z\dfrac{\partial(y)}{\partial y} + xz^2\dfrac{\partial(y)}{\partial y} = 2xyz + x^2z + xz^2$;

$f_z(x,y,z) = \dfrac{\partial(xy^2z+x^2yz+xyz^2)}{\partial z} = xy^2\dfrac{\partial(z)}{\partial z} + x^2y\dfrac{\partial(z)}{\partial z} + xy\dfrac{\partial(z^2)}{\partial z} = xy^2 + x^2y + 2xyz$;

(c) $f(x,y,z) = 2^{xyz}$;

Solução: Temos;

$f_x(x,y,z) = \dfrac{\partial(2^{xyz})}{\partial x} = 2^{xyz}.\ln 2.\dfrac{\partial(xyz)}{\partial x} = yz.2^{xyz}.\ln 2$;

$f_y(x,y,z) = \dfrac{\partial(2^{xyz})}{\partial y} = 2^{xyz}.\ln 2.\dfrac{\partial(xyz)}{\partial y} = xz.2^{xyz}.\ln 2$;

$f_z(x,y,z) = \dfrac{\partial(2^{xyz})}{\partial z} = 2^{xyz}.\ln 2.\dfrac{\partial(xyz)}{\partial z} = xy.2^{xyz}.\ln 2$;

(d) $f(x,y,z) = \sec(x+yz)$;

Solução: Temos;

$f_x(x,y,z) = \dfrac{\partial(\sec(x+yz))}{\partial x} = \sec(x+yz).\operatorname{tg}(x+yz).\dfrac{\partial(x+yz)}{\partial x} = \sec(x+yz).\operatorname{tg}(x+yz)$;

$f_y(x,y,z) = \dfrac{\partial(\sec(x+yz))}{\partial y} = \sec(x+yz).\operatorname{tg}(x+yz).\dfrac{\partial(x+yz)}{\partial y} = z.\sec(x+yz).\operatorname{tg}(x+yz)$;

e

$f_z(x,y,z) = \dfrac{\partial(\sec(x+yz))}{\partial z} = \sec(x+yz).\operatorname{tg}(x+yz).\dfrac{\partial(x+yz)}{\partial z} = y.\sec(x+yz).\operatorname{tg}(x+yz)$;

(e) $f(x,y,z) = \text{arctg}(xyz)$;

Solução: Temos;

$$f_x(x,y,z) = \frac{\partial(\text{arctg}(xyz))}{\partial x} = \frac{1}{1+(xyz)^2}\cdot\frac{\partial(xyz)}{\partial x} = \frac{yz}{1+x^2y^2z^2};$$

$$f_y(x,y,z) = \frac{\partial(\text{arctg}(xyz))}{\partial y} = \frac{1}{1+(xyz)^2}\cdot\frac{\partial(xyz)}{\partial y} = \frac{xz}{1+x^2y^2z^2};$$

e

$$f_z(x,y,z) = \frac{\partial(\text{arctg}(xyz))}{\partial z} = \frac{1}{1+(xyz)^2}\cdot\frac{\partial(xyz)}{\partial z} = \frac{xy}{1+x^2y^2z^2};$$

(f) $f(x,y,z) = z^{xy}$;

Solução: Temos;

$$f_x(x,y,z) = \frac{\partial[z^{xy}]}{\partial x} = \frac{\partial[e^{\ln z^{xy}}]}{\partial x} = \frac{\partial[e^{xy\ln z}]}{\partial x} = e^{xy\ln z}\cdot\frac{\partial(xy\ln z)}{\partial x} = y\cdot z^{xy}\cdot\ln z;$$

$$f_y(x,y,z) = \frac{\partial[z^{xy}]}{\partial y} = \frac{\partial[e^{\ln z^{xy}}]}{\partial y} = \frac{\partial[e^{xy\ln z}]}{\partial y} = e^{xy\ln z}\cdot\frac{\partial(xy\ln z)}{\partial y} = x\cdot z^{xy}\cdot\ln z;$$

e

$$f_z(x,y,z) = \frac{\partial[z^{xy}]}{\partial z} = \frac{\partial[e^{\ln z^{xy}}]}{\partial z} = \frac{\partial[e^{xy\ln z}]}{\partial z} = e^{xy\ln z}\cdot\frac{\partial(xy\ln z)}{\partial z} = \frac{xy\,z^{xy}}{z};$$

(03) Em cada caso, encontre $\frac{\partial u}{\partial x_i}$, $1 \leq i \leq n$, se:

(a) $u(x_1,\dots,x_i,\dots,x_n) = \cos(x_1+\dots+x_i+\dots+x_n)$;

Solução: para cada i, $1 \leq i \leq n$, temos;

$$\frac{\partial u}{\partial x_i}(x_1,\dots,x_i,\dots,x_n) = \frac{\partial[\cos(x_1+\dots+x_i+\dots+x_n)]}{\partial x_i} = -\text{sen}(x_1+\dots+x_i+\dots+x_n)\cdot\frac{\partial(x_1+\dots+x_i+\dots+x_n)}{\partial x_i} =$$

$$= -\text{sen}(x_1+\dots+x_i+\dots+x_n);$$

(b) $u(x_1,\dots,x_i,\dots,x_n) = \left(\sum_{k=1}^{n} x_k\right)^{1/p}$, p inteiro positivo;

Solução: para cada i, $1 \leq i \leq n$, temos:

$$\frac{\partial u}{\partial x_i}(x_1,\dots,x_i,\dots,x_n) = \frac{\partial\left[\left(\sum_{k=1}^{n} x_k\right)^{1/p}\right]}{\partial x_i} = \frac{\partial\left[(x_1+\dots+x_i+\dots+x_n)^{1/p}\right]}{\partial x_i} =$$

$$= \frac{1}{p}(x_1+\dots+x_i+\dots+x_n)^{\frac{1}{p}-1}\cdot\frac{\partial(x_1+\dots+x_i+\dots+x_n)}{\partial x_i}$$

$$= \frac{1}{p}(x_1+\dots+x_i+\dots+x_n)^{\frac{1-p}{p}};$$

(04) Em cada caso, considere que a equação dada define a variável dependente z como uma função das duas variáveis independentes x e y. Então, use derivação implícita para encontrar $\frac{\partial z}{\partial x}$ e $\frac{\partial z}{\partial y}$, onde elas existam.

(a) $xy + z^3x - 2yz = 0$;

Solução: Temos;

$$\frac{\partial(xy+z^3x-2yz)}{\partial x} = \frac{\partial(0)}{\partial x} \therefore \frac{\partial(xy)}{\partial x} + \frac{\partial(z^3x)}{\partial x} - \frac{\partial(2yz)}{\partial x} = 0 \therefore$$

$$\therefore y\frac{\partial(x)}{\partial x} + \left[\frac{\partial(z^3)}{\partial x}.x + \frac{\partial(x)}{\partial x}.z^3\right] - 2y\frac{\partial z}{\partial x} = 0 \therefore$$

$$\therefore y + 3z^2\frac{\partial z}{\partial x}.x + z^3 - 2y\frac{\partial z}{\partial x} = 0 \therefore (3xz^2-2y)\frac{\partial z}{\partial x} = -y-z^3 \therefore \frac{\partial z}{\partial x} = \frac{-y-z^3}{3xz^2-2y};$$

e,

$$\frac{\partial(xy+z^3x-2yz)}{\partial y} = \frac{\partial(0)}{\partial y} \therefore \frac{\partial(xy)}{\partial y} + \frac{\partial(z^3x)}{\partial y} - \frac{\partial(2yz)}{\partial y} = 0 \therefore$$

$$\therefore x\frac{\partial(y)}{\partial y} + x\frac{\partial(z^3)}{\partial y} - 2\frac{\partial(yz)}{\partial y} = 0 \therefore x + x.3z^2.\frac{\partial z}{\partial y} - 2\left[\frac{\partial(y)}{\partial y}.z + y.\frac{\partial z}{\partial y}\right] = 0 \therefore$$

$$\therefore (3z^2x-2y)\frac{\partial z}{\partial y} = -x+2z \therefore \frac{\partial z}{\partial y} = \frac{-x+2z}{3xz^2-2y};$$

(b) $yz + x\ln y = z^2$;

Solução: Temos;

$$\frac{\partial(yz+x\ln y)}{\partial x} = \frac{\partial(z^2)}{\partial x} \therefore \frac{\partial(yz)}{\partial x} + \frac{\partial(x\ln y)}{\partial x} = 2z.\frac{\partial z}{\partial x} \therefore$$

$$\therefore y\frac{\partial z}{\partial x} + \ln y.\frac{\partial(x)}{\partial x} = 2z.\frac{\partial z}{\partial x} \therefore (2z-y)\frac{\partial z}{\partial x} = \ln y \therefore \frac{\partial z}{\partial x} = \frac{\ln y}{2z-y};$$

e,

$$\frac{\partial(yz+x\ln y)}{\partial y} = \frac{\partial(z^2)}{\partial y} \therefore \left[\frac{\partial(y)}{\partial y}.z + \frac{\partial z}{\partial y}.y\right] + x\frac{\partial(\ln y)}{\partial y} = 2z.\frac{\partial z}{\partial y} \therefore$$

$$\therefore z + y\frac{\partial z}{\partial y} + \frac{x}{y} = 2z.\frac{\partial z}{\partial y} \therefore (2z-y)\frac{\partial z}{\partial y} = z + \frac{x}{y} \therefore \frac{\partial z}{\partial y} = \frac{z+\frac{x}{y}}{2z-y};$$

(05) Em cada caso, use o Teorema Fundamental do Cálculo, devido à Leibniz (1646-1716) e à Newton (1643-1727) para encontrar $F_X(X,Y)$ e $F_Y(X,Y)$ se:

ⓐ $F(X,Y)=\int_Y^X e^{\cos t}dt$; ⓑ $F(X,Y)=\int_X^Y \ln \operatorname{sen} t\, dt$;

Solução: iniciamos relembrando o Teorema;

Teorema Fundamental do Cálculo. Se F for contínua em $[a,b]$, então a função g definida em $[a,b]$ por: $g(x)=\int_a^x F(t)dt$, é derivável em (a,b) e $g'(x)=F(x)$;

Vamos então, utilizá-lo na solução de cada alínea:

ⓐ $F_X(X,Y)=\dfrac{\partial\left[\int_Y^X e^{\cos t}dt\right]}{\partial X}=e^{\cos X}$;

e $F_Y(X,Y)=\dfrac{\partial\left[\int_Y^X e^{\cos t}dt\right]}{\partial Y}=\dfrac{\partial\left[-\int_X^Y e^{\cos t}dt\right]}{\partial Y}=\dfrac{-\partial\left[\int_X^Y e^{\cos t}dt\right]}{\partial Y}=-e^{\cos Y}$;

ⓑ $F_X(X,Y)=\dfrac{\partial\left[\int_X^Y \ln \operatorname{sen} t\,dt\right]}{\partial X}=\dfrac{\partial\left[-\int_Y^X \ln \operatorname{sen} t\,dt\right]}{\partial X}=\dfrac{-\partial\left[\int_Y^X \ln \operatorname{sen} t\,dt\right]}{\partial X}=-\ln \operatorname{sen} X$;

e $F_Y(X,Y)=\dfrac{\partial\left[\int_X^Y \ln \operatorname{sen} t\,dt\right]}{\partial Y}=\ln \operatorname{sen} Y$;

(06) Seja $u(X,Y)=\operatorname{sen}\frac{X}{Y}+\ln\frac{Y}{X}$; Mostre que $Y\frac{\partial u}{\partial Y}+X\frac{\partial u}{\partial X}=0$;

Solução: Temos;

$Y\dfrac{\partial u}{\partial Y}=Y.\dfrac{\partial\left[\operatorname{sen}\frac{X}{Y}+\ln\frac{Y}{X}\right]}{\partial Y}=Y.\left[\cos\frac{X}{Y}\cdot\left(\frac{-X}{Y^2}\right)+\frac{1}{\frac{Y}{X}}\cdot\frac{1}{X}\right]=-\frac{X}{Y}\cos\frac{X}{Y}+1$;

$X\dfrac{\partial u}{\partial X}=X.\dfrac{\partial\left[\operatorname{sen}\frac{X}{Y}+\ln\frac{Y}{X}\right]}{\partial X}=X.\left[\cos\frac{X}{Y}\cdot\left(\frac{1}{Y}\right)+\frac{1}{\frac{Y}{X}}\cdot\frac{(-Y)}{X^2}\right]=\frac{X}{Y}\cos\frac{X}{Y}-1$;

Logo, $Y\dfrac{\partial u}{\partial Y}+X\dfrac{\partial u}{\partial X}=-\frac{X}{Y}\cos\frac{X}{Y}+1+\frac{X}{Y}\cos\frac{X}{Y}-1=0$;

07 Seja $u(x,y,z) = x^2y + y^2z + z^2x$; Verifique se: $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2$;

Solução: Temos;

$$\frac{\partial u}{\partial x} = \frac{\partial(x^2y + y^2z + z^2x)}{\partial x} = 2xy + 0 + z^2 = z^2 + 2xy;$$

$$\frac{\partial u}{\partial y} = \frac{\partial(x^2y + y^2z + z^2x)}{\partial y} = x^2 + 2yz + 0 = x^2 + 2yz;$$

e $\frac{\partial u}{\partial z} = \frac{\partial(x^2y + y^2z + z^2x)}{\partial z} = 0 + y^2 + 2xz = y^2 + 2xz;$

Logo, $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = z^2 + 2xy + x^2 + 2yz + y^2 + 2xz =$

$$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2 =$$

$$= (x+y)^2 + 2z(x+y) + z^2 =$$

$$= (x+y+z)^2;$$

08 A lei dos gases para uma massa fixa m de um gás ideal à temperatura absoluta T, pressão P e volume V é $PV = mRT$, onde R é uma constante específica do gás. Então, mostre que:

a) $\frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial T} \cdot \frac{\partial T}{\partial P} = -1$;

Solução: Temos; De, $P = mR\frac{T}{V}$, obtemos: $\frac{\partial P}{\partial V} = mRT \cdot \left(-\frac{1}{V^2}\right) = -\frac{mRT}{V^2}$;

E de, $V = mR\frac{T}{P}$, obtemos: $\frac{\partial V}{\partial T} = \frac{mR}{P} \cdot 1 = \frac{mR}{P}$;

E, por fim, de $T = \frac{1}{mR}PV$, obtemos: $\frac{\partial T}{\partial P} = \frac{V}{mR} \cdot 1 = \frac{V}{mR}$;

Logo, $\frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial T} \cdot \frac{\partial T}{\partial P} = -\frac{mRT}{V^2} \cdot \frac{mR}{P} \cdot \frac{V}{mR} = -\frac{mRT}{PV} = -\frac{mRT}{mRT} = -1$;

b) $T \cdot \frac{\partial P}{\partial T} \cdot \frac{\partial V}{\partial T} = mR$;

Solução: Da alínea a) sabemos que $\frac{\partial V}{\partial T} = \frac{mR}{P}$;

E de $P = mR\frac{T}{V}$, obtemos $\frac{\partial P}{\partial T} = \frac{mR}{V} \cdot 1 = \frac{mR}{V}$;

Logo, $T \cdot \frac{\partial P}{\partial T} \cdot \frac{\partial V}{\partial T} = T \cdot \frac{mR}{V} \cdot \frac{mR}{P} = \frac{mRT}{PV} \cdot mR = \frac{mRT}{mRT} \cdot mR = mR$;