2ª Lista de Exercícios - Cálculo II

① Ache a equação polar a partir da equação cartesiana:

a. $x^2+y^2=a^2$

$$(r\cos\theta)^2+(r\,sen\,\theta)^2=a^2$$
$$r^2\cos^2\theta+r^2 sen^2\theta=a^2$$
$$r^2(\cos^2\theta+sen^2\theta)=a^2$$
$$r^2\cdot 1=a^2 \rightarrow r^2=a^2 \rightarrow r=\sqrt{a^2} \rightarrow \boxed{r=|a|}$$

b. $x^2=6y-y^2$

$$(r\cos\theta)^2=6(r\,sen\,\theta)-(r\,sen\,\theta)^2$$
$$r^2\cos^2\theta=6r\,sen\,\theta-r^2 sen^2\theta$$
$$r^2\cos^2\theta+r^2 sen^2\theta=6r\,sen\,\theta \qquad *\cos^2\theta+sen^2\theta=1$$
$$r^2(\cos^2\theta+sen^2\theta)=6r\,sen\,\theta$$
$$\frac{r^2}{r}=6\,sen\,\theta \rightarrow \boxed{r=6\,sen\,\theta}$$

c. $(x^2+y^2)^2=4(x^2-y^2)$ $\qquad *\cos^2\theta-sen^2\theta=\cos(2\theta)$

$$\left[(r\cos\theta)^2+(r\,sen\,\theta)^2\right]^2=4\left[(r\cos\theta)^2-(r\,sen\,\theta)^2\right]$$
$$(r^2\cos^2\theta+r^2 sen^2\theta)^2=4(r^2\cos^2\theta-r^2 sen^2\theta)$$
$$\left[r^2(\cos^2\theta+sen^2\theta)\right]^2=4r^2(\cos^2\theta-sen^2\theta)$$
$$(r^2\cdot 1)^2=4r^2\cos(2\theta) \rightarrow r^4=4r^2\cos(2\theta) \rightarrow \frac{r^4}{r^2}=4\cos(2\theta)$$
$$\rightarrow r^2=4\cos(2\theta) \rightarrow \boxed{r=2\sqrt{\cos(2\theta)}}$$

d. $(x^2+y^2)^2=x^2-y^2$

$$\left[(r\cos\theta)^2+(r\,sen\,\theta)^2\right]^2=(r\cos\theta)^2-(r\,sen\,\theta)^2$$
$$(r^2\cos^2\theta+r^2 sen^2\theta)^2=r^2\cos^2\theta-r^2 sen^2\theta$$
$$\left[r^2(\cos^2\theta+sen^2\theta)\right]^2=r^2(\cos^2\theta-sen^2\theta)$$
$$(r^2\cdot 1)^2=r^2\cos 2\theta$$
$$r^4=r^2\cos 2\theta$$
$$\frac{r^4}{r^2}=\cos 2\theta \rightarrow r^2=\cos 2\theta$$
$$\boxed{r=\sqrt{\cos 2\theta}}$$

e. $x^2+y^2+x=\sqrt{x^2+y^2}$ $\quad (r\cos\theta)^2+(r\sin\theta)^2+r\cos\theta=\sqrt{(r\cos\theta)^2+(r\sin\theta)^2}$

$$r^2(\cos^2\theta+\sin^2\theta)+r\cos\theta=\sqrt{r^2(\cos^2\theta+\sin^2\theta)}$$

$$r^2\cdot 1+r\cos\theta=\sqrt{r^2\cdot 1}$$

$$r^2+r\cos\theta=r \rightarrow r^2+r\cos\theta-r=0$$

*Princípio do fato zero:
Se $ab=0$ então $a=0$ ou $b=0$

$$r(r+\cos\theta-1)=0$$

$$r+\cos\theta-1=0$$

$$\boxed{r=1+\cos\theta}$$

f. $y=1+3x$

$$r\sin\theta=1+3(r\cos\theta)$$

$$r\sin\theta-3r\cos\theta=1$$

$$r(\sin\theta-3\cos\theta)=1$$

$$\frac{r(\sin\theta-3\cos\theta)}{(\sin\theta-3\cos\theta)}=\frac{1}{(\sin\theta-3\cos\theta)}$$

$$\boxed{r=\frac{1}{(\sin\theta-3\cos\theta)}}$$

g. $y=2$

$$r\sin\theta=2$$

$$r=\frac{2}{\sin\theta} \rightarrow \boxed{r=2\operatorname{cossec}\theta}$$

$$*\ \frac{1}{\sin(t)}=\operatorname{cossec}(t)$$

h. $(x^2+y^2)^{3/2}=y^2$ $\quad \left[(r\cos\theta)^2+(r\sin\theta)^2\right]^{3/2}=(r\sin\theta)^2$

$$\left[r^2(\cos^2\theta+\sin^2\theta)\right]^{3/2}=r^2\sin^2\theta$$

$$(r^2\cdot 1)^{3/2}=r^2\sin^2\theta$$

$$r^{2\cdot 3/2}=r^2\sin^2\theta \rightarrow \frac{r^3}{r^2}=\sin^2\theta \rightarrow \boxed{r=\sin^2\theta}$$

i. $x^4-y^4=2xy$

$$(r\cos\theta)^4-(r\sin\theta)^4=2(r\cos\theta)(r\sin\theta) \qquad *2\cos\theta\sin\theta=\sin(2\theta)$$

$$r^4(\cos^4\theta-\sin^4\theta)=2r^2\cos\theta\sin\theta$$

$$r^4\left[(\cos^2\theta-\sin^2\theta)\cdot(\cos^2\theta+\sin^2\theta)\right]=r^2\sin(2\theta)$$

$$r^4(\cos(2\theta)\cdot 1)=r^2\sin(2\theta)$$

$$r^4\cos(2\theta)-r^2\sin(2\theta)=0$$

$$r^2(r^2\cos(2\theta)-\sin(2\theta))=0$$

$$r^2\cos(2\theta)-\sin(2\theta)=0$$

$$\rightarrow r^2\cos(2\theta)=\sin(2\theta)$$

$$\frac{r^2\cos(2\theta)}{\cos(2\theta)}=\frac{\sin(2\theta)}{\cos(2\theta)}$$

$$r^2=\frac{\sin(2\theta)}{\cos(2\theta)} \rightarrow \boxed{r=\sqrt{\operatorname{tag}(2\theta)}}$$

f. $y^2 = 4(x+1)$ $\quad y^2 = 4x+4$

$y^2+x^2 = 4x+4+x^2$

$r^2 = (x+2)^2$

$r = \sqrt{(x+2)^2}$

$\rightarrow r = x+2$

$r = r\cos\theta + 2$

$r - r\cos\theta = 2$

$r(1-\cos\theta) = 2$

$\rightarrow \boxed{r = \dfrac{2}{(1-\cos\theta)}}$

(2) Ache a equação cartesiana a partir da equação polar:

a. $r^2 = 2\operatorname{sen}2\theta$ $\quad r^2 = 2(2\operatorname{sen}\theta\cos\theta)$

$r^2 = 4\operatorname{sen}\theta\cos\theta$

$r^2 = 4\dfrac{y}{r}\cdot\dfrac{x}{r} \rightarrow r^2 = \dfrac{4xy}{r^2}$

$\rightarrow r^2 \cdot r^2 = 4xy$

$r^4 = 4xy$

$\boxed{(x^2+y^2)^2 = 4xy}$

b. $r^2 = \cos\theta$ $\quad (x^2+y^2) = \cos\theta$

$r\cdot(x^2+y^2) = r\cos\theta$

$r\cdot(x^2+y^2) = x$

$\rightarrow (r\cdot(x^2+y^2))^2 = x^2$

$r^2\cdot(x^2+y^2)^2 = x^2$

$\boxed{(x^2+y^2)^3 = x^2}$

c. $r^2 = \theta$ $\quad (x^2+y^2) = \theta$

$tg(x^2+y^2) = tg\,\theta$

$\rightarrow tg(x^2+y^2) = \dfrac{y}{x} \rightarrow \boxed{y = x\,tg(x^2+y^2)}$

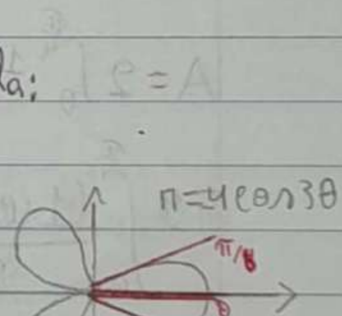
(4) Ache a área limitada pelo gráfico da equação dada:

a. $r = 4\cos 3\theta$

• rosácea de 3 folhas.

$\boxed{A = \int_a^b \tfrac{1}{2} r^2 \, d\theta}$

$a = 0 \quad b = \pi/6$



$A = 6\int_0^{\pi/6} \frac{1}{2}(4\cos 3\theta)^2 d\theta = 6\cdot\frac{1}{2}\cdot 4^2 \int_0^{\pi/6} \cos^2(3\theta)\, d\theta = 3\cdot 16\int_0^{\pi/6}\cos^2(3\theta)\, d\theta =$

$= 48\int_0^{\pi/6} \dfrac{1+\cos(6\theta)}{2}\, d\theta = \dfrac{48}{2}\int_0^{\pi/6} 1\, d\theta + \int_0^{\pi/6}\cos(6\theta)\, d\theta =$

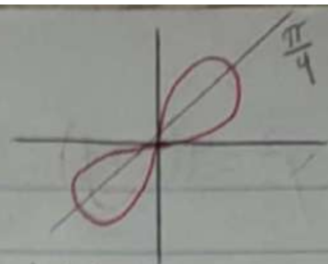
$* \int_0^{\pi/6} 1\, d\theta = \theta\Big|_0^{\pi/6}$

$* \int_0^{\pi/6} \cos(6\theta)\, d\theta = \int_0^{\pi/6} \dfrac{\cos u}{6}\, du = \dfrac{1}{6}\operatorname{sen}(6\theta)\Big|_0^{\pi/6}$

$u = 6\theta$

$\dfrac{du}{6} = d\theta$ $\quad 24\left(\theta + \dfrac{\operatorname{sen}(6\theta)}{6}\right)\Big|_0^{\pi/6} = 24\left(\dfrac{\pi}{6} + \dfrac{\operatorname{sen}(6\frac{\pi}{6})}{6}\right) = 24\left(\dfrac{\pi}{6}+\dfrac{0}{6}\right) = \dfrac{24\pi}{6} = \boxed{4\pi}$

$$\boxed{A=\int_a^b \frac{1}{2} r^2 \, d\theta}$$



b. $r^2 = 4\operatorname{sen} 2\theta \quad a=0 \quad b=\pi/4$

$$A = 4\int_0^{\pi/4} \frac{1}{2}(4\operatorname{sen}2\theta)\,d\theta = 4\cdot\frac{1}{2}\cdot 4\int_0^{\pi/4} \sin(2\theta)\,d\theta =$$

$$= \frac{8}{2}\int_0^{\pi/2} \sin u \, du = 4(-\cos(2\theta))\Big|_0^{\pi/4} = 4\left[(-\cos(2\tfrac{\pi}{4}))-(-\cos 0)\right]$$

$$= 4(0+1) = \boxed{4}$$

5. Ache a área da região limitada pelo gráfico da equação $r=\theta$, de $\theta=0$ até $\theta=\frac{3}{2}\pi$. • É uma espiral.

$$A = \int_0^{\frac{3\pi}{2}} \tfrac{1}{2}\,\theta^2\,d\theta = \frac{1}{2}\int_0^{\frac{3\pi}{2}} \theta^2\,d\theta = \tfrac{1}{2}\left(\frac{\theta^3}{3}\right)_0^{\frac{3\pi}{2}} = \frac{1}{2}\left(\frac{\left(\frac{3\pi}{2}\right)^3}{3}\right) = \frac{1}{2}\cdot\frac{27\pi^3}{8}\cdot\frac{1}{3}$$

$$= \frac{1}{2}\cdot\frac{9\pi^3}{8} = \frac{9\pi^3}{16}$$

6. Ache a área limitada por um laço da curva $r=3\cos 2\theta$.

$$A = 2\int_0^{\frac{\pi}{4}} \tfrac{1}{2}(3\cos 2\theta)^2\,d\theta = \int_0^{\frac{\pi}{4}} 3^2\cos^2(2\theta)\,d\theta = 9\int_0^{\frac{\pi}{4}} \cos^2(2\theta)\,d\theta$$

$$= 9\int_0^{\frac{\pi}{4}} \frac{(1+\cos 4\theta)}{2}\,d\theta = 9\cdot\frac{1}{2}\left[\int_0^{\frac{\pi}{4}} 1\,d\theta + \frac{1}{4}\int_0^{\frac{\pi}{4}} \cos u\,du\right] = \qquad u=4\theta$$

$$\left[\frac{9}{2}\theta + \frac{9}{8}\sin(4\theta)\right]_0^{\frac{\pi}{4}} = \frac{9}{2}\cdot\frac{\pi}{4} + \frac{9}{8}\sin\left(4\frac{\pi}{4}\right) = \frac{9\pi}{8} + 0 = \boxed{\frac{9\pi}{8}}$$