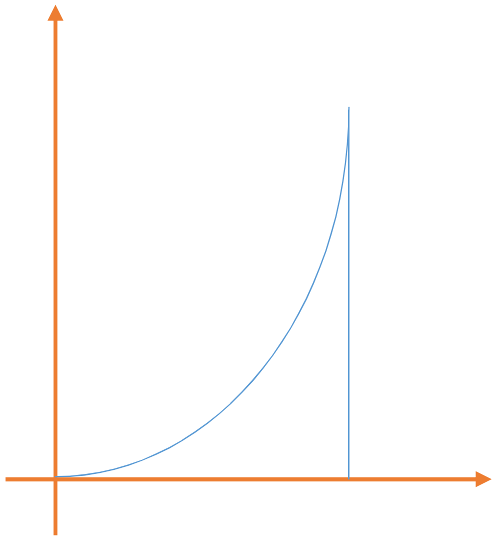
Exercício 7

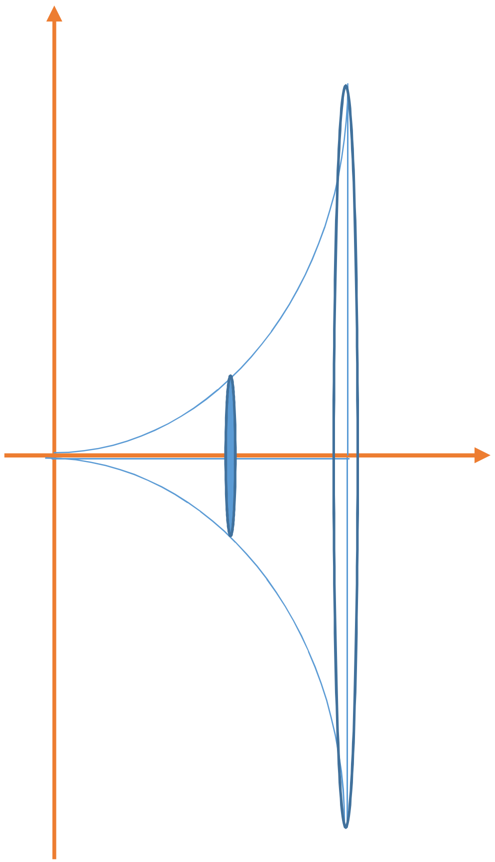
Calcule o volume do sólido gerado pela rotação da região limitada pela curva $y = x^3$ o eixo x e a reta $x = 2$ em torno do(a):

a) Eixo x
b) Reta $x = 4$
c) Reta $y = 8$
d) Eixo y

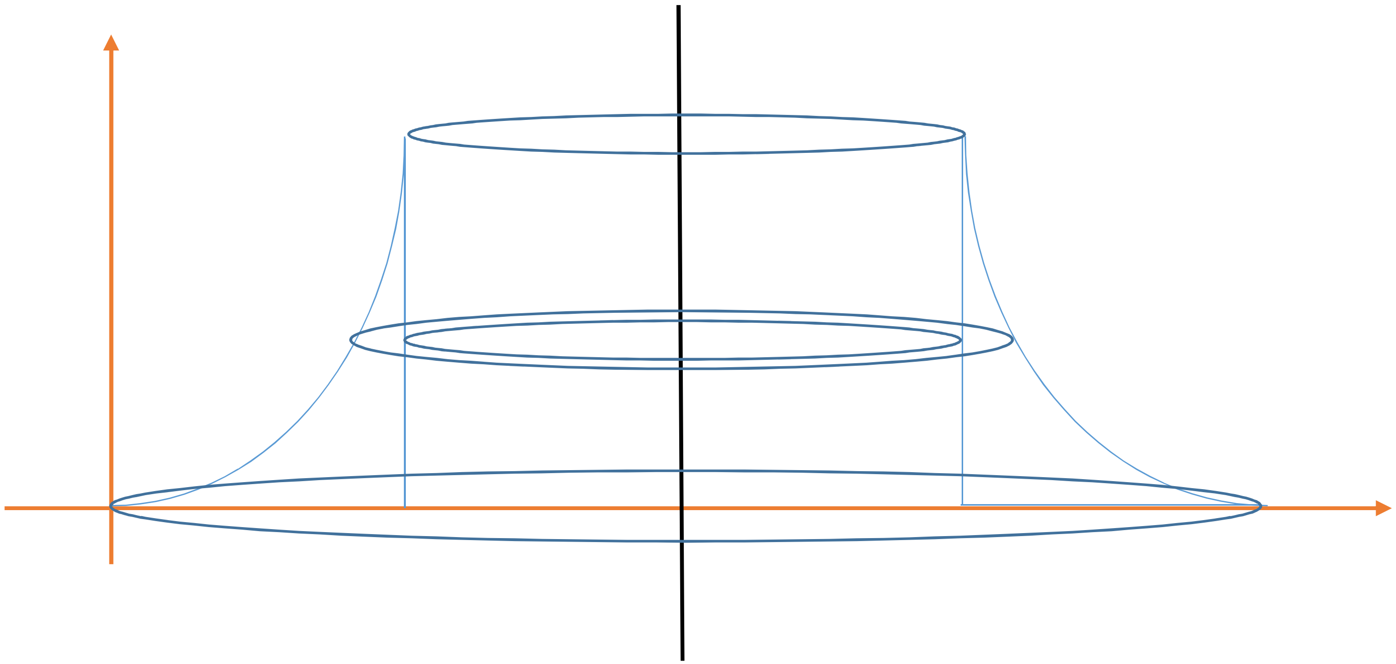


a)

$$V = \pi \int_0^2 (x^3)^2 dx = \pi \int_0^2 x^6 dx = \pi \frac{x^7}{7}\Big|_0^2 = \frac{2^7}{7}\pi = \frac{128}{7}\pi$$



b)



$$V = \pi \int_0^8 \left[\left(4 - y^{\frac{1}{3}}\right)^2 - 4\right] dy = \pi \int_0^8 \left(16 - 8y^{\frac{1}{3}} + y^{\frac{2}{3}} - 4\right) dy =$$

$$\pi\left(12y - 8.\frac{3}{4}.y^{\frac{4}{3}} + \frac{3}{5}.y^{\frac{5}{3}}\right)\Big|_0^8 = \pi\left(12.8 - 6.8^{\frac{4}{3}} + \frac{3}{5}.8^{\frac{5}{3}}\right) =$$

$$\left(96 - 96 + \frac{3}{5}.2^5\right)\pi = \frac{96}{5}\pi$$

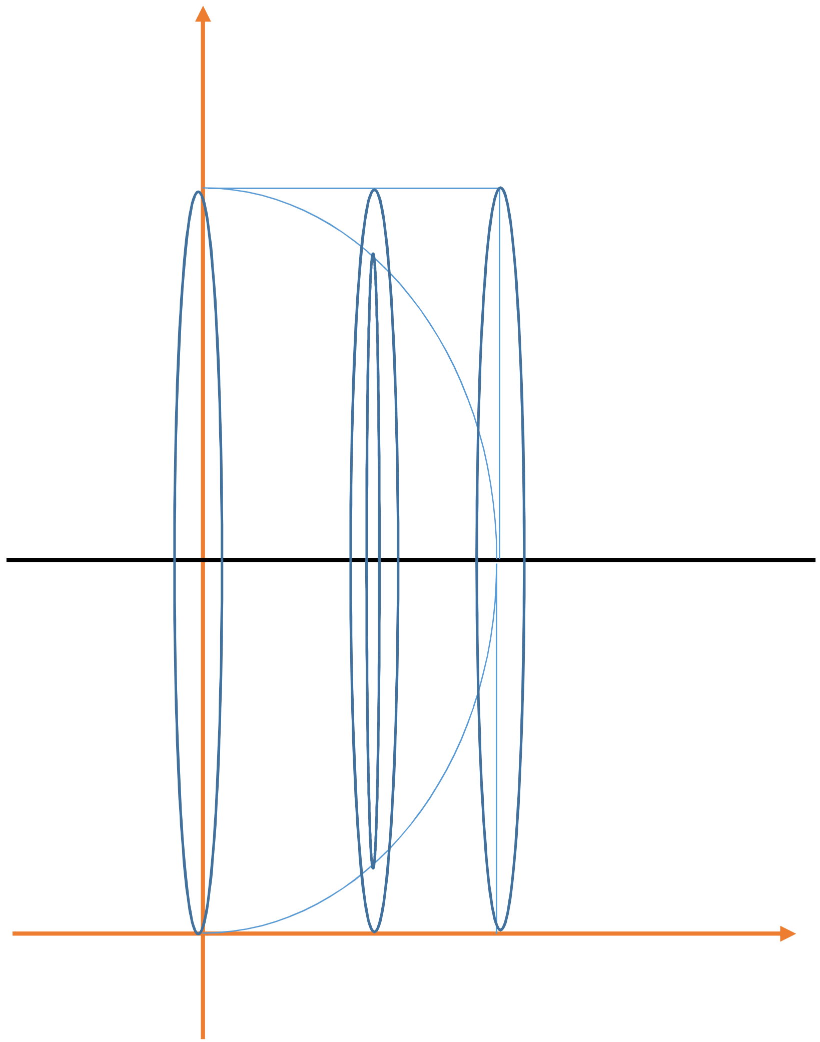
c) $V = \pi \int_0^2 [64 - (8 - x^3)^2] dx =$

$$\pi \int_0^2 (64 - 64 + 16x^3 - x^6) dx =$$

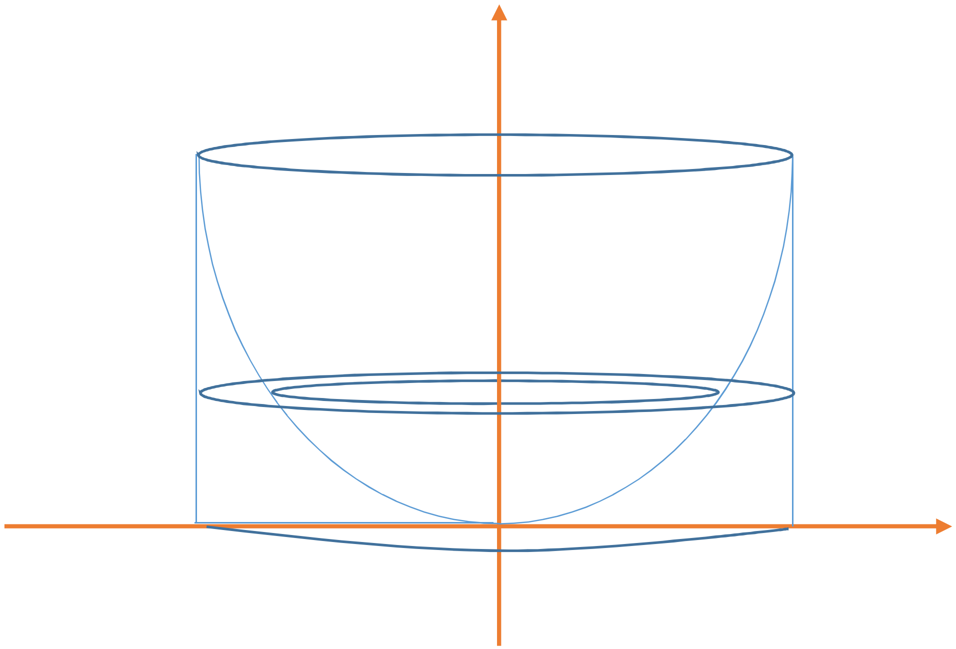
$$\pi \int_0^2 (16x^3 - x^6) dx =$$

$$\pi \left(16\frac{x^4}{4} - \frac{x^7}{7} \right) \Big|_0^2 =$$

$$\pi \left(4.2^4 - \frac{2^7}{7} \right) = \pi \left(64 - \frac{128}{7} \right) = \frac{320}{7} \pi$$



d)



Solução 1

$$V = 2\pi \int_0^2 x.x^3 dx =$$

$$2\pi \int_0^2 x^4 dx = 2\pi \frac{x^5}{5}\Big|_0^2 = \frac{64}{5}\pi$$

Solução 2

$$V = \pi \int_0^8 \left[4 - \left(y^{\frac{1}{3}}\right)^2\right] dy = \pi \int_0^8 \left(4 - y^{\frac{2}{3}}\right) dy = \pi \left(4y - \frac{3}{5} y^{\frac{5}{3}}\right)\Big|_0^8$$

$$\pi \left(32 - \frac{96}{5}\right) = \frac{64}{5}\pi$$

Exercício 1c

Se $G(x) = x.arccotg\ x + \ln\sqrt{1+x^2}$. Calcule $G'(x)$

Solução:

$G'(x) =$

$$1.arccotg\ x - \frac{1}{1+x^2}.x + \frac{1}{\sqrt{1+x^2}}.\frac{1}{2}.\frac{1}{\sqrt{1+x^2}}.2x$$

$$= arccotg\ x - \frac{x}{1+x^2} + \frac{x}{1+x^2} = arccotg\ x$$

Exercício 2 b

$$\int \frac{dx}{\sqrt{15+2x-x^2}}$$

Solução:

$$\int \frac{dx}{\sqrt{15+2x-x^2}} = \int \frac{dx}{\sqrt{15-(x^2-2x)}} =$$

$$\int \frac{dx}{\sqrt{15+1-(x^2-2x+1)}} = \int \frac{dx}{\sqrt{16-(x-1)^2}}$$

Fazendo $u = x - 1$, temos $du = dx$

Daí $\int \frac{dx}{\sqrt{16-(x-1)^2}} = \int \frac{du}{\sqrt{16-u^2}} = arc\ sen\ \frac{u}{4} + c = arc\ sen\ \frac{x-1}{4} + c =$

$$arc\ cos\ \frac{1-x}{4} + c$$

Exercício 3d

$$\int_{-4}^{-2} \frac{dt}{\sqrt{-t^2-6t-5}}$$

Solução:

$$\int_{-4}^{-2} \frac{dt}{\sqrt{-t^2-6t-5}} = \int_{-4}^{-2} \frac{dt}{\sqrt{-5-(t^2+6t)}} =$$

$$\int_{-4}^{-2} \frac{dt}{\sqrt{-5+9-(t^2+6t+9)}} =$$

$$\int_{-4}^{-2} \frac{dt}{\sqrt{4-(t+3)^2}} = arc\ sen\ \frac{t+3}{2} \Big|_{-4}^{-2} =$$

$$arc\ sen\ \frac{1}{2} - arc\ sen\ \frac{-1}{2} = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$$

Exercício 4

Ache a área da região limitada pela curva $y = \frac{8}{x^2+4}$, pelo eixo x , pelo eixo y , e pela reta x = 2

Solução:

$$A = \int_0^2 \frac{8}{x^2+4} dx = 8 \int_0^2 \frac{1}{x^2+4} dx =$$

$$8.\frac{1}{2}.arctg\ \frac{x}{2}\Big|_0^2 =$$

$$4(arctg\ 1 - arctg\ 0) = 4.\frac{\pi}{4} = \pi$$