UNIVERSIDADE FEDERAL DO CEARÁ
CENTRO DE CIÊNCIAS
DEPARTAMENTO DE MATEMÁTICA

2ª Avaliação Parcial – Cálculo Diferencial e Integral II

1. Ache a área da intersecção das regiões limitadas pelos gráficos de $r = 4sen\,\theta$ e $r = 4\cos\theta$.

2. Calcule o limite $\lim\limits_{x\to 0} \frac{x - sen\,x}{tg^3 x}$, se existir

3. Verifique se a integral imprópria $\int_e^{+\infty} \frac{dx}{x(lnx)^2}$ converge ou diverge. No caso de convergência, ache seu valor.

4. Use o polinômio de Maclaurin para calcular $\sqrt[3]{e}$, com precisão até a 2ª casa decimal.

03/06/22

1. • $r = 4\,\text{sen}(\theta)$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2}\left(4\,\text{sen}(\theta)\right)^2 d\theta = \frac{1}{2}\cdot 4^2 \int_0^{\frac{\pi}{4}} \text{sen}(\theta)^2 d\theta = 8\int_0^{\frac{\pi}{4}} \frac{1-\cos(2\theta)}{2}\, d\theta$$

$$= \frac{8}{2}\left[\int_0^{\frac{\pi}{4}} 1\, d\theta - \int_0^{\frac{\pi}{4}} \cos(2\theta)\, d\theta\right] = 4\left[\int_0^{\frac{\pi}{4}} 1\, d\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos(u)\, du\right]$$

$$= 4\left(\theta - \frac{\text{sen}(2\theta)}{2}\right)\Bigg|_0^{\frac{\pi}{4}} = 4\left[\left(\frac{\pi}{4} - \frac{\text{sen}(2\frac{\pi}{4})}{2}\right) - \left(0 - \frac{\text{sen}(2\cdot 0)}{2}\right)\right]$$

$$= 4\left(\frac{\pi}{4} - \frac{1}{2}\right) = \pi - \frac{4}{2} = \pi - 2$$

• $r = 4\cos(\theta)$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}\left(4\cos(\theta)\right)^2 d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{2}\left(4\,\text{sen}(\theta)\right)^2 d\theta = \pi - 2$$

Logo a área é igual a: $A = 2(\pi - 2) = 2\pi - 4$

$$2.\ \lim_{x\to 0} \frac{x-\operatorname{sen}(x)}{tg^3(x)} = \frac{\frac{d}{dx}(x-\operatorname{sen}(x))}{\frac{d}{dx}(tg^3(x))} = \lim_{x\to 0}\left(\frac{1-\cos(x)}{\frac{3\operatorname{sen}^2(x)}{\cos^4(x)}}\right)$$

$$= \lim_{x\to 0}\left((1-\cos(x))\cdot\frac{\cos^4(x)}{3\operatorname{sen}^2(x)}\right) = \lim_{x\to 0}\frac{(1-\cos(x))\cdot\cos^4(x)}{3(1-\cos^2(x))}$$

$$= \lim_{x\to 0}\frac{(1-\cos(x))\cdot\cos^4(x)}{3(1-\cos(x))\cdot(1+\cos(x))} = \lim_{x\to 0}\frac{\cos^4(x)}{3+3\cos(x)} = \frac{\cos^4(0)}{3+3\cos(0)} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

$$\frac{d}{dx}(x-\operatorname{sen}(x)) = 1-\cos(x)$$

$$\frac{d}{dx}(tg^3(x)) = 3tg^2(x)\cdot(tg(x))' = 3tg^2(x)\cdot\sec^2(x)$$

$$= 3\frac{\operatorname{sen}^2(x)}{\cos^2(x)}\cdot\frac{1}{\cos^2(x)} = \frac{3\operatorname{sen}^2(x)}{\cos^4(x)}$$

$$3.\ \int_e^{+\infty}\frac{dx}{x(\ln(x))^2} = \lim_{b\to\infty}\int_e^b\frac{dx}{x(\ln(x))^2} = \lim_{b\to\infty}\left(\frac{-1}{\ln(x)}\right)\Bigg|_e^b =$$

$$= \lim_{b\to\infty}\left(\frac{-1}{\ln(b)}+\frac{1}{\ln(e)}\right) = 0+1 = \boxed{1}$$

$$\int\frac{1}{x\ln(x)^2}\,dx = \int\frac{1}{u^2}\,du = -\frac{1}{(2-1)\cdot u^{2-1}} = -\frac{1}{u} = -\frac{1}{\ln(x)} + C$$

• 4. Então, queremos calcular $f(1/3)$, onde $f(x)=e^x$, com um erro menor que 0,005.

$$R_n(1/3) = \frac{e^c}{(n+1)!} < \frac{e}{(n+1)!} < \frac{3}{(n+1)!} < 0{,}005$$

$$\frac{(n+1)!}{3} > \frac{1}{0{,}005} \rightarrow (n+1)! > \frac{3}{0{,}005} = 600$$

$(n+1)!$ $1!=1$; $2!=2$; $3!=6$; $4!=24$; $5!=120$; $6!=720$

Logo o $n=5$.

Então o valor de $e^{1/3}$, com precisão até a segunda casa decimal é:

$$P_5\left(\frac{1}{3}\right) = f(0) + f'(0)\cdot x + f''(0)\cdot\frac{x^2}{2!} + f^3(0)\cdot\frac{x^3}{3!} + f^4(0)\cdot\frac{x^4}{4!} + f^5(0)\cdot\frac{x^5}{5!}$$

$$= 1 + \frac{1}{3} + \frac{(1/3)^2}{2!} + \frac{(1/3)^3}{3!} + \frac{(1/3)^4}{4!} + \frac{(1/3)^5}{5!} \cong 1{,}39\ldots$$