Theorems from Brezis' *Functional Analysis, Sobolev Spaces and Partial Differential Equations* (first edition)

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1 The Hahn-Banach Theorems. Introduction to Conjugate Convex Functions

Theorem 1.1 (Hahn-Banach)**.** *Let E be a vector space over $\mathbb{R}$, and let $p : E \to \mathbb{R}$ be a Minkowski functional. Let G be a linear subspace of E and let $g : G \to \mathbb{R}$ be a linear functional such that $g(x) \leq p(x)$ for all $x \in G$. There exists a linear functional $f : E \to \mathbb{R}$ such that*

$$f(x) = g(x), \forall x \in G,$$

and

$$f(x) \leq p(x), \forall x \in E.$$

Lemma 1.1 (Zorn)**.** *Every nonempty ordered set that is inductive has a maximal element.*