Definitions used in Brezis' *Functional Analysis, Sobolev Spaces and Partial Differential Equations* (first edition)

Gustaf Bjurstam
bjurstam@kth.se

Preliminaries – not in the book

Definition 0.1. Let E be a vector space over $\mathbb{R}$. A *functional* is a function $f : A \to \mathbb{R}$ where A is some subspace of E.

1 The Hahn-Banach Theorems. Introduction to Conjugate Convex Functions

Definition 1.1. Let E be a vector space over $\mathbb{R}$. A *Minkowski functional* is a function $p : E \to \mathbb{R}$ satisfying

$$p(\lambda x) = \lambda p(x), \qquad \forall x \in E \text{ and } \lambda > 0. \tag{1}$$

$$p(x + y) \leq p(x) + p(y), \qquad \forall x, y \in E. \tag{2}$$

Definition 1.2. Let P be a set with a (partial) order relation $\leq$. A subset $Q \subseteq P$ is *totally ordered* if for any pair (a, b) in Q at least one of $a \leq b$ and $b \leq a$ holds.

Definition 1.3. Let P be a set with a partial order relation $\leq$, and let $Q \subset P$. We say that $c \in P$ is an *upper bound* for Q if $a \leq c$ for all $a \in Q$. We say that $m \in P$ is a *maximal element* of P if there is no element $x \in P \setminus \{m\}$ such that $m \leq x$. If every totally ordered subset Q of P has an upper bound, we call P *inductive*.