

**Definitions used in Brezis' *Functional Analysis, Sobolev Spaces
and Partial Differential Equations* (first edition)**

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Preliminaries – not in the book

Definition 0.1. Let E be a vector space over \mathbb{R} . A *functional* is a function $f : A \rightarrow \mathbb{R}$ where A is some subspace of E .

1 The Hahn-Banach Theorems. Introduction to Conjugate Convex Functions

Definition 1.1. Let E be a vector space over \mathbb{R} . A *Minkowski functional* is a function $p : E \rightarrow \mathbb{R}$ satisfying

$$p(\lambda x) = \lambda p(x), \quad \forall x \in E \text{ and } \lambda > 0. \quad (1)$$

$$p(x + y) \leq p(x) + p(y), \quad \forall x, y \in E. \quad (2)$$

Definition 1.2. Let P be a set with a (partial) order relation \leq . A subset $Q \subseteq P$ is *totally ordered* if for any pair (a, b) in Q at least one of $a \leq b$ and $b \leq a$ holds.