Lab1 - Extreme Values

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1

The plot is shown below.

Figure 1: Part of question 1.

2

The task in the description was performed and the asked for results are presented below.

parameter	value
location	3.8747499
scale	0.1980440
shape	-0.0501095
negative log likelihood	-4.339058

	location	scale	shape
location	1598.6471	-508.7539	136.8932
scale	-508.7539	3040.6365	188.2579
shape	136.8932	188.2579	133.9761

3

	location	scale	shape
location	0.0007802099	0.0001970436	-0.0010740750
scale	0.0001970436	0.0004099808	-0.0007774225
shape	-0.0010740750	-0.0007774225	0.0096538796

4

The result obtained from calculating the inverse of the hessian matrix coincides with the results obtained in the previous question.

5

the fitted values as well as lower and upper endpoints for the 95 % confidence intervals of the 3 parameters of the GEV model are presented below. Here it may be noted that the shaper parameter has a negative estimated value (we will come back to this), however the confidence interval includes zero. This indicates that the shape parameter cannot be significantly considered to be non-zero, suggesting it might not have earned its place in the model. This suggests a model without a shape parameter, which in this context would mean a Gumbel distribution. This possibility will also be discussed later in the report.

	lower	estimate	upper
location	3.8200037	3.8747499	3.9294960
scale	0.1583586	0.1980440	0.2377293
shape	-0.2426841	-0.0501095	0.1424651

6

Using the command given in the instructions results in the same values as in the previous question.

7

The confidence interval for the shape parameter based on profile likelihood was found to be (-0.2178, 0.1702), by using the suggested R-command. The interval still covers zero.

8

The calculations where performed in accordance to the formulas given in the course literature (equation 3.10 and 3.11) where the shape parameter is non-zero. The obtained mean value and variance is presented below. Constructing a 95% confidence interval with these parameters will yield intervals similar to the ones presented in the question 9. Both the variance and mean increases with a longer return level which is to be expected.

return level	mean	variance
10 years	4.296212	0.003026622
100 years	4.688404	0.02522329

9

Confidence intervals for 10- and 100-year return levels were done by using the interface's functions, using the delta method (normal approximation):

return level	lower	upper
10 years	4.188383	4.404041
100 years	4.377120	4.999688

10

The confidence intervals for the 10- and 100-year return level based on the profile likelihood method are presented below. The confidence intervals calculated using the profile likelihood method are quite different from the ones calculated with the delta method. They are not symmetric and skewed towards a higher probability for higher return values. The non-symmetric property is probably reasonable and according to the course literature the confidence interval obtained by the profile likelihood method is often more accurate.

return level	lower	upper
10 years	4.2051	4.4441
100 years	4.4921	5.2595

11 and 12

Theoretical estimate of z_0 and $\nabla \hat{z}_0$ was done by implementing the formulas described at the end of page 56 and the top of page 57 in Coles, and then we arrived at the estimated variance of the upper endpoint by the formula used in question 8. In order to obtain the bounds of the confidence interval by use of the delta method, just as in question 8, we multiplied the estimated standard deviation by the 95 % and 5 % quantiles of the normal distribution.

return level	lower	estimate	upper
z_0	-7.049615	7.826973	22.703561

This is of course highly contradictory, as a wave cannot have negative height, but this is likely due to the high variance of the estimate, and that the confidence interval by this method is symmetrical, not taking into account that the value has to be positive.

13

The Q-Q plot indicates that the model is correct since all quantiles are close to the line $y = x$. The same conclusion is made when viewing the Q-Q plot comparing the quantiles from the simulated model data with the empirically obtained quantiles. The probability density plots are very similar and all return levels are within the confidence interval. Altogether, the plots indicates that the model used is a good fit to the data.

15

As mentioned in question 5, the shape parameter appeared to be insignificant, motivating a check on wether or not a Gumbel distribution model better fits the data. Below are the ML estimates of the two parameters in the Gumbel model:

location	scale
3.8694436	0.194889

16

The estimated covariance matrix of the parameters of the Gumbel model is:

	location	scale
location	0.0006499383	0.0001527080
scale	0.0001527080	0.0003554611

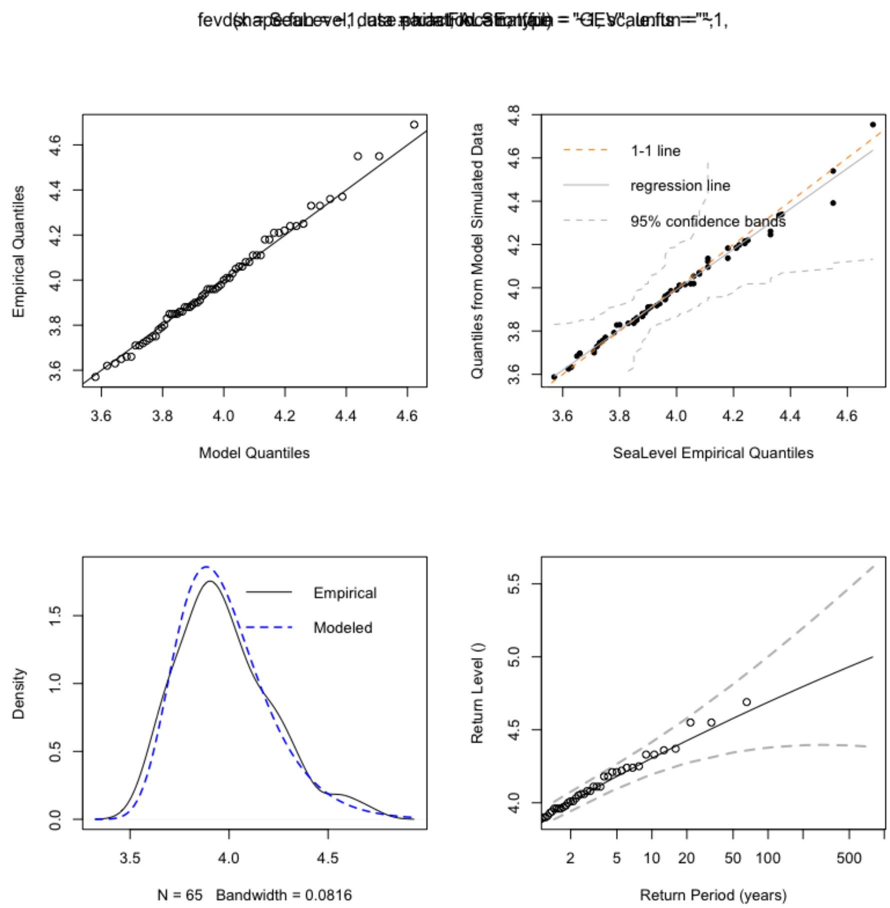

Figure 2: This plot is referred to in question 13.

17

The confidence intervals for the parameters in Gumbel model:

	lower	estimate	upper
location	3.8194765	3.8694436	3.919411
scale	0.1579369	0.1948895	0.231842

18

The plots obtained follows the same reasoning as described in question 13. All of the plots indicates that the model is reasonable to use. When comparing to the previous model the return period looks slightly better while the simulated quantiles looks slightly worse but the differences are subtle and no conclusions can be made and all values are well within the confidence intervals.

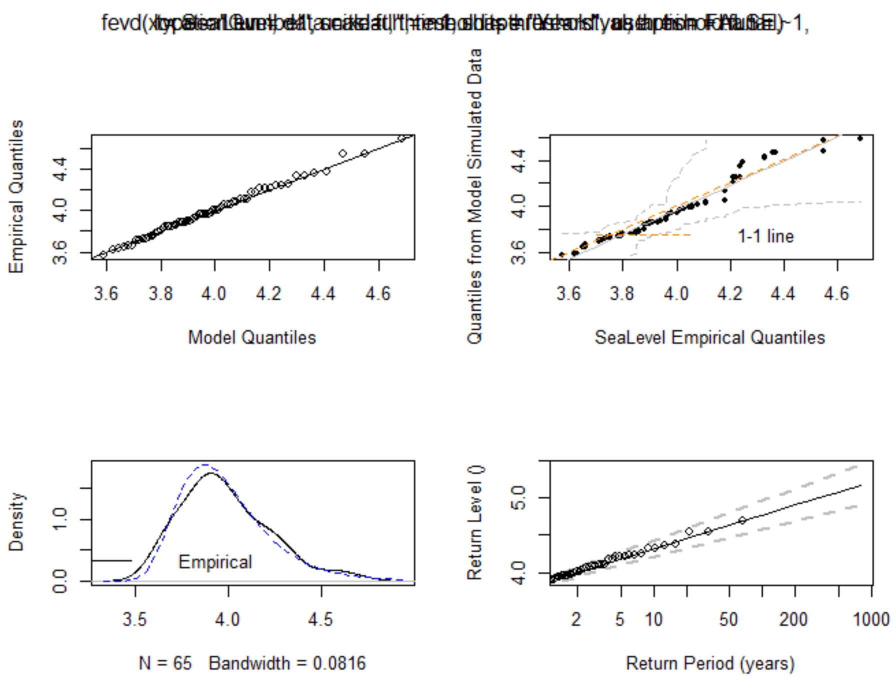

Figure 3: This plot is referred to in question 18

19

With help of the extreme package a likelihood ratio test was performed with $H_0 : \gamma = 0$ and $H_1 : \gamma \neq 0$. The likelihood-ratio obtained was 0.24275. To obtain a 95% confidence interval $\alpha = 0.05$ which gave a chi-square critical value of 3.8415. The likelihood ratio that was reported is far below the critical value. The reported p-value was 0.6222, which is far greater than the specified α. This means that there is not a significant reason to reject the Gumbel model for the larger GEV model, at the cost of extra parameters. The conclusion is that the Gumbel model should not be rejected, and since it is the smaller model, it is preferred.