Lab 3 - Extreme Values

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2.1

The requested scatter plot can be seen in Figure 1.

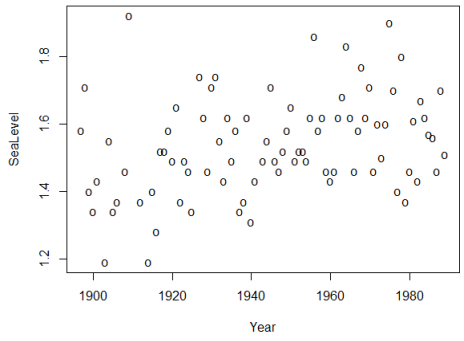

Figure 1: Scatter plot of the data for question 1

2.2

The maximum likelihood estimates of the parameters for models 1 through 6 are presented in Table 1. The interpretation of the table should be fairly straight forward, where the μ_i and ϕ_i are in accordance with what is given in the instructions (and σ is logarithmized in those models where the instructions say so), however it may be noteworthy that when μ_0 or ϕ_0 are the only variables declared for either location or scale, then $\mu_0 = \mu$ and $\phi_0 = \sigma$ is how the table should be interpreted. This only really comes into play for M1 and M6.

	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\phi}_0$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\xi}$
M1	1.48	-	-	0.14	-	-	-0.22
M2	1.39	0.17	-	-1.92	-0.33	-	-0.14
M3	1.34	0.44	-0.25	-1.92	-0.37	-	-0.11
M4	1.4	0.18	0.07	-1.9	-0.42	0.27	-0.22
M5	1.4	0.18	0.06	-2.11	0.27	-	-0.19
M6	1.38	0.19	0.05	0.12	-	-	-0.15

Table 1: The table for question 2.2

2.3

In Table 2, a series of Likelihood Ratio tests are presented. We started by checking M1 against M2, as M1 is nested in M2. Seeing as this yielded a preference for M2, we went on to test M2 against M3. Only when M2 was pitted against M4 could we reject M2. By continuously making sure we tested nested models against bigger models, we can finally conclude that M5 appears to perform the best in this test.

H_0	H_1	λ_{LR}	Critical chi-square	Rejection
M_1	M_2	13.4	5.99	Yes
M_2	M_3	1.8	3.84	No
M_2	M_4	13.4	5.99	Yes
M_5	M_4	3.2	3.84	No
M_1	M_6	20.7	5.99	Yes
M_6	M_5	4.8	3.84	Yes

Table 2: Chain of likelihood ratio tests. Final conclusion is that M5 performs the best.

3.1

Figure 2 contains the requested scatter plot for question 3.1.

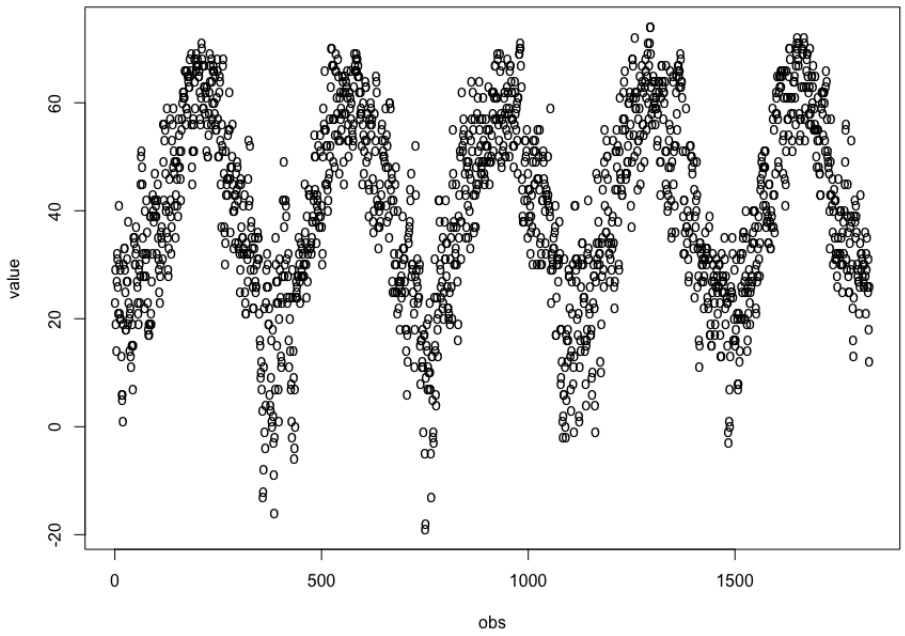

Figure 2: Scatter plot of the data for question 3.1

3.2

To answer question 3.2, we have recreated Table 7.1 from the course book with the data from wooster, and present it in Table 3. What each model contains should be clear from their description in the table, p signifies how many parameters each model has, and l is the log-likelihood arrived at from the estimation of each model. As in question 2.3, we conducted a series of Likelihood Ratio tests, pitting nested models against bigger models. The procedure was essentially analogous to the one in 2.3 and the results are collected in Table 4. Going simply by the LR-test, it is concluded that model 6 is the best model of the available models in Table 3.

Model	p	ℓ
1. Time-homogeneous	3	-140.5
2. As 1. but periodic in μ	5	74.0
3. As 2. but periodic in $\log \sigma$	7	157.2
4. As 3. but periodic in ξ	9	159.1
5. As 3. plus linear trend in μ and $\log \sigma$	9	159.3
6. As 3. with separate ξ for each season	10	164.4

Table 3: Table for question 3.2.

H_0	H_1	λ_{LR}	Critical chi-square	Rejection
M_1	M_2	429.1	5.99	Yes
M_2	M_3	166.4	5.99	Yes
M_3	M_4	3.8	5.99	No
M_3	M_5	4.1	5.99	No
M_3	M_6	14.4	9.4877	Yes

Table 4: LR test for question 3.2

3.3

The estimation of the parameters for each model is found in Table 5. From the previous task it is clear that M_1 and M_2 are the worst models and the estimation of parameters are very different from the other models. No plots of these models are presented since these are clearly a very bad fit. The other models have fairly similar estimations of parameter values.

	location				scale				shape				
	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\phi}_0$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\xi}_0$	$\hat{\xi}_1$	$\hat{\xi}_2$	$\hat{\xi}_3$	$\hat{\xi}_4$
M1	381.2	-	-	-	313.6	-	-	-	0.7	-	-	-	-
M2	346.5	7.4	22.8	-	445.3	-	-	-	1.2	-	-	-	-
M3	-15.3	9.6	28.5	-	0.5	0.1	0.5	-	-0.4	-	-	-	-
M4	-15.4	8.7	29	-	0.5	-0.2	0.9	-	-0.3	-0.1	-	-	-
M5	-15.4	8.6	28.3	-2.3	0.5	0.04	0.5	-0.1	-0.3	-	-	-	-
M6	-15.6	9.2	28.8	-	0.5	0.05	0.8	-	-0.3	0.06	-0.1	-0.2	-0.05

Table 5: The estimation of the parameters for each model in question 3.3.

Figures 3, 4 and 5 depict fit diagnostics plots produced by the interface in2extRemes for models 3, 5 and 6. The LR test discussed above quickly dismissed model 1, 2 and 4, which is why they are not presented here. For all of these models the return time seem to follow a periodicity which is to be expected. Overall, the plots indicates that all 3 of these models are reasonable.

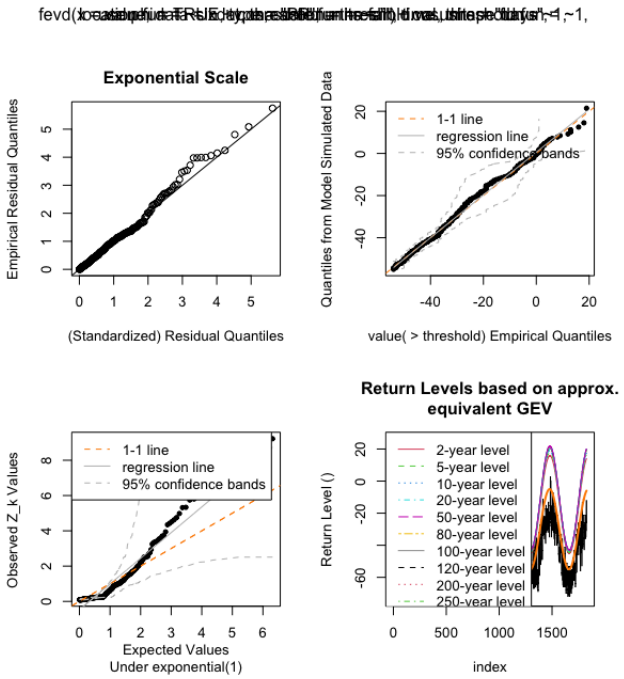

Figure 3: Fit diagnostics plot for model 3.

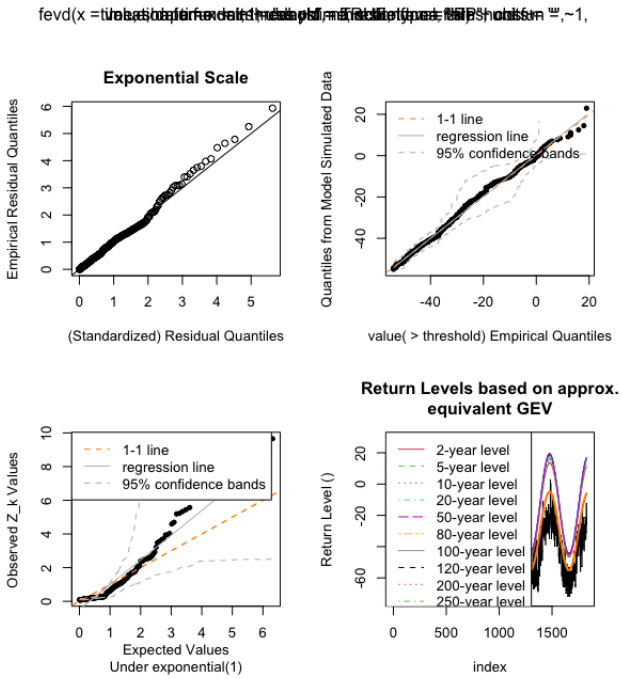

Figure 4: Fit diagnostics plot for model 5.

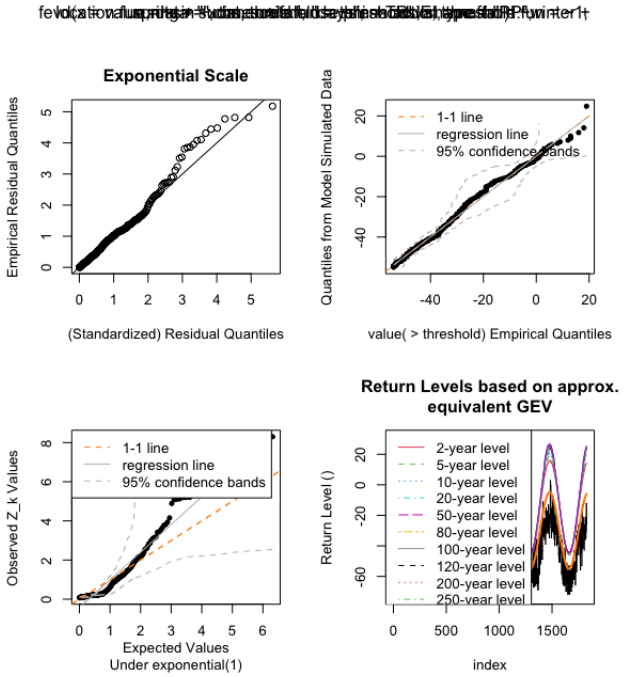

Figure 5: Fit diagnostics plot for model 6.

3.4

Let us denote M7 as M6 but without the default constant term for shape parameter. A Likelihood Ratio test revealed that M6 cannot be preferred over M7 on a 95 % confidence level, hence M7 is deemed the best candidate. Further, since M6 was the best candidate of our previous models, M7 is the best of all tested models for the wooster data.