HA 1 FMSN50 Sundell & Ekman

Gustaf Sundell (gu0147su-s)
Agnes Ekman (ag8720ek-s)

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1 Random number generation

In this first section, we will explore the conditional probability density function and cumulative distribution function of an arbitrary stochastic variable $X \in \mathbb{R}$ with pdf $f_X(x)$ and cdf $F_X(x)$, where the condition is that $X \in I$, where $I = (a, b)$ is an open interval for some real numbers a and b.

1.1 Conditional distribution

As X is a random variable in $\mathbb{R}$, and $P(X \in I) > 0$ was given, it is safe to assume that $a < b$. The conditional distribution function may be found by dividing the probability into cases:

$$\mathbb{P}(X \le x | X \in I) = F_{X|X\in I}(x) = \begin{cases} 0, \ if \ x \le a. \\ 1, \ if \ x \ge b. \\ \frac{F_X(x) - F_X(a)}{F_X(b) - F_X(a)}, \ if \ x \in I \end{cases} \tag{1}$$

And the corresponding conditional probability density function is given by the derivative of the former with respect to x:

$$f_{X|X\in I}(x) = \frac{d}{dx} F_{X|X\in I}(x) = \begin{cases} 0, \ if \ x \le a. \\ 0, \ if \ x \ge b. \\ \frac{f_X(x)}{F_X(b) - F_X(a)}, \ if \ x \in I \end{cases} \tag{2}$$

1.2 Inverse of conditional distribution

Let $u = F_{X|X\in I}(x)$, obviously being bounded in $[0, 1]$. The inverse may then be found by the same reasoning as in 1.1, solving for x:

$$x = F^{-1}_{X|X\in I}(u) \begin{cases} \leq a, \ if \ u = 0. \\ \geq b, \ if \ u = 1. \\ = F_X^{-1}((F_X(b) - F_X(a))u + F_X(a)), \ if \ u \in (0,1). \end{cases} \tag{3}$$

Where $F_X^{-1}(x)$ denotes the inverse of the given, known density function $F_X(x)$, and it is assumed that its inverse is also known. Note that it may be a generalized inverse, as discussed and defined in Lecture 2, slide 17. This may be used in a very neat way to simulate X, when one is only interested in $X|X \in I$, something very applicable to stratified sampling. This will be explicitly implemented and explained in section 2.3, regarding task 2.a). The idea behind stratified sampling is dividing all the values that X can assume into disjoint sets. Seeing as $X \in \mathbb{R}$, the disjoint sets will be disjoint intervals. Following the calculations done here, the sets may be constructed as the 3 disjoint subsets in $\mathbb{R}$:

$$\begin{aligned} A_1 &= \{x : x \leq a\} \\ A_2 &= \{x : x \in I\} \\ A_3 &= \{x : x \geq b\} \end{aligned} \tag{4}$$

After this, when estimating τ, with the notation $f_i(x) = f_{X|X\in A_i}(x)$ and $p_i = \mathbb{P}(X \in A_i)$, one may estimate it according to the following:

$$\tau = \mathbb{E}_{f_X}(\phi(X)) = \sum_{i=1}^{3} p_i \mathbb{E}_{f_i}(\phi(X)) \tag{5}$$

Where, as follows from the definition of the variable's cdf, $\mathbb{P}(X \in I) = F_X(b) - F_X(a)$.

2 Power production of a wind turbine

2.1 General setup

In this section, a comparison between different approaches to simulating the power produced by a windmill will be presented. For all approaches, the underlying distributional assumption of the stochastic wind was a Weibull distribution, as stated in the instructions for Home assignment 1, page 2. Therefore the wind has the following probability density function:

$$f(v) = \frac{k}{\lambda}\left(\frac{v}{\lambda}\right)^{k-1} e^{-\left(\frac{v}{\lambda}\right)^k}, v \geq 0 \tag{6}$$

and the cumulative density function:

$$F(v) = 1 - e^{-\left(\frac{v}{\lambda}\right)^k}, v \geq 0 \tag{7}$$

, where λ and k vary between months, as specified in the table on page 2 in the instructions for Home assignment 1. The cumulative density function was found by integrating over the domain of V.

The windmill was further assumed to (theoretically) produce power according to the following equation, found on page 1 of the Home assignment 1 instructions:

$$P_{tot}(v) = \frac{1}{8}\rho\pi d^2 v^3 \tag{8}$$

, where $\rho[kg/m^3]$ is the air density and $d[m]$ is the rotor diameter.

The following simulations use the specifications of the wind turbine Vestas V164, with a rotor diameter of 164 m, and the air density by the sea level: $1.225kg/m^3$. The cut-in and cut-off wind speeds were 3.5 and $25m/s$ respectively, meaning that the windmill produces no power when the wind speed lies outside the interval $[3.5, 25]$. As part of the instructions to this project was a MATLAB function downloaded as $powercurve\_V164.mat$, a MATLAB object with built-in function to calculate the actual power produced by the Vestas V164 for given winds. This will from now on be denoted as $P(v)$, and should not be confused with the aforementioned $P_{tot}(v)$.

In all simulation approaches below, the wind was simulated for each month separately, since the underlying distribution parameters λ and k vary between months. The power produced for each wind sample was then computed according to the equation and specifications above. Finally, the mean and standard deviation were computed to form the confidence interval:

$$I = \mu \pm \lambda_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{9}$$

, where μ is the mean, σ the standard deviation, $\lambda_{\alpha/2}$ the $\frac{\alpha}{2}$-quantile of the Normal distribution, obtaining a confidence interval with significance level $1-\alpha$, noting that the Normal distribution is symmetric. In all approaches $\alpha = 0.05$ was set, and the sample size set to $n = 10000$.

2.2 Standard Monte Carlo sampling

As a first approach to simulating the power generated from the windmill, the standard Monte Carlo method was implemented. This is of course the standard setup presented in lecture 1, slides 7 and 8, with the notation of $X = V =$ stochastic wind, $\Phi(X) = P(V) =$ power curve of the Vestas V164 as a function of the stochastic wind, and $f_X(x) = f(v)$ as in equation (6), the Weibull distribution with varying λ and k across the 12 months of the year. The wind samples were simply drawn from the Weibull distribution stated above, by using MATLAB's built-in function *wblrnd*, instead of implementing the inverse method using an analytically derived inverse function, and the corresponding confidence interval of the power was computed for each month, using the P(v)-function and equation (9). The results are found in table 1.

2.3 Truncated Monte Carlo sampling

In the truncated simulation approach, wind samples were only generated within the power producing interval, $[3.5, 25]$ m/s.

This approach is an implementation of stratified sampling, which was presented in section 1.2. Following the notation introduced in equations (4) and (5), it may be noted that A_2 here corresponds to winds within the power-producing interval $[3.5, 25]$ and A_1 and A_3 corresponds to winds below and above the interval, respectively. When calculating the expected power produced, using stratified sampling in this way has some clear benefits, as $\mathbb{E}_{f_i}(P(V)) = 0$ for $i = 1$ and $i = 3$, using the same notation as in equations (4) and (5) and section 2.2. The scheme for calculating τ using the truncated sampling method therefore reduces to:

$$\tau = \mathbb{E}_f(P(V)) = \sum_{i=1}^{3} p_i \mathbb{E}_{f_i}(P(V)) = p_2 \mathbb{E}_{f_2}(P(V)) \tag{10}$$

Finally, relying on the notation in sections 1.1 and 1.2 with the general $f_X(x)$ and $F_X(x)$ substituted for their Weibull counterparts found in (6) and (7) respectively. Realizing that $p_2 = P(V \in [3.5, 25]) = F(25) - F(3.5)$, the setup for the truncated sampling is complete.

First, the samples were drawn from the uniform distribution, generating n probability samples between 0 and 1. The probability samples were then filtered through the inverse of the wind's cumulative distribution function conditional on $V \in [3.5, 25]$ (following the scheme presented in equation (3)) to generate the wind samples. I.e.

$$\begin{aligned} u_i &\sim U(0,1) \\ v_i &= F^{-1}_{V|V\in[3.5,25]}(u_i) \end{aligned} \tag{11}$$

These samples were then used to form the confidence interval, according to the general procedure described above. The results are found in table 2.

2.4 Importance sampling

The truncated simulation approach using stratified sampling, introduced above in section 2.3, is a special case of the more general importance sampling approach where the samples are drawn from another distribution than the assumed underlying one. The density function, $f(v)$, and the integrand, $P(v)$, are dissimilar. This means that a large part of the mass of the density function lies outside the power producing interval of the wind, resulting in many wasted samples.

An importance sampling approach opens up to the possibility of drawing wind samples from a distribution with a larger mass in the power producing wind interval,$g(v)$, reducing the amount of wasted samples and consequently also reducing the variance, see lecture 4 slide 14. $P(v)$ is then multiplied by the ratio between $f(v)$ and $g(v)$ to restore the distributional properties of the wind, see lecture 4 slide 15, before computing the mean, standard deviation, and confidence interval.

The two most important properties of $g(v)$ are: 1. $g(v)$ has a large mass in the power producing interval, and 2. $\frac{P(v)f(v)}{g(v)}$ is close to constant, see lecture 4 slides 14 and 17 . One function, $g(v)$, which fulfill these properties is the $\Gamma(7,2)$ function. Though it might be bold to argue that $\frac{P(v)f(v)}{g(v)}$ is close to constant when using $g(v) = \Gamma(7,2)$, it is safe to say that it does not diverge for any of the monthly combinations of λ:s and k:s, which is enough to secure the efficiency of the method at least in this particular case.

After choosing $g(v)$, the wind samples were drawn from the $\Gamma(7,2)$ distribution. Then, tau was computed as:

$$\tau = \mathbb{E}_g \left(\frac{P(v)f(v)}{g(v)} \right) \tag{12}$$

, and used to form the confidence interval according to the same general procedure as before. The results are found in table 3.

2.5 Antithetic sampling

As a final approach, antithetic sampling was used. Here, probability samples were drawn from a uniform distribution on the interval $[0,1]$, as in the truncated sampling case. Then, a set of antithetic variables was created trough a transformation of u, where u denotes the uniformly drawn samples, with the purpose to reduce the variance. Since $\Phi = P$ is a monotone function, the transformation $T(u) = 1 - u$ fulfills the purpose, see Lecture 4 slide 21. Hence, the antithetic variable $\tilde{u}$ was introduced as $\tilde{u}_i = 1 - u_i$. The resulting antithetic variables, $\tilde{u}$, are also uniformly distributed on the interval $[0,1]$, see lecture 4 (handwritten slide after slide 21) but naturally negatively correlated with the original samples, u. From these two sets of uniform samples, two sets of wind were generated following the exact same scheme as for the truncated samples, namely conditioning the winds on being in the interval $[3.5, 25]$

$$v_i = F^{-1}_{V|V\in[3.5,25]}(u_i) \tag{13}$$

and

$$\tilde{v}_i = F^{-1}_{V|V\in[3.5,25]}(\tilde{u}_i) \tag{14}$$

The power produced based on the wind samples and the antithetic variables were then added and normalized by a division of two:

$$\tilde{P}_i = \frac{P(v_i) + P(\tilde{v}_i)}{2} \tag{15}$$

The mean and standard deviation were then computed based on $\tilde{P}_i$. Finally the confidence interval was computed according to equation 9 as usual. The results are found in table 4.

2.6 Comparison of the different approaches

As a primary comparison variable, the width of the confidence interval was used, i.e.:

$$w = 2\lambda_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{16}$$

To test the bias of the truncated, importance, and antithetic sampling approaches, the monthly mean values were compared to the confidence intervals of the standard Monte Carlo approach. The standard MC approach was used as a benchmark since it used wind samples drawn from the raw Weibull distribution, which was the assumed underlying distribution of the wind. Hence, the standard MC approach is unbiased but with a large variability since part of the samples are outside the power producing wind speed.

The results are presented in Table 5, where, for each method and month, the width of the confidence interval is presented. The width of the confidence interval is a good indicator of how well behaved the simulation method performed, as high certainty is desirable. The column titled "Inside?" shows whether or not the confidence interval for each method overlaps the one given by the standard MC method, which is assumed to be unbiased. Being inside is clearly desirable as being outside suggests a bias. As can be seen in Table **??**, all methods seem to be unbiased.

The values in the table suggest that antithetic sampling was the method proving most reliable, as it had the most narrow confidence interval in all months. Notably, checking in MATLAB, even the means for each month with antithetic sampling fall inside the confidence interval of the standard MC, whereas truncated and importance sampling methods yield means that fall outside. Importance sampling shows promise, with the second lowest width in all months. Notably, this could probably be improved further by alternating $g(v)$ to fit the behaviour of the wind for each month, and in this project only one $g(v)$ was used to fit all. Also, the variance generated by the importance sampling could probably be further reduced if a function $g(v)$ was found which yields a more constant ratio $\frac{\Phi f(v)}{g(v)}$ than the $g(v)$ that was used in section 2.4.

Finally, the truncated method manages to reduce the interval width from the standard MC method.

2.7 Probability of delivering power

The probability that the turbine delivers power is equivalent to the probability that the wind speed is within the interval I, yielding the probability expression:

$$\mathbb{P}(P(v) > 0) = \mathbb{P}(V \in I) = F_V(25) - F_V(3.5) \tag{17}$$

2.8 Average power coefficient

The expected value of the total wind power can be computed analytically, using the total amount of wind power for some wind and the probability density function of the wind:

$$P_{tot}(v) = \frac{1}{8}\rho\pi d^2 v^3 \tag{18}$$

$$f(v) = \frac{k}{\lambda}\left(\frac{v}{\lambda}\right)^{k-1} e^{-\left(\frac{v}{\lambda}\right)^k} \tag{19}$$

$$\begin{aligned}
\mathbb{E}P_{tot}(V) = \int_0^\infty P_{tot}(v)f(v)dv = \int_0^\infty \frac{1}{8}\rho\pi d^2 v^3 \frac{k}{\lambda}\left(\frac{v}{\lambda}\right)^{k-1} e^{-\left(\frac{v}{\lambda}\right)^k} dv = \\
= \left[u = v^k, du = kv^{k-1}dv\right] = \frac{1}{8}\rho\pi d^2 \int_0^\infty \lambda^{3-k}\left(\frac{u}{\lambda^k}\right)^{\frac{3}{k}} e^{-\left(\frac{u}{\lambda^k}\right)} du = \\
= \left[z = \frac{u}{\lambda^k}, dz = \frac{1}{\lambda^k}du\right] = \frac{1}{8}\rho\pi d^2 \int_0^\infty \lambda^3 z^{\frac{3}{k}} e^{-z} dz = \\
= \frac{1}{8}\rho\pi d^2 \lambda^3 \Gamma\left(1 + \frac{3}{k}\right)
\end{aligned} \tag{20}$$

Since the estimation of the expected value of the actual wind power had the smallest variance when simulated using antithetic sampling, see section 2.6, that value was used together with the analytical total wind power to form the average power coefficient:

$$\frac{\mathbb{E}P(V)}{\mathbb{E}P_{tot}(V)} = \frac{\mathbb{E}_{anti}P(V)}{\frac{1}{8}\rho\pi d^2\lambda^3\Gamma\left(1+\frac{3}{k}\right)} \tag{21}$$

The built-in *gamma* function in MATLAB was used to compute the denominator, and the result is presented in table 6.

2.9 Capacity and availability

The capacity factor is defined as the ratio of the actual output over a time period and the maximum possible output during that time, see instructions for Handout 1 page 3. This is equivalent to the ratio of the average output over a time period and the average maximum possible output during that time, see equation 22.

$$CF_m = \frac{\sum_{i=1}^{n} P(V)_{m,i}}{9.5 * 10^6 * n} = \frac{\mathbb{E}P(V)_m}{9.5 * 10^6} \tag{22}$$

Since the antithetic sampling resulted in the most narrow confidence interval, see section 2.6, the mean values generated through that sampling method were used to compute the capacity factor. Finally, the yearly mean was computed by averaging over the twelve months:

$$\mathbb{E}CF = \frac{\sum_{m=1}^{12} CF_m}{12} = 0.3936 \tag{23}$$

The availability factor was obtained by utilizing the probabilities for producing power in each month, found by equation (17) for each month with corresponding parameters, and then averaging the probability over the 12 months. The result was 0.8501.

The recommendations given in the instructions are that the capacity factor should be between 20 and 40 %, and that the availability factor should be above 90 %. In this case, the availability factor is slightly below the recommended level, whereas the capacity factor is on the high end of the recommended interval. This indicates that it is not completely unreasonable to build a wind power plant on this location.

3 Combined power production of two wind turbines

3.1 General setup

In this part, two windmills were modeled, affected by "two winds" V_1 and V_2, both stochastic. The bivariate cumulative distribution function and probability density function for the two winds are, for $v_1 \geq 0$ and $v_2 \geq 0$:

$$F(v_1, v_2) = F(v_1)F(v_2)\left(1 + \alpha\left(1 - F(v_1)^p\right)^q \left(1 - F(v_2)^p\right)^q \right. \tag{24}$$

$$f(v_1, v_2) = f(v_1)f(v_2)\Big(1 + \alpha\left(1 - F(v_1)^p\right)^{q-1}\left(1 - F(v_2)^p\right)^{q-1} ...$$
$$... \left(F(v_1)^p (1 + pq) - 1\right)\left(F(v_2)^p (1 + pq) - 1\right)\Big)$$

This is clearly a non-trivial distribution to sample from, and importance sampling might simplify this with the right choice of $g(v)$, since the samples are then drawn from $g(v_1, v_2)$ instead of from $f(v)$. An intuitive approach is to sample v_1 and v_2 as two independent Weibull distributions with the same λ and k as in f, yielding the joint distribution:

$$g(v_1, v_2) = f(v_1)f(v_2) \tag{25}$$

with f again being the Weibull pdf.

When moving forward to compute the covariance of $P(v_1)$ and $P(v_2)$, and the variance and mean of the total power production $P(v_1) + P(v_2)$, there are three relevant Φ-functions:

$$\Phi_{SUM} = P(v_1) + P(v_2) \tag{26}$$
$$\Phi_{PROD} = P(v_1)P(v_2) \tag{27}$$
$$\Phi_{single} = P(v) \tag{28}$$

Hence, the critical ratio $\frac{\Phi(v_1,v_2)f(v_1,v_2)}{g(v_1,v_2)}$, was plotted for the first two Φ-functions, corresponding to the sum and product respectively, to make sure that neither of them diverged. No divergence was found and $g(v_1, v_2)$ was hence considered a sufficiently decent choice for importance sampling, and the wind samples v_1 and v_2 were drawn from the one-dimensional Weibull distribution. Note that Φ_{single} corresponds to the critical ratio:

$$\Phi_{single}\frac{f(v)}{g(v)} = P(v), \text{ since } f(v) = g(v) \tag{29}$$

3.2 Expected Power

The expectation of the sum of the two power reduces to a linear problem, as the expectation is a linear operator, and the two variables are identically distributed.

Even though $P(v)$ is a non-linear transformation, applying it to two identically distributed variables yields the same expectation. Hence the expectation of the v_1 sample, drawn in section 3.1, was taken and doubled to compute the mean of the total power:

$$\mathbb{E}(P(V_1) + P(V_2)) = \mathbb{E}P(V_1) + \mathbb{E}P(V_2) = 2\mathbb{E}P(V_1) = 7.4295 * 10^6 \tag{30}$$

3.3 Covariance

The covariance of $P(V_1)$ and $P(V_2)$ may be expressed as:

$$\mathbb{C}(P(V_1), P(V_2)) = \mathbb{E}(P(V_1)P(V_2)) - \mathbb{E}(P(V_1))\mathbb{E}(P(V_2)) = 6.5037 * 10^{12} \tag{31}$$

Where the expectation of the product was calculated according to

$$\mathbb{E}(P(V_1)P(V_2)) = \mathbb{E}_g\left(\Phi_{prod}(v_1, v_2)\frac{f(v_1, v_2)}{g(v_1, v_2)}\right) \tag{32}$$

and the separate expectations were calculated according to section 3.2

3.4 Power production variance

The variance and standard deviation of the total power produced by the two windmills was computed as:

$$\begin{aligned} V\left(P(v_1) + P(v_2)\right) &= V_g\left(\Phi_{SUM}(v_1, v_2)\frac{f(v_1, v_2)}{g(v_1, v_2}\right) = \\ &= 9.5885 * 10^{13} \end{aligned} \tag{33}$$

$$SD\left(P(v_1) + P(v_2)\right) = \sqrt{V\left(P(v_1) + P(v_2)\right)} = 9.7921 * 10^6 \tag{34}$$

3.5 Capacity probability

The classical definition of probability of a case, H, is:

$$\mathbb{P}(H) = \frac{number\ of\ favorable\ cases}{number\ of\ all\ cases\ possible} \tag{35}$$

Hence, the best estimate of the probability of the two windmills producing a total of more than 9.5 MW, given the importance sampling described above, is simply the number of samples where they produced more than 9.5 MW divided by the total number of samples, n. Similarly, the best estimate of the probability of the two windmills producing a total of less than 9.5 MW is the number of samples where they produced less than 9.5 MW divided by the total number of samples, n.

The estimates of these two probabilities will naturally sum up to one, since the sum of the number of samples where the production is larger than 9.5 and the number of samples where the production is lower than 9.5 is equal to the total number of samples. The only time that this would not hold is if one or more samples generated a power production equal to 9.5, since these would not be included into either of the two sets. However, the probability of drawing a sample generating that exact power is zero, so there is no need to take that into consideration.

The estimates of the probabilities of producing more than and less than 9.5, based on the bivariate importance sampling, was:

$$\hat{\mathbb{P}}_{more} = 0.7932\hat{\mathbb{P}}_{less} = 0.2068 \tag{36}$$

Since there are two possible outcomes, more than and less than 9.5 MW, the overall outcome can be viewed as a binomial distribution. The standard deviation of the probability estimations will hence behave as that of the binomial distribution:

$$\sigma(\hat{\mathbb{P}}) = \sqrt{\frac{npq}{n^2}} = \sqrt{\frac{pq}{n}} \tag{37}$$

, where n is the sample size, p is the probability of producing more than 9.5 MW and q is the probability of producing less than 9.5 MW. Finally, the confidence intervals for the two probabilities can be constructed as:

$$I_{more} = \hat{\mathbb{P}}_{more} \pm \lambda_{\alpha/2}\sqrt{\frac{pq}{n}} = [0.7853,\ 0.8011] \tag{38}$$

$$I_{less} = \hat{\mathbb{P}}_{less} \pm \lambda_{\alpha/2}\sqrt{\frac{pq}{n}} = [0.1989,\ 0.2147] \tag{39}$$

Note that, naturally, the lower limit of the first confidence interval and the upper limit of the second confidence interval sum up to one and vice versa, see equations (39) and (40);

4 Tables

	lwr	fit	upr
January	4.6081	4.6800	4.7520
February	4.0056	4.0755	4.1453
March	3.7957	3.8639	3.9320
April	2.9924	3.0555	3.1186
May	2.8181	2.8801	2.9420
June	2.9990	3.0620	3.1250
July	2.8154	2.8771	2.9388
August	3.0523	3.1163	3.1803
September	3.7536	3.8217	3.8898
October	4.1479	4.2194	4.2910
November	4.5401	4.6120	4.6840
December	4.5481	4.6199	4.6916

Table 1: Approximate 95 % confidence interval for the expected amount of power generated by the wind turbine, estimated using standard MC simulation.

	lwr	fit	upr
January	4.5756	4.6439	4.7121
February	4.1356	4.2030	4.2704
March	3.7296	3.7956	3.8615
April	2.9220	2.9842	3.0464
May	2.8156	2.8768	2.9380
June	3.0080	3.0703	3.1327
July	2.8160	2.8773	2.9386
August	3.0002	3.0629	3.1256
September	3.6591	3.7250	3.7908
October	4.1733	4.2419	4.3104
November	4.6302	4.6987	4.7671
December	4.6139	4.6821	4.7504

Table 2: Approximate 95 % confidence interval for the expected amount of power generated by the wind turbine, estimated using truncated simulation.

	lwr	fit	upr
January	4.6188	4.6487	4.6785
February	4.1063	4.1402	4.1740
March	3.8153	3.8513	3.8874
April	2.9792	3.0155	3.0519
May	2.8570	2.8716	2.9082
June	3.0474	3.0835	3.1196
July	2.8570	2.8937	2.9305
August	3.0563	3.0923	3.1283
September	3.7316	3.7683	3.8050
October	4.1759	4.2054	4.2349
November	4.6648	4.6945	4.7241
December	4.6298	4.6596	4.6894

Table 3: Approximate 95 % confidence interval for the expected amount of power generated by the wind turbine, estimated using importance sampling.

	lwr	fit	upr
January	4.6570	4.6609	4.6648
February	4.1384	4.1450	4.1517
March	3.8159	3.8258	3.8357
April	2.9988	3.0154	3.0319
May	2.8594	2.8769	2.8944
June	3.0543	3.0703	3.0863
July	2.8582	2.8756	2.8931
August	3.0685	3.0846	3.1007
September	3.7572	3.7677	3.7781
October	4.2218	4.2271	4.2323
November	4.6547	4.6586	4.6626
December	4.6530	4.6570	4.6609

Table 4: Approximate 95 % confidence interval for the expected amount of power generated by the wind turbine, estimated using antithetic sampling.

	Standard MC		Truncated MC		Importance sampling		Antithetic Sampling	
Month	CI width	Inside?	CI width	Inside?	CI width	Inside?	CI width	Inside?
January	1.4381	Yes	1.3650	Yes	0.5969	Yes	0.0787	Yes
February	1.3967	Yes	1.3473	Yes	0.6766	Yes	0.1342	Yes
March	1.3632	Yes	1.3188	Yes	0.7209	Yes	0.1978	Yes
April	1.2622	Yes	1.2445	Yes	0.7264	Yes	0.3304	Yes
May	1.2389	Yes	1.2237	Yes	0.7316	Yes	0.3484	Yes
June	1.2606	Yes	1.2464	Yes	0.7219	Yes	0.3235	Yes
July	1.2338	Yes	1.2266	Yes	0.7352	Yes	0.3478	Yes
August	1.2796	Yes	1.2542	Yes	0.7197	Yes	0.3212	Yes
September	1.3624	Yes	1.3173	Yes	0.7345	Yes	0.2108	Yes
October	1.4306	Yes	1.3705	Yes	0.5896	Yes	0.1051	Yes
November	1.4397	Yes	1.3698	Yes	0.5937	Yes	0.0780	Yes
December	1.4352	Yes	1.3649	Yes	0.5966	Yes	0.0788	Yes

Table 5: Presenting output of simulation methods on each month. "Inside?" refers to whether or not the methods confidence interval overlaps with that yielded by the standard method for each month. Please note that the numbers in the "CI width"-columns are expressed in 10^5 MW.

	lwr	fit	upr
January	0.2273	0.2275	0.2277
February	0.2636	0.2641	0.2645
March	0.2849	0.2857	0.2864
April	0.3217	0.3234	0.3252
May	0.3309	0.3329	0.3350
June	0.3156	0.3173	0.3189
July	0.3308	0.3328	0.3348
August	0.3171	0.3188	0.3204
September	0.2899	0.2907	0.2915
October	0.2389	0.2392	0.2395
November	0.2272	0.2274	0.2276
December	0.2271	0.2273	0.2275

Table 6: Approximate 95 % confidence interval for the ratio between actually produced power and theoretically produced power.