SciencesPo
DEPARTMENT OF ECONOMICS

ScPoEconometrics

Differences-in-Differences and Regression Discontinuity Design

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- *Standard error* of regression coefficients
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Regression Discontinuity Design

- Life is full of random rules
- Exploits knowledge of assignment rule
- *Empirical application:* effect of ***alchol consumption*** on ***mortality***

Program Evaluation

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  - ***regression discontinuity designs (RDD)***.
- These methods are used to identify **causal relationships** between treatments and outcomes.
- In this lecture, we will cover two popular and rigorous program evaluation methods:
  1. **differences-in-differences**,
  2. **regression discontinuity designs**.

Differences-in-Differences

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DiD Requirements:

- 2 time periods: before and after treatment.
- 2 groups:
  - *control group:* never receives treatment,
  - *treatment group:* initially untreated and then fully treated.
- Under certain assumptions, control group can be used as the counterfactual for treatment group

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- Seminal 1994 paper by prominent labor economists David Card and Alan Krueger entitled "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania"
- Estimates the effect of an increase in the minimum wage on the employment rate in the fast-food industry. Why this industry?

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Pennsylvania and New Jersey are ***very similar***: similar institutions, similar habits, similar consumers, similar incomes, similar weather, etc.

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Let's take a closer at their data

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  # install package that contains the cleaned data
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  # load package
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  # load data
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```r
ck1994 %>%
  select(sheet,chain,state,observation,empft,emppt) %>
  head()
```

```
## # A tibble: 6 x 6
##   sheet chain  state        observation   empft emppt
##   <chr> <chr>  <chr>        <chr>         <dbl> <dbl>
## 1 46    bk     Pennsylvania February 1992  30    15
## 2 49    kfc    Pennsylvania February 1992   6.5   6.5
## 3 506   kfc    Pennsylvania February 1992   3     7
## 4 56    wendys Pennsylvania February 1992  20    20
## 5 61    wendys Pennsylvania February 1992   6    26
## 6 62    wendys Pennsylvania February 1992   0    31
```

Card and Krueger DiD: Tabular Results

Average Employment Per Store Before and After the Rise in NJ Minimum Wage

Variables	Pennsylvania	New Jersey
FTE employment before	23.33	20.44
FTE employment after	21.17	21.03
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Differences-in-differences causal estimate: $0.59 - (-2.17) = 2.76$

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Let's look at these results graphically.

DiD Graphically

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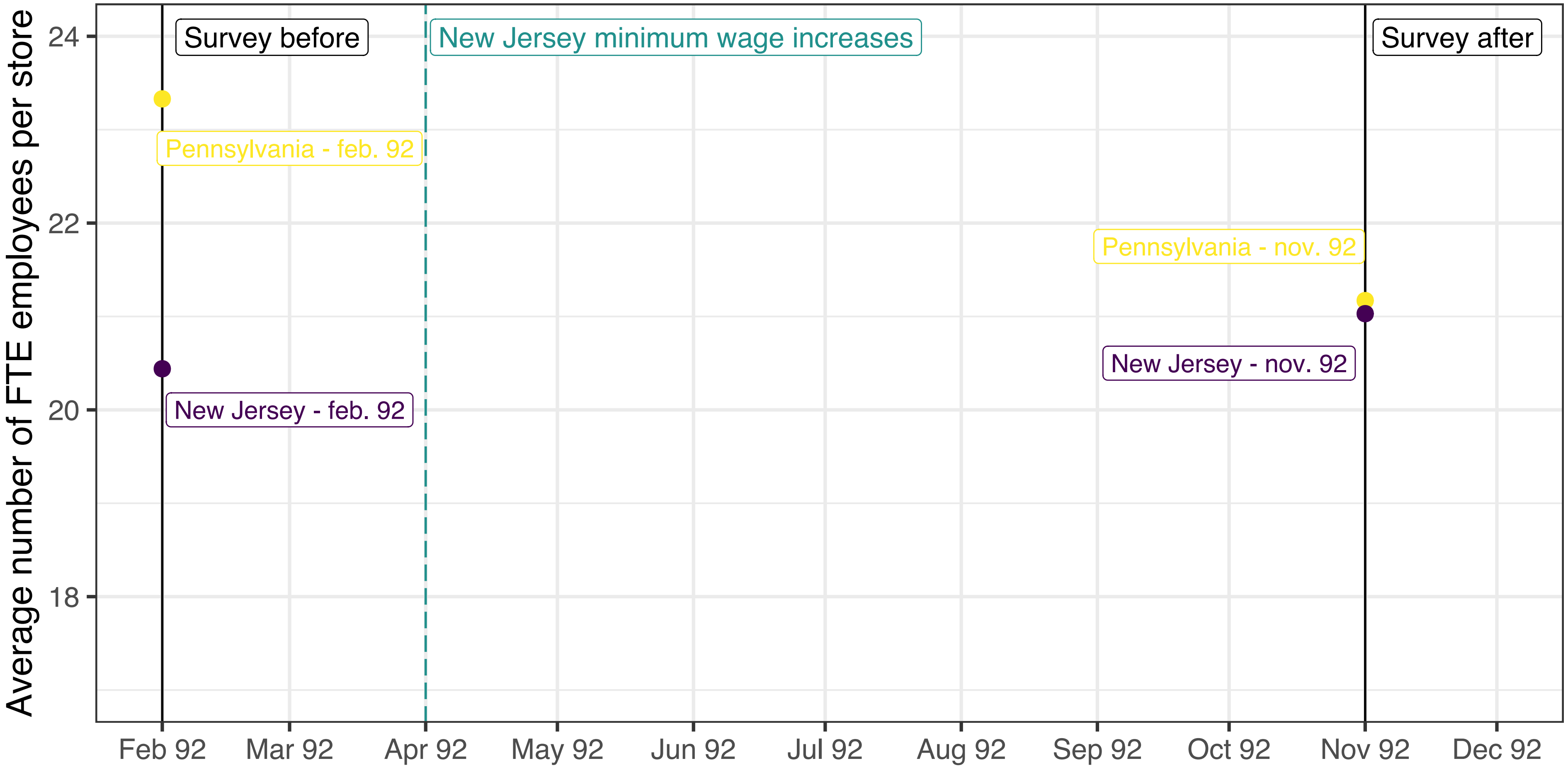

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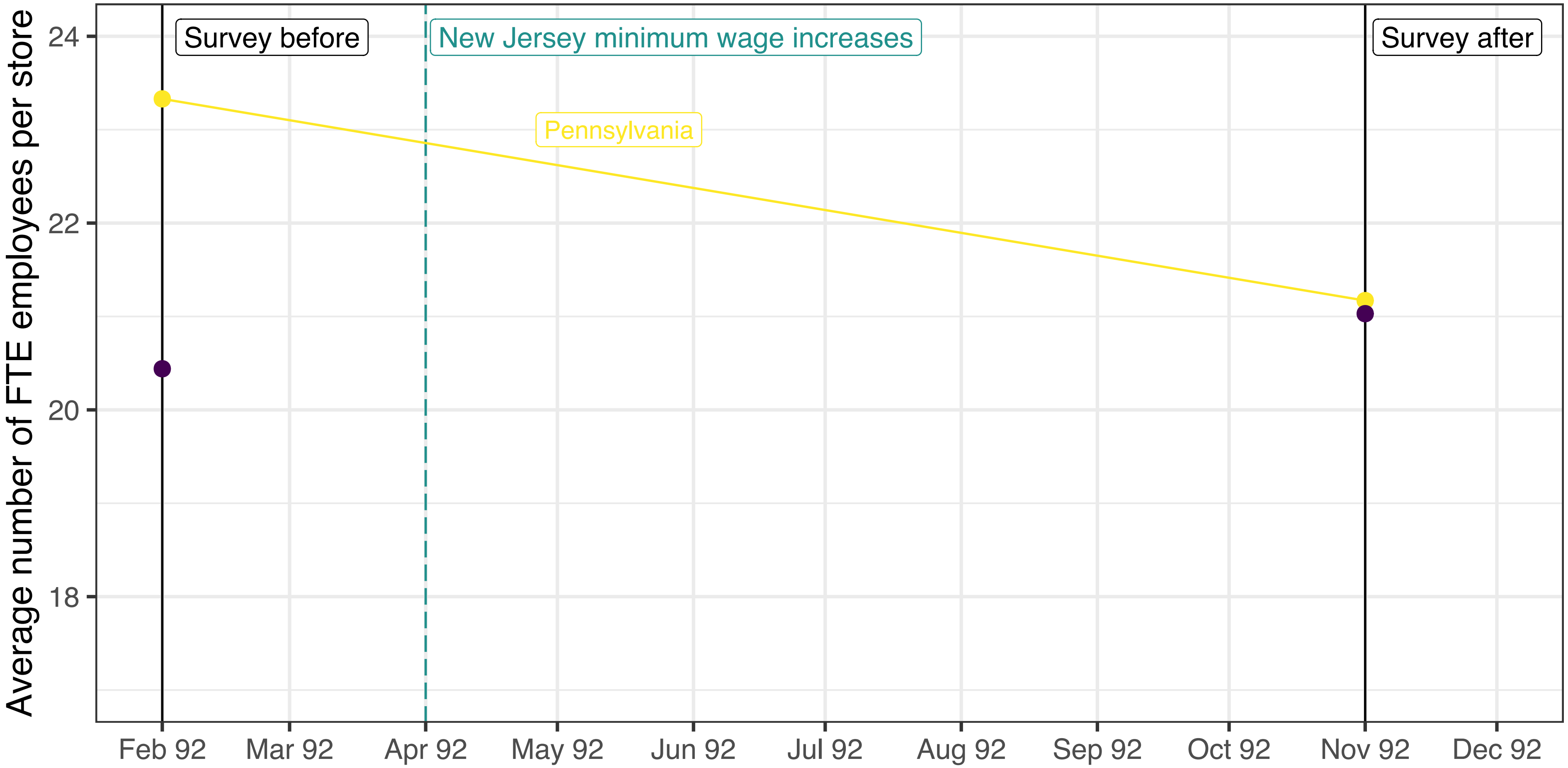

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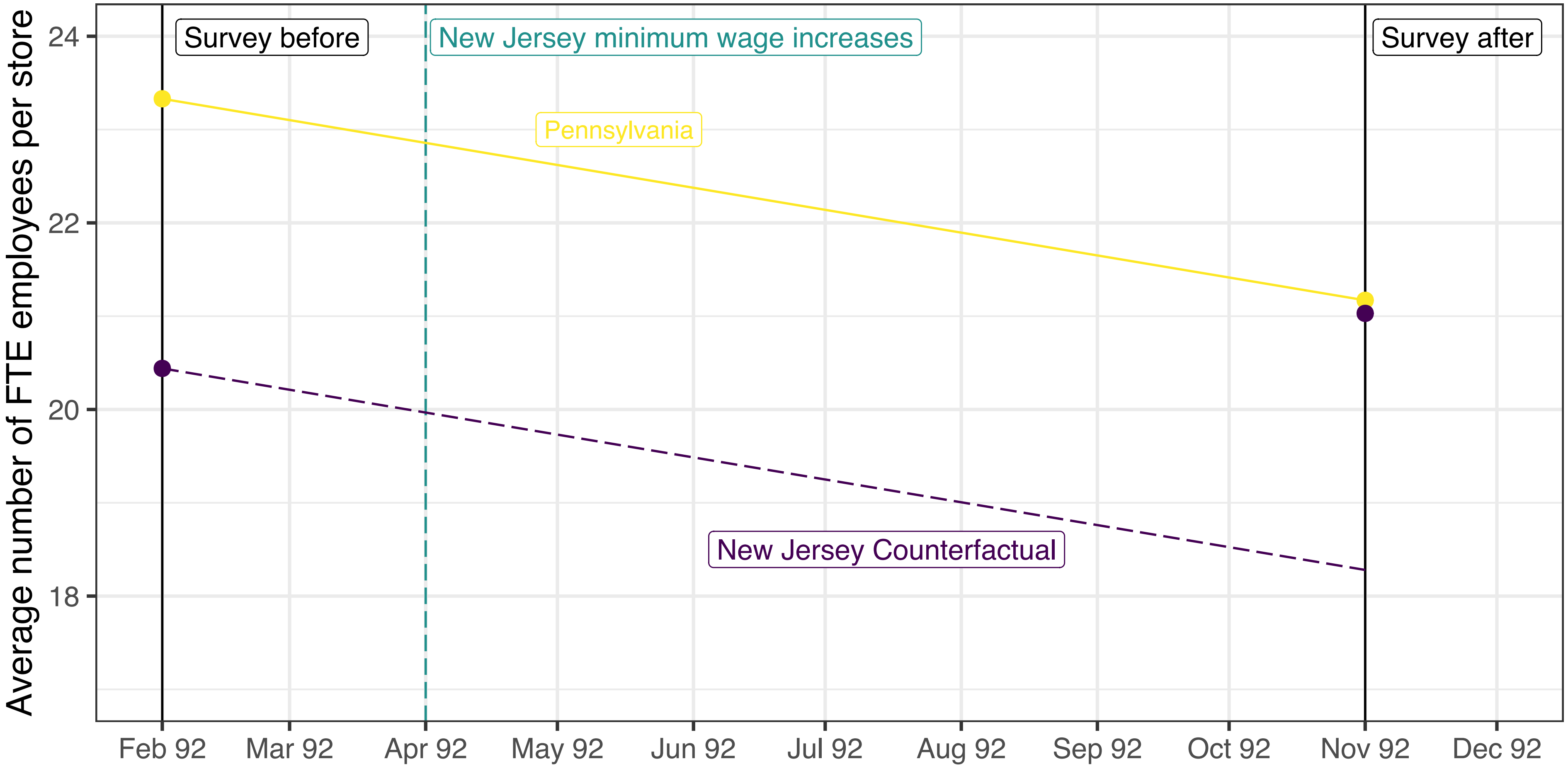

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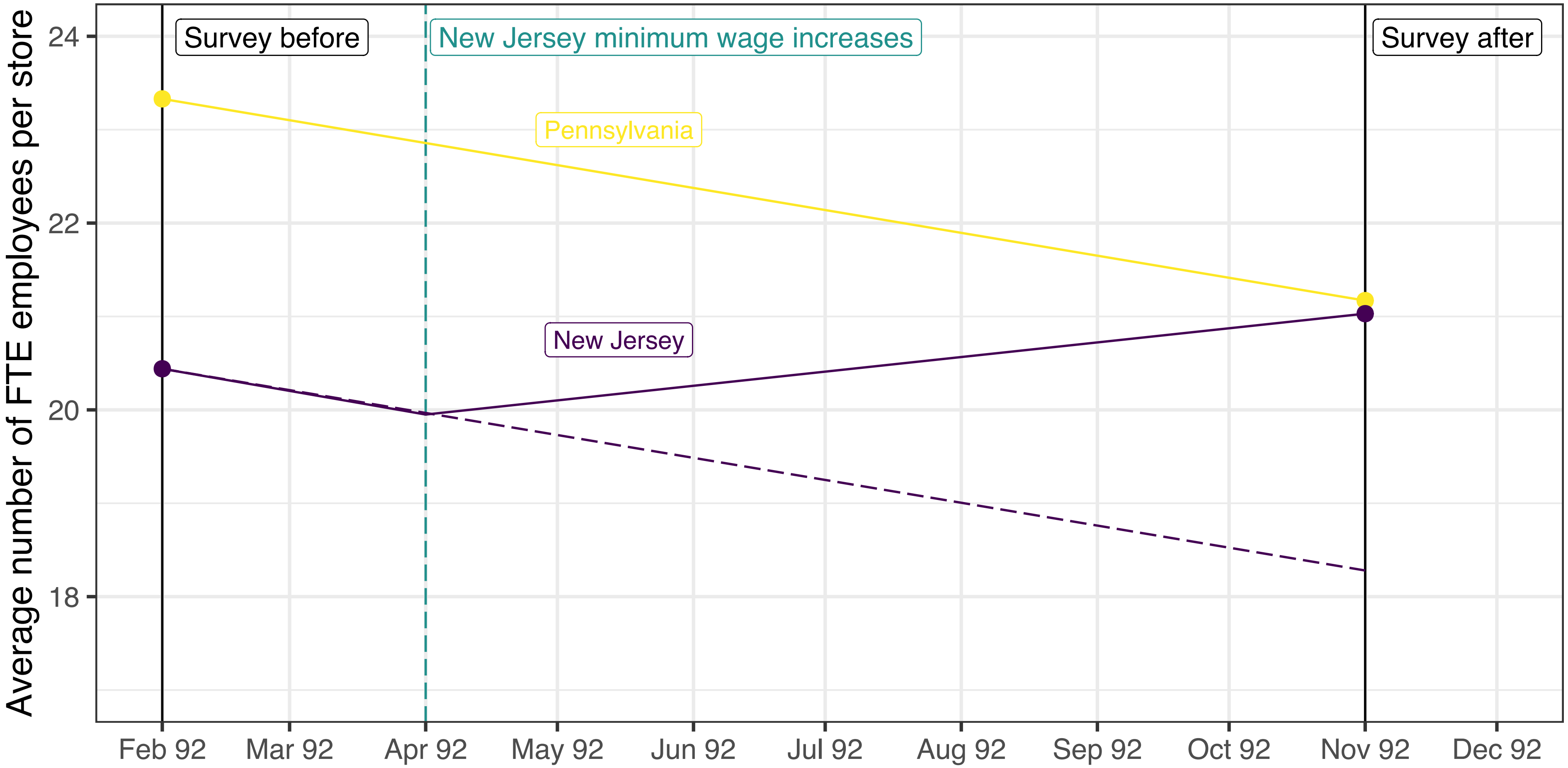

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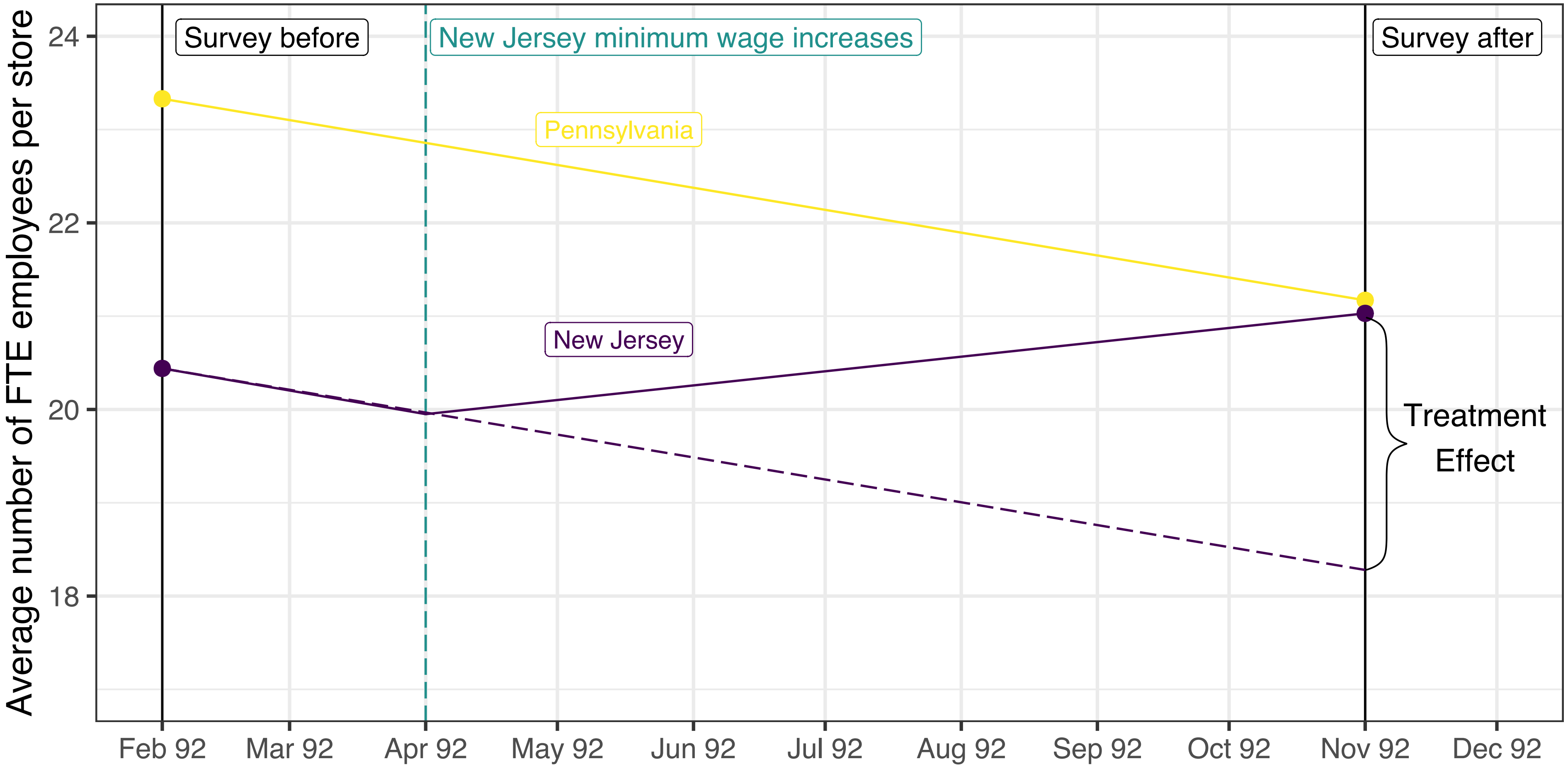

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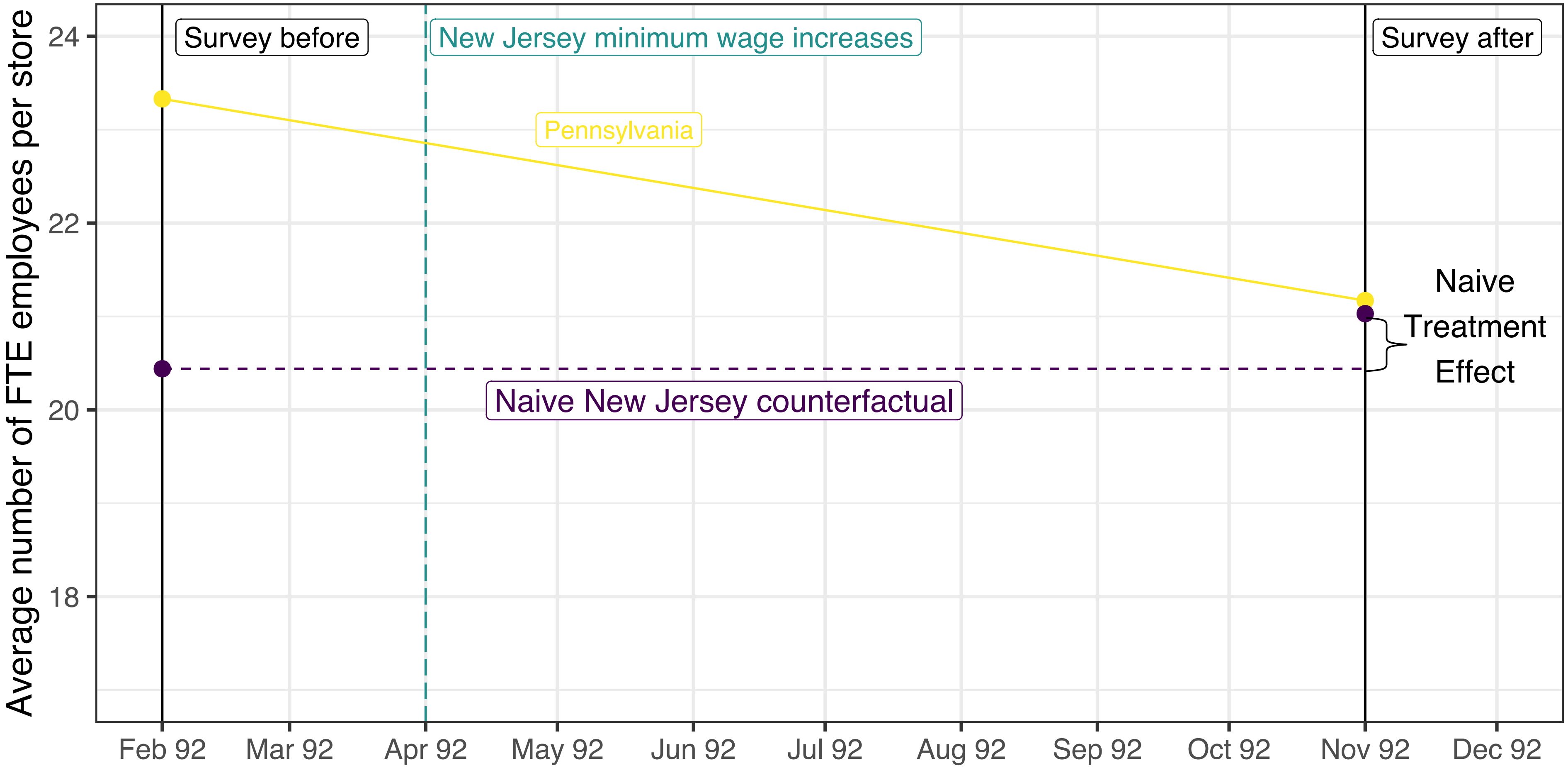

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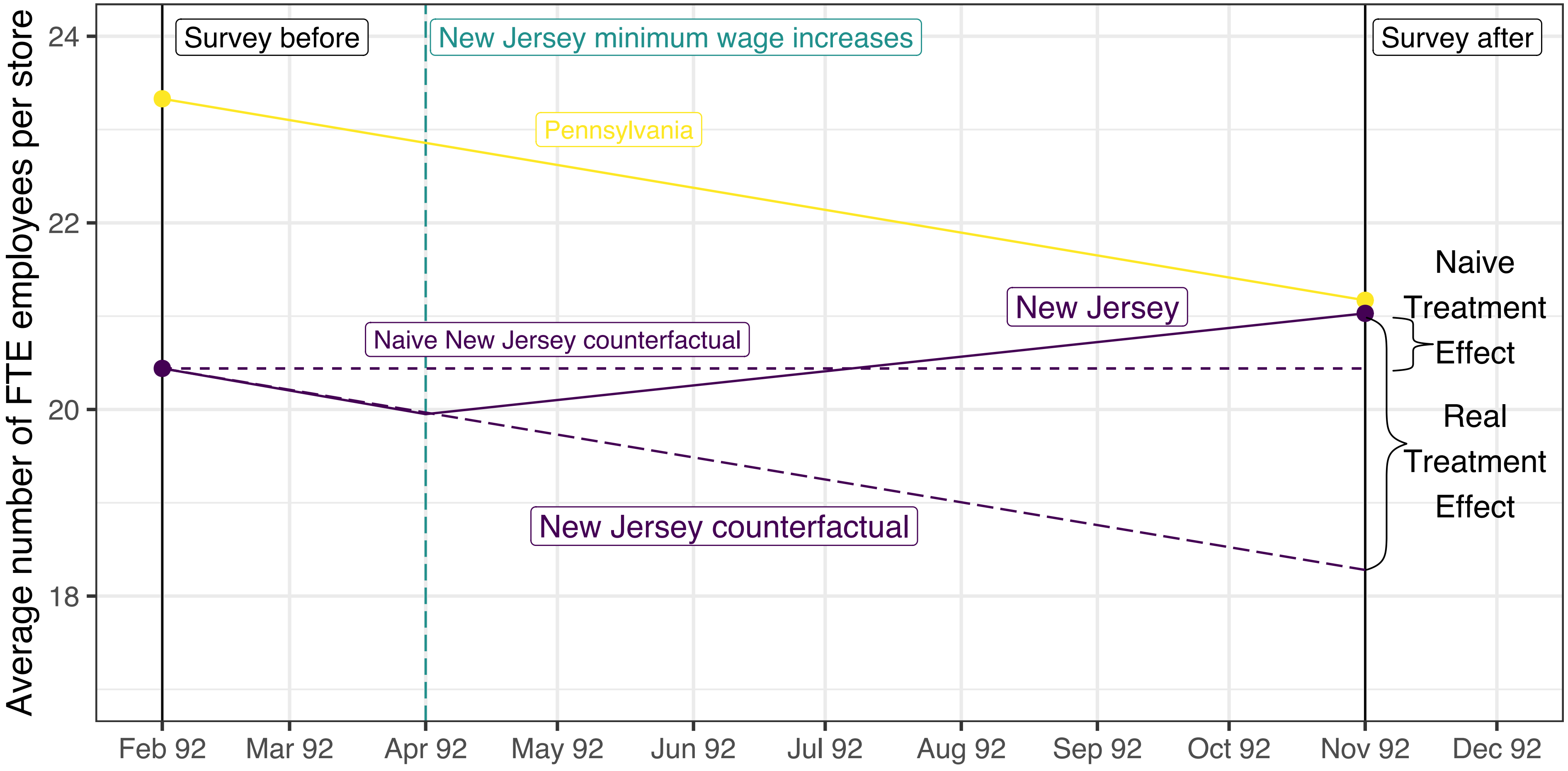

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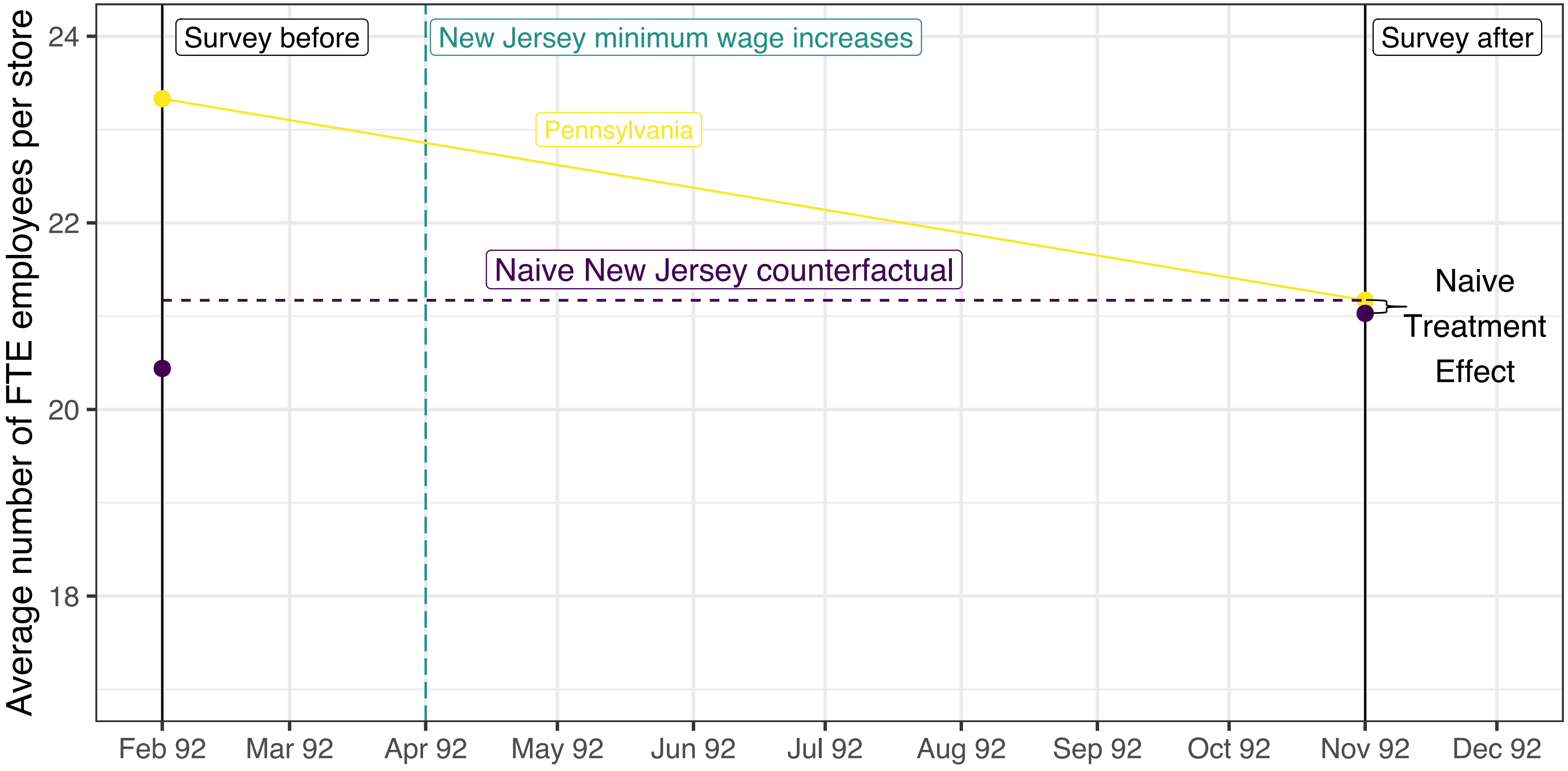

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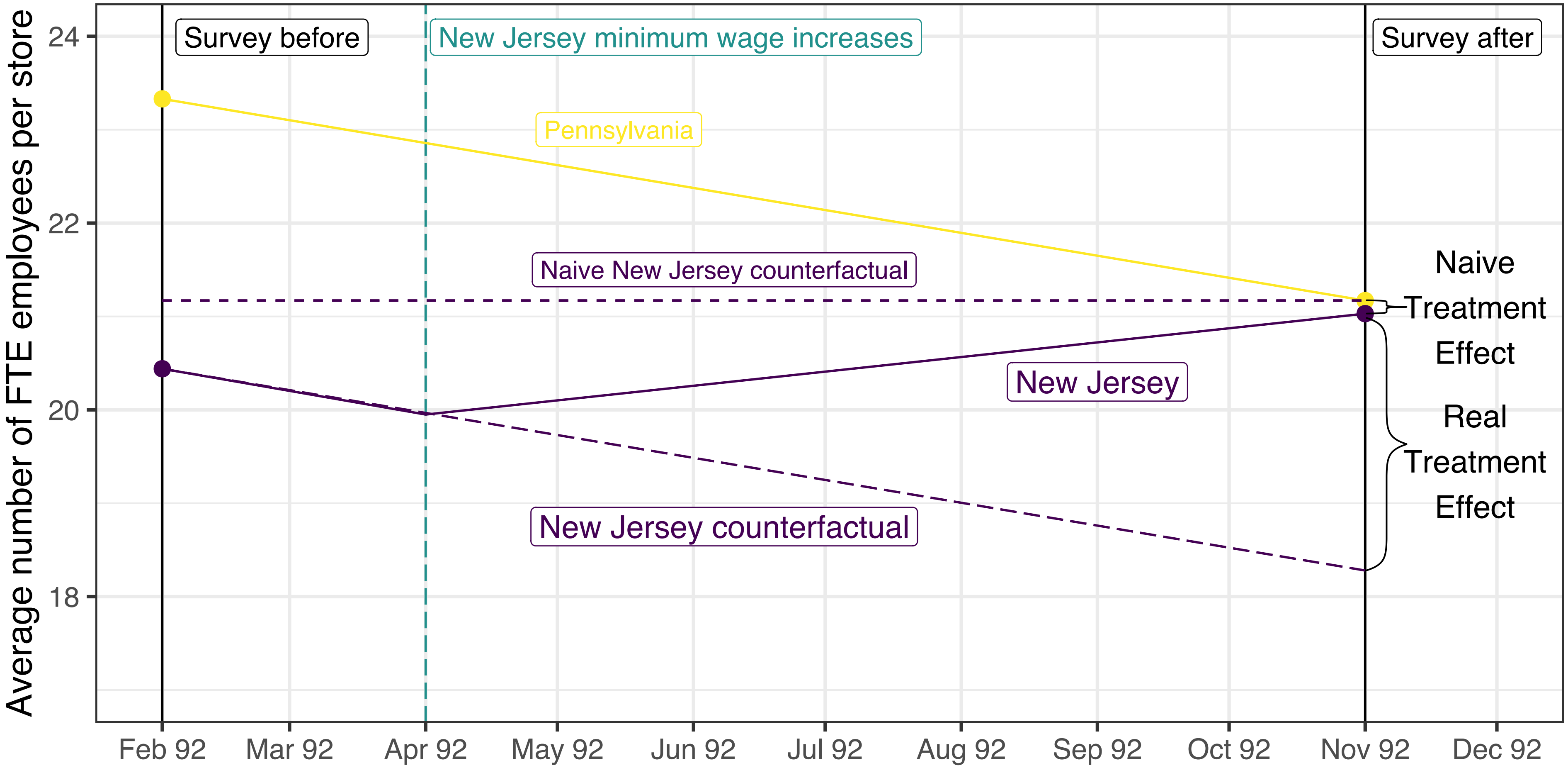

Estimation

DiD in Regression Form

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- There are more data points before and after the policy change

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2. **Post-treatment periods dummy variables**: $POST_t$ where the t subscript reminds us that this variable varies over time
3. **Interaction term between the two**: $TREAT_s \times POST_t$ 👉 the ***coefficient on this term is the DiD causal effect***!

DiD in Regression Form

Treatment dummy variable

$$TREAT_s = \begin{cases} 0 & \text{if } s = \text{Pennsylvania} \\ 1 & \text{if } s = \text{New Jersey} \end{cases}$$

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- Let's put all these ingredients together:

$$EMP_{st} = \alpha + \beta TREAT_s + \gamma POST_t + \delta(TREAT_s \times POST_t) + \varepsilon_{st}$$

- δ: causal effect of the minimum wage increase on employment

Understanding the Regression

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$$[\mathbb{E}(EMP_{st} \mid TREAT_s = 1, POST_t = 1) - \mathbb{E}(EMP_{st} \mid TREAT_s = 1, POST_t = 0)] -$$
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Understanding the Regression

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In table form:

	Pre mean	**Post mean**	**Δ(post - pre)**
Pennsylvania (PA)	α	$\alpha + \gamma$	γ
New Jersey (NJ)	$\alpha + \beta$	$\alpha + \beta + \gamma + \delta$	$\gamma + \delta$
Δ(NJ - PA)	β	$\beta + \delta$	δ

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This table generalizes to other settings by substituting *Pennsylvania* with *Control* and *New Jersey* with *Treatment*

Task 2 (10 minutes)

1. Create a full-time equivalent (FTE) employees variable called `empfte` equal to `empft` + 0.5*`emppt` + `nmgrs`. `empft` and `emppt` correspond respectively to the number of full-time and part-time employees. `nmgrs` corresponds to the number of managers. This is how Card and Krueger compute their full-time equivalent (FTE) employment variable (p.775 of the paper).

2. Create a dummy variable, `treat`, equal to `FALSE` if `state` is Pennsylvania and `TRUE` if New Jersey.

3. Create a dummy variable, `post`, equal to `FALSE` if `observation` is February 1992 and `TRUE` otherwise.

4. Estimate the following regression model. Do you obtain the same results as in slide 9 (≈ 2.76)?

$$empfte_{st} = \alpha + \beta treat_s + \gamma post_t + \delta(treat_s \times post_t) + \varepsilon_{st}$$

Identifying Assumptions

DiD Crucial Assumption: Parallel Trends

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- This assumption states that Pennsylvania's fast-food employment trend between February and November 1992 provides a reliable counterfactual employment trend New Jersey's fast-food industry *would have experienced* had New Jersey not increased its minimum wage.

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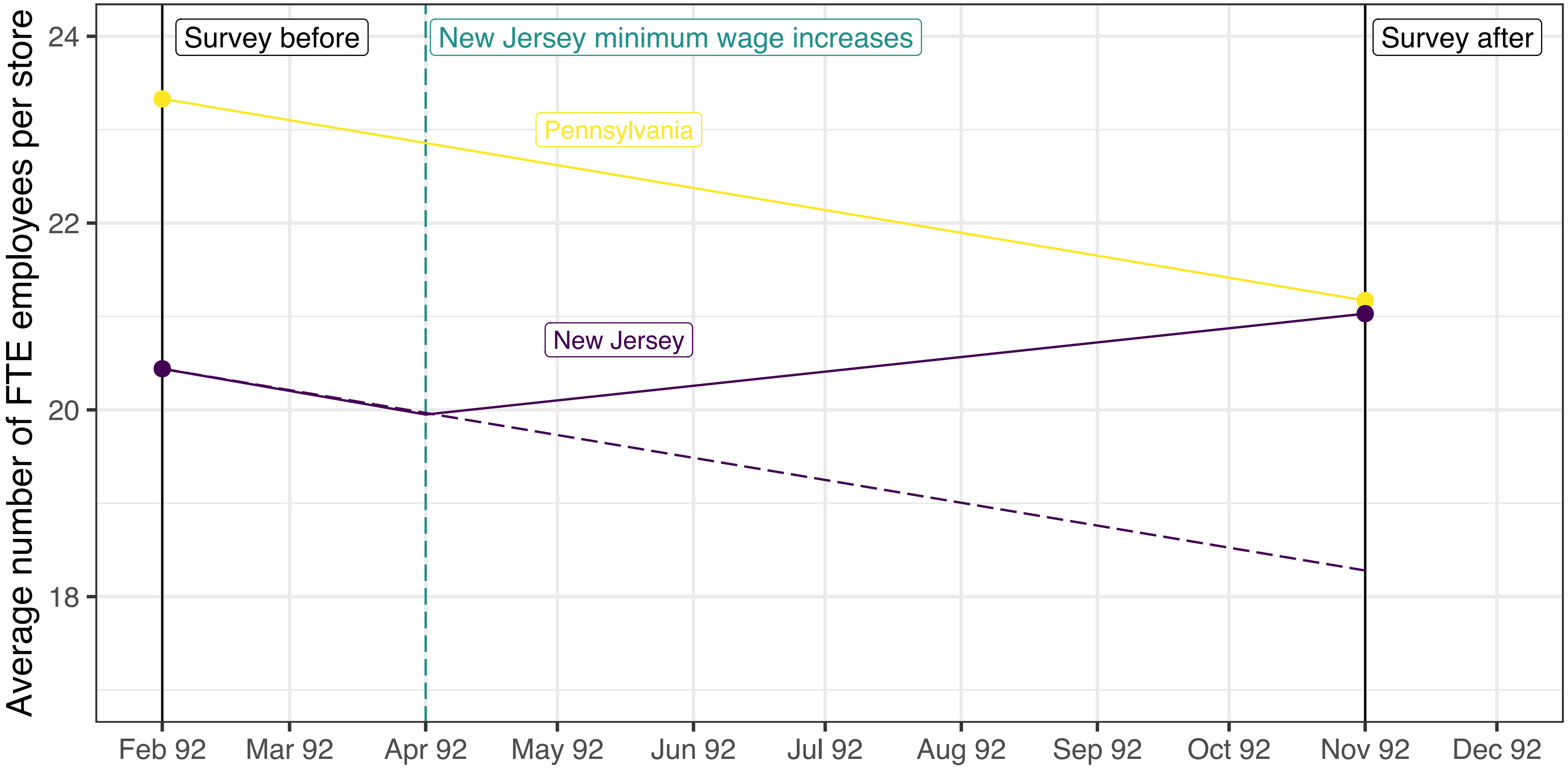
- This assumption states that Pennsylvania's fast-food employment trend between February and November 1992 provides a reliable counterfactual employment trend New Jersey's fast-food industry *would have experienced* had New Jersey not increased its minimum wage.

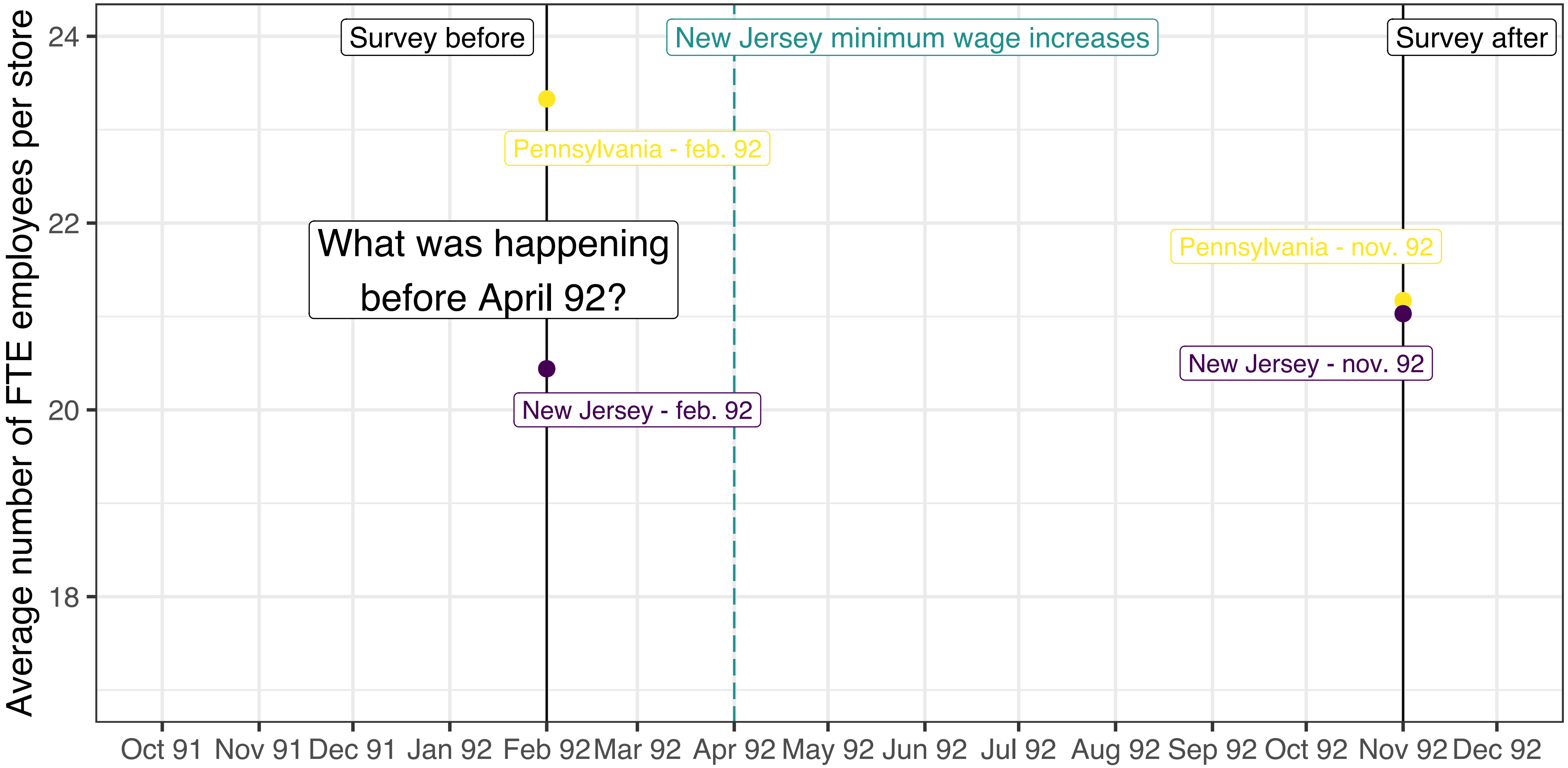
- Impossible to completely validate or invalidate this assumption.

- *Intuitive check:* compare trends before policy change (and after policy change if no expected medium-term effects)

Parallel Trends: Graphically



Checking the parallel trends assumption



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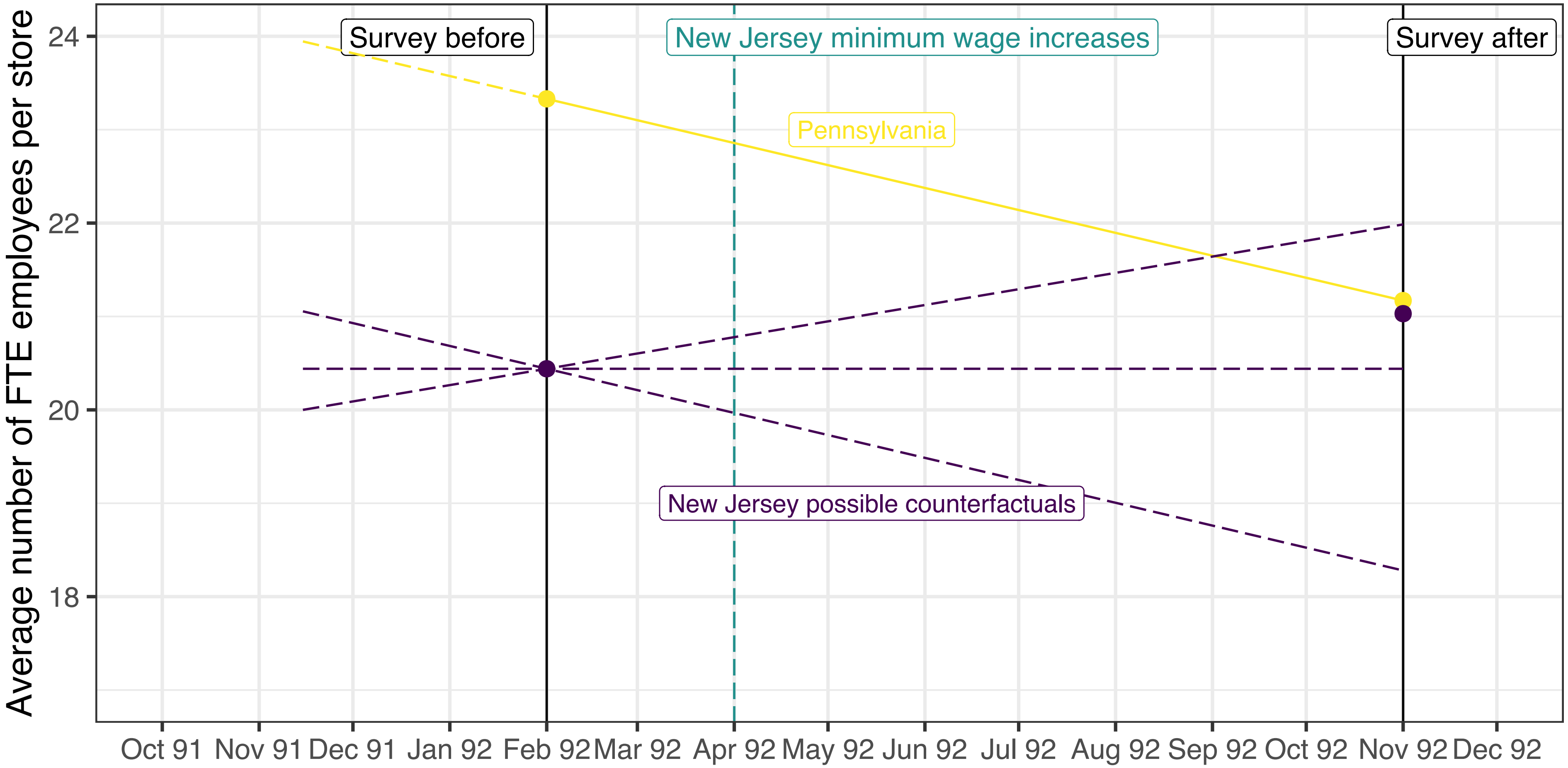
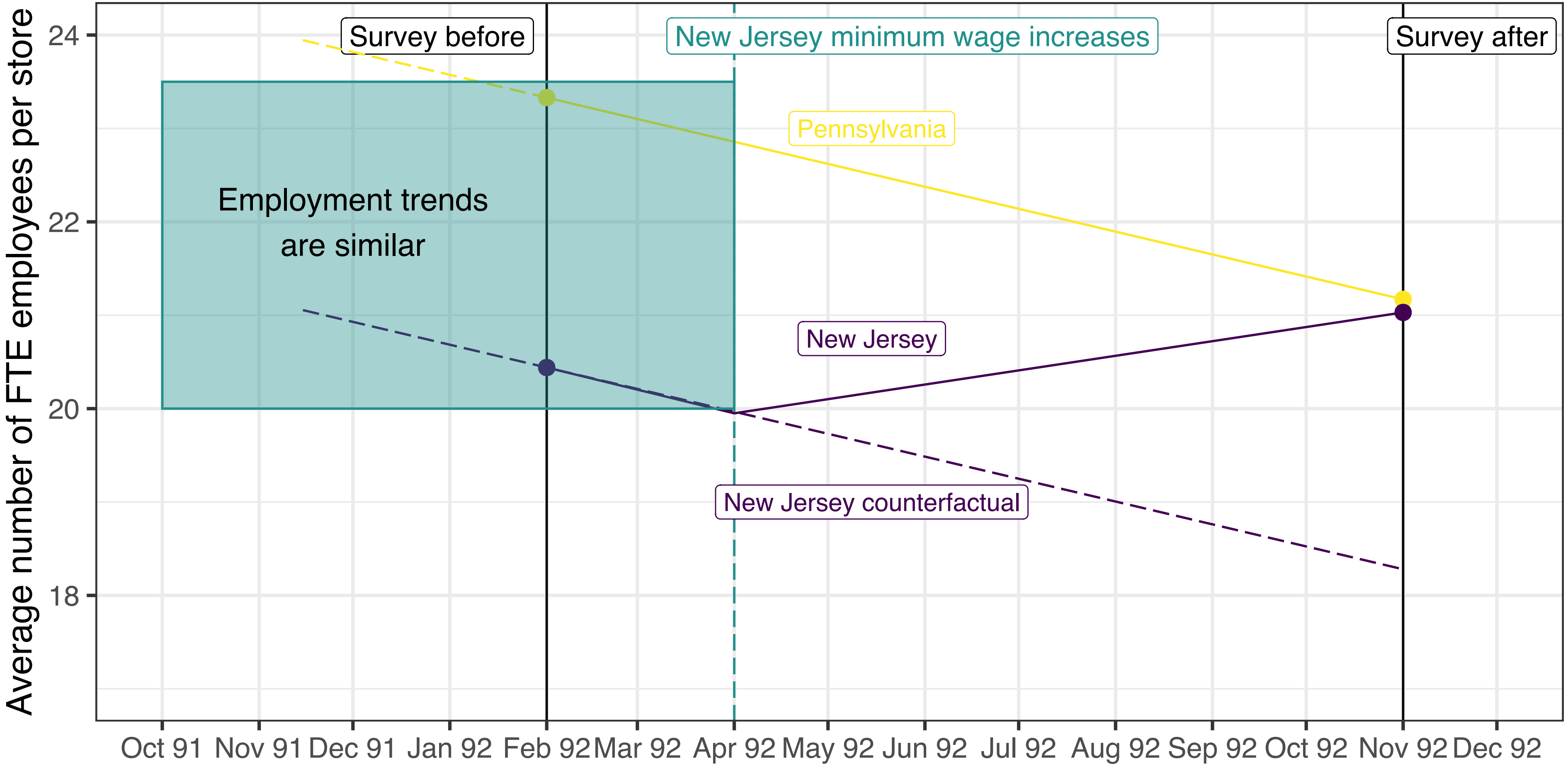


Parallel trends assumption → Verified ✅



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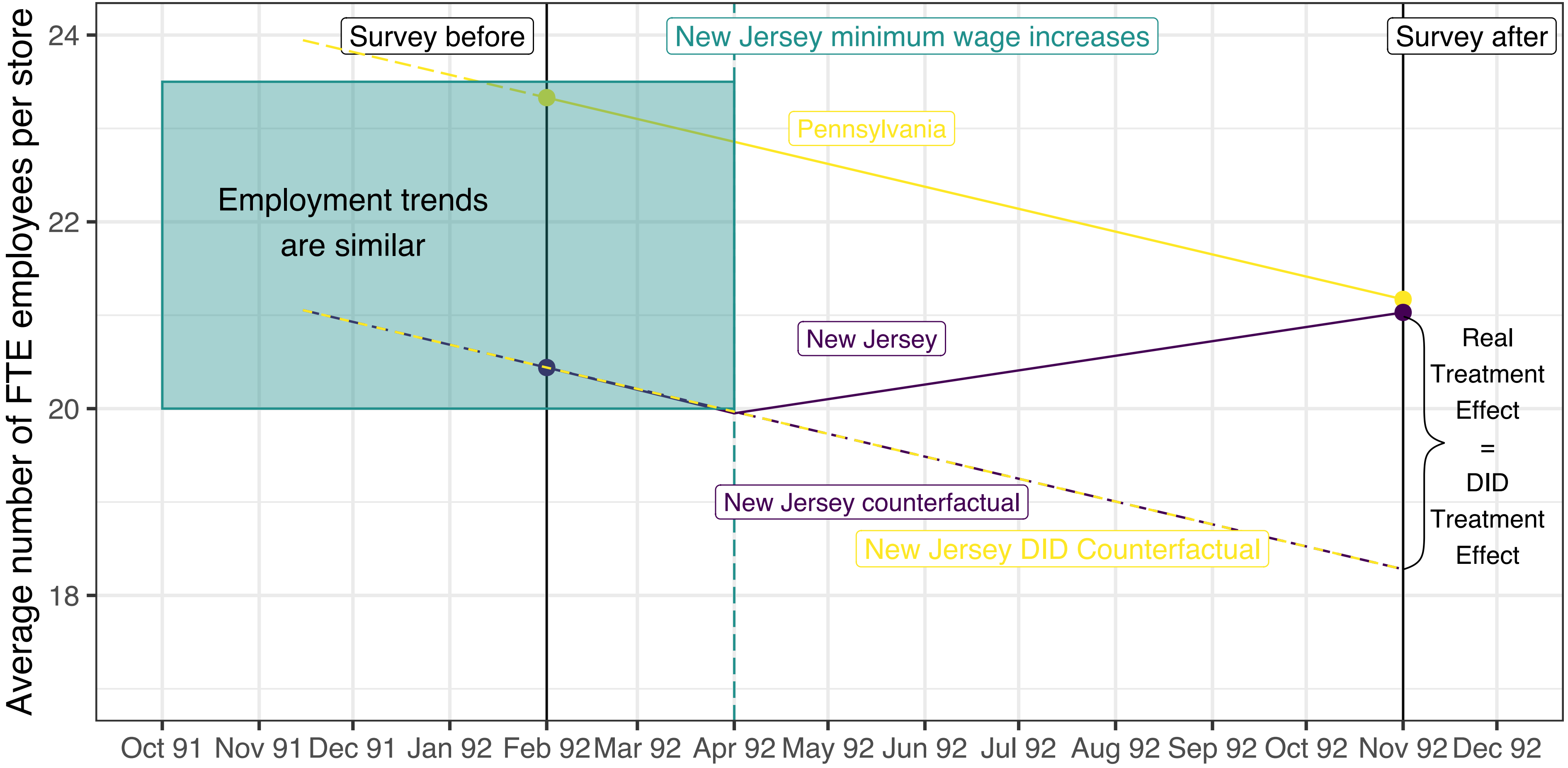

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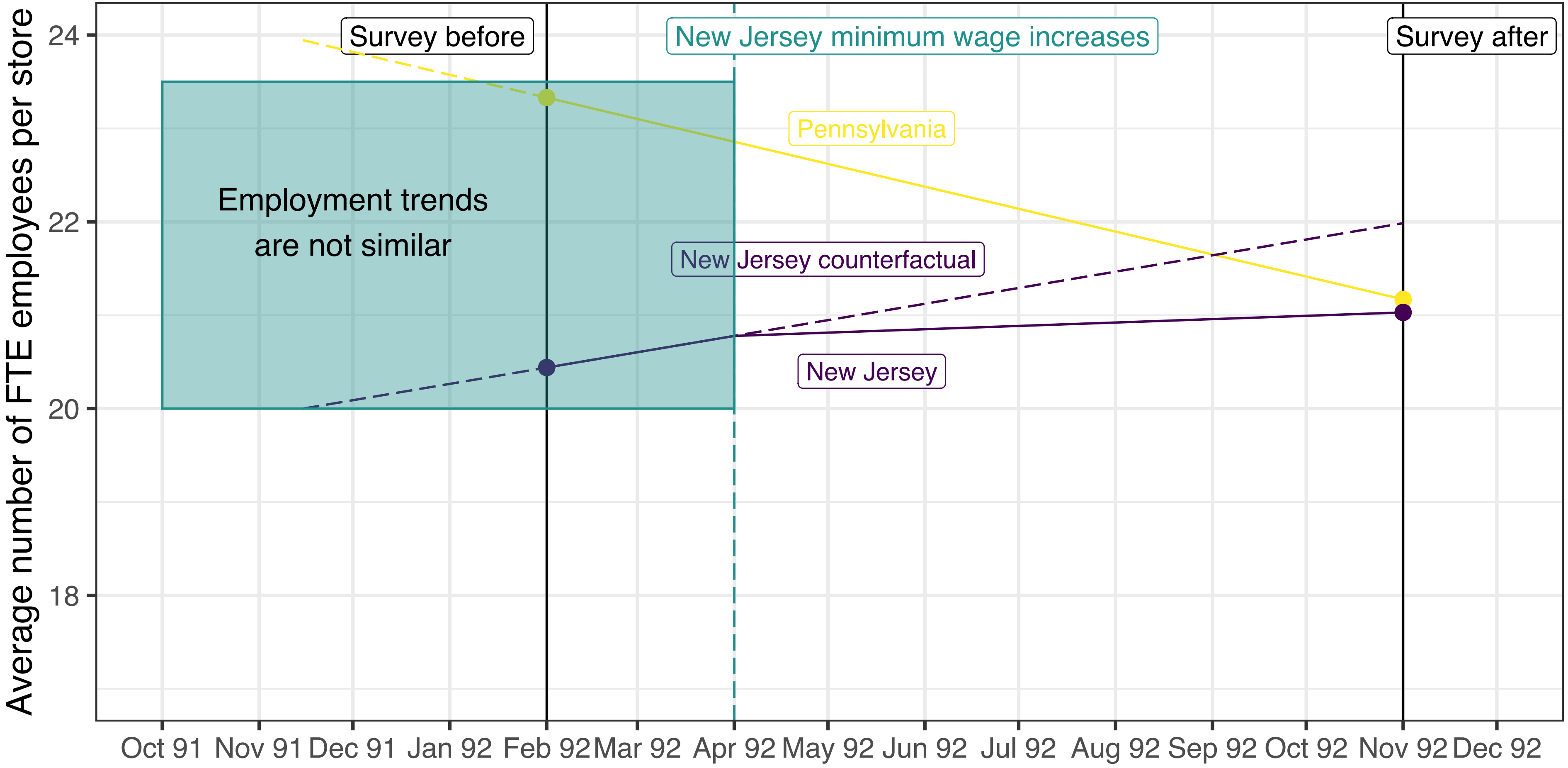

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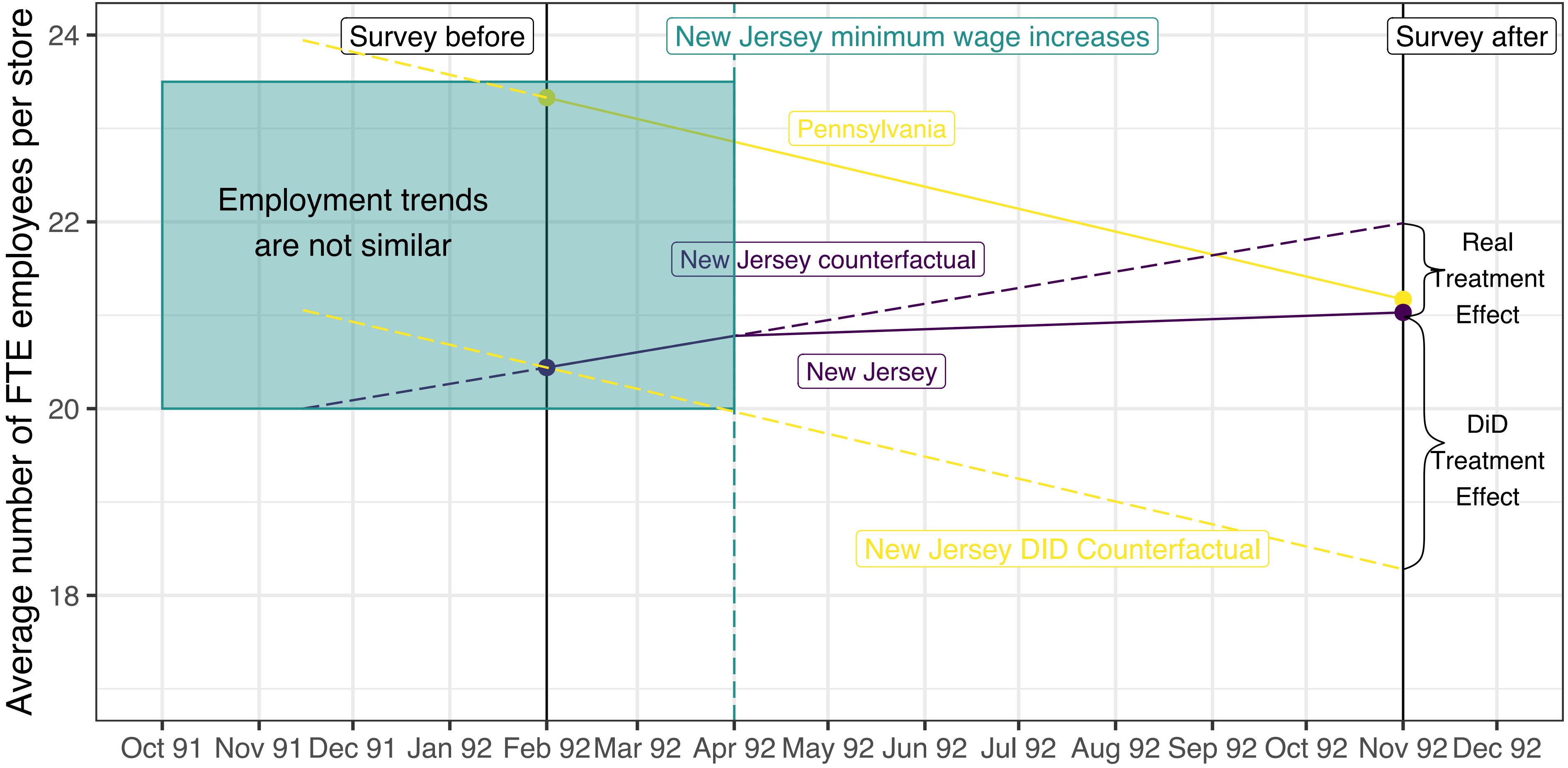

Regression Discontinuity Design

Regression Discontinuity Design (RDD)

- Very common research design in applied research because it provides credible causal estimates.

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- **RDD** exploits this precise information about allocation to treatment!

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We will focus our analysis on the following discontinuity:

- In the US, the legal drinking age is 21 years old (Carpenter and Dobkin, 2009).

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  - Because there may be unobserved selection into alcohol consumption that may also be a determinant of mortality.
- In the US, alcohol consumption is prohibited before the age of 21.
- Debate on whether the minimum legal drinking age (MLDA) should be lowered to 18, as was the case in the Vietnam-era.

Key Terms and Intuition

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- 👉 *Regression discontinuity design* exploits this allocation to treatment!

Carpenter and Dobkin's data

- Let's take a closer at the data used in the paper

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##   agecell   all internal external alcohol homicide suicide
##     <dbl> <dbl>    <dbl>    <dbl>   <dbl>    <dbl>   <dbl>
## 1    19.1  92.8     16.6     76.2   0.639     16.3    11.2
## 2    19.2  95.1     18.3     76.8   0.677     16.9    12.2
## 3    19.2  92.1     18.9     73.2   0.866     15.2    11.7
## 4    19.3  88.4     16.1     72.3   0.867     16.7    11.3
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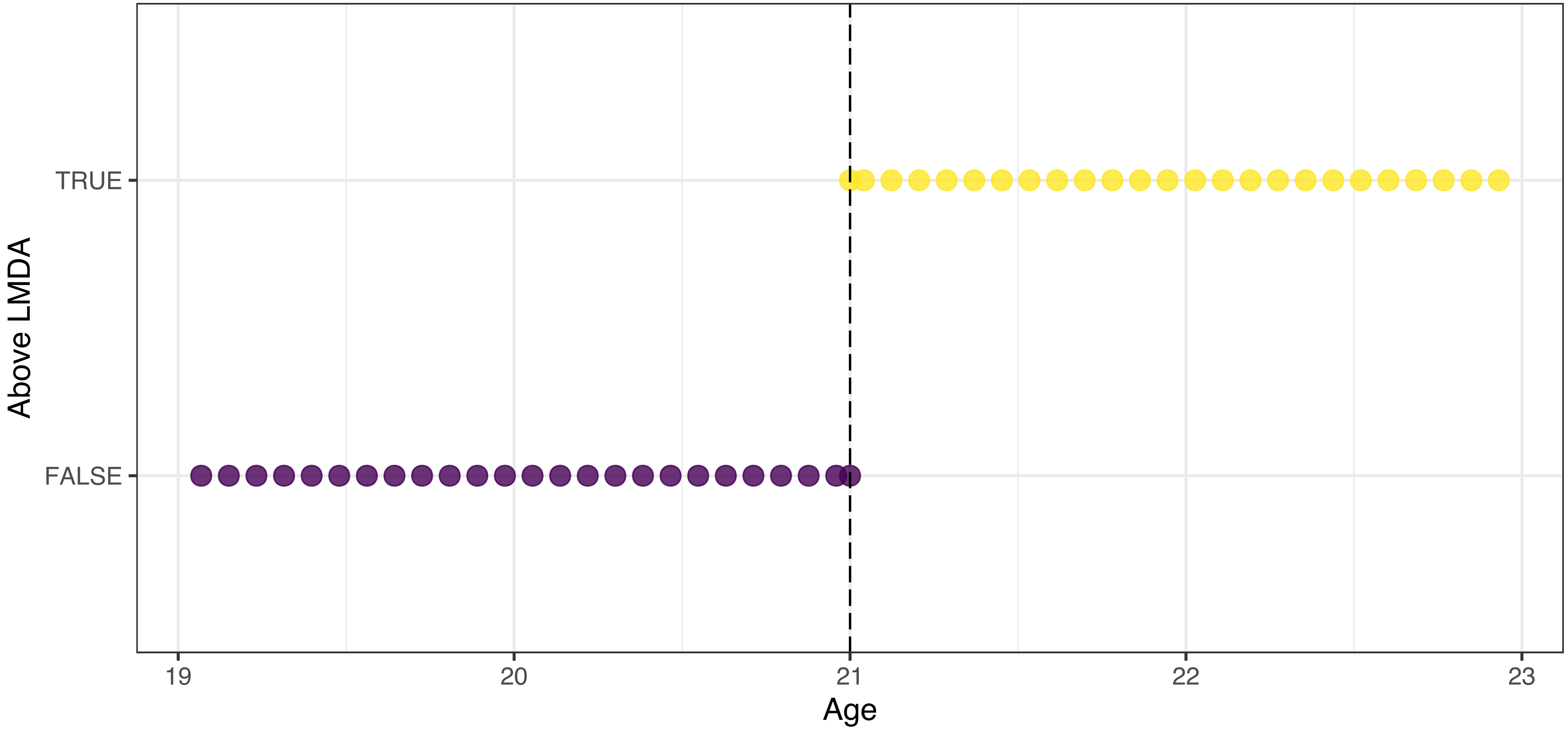
- This dataset contains aggregate death rates (and their causes) for different age groups (`agecell`) between 19 and 23 years old.

Sharp Discontinuity at Cutoff



At the threshold, the probability of being treated jumps from 0 to 1.

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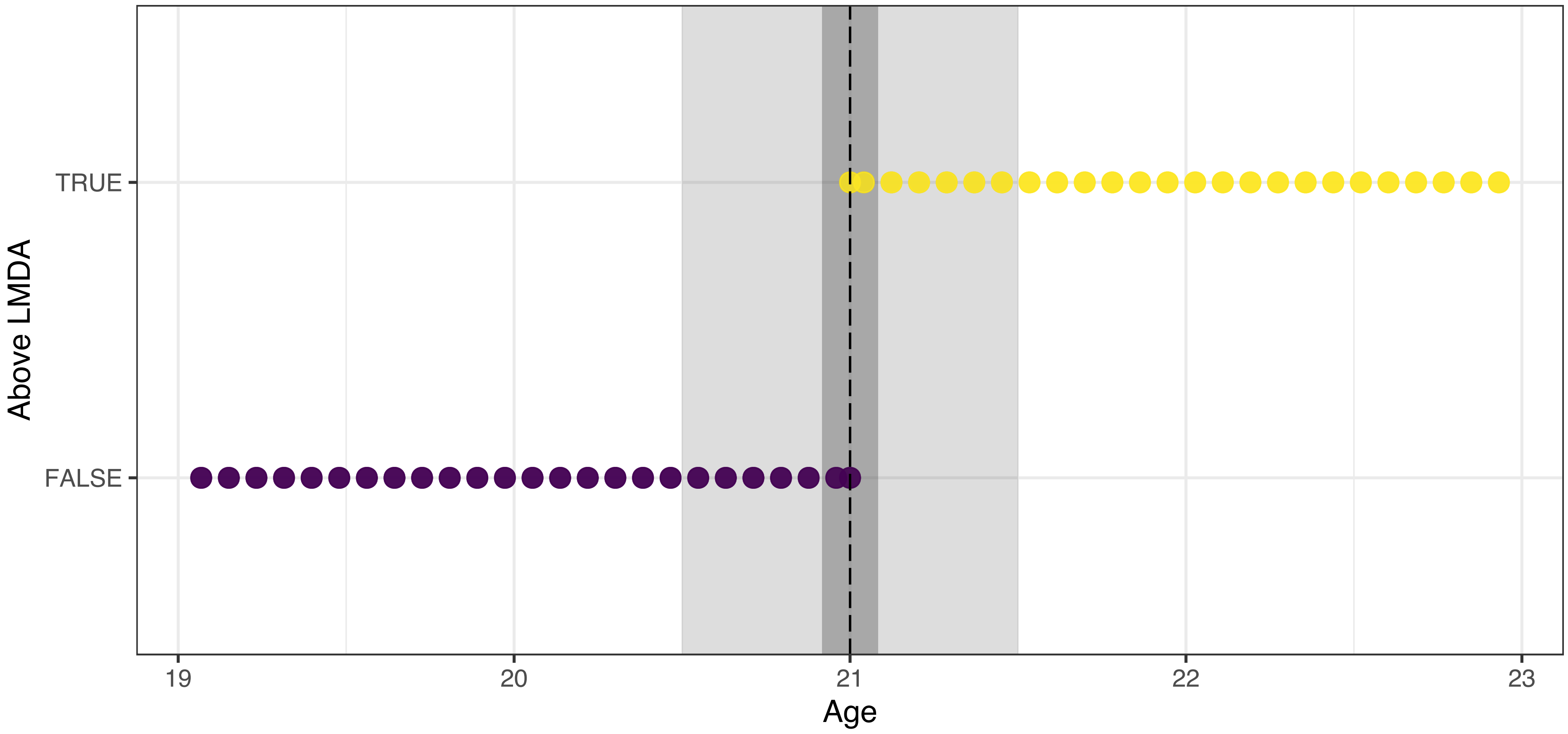

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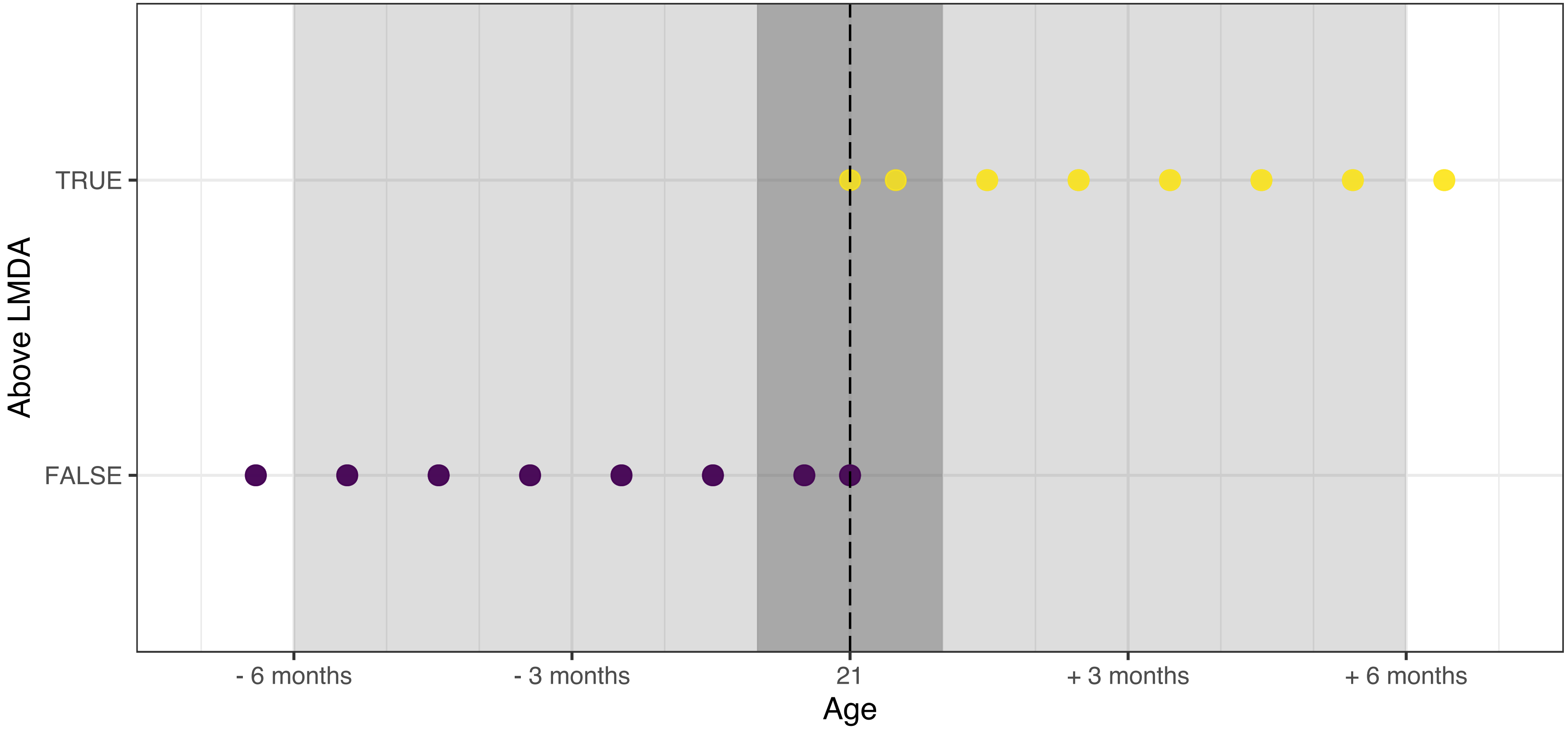

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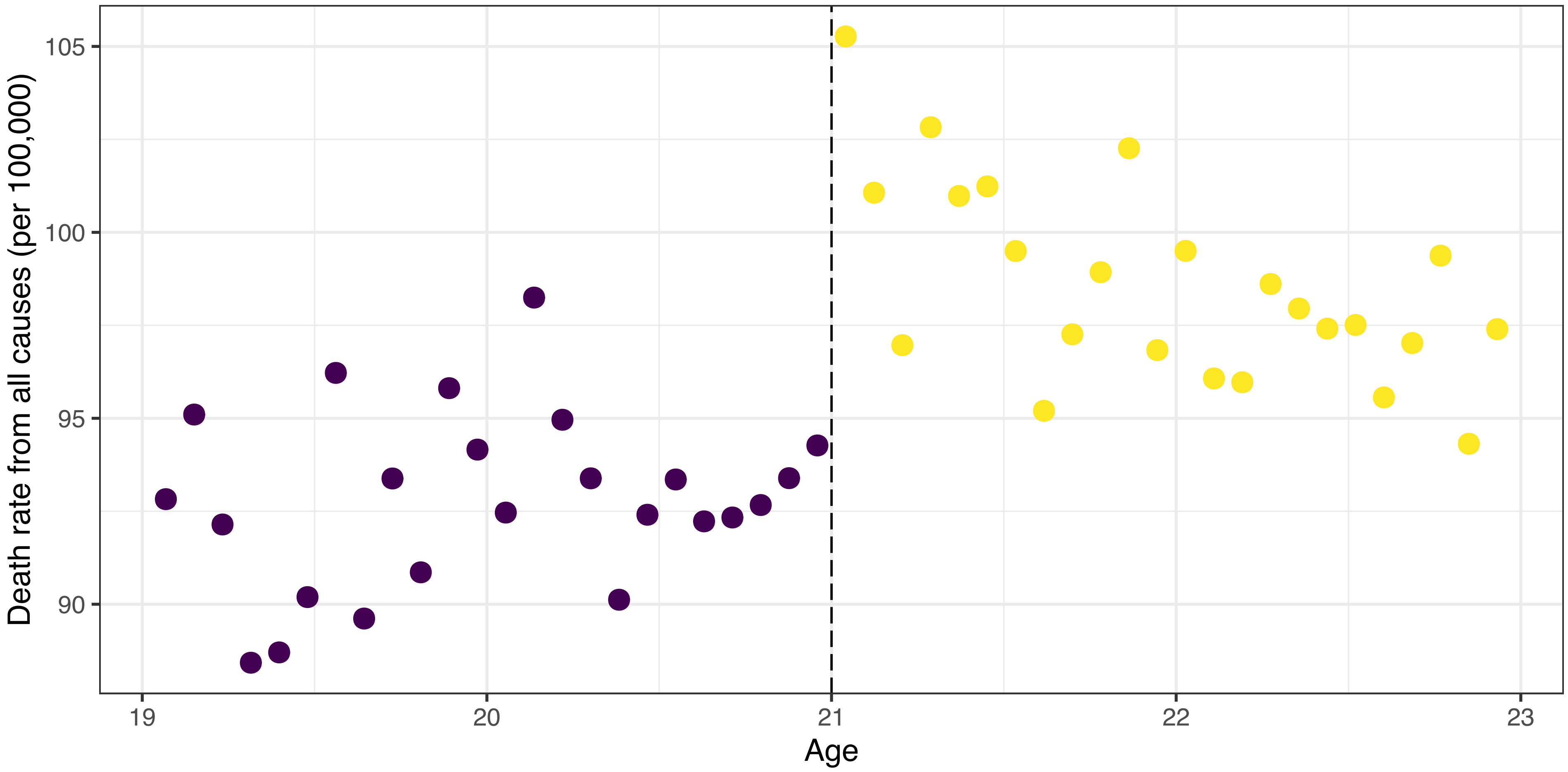
2. Treatment status is a **discontinuous** function of $a \rightarrow$ there is some cutoff level

Graphical Results: All Death Rates



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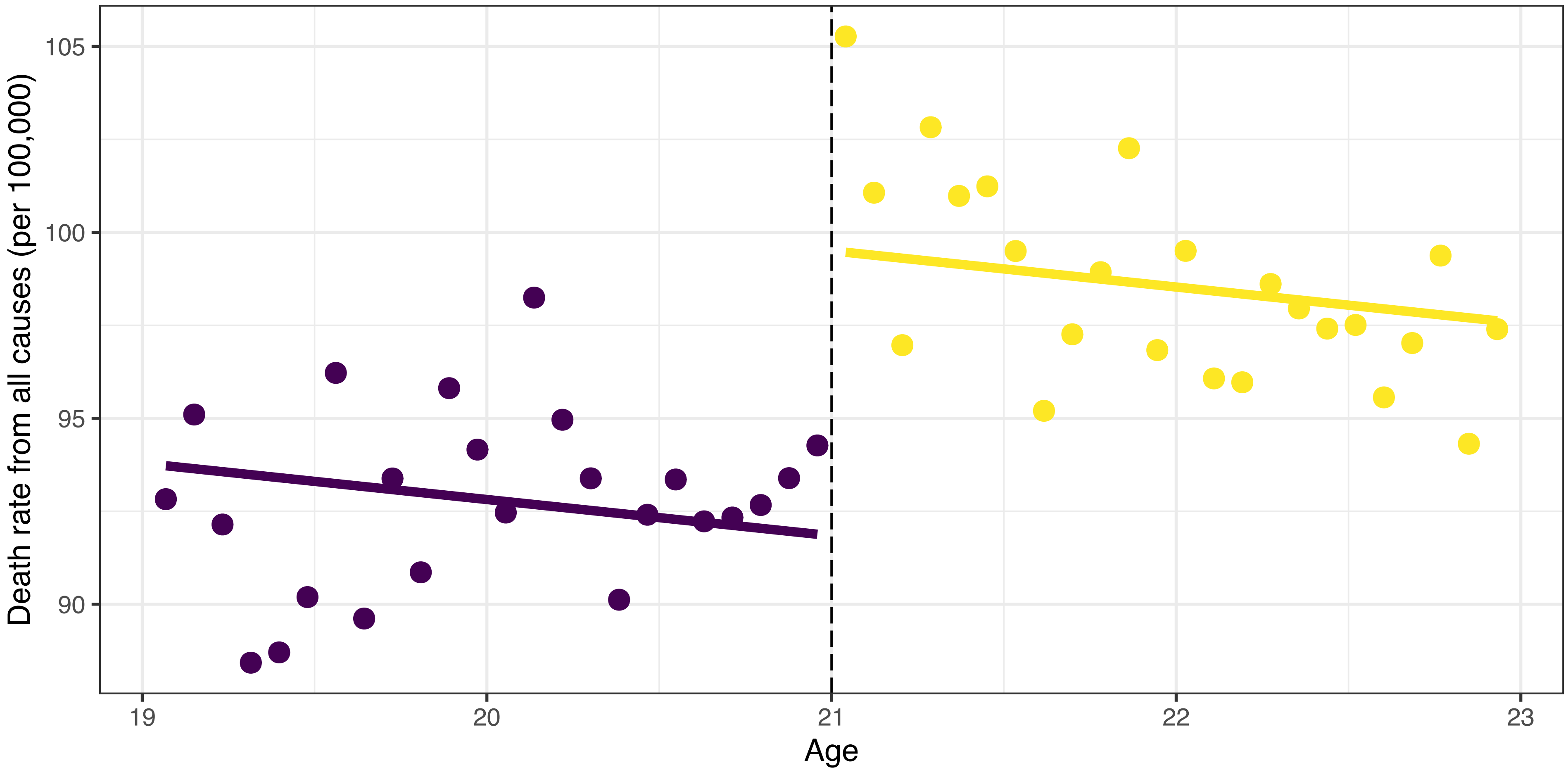

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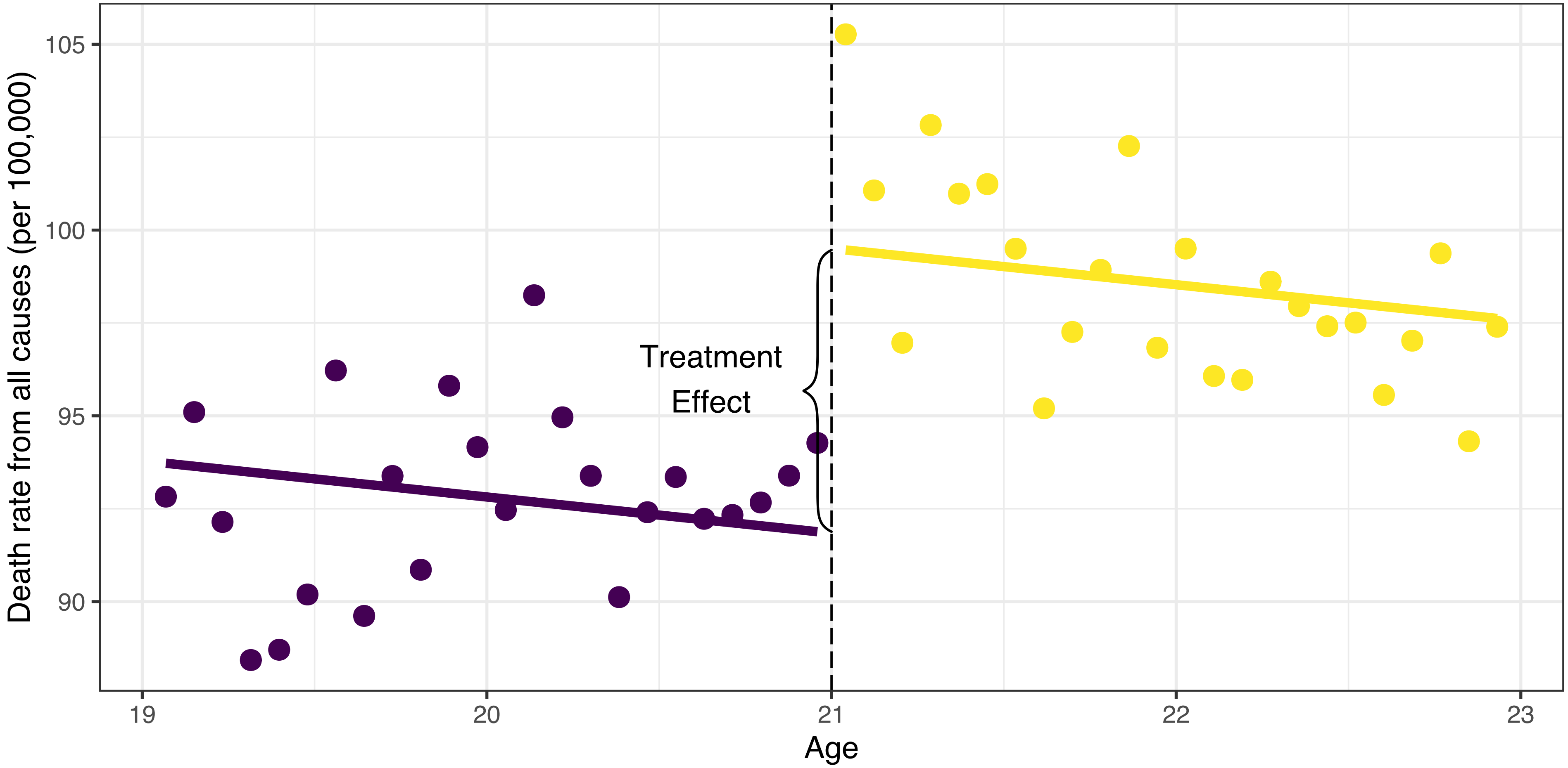

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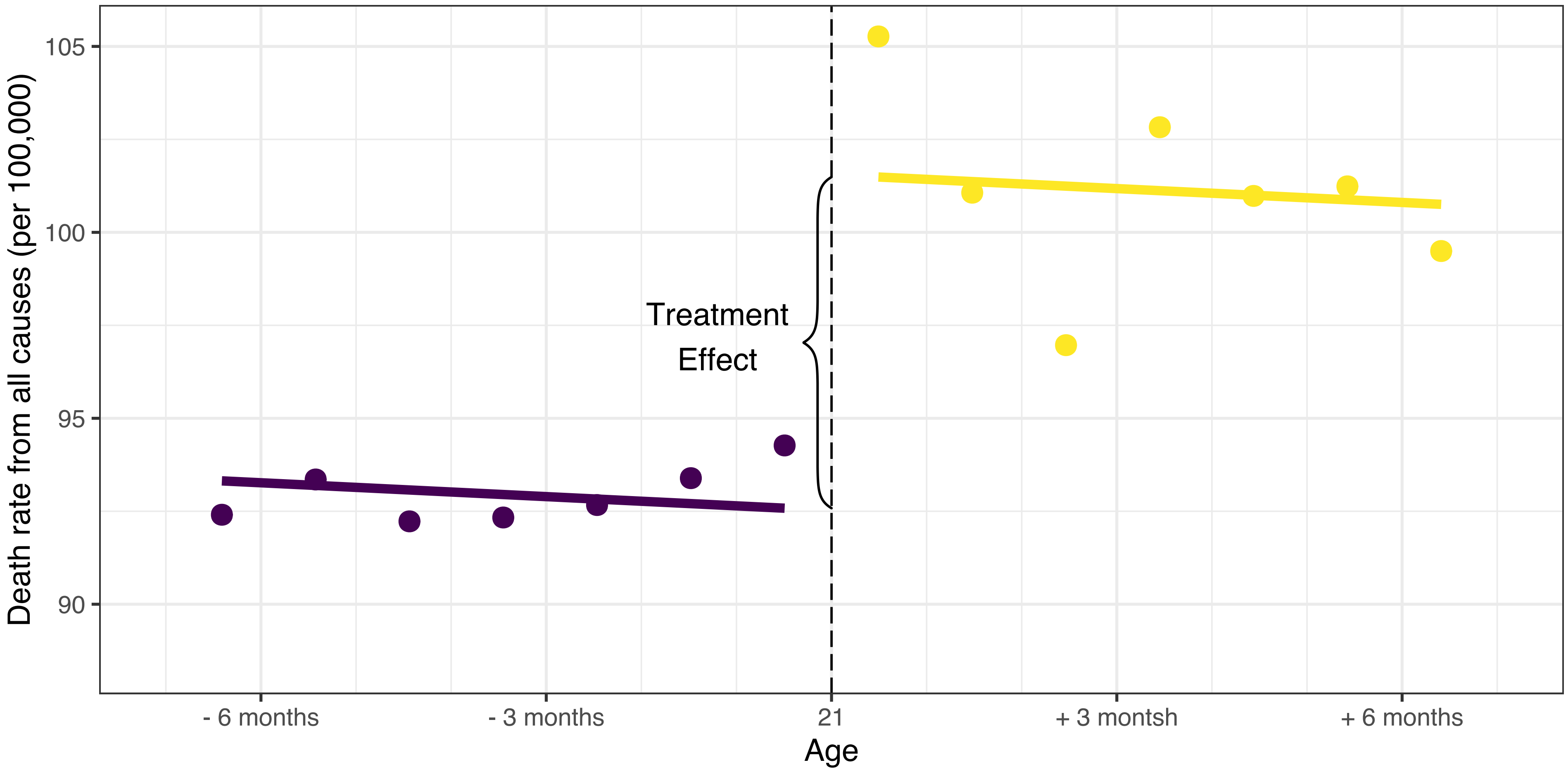

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- Using the 21 year old alcohol restriction age in the RD context will only tell you the effect of this restriction on death rates but not the general effect of alcohol consumption.
- One may easily argue that all results from quantitative empirical analyses have a local nature.

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- The RDD estimator exploits a discontinuity at $a = 21$ in the conditional expectation function:

$$\underbrace{\lim_{c \to 21^+} \mathbb{E}[DEATHRATE_a | a = c]}_{\alpha+\delta} - \underbrace{\lim_{c \to 21^-} \mathbb{E}[DEATHRATE_a | a = c]}_{\alpha} = \delta$$

Task 2 (5 minutes)

1. Estimate the following model on `all` death causes.

$$DEATHRATE_a = \alpha + \delta D_a + \beta a + \varepsilon_i,$$

   Does the RDD coefficient correspond to the graphical illustration?

2. How do you interpret each coefficient?

3. What is the causal effect of legal access to alcohol on death rates?

Estimation #1: Simple Linear Model

$$DEATHRATE_a = \alpha + \delta D_a + \beta a + \varepsilon_a,$$

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  mutate(over21 = (agecell >= 21),
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rdd <- lm(all ~ agecell_21 + over21, mlda)

library(broom)
tidy(rdd)
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## # A tibble: 3 x 5
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Interpretation:

On average, the MLDA increases death rates from all causes by 7.66 percentage points.

This is a big effect considering the average death rate for individuals between 19 and 22 is:

```
mean(mlda$all, na.rm = TRUE)
```

```
## [1] 95.67272
```

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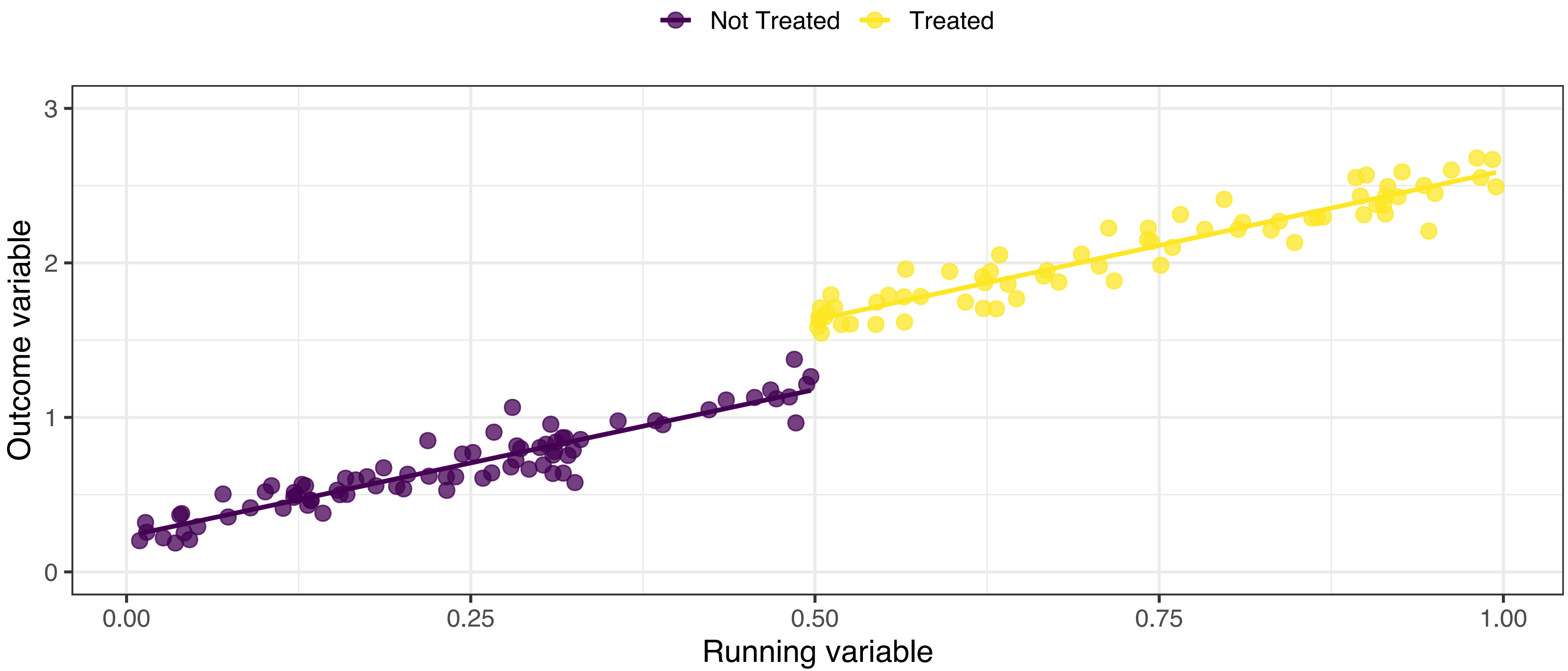
  $\rightarrow$ an insufficiently flexible specification runs the risk of mistaking nonlinearity for treatment effect;

  $\rightarrow$ an overly flexible specification reduces precision and runs the risk of overfitting.

Simulations - Linear Relationship and Clear Discontinuity



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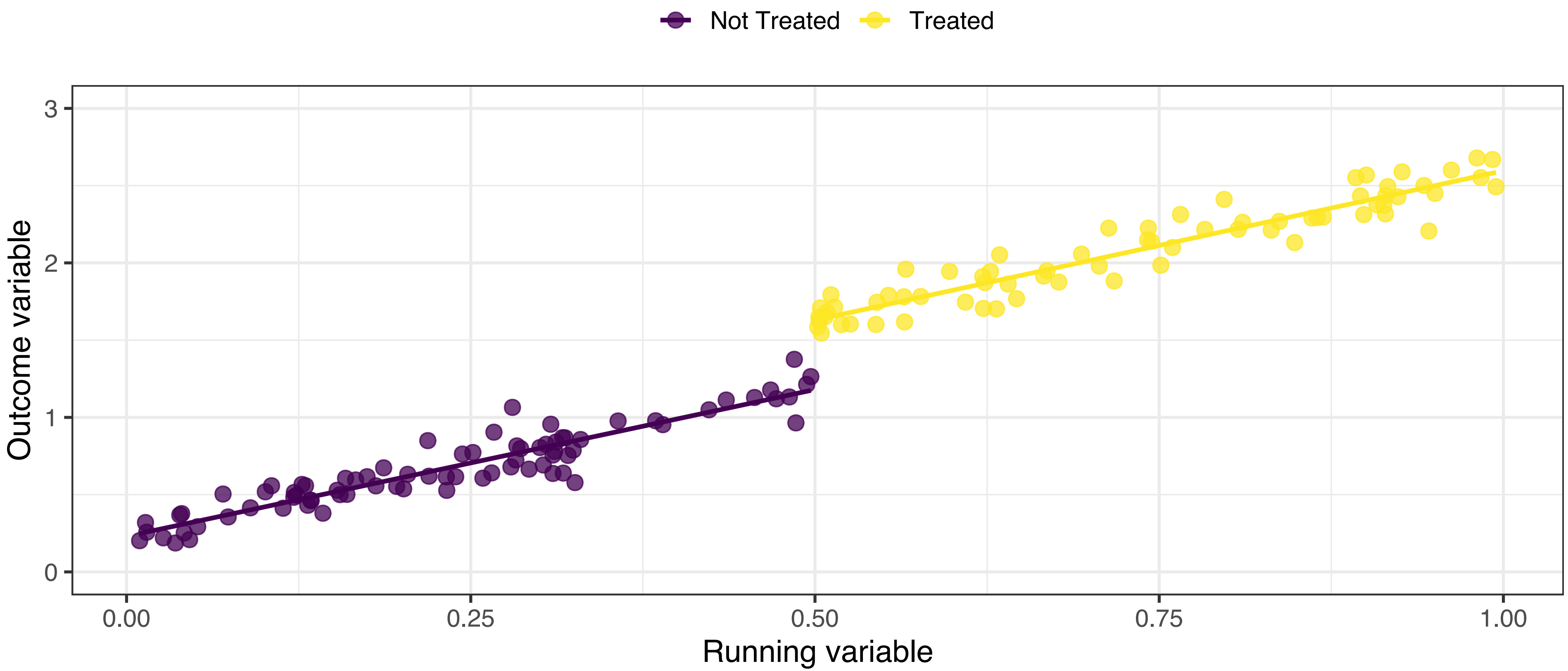

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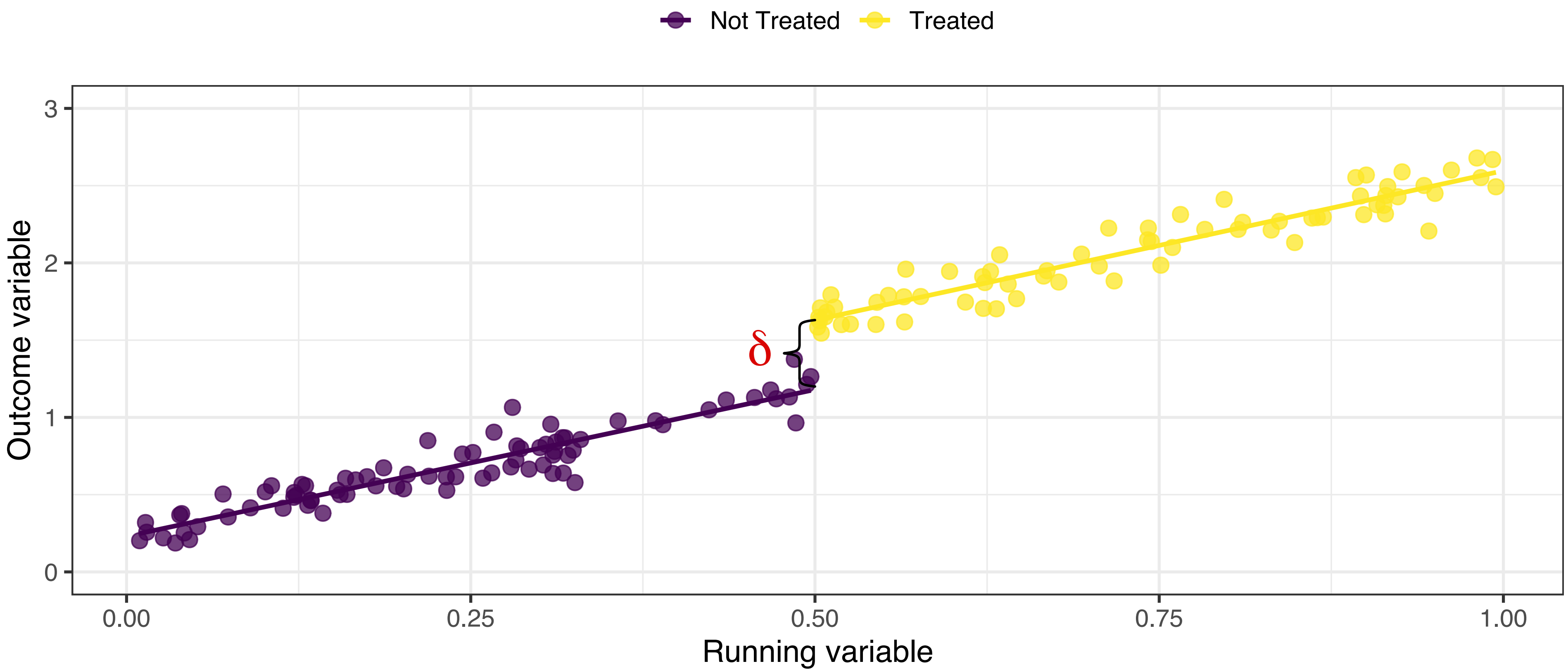

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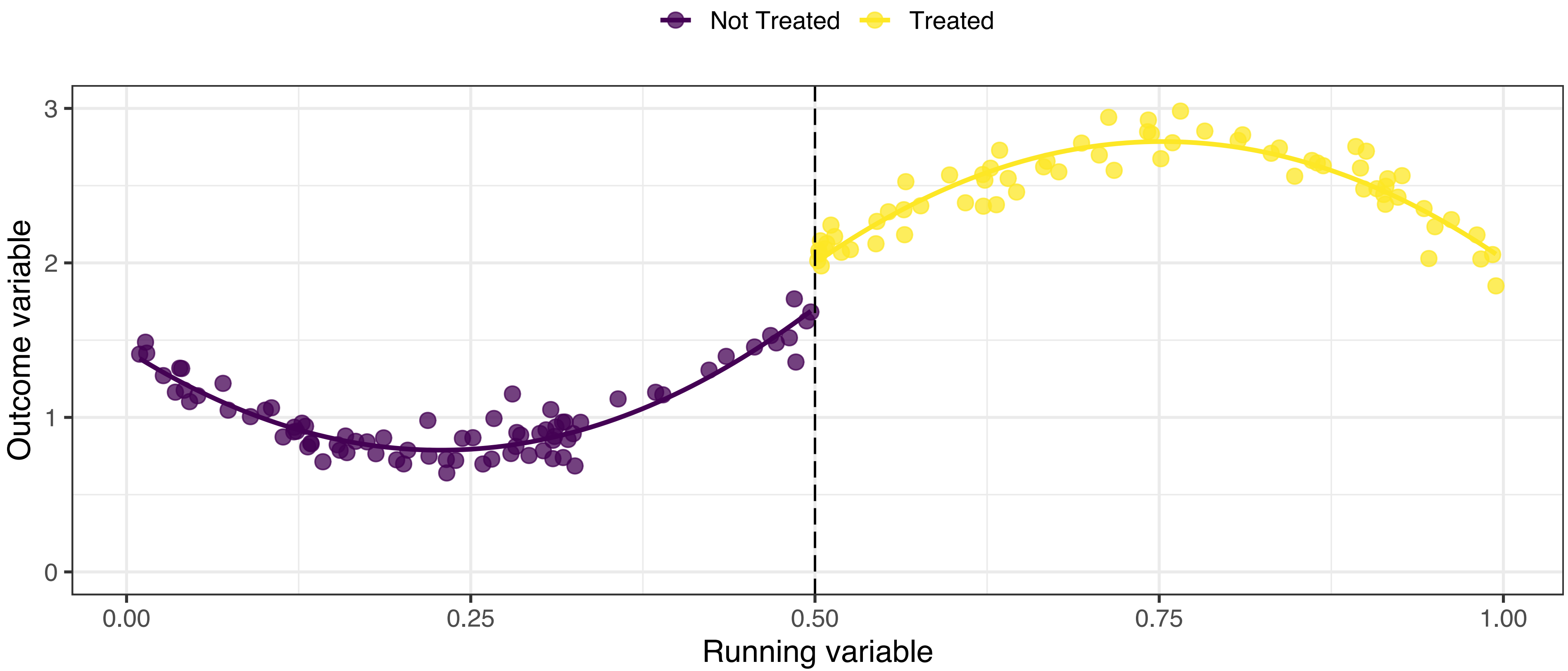

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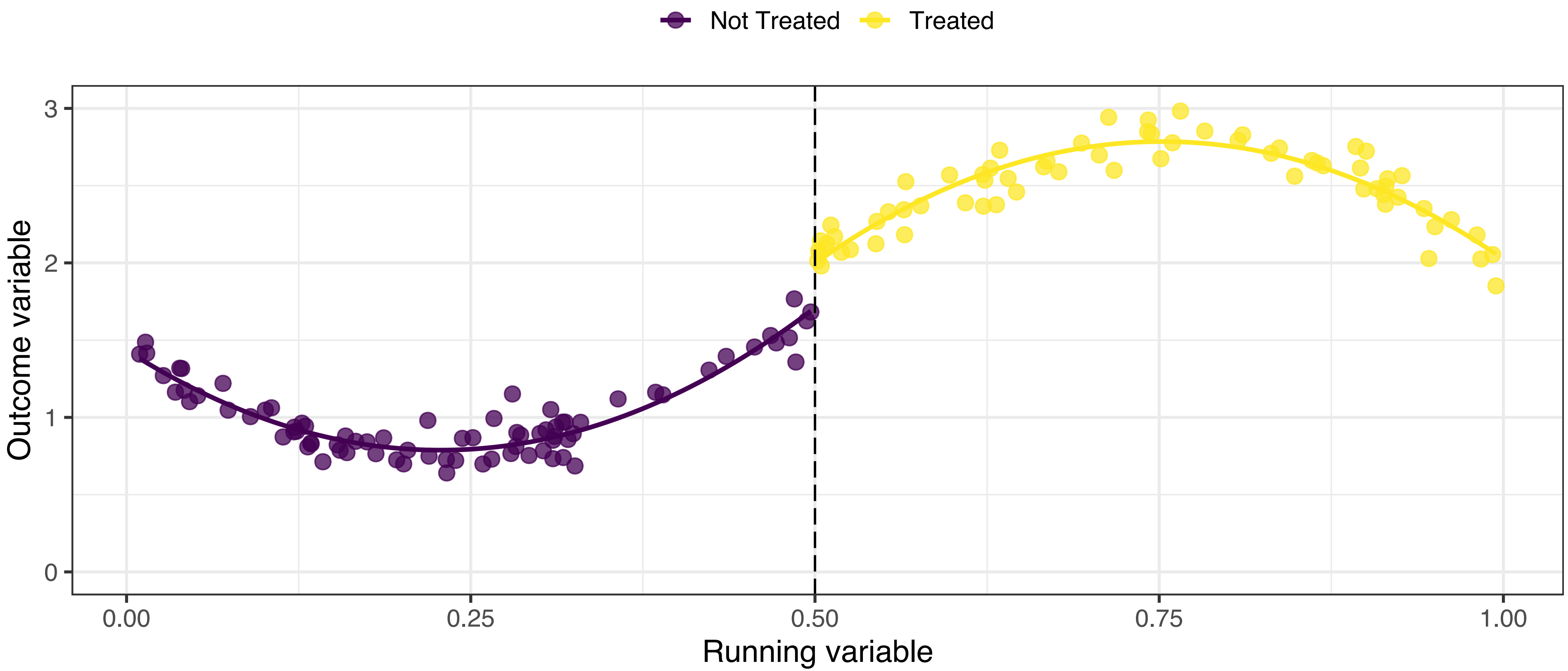

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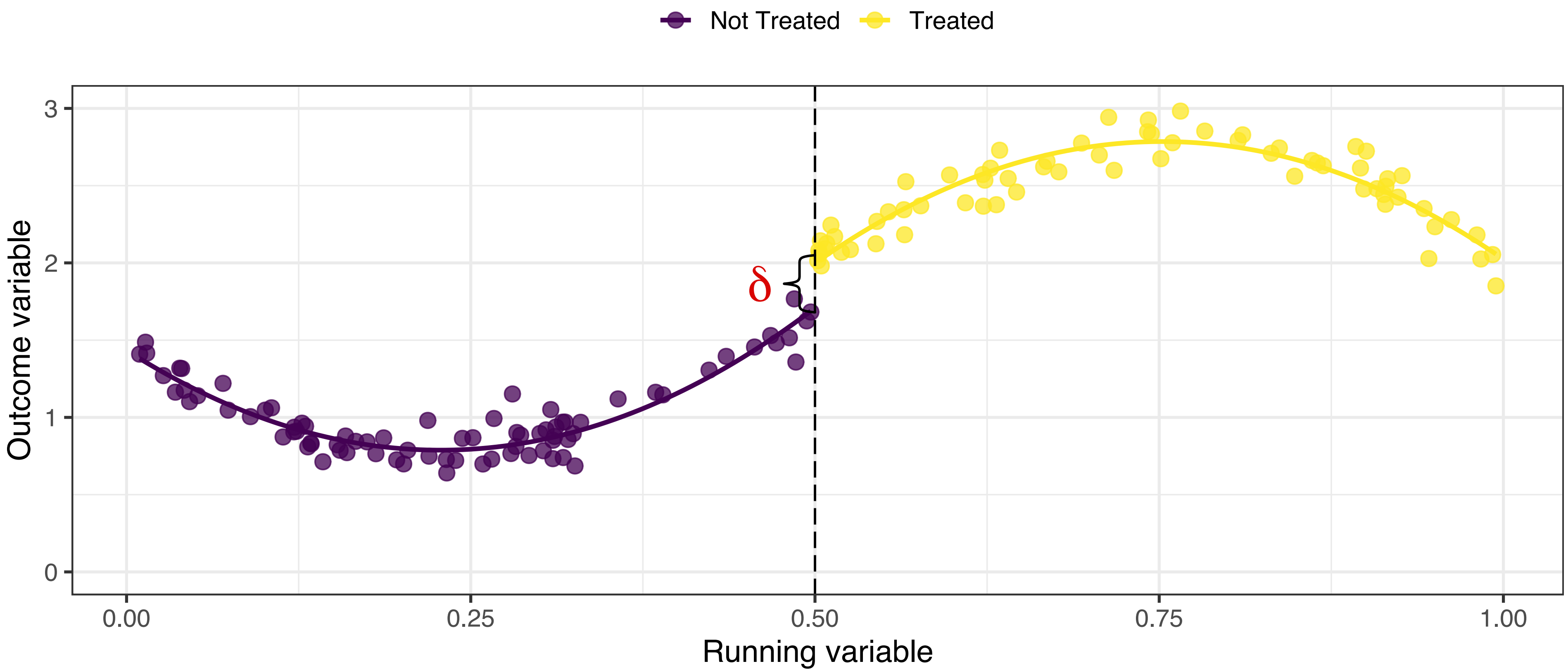

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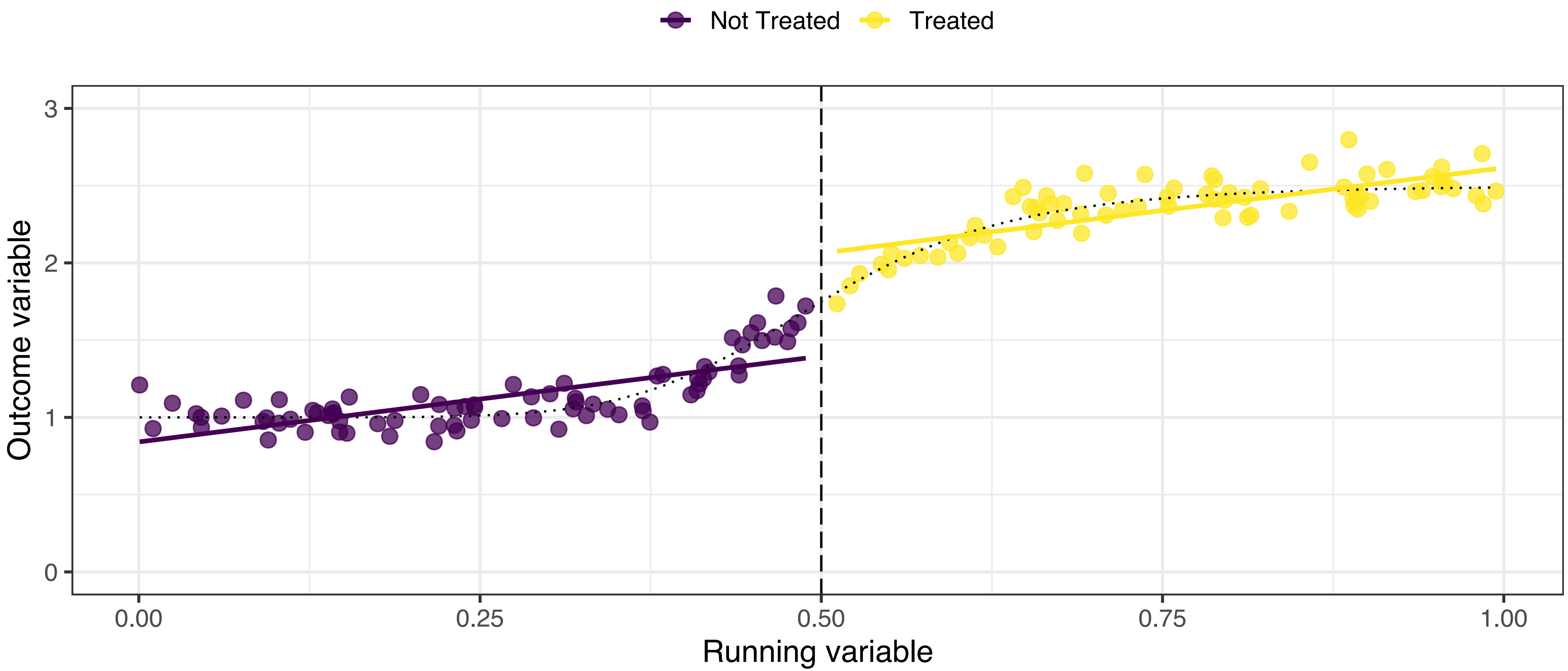
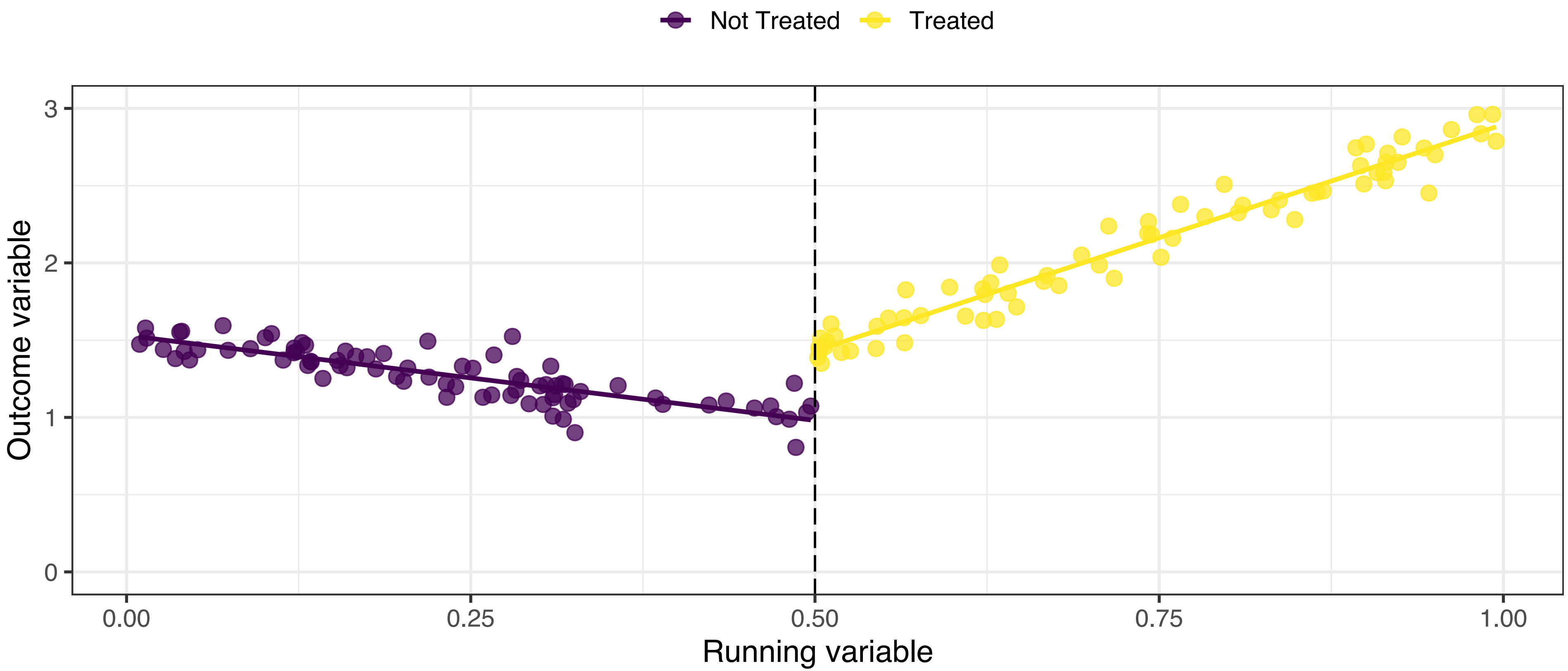
Simulations - Linear Relationship but NO Discontinuity



Simulations - Different (Linear) Slopes



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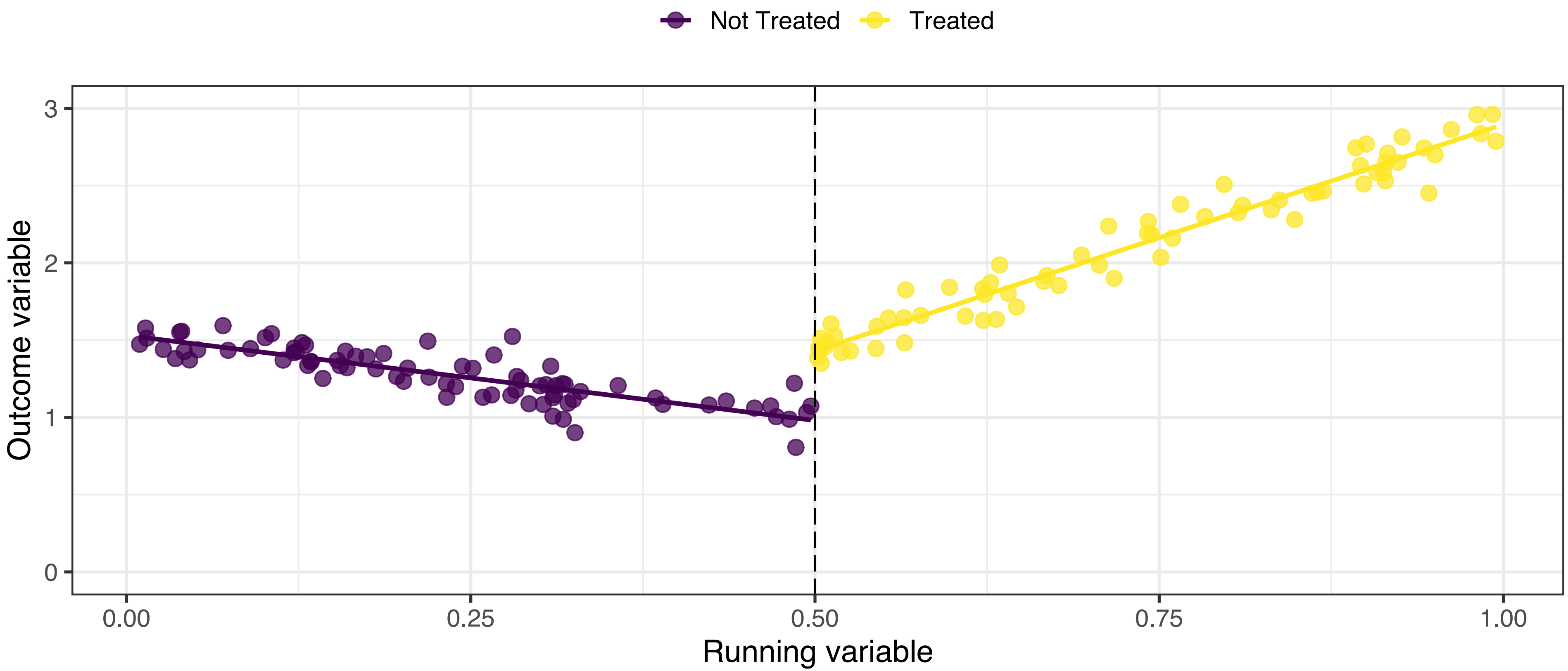

$$outcome_i = \alpha + \delta treatment_i + \beta(running_i - cutoff) +$$
$$\color{red}{\gamma treatment_i * (running_i - cutoff)} + e_i,$$

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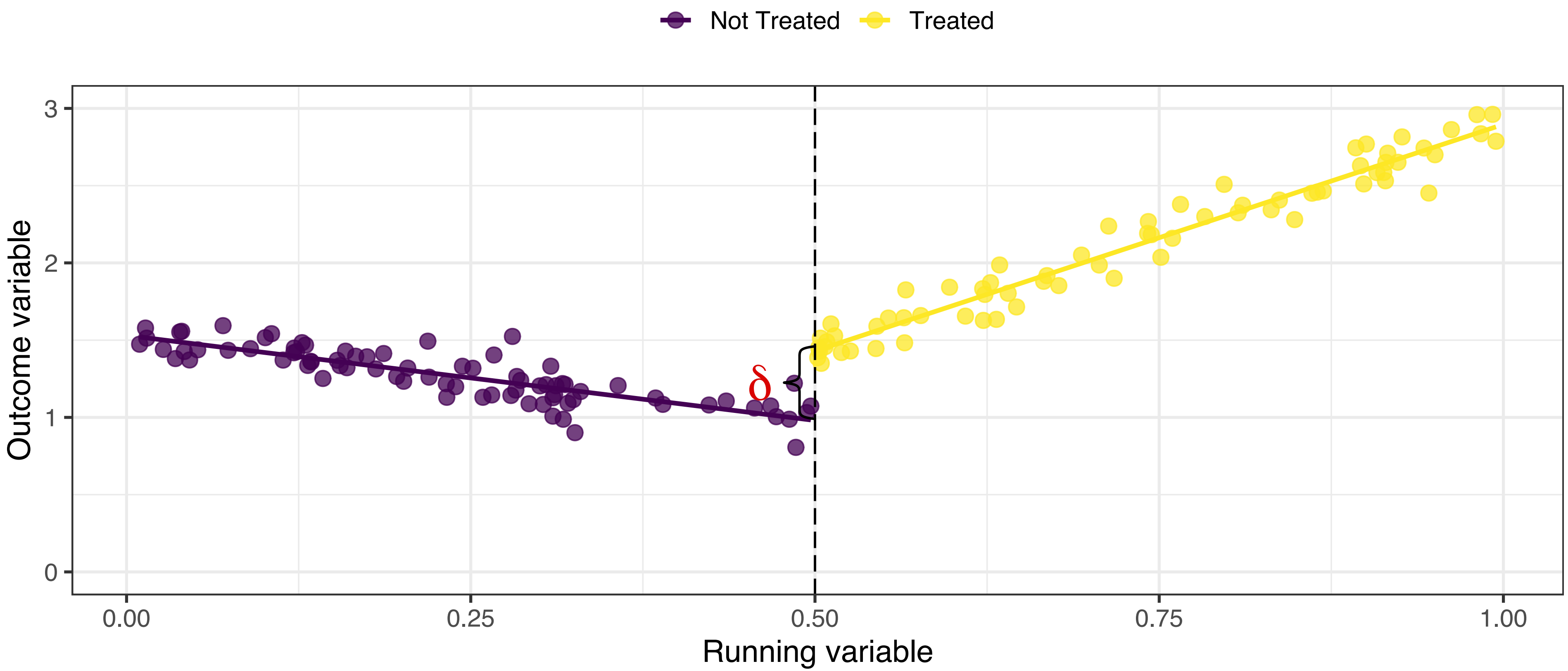

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- Coefficients across models shouldn't vary too much.
- Should we expect the relationship between the outcome variable and the running variable to be nonlinear? Should we expect it to differ around the cutoff?
- Gelman and Imbens (2019), "Why High-Order Polynomials Should Not Be Used in Regression Discontinuity Designs":
  "We recommend researchers [...] use estimators based on local linear or quadratic polynomials or other smooth functions."

Task 3 (10 minutes)

1. Estimate the following *quadratic* model on `all` death causes. Does the RDD coefficient differ from the linear model?

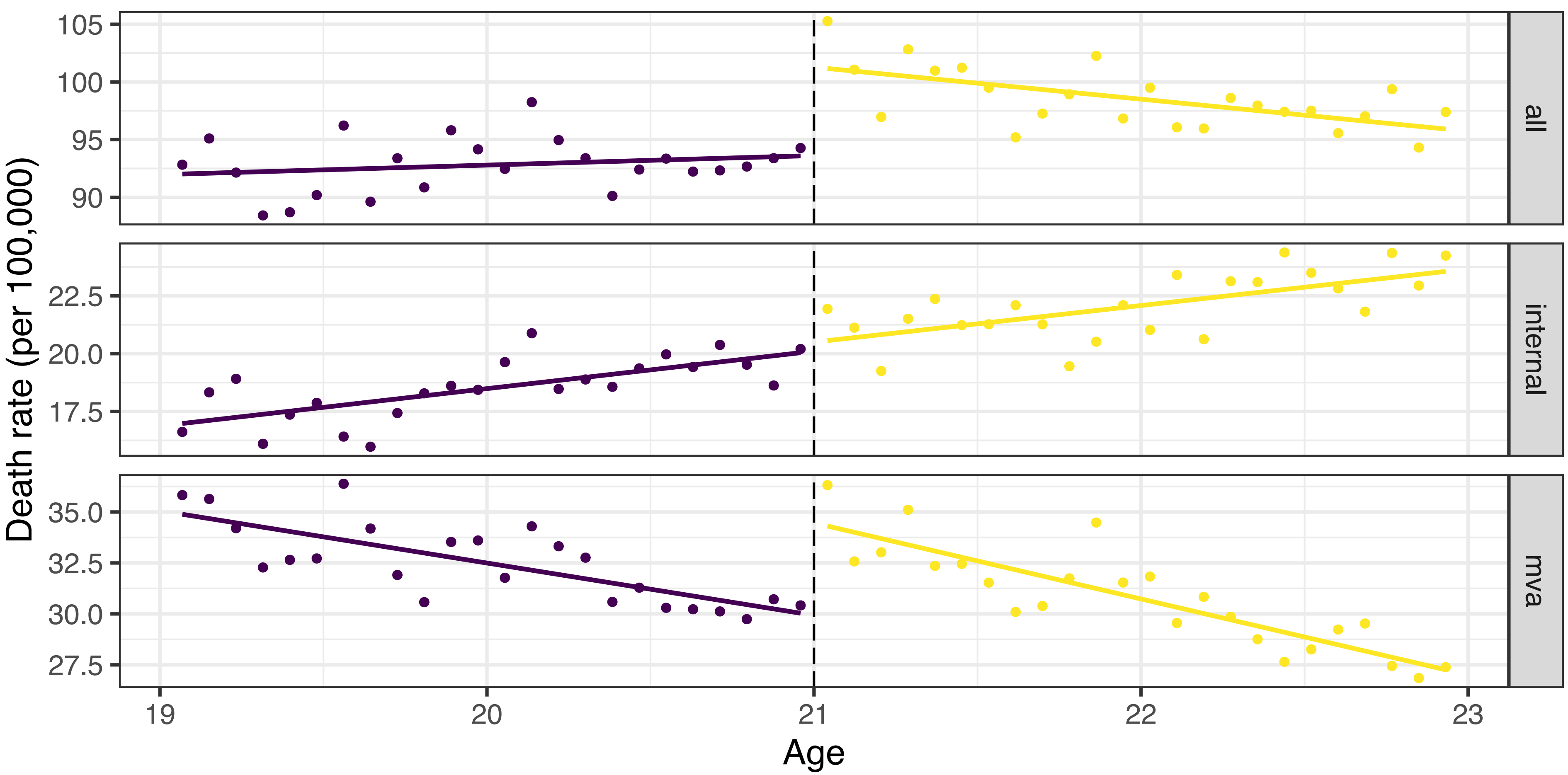
$$DEATHRATE_a = \alpha + \delta D_a + \beta a + \beta a^2 + \varepsilon_a,$$

2. Recall that the regression model allowing for different slopes on each side of the cutoff is:

$$DEATHRATE_a = \alpha + \delta D_a + \beta(a - 21) + \gamma D_a * (a - 21) + \varepsilon_a,$$

   - Why do we need to substract the 21 from `a`? (Hint: compute $\mathbb{E}(DEATHRATE_a | a = 21)$)
   - Should we expect the relationship between death rates and age to change at 21?
   - Estimate this model. How different is the RDD coefficient from the other models you have estimated?

Graphical Representation of the Regression Results



Nonparametric Estimation

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Luckily there's an `R` package that chooses these settings optimally based on fancy algorythms: `rdrobust`.

Identifying Assumptions

RDD Assumptions

> *Key assumption*: ***Potential outcomes are smooth at the threshold.***

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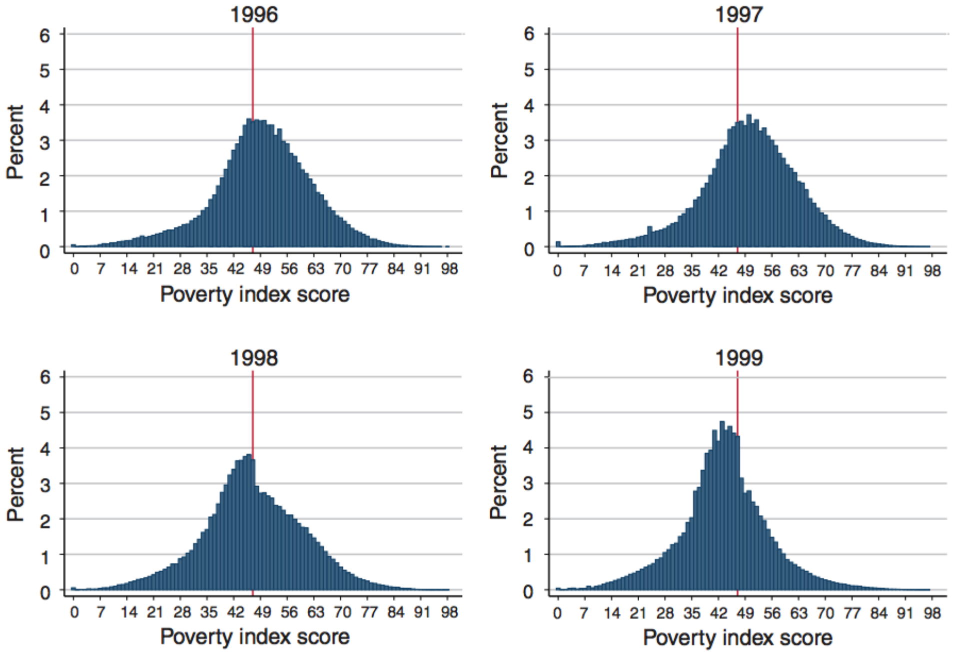
- Assumption is violated if people can manipulate the running variable because they know the cutoff value.
  - Knowing the cutoff value in itself does not violate the assumption, only ability to manipulate running variable does.

Example of Manipulation: Camacho and Conover (2011)

What happens when threshold for eligibility to social assistance programs becomes known?



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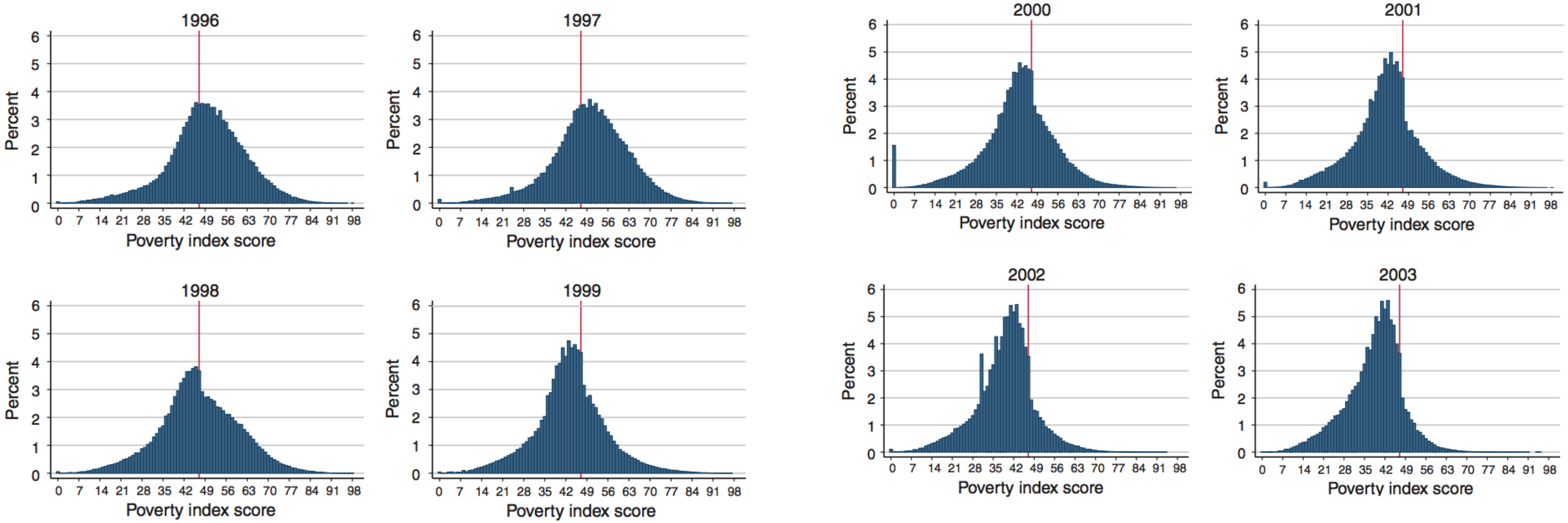

Noncompliance

What if the running variable does not *fully* determine assignment to treatment?

$\rightarrow$ ***Fuzzy RDD***

- Even if all observations that satisfy the treatment condition are not treated, there is still a jump in the probability of being treated.
- For you, just know that problem of imperfect determination of allocation to treatment can still be solved

5 Steps for Conducting RDD in Practice[1]

Step #1: ***Is assignment to treatment rule-based?***

[1] Taken from Andrew Heiss' wonderful course on RDD.

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Step #5: *How big is the gap?*

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END

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