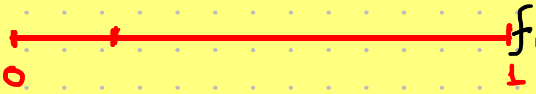
# Variáveis Aleatórias Contínuas

Contínuas x Discretas
↳ assume valores inteiros
↳ assume valores reais



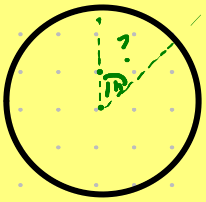

$$f(x) = \mathbb{P} = \begin{cases} \frac{c}{x^2} = \frac{1}{\infty} = 0 \ (?) & p/\ x \geq 10 \\ 0 & p/\ x < 10 \end{cases}$$

AO INVÉS DE PENSARMOS NA PROBABILIDADE DE 1 PT ESPECÍFICO, PENSAMOS NA PROBABILIDADE DE UM INTERVALO.

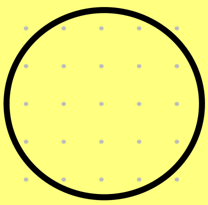
$P(x=1)$ ———— $P(0{,}95 < X < 1{,}05)$

EXEMPLO 7.1

RELÓGIO MECÂNICO

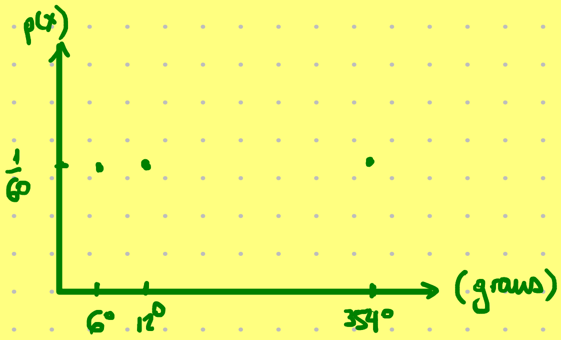


RELÓGIO ELÉTRICO



$P(x) = \frac{1}{\infty} = 0$ ?

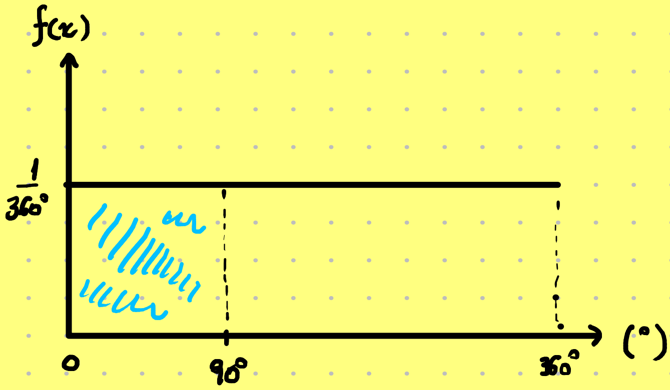
$P(x) = \frac{1}{60}$




(MODELO DISCRETO)

$P(0 \leq X \leq 90°) = \frac{90°}{360°} = \frac{1}{4}$

$P(0{,}99° \leq X \leq 1{,}01°) = \frac{0{,}02}{360} = \dots$




área ... integral

$f(x)$: FUNÇÃO DENSIDADE DE PROBABILIDADE (FDP)

- $f(x) \geq 0$ p/ $\forall x \in [a,b]$
- $\int_a^b f(x)\,dx = 1$

Exercício 4)

$$f(x) = \begin{cases} \frac{c}{x^2} & p/ \ x \geq 10 \\ 0 & p/ \ x < 10 \end{cases}$$

ENCONTRAR O VALOR DE $c$ QUE FAZ A $f(x)$ UMA FDP.

•) $f(x) \geq 0 \ \therefore \ c \geq 0$

•) $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \cancel{\int_{-\infty}^{10} 0\,dx}^{\,0} + \int_{10}^{+\infty} \frac{c}{x^2}\,dx = 1$$

$$\therefore \lim_{t\to+\infty} \int_{10}^{t} \frac{c}{x^2}\,dx = 1 \ \therefore \ \lim_{t\to+\infty} -cx^{-1}\Big|_{10}^{t} = 1$$

$$\therefore \lim_{t\to+\infty} \cancel{\frac{-c}{t}}^{\,0} - \left(-\frac{c}{10}\right) = 1$$

$$\therefore \lim_{t\to+\infty} \frac{c}{10} = 1$$

$$\therefore \frac{c}{10} = 1 \ \therefore \ \boxed{c = 10}$$

$$P(X>15) = \int_{15}^{+\infty} \frac{10}{x^2}\,dx = \lim_{t\to+\infty} \int_{15}^{t} 10x^{-2}\,dx = \lim_{t\to+\infty} -10x^{-1}\Big|_{15}^{t} = \lim_{t\to+\infty} \frac{-10}{x}\Big|_{15}^{t}$$

$$\lim_{t\to+\infty} \cancel{\frac{-10}{t}}^{\,0} - \left(-\frac{10}{15}\right)$$

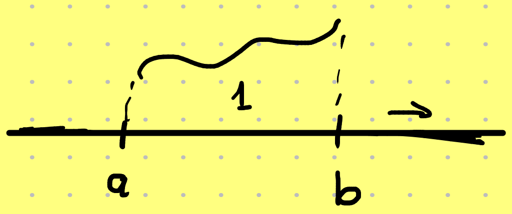
$$= \frac{10}{15} = \frac{2}{3}$$

$$f(x) = \begin{cases} \frac{10}{x^2} & p/ \ x \geq 10 \\ 0 & p/ \ x < 10 \end{cases}$$

## VALOR MÉDIO DE UMA VA ALEATÓRIA

$$E(x) = \int_a^b x f(x)\,dx$$ $p(x)$, mais geralmente $$E(x) = \int_{-\infty}^{+\infty} x f(x)\,dx$$

$\Sigma$



RELÓGIO ELÉTRICO

$$f(x) = \frac{1}{360}$$

$$E(x) = \int_0^{360} x f(x)\, dx = \int_0^{360} x \cdot \frac{1}{360}\, dx = \frac{1}{360} \frac{x^2}{2} \Big|_0^{360} = \frac{\cancel{360}^2}{\cancel{360} \cdot 2} = 180°$$

VARIÂNCIA DE UMA VA CONTÍNUA

$$Var(X) = E\big((X - E(X))^2\big) = \int_{-\infty}^{+\infty} (x - E(x))^2 f(x)\, dx$$

$E(X) = ?$ $E(X^2) = ?$

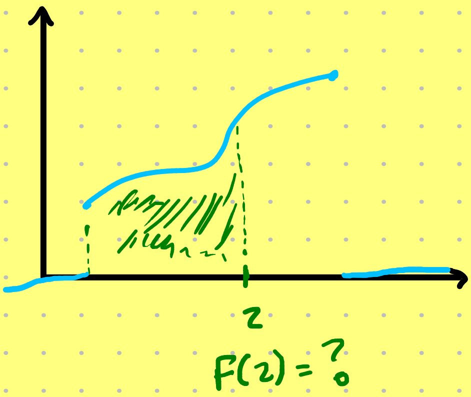
$$Var(x) = E(x^2) - E^2(x) = \ldots$$

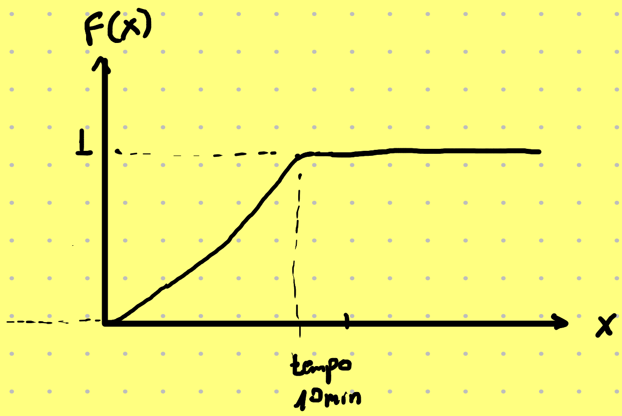
FUNÇÃO ACUMULADA

- DISCRETAS $\mathbb{P}(X \leq x)$
- CONTÍNUO: $F(x) = \int_{-\infty}^{x} f(t)\, dt$

$f(x) = x^2$
$f(u) = u^2$
$f(t) = t^2$

$X$: tempo do último corredor




$$\lim_{X \to +\infty} F(X) = 1 \qquad \lim_{X \to -\infty} F(X) = 0$$

$$F'(x) = f(x)$$