Distribuição Conjunta

x 2 ou + vals

Exemplo. 8.1

Família, 3 crianças

X = nº rapazes

$Y = \begin{cases} 1, \text{ se homem} \\ 0, \text{ se mulher} \end{cases}$

Z = nº variações

Casos	Prob	x	y	z
HHH	1/8	3	1	0
HHM	1/8	2	1	1
HMH	1/8	2	1	2
MHH	1/8	2	0	1
HMM	1/8	1	1	1
MHM	1/8	1	0	2
MMH	1/8	1	0	1
MMM	1/8	0	0	0

Tabela de dist. conjunta (X, Y)

(x,y)	$P(x,y)$
(0,0)	1/8
(1,0)	2/8
(1,1)	1/8
(2,0)	1/8
(2,1)	2/8
(3,1)	1/8

y \ x	0	1	2	3	P(Y)
0	1/8	2/8	1/8	0	1/2
→ 1	0	1/8	2/8	1/8	1/2
P(X)	1/8	3/8	3/8	1/8	1

dist marginal de X

dist. marginal de Y

Prob Condicional

$$P(X=3 \mid Y=1) = \frac{P(X=3, Y=1)}{P(Y=1)} = \frac{1/8}{1/2} = 1/4$$

$$\sum_{i=0}^{3} P(X = x_i \mid Y = 1) = 1$$

Esperança na prob. condicional

dist $X \mid Y=1$ $\qquad P(Y=1) = 1/2$

x	0	1	2	3
$P(x \mid Y=1)$	0	1/4	1/2	1/4

$$E(X \mid Y=1) = 0\cdot 0 + 1\cdot\frac{1}{4} + 2\cdot\frac{1}{2} + 3\cdot\frac{1}{4} = 2$$

$$\frac{2/8}{1/2}$$

Independência

- A, B independentes $\therefore$ $P(A \mid B) = P(A)$

$$\frac{P(A \cap B)}{P(B)} = P(A) \therefore \boxed{P(A \cap B) = P(A)\cdot P(B)}$$

Tabela dupla entrada entre Y, Z

Y \ Z	0	1	2	$P(Y)$
0	1/8	2/8	1/8	1/2
1	1/8	2/8	1/8	1/2
$P(Z)$	1/4	1/2	1/4	1

Exatos	Prob	Y	Z
HHH	1/8	1	0
HHM	1/8	1	1
HMH	1/8	1	2
MHH	1/8	0	1
HMM	1/8	1	1
MHM	1/8	0	2
MMH	1/8	0	1
MMM	1/8	0	0

$$P(Z=1 \cap Y=1) = \frac{2}{8}$$

$$P(Z=1)\cdot P(Y=1) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} = \frac{2}{8} \leftarrow$$

Y, Z são independentes?

* Para Y, Z serem independentes p/ cada "TODOS" (Y_i, Z_j)

$$P(Y=Y_i \cap Z=Z_j) = P(Y=Y_i)\cdot P(Z=Z_j)$$

1)

a) {coroa, 1}, {coroa, 2}, ..., {coroa, 6}

{coroa, 1}, {coroa, 2}, ..., {coroa, 6}

b)

(x, y)	P(x, y)
(0, 1)	1/12
(0, 2)	
⋮	
(0, 6)	
(1, 1)	
(1, 2)	
⋮	
(1, 6)	1/12

$\frac{6}{12} = \frac{1}{2}$

$\frac{6}{12} = \frac{1}{2}$

x \ y	1	2	3	4	5	6	P(x)
0	1/12	1/12	⋯	⋯	1/12	1/12	1/2
1	1/12	1/12	⋯	⋯	1/12	1/12	1/2
P(y)	1/6	1/6	⋯	⋯	1/6	1/6	

c) São independentes

d) $P(x = 1) = 1/2$

$P(x \leq 1) = 1$

$P(x < 1) = 1 - 1/2 = 1/2 = P(x = 0)$

$P(x = 2, y = 3) = 0$

$P(x \geq 0, y \leq 4) = 8/12$

$P(x = 0, y \geq 1) = P(x = 0) = 1/2$