Retomando ...

X = número de meninos

$Y = \begin{cases} 1, & 1^\circ \text{ filho for homem} \\ 0, & 1^\circ \text{ filha for mulher} \end{cases}$

Z = variação no sexo

Ex: HMM $\rightarrow$ $X = 1$, $Y = 1$, $Z = 1$

- dist. marginal
- tabela conjunta

$X \backslash Y$	1	2	$P(x)$
			0
			0
$P(Y)$	0	0	1

- Prob. condicional

$X \backslash Y$	1	2	$P(x)$
2		$P(2,2)$	
$P(Y)$		$P(Y=2)$	1

$$\mathbb{P}(X=2 \mid Y=2) = \frac{P(X=2, Y=2)}{P(Y=2)} = \frac{?}{?} =$$

- Esperança Condicional

$$E(X \mid Y = y_j) = \sum_{i=1}^{n} x_i \, P(X = x_i \mid Y = y_j) = \, ?$$

$1, 2, \ldots$

Independência

$$\mathbb{P}(X = x_i, Y = y_j) = \mathbb{P}(X = x_i) \cdot \mathbb{P}(Y = y_j) \quad \therefore \quad X, Y \text{ são independentes}$$

$\forall i, j$

$X, Y \therefore$ $X+Y$, XY, X^2Y ... $E(\ ) = ?$ $Var(\ ) = ?$

$E(X+Y) = E(X) + E(Y)$?

(x_i, y_j)	$X+Y$	XY	$p(x_i, y_j)$
(0,0)	0	0	1/8
(0,1)	1	0	0
(1,0)	1	0	2/8
(1,1)	2	1	1/8
(2,0)	2	0	1/8
(2,1)	3	2	2/8
(3,0)	3	0	0
(3,1)	4	3	1/8

$X+Y$	0	1	2	3	4
$P(X+Y)$	1/8	2/8	2/8	2/8	1/8

$$E(X+Y) = 0 + \frac{2}{8} + \frac{4}{8} + \frac{6}{8} + \frac{4}{8} = 2$$

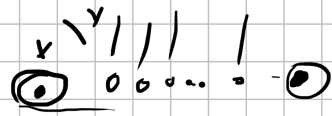
Veja o pdf da monitoria anterior.

$E(X) = 1{,}5$ e $E(Y) = 0{,}5$

Sim! Aconteceu :)

Teorema 5.1 Se X é uma va c/ valores $x_1, x_2, \dots, x_n$ e probs $p(x_1), p(x_2), \dots, p(x_n)$ e Y idem, então

$$E(X+Y) = E(X) + E(Y)$$



Demonst.

$$E(X+Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} (x_i + y_j)\cdot p(x_i, y_j) = \sum_{i=1}^{n}\sum_{j=1}^{m} x_i (p(x_i, y_j)) + y_j (p(x_i, y_j))$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{m} x_i\, p(x_i, y_j) + \sum_{j=1}^{m}\sum_{i=1}^{n} y_j\, p(x_i, y_j) = \sum_{i=1}^{n} x_i \underbrace{\sum_{j=1}^{m} p(x_i, y_j)}_{p(x_i)} + \sum_{j=1}^{m} y_j\, p(y_j)$$

$$= E(X) + E(Y)$$

(x_i, y_j)	$X+Y$	XY	$p(x_i, y_j)$
(0,0)	0	0	1/8
(0,1)	1	0	0
(1,0)	1	0	2/8
(1,1)	2	1	1/8
(2,0)	2	0	1/8
(2,1)	3	2	2/8
(3,0)	3	0	0
(3,1)	4	3	1/8

XY	0	1	2	3
$P(XY)$	4/8	1/8	2/8	1/8

$$E(XY) = \frac{1}{8} + \frac{4}{8} + \frac{3}{8} = 1$$

Veja o pdf da monitoria anterior.

$E(X) = 1{,}5$ e $E(Y) = 0{,}5$

$E(XY) \neq E(X)\cdot E(Y)$:(

$P(X \cap Y) = P(X)\cdot P(Y)$ $(\forall\, i, j)$

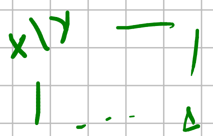
Teorema 5.2 Se X e Y são va independentes, então

$$E(XY) = E(X)E(Y)$$

Demonstração. (Ao leitor)

dica $p(x_i, y_j) = p(x_i)\cdot p(y_j)$

Situação



$E(XY) = E(X)\,E(Y) \Rightarrow X, Y$ são independentes

A recíproca não é verdadeira! :(

Covariância

$$Cov(X,Y) = E\big((X-E(X))(Y-E(Y))\big)$$

$$= E\big(XY - XE(Y) - YE(X) + E(X)E(Y)\big) = E(XY) + E(-XE(Y)) + E(-YE(X))$$

$$+ E(X)E(Y) = E(XY) - E(Y)E(X) - \cancel{E(X)E(Y)} + \cancel{E(X)E(Y)} = E(XY) - E(Y)E(X)$$

$$\boxed{\text{Covariância} = E(XY) - E(X)E(Y)}$$

- Se $Cov(X,Y) = 0 \therefore X, Y$ são não correlacionados