

## Exercício 2

$y \backslash x$	1	2	3	$P(y)$
0	0,1	0,1	0,1	0,3
1	0,2	0	0,3	0,5
2	0	0,1	0,1	0,2
$P(x)$	0,3	0,2	0,5	1

$$E(x) = 1 \cdot 0,3 + 2 \cdot 0,2 + 3 \cdot 0,5 = 2,2$$

$$E(y) = 0 \cdot 0,3 + 1 \cdot 0,5 + 2 \cdot 0,2 = 0,9$$

$$E(x^2) = 1 \cdot 0,3 + 4 \cdot 0,2 + 9 \cdot 0,5 = 5,6$$

$$\text{Var}(x) = E(x^2) - E^2(x) = 5,6 - 2,2^2 = 0,76$$

$$E(y^2) = 0 \cdot 0,3 + 1 \cdot 0,5 + 4 \cdot 0,2 = 1,3$$

$$\text{Var}(y) = E(y^2) - E^2(y) = 1,3 - 0,9^2 = 0,49$$

Note que  $P(x=1, y=0) = 0,1$ , mas  $P(x=1) \cdot P(y=0) = 0,3 \cdot 0,3 = 0,09 \neq 0,1$ .

Então  $X$  e  $Y$  não são independentes

$$\begin{aligned} P(X=1 | Y=0) \\ &= \frac{P(X=1, Y=0)}{P(Y=0)} \\ &= \frac{0,1}{0,3} = \frac{1}{3} \end{aligned}$$

$y \backslash x$	1	2	3	$P(y)$
0	0,1	0,1	0,1	0,3
1	0,2	0	0,3	0,5
2	0	0,1	0,1	0,2
$P(x)$	0,3	0,2	0,5	1

$$P(Y=2 | X=3) = \frac{P(Y=2, X=3)}{P(X=3)} = \frac{0,1}{0,5} = \frac{1}{5}$$

$$e) P(X \leq 2) = 1 - P(X > 2) = 1 - P(X=3) = 1 - 0,5 = 0,5$$

$$P(X=2, Y \leq 1) = 0,1$$

## Exercício 3

$y \backslash x$	-1	0	1	$P(y)$
-1	1/12	0	1/12	1/6
0	2/12	0	2/12	1/3
1	1/4	0	1/4	1/2
$P(x)$	1/2	0	1/2	1

$$\begin{aligned} a &= \frac{1}{12b} = \frac{1}{12 \cdot \frac{1}{2}} = \frac{1}{6} \\ ab &= \frac{1}{12} \\ bd &= \frac{1}{4} \\ cd &= \frac{1}{4} \\ a+d &= \frac{2}{3} \\ b(a+d) &= \frac{1+3}{12} = \frac{4}{12} = \frac{1}{3} \\ \frac{2}{3}b &= \frac{1}{3} \therefore b = \frac{1}{2} \end{aligned}$$

$$E(X) = -1 \cdot \frac{1}{2} + 0 \cdot 0 + 1 \cdot \frac{1}{2} = 0$$

$$E(Y) = -1 \cdot \frac{1}{6} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{2} = \frac{1}{3}$$

$y \backslash x$	-1	0	1	$P(y)$
-1	$1/12$	0	$1/12$	$1/6$
0	$2/12$	0	$2/12$	$1/3$
1	$1/4$	0	$1/4$	$1/2$
$P(x)$	$1/2$	0	$1/2$	1

$x$	$P(x   y=0)$
-1	$1/2$
0	0
1	$1/2$

$$P(X=-1 | Y=0) = \frac{2/12}{1/3} = \frac{6}{12} = \frac{1}{2}$$

$$P(X=0 | Y=0) = \frac{0}{1/3} = 0$$

$$P(Y=-1 | X=1) = \frac{1/12}{1/2} = \frac{1}{6}$$

$$P(Y=0 | X=1) = \frac{2/12}{1/2} = \frac{4}{12} = \frac{2}{6}$$

$y$	$P(y   x=1)$
-1	$1/6$
0	$2/6$
1	$3/6$

#### Exercício 4

$y \backslash x$	1	2	3	$P(y)$
0	$0,1$	$0,1$	$0,1$	$0,3$
1	$0,2$	0	$0,3$	$0,5$
2	0	$0,1$	$0,1$	$0,2$
$P(x)$	$0,3$	$0,2$	$0,5$	1

$x, y$	$x+y$	$(x, y)$	$P(x, y)$
0	1	(0,1)	$0,1$
0	2	(0,2)	$0,1$
0	3	(0,3)	$0,1$
1	2	(1,1)	$0,2$
2	3	(1,2)	0
3	4	(1,3)	$0,3$
4	4	(2,1)	0
4	4	(2,2)	$0,1$
6	5	(2,3)	$0,1$

$xy$	$P(xy)$
0	$0,3$
1	$0,2$
2	0
3	$0,3$
4	$0,1$
6	$0,1$

$x+y$	$P(x+y)$
1	$0,1$
2	$0,3$
3	$0,1$
4	$0,4$
5	$0,1$

$$E(x+y) = E(x) + E(y) = 2,2 + 0,9 = 3,1$$

$$E(xy) = 0 \cdot 0,3 + 1 \cdot 0,2 + 2 \cdot 0 + 3 \cdot 0,3 + 4 \cdot 0,1 + 6 \cdot 0,1 = 2,1$$

$$E((xy)^2) = \cdot$$

$$\text{Var}(xy) = \cdot - 2,1^2$$

$$\text{Var}(x+y) = E((x+y)^2) - 3,1^2$$

$$\text{Prob 5} - E(x+y) = E(x) + E(y) \checkmark$$

$$\text{Var}(x+y) = \text{Var}(x) + \text{Var}(y) \checkmark$$