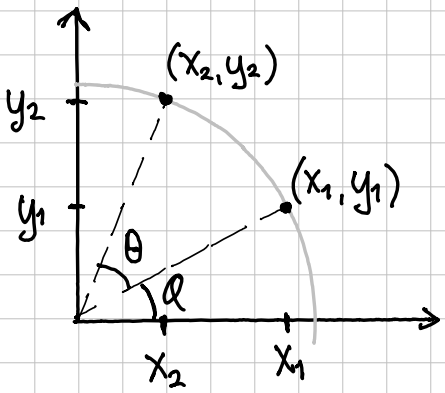
CORDIC Algorithm



$x_1 = \cos\alpha$ $\quad y_1 = \sin\alpha$

$x_2 = \cos(\alpha+\theta)$ $\quad y_2 = \sin(\alpha+\theta)$

$$\Delta x = \cos\alpha - \cos(\alpha+\theta)$$
$$= \cos\alpha - \cos\alpha\cos\theta + \sin\alpha\sin\theta$$
$$= x_1(1-\cos\theta) + y_1\sin\theta$$

$$\Delta y = \sin(\alpha+\theta) - \sin\alpha$$
$$= \cos\alpha\sin\theta + \cos\theta\sin\alpha - \sin\alpha$$
$$= x_1\sin\theta + y_1(\cos\theta - 1)$$

$$x_2 = x_1 - \Delta x$$
$$x_2 = x_1 - (x_1(1-\cos\theta) + y_1\sin\theta)$$
$$x_2 = x_1\cos\theta - y_1\sin\theta$$

$$y_2 = y_1 + x_1\sin\theta + y_1(\cos\theta - 1)$$
$$y_2 = x_1\sin\theta + y_1\cos\theta$$

$$x_2 = x_1 \cos\theta - y_1 \sin\theta \qquad y_2 = x_1 \sin\theta + y_1 \cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} \qquad \sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} \qquad \tan\theta = \pm 2^{-i}$$

$$x_2 = x_1 \cdot \frac{1}{\sqrt{1+\tan^2\theta}} - y_1 \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} \qquad y_2 = x_1 \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} + y_1 \frac{1}{\sqrt{1+\tan^2\theta}}$$

$$x_2 = \frac{1}{\sqrt{1+\tan^2\theta}} (x_1 - y_1 \tan\theta) \qquad y_2 = \frac{1}{\sqrt{1+\tan^2\theta}} (x_1 \tan\theta + y_1)$$

$$x_{i+1} = \frac{1}{\sqrt{1+\tan^2\theta}} (x_i - y_i \tan\theta) \qquad y_{i+1} = \frac{1}{\sqrt{1+\tan^2\theta}} (x_i \tan\theta + y_i)$$

$$x_{i+1} = \frac{1}{\sqrt{1+2^{-2i}}} (x_i \pm 2^{-i} y_i) \qquad y_{i+1} = \frac{1}{\sqrt{1+2^{-2i}}} (y_i \pm 2^{-i} x_i)$$

We can scale x and y after n iterations:

$$K(n) = \prod_{i=0}^{n-1} K_i = \prod_{i=0}^{n-1} \frac{1}{\sqrt{1+2^{-2i}}}$$

$$x_{i+1} = x_i \pm 2^{-i} y_i \qquad y_{i+1} = y_i \pm 2^{-i} x_i$$

$$\cos\theta \approx x = K(n) x_n \qquad \sin\theta \approx y = K(n) y_n$$

To decide rotation direction we keep track of rotated angle at each iteration

$\theta_0 = \theta$ $\qquad$ $\theta_{i+1} < 0 \Rightarrow \sigma_{i+1} = -1$

$\theta_{i+1} = \theta_i - \sigma_i \gamma_i$ $\qquad$ $\theta_{i+1} > 0 \Rightarrow \sigma_{i+1} = 1$

γ_i is rotated angle at iteration i

$$x_{i+1} = x_i - \sigma_i y_i 2^{-i} \qquad x_0 = 1$$

$$y_{i+1} = y_i + \sigma_i x_i 2^{-i} \qquad y_0 = 0$$

$$\theta_{i+1} = \theta_i - \sigma_i \gamma_i \qquad \theta_0 = \theta$$

$$\sigma_i = \begin{cases} 1 & \theta_{i+1} \geq 0 \\ -1 & \theta_{i+1} < 0 \end{cases} \qquad \sigma_0 = \begin{cases} 1 & \theta_0 \geq 0 \\ -1 & \theta_0 < 0 \end{cases}$$

$$K(n) = \prod_{i=0}^{n-1} \frac{1}{\sqrt{1+2^{-2i}}}$$

$\cos\theta \approx K(n) x_n$ $\qquad$ $\sin\theta \approx K(n) y_n$