Large N

POLI SCI 210

Introduction to Empirical Methods in Political Science

Be ready for next week!

NO EMPS CHAPTER ASSIGNED

Lecture reading

- Huntinton-Klein, Nick. 2022. *The Effect: An Introduction to Research Design and Causality*. Chapman & Hall. Chapter 18
- Cattaneo, Matias D., Nicolás Idrobo, and Rocío Titiunik. 2020. *A Practical Introduction to Regression Discontinuity Designs: Foundations*. Cambridge University Press. Chapters 1-4

Discussion section (read at least on very carefully)

- García-Montoya, Laura, Ana Arjona, and Matthew Lacombe. 2022. "Violence and Voting in the United States: How School Shootings Affect Elections." *American Political Science Review* 116 (3): 807-826
- Ademi, Ubeydullah and Firat Kimya. 2024. "Democratic transition and party polarization: A fuzzy regression discontinuity design approach." *Party Politics* 30 (4):736-749

AI Prompts

- (Linear) regression
- Bivariate vs. multivariate regression
- Ordinary least squares (OLS)
- OLS regression assumptions (ask me why I did not mention them in class)
- How to choose what variables to include
- Covariance vs. correlation vs. regression

Last week

- Experiments to learn about *cause and effect*
- **Broadly:** Summarizing relationships between two variables (difference in means between treatment and control)
- **This week:** A more general method to summarize relationships between two (or more) variables
- **Tuesday:** *Bivariate* relationships
- **Thursday:** *Multivariate* relationships

An experiment has two variables

- **Y:** Observed outcome
- **D:** Treatment assignment (0: control, 1: treatment)

Y can be any kind of variable (numerical, categorical)

D is categorical because it denotes group membership

More general names

Y
Outcome variable
Response variable
Dependent variable
Thing you ***want*** to explain

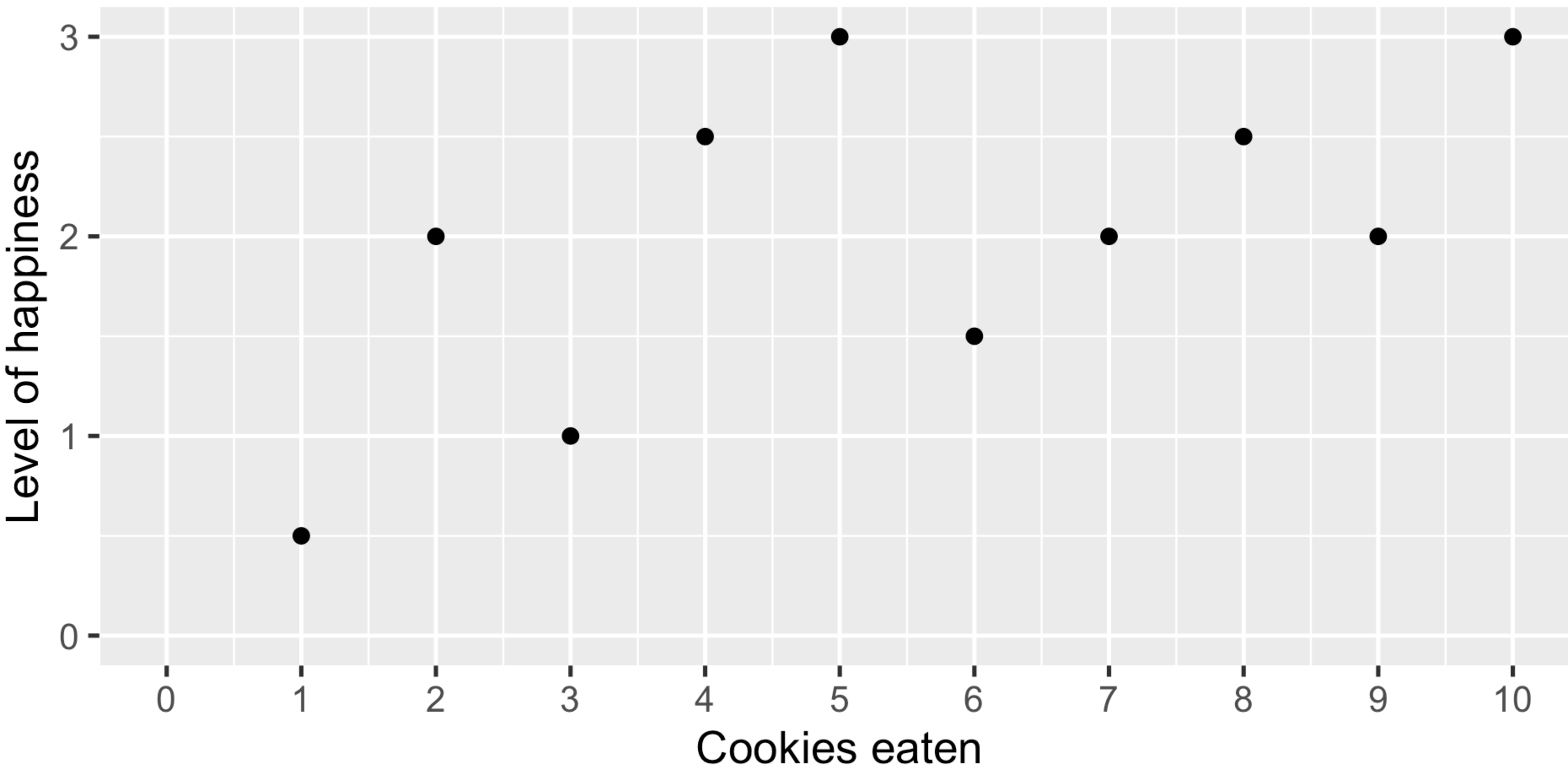
More general names

Y	X
Outcome variable	Explanatory variable
Response variable	Predictor variable
Dependent variable	Independent variable
Thing you ***want*** to explain	Thing you ***use*** to explain

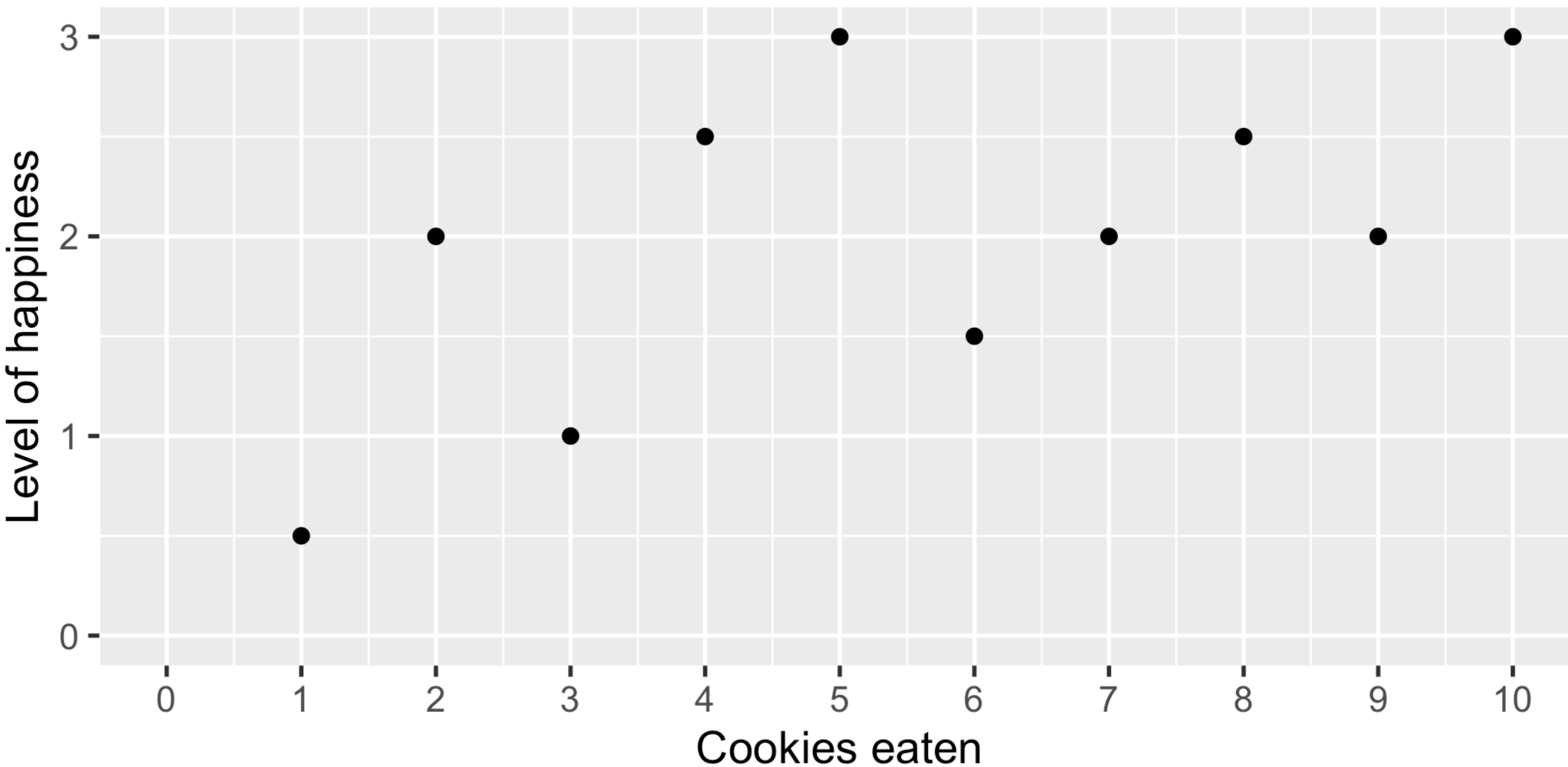
X and **Y** can now be any type (numerical, categorical)

That means we can't just compare means

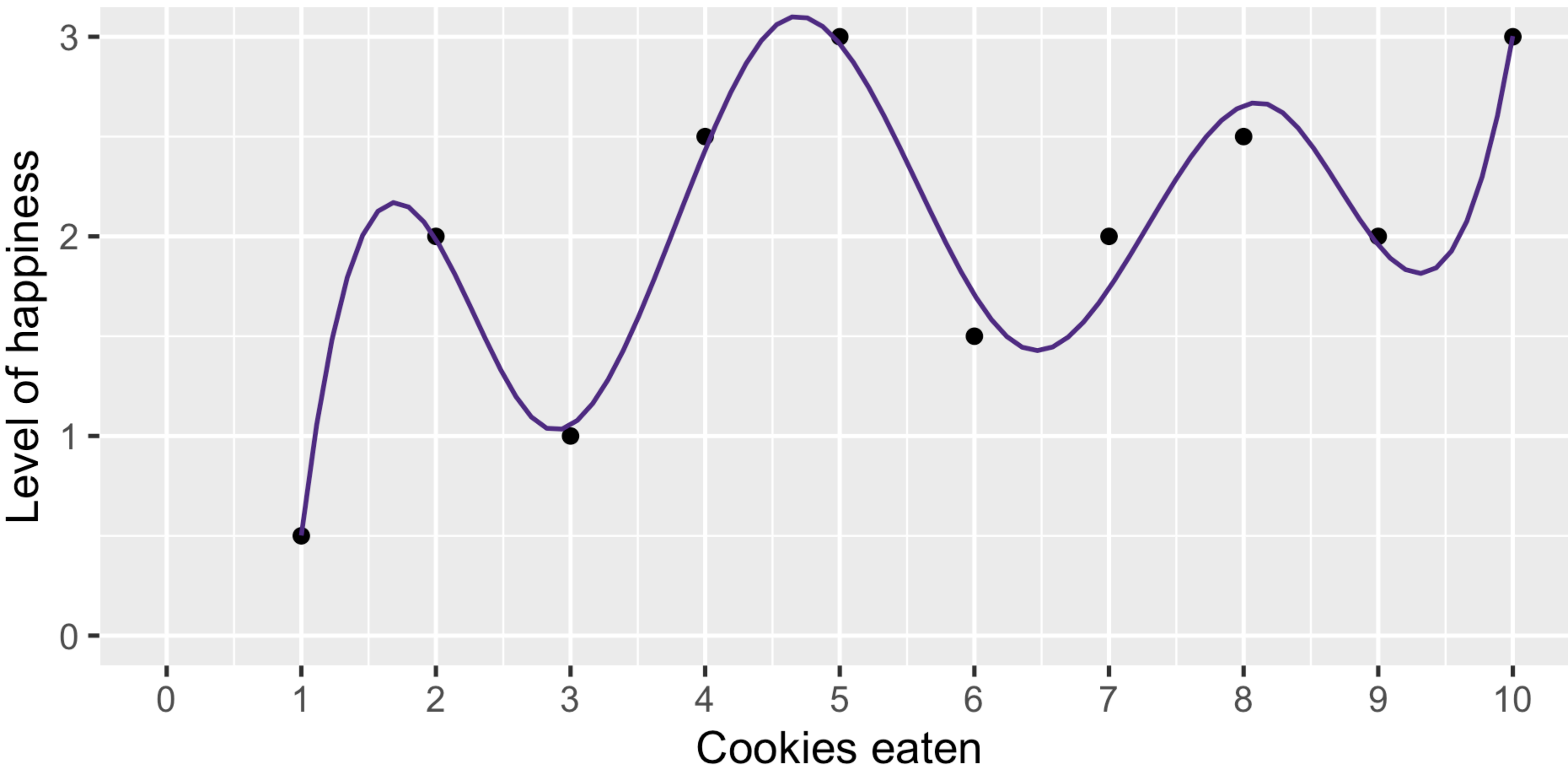
Example



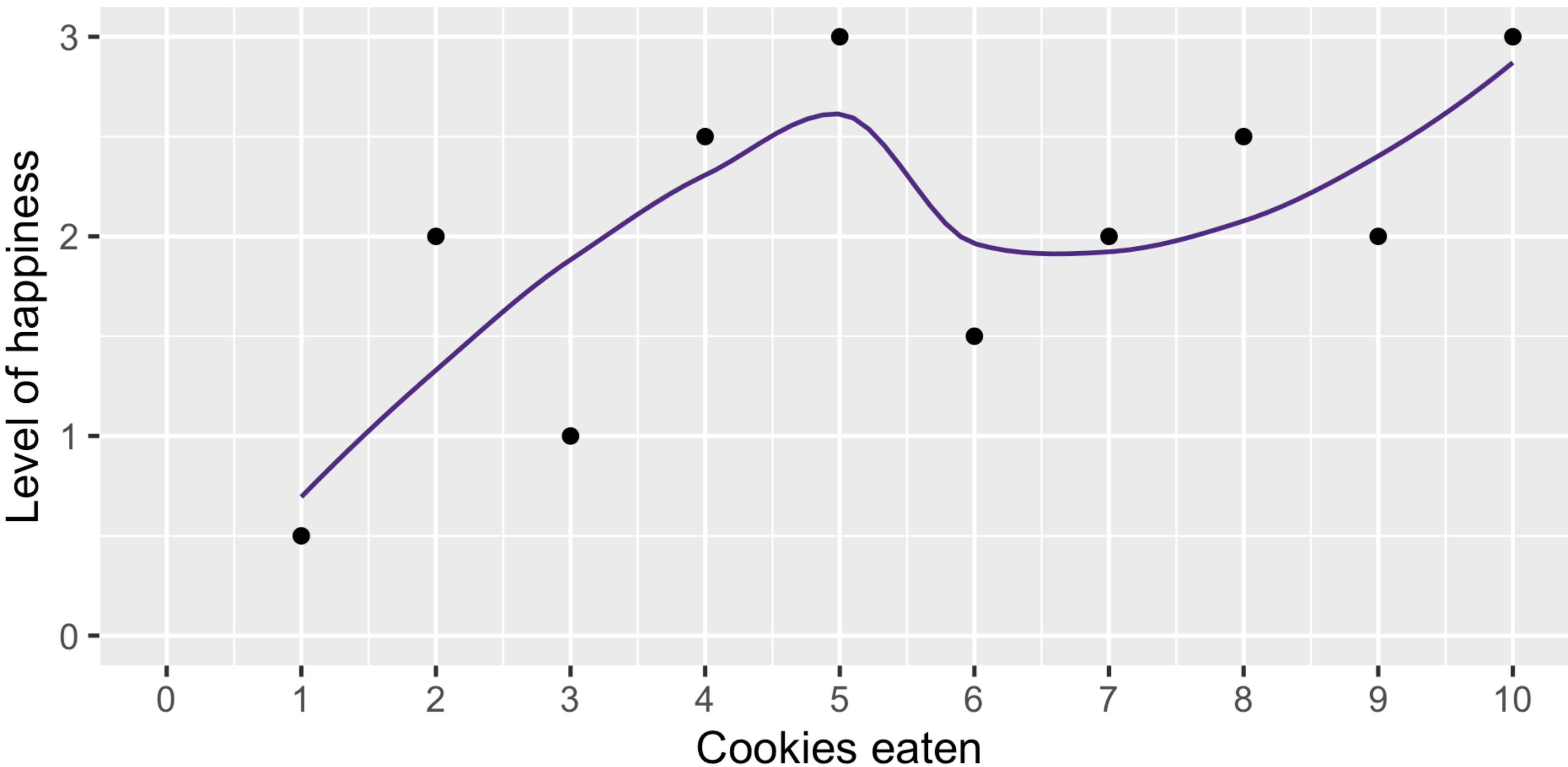
How to summarize?



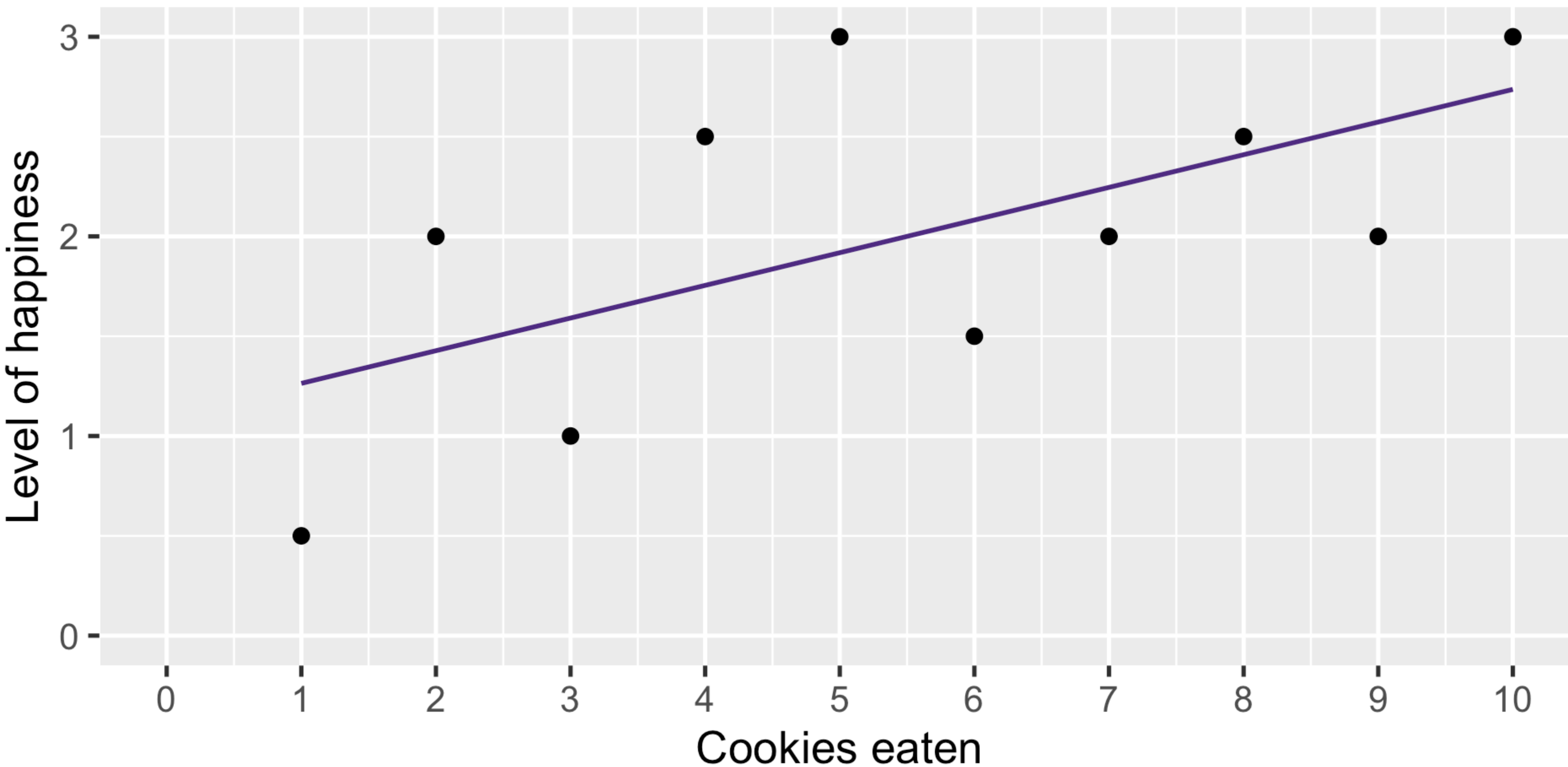
Connect with a line



Maybe smoother?



A straight line?



Straight lines are good

They can be written as a **linear equation**

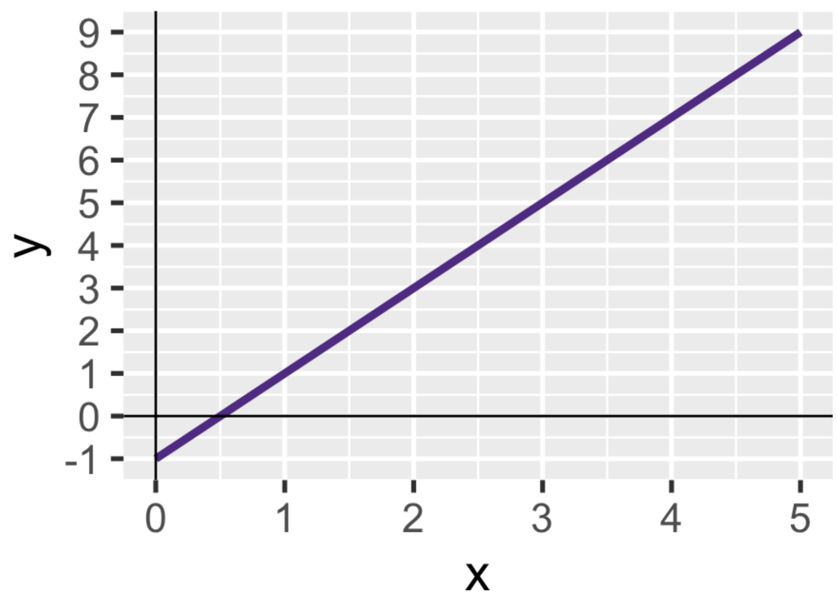
$$y = mx + b$$

y	Outcome variable
x	Explanatory variable
m	Slope ($\frac{\text{rise}}{\text{run}}$)
b	y-intercept

This is the *smallest number of parameters* to draw a line

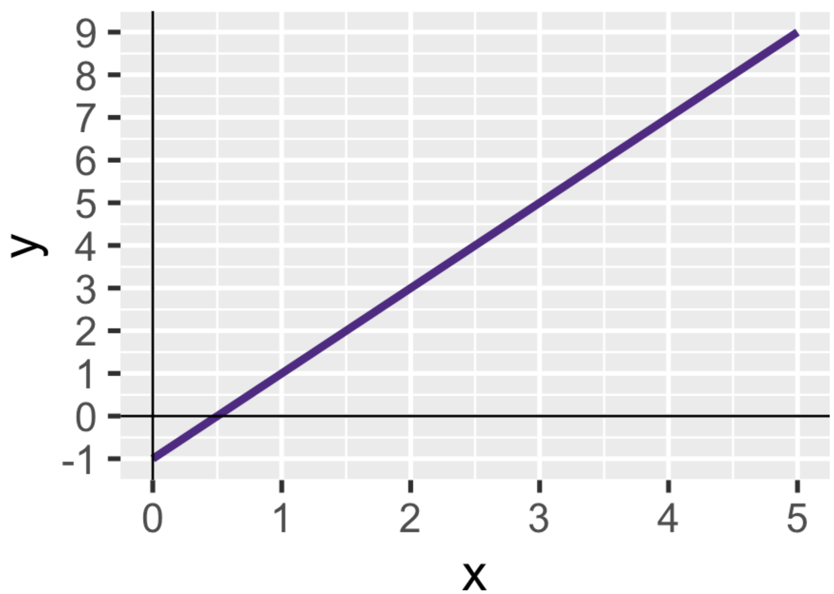
Slopes and intercepts

$$y = 2x - 1$$

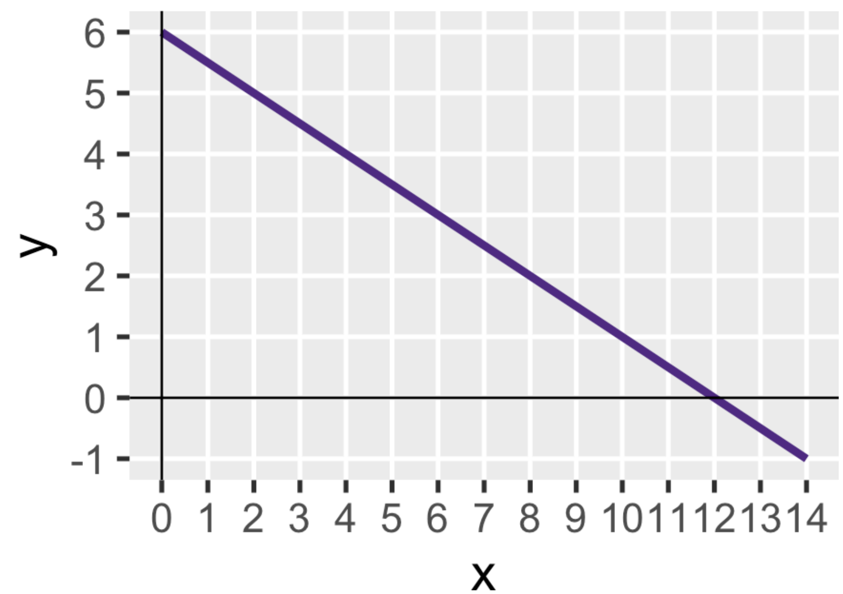


Slopes and intercepts

$y = 2x - 1$



$y = -0.5x + 6$



We can think of *intercept* and *slope* as **estimands** or **inferential targets**

Drawing lines in statistics

$$y = mx + b$$

Drawing lines in statistics

$$\widehat{y} = \widehat{\beta}_0 + \widehat{\beta}_1 x_1$$

y	$\widehat{y}$	Outcome variable
x	x_1	Explanatory variable
m	$\widehat{\beta}_1$	Slope
b	$\widehat{\beta}_0$	y-intercept

You may see this equation with an extra error term ε in some textbooks

What are we doing?

- **Before:** Assume there is a *true parameter* that we do not observe (e.g. population mean, ATE)
- **Now:** Assume there is a *true line* that best describes the relationship between X and Y
- There is a **best linear predictor** that we want to *estimate*

Which line is a better summary?

Show the Line

By default, R will show the line defined as Line #1 below.

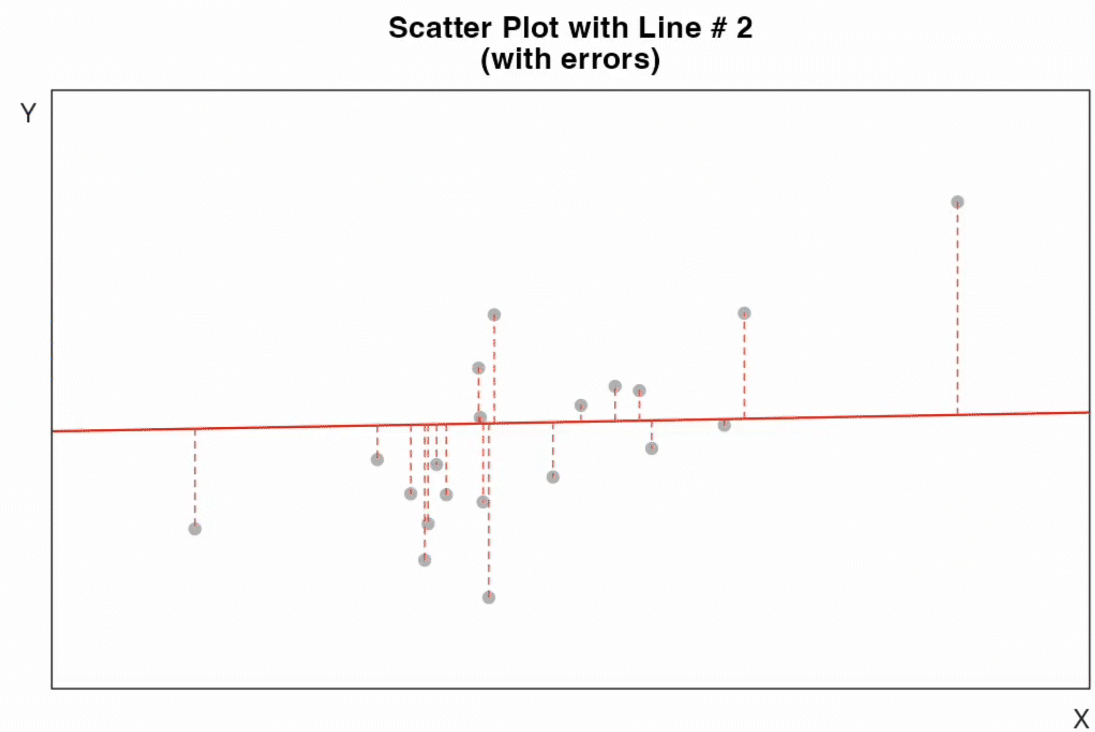
Show the Errors

The errors are the vertical distances between the dots and the line.

Choose A Different Line:

Line #2

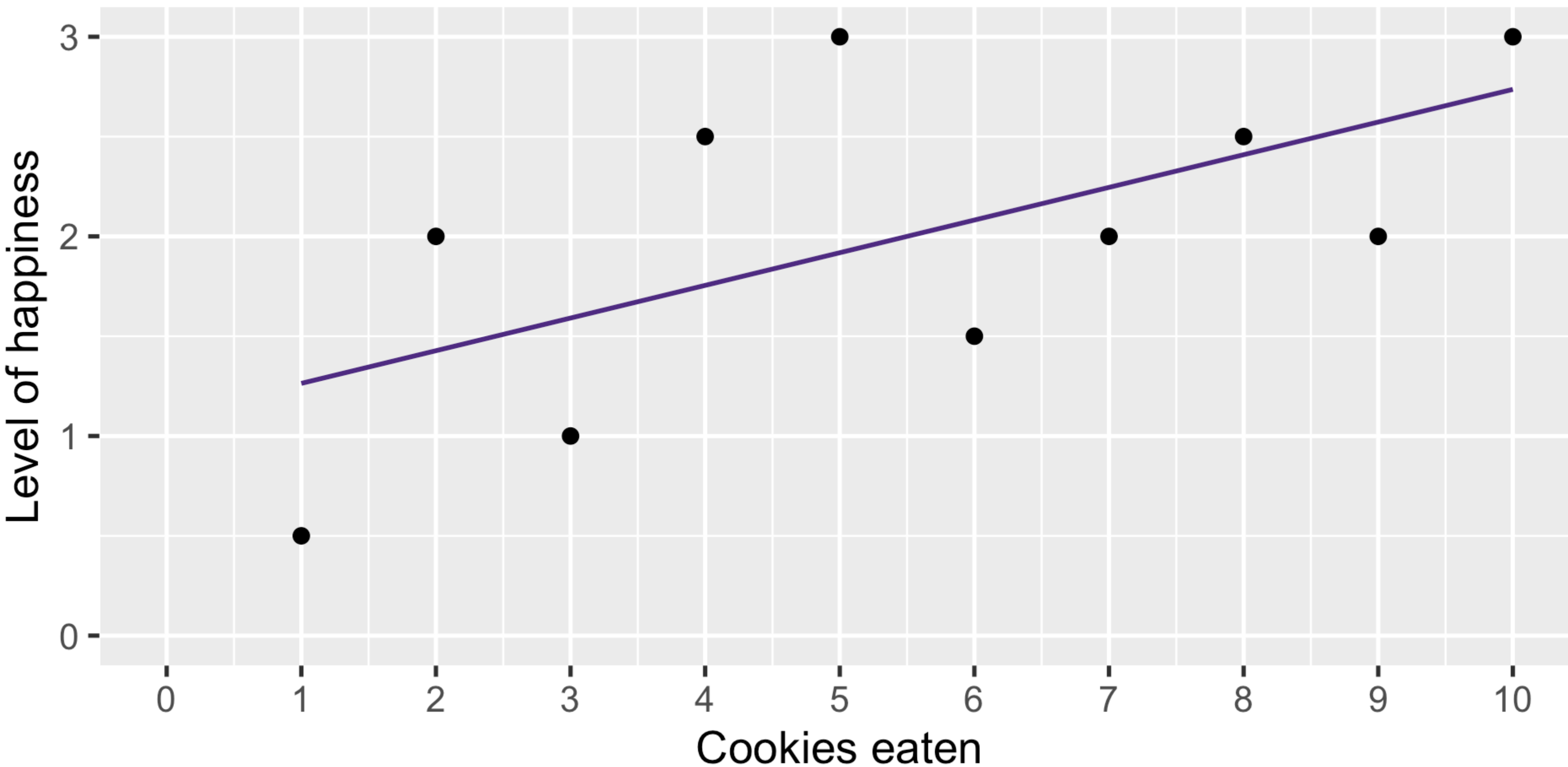
- Line #1
- Line #2
- Line #3



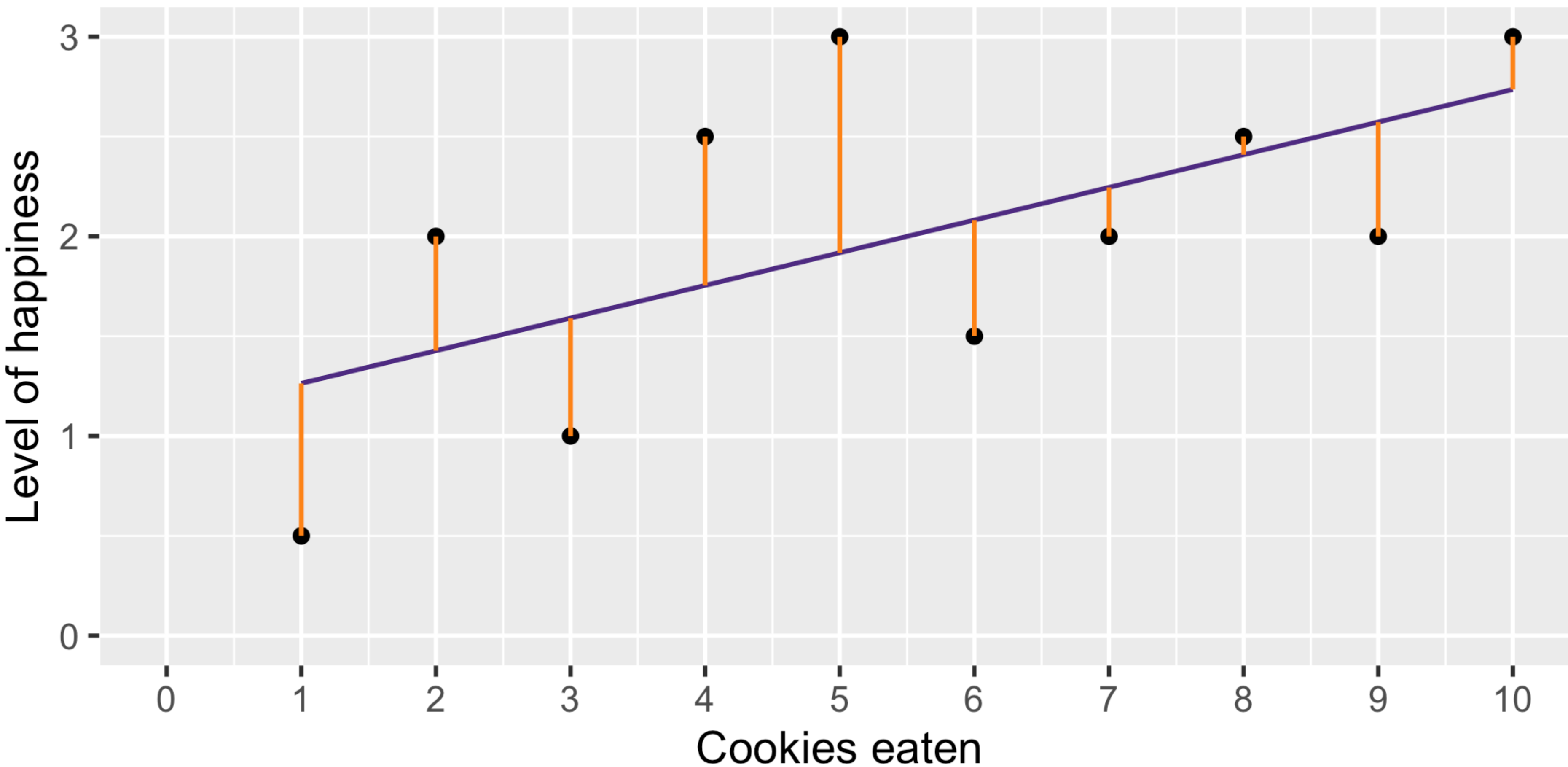
More formally

- The **best linear predictor** is the line that minimizes the distance of each observation to the line
- That distance is known as **residual** or **error**

Visualizing residuals



Visualizing residuals



More formally

- The **best linear predictor** is the line that minimizes the distance of each observation to to the line
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$$e_i = (y_i - \widehat{y}_i)$$

More formally

- The **best linear predictor** is the line that minimizes the distance of each observation to to the line
- That distance is know as a **residual** or **error**

$$e_i = (y_i - (b_0 + b_1 x_{1i}))$$

Minimizing residuals

We want to find a **vector of coefficients** $(\widehat{\beta}_0, \widehat{\beta}_1)$ that minimizes the **sum of squared residuals**

$$SSR = \sum_{i=1}^{n} e_i^2$$

We could try many lines until we find the the smallest SSR

Or use a method called **Ordinary Least Squares** (OLS)

OLS regression

Estimand

$$\alpha = E[Y] - \frac{\mathrm{Cov}[X,Y]}{V[X]} E[X] \qquad \beta = \frac{\mathrm{Cov}[X,Y]}{V[X]}$$

Estimator

$$\widehat{\alpha} = \overline{Y} - \frac{\overline{XY} - \overline{X} \cdot \overline{Y}}{\overline{X^2} - \overline{X}^2} \overline{X} \qquad \widehat{\beta} = \frac{\overline{XY} - \overline{X} \cdot \overline{Y}}{\overline{X^2} - \overline{X}^2}$$

$\widehat{\alpha}$: intercept; $\widehat{\beta}$: slope

Back to cookies

$$\widehat{y} = \widehat{\beta}_0 + \widehat{\beta}_1 x_1$$

Back to cookies

$$\widehat{\text{happiness}} = \beta_0 + \beta_1 \text{cookies}$$

Back to cookies

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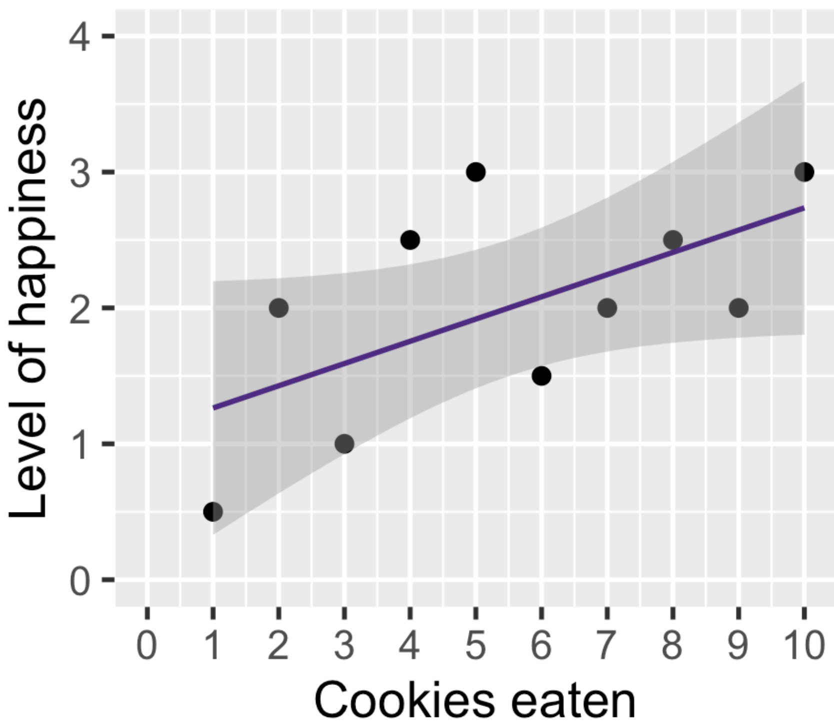
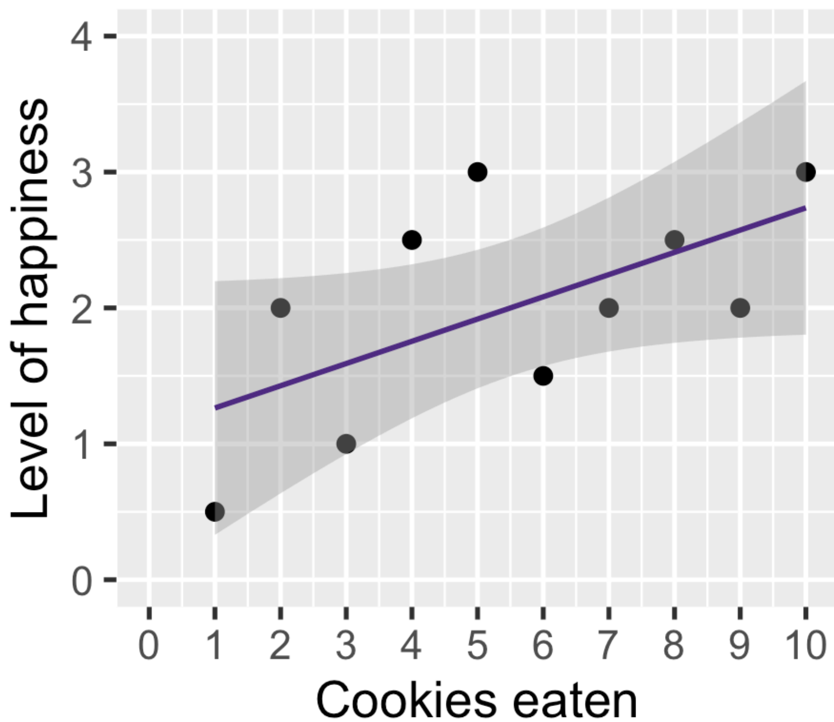


Back to cookies

$$\widehat{\text{happiness}} = \beta_0 + \beta_1 \text{cookies}$$

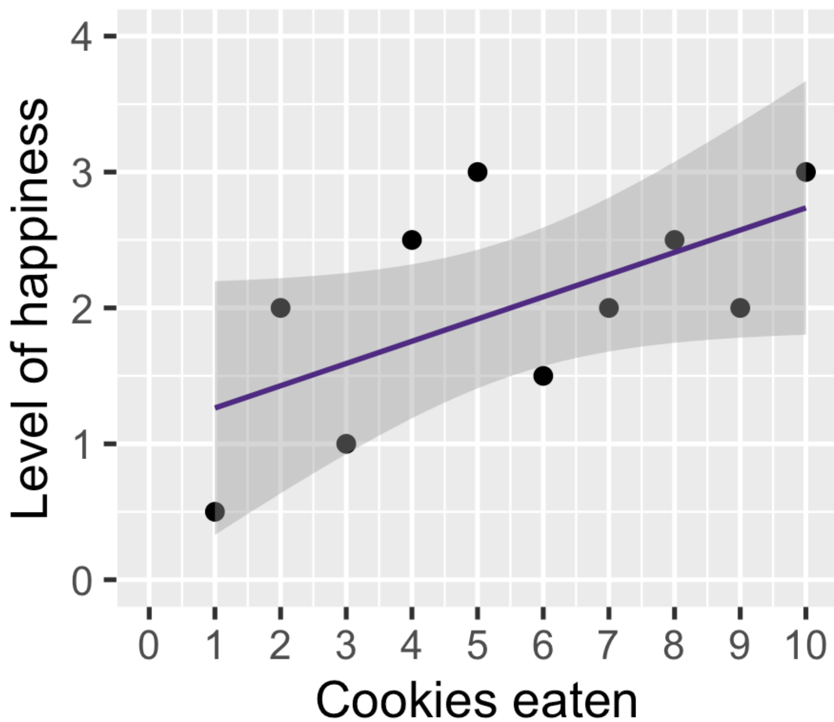


	happinness
(Intercept)	1.100*
	(0.470)
cookies	0.164+
	(0.076)
Num.Obs.	10
R2	0.368

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

Back to cookies

$$\widehat{\text{happiness}} = \beta_0 + \beta_1 \text{cookies}$$



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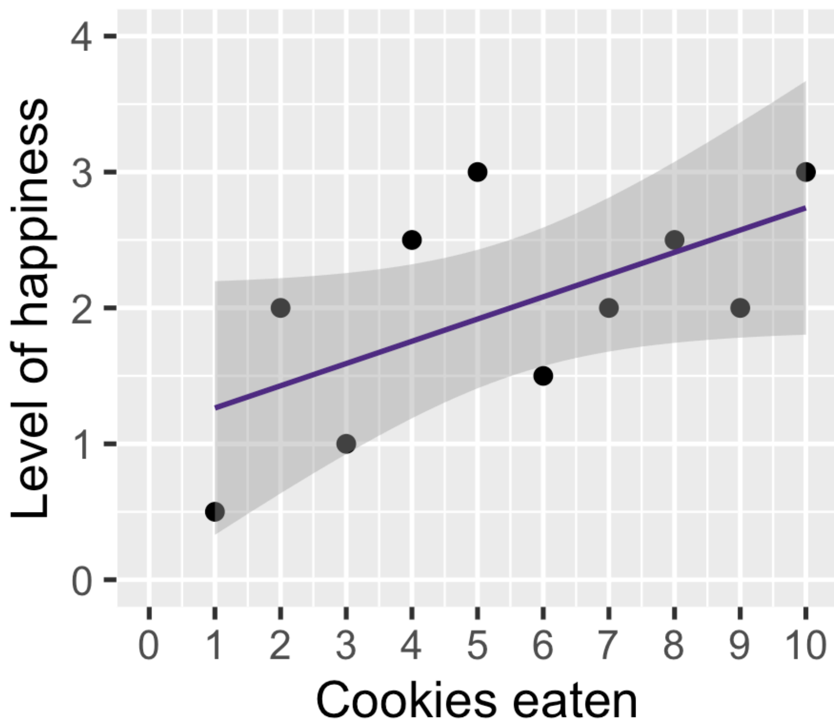
Back to cookies

$$\widehat{\text{happiness}} = 1.10 + 0.16 \cdot \text{cookies}$$



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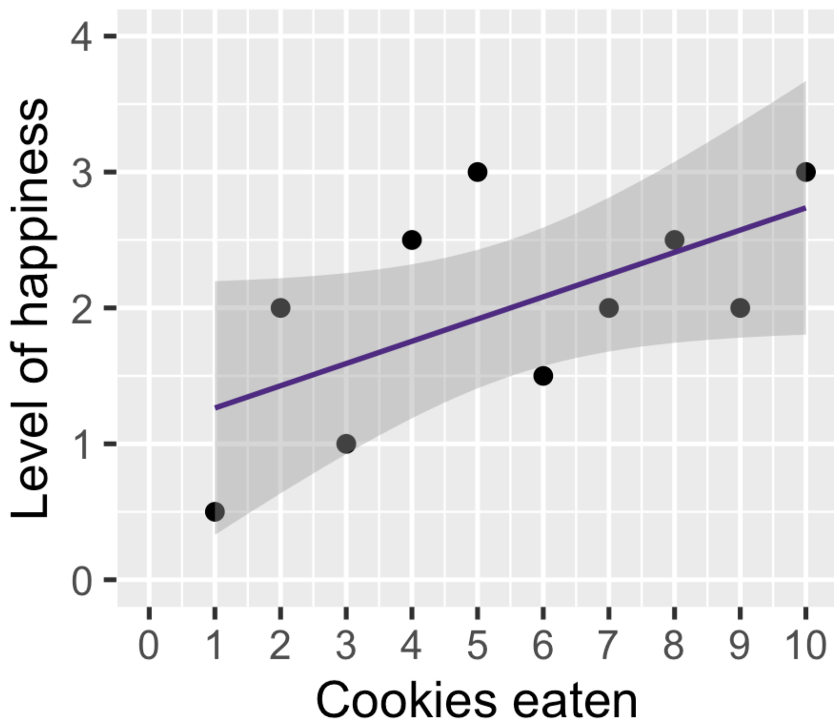

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On average

Back to cookies

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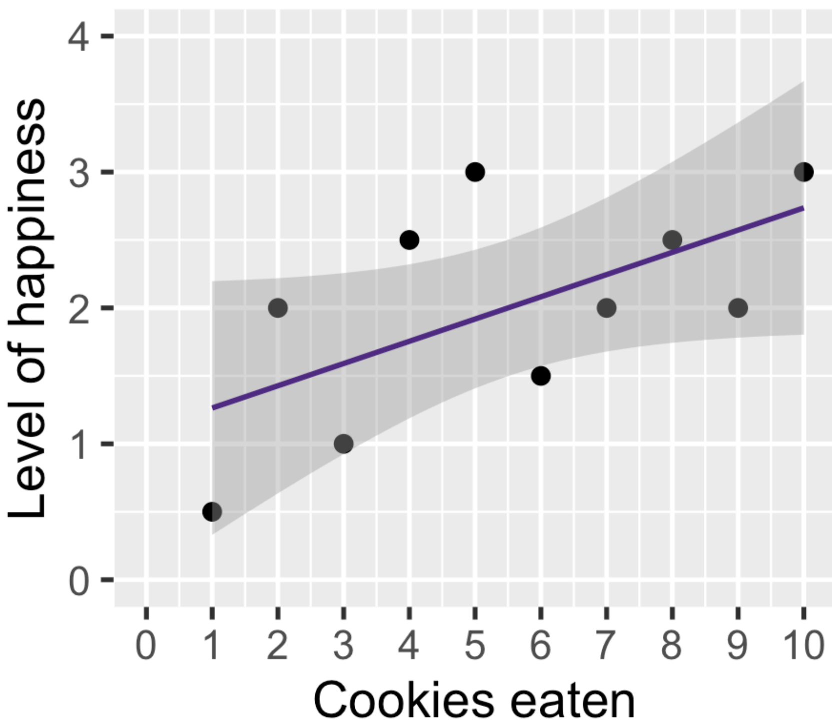

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(Intercept)	1.100*
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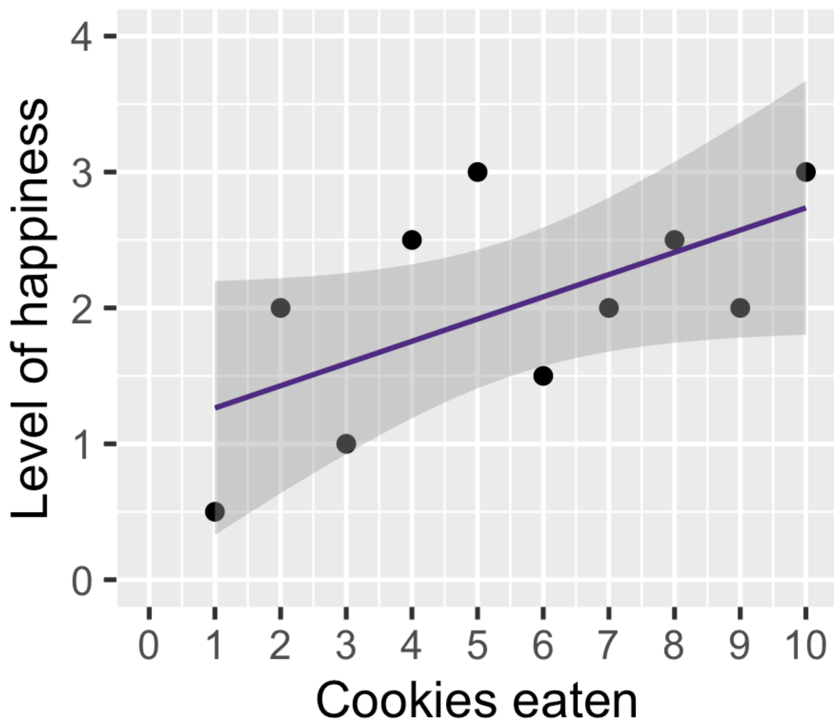
On average, one additional cookie

Back to cookies

$$\widehat{\text{happiness}} = 1.10 + 0.16 \cdot \text{cookies}$$



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(Intercept)	1.100*
	(0.470)
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Num.Obs.	10
R2	0.368
* p < 0.05	

On average, one additional cookie increases happiness by 0.16 points

Regression and correlation

Informally, we use regression coefficients (slopes) to determine whether two variables are **correlated**

Technically, they are related but on a different scale

Regression coefficient: $\beta = \frac{\text{Cov}[X,Y]}{V[X]}$

Correlation: $\rho = \frac{\text{Cov}[X,Y]}{SD[X]SD[Y]}$

By the way, $\text{Cov}[X, Y] = E[(X - E[X])(Y - E[Y])]$

Regression and correlation

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Regression coefficient: $\beta = \frac{\text{Cov}[X,Y]}{V[X]} \Rightarrow$ in units of Y (happiness)

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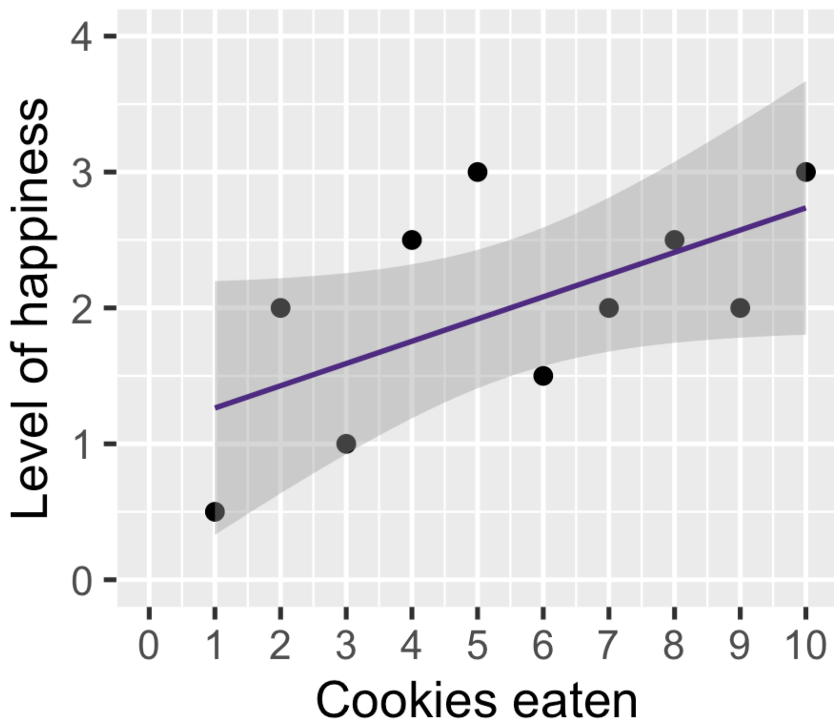
Correlation: $\rho = \frac{\text{Cov}[X,Y]}{SD[X]SD[Y]} \Rightarrow$ [-1, 1] scale

By the way, $\text{Cov}[X, Y] = E[(X - E[X])(Y - E[Y])]$

With cookies again

$$\widehat{\text{happiness}} = 1.10 + 0.16 \cdot \text{cookies}$$



- **On average**, one additional cookie increases happiness by 0.16 points
- Corresponds to a correlation of 0.61

Helpful for comparison

Is 0.16 happiness points per cookie a lot?

We cannot tell without a point of reference

But correlation is a reference on its own:

Absolute magnitude	Effect
0.1	Small
0.3	Moderate
0.5	Large

Summary

- Lines are a convenient way to summarize bivariate relationships
- We can treat line-fitting as an estimation problem
- OLS regression has good statistical properties (minimizes SSR)
- Regression and correlation are related but different
- Many different kinds of regression models!

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POLI SCI 210

Introduction to Empirical Methods in Political Science

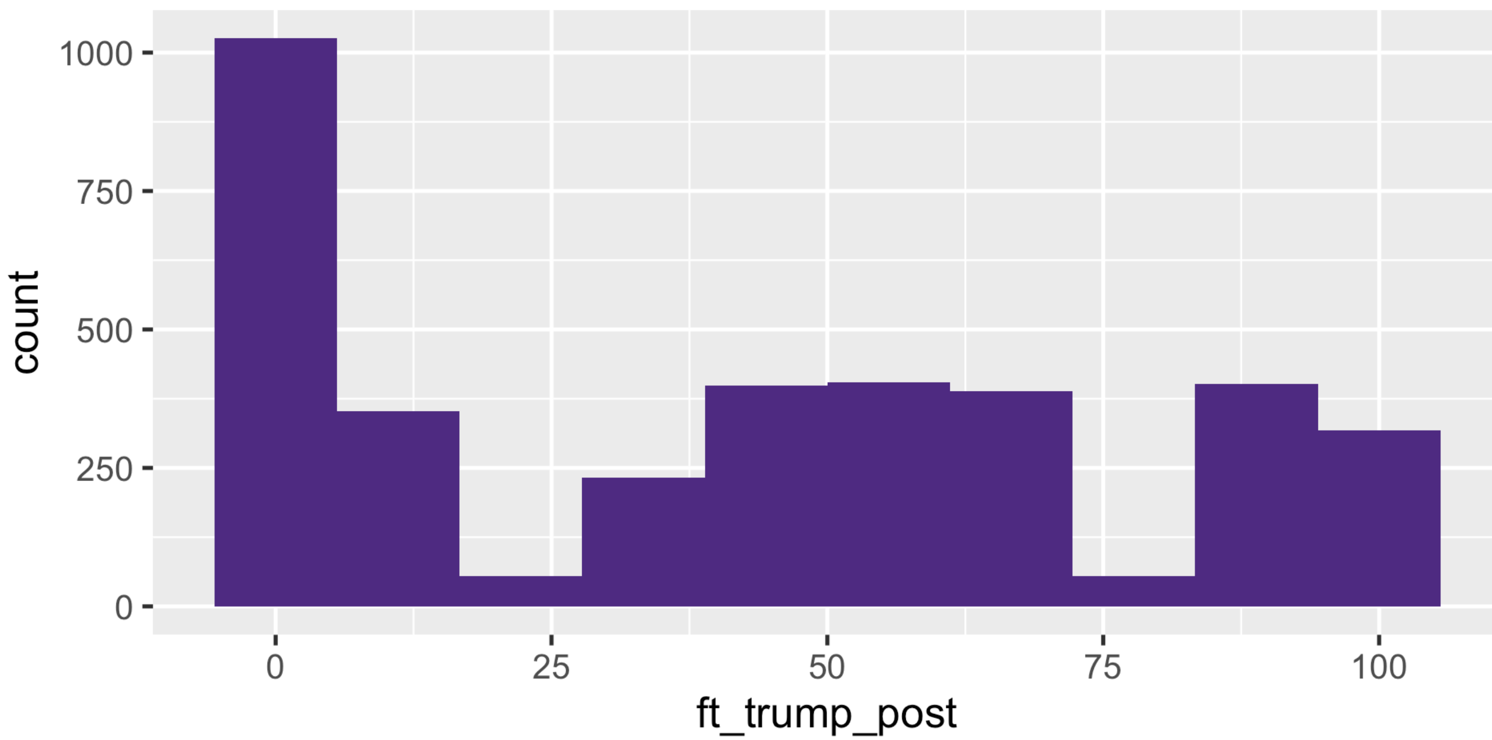
Last time

- **Bivariate regression** as a method to understand relationship between X and Y
- **Today:** More variables! (and why you would want that)
- **Multivariate regression**

Running example: ANES 2016 data

Outcome variable

- `ft_trump_post`: Post-election feeling thermometer toward Trump



Running example: ANES 2016 data

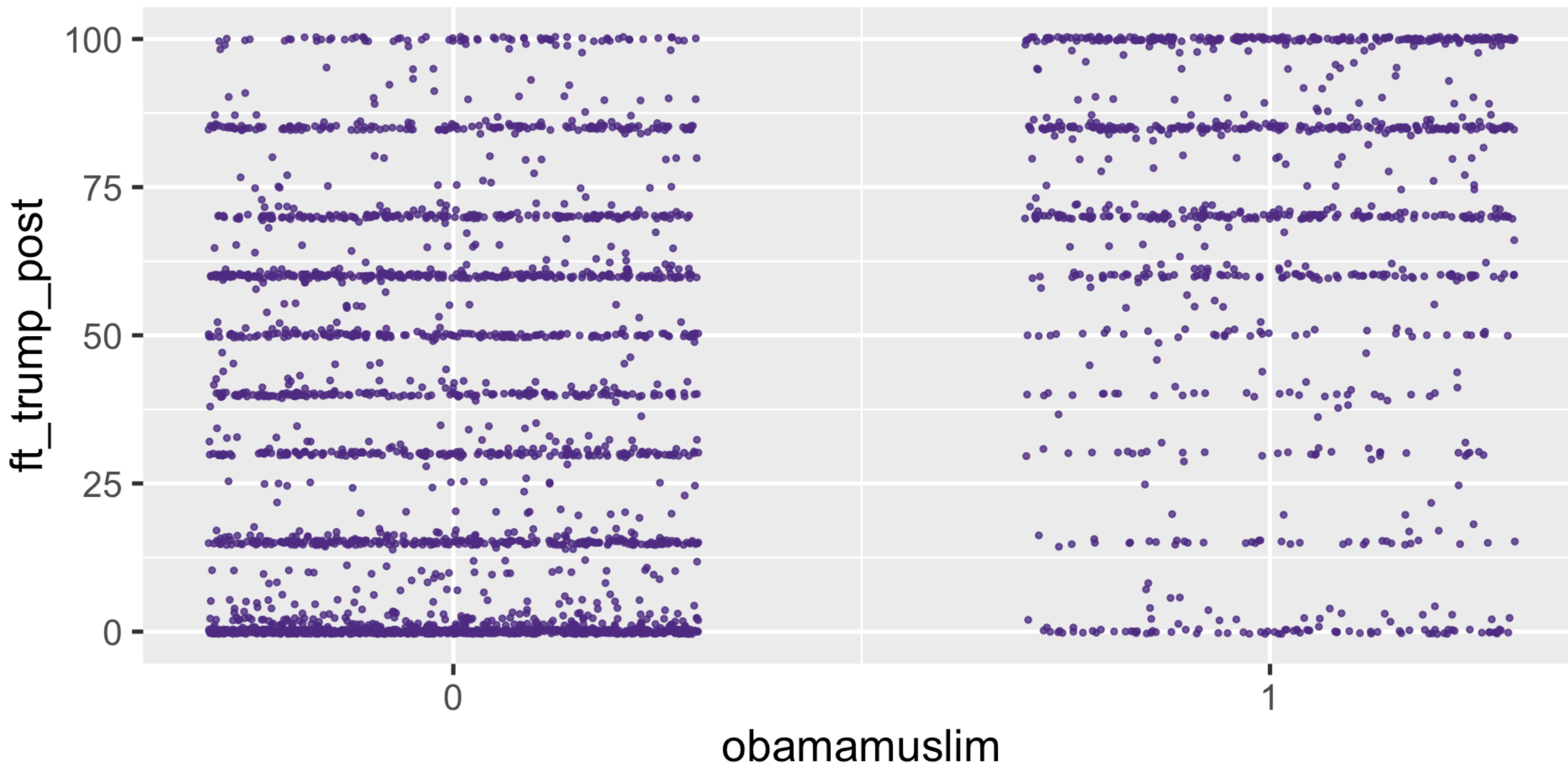
Explanatory variables

- `women_at_home`: Believe women should stay home
- `obamamuslim`: Believe Obama is a Muslim
- `age`: Age in years
- `age0`: Age in years (starting with 18 = 0)
- `educ_hs`: Any kind of post-secondary education
- `republican`: Identifies with Republican party (including leaners)

Running example: ANES 2016 data

```
# A tibble: 4,270 × 7
   ft_trump_post women_at_home obamamuslim   age  age0 educ_hs republican
           <dbl>         <dbl>       <dbl> <dbl> <dbl>   <dbl>      <dbl>
 1            85             0           0    29    11       1          1
 2            60             0           1    26     8       1          1
 3            70             1           1    23     5       1          0
 4            60             1           0    58    40       1          1
 5            15             0           1    38    20       1          0
 6            65             0           0    60    42       1          1
 7            50             0           0    58    40       0          0
 8            85             1           1    56    38       1          0
 9            70             0           0    45    27       1          1
10            60             0           0    30    12       1          0
# ℹ 4,260 more rows
```

Main relationship



Regression as conditional means

$$\widehat{\texttt{ft\_trump\_post}} = \beta_0 + \beta_1 \cdot \texttt{obamamuslim}$$

term	estimate	std.error	p.value
(Intercept)	32.17	0.61	0
obamamuslim	35.04	1.15	0

Regression as conditional means

$$\widehat{\texttt{ft\_trump\_post}} = 32.17 + 35.04 \cdot \texttt{obamamuslim}$$

term	estimate	std.error	p.value
(Intercept)	32.17	0.61	0
obamamuslim	35.04	1.15	0

- What is the average feeling thermometer for someone who *does not* believe Obama is a Muslim?
- What is the average feeling thermometer for someone who *does* believe Obama is a Muslim?

What can we say with regression?

- **Level 1:** Description of *conditional means*
- **Level 2:** Statistical inference (needs CIs or p-values)
- **Level 3:** Causal inference (needs **assumptions**)

What do we need to assume to make causal claims?

Berk, Richard. 2010. "What You Can and Can't Properly do with Regression." *Journal of Quantitative Criminology* 26: 481-487

Strategy 1: Random assignment

If treatment D is *randomly assigned*

- Potential outcomes are *independent* from treatment: $(Y(0), Y(1)) \perp\!\!\!\perp D$
- ATE $E[\tau_i]$ is **point-identified**
- Estimate with *difference in means* between treatment and control
- Bivariate regression yields the same result

What if random assignment is not possible?

Return to ANES 2016

We **found** that those who believe Obama is a Muslim were, on average, 35 points more favorable toward Trump

We want to **claim** this is because:

$$\text{Belief in conspiracies} \Rightarrow \text{Support for Trump}$$

What prevents us from making such a claim?

- Reverse causation
- Omitted variable bias
- Selection bias

Strategy 2: Ignorability

We want to be able to **ignore** the role of *potential confounders*

We usually do this by presenting a **controlled** comparison

So we can say that our *explanatory variable* is distributed in a way that is **conditionally independent**

Conditional independence: $(Y(0), Y(1)) \perp\!\!\!\perp D | \mathbf{X}$

We now distinguish between:

- *Explanatory* variable (D)
- *Control* variables or *covariates* (X)

There is strong and weak ignorability. For our purposes they are the same

Another way to think about it

There *is* a causal effect to be found in **observational data**

But without random assignment, the effect is *contaminated* by potential confounders

We want to **adjust** or **control** for these variables

Multivariate regression

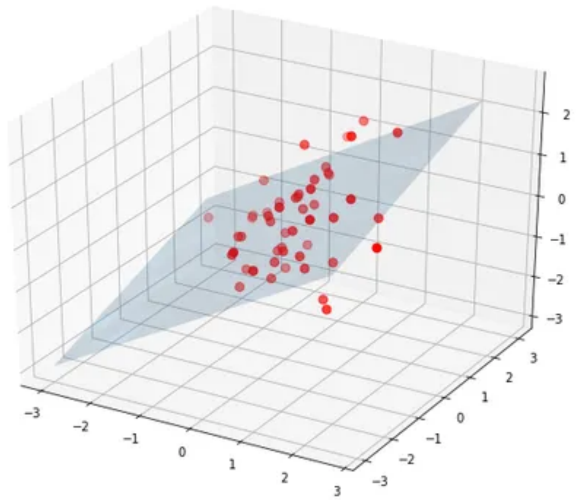
The linear model setup is flexible

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \ldots + \hat{\beta}_K x_K$$

You can technically put *whatever* you want in a regression as long as $\text{observations} > \text{variables}$

But for *statistical* or *causal inference*, anything with more than 2-3 control variables doesn't make much sense

Increasing dimensions



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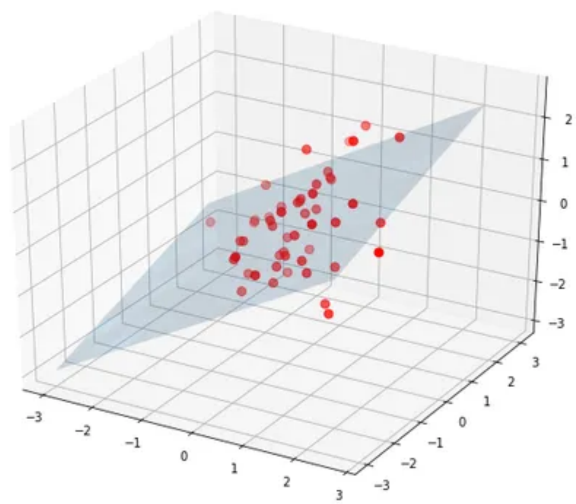


More dimensions → more likely to see:

- **Extrapolation:** Fitting line beyond actual range
- **Interpolation:** Gaps within actual range

Illusion of learning from **empty space**!

Practice

We argue conspiracy beliefs $\Rightarrow$ support Trump

Some alternative explanations:

- Differences in education
- Differences in age
- Partisan motivated reasoning

Models

Estimate the following models:

$$\widehat{\texttt{ft\_trump\_post}} = \beta_0 + \beta_1 \texttt{obamamuslim} \text{ (Baseline)}$$

$$\widehat{\texttt{ft\_trump\_post}} = \beta_0 + \beta_1 \texttt{obamamuslim} + \beta_2 \texttt{educ\_hs}$$

$$\widehat{\texttt{ft\_trump\_post}} = \beta_0 + \beta_1 \texttt{obamamuslim} + \beta_2 \texttt{age}$$

$$\widehat{\texttt{ft\_trump\_post}} = \beta_0 + \beta_1 \texttt{obamamuslim} + \beta_2 \texttt{republican}$$

$$\widehat{\texttt{ft\_trump\_post}} = \beta_0 + \beta_1 \texttt{obamamuslim} + \beta_2 \texttt{educ\_hs} + \beta_3 \texttt{age} + \beta_4 \texttt{republican}$$

Results

	(1)	(2)	(3)	(4)	(5)
(Intercept)					
obamamuslim					
educ_hs					
age					
republican					
Num.Obs.					
R2					
* p < 0.05					

Results

	(1)	(2)	(3)	(4)	(5)
(Intercept)	32.168*				
	(0.611)				
obamamuslim	35.037*				
	(1.147)				
educ_hs					
age					
republican					
Num.Obs.	3632				
R2	0.204				

* p < 0.05

Results

	(1)	(2)	(3)	(4)	(5)
(Intercept)	32.168*	32.763*			
	(0.611)	(2.104)			
obamamuslim	35.037*	34.852*			
	(1.147)	(1.154)			
educ_hs		-0.557			
		(2.135)			
age					
republican					
Num.Obs.	3632	3601			
R2	0.204	0.203			

* p < 0.05

Results

	(1)	(2)	(3)	(4)	(5)
(Intercept)	32.168*	32.763*	23.090*		
	(0.611)	(2.104)	(1.574)		
obamamuslim	35.037*	34.852*	34.715*		
	(1.147)	(1.154)	(1.162)		
educ_hs		-0.557			
		(2.135)			
age			0.185*		
			(0.030)		
republican					
Num.Obs.	3632	3601	3536		
R2	0.204	0.203	0.214		

* p < 0.05

Results

	(1)	(2)	(3)	(4)	(5)
(Intercept)	32.168*	32.763*	23.090*	20.367*	
	(0.611)	(2.104)	(1.574)	(0.581)	
obamamuslim	35.037*	34.852*	34.715*	22.742*	
	(1.147)	(1.154)	(1.162)	(0.996)	
educ_hs		-0.557			
		(2.135)			
age			0.185*		
			(0.030)		
republican				37.533*	
				(0.913)	
Num.Obs.	3632	3601	3536	3616	
R2	0.204	0.203	0.214	0.459	

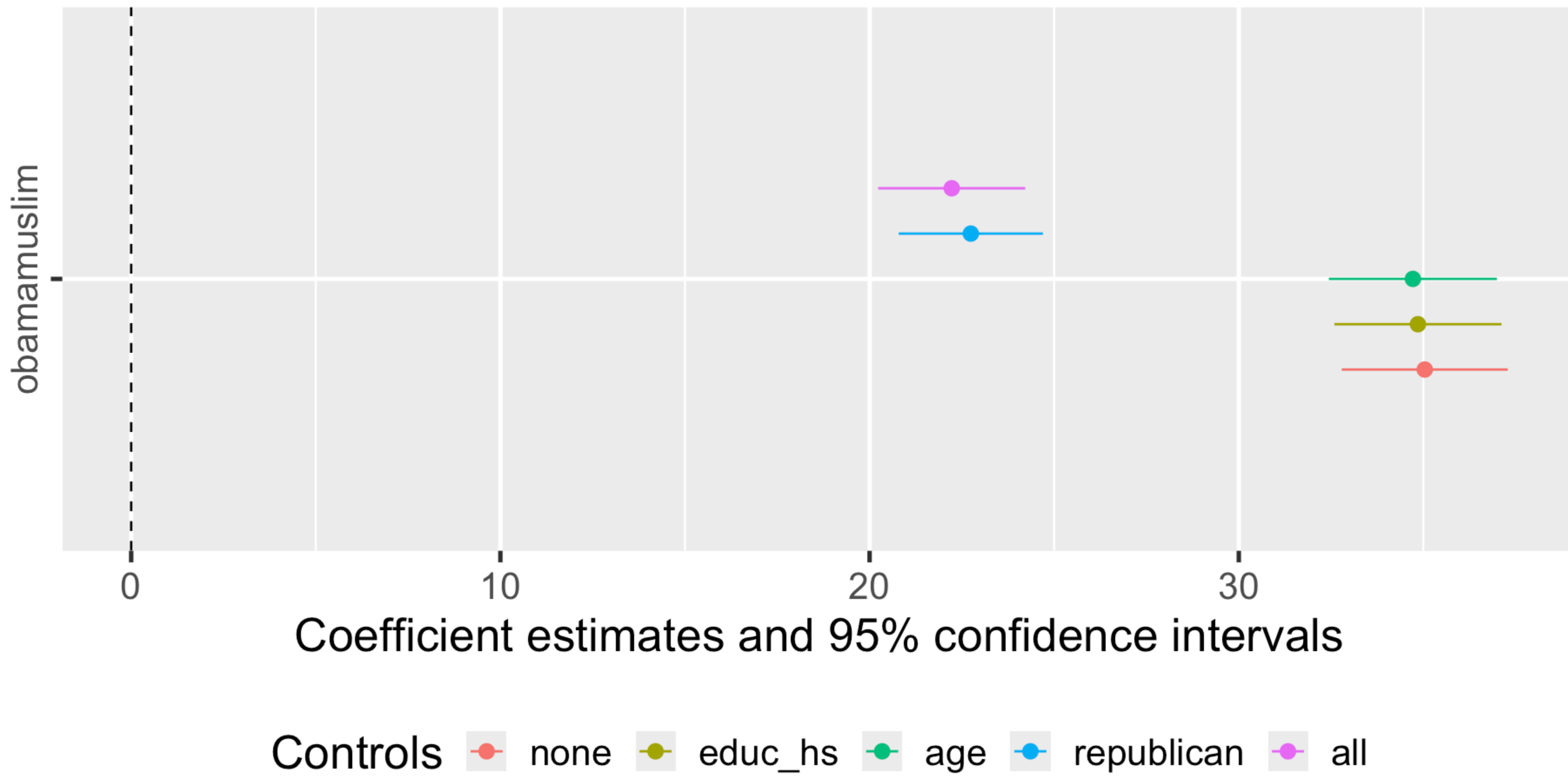
* p < 0.05

Results

	(1)	(2)	(3)	(4)	(5)
(Intercept)	32.168*	32.763*	23.090*	20.367*	21.338*
	(0.611)	(2.104)	(1.574)	(0.581)	(2.157)
obamamuslim	35.037*	34.852*	34.715*	22.742*	22.225*
	(1.147)	(1.154)	(1.162)	(0.996)	(1.016)
educ_hs		-0.557			-6.216*
		(2.135)			(1.788)
age			0.185*		0.101*
			(0.030)		(0.025)
republican				37.533*	37.539*
				(0.913)	(0.933)
Num.Obs.	3632	3601	3536	3616	3496
R2	0.204	0.203	0.214	0.459	0.462

* p < 0.05

Visualizing



Everything else constant

Plug-in coefficients in equations:

$$\widehat{\texttt{ft\_trump\_post}} = 32.17 + 35.04 \cdot \texttt{obamamuslim}$$

$$\widehat{\texttt{ft\_trump\_post}} = 32.76 + 34.85 \cdot \texttt{obamamuslim} - 0.56 \cdot \texttt{educ\_hs}$$

$$\widehat{\texttt{ft\_trump\_post}} = 23.09 + 34.72 \cdot \texttt{obamamuslim} + 0.19 \cdot \texttt{age}$$

$$\widehat{\texttt{ft\_trump\_post}} = 20.367 + 22.74 \cdot \texttt{obamamuslim} + 37.53 \cdot \texttt{republican}$$

$$\widehat{\texttt{ft\_trump\_post}} = 21.34 + 22.23 \cdot \texttt{obamamuslim} - 6.22 \cdot \texttt{educ\_hs} + 0.10 \cdot \texttt{age} + 37.54 \cdot \texttt{republican}$$

Coefficients now need to be interpreted as **marginal means** or **marginal slopes**

These only make sense if you think *at least one variable* is a **focal point**

Interactions

What if we believed the effect of `obamamuslim` varies depending on attitudes about gender roles?

Model:

$$\widehat{\texttt{ft\_trump\_post}} = \beta_0 + \beta_1 \texttt{obamamuslim} + \beta_2 \texttt{women\_at\_ho}$$

$$\beta_3 \texttt{obamamuslim} \times \texttt{women\_at\_home}$$

Interactions

Model:

$$\widehat{\texttt{ft\_trump\_post}} = \beta_0 + \beta_1 \texttt{obamamuslim} + \beta_2 \texttt{women\_at\_ho}$$
$$\beta_3 \texttt{obamamuslim} \times \texttt{women\_at\_home}$$

term	estimate	std.error	p.value
(Intercept)	27.74	0.73	0.00
obamamuslim	34.95	1.51	0.00
women_at_home	14.17	1.30	0.00
interaction	-4.74	2.31	0.04

Summary

- Regression is a way to estimate conditional means
- Multivariate regression needs “everything else constant” interpretation
- Coefficients are now **marginal means** or **marginal slopes**
- Only makes sense from a causal inference perspective