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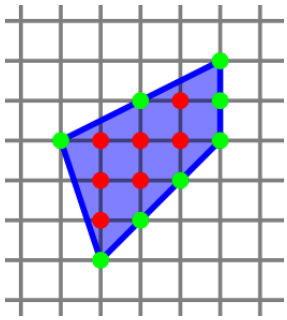
Pick's theorem

In geometry, **Pick's theorem** provides a formula for the area of a simple polygon with integer vertex coordinates, in terms of the number of integer points within it and on its boundary. The result was first described by Georg Alexander Pick in 1899.[1] It was popularized in English by Hugo Steinhaus in the 1950 edition of his book *Mathematical Snapshots*.[2][3] It has multiple proofs, and can be generalized to formulas for certain kinds of non-simple polygons.



$i = 7$, $b = 8$,
$A = i + \frac{b}{2} - 1 = 10$

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Formula

Suppose that a polygon has integer coordinates for all of its vertices. Let i be the number of integer points that are interior to the polygon, and let b be the number of integer points on its boundary (including vertices as well as points along the sides of the polygon). Then the area A of this polygon is:[4][5][6][7]

$$A = i + \frac{b}{2} - 1.$$

The example shown has $i = 7$ interior points and $b = 8$ boundary points, so its area is $A = 7 + \frac{8}{2} - 1 = 10$ square units.

Proofs

Via Euler's formula