Synthese (2013) 190:3307–3324
DOI 10.1007/s11229-012-0165-0

Verisimilitude and belief change for nomic conjunctive theories

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Received: 8 May 2011 / Accepted: 30 July 2012 / Published online: 8 September 2012

Abstract In this paper, we address the problem of truth approximation through theory change, asking whether revising our theories by newly acquired data leads us closer to the truth about a given domain. More particularly, we focus on "nomic conjunctive theories", i.e., theories expressed as conjunctions of logically independent statements concerning the physical or, more generally, nomic possibilities and impossibilities of the domain under inquiry. We define both a comparative and a quantitative notion of the verisimilitude of such theories, and identify suitable conditions concerning the (partial) correctness of acquired data, under which revising our theories by data leads us closer to "the nomic truth", construed as the target of scientific inquiry. We conclude by indicating some further developments, generalizations, and open issues arising from our results.

Keywords Nomic verisimilitude · Truthlikeness · Truth approximation · Belief change · Belief revision · AGM

The intuitive idea underlying the notion of verisimilitude can be expressed as follows: a theory is highly verisimilar, or "close to the whole truth", if it says many things about the domain under investigation and many of those things are (almost) exactly true. The first formal definition of verisimilitude was proposed by Popper (1963).

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According to Popper, a theory is more verisimilar than another if the former implies more true sentences and fewer false sentences than the latter. Notwithstanding its intuitive appeal, Popper's definition was shown to be untenable by Tichý (1974) and Miller (1974), who independently proved that, according to this definition, a false theory can never be closer to the truth than another (true or false) theory. The Tichý-Miller theorem opened the way to the post-Popperian approaches to verisimilitude, which have emerged since 1975. Such approaches escape the strictures pointed out by Tichý and Miller, allowing for a comparison of at least some false theories with regard to their closeness to the truth.[1]

Some of the earliest attempts to explicate the concept of verisimilitude were developed with respect to theories expressed in propositional languages (see, for instance, Tichý 1974). However, propositional languages are usually considered exceedingly simple and at most useful for providing toy examples of scientific theories. Hence, post-Popperian approaches to verisimilitude typically deal with theories expressed in predicate languages. Nonetheless, a revival of interest for propositional languages and theories has recently involved, somewhat surprisingly, a number of verisimilitude theorists, among which Zwart (2001), Niiniluoto (2003), Burger and Heidema (2005), Gemes (2007), and Schurz and Weingartner (2010). In the following, we present a notion of verisimilitude for propositional languages which can be fruitfully generalized to more complex languages.

In Sect. 1, we introduce a comparative definition of verisimilitude applicable to so called *conjunctive theories*, i.e., theories expressed as conjunctions of logically independent statements concerning the domain under inquiry. The notion of conjunctive theory, defined within propositional languages, can be extended to richer kinds of languages, such as predicate and "nomic" languages. To this aim, in Sect. 2 we consider Kuipers' notions of nomic theory and *nomic verisimilitude* (Kuipers 2000), and show how our comparative notion of verisimilitude can be applied to nomic theories.[2] A basic problem in the analysis of rational theory change is *theory revision*, i.e., the problem of revising our theories in response to newly acquired data. In Sect. 3, we deal with the problem of *nomic* theory change (Kuipers 2011a,b) and prove that, under suitably defined conditions, changing nomic theories in response to data leads those theories closer to the truth. These results are strengthened in Sect. 4, where we present the so called *contrast measures* of verisimilitude, recently proposed by Cevolani et al. (2010), and apply them to the analysis of nomic truth approximation through theory change. Finally, in Sect. 5, we briefly survey some possible extensions of our approach, discuss its relations with other accounts of (nomic) verisimilitude, and suggest some directions for further research.

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[1] An excellent survey of the main post-Popperian accounts of verisimilitude can be found in Niiniluoto (1998). See also Zwart (2001).

[2] The debate on nomic verisimilitude was triggered by Leonard J. Cohen's suggestion (Cohen 1980) that the main goal of scientific inquiry is *legisimilitude*, i.e., closeness to truth about natural necessity (see also Cohen 1987). Earlier attempts to offer an adequate formal notion of nomic verisimilitude, or approximation to the physically necessary truth, include Oddie (1982; 1986, Sect. 5.4), Kuipers (1982; 1987), and Niiniluoto (1983; 1987, Chap. 11).

1 The basic feature approach to verisimilitude

In a number of recent papers, Cevolani et al. have developed a *basic feature approach*—for short, *BF-approach*—to verisimilitude.[3] Suppose that the basic features of the domain under inquiry $\mathcal{U}$ ("the world") are described by a language $\mathcal{L}$. Then, "the whole truth"—or simply "the truth"—about $\mathcal{U}$ in $\mathcal{L}$ can be construed as the most complete true description (in $\mathcal{L}$) of the basic features of $\mathcal{U}$. Given a theory T in $\mathcal{L}$, the basic content of T can be seen as the information conveyed by T about the basic features of $\mathcal{U}$. Then, according to the BF-approach, the verisimilitude of T is interpreted in terms of the balance of true and false information conveyed by T about the basic features of $\mathcal{U}$.

The BF-approach may be illustrated assuming that the world $\mathcal{U}$ is described by a propositional language $\mathcal{L}_n$ with n atomic sentences $p_1, \ldots, p_n$. The basic features of $\mathcal{U}$ are then described by the so-called literals of $\mathcal{L}_n$. A literal is either an atomic sentence p_i or the negation $\neg p_i$ of an atomic sentence; thus, for any atomic sentence p_i, there is a pair of literals $(p_i, \neg p_i)$ whose elements are said to be the dual of each other. Of course, the set $\mathcal{B} = \{p_1, \neg p_1, \ldots, p_n, \neg p_n\}$ of the literals of $\mathcal{L}_n$ contains $2n$ members. In the following, a literal of $\mathcal{L}_n$ will be denoted by "$\pm p_i$", where $\pm$ is either empty or "$\neg$"; p_i is called the *positive* literal with p_i and $\neg p_i$ is called the *negative* literal with p_i. A constituent C of $\mathcal{L}_n$ is defined as a conjunction of n literals, one for each atomic sentence.[4] A constituent will thus have the following form:

$$\pm p_1 \wedge \cdots \wedge \pm p_n \tag{1}$$

One can check that there are exactly $q = 2^n$ constituents and that only one is *true*. The true constituent, denoted by "$C_\star$", can be identified with "the (whole) truth" in $\mathcal{L}_n$. A *conjunctive theory* T of $\mathcal{L}_n$—"c-theory", for short—is a conjunction of k literals concerning k different atomic sentences. A c-theory will thus have the following form:

$$\pm p_{i_1} \wedge \cdots \wedge \pm p_{i_k} \tag{2}$$

where $0 \leq k \leq n$. A tautological c-theory, denoted by "$\top$", is a c-theory such that $k = 0$, whereas a c-theory with $k = n$ is a constituent.[5]

A literal $\pm p_i$ occurring as a conjunct of a c-theory T will be called a basic (b-)claim of T.[6] The set T^b of all the b-claims of a c-theory T will be referred to as the basic (b-)content of T. The set formed by the duals of the b-claims of T will be denoted by "T^d"; of course, $|T^b| = |T^d|$. Finally, the set of the atomic sentences p_i of $\mathcal{L}_n$

[3] For early motivation see Festa (2007) and, for some applications, Cevolani et al. (2010) and Cevolani et al. (2011). It is now clear that the key ideas underlying the BF-approach have been anticipated by Kuipers (1982).

[4] See, for instance, Hintikka (1973, p. 152).

[5] A c-theory is called "descriptive statement" (or D-statements) by Kuipers (1982, pp. 348–349), and (propositional) "quasi-constituent" by Oddie (1986, p. 86). One can easily see that the class of c-theories expressible within $\mathcal{L}_n$ contains 3^n members.

[6] This terminology is inspired by Carnap (1950, p. 67), who calls the literals of $\mathcal{L}_n$ "basic sentences".

such that neither p_i nor $\neg p_i$ is a b-claim of T will be denoted by "$T^?$". Note that $T^?$ corresponds to the set of the basic features of $\mathcal{U}$ about which T does not say anything, that is, remains silent. Indeed, the symbol "$T^?$" is motivated by the fact that $T^?$ can be seen as the "question mark area" of T.

Given a constituent C and a c-theory T, we say that a b-claim $\pm p_i$ of T is true in C just in case $\pm p_i \in C^b$; otherwise, it is false in C. Accordingly, T^b can be partitioned into two subsets with respect to C: 1) the set $t(T, C) \overset{\text{df}}{=} T^b \cap C^b$ of the b-claims of T which are true in C, and 2) the set $f(T, C) \overset{\text{df}}{=} T^b \setminus C^b$ of the b-claims of T which are false in C. We shall say that $t(T, C)$ is the true b-content of T with respect to C, while $f(T, C)$ is the false b-content of T with respect to C. Given a non-tautological c-theory T, we will say that T is (completely) true in C just in case $t(T, C) = T^b$, that T is false in C when T is not true in C and that T is completely false in C when $t(T, C) = \emptyset$.

The verisimilitude of T can be understood as the similarity, or closeness, of T to the true constituent $C_\star$. The key intuition underlying the BF-approach is that T is highly verisimilar if T tells many things about the basic features of $\mathcal{U}$, as described by $C_\star$, and many of those things are true. Calling each true b-claim of T a *match*, and each false b-claim a *mistake* of T, we may say that T is highly verisimilar if T makes many matches and few mistakes about $C_\star$, i.e., if T^b contains many true b-claims and few false b-claims. This intuition suggests the following comparative notion of verisimilitude for c-theories:[7]

Definition 1 Given two c-theories T_1 and T_2, T_2 is *more verisimilar* than T_1—in symbols, $T_2 >_{vs} T_1$—iff one of the following conditions holds:

$$\begin{aligned} &t(T_1, C_\star) \subset t(T_2, C_\star) \quad \text{and} \quad f(T_2, C_\star) \subseteq f(T_1, C_\star) \text{ or} \\ &t(T_1, C_\star) \subseteq t(T_2, C_\star) \quad \text{and} \quad f(T_2, C_\star) \subset f(T_1, C_\star) \end{aligned}$$

In words, T_2 is more verisimilar than T_1 either if T_2 makes more matches, but no more mistakes, than T_1 or if T_2 makes fewer mistakes than T_1, and at least the same matches.[8]

It is worth noting that, besides proper mistakes, a c-theory T will typically also commit omissions or "errors of ignorance", corresponding to the members of $T^?$.[9] In

[7] Definition 1 is equivalent to Kuipers1982's definition of "descriptive verisimilitude" (cf. Kuipers 1982, Definition 2 on p. 355).

[8] One may note that Definition 1 is structurally identical to Popper's comparative definition of verisimilitude (Popper 1963, p. 233). The crucial difference is that, while Popper's definition is stated in terms of the "truth content" and the "falsity content" of logically closed theories, defined as the classes of their true and false logical consequences, respectively, our Definition 1 is stated in terms of the "true b-content" and "false b-content" of c-theories. Since c-theories are conjunctions of "elementary" (atomic and logically independent) truths and falsehoods, our proposal essentially amounts to restricting Popper's set-theoretical comparisons of truth and falsity contents to the sets of elementary truths and falsehoods. Moreover, it should be noted that Definition 1 agrees with all the intuitive judgements of comparative verisimilitude for propositional c-theories stated by Tichy (1974, p. 159; 1976, p. 26) so that it may be seen as a compact reformulation of Tichy's intuitions.

[9] The expression "error of ignorance" is borrowed from Niiniluoto (1987), p. 159. Following Kuipers (1999), such errors of ignorance may also be called the *lacunae* of T (cf. also Aliseda 2005).

contrast, a constituent C commits no errors of ignorance, since in this case $C^? = \emptyset$. Hence, one can prove that, given two constituents C_1 and C_2,

$$C_2 >_{vs} C_1 \text{ iff } t(C_1, C_\star) \subset t(C_2, C_\star). \tag{3}$$

Thus, as far as constituents are concerned, Definition 1 reduces to the claim that C_2 is closer to the truth than C_1 if and only if all matches of C_1 are also matches of C_2 (cf. also Kuipers 2000, pp. 150–151).

2 Comparative verisimilitude for nomic conjunctive theories

The intuitive idea underlying the notion of *nomic verisimilitude* introduced by Kuipers (2000) can be expressed as follows. Given a domain of inquiry $\mathcal{U}$, let $\mathcal{C} = \{c_1, \ldots, c_n\}$ be the set of all relevant *conceptual possibilities* which might occur within $\mathcal{U}$.[10] $\mathcal{C}$ may be construed as the conceptual frame of a given scientific inquiry, including the relevant kinds of objects, events or states of natural systems or artifacts under investigation. As an example, $\mathcal{C}$ might include four kinds of object: “black raven”, “black non-raven”, “non-black raven” and “non-black non-raven”. One may assume that there is a unique subset $P_\star$ of $\mathcal{C}$, including all the *nomic possibilities* of $\mathcal{U}$, i.e., all conceptual possibilities which are realizable within $\mathcal{U}$.[11] Here, “nomic possibilities” may assume different meanings, depending on the particular context: e.g., $P_\star$ may contain the physical, chemical, biological, psychological or socio-economical possibilities of $\mathcal{C}$. The set $I_\star \overset{\text{df}}{=} \mathcal{C} \setminus P_\star$ is then the set of the *nomic impossibilities* of $\mathcal{U}$, i.e., of those conceptual possibilities which are unrealizable in $\mathcal{U}$.[12] For example, $P_\star$ might contain the conceptual possibilities “black raven”, “black non-raven”, and “non-black non-raven”, whereas $I_\star$ might contain the conceptual possibility “non-black raven”. Since scientists usually aim at understanding and discovering the nomic features of the world, $P_\star$ can be construed as the target, or “the great unknown”, of scientific inquiry (Kuipers 2000, pp. 143 ff.). Accordingly, $P_\star$ may be construed as the (whole) nomic truth about $\mathcal{U}$ and the *nomic verisimilitude* of a nomic theory T may be defined as the similarity of T to the nomic truth $P_\star$. The problem of nomic verisimilitude thus amounts to the explication of the idea that a given theory T is closer to $P_\star$ than another.

An answer to this problem is provided, in a quite natural way, by the BF-approach to verisimilitude presented in Sect. 1. Here, the conceptual possibilities in $\mathcal{C}$ can be expressed within an appropriate *nomic propositional language*. The basic idea is that, since the conceptual possibilities in $\mathcal{U}$ are logically mutually independent, they can be adequately described by the atomic sentences of a suitably interpreted propositional language. More precisely, given a finite conceptual frame $\mathcal{C} = \{c_1, \ldots, c_n\}$, a nomic (propositional) language $\mathcal{L}_n$ is a propositional language where each atom p_i says that the corresponding conceptual possibility c_i is a nomic possibility of $\mathcal{U}$. Consequently, a

[10] For the sake of simplicity, in the following we shall assume that $\mathcal{C}$ is finite.

[11] This assumption is called “the nomic postulate” by Kuipers (2000), pp. 147 ff.

[12] For this reason, the members of $I_\star$ may be called the “(merely) virtual possibilities” of $\mathcal{C}$ (Kuipers 2000, p. 147).

positive literal p_i of $\mathcal{L}_n$ claims that the conceptual possibility c_i is a nomic possibility of $\mathcal{C}$ (i.e., that $c_i \in P_\star$), whereas a negative literal $\neg p_i$ claims that c_i is a nomic impossibility of $\mathcal{C}$ (i.e., that $c_i \notin P_\star$ or, equivalently, $c_i \in I_\star$). A nomic constituent of $\mathcal{L}_n$ tells, for each conceptual possibility c_i of $\mathcal{C}$, whether c_i is a nomic possibility or not, i.e., whether p_i is true or false. Hence, the true nomic constituent $C_\star$ of $\mathcal{L}_n$ can be taken as representing the truth about $\mathcal{U}$, since it specifies all the nomic possibilities and impossibilities of $\mathcal{C}$. Indeed, $P_\star = \{c_i : p_i \in C_\star^b\}$ is the set of nomic possibilities, and $I_\star = \{c_i : \neg p_i \in C_\star^b\}$ the set of the nomic impossibilities.

Whereas a nomic constituent provides a complete list of the nomic possibilities and impossibilities of $\mathcal{C}$, a c-theory T of $\mathcal{L}_n$—i.e., a *nomic c-theory*—typically gives an incomplete list of such possibilities and impossibilities. A positive b-claim p_i of a nomic c-theory T may be called a *possibility claim* of T, since it says that the conceptual possibility c_i is a nomic possibility; likewise, a negative b-claim $\neg p_i$ may be called an *impossibility claim* of T, since it says that the conceptual possibility c_i is a nomic impossibility. Moreover, it should be noted that the members of $T^?$ correspond to conceptual possibilities about which T does not make any claim.

A comparative definition of what it means for a nomic c-theory T_2 to be closer to the nomic truth $P_\star$ than a nomic c-theory T_1 is immediately obtained by applying Definition 1 to propositional nomic languages. However, the notion of matches and mistakes is now interpreted in nomic terms. This implies that there are two kinds of matches, and two kinds of mistakes, possibly made by a nomic c-theory T. A match of the first kind is a true possibility claim of T, corresponding to a nomic possibility correctly identified by T. Likewise, a match of the second kind is a true impossibility claim of T, corresponding to a nomic impossibility correctly excluded by T. Conversely, a mistake of the first kind is a false possibility claim of T, corresponding to a nomic impossibility erroneously admitted by T as a nomic possibility; while a mistake of the second kind is a false impossibility claim of T, corresponding to a nomic possibility erroneously excluded by T as a nomic impossibility. According to Definition 1, T_2 is more verisimilar than T_1 if, for both kinds of matches and mistakes, either T_2 makes more matches, but no more mistakes, than T_1 or T_2 makes fewer mistakes than T_1, and at least the same matches.

3 Changing nomic theories in response to data

A basic problem in the analysis of rational theory change is given by theory revision, i.e., by the revision of our theories in response to new evidence. This can be seen as a particular case of the general problem of *belief change*, which has been intensively investigated by logicians starting at least from Alchourrón et al. (1985). Within their approach—the so called "AGM approach"—a belief system is confronted with a "(doxastic) input", which may cause some new information to be added to the system, or some old information to be withdrawn from it. In the AGM approach, the belief system is represented by a logically closed set of sentences in a propositional language, called a belief set or a theory, and a doxastic input by a single sentence to be added to the belief set. Cevolani et al. (2011) have studied the case of "conjunctive theory change", where both the belief system and the input are expressed in the form of c-theories. As shown below, their analysis, summarized in Sect. 3.1, can be easily applied also to the problem of nomic theory change (Sect. 3.2).

3.1 Conjunctive theory change

The problem of conjunctive theory change amounts to the analysis of how a given c-theory T changes in response to some newly acquired information represented by input A, also expressed in the form of a c-theory. Two cases have to be distinguished. First, A may be logically compatible with T: in this case, the result of changing T by A is represented by the c-theory $T + A$, called the *expansion* of T by A. Second, A may be logically incompatible with T: in this case, the result of changing T by A is represented by the c-theory $T * A$, called the *revision* of T by A.

Both the expansion and the revision of T by A can be analyzed by distinguishing three "parts" of input A (cf. Fig. 1):

(i) the *overlapping* part of A (with respect to T), i.e., the conjunction of the elements of $A^b \cap T^b$, denoted by A_{oT};
(ii) the *conflicting* part of A (with respect to T), i.e., the conjunction of the elements of $A^b \cap T^d$, denoted by A_{cT};
(iii) the *excess* part of A (with respect to T), i.e., the conjunction of the elements of $A^b \cap T^?$, denoted by A_{xT}.

The "new" information conveyed by A with respect to T can be specified in terms of the three parts of A just described. In fact, the overlapping part of A only conveys "old" information, already expressed by the b-claims of T. In contrast, both the conflicting and the excess part of A conveys some "new" information with respect to T, in the sense that they are not entailed by T. For this reason, we may call the conjunction of the two non-overlapping parts of A—i.e., $A_{cT} \wedge A_{xT}$—the *new part* of A (with respect to T).

Let us consider, first of all, the case where A is logically compatible with T. In such case, the conflicting part of A with respect to T is tautological. Given that the overlapping part of A is contained in T, the new part of A is then given by its excess part. The result of changing T by A is the new c-theory $T + A$, called the *expansion* of T by A, which can be expressed as follows:

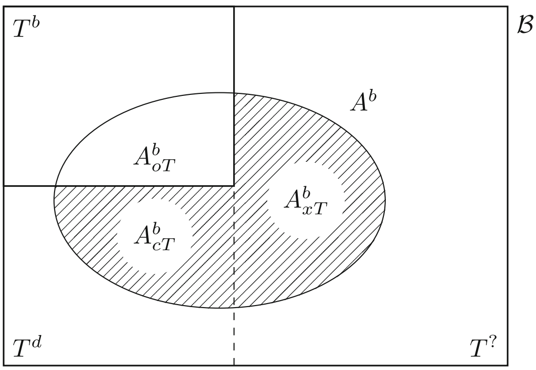


Fig. 1 The b-contents (indicated by a *superscript b*) of the overlapping part (A_{oT}) and of the new part (*shaded*), further partitioned in the conflicting (A_{cT}) and the excess (A_{xT}) part, of input A with respect to c-theory T

$$T + A = T \wedge A \tag{4}$$

or, equivalently, as $(T + A)^b = T^b \cup A^b$. If also the excess part of A is tautological, then $T + A$ is trivially equivalent to T. In contrast, a *genuine* expansion of T by A fills, so to speak, some of the lacunae of T by transforming some of its omissions into b-claims of $T + A$.

Let us then consider the case where T and A are incompatible. In such case, T must be revised in order to accommodate the new information conveyed by A. In fact, the conflicting part of A contradicts some of the b-claims of T, i.e., the members of $T^b \cap A^d$. When revising T by A, such b-claims must first be removed from T, in order to accept their negations without inconsistency. The result of changing T by A is the c-theory $T * A$, called the *revision* of T by A, which can be expressed as follows:

$$T * A = T_{xA} \wedge A \tag{5}$$

or, equivalently, as $(T * A)^b = (T^b \backslash A^d) \cup A^b$. Note that, if T and A are compatible, i.e., if the conflicting part of A is tautological, then the revision is reduced to the expansion of T by A. In contrast, a *genuine* revision of T by A corrects, so to speak, some of the mistakes of T and possibly fills some of its lacunae. It is worth noting that the overlapping part of A is "irrelevant" to the result of expanding or revising T by A. In fact, it follows from (4) and (5) that:

$$T + A = T + A_{xT} \quad \text{and} \quad T * A = T * (A_{cT} \wedge A_{xT}) \tag{6}$$

In words, only the new part of A does really matter when T is changed with respect to A.

3.2 Nomic theory change guided by correct data

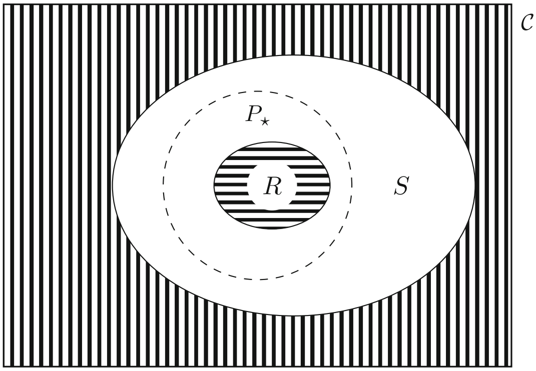
The problem of *nomic theory change*, i.e., of changing a nomic theory in response to newly acquired data, has recently been investigated by Kuipers (2011a, 2011b). In an empirical inquiry about $\mathcal{U}$, two kinds of *data* are typically available:

1. Data of the first kind are *directly* provided by experiments and observations, which may *realize* some conceptual possibilities of $\mathcal{C}$, thus showing that they are nomic possibilities. In fact, only nomic possibilities can be realized and, hence, recorded as experimental results.
2. Data of the second kind are *inductively* accepted starting from data of the first kind. Here, some conceptual possibilities of $\mathcal{C}$ are ruled out as nomically impossible by inductive generalizations made on the basis of experimental and observational evidence.[13]

For example, observing a black raven provides evidence of the first kind, by showing that this kind of bird is a nomic possibility of $\mathcal{U}$. On the other hand, an inductive

[13] It should be noted that, for our present purposes, we don't need to commit ourselves to any particular view of the "inductive jump" that brings us from the available evidence to inductive generalizations.

Fig. 2 The *correct data hypothesis* $R \subseteq P_\star \subseteq S$. Note that R contains only nomic possibilities (*horizontal shading*), while S excludes only nomic impossibilities (*vertical shading*)



generalization such as "all ravens are black", claiming that a non-black raven is a nomic impossibility in $\mathcal{U}$, is an example of data of the second kind.

These two kinds of data—obtained, directly or indirectly, from experimental and observational evidence—correspond to two different subsets of $\mathcal{U}$. In the following, the set of all the *established conceptual possibilities*—i.e., of all those possibilities of $\mathcal{C}$ that have been realized, through experiments or observations, up to a given time—will be denoted by R. In short, R will be called "the evidence" about $\mathcal{U}$. Assuming that there are no experimental or observational errors, and recalling that only nomic possibilities can be realized, R will be a subset of $P_\star$. Moreover, the set of conceptual possibilities which are compatible with the *strongest inductive generalization* accepted up to that moment will be denoted by S; accordingly, the complement of S will include precisely the conceptual possibilities which are ruled out by S as nomically possible. Assuming that the inductive generalization is correct, $P_\star$ will be a subset of S, i.e., S will exclude only nomic impossibilities. In the following, we will assume that both the evidence and the inductive generalizations are *correct* in the sense that: $R \subseteq P_\star \subseteq S$. Such assumption, to be called the *correct data hypothesis*—or *CD-hypothesis* for short (Kuipers, 2000, p. 157)—is graphically represented in Fig. 2.

Nomic theory change can be analyzed in terms of the expansion and revision of a nomic theory by the input represented by data R and S. Within the present framework, both the theory to be changed and the input are represented in the form of nomic c-theories. In fact, given a propositional nomic language $\mathcal{L}_n$, the data R and S can be jointly represented as a nomic c-theory—to be called R/S—claiming that all the conceptual possibilities in R are nomic possibilities and all the conceptual possibilities outside S are nomic impossibilities. Thus, R/S may be expressed in the following form:

$$R/S \overset{\mathrm{df}}{=} \bigwedge_{c_i \in R} p_i \wedge \bigwedge_{c_i \notin S} \neg p_i \tag{7}$$

It follows that the result of changing T by R/S can be represented by the expansion $T + R/S$ of T by R/S, when R/S is compatible with T, or by the revision $T * R/S$ of T by R/S, when R/S is incompatible with T.

An interesting question concerning the relationship between nomic theory change and nomic truth approximation is the following: are the expansion $T + R/S$ and the revision $T * R/S$ of T by correct data R/S more verisimilar than T itself?[14] To answer this question, let us note that, since we are assuming that the CD-hypothesis holds, R/S is a *true* nomic c-theory. In turn, this implies that, if R/S is compatible with T, the excess part of R/S with respect to T will convey some new true information about the world. Thus, any genuine expansion of T by R/S will make more matches and no more mistakes than T, i.e., will be more verisimilar than T according to Definition 1. Moreover, if R/S is incompatible with T, the revision of T by R/S will correct some of the mistakes of T and possibly also add some extra true information. In any case, $T * R/S$ will make more matches and fewer mistakes than T. According to Definition 1, this means that any genuine revision of T by R/S is more verisimilar than T. In sum, all genuine expansions and revisions of T by R/S improve the verisimilitude of T under the CD-hypothesis:

Theorem 1 *If the CD-hypothesis holds, then* $T + R/S >_{vs} T$ *and* $T * R/S >_{vs} T$.

In other words, expanding or revising a theory T by *correct* data R/S always leads to theories which are closer than T to the whole nomic truth.

4 Contrast measures of verisimilitude and nomic theory change

In Sect. 3 we showed that nomic theory change guided by correct data leads to nomic truth approximation, when the latter concept is explicated in terms of the comparative notion of verisimilitude introduced in Definition 1. It should be noted, however, that such notion doesn't allow one to compare two arbitrary c-theories with respect to their verisimilitude, but only those c-theories whose true and false b-contents are set-theoretically comparable in terms of the definition. In other words, Definition 1 only specifies a partial order over the whole class of c-theories. For instance, if $C_\star = p_1 \wedge p_2 \wedge p_3$ is the truth in language $\mathcal{L}_3$, then we cannot say that $T_2 \equiv p_1 \wedge p_2$ is more verisimilar than $T_1 \equiv \neg p_1 \wedge p_3$—i.e., that $T_2 >_{vs} T_1$—since $t(T_1, C_\star) = \{p_3\}$ and $t(T_2, C_\star) = \{p_1, p_2\}$ are not comparable on the basis of Definition 1. In order to compare two arbitrary c-theories with respect to their verisimilitude, one needs to introduce an appropriate *verisimilitude measure*. In what follows, we will introduce a family of such measures (Sect. 4.1) and we will show how they can be applied to a refined analysis of the relationships between nomic theory change and nomic truth approximation (Sect. 4.2). More precisely, we will prove that nomic truth approximation can be achieved through theory change guided not only by correct but also by *partially* correct data.

4.1 Contrast measures of verisimilitude

In principle, many different verisimilitude measures for c-theories can be defined. Here, we will require that a verisimilitude measure *Vs* coheres with the comparative notion of verisimilitude introduced in Definition 1, in the following sense:

[14] See also Niiniluoto (1999, 2010, 2011) and Schurz (2011) on the surprisingly complex relationships between truth approximation and AGM theory change.

Coherence. For any pair of c-theories T_1 and T_2, if $T_2 >_{vs} T_1$ then $Vs(T_2) > Vs(T_1)$.

A family of verisimilitude measures of this kind is provided by so-called *contrast measures*, which are briefly described below.[15]

Let us recall from Sect. 1 that, given a constituent C and a c-theory T, $t(T, C)$ and $f(T, C)$ are the true and false b-content, respectively, of T with respect to C. We can now define the degree of true b-content $cont_t(T, C)$ and the degree of false b-content $cont_f(T, C)$ of T with respect to C as follows:

$$cont_t(T, C) \stackrel{\text{df}}{=} \frac{|t(T, C)|}{n} \quad \text{and} \quad cont_f(T, C) \stackrel{\text{df}}{=} \frac{|f(T, C)|}{n} \tag{8}$$

Note that $0 \leq cont_t(T, C), cont_f(T, C) \leq 1$ and $cont_t(T, C) + cont_f(T, C) = \frac{|T^b|}{n}$. In words, $cont_t(T, C)$ is the normalized number of matches, and $cont_f(T, C)$ the normalized number of mistakes, that T makes with respect to C. Accordingly, $cont_t(T, C)$ may be construed as the overall *reward* attributed to the matches of T and $-cont_f(T, C)$ as the overall *penalty* attributed to the mistakes of T. A contrast measure of similarity between T and C is a weighted average of the prize due to T's degree of true b-content and of the penalty due to T's degree of false b-content:

$$s_\phi(T, C) \stackrel{\text{df}}{=} cont_t(T, C) - \phi cont_f(T, C) \tag{9}$$

where $\phi > 0$. Intuitively, different values of ϕ reflect the relative weight assigned to truths and falsities, i.e., to the matches and mistakes of T with respect to C. The verisimilitude $Vs_\phi(T)$ of T can be construed as the similarity $s(T, C_\star)$ of T to the truth, i.e., to the true constituent $C_\star$ of $\mathcal{L}_n$. Hence,

$$Vs_\phi(T) = cont_t(T, C_\star) - \phi cont_f(T, C_\star) \tag{10}$$

One can easily check that Vs_ϕ satisfies the condition of coherence introduced above, i.e. that, given two c-theories T_1 and T_2, if $T_2 >_{vs} T_1$ then $Vs_\phi(T_2) > Vs_\phi(T_1)$.

4.2 Nomic theory change guided by differentially verisimilar data

The verisimilitude measure Vs_ϕ defined above can be appropriately applied also to propositional nomic languages. This allows one to compare the degrees of verisimilitude $Vs_\phi(T_1)$ and $Vs_\phi(T_2)$ of two arbitrary nomic c-theories T_1 and T_2. Moreover, one can see that the coherence of Vs_ϕ with the comparative notion $>_{vs}$ immediately implies the following quantitative counterpart of Theorem 1:

Theorem 2 *If the CD-hypothesis holds then* $Vs_\phi(T + R/S) > Vs_\phi(T)$ *and* $Vs_\phi(T * R/S) > Vs_\phi(T)$.

[15] See Cevolani et al. (2011). The expression "contrast measures" refers to the fact that these measures can be seen as an application of the contrast model of similarity introduced by Tversky (1977) in his study of the similarity between psychological stimuli.

In words, any genuine expansion or revision of T by *correct* data R/S leads to a theory whose degree of verisimilitude, as measured by Vs_ϕ, is higher than that of T.

Apart from reproducing the results obtained on the basis of the comparative notion $>_{vs}$, the verisimilitude measure Vs_ϕ allows for a more refined analysis of nomic truth approximation through nomic theory change. In particular, we will show that further results can be obtained for the case where nomic theories are changed, not in response to (entirely) correct data, but in response to *partially* correct data. This amounts to considering the problem of expanding and revising a nomic c-theory T by a nomic c-theory R/S which is not true, but only *verisimilar* in some suitably defined sense.

The classificatory notion of a *verisimilar* c-theory can be defined on the basis of any appropriate measure of the verisimilitude of c-theories. Indeed, one may say that a c-theory is verisimilar if and only if its verisimilitude exceeds some fixed threshold. Although one can select, in principle, any degree of verisimilitude as the fixed threshold, as far as contrast measures are concerned a plausible choice is given by the degree of verisimilitude of a tautological c-theory $\top$. In fact, since $\top$ makes no claim at all about the world (i.e., $\top^b = \emptyset$), it is easy to check that, for any contrast measure Vs_ϕ, $Vs_\phi(\top) = 0$. The verisimilitude $Vs_\phi(T)$ of a non-tautological c-theory T is higher or lower than 0 depending on the number of the matches and mistakes of T, and on their relative weight. Accordingly, we introduce the following definition of *verisimilar* c-theories and of c-theories which are *distant from the truth*—or *t-distant* for short:

Definition 2 Given a c-theory T, we say that:

(i) T is verisimilar if and only if $Vs_\phi(T) > 0$;
(ii) T is t-distant if and only if $Vs_\phi(T) < 0$.

Coming back to the problem of nomic theory change, at first sight one might believe that expanding or revising a nomic c-theory T by verisimilar data R/S should lead to theories $T + R/S$ and $T * R/S$ which are closer to the truth than T. However, on closer inspection, this cannot be expected in general, since R/S might be verisimilar only due to its overlapping part with respect to T, whereas its new part is t-distant. In this case, since what really does matter is the verisimilitude of the new part of R/S, the expansion and the revision of T by R/S might be in fact less verisimilar than T. For the sake of illustration, let us consider the contrast measure of verisimilitude Vs_1, with $\phi = 1$, defined as follows:

$$Vs_1(T) \stackrel{\text{df}}{=} cont_t(T, C_\star) - cont_f(T, C_\star) \tag{11}$$

Given a nomic c-theory T, $Vs_1(T)$ simply amounts to the difference between the number of matches and the number of mistakes of T, normalized by n. Note that R/S is verisimilar with respect to Vs_1 just in case R/S makes more true than false b-claims, and is t-distant in the opposite case.

To see why the revision of T by verisimilar data R/S may be less verisimilar than T, let us consider the following example.

Example 1 Suppose that $p_1, \ldots, p_{10}$ are true literals of a given nomic propositional language $\mathcal{L}_n$. Then, given c-theory $T = p_1 \wedge \cdots \wedge p_5 \wedge \neg p_6$ and data $R/S =$

$p_2 \wedge \cdots \wedge p_6 \wedge \neg p_7 \wedge \cdots \wedge \neg p_{10}$, one can see that $Vs_1(T) = \frac{4}{n}$ and $Vs_1(R/S) = \frac{1}{n}$. Hence, R/S is verisimilar. However, one can check that the revision of T by R/S—i.e., $T * R/S = p_1 \wedge \cdots \wedge p_6 \wedge \neg p_7 \wedge \cdots \wedge \neg p_{10}$—is less verisimilar than T, since $Vs_1(T * R/S) = \frac{2}{n}$. This is due to the fact that the high verisimilitude of R/S is mainly due to its overlapping part with respect to T, i.e., to $R/S_{oT} = p_2 \wedge \cdots \wedge p_5$, and this part plays no role in the revision of T.

Example 1 might suggest that if the *new part* of R/S is verisimilar, then the revision of T by R/S is more verisimilar than T. However, even this weaker expectation can not be fulfilled in general, as the following exam shows.

Example 2 Suppose again that $p_1, \ldots, p_{10}$ are all true in $\mathcal{L}_n$. Then, given c-theory $T = p_1 \wedge \cdots \wedge p_4 \wedge \neg p_5$ and data $R/S = \neg p_2 \wedge \cdots \wedge \neg p_5 \wedge p_6 \wedge \cdots \wedge p_{10}$, one can see that $Vs_1(T) = \frac{3}{n}$ and $Vs_1(R/S) = \frac{1}{n}$. Moreover, one can see that R/S is verisimilar and conveys only new information with respect to T, since the overlapping part of R/S is tautological. However, one can check that $T * R/S = p_1 \wedge \neg p_2 \wedge \cdots \wedge \neg p_5 \wedge p_6 \wedge \cdots \wedge p_{10}$ is less verisimilar than T, since $Vs_1(T * R/S) = \frac{2}{n}$. This is due to the fact that R/S is verisimilar only because its excess part is highly verisimilar, while its conflicting part is t-distant, thus making the revision of T by R/S less verisimilar than T.

Examples 1 and 2 show that, in the case where data are not entirely correct, i.e., R/S is not true, no condition as simple as the CD-hypothesis can be specified according to which the revision of T by R/S is more verisimilar than T. In particular, neither the fact that R/S is verisimilar, nor the fact that its new part with respect to T is verisimilar, is sufficient to guarantee that $T * R/S$ is more verisimilar than T. However, one can consider the *differentially verisimilar data hypothesis*—or DVD-hypothesis for short—according to which the "relevant parts" of R/S, and not R/S itself, are verisimilar. These relevant parts are the conflicting and the excess parts of R/S with respect to T:

Definition 3 Given a nomic c-theory T, R/S is *differentially verisimilar* with respect to T if and only if both $Vs_\phi(R/S_{cT}) > 0$ and $Vs_\phi(R/S_{xT}) > 0$.

One can then prove that, under the DVD-hypothesis, any genuine expansion or revision of T by R/S is more verisimilar than T:

Theorem 3 *If the DVD-hypothesis holds, then* $Vs_1(T + R/S) > Vs_1(T)$ *and* $Vs_1(T * R/S) > Vs_1(T)$.[16]

[16] Proof. First of all, note that Vs_1 (and, more generally Vs_ϕ) is additive in the following sense: given a c-theory T, $Vs_1(T) = \sum_{p_i \in T^b} Vs_1(p_i)$. In turn, this implies that, for any T and R/S, $Vs_1(T) = Vs_1(T_{oR/S}) + Vs_1(T_{cR/S}) + Vs_1(T_{xR/S})$ and $Vs_1(R/S) = Vs_1(R/S_{oT}) + Vs_1(R/S_{cT}) + Vs_1(R/S_{xT})$. Consider first the case of a genuine expansion of T by R/S. From (4), $T + R/S = T \wedge R/S_{xT}$, and since the DVD-hypothesis holds, $Vs_1(R/S_{xT}) > 0$. It follows that $Vs_1(T + R/S) = Vs_1(T) + Vs_1(R/S_{xT}) > Vs_1(T)$, as required. For what concerns genuine revisions of T by R/S, recall from (5) that $T * R/S = T_{xR/S} \wedge R/S$. Thus, $Vs_1(T * R/S) > Vs_1(T)$ iff $Vs_1(T_{xR/S} \wedge R/S) > Vs_1(T)$ iff $Vs_1(T_{xR/S}) + Vs_1(R/S_{oT}) + Vs_1(R/S_{cT}) + Vs_1(R/S_{xT}) > Vs_1(T_{oR/S}) + Vs_1(T_{cR/S}) + Vs_1(T_{xR/S})$ iff, since $Vs_1(R/S_{oT}) = Vs_1(T_{oR/S})$ by definition, $Vs_1(R/S_{cT}) + Vs_1(R/S_{xT}) > Vs_1(T_{cR/S})$. The DVD-hypothesis implies that both $Vs_1(R/S_{cT}) > 0$ and $Vs_1(R/S_{xT}) > 0$. Since $Vs_1(R/S_{cT}) > 0$, R/S_{cT} makes more matches than mistakes; then, by definition, $T_{cR/S}$ makes more mistakes than matches, i.e., $Vs_1(T_{cR/S}) < 0$. It follows that $Vs_1(T * R/S) > Vs_1(T)$.

In other words, if the DVD-hypothesis holds—i.e., if R/S is differentially verisimilar with respect to T, even if not true—then expansions and revisions of T by R/S will be more verisimilar than T. It is worth noting that, although Theorem 3 is formulated in terms of the particular measure Vs_1, a general version of this result can be proved, with appropriate modifications, for all contrast measures Vs_ϕ with $\phi \neq 1$.[17]

5 Comparisons and extensions of the present approach to (nomic) verisimilitude

In this paper, both a comparative and a quantitative notion of verisimilitude for nomic conjunctive theories has been defined and applied to the problem of nomic truth approximation through theory change. In particular, we identified suitable conditions concerning the (partial) correctness of acquired data, under which revising our theories by data leads us closer to the target of scientific inquiry, construed as the nomic truth about a given domain. Such results suggest some interesting directions for further research. In particular, we will point out some conceptual relations between our BF-approach and other approaches to (nomic) verisimilitude, and suggest some possible extensions and generalizations of the BF-approach.

Conceptual relations between the BF-approach and the content, consequence, and likeness approaches to verisimilitude. During the last 30 years, theories of verisimilitude have been commonly classified into two rival camps, i.e., the content approach and the likeness approach. The problem as to whether these approaches are compatible, and can be combined in a fruitful way, was first raised by Zwart (2001) and then investigated in a number of papers in this journal: see Zwart and Franssen (2007); Schurz and Weingartner (2010), and Oddie (2011). Taking a cue from Schurz and Weingartner, Oddie argues that the traditional content/likeness dichotomy should be replaced by a more adequate classification, which takes into account a third, distinct approach, the so-called consequence approach. After providing an unified characterization of these three approaches, Oddie proves that they cannot be combined in a fruitful way, i.e., without trivializing the notion of verisimilitude. With reference to such results about the impossibility of reconciliation of the three approaches, an anonymous referee for this journal asks to specify the conceptual relations between our BF-approach and (the various desiderata characterizing) the content, consequence, and likeness approaches to verisimilitude. Here, we can only make some short remarks on this issue.

First of all, it should be recalled that the verisimilitude ordering—for short, $>_{vs}$-ordering—introduced in Definition 1 (see Sect. 1), does not apply to any kind of theory but only to c-theories. This means that, in principle, the $>_{vs}$-ordering might be in agreement with different, and mutually incompatible, "general" approaches to verisimilitude, applicable to any kind of theory. In other words, it might be the case

[17] It is perhaps worth mentioning that, as noted by an anonymous referee for this journal, Vs_ϕ is closely related to the verisimilitude measure for propositional constituents proposed by Tichý (1976). More precisely, one can prove that, for $\phi = 1$, Vs_ϕ is a special case of Tichy's measure (cf. Cevolani and Festa 2012).

that the $>_{vs}$-ordering of c-theories is, so to speak, a common *fragment* of different, and mutually incompatible, general verisimilitude orderings of theories.

In this connection, it is interesting to note that the $>_{vs}$-ordering is indeed compatible with all the three approaches to verisimilitude analyzed by Oddie, as the following quick remarks should make clear.[18] First, the $>_{vs}$-ordering is compatible with the consequence approach as characterized by Oddie (2011, Sect. 4). More precisely, the $>_{vs}$-ordering can be seen as a special case of a version of the consequence approach proposed by Schurz and Weingartner (2010) under the label of *relevant element* approach.[19] Secondly, as far as the likeness approach is concerned (Oddie 2011, Sects. 5 and 6), one can prove two closely related results: (1) the average likeness measure of verisimilitude, suggested by Tichý (1974) and Oddie (1986), coheres with the $>_{vs}$-ordering, in the sense of coherence defined at the beginning of Sect. 4.1, and (2) the $>_{vs}$-ordering is compatible with all the desiderata for the likeness approach suggested by Oddie. Thirdly, let us consider the relations between the $>_{vs}$-ordering and the content approach (Oddie 2011, Sect. 3). Oddie argues that a necessary ingredient of any version of the content approach is the following condition, that he calls *the strong value of content for truths*: if T_1 and T_2 are true, and T_1 strictly entails T_2, then T_1 is strictly more verisimilar than T_2. A glance at the first clause of Definition 1 is sufficient to show that the $>_{vs}$-ordering for c-theories fulfills the above condition.

Extensions of the BF-approach to different kinds of language. As suggested by Cevolani et al. (2011), the notion of conjunctive theory can be appropriately defined within many kinds of languages besides propositional languages. In this connection, one should note that our approach can be generalized to any language characterized by a suitable notion of *constituent*—where a constituent is informally defined as a maximally informative conjunction of elementary (atomic and logically independent) claims about the world. In such "languages with constituents", in fact, a c-theory can be conveniently defined as a "fragment" of a constituent, i.e., in the terminology adopted by Oddie (1986), as a *quasi-constituent*. This suggests that the BF-approach to verisimilitude can be easily extended to any language with constituents. A further extension, discussed by Festa (2012), deals with the verisimilitude of theories defined within quantitative languages, including quantitative, statistical and tendency hypotheses.

Extensions of the BF-approach to nomic verisimilitude. Some extensions of the present approach to nomic verisimilitude are worth mentioning. First, one doesn't need to assume that the set of conceptual possibilities characterizing the target domain $\mathcal{U}$ is finite. Indeed, Kuipers (2011b) offers a generalization of the present approach to the case of a possibly infinite universe, where theoretical claims are interpreted in several ways, notably as nomic states in and out of equilibrium and as instantiated and non-instantiated "Q-predicates" of a monadic language (cf. also Festa 2007 and

[18] For a more extended analysis, and a proof of the claims made in this subsection, see Cevolani and Festa (2012).

[19] On the relations between the BF-approach and the relevant element approach, see Schurz (2011, pp. 206 ff.).

Niiniluoto 2011). Second, the present approach is based on Kuipers' (2000) theory of nomic verisimilitude. Although a comparison with other proposals in the literature is beyond the scope of this paper, it is worth noting that the BF-approach might be fruitfully applied also in contexts different from the present one. Indeed, current theories of legisimilitude typically construe nomic verisimilitude as similarity to the true constituent $C_\star$ either of a second-order language (Oddie, 1986, Sect. 5.4) or of a first-order modal language (Niiniluoto, 1987, Chap. 11), where $C_\star$ expresses the relevant nomic relations holding amongst the conceptual possibilities of the domain. As far as these relations can be expressed as conjunctions of elementary statements about the domain, the BF-approach can be applied to evaluate the verisimilitude of different nomic theories, along the lines suggested above. In particular, as noted by an anonymous referee for this journal, Oddie's approach to nomic verisimilitude, which is based on the idea of reducing truths about nomic necessity to certain basic propositions about relations between universals in a second-order framework, is not entirely dissimilar to the approach outlined in the present paper.

Generalized contrast measures of nomic verisimilitude. The Vs_ϕ measures introduced above (Sect. 4.1) are only a subset of the much richer class of contrast measures of verisimilitude, discussed in full details by Cevolani and Festa (2012). In particular, a peculiar feature of Vs_ϕ is that, when evaluating the verisimilitude $Vs_\phi(T)$ of a nomic c-theory T, the matches and mistakes of T concerning both nomic *possibilities* and nomic *impossibilities* are equally weighted. More precisely, if p_i is a true possibility claim and $\neg p_j$ is a true impossibility claim of T, each of them is simply counted as a true b-claim of T in the calculation of $Vs_\phi(T)$. Likewise, both the false possibility claim p_j and the false impossibility claim $\neg p_i$ will be equally evaluated as false b-claims of T. However plausible this assumption may be in general, it is not necessarily appropriate in all contexts. For instance, suppose that in a given conceptual frame $\mathcal{C}$, nomic possibilities are more important or more relevant than nomic impossibilities; or that nomic possibilities are very rare as compared to the nomic impossibilities of $\mathcal{U}$. This suggests that the (true and false) possibility claims of a theory should weigh more than its (true and false) impossibility claims in assessing its verisimilitude. More generally, matches and mistakes concerning nomic possibilities should be differently weighted, in evaluating the verisimilitude of nomic theories, with respect to matches and mistakes concerning nomic impossibilities. In cases like this, contrast measures can be introduced that treat nomic possibilities and impossibilities in a different manner as far as the evaluation of the verisimilitude of nomic c-theories is concerned. Generalized contrast measures of this kind are discussed both by Cevolani and Festa (2012) and by Kuipers (2011b).

Refined measures of nomic verisimilitude. An interesting question concerns the possibility of extending our results about truth approximation through theory change to different kinds of verisimilitude measures besides contrast measures. In this connection, it is an open problem whether our results, or at least some of them, can be extended to other measures which, like contrast measures Vs_ϕ, cohere with the comparative ordering of Definition 1.

An anonymous referee for this journal has noted that our approach can not account for the fact that one mistake may be less severe than another. For instance, two quantitative theories may wrongly exclude the same conceptual possibility as a nomic impossibility on the basis of strongly different estimates of the relevant quantities involved. In such case, however, our approach can not differentiate between the two theories as far as their nomic verisimilitude is concerned, since the latter depends only on the number of their mistakes (and matches), and not on the severity of such mistakes. In order to account for cases like this, Kuipers (2000, Ch. 10) has introduced a *refined* comparative notion of nomic verisimilitude, based on a ternary likeness relation between conceptual possibilities. Building on this refined account, Kuipers (2011a) has also shown that theory revision can be refined to so-called partial meet revision (Alchourrón et al. 1985) in such a way that, assuming correct data, it leads to an at least as successful theory and therefore, due to a "success theorem" (Kuipers, 2000, pp. 160, 260 ff.), to a potentially more verisimilar theory. Hence, it is a valuable challenge to find out whether the conjunctive approach of nomic theory revision, as elaborated in Sect. 3, can be refined in a similar way, with a similar result (Kuipers 2012).

Acknowledgments We thank an anonymous referee for this journal for very helpful comments on a previous draft of this paper.

References

Alchourrón, C., Gärdenfors, P., & Makinson, D. (1985). On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic, 50*, 510–530.

Aliseda, A. (2005). Lacunae, empirical progress and semantic tableaux. In R. Festa, A. Aliseda, & J. Peijnenburg (Eds.), *Confirmation, empirical progress, and truth approximation* (pp. 141–161). Amsterdam/New York: Rodopi.

Burger, I. C., & Heidema, J. (2005). For better, for worse: Comparative orderings on states and theories. In R. Festa, A. Aliseda, & J. Peijnenburg (Eds.), *Confirmation, empirical progress, and truth approximation. Essays in debate with Theo Kuipers* (pp. 459–488). Amsterdam: Rodopi.

Carnap, R. (1950). *Logical foundations of probability*. Chicago: University of Chicago Press.

Cevolani, G., Crupi, V., & Festa, R. (2010). The whole truth about Linda: Probability, verisimilitude and a paradox of conjunction. In M. D'Agostino, G. Giorello, F. Laudisa, T. Pievani, & C. Sinigaglia (Eds.), *New essays in logic and philosophy of science* (pp. 603–615). London: College Publications.

Cevolani, G., Crupi, V., & Festa, R. (2011). Verisimilitude and belief change for conjunctive theories. *Erkenntnis, 75*(2), 183–202.

Cevolani, G., & Festa, R. (2012). Features of verisimilitude. In preparation.

Cohen, L. J. (1980).What has science to do with truth? *Synthese, 45*(3), 489–510.

Cohen, L. J. (1987). Verisimilitude and legisimilitude. In T. A. F. Kuipers (Ed.), *What is closer-to-the-truth?* (pp. 129–144). Amsterdam: Rodopi.

Festa, R. (2007). Verisimilitude, qualitative theories, and statistical inferences. In S. Pihlström, P. Raatikainen, & M. Sintonen (Eds.), *Approaching truth: Essays in honour of Ilkka Niiniluoto.* (pp. 143–178). London: College Publications.

Festa, R. (2012). On the verisimilitude of tendency hypotheses. In D. Dieks, W. J. Gonzalez, S. Hartmann, M. Stöltzner, & M. Weber (Eds.), *Probabilities, laws, and structures, Vol. 3 of the philosophy of science in a European perspective.* (pp. 43–55). Dordrecht: Springer.

Gemes, K. (2007).Verisimilitude and content. *Synthese, 154*(2), 293–306.

Hintikka, J. (1973). *Logic, language-games and information*. Oxford: Oxford University Press.

Kuipers, T. A. F. (1982). Approaching descriptive and theoretical truth *Erkenntnis, 18*, 343–378.

Kuipers, T. A. F. (1987). A structuralist approach to truthlikeness. In T. A. F. Kuipers (Ed.), *What is closer-to-the-truth?* (pp. 79–99). Amsterdam: Rodopi.
Kuipers, T. A. F. (1999). Abduction aiming at empirical progress or even truth approximation leading to a challenge for computational modelling *Foundations of Science, 4*, 307–323.
Kuipers, T. A. F. (2000). *From instrumentalism to constructive realism*. Dordrecht: Kluwer Academic Publishers.
Kuipers, T. A. F. (2011a). Basic and refined nomic truth approximation by evidence-guided belief revision in AGM-terms *Erkenntnis, 75*, 223–236.
Kuipers, T. A. F. (2011b). Dovetailing belief base revision with (basic) truth approximation. (to appear).
Kuipers, T. A. F. (2012). Empirical progress and truth approximation revisited. Manuscript.
Miller, D. (1974). Popper's qualitative theory of verisimilitude *The British Journal for the Philosophy of Science, 25*(2), 166–177.
Niiniluoto, I. (1983).Verisimilitude vs legisimilitude *Studia Logica, 17*, 315–329.
Niiniluoto, I. (1987). *Truthlikeness*. Dordrecht: Reidel.
Niiniluoto, I. (1998). Verisimilitude: The third period *The British Journal for the Philosophy of Science, 49*(1), 1–29.
Niiniluoto, I. (1999). Belief revision and truthlikeness. In B. Hansson, S. Halldén, N.-E. Sahlin, & W. Rabinowicz (Eds.), *Internet Festschrift for Peter Gärdenfors*. Lund: Department of Philosophy, Lund University.
Niiniluoto, I. (2003). Content and likeness definitions of truthlikeness. In J. Hintikka, T. Czarnecki, K. Kijania-Placek, A. Rojszczak, & T. Placek (Eds.), *Philosophy and logic: In search of the Polish tradition. Essays in honor of Jan Woleński on the occasion of his 60th birthday.* (pp. 27–35). Dordrecht: Kluwer Academic Publishers.
Niiniluoto, I. (2010). Theory change, truthlikeness, and belief revision. In M. Suárez, M. Dorato, & M. Rèdei (Eds.), *EPSA epistemology and methodology of science: Launch of the European philosophy of science association* (pp. 189–199). Dordrecht: Springer.
Niiniluoto, I. (2011). Revising beliefs towards the truth *Erkenntnis, 75*(2), 165–181.
Oddie, G. (1982). Cohen on verisimilitude and natural necessity *Synthese, 51*, 355–379.
Oddie, G. (1986). *Likeness to truth*. Dordrecht: Reidel.
Oddie, G. (2011). The content, consequence and likeness approaches to verisimilitude: Compatibility, trivialization, and underdetermination. Synthese. http://dx.doi.org/10.1007/s11229-011-9930-8.
Popper, K. R. (1963). *Conjectures and refutations: The growth of scientific knowledge* (3rd ed.). London: Routledge and Kegan Paul.
Schurz, G. (2011).Verisimilitude and belief revision. With a focus on the relevant element account *Erkenntnis, 75*(2), 203–221.
Schurz, G., & Weingartner, P. (2010). Zwart and Franssen's impossibility theorem holds for possible-world-accounts but not for consequence-accounts to verisimilitude *Synthese, 172*, 415–436.
Tichý, P. (1974). On Popper's definitions of verisimilitude *The British Journal for the Philosophy of Science, 25*(2), 155–160.
Tichý, P. (1976). Verisimilitude redefined *The British Journal for the Philosophy of Science, 27*, 25–42.
Tversky, A. (1977). Features of similarity *Psychological Review, 84*, 327–352.
Zwart, S. D. (2001). *Refined verisimilitude*. Dordrecht: Kluwer Academic Publishers.
Zwart, S., & Franssen, M. (2007). An impossibility theorem for verisimilitude *Synthese, 158*, 75–92.