Lista de exercícios de Introdução à Redes Booleanas Probabilisticas

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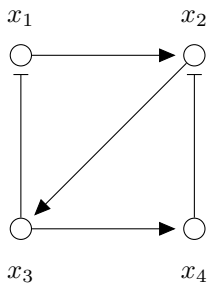
Exercício 1. Dada a rede booleana abaixo:



(1) Monte a matriz de interação.

R:

$$\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} \end{array}$$

(2) Para cada gene, encontre sua expressão booleana

R:

Para x_1:

$x_1(t)$	$x_3(t)$	$x_1(t+1)$
0	0	0
0	1	0
1	0	1
1	1	0

Portanto, $x_1(t+1) = x_1(t)\bar{x}_3(t)$

Para x_2:

$x_2(t)$	$x_1(t)$	$x_4(t)$	$x_2(t+1)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Portanto, $x_2(t+1) = x_1(t)\bar{x}_2(t)\bar{x}_4(t) + \bar{x}_1(t)x_2(t)\bar{x}_4(t) + x_1(t)x_2(t)\bar{x}_4(t) + x_1(t)x_2(t)x_4(t)$

Para x_3:

$x_3(t)$	$x_2(t)$	$x_3(t+1)$
0	0	0
0	1	1
1	0	1
1	1	1

Portanto, $x_3(t+1) = x_2(t) + x_3(t)$

Para x_4:

$x_4(t)$	$x_3(t)$	$x_4(t+1)$
0	0	0
0	1	1
1	0	1
1	1	1

Portanto, $x_4(t+1) = x_3(t) + x_4(t)$

Exercício 2. Monte a tabela de probabilidade condicional para a rede do exercício 1 usando o modelo de PBNs de αs e βs

R:

Para x_1:

$x_1(t)$	$x_3(t)$	$P(x_1(t+1)=0 \mid x_1(t), x_3(t))$	$P(x_1(t+1)=1 \mid x_1(t), x_3(t))$
X	1	$\frac{e^{\beta}}{e^{\beta}+e^{-\beta}}$	$\frac{e^{-\beta}}{e^{\beta}+e^{-\beta}}$
0	0	$\frac{1}{1+e^{-\alpha}}$	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$
1	0	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$	$\frac{1}{1+e^{-\alpha}}$

Para x_2:

$x_2(t)$	$x_1(t)$	$x_4(t)$	$P(x_2(t+1)=0 \mid x_1(t), x_2(t), x_4(t))$	$P(x_2(t+1)=1 \mid x_1(t), x_2(t), x_4(t))$
X	1	0	$\frac{e^{-\beta}}{e^{\beta}+e^{-\beta}}$	$\frac{e^{\beta}}{e^{\beta}+e^{-\beta}}$
X	0	1	$\frac{e^{\beta}}{e^{\beta}+e^{-\beta}}$	$\frac{e^{-\beta}}{e^{\beta}+e^{-\beta}}$
0	0	0	$\frac{1}{1+e^{-\alpha}}$	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$
1	0	0	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$	$\frac{1}{1+e^{-\alpha}}$
0	1	1	$\frac{1}{1+e^{-\alpha}}$	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$
1	1	1	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$	$\frac{1}{1+e^{-\alpha}}$

Para x_3:

$x_3(t)$	$x_2(t)$	$P(x_3(t+1)=0\|x_2(t),x_3(t))$	$P(x_3(t+1)=1\|x_2(t),x_3(t))$
X	1	$\frac{e^{-\beta}}{e^{\beta}+e^{-\beta}}$	$\frac{e^{\beta}}{e^{\beta}+e^{-\beta}}$
0	0	$\frac{1}{1+e^{-\alpha}}$	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$
1	0	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$	$\frac{1}{1+e^{-\alpha}}$

Para x_4:

$x_4(t)$	$x_3(t)$	$P(x_4(t+1)=0\|x_3(t),x_4(t))$	$P(x_4(t+1)=1\|x_3(t),x_4(t))$
X	1	$\frac{e^{-\beta}}{e^{\beta}+e^{-\beta}}$	$\frac{e^{\beta}}{e^{\beta}+e^{-\beta}}$
0	0	$\frac{1}{1+e^{-\alpha}}$	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$
1	0	$\frac{e^{-\alpha}}{1+e^{-\alpha}}$	$\frac{1}{1+e^{-\alpha}}$

Exercício 3. Mostre a tabela de transição de estados para a PBN do último exercício

R:

	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
0000	0.82	0.041	0.041	2.04e-03	0.041	2.04e-03	2.04e-03	1.02e-04	0.041	2.04e-03	2.04e-03	1.02e-04	2.04e-03	1.02e-04	1.02e-04	5.06e-06
0001	0.043	0.86	2.14e-03	0.043	1.06e-04	2.14e-03	5.30e-06	1.06e-04	2.14e-03	0.043	1.06e-04	2.14e-03	5.30e-06	1.06e-04	2.64e-07	5.30e-06
0010	1.11e-04	0.045	2.24e-03	0.9	5.55e-06	2.24e-03	1.11e-04	0.045	2.76e-07	1.11e-04	5.55e-06	2.24e-03	1.38e-08	5.55e-06	2.76e-07	1.11e-04
0011	1.17e-04	0.047	2.34e-03	0.95	2.89e-07	1.17e-04	5.81e-06	2.34e-03	2.89e-07	1.17e-04	5.81e-06	2.34e-03	7.17e-10	2.89e-07	1.44e-08	5.81e-06
0100	1.06e-04	5.30e-06	0.043	2.14e-03	2.14e-03	1.06e-04	0.86	0.043	5.30e-06	2.64e-07	2.14e-03	1.06e-04	1.06e-04	5.30e-06	0.043	2.14e-03
0101	1.11e-04	2.24e-03	0.045	0.9	2.76e-07	5.55e-06	1.11e-04	2.24e-03	5.55e-06	1.11e-04	2.24e-03	0.045	1.38e-08	2.76e-07	5.55e-06	1.11e-04
0110	2.89e-07	1.17e-04	1.17e-04	0.047	5.81e-06	2.34e-03	2.34e-03	0.95	7.17e-10	2.89e-07	2.89e-07	1.17e-04	1.44e-08	5.81e-06	5.81e-06	2.34e-03
0111	6.08e-06	2.45e-03	2.45e-03	0.99	1.51e-08	6.08e-06	6.08e-06	2.45e-03	1.51e-08	6.08e-06	6.08e-06	2.45e-03	3.74e-11	1.51e-08	1.51e-08	6.08e-06
1000	1.06e-04	5.30e-06	5.30e-06	2.64e-07	0.043	2.14e-03	2.14e-03	1.06e-04	2.14e-03	1.06e-04	1.06e-04	5.30e-06	0.86	0.043	0.043	2.14e-03
1001	2.04e-03	0.041	1.02e-04	2.04e-03	1.02e-04	2.04e-03	5.06e-06	1.02e-04	0.041	0.82	2.04e-03	0.041	2.04e-03	0.041	1.02e-04	2.04e-03
1010	2.89e-07	1.17e-04	5.81e-06	2.34e-03	1.17e-04	0.047	2.34e-03	0.95	7.17e-10	2.89e-07	1.44e-08	5.81e-06	2.89e-07	1.17e-04	5.81e-06	2.34e-03
1011	1.11e-04	0.045	2.24e-03	0.9	5.55e-06	2.24e-03	1.11e-04	0.045	2.76e-07	1.11e-04	5.55e-06	2.24e-03	1.38e-08	5.55e-06	2.76e-07	1.11e-04
1100	2.76e-07	1.38e-08	1.11e-04	5.55e-06	1.11e-04	5.55e-06	0.045	2.24e-03	5.55e-06	2.76e-07	2.24e-03	1.11e-04	2.24e-03	1.11e-04	0.9	0.045
1101	2.64e-07	5.30e-06	1.06e-04	2.14e-03	5.30e-06	1.06e-04	2.14e-03	0.043	5.30e-06	1.06e-04	2.14e-03	0.043	1.06e-04	2.14e-03	0.043	0.86
1110	1.51e-08	6.08e-06	6.08e-06	2.45e-03	6.08e-06	2.45e-03	2.45e-03	0.99	3.74e-11	1.51e-08	1.51e-08	6.08e-06	1.51e-08	6.08e-06	6.08e-06	2.45e-03
1111	2.89e-07	1.17e-04	1.17e-04	0.047	5.81e-06	2.34e-03	2.34e-03	0.95	7.17e-10	2.89e-07	2.89e-07	1.17e-04	1.44e-08	5.81e-06	5.81e-06	2.34e-03

Exercício 4. Faça um programa que recebe $n > 0$, α, β e a matriz de que representa a rede e devolva a matriz de transição.

Exercício 5. Faça um programa que recebe $n > 0$, uma probabilidade de inversão de bits p e a matriz de que representa a rede e devolva a matriz de transição.

Exercício 6. Faça um programa que receba a matriz de transição e devolva a matriz estacionária.

Exercício 7. Faça um programa que receba a matriz de transição e devolva as probabilidades de fluxo.

Exercício 8. Faça um programa que receba $n > 0$, α, β e a matriz que representa a rede e devolva a matriz de fluxo total.

Exercício 9. Reproduza os resultados do paper "Generating Boolean networks with a prescribed attractor structure".