# Identification of cell signaling pathways based on biochemical reaction kinetics repositories

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# Introduction

Cell signaling allows cells to respond to signals that come from its environment changing its behaviour accordingly.

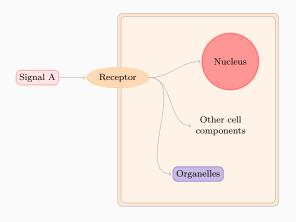
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Understanding the functioning of cell signaling is important in many biological areas.



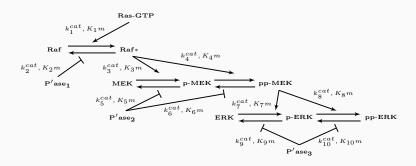
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We call the path of a signal a cell signaling pathway.

#### **Cell Signaling Pathways**

A cell signaling network can be characterized by a sequence of chemical reactions



# Mathematical Models of Signaling Networks

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Using biochemical kinetics, we can model the concentration change of chemical species over time of a pathway.

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As the input, a description of a biological experiment and a set of experimental measurements are given. A possible output to the problem is composed by:

- a model composed by a set of chemical reactions that are relevant for the biological experiment;
- information about the reaction rate constants of the model.

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Hence, it is desirable to construct a method that can systematically modify these models and choose the one that better represents the experiment.

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On her work, the problem of identification of cell signaling pathways is treated as a feature selection problem.

#### Feature Selection for Identification of Signaling Pathways

The methodology proposed by Wu defines the set of features as a set of chemical reactions that can be added to a starting model.

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The methodology proposed by Wu defines the set of features as a set of chemical reactions that can be added to a starting model. This set of chemical reactions is fetched from KEGG and stored in a database of interactions.

#### Wu's Search Algorithm for Feature Selection

The search algorithm used by Wu is the Sequential Forward Selection (SFS).

#### Wu's Cost Function for Feature Selection

Wu defines the cost function as the minimum distance between experimental and simulated data.

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- the database of interactions used could be more nearly complete;
- the search algorithm could also consider removing interactions;
- the cost function could implement a proper penalization of models;

#### What we Propose on this Project

We proposed to create a methodology that uses a feature selection approach for identification of signaling pathways, tackling the difficulty of penalizing complex models.

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We used Bayesian approaches of model selection that allow us to create estimates of p(M|D) or p(D|M).

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- Observe the surface induced by the cost function over the search space.

# **Fundamental Concepts**

Kinetics Modeling of Chemical Reactions

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- Michaelis-Menten enzymatic kinetics.

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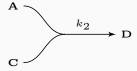
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has rate of:

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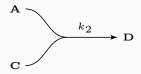
## **Kinetic Modeling of Second Order Iteration**

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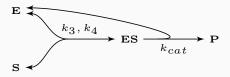


has rate of:

$$k_2[A][C]$$
.

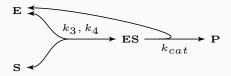
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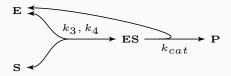
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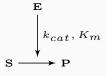
An enzymatic reaction:



Can be divided in two first order reactions plus a second order reaction. However, with the appropriate assumptions, it is possible to use a Michaelis-Menten simplification of this reaction.

#### Michaelis-Menten Kinetics

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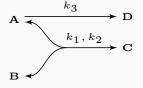
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$$\mathbf{S} \xrightarrow{\mathbf{E}} k_{cat}, K_m$$

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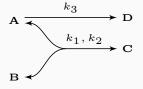
$$k_{cat} \frac{[E][S]}{K_M + [S]}.$$

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- C  $\longrightarrow$  A + B, with rate  $k_2[C]$ ,
- A  $\longrightarrow$  D, with rate  $k_3[A]$ .

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Then, the differential equation that models the concentration change of A is:

$$\frac{d[A]}{dt} = -k_1[A][B] + k_2[C] - k_3[D].$$

Bayesian Methods for Biochemical Model Selection

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- Approximate Bayesian Computation;
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For both methods, we resort to Metropolis-Hastings algorithm to generate samples of distributions.

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- 5. Increase *t* by one and repeat from Step 2 if not reached iteration limit.

# Model Selection

Marginal Likelihood Estimation

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$$p(\mathbf{D}|M, \boldsymbol{\theta}) = p_{\mathcal{N}_{(\vec{0}, \Sigma)}}(\phi(M, \boldsymbol{\theta}) - \boldsymbol{D}).$$

Where  $\phi(M, \theta)$  is the simulated observation.

#### Marginal Likelihood of Data

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Calculating this integral is hard, therefore we resort to estimating another integral.

#### **Power-posterior distributions**

We define a power-posterior distribution as:

$$p_{\beta}(\boldsymbol{\theta}) = \frac{p(\boldsymbol{D}|\boldsymbol{\theta}, M)^{\beta} p(\boldsymbol{\theta}|M)}{\int_{\boldsymbol{\Theta}} p(\boldsymbol{D}|\boldsymbol{\theta}, M)^{\beta} p(\boldsymbol{\theta}|M) d\boldsymbol{\theta}},$$

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$$p_{\beta=1}(\theta) = \frac{p(\boldsymbol{D}, \theta|M)}{\int_{\Theta} p(\boldsymbol{D}, \theta|M) d\theta} = \frac{p(\theta|\boldsymbol{D}, M)p(\boldsymbol{D}|M)}{p(\boldsymbol{D}|M)} = p(\theta|\boldsymbol{D}, M).$$

## The Thermodynamic Integral

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$$\ln p(\mathbf{D}|M) = \int_0^1 \mathbb{E}_{p_{\beta}(\boldsymbol{\theta})}[\ln p(\mathbf{D}|\boldsymbol{\theta}, M)]d\beta.$$

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$$\sum_{t=0}^{T-1} (\beta_{t+1} - \beta_t) \frac{\mathbb{E}_{p_{\beta_{t+1}}(\boldsymbol{\theta})}[\log p(\boldsymbol{D}|M,\boldsymbol{\theta})] + \mathbb{E}_{p_{\beta_t}(\boldsymbol{\theta})}[\log p(\boldsymbol{D}|M,\boldsymbol{\theta})]}{2}$$

To produce the estimates of

$$\mathbb{E}_{p_{\beta_t}(\boldsymbol{\theta})}[\log p(\boldsymbol{D}|M,\boldsymbol{\theta})]$$
 for  $t \in \{0,\ldots,T\}$ 

we need to produce samples of the power-posteriors  $p_{eta_t}( heta)$ .

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On the second sampling step, called posterior shaped burn-in, we use the covariance of the current sample times some constant as the covariance of the jump distribution.

On the third step, we perform the Populational Monte Carlo Markov Chain sampling. This algorithm allows us to mix samples from different power posteriors.

Ranking with Approximate Bayesian Computation

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- 4. Repeat until some iteration limit.

The result of the algorithm is a sample of the distribution

$$p(\theta, M|d(\phi(M, \theta), \mathbf{D}) < \epsilon).$$

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We can use the accepted parameters of tolerance  $\epsilon$  and model M to estimate

$$p(M|d(\phi(M, \boldsymbol{\theta}), \boldsymbol{D}) < \epsilon).$$

# Development of SigNetMS

The SigNetMS Software

To choose a cost function, we needed to compare the ABC-SMC and Marginal Likelihood approaches for model selection.

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- ABC-SysBio is a software that implements ABC-SMC
- BioBayes is a software that implements the estimation of the marginal likelihood.

However, the usage of BioBayes in our context was cumbersome.

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# The SigNetMS software

SigNetMS is a Python package and command line software that estimates the marginal likelihood of data given a model, p(D|M).

# The input expected by SigNetMS

The input to SigNetMS includes:

- An SBML file model;
- An XML file with prior distributions of parameters;
- An XML file with experimental data;

## The output produced by SigNetMS

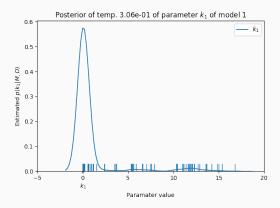
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 $Fast\ integration\ and\ parameter\ sampling$ 

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For each step we need to evaluate the likelihood function, and numerically integrate the system. That makes sampling the most time consuming procedure of SigNetMS.

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The first implementation of SigNetMS did not cope with larger instances of model selection. We tackled this problem in two ways:

- change the representation of the system of ordinary differential equations;
- implement parallelization.

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We used SymPy to provide automatically generated code that allowed us to create a C function to represent the system of ODEs.

# Comparing the representation of the system of ordinary differential equations

	Average time (seconds) to perform a sequence of integrations	
Number of Integrations	String Evaluation	sympy.autowrap
10	2.98	0.9
100	35.3	6.6
200	72.1	13.1
400	139.1	26.9

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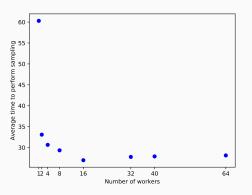
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# **Experiments and Results**

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- Comparison between ABC-SysBio and SigNetMS
- Solving model selection as a feature selection instance

 ${\tt Comparison \ between \ ABC-SySBio \ and \ SigNetMS}$ 

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# Choosing a software for model selection

ABC-SysBio and SigNetMS use different Bayesian approaches for model selection. The first creates an estimate of p(M|D), and the second creates an estimate of p(D|M) (the marginal likelihood).

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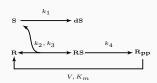
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- For one of the models, choose a set of parameter values and time steps and simulate data.
- Add Gaussian noise to the simulations. Repeat two more times to generate three observations of the system.
- Neglect chosen parameter values and define prior distributions for every parameter.
- Rank the four models.

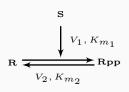
#### The instance

This instance is originally from Vyshemirsky and Girolami (2007), in which they present results of Annealing Melting Integration.

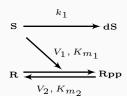
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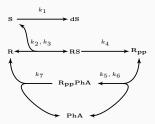
The "correct" model



The incorrect model

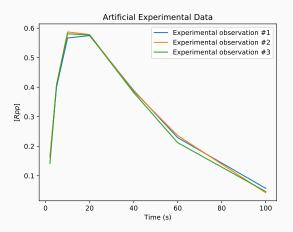


The simplification model



The generalization model

### The instance



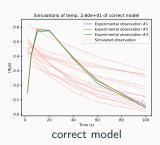
The ABC-SysBio software returned the following ranking of models:

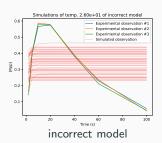
1. incorrect model

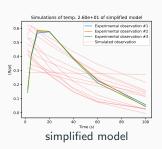
- 1. incorrect model
- 2. simplification model

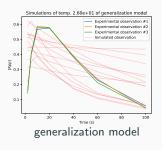
- 1. incorrect model
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- 3. generalization model

- 1. incorrect model
- 2. simplification model
- 3. generalization model
- 4. correct model









The ranking returned by SigNetMS on the first experiment is:

1. correct model

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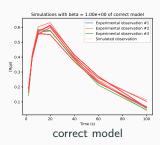
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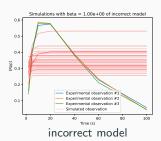
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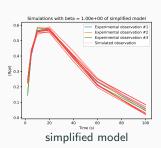
- 1. correct model
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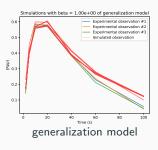
The ranking returned by SigNetMS on the first experiment is:

- 1. correct model
- 2. simplification model
- 3. generalization model
- 4. incorrect model









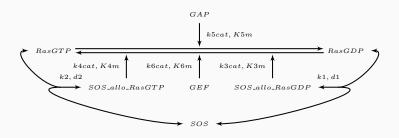
Simulations generated by the correct model  $% \left( 1\right) =\left( 1\right) \left( 1\right) \left($ 

 ${\tt Model \ selection \ as \ a \ feature \ selection \ problem}$ 

After defining that SigNetMS is our software choice for a cost function, we are able to experiment the approach of solving a model selection instance as a feature selection problem.

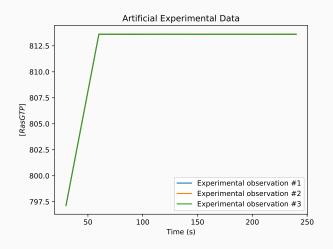
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The concentration of activated Ras was measured at the time steps of 30, 60, 90, 120, 150, 180, 210, and 240 seconds.

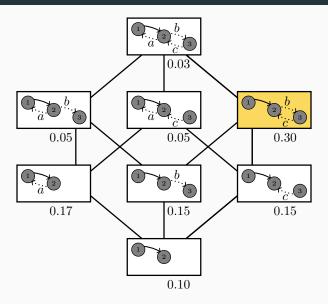
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The feature selection problem consists in finding the best subset of a set of features, S, given a cost function c.

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If we define the set of feature as a set of reactions, we can create a feature selection instance that represents a model selection instance.

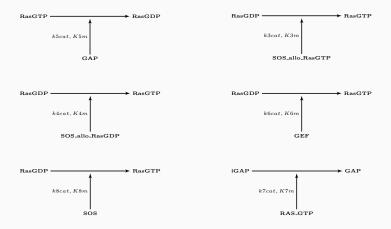


In the instance we prepared, the base model has zero reactions,

In the instance we prepared, the base model has zero reactions, and the set of features  ${\cal S}$  is composed by 10 reactions,

In the instance we prepared, the base model has zero reactions, and the set of features S is composed by 10 reactions, 8 of them present on the correct model.





The search space  $\mathcal{P}(S)$ , has  $2^{10}$ . Therefore, a heuristic is necessary to traverse the space.

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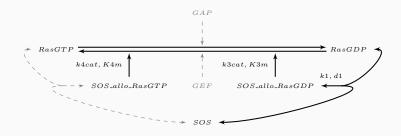
In the SFS procedure, we start from the bottom of the search space.

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Characteristic Vector	Score	Cost function time (seconds)
000000000	330721.05	851.3
0010000000	245681.93	1083.4
0010010000	211.62	4257.4
0011010000	-1.32	5007.71
0011011000	-4.27	4458.7
0111011000	-7.90	5035.7

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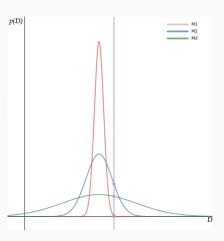


Simulations generated by the found model

Simulations generated by the correct model  $% \left( 1\right) =\left( 1\right) \left( 1\right) \left($ 

In this experiment, we experienced a known feature of marginal likelihood approaches:

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# **Conclusions**

The main contributions of this work are:

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- the implementation of the SigNetMS software;
- the comparison between SigNetMS and ABC-SysBio;
- the experimentation of feature selection on model selection using a marginal likelihood approach to define the cost function.

We also suggest a few topics for future related work:

efficiency improvements on SigNetMS;

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- treatment of numerical instabilities on numerical integrations of SigNetMS;
- solving the model selection problem as a U-Curve problem;
- experimentation on heterogeneous conditions of experimental measurements;
- application of the methodology on real instances.

Thank you!