

Identification of cell signaling pathways based on biochemical reaction kinetics repositories

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Laboratório Especial de Ciclo Celular, Instituto Butantan
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Introduction

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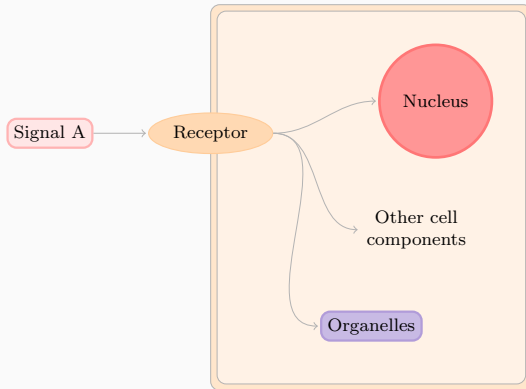
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Understanding the functioning of cell signaling is important in many biological areas.

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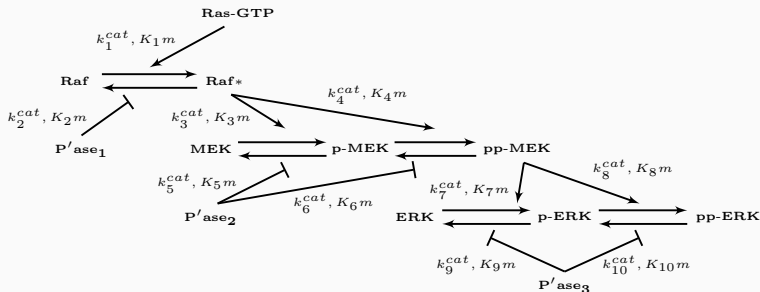
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We call the path of a signal a **cell signaling pathway**.

Cell Signaling Pathways

A cell signaling network can be characterized by a sequence of chemical reactions



Mathematical Models of Signaling Networks

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Using biochemical and enzymatic kinetics, we can model the concentration change of chemical species over time of a pathway.

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- a model composed by a set of chemical reactions that are relevant for the biological experiment;
- information about the reaction rate constants of the model.

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One can search for the set of chemical reactions relevant for a biological experiment in repositories like the Kyoto Encyclopedia of Genes and Genomes (KEGG). However, the pathway maps from KEGG may be incomplete or have impertinent reactions for the biological experiment of interest.

Hence, it is desirable to construct a method that can systematically modify these models and choose the one that better represents the experiment.

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On her work, the problem of identification of cell signaling pathways is treated as a feature selection problem.

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Wu's Search Algorithm for Feature Selection

The search algorithm used by Wu is the Sequential Forward Selection (SFS).

Wu's Cost Function for Feature Selection

Wu defines the cost function as the minimum distance between experimental and simulated data.

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- the database of interactions used could be more nearly complete;
- the search algorithm could also consider removing interactions;
- the cost function could implement a proper penalization of models;

What we Propose on this Project

We propose to create a methodology that uses a feature selection approach for identification of signaling pathways, tackling the difficulty of penalizing complex models.

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We intend to use Bayesian approaches of model selection that allow us to create estimates of $p(M|D)$ or $p(D|M)$.

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- Observe the surface induced by the cost function over the search space.

Fundamental Concepts

Kinetics Modeling of Chemical Reactions

Mathematical Modeling of Reactions

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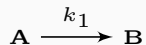
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In this project we use three possible models of kinetics of an interaction:

- first order interaction kinetics;
- second order interaction kinetics;
- Michaelis-Menten enzymatic kinetics.

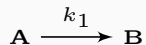
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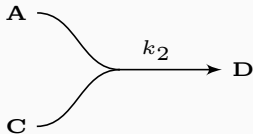


has rate of:

$$k_1[\mathbf{A}].$$

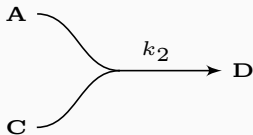
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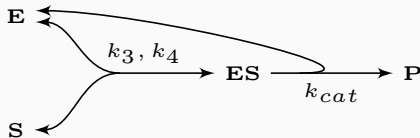


has rate of:

$$k_2[A][C].$$

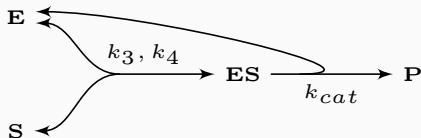
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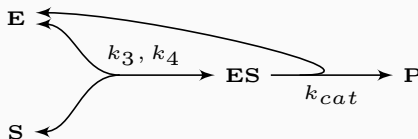
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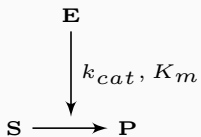
An enzymatic reaction:



Can be divided in two first order reactions plus a second order reaction. However, with the appropriate assumptions, it is possible to use a Michaelis-Menten simplification of this reaction.

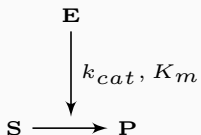
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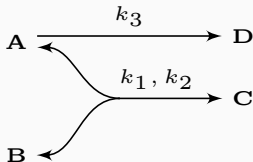


and it has rate of:

$$k_{cat} \frac{[\mathbf{E}][\mathbf{S}]}{K_M + [\mathbf{S}]}.$$

Kinetics of a System of Reactions

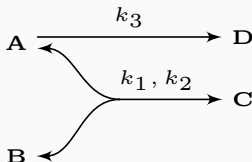
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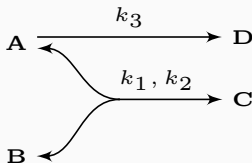


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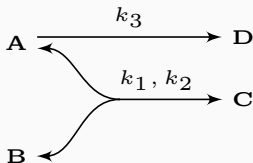


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- $A \longrightarrow D$, with rate $k_3[A]$.

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Then, the differential equation that models the concentration change of A is:

$$\frac{d[A]}{dt} = -k_1[A][B] + k_2[C] - k_3[A].$$

Bayesian Methods for Biochemical Model Selection

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For both methods, we resort to Metropolis-Hastings algorithm to generate samples of distributions.

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5. Increase t by one and repeat from Step 2 if not reached iteration limit.

Model Selection

Ranking with Marginal Likelihood Estimation

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$$p(D|M, \theta) = p_{\mathcal{N}_{(\vec{0}, \Sigma)}}(\phi(M, \theta) - D).$$

Where $\phi(M, \theta)$ is the simulated observation.

Marginal Likelihood of Data

We can marginalize the likelihood to obtain:

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Calculating this integral is hard, therefore we resort to estimating another integral.

Power-posterior distributions

We define a power-posterior distribution as:

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and that

$$p_{\beta=1}(\theta) = \frac{p(D, \theta|M)}{\int_{\Theta} p(D, \theta|M) d\theta} = \frac{p(\theta|D, M)p(D|M)}{p(D|M)} = p(\theta|D, M).$$

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$$\ln p(D|M) = \int_0^1 \mathbb{E}_{p_\beta(\theta)} [\ln p(D|\theta, M)] d\beta.$$

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$$\sum_{t=0}^{T-1} (\beta_{t+1} - \beta_t) \frac{\mathbb{E}_{p_{\beta_{t+1}}(\theta)}[\log p(D|M, \theta)] + \mathbb{E}_{p_{\beta_t}(\theta)}[\log p(D|M, \theta)]}{2}$$

Estimating the Thermodynamic Integral

To produce the estimates of

$$\mathbb{E}_{p_{\beta_t}(\theta)}[\log p(D|M, \theta)] \text{ for } t \in \{0, \dots, T\}$$

we need to produce samples of the power-posteriors $p_{\beta_t}(\theta)$.

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Sampling from the Power-posteriors

On the second sampling step, called **posterior shaped burn-in**, we use the covariance of the current sample times some constant as the covariance of the jump distribution.

Sampling from the Power-posteriors

On the third step, we perform the **Populational Monte Carlo Markov Chain** sampling. This algorithm allows us to mix samples from different power posteriors.

Ranking with Approximate Bayesian Computation

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4. Repeat until some iteration limit.

The result of the algorithm is a sample of the distribution

$$p(\theta, M | d(\phi(M, \theta), D) < \epsilon).$$

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We can use the accepted parameters of tolerance ϵ and model M to estimate

$$p(M|d(\phi(M, \theta)) < \epsilon).$$

Development of SigNetMS

The SigNetMS Software

A software that estimates the marginal likelihood

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- ABC-SysBio is a software that implements ABC-SMC
- BioBayes is a software that implements the estimation of the marginal likelihood.

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Therefore, we decided to implement **SigNetMS**.

The SigNetMS software

SigNetMS is a Python package and command line software that estimates the marginal likelihood of a model given experimental data.

The input expected by SigNetMS

The input to SigNetMS includes:

- An SBML file model;
- An XML file with prior distributions of parameters;
- An XML file with experimental data;

The output produced by SigNetMS

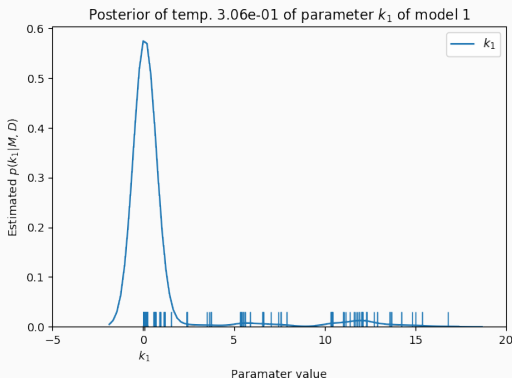
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Fast integration and parameter sampling

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For each step we need to evaluate the likelihood function, and **numerically integrate the system**. That makes sampling the most time consuming procedure of SigNetMS.

Our first implementation was not very efficient

The first implementation of SigNetMS did not cope with larger instances of model selection.

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The first implementation of SigNetMS did not cope with larger instances of model selection. We tackled this problem in two ways:

- change the representation of the system of ordinary differential equations;
- implement parallelization.

Changing the representation of the system of ordinary differential equations

In the first implementation of SigNetMS, systems of EDOs were represented as an array of strings.

Changing the representation of the system of ordinary differential equations

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We used SymPy to provide automatically generated code that allowed us to create a C function to represent the system of ODEs.

Comparing the representation of the system of ordinary differential equations

| Number of Integrations | Average time (seconds) to perform a sequence of integrations | |
|------------------------|--|-----------------------------|
| | String Evaluation | <code>sympy.autowrap</code> |
| 10 | 2.98 | 0.9 |
| 100 | 35.3 | 6.6 |
| 200 | 72.1 | 13.1 |
| 400 | 139.1 | 26.9 |

Parallelizing the sampling of multiple power posteriors

The first two phases of the sampling procedure occurs independently between different power posteriors.

Parallelizing the sampling of multiple power posteriors

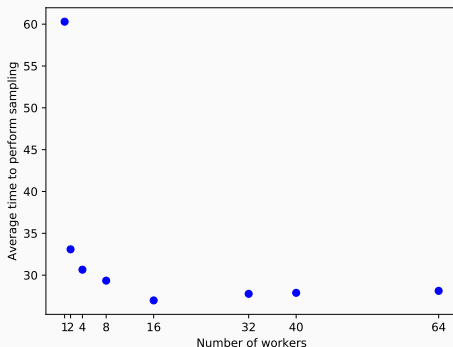
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We used the map pattern to parallelize the sampling of different power posterior distributions.

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Experiments and Results

We prepared two experiments in this work:

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- Comparison between ABC-SysBio and SigNetMS
- Solving model selection as a feature selection instance

Comparison between ABC-SySBio and SigNetMS

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Choosing a software for model selection

ABC-SysBio and SigNetMS use different Bayesian approaches for model selection. The first creates an estimate of $p(M|D)$, and the second creates an estimate of $p(D|M)$ (the marginal likelihood).

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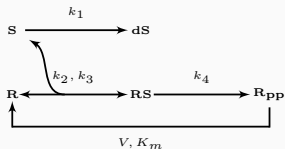
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- Rank the four models.

In this presentation we show results of the first experiment only.

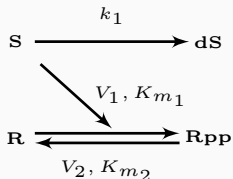
The instance

This instance is originally from Vyshemirsky and Girolami (2007), in which they present results of Annealing Melting Integration.

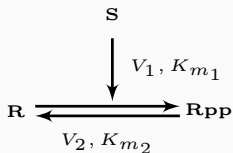
The instance



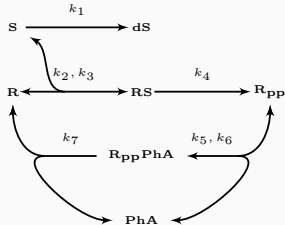
The "correct" model



The simplification model

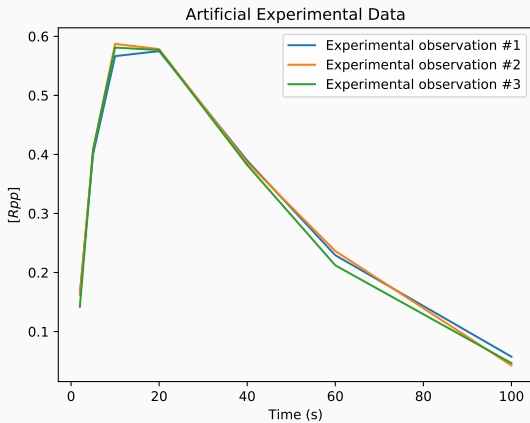


The incorrect model



The generalization model

The instance



The ABC-SysBio software returned the following ranking of models:

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2. simplification model

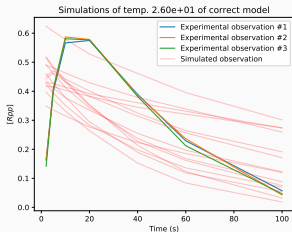
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1. incorrect model
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3. generalization model

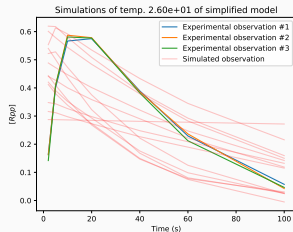
The ABC-SysBio software returned the following ranking of models:

1. incorrect model
2. simplification model
3. generalization model
4. correct model

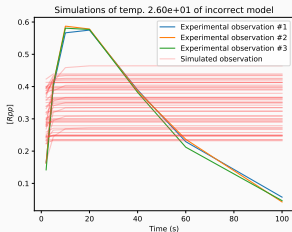
Results on ABC-SysBio



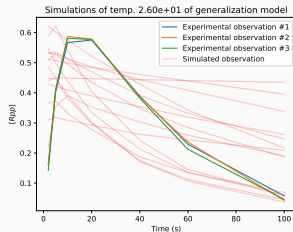
correct model



simplified model



incorrect model



generalization model

The ranking returned by SigNetMS on the first experiment is:

1. correct model

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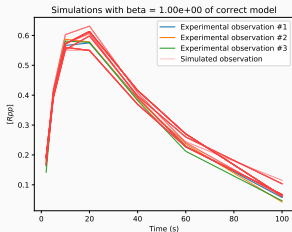
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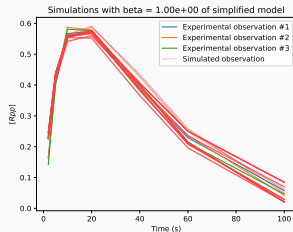
The ranking returned by SigNetMS on the first experiment is:

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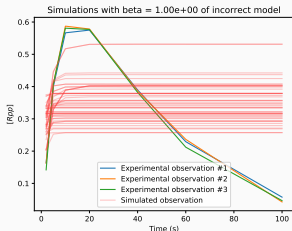
Results on SigNetMS



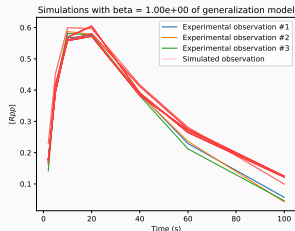
correct model



simplified model



incorrect model



generalization model

Simulations generated by the correct model

Model selection as a feature selection problem

Model selection as a feature selection problem

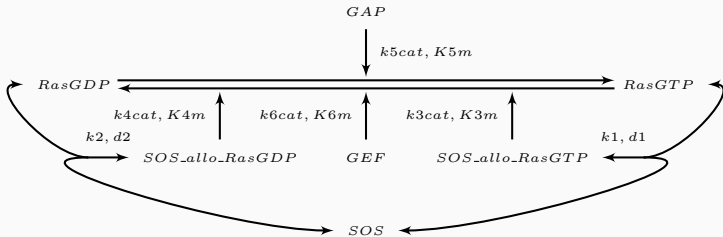
After defining that SigNetMS is our software choice for a cost function, we are able to experiment the approach of solving a model selection instance as a feature selection problem.

The model selection instance

We proposed a Ras switch pathway to experiment on.

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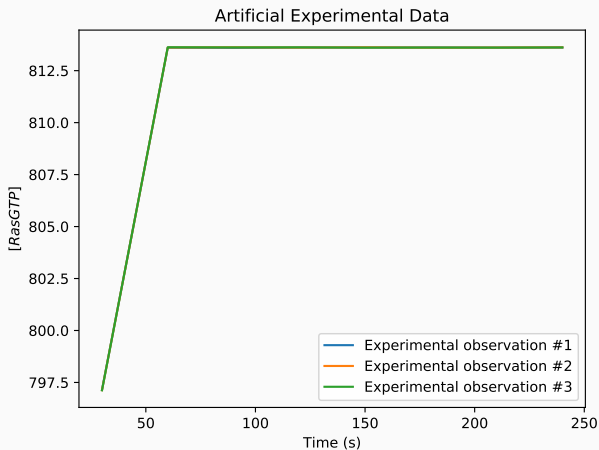


The model selection instance

The concentration of activated Ras was measured at the time steps of 30, 60, 90, 120, 150, 180, 210, and 240 seconds.

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Model selection as a feature selection problem

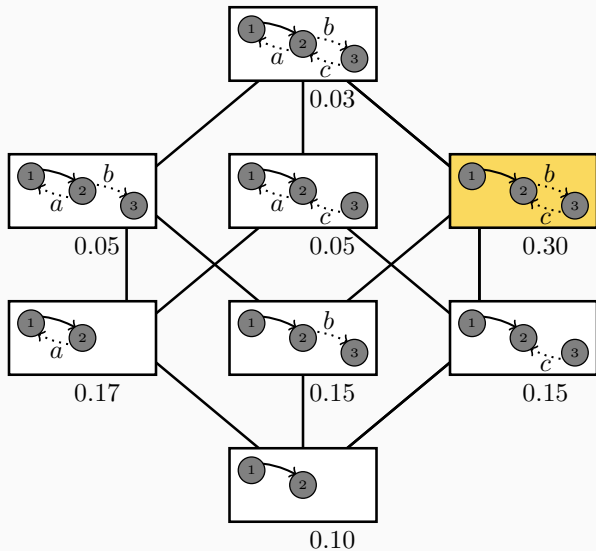
The feature selection problem consists in finding the best subset of a set of features, S , given a cost function c .

Model selection as a feature selection problem

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If we define the set of feature as a set of reactions, we can create a feature selection instance that represents a model selection instance.

Model selection as a feature selection problem



The set of features of our experiment

In the instance we prepared, the base model has zero reactions,

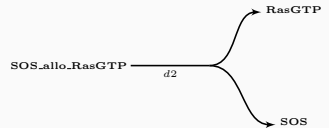
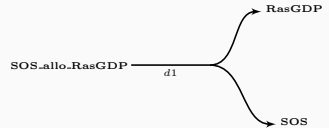
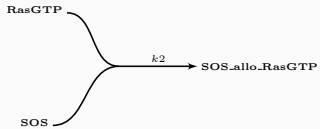
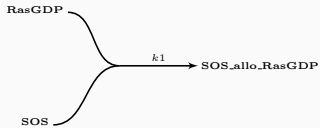
The set of features of our experiment

In the instance we prepared, the base model has zero reactions, and the set of features S is composed by 10 reactions,

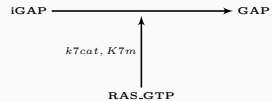
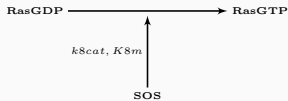
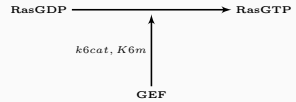
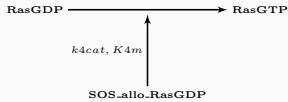
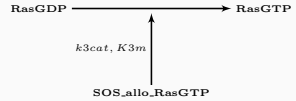
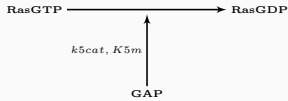
The set of features of our experiment

In the instance we prepared, the base model has zero reactions, and the set of features S is composed by 10 reactions, 8 of them present on the correct model.

The set of features of our experiment



The set of features of our experiment



Finding a solution

The search space $\mathcal{P}(S)$, has 2^{10} . Therefore, a heuristic is necessary to traverse the space.

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Finding a solution

In the SFS procedure, we start from the bottom of the search space.

Finding a solution

In the SFS procedure, we start from the bottom of the search space. And for every iteration, we select the best adjacent model that has one more reaction.

Results of the search

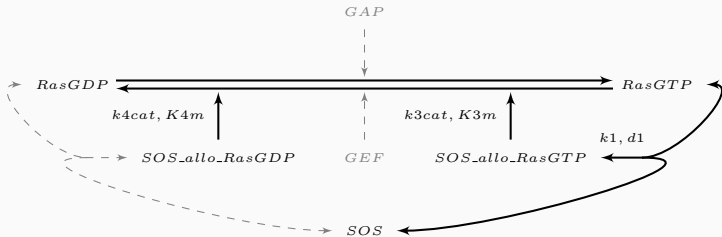
| Characteristic Vector | Score | Cost function time (seconds) |
|-----------------------|-----------|------------------------------|
| 0000000000 | 330721.05 | 851.3 |
| 0010000000 | 245681.93 | 1083.4 |
| 0010010000 | 211.62 | 4257.4 |
| 0011010000 | -1.32 | 5007.71 |
| 0011011000 | -4.27 | 4458.7 |
| 0111011000 | -7.90 | 5035.7 |

Results of the search

The found model is contained in the correct model:

Results of the search

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Results of the search

Simulations generated by the found model

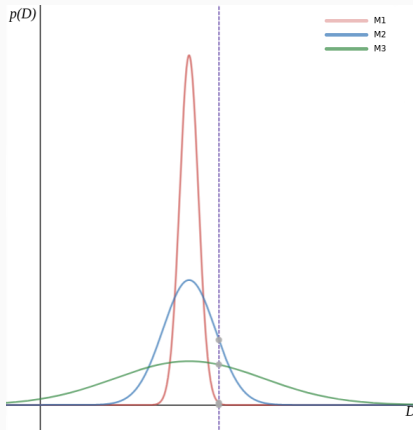
Simulations generated by the correct model

Results of the search

In this experiment, we experienced a known feature of marginal likelihood approaches:

Results of the search

In this experiment, we experienced a known feature of marginal likelihood approaches: **intermediate complexity models are preferred**.



Conclusions

Thank you!