In[*]:= PROVA 2 COMPUTAÇÃO GRÁFICA QUESTÃO 1

```
ln[*]:= P = \{0, 1, 0, 0\}
      \mbox{quaternion} \left[ \, \left\{ \, x_{-} , \, y_{-} , \, z_{-} , \, w_{-} \, \right\} \, \right] \; := \; \left\{ \, x^2 - y^2 - z^2 - w^2 , \, 2 \, x \, y , \, 2 \, x \, z \, , \, 2 \, x \, w \, \right\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n + + ] ]
      iteration[
        {0,
         0,
         0,
         0}]
Out[\bullet]= {0, 1, 0, 0}
      n= 1, q= \{0, 1, 0, 0\}, q= 1.
      n= 2,q= \{-1, 1, 0, 0\}, q= 1.41421
      n= 3, q= \{0, -1, 0, 0\}, q= 1.
      n= 4, q= \{-1, 1, 0, 0\}, \q = 1.41421
      n = 5, q = \{0, -1, 0, 0\}, q = 1.
      n = 6, q = \{-1, 1, 0, 0\}, q = 1.41421
      n= 7, q= \{0, -1, 0, 0\}, q= 1.
      n= 8, q= \{-1, 1, 0, 0\}, q = 1.41421
      n= 9, q= \{0, -1, 0, 0\}, q= 1.
      n= 10, q= \{-1, 1, 0, 0\}, q = 1.41421
      n= 11,q= \{0,-1,0,0\}, q= 1.
      n= 12, q= \{-1, 1, 0, 0\}, q= 1.41421
      n= 13, q= \{0, -1, 0, 0\}, q= 1.
      n= 14, q= \{-1, 1, 0, 0\}, q= 1.41421
      n= 15, q= \{0, -1, 0, 0\}, q= 1.
      n= 16, q= \{-1, 1, 0, 0\}, q= 1.41421
      n= 17, q= \{0, -1, 0, 0\}, q= 1.
      n= 18, q= \{-1, 1, 0, 0\}, q= 1.41421
      n= 19, q= \{0, -1, 0, 0\}, q= 1.
      n= 20, q= \{-1, 1, 0, 0\}, q= 1.41421
```

+

+

IN[*]:= FAZENDO A ITEREÇÃO POR 20 PASSOS A SEQUENCIA 0 + 1 i NÃO AUMENTA EM MÓDULO,
DESTE MODO O PONTO 0 + 1 i É UM PONTO PRISIONEIRO

QUESTÃO 2

```
ln[-]:= P = \{0.5, 0.1, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.5, 0.1, 0, 0}
       n= 1,q= \{0.5, 0.1, 0, 0\}, q= 0.509902
       n= 2,q= \{0.74, 0.2, 0., 0.\}, q= 0.766551
       n= 3,q= \{1.0076, 0.396, 0., 0.\}, q= 1.08262
       n= 4, q= \{1.35844, 0.898019, 0., 0.\}, q= 1.62844
       n= 5,q= \{1.53893, 2.53981, 0., 0.\}, q= 2.96967
       n=6,q=\{-3.58236, 7.91717, 0., 0.\},\q\=8.68993
       n= 7, q= \{-49.3482, -56.6243, 0., 0.\}, \q = 75.1103
       n= 8, q= \{-770.564, 5588.72, 0., 0.\}, q = 5641.59
       n= 9,q= \{-3.064 \times 10^7, -8.61293 \times 10^6, 0., 0.\},\q\= 3.18276 \times 10^7
       n= 10,q= \{8.64628 \times 10^{14}, 5.27801 \times 10^{14}, 0., 0.\}, \q\= 1.01299 \times 10<sup>15</sup>
       n= 11,q= \{4.69008 \times 10^{29}, 9.12703 \times 10^{29}, 0., 0.\}, \q\= 1.02616×10<sup>30</sup>
       n= 12,q= \{-6.13058 \times 10^{59}, 8.5613 \times 10^{59}, 0., 0.\},\q\= 1.05299\times 10^{60}
       n= 13,q= \{-3.57118 \times 10^{119}, -1.04971 \times 10^{120}, 0., 0.\}, \q\= 1.1088 \times 10^{120}
       n= 14,q= \{-9.74366 \times 10^{239}, 7.49744 \times 10^{239}, 0., 0.\}, \q\= 1.22943 \times 10^{240}
       n= 15,q= \{3.872729684870037 \times 10^{479}, -1.461049105567651 \times 10^{480}, 0., 0.\}
        \sqrt{q} = 1.511504165062329 \times 10^{480}
       n= 16,q= \{-1.984684136759296 \times 10^{960}, -1.131649648436931 \times 10^{960}, 0., 0.\}
        \q = 2.284644841000767 \times 10^{960}
       n= 17,q= \{2.658340195896561 \times 10^{1920}, 4.491934211244023 \times 10^{1920}, 0., 0.\}
        \sqrt{q} = 5.219602049511422 \times 10^{1920}
       n= 18,q= \{-1.311070036102510 \times 10^{3841}, 2.388217854214581 \times 10^{3841}, 0., 0.\}
        \sqrt{q} = 2.724424555526384 \times 10^{3841}
       n= 19,q= \{-3.984679879623459 \times 10^{7682}, -6.262241736691538 \times 10^{7682}, 0., 0.\}
        \sqrt{q} = 7.422489158755133 \times 10^{7682}
       n= 20,q= \left\{-2.333799782568543\times10^{15\,365}\text{, }4.990605729906608\times10^{15\,365}\text{, }0.\text{, }0.\right\}
        \sqrt{q} = 5.509334531183748 \times 10^{15365}
```

APÓS 20 PASSOS O MÓDULO CRESCE TENDENDO PARA INFINITO, LOGO 0.5 + 0.1 i NÃO É PRISIONEIRO **QUESTÃO 3**

```
ln[-]:= P = \{0.2, 0.2, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, z_{w_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
      iteration[
       {0,
        0,
        0,
        0}]
Out[*]= {0.2, 0.2, 0, 0}
      n= 1,q= \{0.2, 0.2, 0, 0\}, q= 0.282843
      n= 2, q= \{0.2, 0.28, 0., 0.\}, q= 0.344093
      n= 3, q= \{0.1616, 0.312, 0., 0.\}, q= 0.351367
      n=4,q=\{0.128771,0.300838,0.,0.\},\q=0.327239
      n=5,q=\{0.126078,0.277478,0.,0.\},\q=0.304778
      n = 7, q = \{0.146411, 0.274998, 0., 0.\}, q = 0.311545
      n= 8,q= \{0.145812, 0.280525, 0., 0.\}, q= 0.316158
      n= 10, q= \{0.140909, 0.280353, 0., 0.\}, q= 0.313773
      n= 11, q= \{0.141258, 0.279009, 0., 0.\}, q= 0.312729
      n= 12, q= \{0.142108, 0.278824, 0., 0.\}, q= 0.31295
      n= 13, q= \{0.142452, 0.279246, 0., 0.\}, q= 0.313482
      n= 14, q= \{0.142314, 0.279558, 0., 0.\}, q= 0.313697
      n= 16,q= {0.142033, 0.279454, 0., 0.},\q\= 0.313477
      n = 17, q = \{0.142079, 0.279383, 0., 0.\}, q = 0.313435
      n= 18,q= {0.142131, 0.279389, 0., 0.},q = 0.313464
      n= 19, q= \{0.142143, 0.27942, 0., 0.\}, q= 0.313497
      n = 20, q = \{0.142129, 0.279435, 0., 0.\}, q = 0.313504
      PONTO 0.2 + 0.2 i É UM PONTO PRISIONEIRO
ln[169] = P = \{-0.3, 0.2, 0, 0\}
                                                                                             +
      quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
      iteration[
       {0,
        0,
        0,
        0}]
Out[169]= \{-0.3, 0.2, 0, 0\}
```

```
n= 1,q= \{-0.3, 0.2, 0, 0\}, q= 0.360555
     n= 2, q= \{-0.25, 0.08, 0., 0.\}, q= 0.262488
     n= 3, q= \{-0.2439, 0.16, 0., 0.\}, q= 0.291697
     n=4,q=\{-0.266113, 0.121952, 0., 0.\}, q=0.292726
     n=5,q=\{-0.244056,0.135094,0.,0.\},\q=0.278951
     n = 6, q = \{-0.258687, 0.134059, 0., 0.\}, \\ q = 0.29136
     n=7,q=\{-0.251053, 0.130641, 0., 0.\},\q=0.28301
     n=8,q=\{-0.25404, 0.134404, 0., 0.\}, q=0.287403
     n=9,q=\{-0.253528, 0.131712, 0., 0.\}, q=0.2857
     n=10,q=\{-0.253071, 0.133215, 0., 0.\}, q=0.285992
     n= 11,q= {-0.253701, 0.132574, 0., 0.},q= 0.286252
     n= 12,q= \{-0.253212, 0.132731, 0., 0.\}, q= 0.285891
     n= 13, q= \{-0.253501, 0.132782, 0., 0.\}, q= 0.286171
     n= 14,q= {-0.253368, 0.132679, 0., 0.},q= 0.286005
     n=15,q=\{-0.253408, 0.132767, 0., 0.\}, q=0.286082
     n= 16, q= \{-0.253411, 0.132712, 0., 0.\}, q= 0.286059
     n= 17, q= \{-0.253395, 0.132739, 0., 0.\}, q= 0.286057
     n= 18, q= \{-0.25341, 0.132729, 0., 0.\}, q= 0.286066
     n= 19, q= \{-0.2534, 0.13273, 0., 0.\}, q= 0.286058
     n=20,q=\{-0.253406,0.132732,0.,0.\},\q\=0.286063
     O PONTO - 0.3 + 0.3 i É PRISIONEIRO
ln[ \circ ] := P = \{-0.6, 0.4, 0, 0\}
     quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
     iteration[
       {0,
       0,
        0,
        0}]
Out[\circ]= {-0.6, 0.4, 0, 0}
```

```
n= 1, q= \{-0.6, 0.4, 0, 0\}, \q = 0.72111
n= 2, q= \{-0.4, -0.08, 0., 0.\}, \q = 0.407922
n= 3, q= \{-0.4464, 0.464, 0., 0.\}, \q = 0.64387
n=4,q=\{-0.616023, -0.0142592, 0., 0.\}, q=0.616188
n=5,q=\{-0.220719,0.417568,0.,0.\},\q=0.472313
n=6,q=\{-0.725646,0.21567,0.,0.\},\q\=0.757018
n=7,q=\{-0.119951, 0.0870003, 0., 0.\},\q=0.14818
n=8,q=\{-0.593181, 0.379128, 0., 0.\}, q=0.70399
n=9,q=\{-0.391875, -0.0497834, 0., 0.\}, q=0.395024
n= 10,q= {-0.448912, 0.439018, 0., 0.},\q\= 0.6279
n= 11,q= {-0.591214, 0.00583894, 0., 0.},\q\= 0.591243
n= 12, q= \{-0.2505, 0.393096, 0., 0.\}, q= 0.466127
n= 13, q= \{-0.691774, 0.203059, 0., 0.\}, \q = 0.720961
n= 14,q= {-0.162681, 0.119058, 0., 0.},\q\= 0.201594
n=15,q=\{-0.58771, 0.361263, 0., 0.\},\q=0.689865
n= 16, q= \{-0.385108, -0.0246354, 0., 0.\}, \q = 0.385896
n= 17,q= {-0.452298, 0.418975, 0., 0.},q = 0.616533
n= 18, q= \{-0.570966, 0.0209969, 0., 0.\}, q= 0.571352
n= 19, q= \{-0.274439, 0.376023, 0., 0.\}, q= 0.465521
n=20,q=\{-0.666077, 0.193609, 0., 0.\}, q=0.693644
APÓS 20 PASSOS, A SEQUENCIA DA ITERAÇÃO NÃO AUMENTA EM MÓDULO,
PORTANTO O PONTO P = -0.6 + 0.4 i \in UM PONTO PRISIONEIRO (P)
```

```
ln[\cdot]:= P = \{-1.3, 0.1, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
          n++]]
      iteration[
       {0,
        0,
        0,
        0}]
Out[\bullet]= {-1.3, 0.1, 0, 0}
```

```
n= 1, q= \{-1.3, 0.1, 0, 0\}, q= 1.30384
      n= 2,q= \{0.38, -0.16, 0., 0.\}, \q = 0.412311
      n= 3,q= \{-1.1812, -0.0216, 0., 0.\}, q= 1.1814
      n=4,q=\{0.0947669, 0.151028, 0., 0.\},\q=0.178298
      n=5,q=\{-1.31383, 0.128625, 0., 0.\}, q=1.32011
      n=6,q=\{0.409601, -0.237982, 0., 0.\}, q=0.473718
      n= 7, q= \{-1.18886, -0.0949556, 0., 0.\}, q= 1.19265
      n= 8, q= \{0.104377, 0.325778, 0., 0.\}, q= 0.342091
      n= 9, q= \{-1.39524, 0.168007, 0., 0.\}, \q = 1.40532
      n= 10,q= {0.61846, -0.36882, 0., 0.},\q\= 0.720084
      n= 11,q= \{-1.05354, -0.356201, 0., 0.\}, q= 1.11212
      n= 12, q= \{-0.316941, 0.850541, 0., 0.\}, q= 0.907673
      n= 13, q= \{-1.92297, -0.439142, 0., 0.\}, q= 1.97247
      n= 14, q= \{2.20496, 1.78891, 0., 0.\}, q= 2.83937
      n= 15, q= \{0.361643, 7.98895, 0., 0.\}, q= 7.99713
      n= 16, q= \{-64.9926, 5.87829, 0., 0.\}, q= 65.2579
      n= 17, q= \{4188.18, -763.991, 0., 0.\}, q= 4257.29
      n= 18,q= \{1.69572 \times 10^7, -6.39947 \times 10^6, 0., 0.\}, \q\= 1.81245 \times 10^7
      n= 19,q= \{2.46593 \times 10^{14}, -2.17034 \times 10^{14}, 0., 0.\}, \q\= 3.28499 \times 10<sup>14</sup>
      n= 20,q= \{1.37043 \times 10^{28}, -1.07038 \times 10^{29}, 0., 0.\}, \q\= 1.07912 \times 10^{29}
      O PONTO - 1.3 + 0.1 i APÓS 18 ITERAÇÕES TENDE AO INFINITO, LOGO NÃO É PONTO PRISIONEIRO P
ln[ \circ ] := P = \{-1.7, 0.1, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
          n++]]
      iteration[
       {0,
        0,
        0,
        0}]
Out[\circ]= {-1.7, 0.1, 0, 0}
```

```
n= 1, q= \{-1.7, 0.1, 0, 0\}, q= 1.70294
      n= 2,q= \{1.18, -0.24, 0., 0.\}, q= 1.20416
      n= 3,q= \{-0.3652, -0.4664, 0., 0.\}, \q = 0.592368
      n= 4, q= \{-1.78416, 0.440659, 0., 0.\}, q= 1.83777
      n= 5, q= \{1.28904, -1.47241, 0., 0.\}, q= 1.95694
      n=6,q=\{-2.20637, -3.69599, 0., 0.\}, q=4.30446
      n= 7, q= \{-10.4923, 16.4094, 0., 0.\}, q= 19.4771
      n=8,q=\{-160.88, -344.244, 0., 0.\}, q=379.982
      n= 9, q= \{-92622.8, 110764., 0., 0.\}, q= 144387.
      n= 10,q= \{-3.68972 \times 10^9, -2.05186 \times 10^{10}, 0., 0.\}, \q\= 2.08477 \times 10^{10}
      n= 11,q= \{-4.07398 \times 10^{20}, 1.51416 \times 10^{20}, 0., 0.\}, \q\= 4.34626 \times 10^{20}
      n= 12,q= \{1.43046 \times 10^{41}, -1.23373 \times 10^{41}, 0., 0.\},\q\= 1.889×10<sup>41</sup>
      n= 13,q= \{5.24138 \times 10^{81}, -3.52961 \times 10^{82}, 0., 0.\}, \q\= 3.56831×10<sup>82</sup>
      n= 14,q= \{-1.21834 \times 10^{165}, -3.70001 \times 10^{164}, 0., 0.\}, \q\= 1.27329 \times 10^{165}
      n= 15,q= \{1.347457708064954 \times 10^{330}, 9.01574774938275 \times 10^{329}, 0., 0.\}
        \sqrt{q} = 1.621258569700916 \times 10^{330}
      n= 16,q= \{1.002805200218656 \times 10^{660}, 2.429667759775010 \times 10^{660}, 0., 0.\}
        \q = 2.628479349828660 \times 10^{660}
      n= 17,q= \{-4.897667153304539 \times 10^{1320}, 4.872966928611985 \times 10^{1320}, 0., 0.\}
        \sqrt{q} = 6.908903692475696 \times 10^{1320}
      n= 18,q= \{2.413368572120610 \times 10^{2639}, -4.773234013080444 \times 10^{2641}, 0., 0.\}
        \sqrt{q} = 4.773295023190431 \times 10^{2641}
      n= 19,q= \{-2.278318050884155 \times 10^{5283}, -2.303914590909096 \times 10^{5281}, 0., 0.\}
        \q = 2.278434537841453 \times 10^{5283}
      n = 20, q = \{5.190202338740357 \times 10^{10566}, 1.049810040032716 \times 10^{10565}, 0., 0.\}
        \sqrt{q} = 5.191263943228798 \times 10^{10566}
      O PONTO - 1.7 + 0.1 i APÓS 18 ITERAÇÕES TENDE AO INFINITO, LOGO NÃO É PONTO PRISIONEIRO P
ln[\cdot]:= P = \{0.2, -0.2, 0, 0\}
                                                                                                                        +
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {0.2, -0.2, 0, 0}
```

```
n= 1, q= \{0.2, -0.2, 0, 0\}, q= 0.282843
     n= 2,q= \{0.2, -0.28, 0., 0.\}, q= 0.344093
     n= 3,q= \{0.1616, -0.312, 0., 0.\}, q= 0.351367
     n=4,q=\{0.128771, -0.300838, 0., 0.\}, q=0.327239
     n=5,q=\{0.126078,-0.277478,0.,0.\},\q=0.304778
     n=6,q=\{0.138902, -0.269968, 0., 0.\}, q=0.303605
     n= 7, q= \{0.146411, -0.274998, 0., 0.\}, q= 0.311545
     n= 8, q= \{0.145812, -0.280525, 0., 0.\}, q= 0.316158
     n= 9, q= \{0.142567, -0.281808, 0., 0.\}, q= 0.315818
     n=10,q=\{0.140909, -0.280353, 0., 0.\}, q=0.313773
     n= 11,q= {0.141258, -0.279009, 0., 0.},\q\= 0.312729
     n= 12, q= \{0.142108, -0.278824, 0., 0.\}, q= 0.31295
     n= 13, q= \{0.142452, -0.279246, 0., 0.\}, q= 0.313482
     n= 14, q= \{0.142314, -0.279558, 0., 0.\}, q= 0.313697
     n= 15, q= \{0.142101, -0.27957, 0., 0.\}, q= 0.313611
     n= 16, q= \{0.142033, -0.279454, 0., 0.\}, q= 0.313477
     n= 17, q= \{0.142079, -0.279383, 0., 0.\}, q= 0.313435
     n= 18, q= \{0.142131, -0.279389, 0., 0.\}, q= 0.313464
     n= 19, q= \{0.142143, -0.27942, 0., 0.\}, q= 0.313497
     n=20,q=\{0.142129,-0.279435,0.,0.\},\q=0.313504
     O PONTO 0.2 - 0.2 i É PRISIONEIRO DE P
ln[ \circ ] := P = \{-0.3, -0.2, 0, 0\}
     quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}]
Out[\circ]= {-0.3, -0.2, 0, 0}
```

Out[\circ]= {-0.6, -0.4, 0, 0}

 $n= 1, q= \{-0.6, -0.4, 0, 0\}, q= 0.72111$

```
n= 2, q= \{-0.4, 0.08, 0., 0.\}, q= 0.407922
     n= 3,q= \{-0.4464, -0.464, 0., 0.\}, \q = 0.64387
     n=4,q=\{-0.616023, 0.0142592, 0., 0.\}, q=0.616188
     n=5,q=\{-0.220719, -0.417568, 0., 0.\}, q=0.472313
     n=6,q=\{-0.725646, -0.21567, 0., 0.\}, \q = 0.757018
     n=7,q=\{-0.119951, -0.0870003, 0., 0.\},\q=0.14818
     n= 8, q= \{-0.593181, -0.379128, 0., 0.\}, \q = 0.70399
     n = 9, q = \{-0.391875, 0.0497834, 0., 0.\}, \\ | q = 0.395024
     n=10,q=\{-0.448912, -0.439018, 0., 0.\}, q=0.6279
     n=11,q=\{-0.591214, -0.00583894, 0., 0.\}, q=0.591243
     n= 12, q= \{-0.2505, -0.393096, 0., 0.\}, q= 0.466127
     n= 13, q= \{-0.691774, -0.203059, 0., 0.\}, q= 0.720961
     n= 14, q= \{-0.162681, -0.119058, 0., 0.\}, q= 0.201594
     n=15,q=\{-0.58771, -0.361263, 0., 0.\}, q=0.689865
     n= 16, q= \{-0.385108, 0.0246354, 0., 0.\}, q= 0.385896
     n=17,q=\{-0.452298, -0.418975, 0., 0.\}, q=0.616533
     n= 18, q= \{-0.570966, -0.0209969, 0., 0.\}, \q = 0.571352
     n=19,q=\{-0.274439, -0.376023, 0., 0.\}, q=0.465521
     n=20,q=\{-0.666077, -0.193609, 0., 0.\}, q=0.693644
     O PONTO - 0.6 - 0.4 i É PRISIOEIRO DE (P)
ln[ \circ ] := P = \{-1.3, -0.1, 0, 0\}
     quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
     iteration[
       {0,
       0,
        0,
        0}]
Out[\circ]= {-1.3, -0.1, 0, 0}
```

```
n= 1, q= \{-1.3, -0.1, 0, 0\}, q= 1.30384
      n= 2,q= \{0.38, 0.16, 0., 0.\}, q= 0.412311
      n= 3,q= \{-1.1812, 0.0216, 0., 0.\}, q= 1.1814
      n=4,q=\{0.0947669, -0.151028, 0., 0.\}, q=0.178298
      n=5,q=\{-1.31383, -0.128625, 0., 0.\}, q=1.32011
      n=6,q=\{0.409601,0.237982,0.,0.\},\q=0.473718
      n= 7, q= \{-1.18886, 0.0949556, 0., 0.\}, q= 1.19265
      n= 8, q= \{0.104377, -0.325778, 0., 0.\}, q= 0.342091
      n=9,q=\{-1.39524, -0.168007, 0., 0.\}, q=1.40532
      n=10,q=\{0.61846,0.36882,0.,0.\},\q\=0.720084
      n= 11,q= \{-1.05354, 0.356201, 0., 0.\}, q= 1.11212
      n= 12, q= \{-0.316941, -0.850541, 0., 0.\}, q= 0.907673
      n= 13, q= \{-1.92297, 0.439142, 0., 0.\}, q= 1.97247
      n= 14, q= \{2.20496, -1.78891, 0., 0.\}, q= 2.83937
      n= 15, q= \{0.361643, -7.98895, 0., 0.\}, q= 7.99713
      n= 16, q= \{-64.9926, -5.87829, 0., 0.\}, q= 65.2579
      n= 17,q= {4188.18, 763.991, 0., 0.},\q\= 4257.29
      n= 18,q= \{1.69572 \times 10^7, 6.39947 \times 10^6, 0., 0.\}, q= 1.81245 \times 10^7
      n= 19,q= \{2.46593 \times 10^{14}, 2.17034 \times 10^{14}, 0., 0.\}, \q\= 3.28499 \times 10^{14}
      n= 20,q= \{1.37043 \times 10^{28}, 1.07038 \times 10^{29}, 0., 0.\}, q= 1.07912 \times 10^{29}\}
      APÓS 17 ITERAÇÕES O PONTO - 1.3 - 0.1 i ESCAPA AO INFINITO
ln[ \circ ] := P = \{-1.7, -0.1, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
      iteration[
       {0,
        0,
        0,
        0}]
Out[\circ]= {-1.7, -0.1, 0, 0}
```

```
n= 1, q= \{-1.7, -0.1, 0, 0\}, q= 1.70294
      n= 2,q= \{1.18, 0.24, 0., 0.\}, q= 1.20416
      n= 3,q= \{-0.3652, 0.4664, 0., 0.\}, q= 0.592368
      n=4,q=\{-1.78416, -0.440659, 0., 0.\}, q=1.83777
      n= 5, q= \{1.28904, 1.47241, 0., 0.\}, q= 1.95694
      n=6,q=\{-2.20637, 3.69599, 0., 0.\}, q=4.30446
      n= 7, q= \{-10.4923, -16.4094, 0., 0.\}, q= 19.4771
      n=8,q=\{-160.88,344.244,0.,0.\},\q=379.982
      n=9,q=\{-92622.8, -110764., 0., 0.\}, q=144387.
      n= 10,q= \{-3.68972 \times 10^9, 2.05186 \times 10^{10}, 0., 0.\},\q\= 2.08477 \times 10^{10}
      n= 11,q= \{-4.07398 \times 10^{20}, -1.51416 \times 10^{20}, 0., 0.\},\q\= 4.34626 \times 10^{20}
      n= 12,q= \{1.43046 \times 10^{41}, 1.23373 \times 10^{41}, 0., 0.\}, \q\= 1.889×10<sup>41</sup>
      n= 13,q= \{5.24138 \times 10^{81}, 3.52961 \times 10^{82}, 0., 0.\}, \q\= 3.56831×10<sup>82</sup>
      n= 14,q= \{-1.21834 \times 10^{165}, 3.70001 \times 10^{164}, 0., 0.\}, \q\= 1.27329 \times 10<sup>165</sup>
      n= 15,q= \{1.347457708064954 \times 10^{330}, -9.01574774938275 \times 10^{329}, 0., 0.\}
        \sqrt{q} = 1.621258569700916 \times 10^{330}
      n= 16,q= \{1.002805200218656 \times 10^{660}, -2.429667759775010 \times 10^{660}, 0., 0.\}
        \q = 2.628479349828660 \times 10^{660}
      n= 17,q= \{-4.897667153304539 \times 10^{1320}, -4.872966928611985 \times 10^{1320}, 0., 0.\}
        \sqrt{q} = 6.908903692475696 \times 10^{1320}
      n= 18,q= \{2.413368572120610 \times 10^{2639}, 4.773234013080444 \times 10^{2641}, 0., 0.\}
       \sqrt{q} = 4.773295023190431 \times 10^{2641}
      n= 19,q= \{-2.278318050884155 \times 10^{5283}, 2.303914590909096 \times 10^{5281}, 0., 0.\}
        \sqrt{q} = 2.278434537841453 \times 10^{5283}
      n= 20,q= \{5.190202338740357 \times 10^{10566}, -1.049810040032716 \times 10^{10565}, 0., 0.\}
        \sqrt{q} = 5.191263943228798 \times 10^{10566}
      APÓS 9 ITERAÇÕES O PONTO - 1.7 - 0.1 i ESCAPA AO INFINITO
      QUESTÃO 5
ln[\cdot]:= P = \{-2.4, 1.4, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, z_{w_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
      iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[\bullet]= {-2.4, 1.4, 0, 0}
```

Out[\circ]= {-2.4, 1.2, 0, 0}

0, 0, 0}] $n= 1,q= \{-2.4, 1.2, 0, 0\}, q= 2.68328$

```
n= 2,q= \{1.92, -4.56, 0., 0.\}, q= 4.94773
       n= 3,q= \{-19.5072, -16.3104, 0., 0.\}, \q = 25.4275
       n= 4, q= \{112.102, 637.54, 0., 0.\}, q= 647.321
       n=5,q=\{-393893., 142940., 0., 0.\}, q=419027.
       n= 6,q= \{1.3472 \times 10^{11}, -1.12606 \times 10^{11}, 0., 0.\}, \q\= 1.75584 \times 10^{11}
       n= 7,q= \{5.46938 \times 10^{21}, -3.03407 \times 10^{22}, 0., 0.\}, \q\= 3.08297\times 10^{22}
       n=8,q=\{-8.90642\times10^{44},-3.31889\times10^{44},0.,0.\},\q\ge 9.5047\times10^{44}\}
       n = 9,q = \{6.83093 \times 10^{89}, 5.91189 \times 10^{89}, 0., 0.\}, q = 9.03394 \times 10^{89}\}
       n= 10,q= \{1.17111 \times 10^{179}, 8.07675 \times 10^{179}, 0., 0.\},\q\= 8.16121 \times 10^{179}
       n= 11,q= \{-6.386230150888532 \times 10^{359}, 1.891758440540132 \times 10^{359}, 0., 0.\}
        |q| = 6.660531926015563 \times 10^{359}
       n= 12,q= \{3.720518554276293\times10^{719}, -2.416240958235052\times10^{719}, 0., 0.\}
        \sqrt{q} = 4.436268553747259 \times 10^{719}
       n= 13,q= \{8.004037944461517 \times 10^{1438}, -1.797933863343168 \times 10^{1439}, 0., 0.\}
        \q = 1.968047868096680 \times 10^{1439}
       n= 14,q= \{-2.591919942792292 \times 10^{2878}, -2.878146172766201 \times 10^{2878}, 0., 0.\}
        \sqrt{q} = 3.873212411119887 \times 10^{2878}
       n= 15,q= \{-1.565676401964332 \times 10^{5756}, 1.491984892692805 \times 10^{5757}, 0., 0.\}
        \q = 1.500177438165313 \times 10^{5757}
       n= 16,q= \{-2.201505494066881 \times 10^{11514}, -4.671931077152820 \times 10^{11513}, 0., 0.\}
       \q = 2.250532345980240 \times 10^{11514}
       n= 17,q= \{4.628357040509997 \times 10^{23028}, 2.057056386850747 \times 10^{23028}, 0., 0.\}
        \sqrt{q} = 5.064895840303323 \times 10^{23028}
       n= 18,q= \{1.719020791575501 \times 10^{46057}, 1.904158282161342 \times 10^{46057}, 0., 0.\}
        ,\q\= 2.565316987312191 \times 10^{46057}
       n= 19,q= \{-6.707862816547707 \times 10^{92113}, 6.546575354972072 \times 10^{92114}, 0., 0.\}
        \sqrt{q} = 6.580851245392495 \times 10^{92114}
       n= 20,q= \{-4.240769464267148 \times 10^{184229}, -8.782705879868954 \times 10^{184228}, 0., 0.\}
        \sqrt{q} = 4.330760311398395 \times 10^{184229}
ln[-]:= P = \{-2.4, 0.7, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.4, 0.7, 0, 0}
```

```
n= 2,q= \{2.87, -2.66, 0., 0.\}, q= 3.91312
       n= 3,q= \{-1.2387, -14.5684, 0., 0.\}, q= 14.621
       n=4,q=\{-213.104, 36.7918, 0., 0.\},\q=216.257
       n=5,q=\{44\,057.2,-15\,680.2,0.,0.\},\q=46\,764.4
       n = 6,q = \{1.69517 \times 10^9, -1.38166 \times 10^9, 0., 0.\}, q = 2.18691 \times 10^9
       n= 7,q= \{9.64632 \times 10^{17}, -4.68428 \times 10^{18}, 0., 0.\},\q\= 4.78258 \times 10^{18}
       n= 8,q= \{-2.1012 \times 10^{37}, -9.03722 \times 10^{36}, 0., 0.\}, q = 2.2873 \times 10^{37}\}
       n = 9,q = \{3.59833 \times 10^{74}, 3.7978 \times 10^{74}, 0., 0.\}, q = 5.23175 \times 10^{74}\}
       n= 10,q= \{-1.47531 \times 10^{148}, 2.73315 \times 10^{149}, 0., 0.\}, \q\= 2.73713 \times 10^{149}
       n= 11,q= \left\{-7.44832\times10^{298}, -8.06447\times10^{297}, 0., 0.\right\},\q\= 7.49186\times10^{298}
       n= 12,q= \{5.482717792520735 \times 10^{597}, 1.201336439691409 \times 10^{597}, 0., 0.\}
        \sqrt{q} = 5.612789291765180 \times 10^{597}
       n= 13,q= \{2.861698515109301 \times 10^{1195}, 1.317317734539920 \times 10^{1195}, 0., 0.\}
        \q = 3.150340363375387 \times 10^{1195}
       n= 14,q= \{6.453992377645391 \times 10^{2390}, 7.539532409720072 \times 10^{2390}, 0., 0.\}
        \sqrt{q} = 9.92464440511216 \times 10^{2390}
       n= 15,q= \{-1.519053134651456 \times 10^{4781}, 9.73201694066875 \times 10^{4781}, 0., 0.\}
        n= 16,q= \{-9.24046313075691 \times 10^{9563}, -2.956690168040786 \times 10^{9563}, 0., 0.\}
        \q = 9.70196761593579 \times 10^{9563}
       n= 17,q= \{7.664414212108867 \times 10^{19127}, 5.464237297370464 \times 10^{19127}, 0., 0.\}
        \sqrt{q} = 9.41281756206668 \times 10^{19127}
       n= 18,q= \{2.888535597280191 \times 10^{38255}, 8.376035600060306 \times 10^{38255}, 0., 0.\}
        ,\q\= 8.860113445675085 \times 10^{38255}
       n= 19,q= \{-6.181433447672278\times10^{76511}, 4.838895398972067\times10^{76511}, 0., 0.\}
        \sqrt{q} = 7.850161027023243 \times 10^{76511}
       n= 20,q= \{1.479521078580855 \times 10^{153023}, -5.982261973798685 \times 10^{153023}, 0., 0.\}
        ,\q\= 6.162502815019462 \times 10^{153023}
ln[ \circ ] := P = \{-2.4, 0.5, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.4, 0.5, 0, 0}
```

 $n= 1, q= \{-2.4, 0.7, 0, 0\}, q= 2.5$

 $n= 1,q= \{-2.4, 0.5, 0, 0\}, q= 2.45153$

```
n= 2, q= \{3.11, -1.9, 0., 0.\}, q= 3.64446
       n= 3,q= \{3.6621, -11.318, 0., 0.\}, \q = 11.8957
       n=4,q=\{-117.086, -82.3953, 0., 0.\}, q=143.172
       n= 5, q= \{6917.78, 19295.2, 0., 0.\}, q= 20497.8
       n=6,q=\{-3.24449\times10^8, 2.6696\times10^8, 0., 0.\}, q=4.2016\times10^8
       n= 7,q= \{3.39995 \times 10^{16}, -1.7323 \times 10^{17}, 0., 0.\}, \q\= 1.76535 \times 10^{17}
       n= 8,q= \{-2.88526 \times 10^{34}, -1.17794 \times 10^{34}, 0., 0.\}, q= 3.11645 \times 10^{34}\}
       n = 9,q = \{6.93715 \times 10^{68}, 6.79734 \times 10^{68}, 0., 0.\}, q = 9.71225 \times 10^{68}\}
       n= 10,q= \{1.92016 \times 10^{136}, 9.43083 \times 10^{137}, 0., 0.\}, q= 9.43278 \times 10^{137}\}
       n= 11,q= \{-8.89036 \times 10^{275}, 3.62174 \times 10^{274}, 0., 0.\},\q\= 8.89773 \times 10^{275}
       n= 12,q= \{7.890732392868608 \times 10^{551}, -6.439721240970794 \times 10^{550}, 0., 0.\}
        \sqrt{q} = 7.916966451392860 \times 10^{551}
       n= 13,q= \{6.184895759925184 \times 10^{1103}, -1.016282339943445 \times 10^{1103}, 0., 0.\}
        \sqrt{q} = 6.267835779248005 \times 10^{1103}
       n= 14,q= \{3.722010576665960 \times 10^{2207}, -1.257120067040612 \times 10^{2207}, 0., 0.\}
        \q = 3.928576535562145 \times 10^{2207}
       n= 15,q= \{1.227301186985708 \times 10^{4415}, -9.35802837132836 \times 10^{4414}, 0., 0.\}
        ,\q\ =\ 1.543371359576946\times 10^{4415}
       n= 16,q= \{6.305412535906624 \times 10^{8829}, -2.297023865595444 \times 10^{8830}, 0., 0.\}
        \sqrt{q} = 2.381995153562392 \times 10^{8830}
       n= 17,q= \left\{-4.878736366635353\times10^{17660}, -2.896736615480441\times10^{17660}, 0., 0.\right\}
        \sqrt{q} = 5.673900911594721 \times 10^{17660}
       n= 18,q= \{1.541098551566525 \times 10^{35321}, 2.826482854101728 \times 10^{35321}, 0., 0.\}
        ,\q\= 3.219315155459541 \times 10^{35321}
       n= 19,q= \{-5.614020578890608 \times 10^{70642}, 8.711777264967578 \times 10^{70642}, 0., 0.\}
        \sqrt{q} = 1.036399007017149 \times 10^{70643}
       n= 20,q= \{-4.437783605419874 \times 10^{141285}, -9.78161936884786 \times 10^{141285}, 0., 0.\}
        ,\q\= 1.074122901746132 \times 10^{141286}
ln[ \circ ] := P = \{-2.4, 0.4, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.4, 0.4, 0, 0}
```

```
n= 1, q= \{-2.4, 0.4, 0, 0\}, q= 2.43311
       n= 2,q= \{3.2, -1.52, 0., 0.\}, q= 3.54265
       n= 3,q= \{5.5296, -9.328, 0., 0.\}, q= 10.8438
       n=4,q=\{-58.8351, -102.76, 0., 0.\}, q=118.411
       n=5,q=\{-7100.49, 12092.2, 0., 0.\}, q=14022.8
       n = 6,q = \{-9.58047 \times 10^7, -1.71721 \times 10^8, 0., 0.\}, \q = 1.96639 \times 10^8
       n= 7,q= \{-2.03097 \times 10^{16}, 3.29034 \times 10^{16}, 0., 0.\},\q\= 3.86668 \times 10^{16}
       n= 8,q= \{-6.70153 \times 10^{32}, -1.33652 \times 10^{33}, 0., 0.\},\q\= 1.49512×10<sup>33</sup>
       n= 9,q= \left\{-1.33717\times10^{66}\text{, }1.79134\times10^{66}\text{, }0.\text{, }0.\right\}\text{,}q=\text{ }2.23538\times10^{66}
       n= 10,q= \{-1.42087 \times 10^{132}, -4.79067 \times 10^{132}, 0., 0.\}, \q\= 4.99694 \times 10^{132}
       n= 11,q= \{-2.09317 \times 10^{265}, 1.36139 \times 10^{265}, 0., 0.\},\q\= 2.49694\times10<sup>265</sup>
       n= 12,q= \{2.527971569102691 \times 10^{530}, -5.699207888961210 \times 10^{530}, 0., 0.\}
        \sqrt{q} = 6.234710162933736 \times 10^{530}
       n= 13,q= \{-2.609033030740617 \times 10^{1061}, -2.881487101939941 \times 10^{1061}, 0., 0.\}
        \sqrt{q} = 3.887161081578921 \times 10^{1061}
       n= 14,q= \{-1.495914563150672 \times 10^{2122}, 1.503579005322872 \times 10^{2123}, 0., 0.\}
        \sqrt{q} = 1.511002127414181 \times 10^{2123}
       n= 15,q= \{-2.238372221445256 \times 10^{4246}, -4.498451461820174 \times 10^{4245}, 0., 0.\}
        \sqrt{q} = 2.283127429050181 \times 10^{4246}
       n= 16,q= \{4.807949546194248 \times 10^{8492}, 2.013841758331616 \times 10^{8492}, 0., 0.\}
        \q = 5.212670857281289 \times 10^{8492}
       n= 17,q= \{1.906082021114930 \times 10^{16985}, 1.936489913615504 \times 10^{16985}, 0., 0.\}
        \sqrt{q} = 2.717193746634965 \times 10^{16985}
       n= 18,q= \{-1.168445143170074 \times 10^{33969}, 7.382217216825830 \times 10^{33970}, 0., 0.\}
        ,\q\= 7.383141856752157 \times 10^{33970}
       n= 19,q= \left\{-5.448347839587373\times10^{67941}, -1.725143170565328\times10^{67940}, 0., 0.\right\}
        \q = 5.451078367692569 \times 10^{67941}
       n= 20,q= \{2.965473299154691 \times 10^{135883}, 1.879836013265703 \times 10^{135882}, 0., 0.\}
         ,\q\= 2.971425537072588 \times 10^{135883}
ln[ \circ ] := P = \{-2.4, 0.2, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.4, 0.2, 0, 0}
```

 $n= 1,q= \{-2.4, 0.2, 0, 0\}, q= 2.40832$

```
n= 2,q= \{3.32, -0.76, 0., 0.\}, q= 3.40588
       n= 3,q= \{8.0448, -4.8464, 0., 0.\}, q= 9.39183
       n= 4,q= \{38.8312, -77.7766, 0., 0.\}, \q = 86.9314
       n=5,q=\{-4543.74, -6040.12, 0., 0.\}, q=7558.35
       n=6,q=\{-1.58375\times10^7, 5.48895\times10^7, 0., 0.\}, q=5.71287\times10^7
       n= 7,q= \{-2.76203 \times 10^{15}, -1.73862 \times 10^{15}, 0., 0.\}, \q\= 3.26369 \times 10^{15}
       n= 8,q= \{4.60601 \times 10^{30}, 9.60428 \times 10^{30}, 0., 0.\}, q= 1.06516 \times 10^{31}
       n=9,q=\{-7.10268\times10^{61}, 8.84748\times10^{61}, 0., 0.\}, q=1.13457\times10^{62}\}
       n= 10,q= \{-2.78299 \times 10^{123}, -1.25682 \times 10^{124}, 0., 0.\}, \q\= 1.28726 \times 10^{124}
       n= 11,q= \{-1.50214 \times 10^{248}, 6.99542 \times 10^{247}, 0., 0.\}, \q\= 1.65704 \times 10^{248}
       n= 12,q= \{1.767055912188359 \times 10^{496}, -2.101617338541634 \times 10^{496}, 0., 0.\}
        \sqrt{q} = 2.745775306622640 \times 10^{496}
       n= 13,q= \{-1.294308840858984 \times 10^{992}, -7.427350686455117 \times 10^{992}, 0., 0.\}
        \sqrt{q} = 7.539282034458655 \times 10^{992}
       n= 14,q= \{-5.349030284405958 \times 10^{1985}, 1.922657131527780 \times 10^{1985}, 0., 0.\}
        \q = 5.684077359511103 \times 10^{1985}
       n= 15,q= \{2.491551453807745 \times 10^{3971}, -2.056870244614237 \times 10^{3971}, 0., 0.\}
        \sqrt{q} = 3.230873542890671 \times 10^{3971}
       n= 16,q= \{1.977113443792054 \times 10^{7942}, -1.024959609652499 \times 10^{7943}, 0., 0.\}
        \q = 1.043854385015092 \times 10^{7943}
       n= 17,q= \{-1.011452425722770 \times 10^{15886}, -4.052922847175623 \times 10^{15885}, 0., 0.\}
        \sqrt{q} = 1.089631977115235 \times 10^{15886}
       n= 18,q= \{8.587741734488940 \times 10^{31771}, 8.198677290086038 \times 10^{31771}, 0., 0.\}
        ,\q\= 1.187297845552057 \times 10^{31772}
       n= 19,q= \{6.530998791310567 \times 10^{63542}, 1.408162462633571 \times 10^{63544}, 0., 0.\}
        \sqrt{q} = 1.409676174052556 \times 10^{63544}
       n= 20,q= \{-1.978656126649033 \times 10^{127088}, 1.839341468285753 \times 10^{127087}, 0., 0.\}
        ,\q\= 1.987186915691453 \times 10^{127088}
ln[ \circ ] := P = \{-2.4, -0.2, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.4, -0.2, 0, 0}
```

Out[\circ]= {-2.4, -0.4, 0, 0}

{0, 0, 0, 0, $n= 1, q= \{-2.4, -0.4, 0, 0\}, q= 2.43311$

```
n= 2,q= \{3.2, 1.52, 0., 0.\}, q= 3.54265
       n= 3,q= \{5.5296, 9.328, 0., 0.\}, q= 10.8438
       n= 4,q= \{-58.8351, 102.76, 0., 0.\}, \q = 118.411
       n=5,q=\{-7100.49, -12092.2, 0., 0.\}, q=14022.8
       n=6,q=\{-9.58047\times10^7, 1.71721\times10^8, 0., 0.\}, q=1.96639\times10^8
       n= 7,q= \{-2.03097 \times 10^{16}, -3.29034 \times 10^{16}, 0., 0.\}, \q\= 3.86668 \times 10^{16}
       n= 8,q= \{-6.70153 \times 10^{32}, 1.33652 \times 10^{33}, 0., 0.\}, \q\= 1.49512 \times 10<sup>33</sup>
       n = 9,q = \{-1.33717 \times 10^{66}, -1.79134 \times 10^{66}, 0., 0.\}, q = 2.23538 \times 10^{66}\}
       n= 10,q= \{-1.42087 \times 10^{132}, 4.79067 \times 10^{132}, 0., 0.\}, \q\= 4.99694 \times 10<sup>132</sup>
       n= 11,q= \{-2.09317 \times 10^{265}, -1.36139 \times 10^{265}, 0., 0.\},\q\= 2.49694\times10<sup>265</sup>
       n= 12,q= \{2.527971569102691 \times 10^{530}, 5.699207888961210 \times 10^{530}, 0., 0.\}
        \sqrt{q} = 6.234710162933736 \times 10^{530}
       n= 13,q= \{-2.609033030740617 \times 10^{1061}, 2.881487101939941 \times 10^{1061}, 0., 0.\}
        \sqrt{q} = 3.887161081578921 \times 10^{1061}
       n= 14,q= \{-1.495914563150672 \times 10^{2122}, -1.503579005322872 \times 10^{2123}, 0., 0.\}
        \sqrt{q} = 1.511002127414181 \times 10^{2123}
       n= 15,q= \{-2.238372221445256 \times 10^{4246}, 4.498451461820174 \times 10^{4245}, 0., 0.\}
        \sqrt{q} = 2.283127429050181 \times 10^{4246}
       n= 16,q= \{4.807949546194248 \times 10^{8492}, -2.013841758331616 \times 10^{8492}, 0., 0.\}
        \q = 5.212670857281289 \times 10^{8492}
       n= 17,q= \{1.906082021114930 \times 10^{16985}, -1.936489913615504 \times 10^{16985}, 0., 0.\}
        \sqrt{q} = 2.717193746634965 \times 10^{16985}
       n= 18,q= \{-1.168445143170074 \times 10^{33969}, -7.382217216825830 \times 10^{33970}, 0., 0.\}
        ,\q\= 7.383141856752157 \times 10^{33970}
       n= 19,q= \{-5.448347839587373 \times 10^{67941}, 1.725143170565328 \times 10^{67940}, 0., 0.\}
        \q = 5.451078367692569 \times 10^{67941}
       n= 20,q= \{2.965473299154691 \times 10^{135883}, -1.879836013265703 \times 10^{135882}, 0., 0.\}
        ,\q\= 2.971425537072588 \times 10^{135883}
ln[ \circ ] := P = \{-2.4, -0.5, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.4, -0.5, 0, 0}
```

```
n= 1, q= \{-2.4, -0.5, 0, 0\}, q= 2.45153
       n= 2,q= \{3.11, 1.9, 0., 0.\}, q= 3.64446
       n= 3,q= \{3.6621, 11.318, 0., 0.\}, q= 11.8957
       n=4,q=\{-117.086, 82.3953, 0., 0.\}, q=143.172
       n=5,q=\{6917.78, -19295.2, 0., 0.\}, q=20497.8
       n=6,q=\{-3.24449\times10^8, -2.6696\times10^8, 0., 0.\}, q=4.2016\times10^8
       n= 7,q= \{3.39995 \times 10^{16}, 1.7323 \times 10^{17}, 0., 0.\}, \q\= 1.76535 \times 10^{17}
       n= 8,q= \{-2.88526 \times 10^{34}, 1.17794 \times 10^{34}, 0., 0.\}, q= 3.11645 \times 10^{34}\}
       n = 9,q = \{6.93715 \times 10^{68}, -6.79734 \times 10^{68}, 0., 0.\}, q = 9.71225 \times 10^{68}\}
       n= 10,q= \{1.92016 \times 10^{136}, -9.43083 \times 10^{137}, 0., 0.\}, \q\= 9.43278 \times 10<sup>137</sup>
       n= 11,q= \left\{-8.89036\times10^{275}, -3.62174\times10^{274}, 0., 0.\right\},\q\= 8.89773\times10^{275}
       n= 12,q= \{7.890732392868608 \times 10^{551}, 6.439721240970794 \times 10^{550}, 0., 0.\}
        \sqrt{q} = 7.916966451392860 \times 10^{551}
       n= 13,q= \{6.184895759925184 \times 10^{1103}, 1.016282339943445 \times 10^{1103}, 0., 0.\}
        \sqrt{q} = 6.267835779248005 \times 10^{1103}
       n= 14,q= \{3.722010576665960 \times 10^{2207}, 1.257120067040612 \times 10^{2207}, 0., 0.\}
        \q = 3.928576535562145 \times 10^{2207}
       n= 15,q= \{1.227301186985708 \times 10^{4415}, 9.35802837132836 \times 10^{4414}, 0., 0.\}
        ,\q\ =\ 1.543371359576946\times 10^{4415}
       n= 16,q= \{6.305412535906624 \times 10^{8829}, 2.297023865595444 \times 10^{8830}, 0., 0.\}
        \q = 2.381995153562392 \times 10^{8830}
       n= 17,q= \{-4.878736366635353 \times 10^{17660}, 2.896736615480441 \times 10^{17660}, 0., 0.\}
        \sqrt{q} = 5.673900911594721 \times 10^{17660}
       n= 18,q= \{1.541098551566525 \times 10^{35321}, -2.826482854101728 \times 10^{35321}, 0., 0.\}
        ,\q\= 3.219315155459541 \times 10^{35321}
       n= 19,q= \{-5.614020578890608 \times 10^{70642}, -8.711777264967578 \times 10^{70642}, 0., 0.\}
        \q = 1.036399007017149 \times 10^{70643}
       n= 20,q= \left\{-4.437783605419874\times10^{141285}, 9.78161936884786\times10^{141285}, 0., 0.\right\}
        ,\q\= 1.074122901746132 \times 10^{141286}
ln[ \circ ] := P = \{-2.4, -0.7, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.4, -0.7, 0, 0}
```

```
n= 1,q= \{-2.4, -0.7, 0, 0\}, q= 2.5
       n= 2,q= \{2.87, 2.66, 0., 0.\}, q= 3.91312
       n= 3,q= \{-1.2387, 14.5684, 0., 0.\}, q= 14.621
       n=4,q=\{-213.104, -36.7918, 0., 0.\}, q=216.257
       n= 5, q= \{44057.2, 15680.2, 0., 0.\}, q= 46764.4
       n = 6,q = \{1.69517 \times 10^9, 1.38166 \times 10^9, 0., 0.\}, q = 2.18691 \times 10^9\}
       n= 7,q= \{9.64632 \times 10^{17}, 4.68428 \times 10^{18}, 0., 0.\}, \q\= 4.78258 \times 10^{18}
       n= 8,q= \{-2.1012 \times 10^{37}, 9.03722 \times 10^{36}, 0., 0.\}, q= 2.2873 \times 10^{37}
       n = 9,q = \{3.59833 \times 10^{74}, -3.7978 \times 10^{74}, 0., 0.\}, q = 5.23175 \times 10^{74}\}
       n= 10,q= \{-1.47531 \times 10^{148}, -2.73315 \times 10^{149}, 0., 0.\}, \q\= 2.73713 \times 10^{149}
       n= 11,q= \{-7.44832 \times 10^{298}, 8.06447 \times 10^{297}, 0., 0.\},\q\= 7.49186 \times 10^{298}
       n= 12,q= \{5.482717792520735 \times 10^{597}, -1.201336439691409 \times 10^{597}, 0., 0.\}
        \sqrt{q} = 5.612789291765180 \times 10^{597}
       n= 13,q= \{2.861698515109301\times10^{1195}, -1.317317734539920\times10^{1195}, 0., 0.\}
        \q = 3.150340363375387 \times 10^{1195}
       n= 14,q= \{6.453992377645391 \times 10^{2390}, -7.539532409720072 \times 10^{2390}, 0., 0.\}
        \sqrt{q} = 9.92464440511216 \times 10^{2390}
       n= 15,q= \{-1.519053134651456 \times 10^{4781}, -9.73201694066875 \times 10^{4781}, 0., 0.\}
        ,\q\ =\ 9.84985665679242\times 10^{4781}
       n= 16,q= \{-9.24046313075691 \times 10^{9563}, 2.956690168040786 \times 10^{9563}, 0., 0.\}
        ,\q\ = 9.70196761593579 \times 10^{9563}
       n= 17,q= \{7.664414212108867 \times 10^{19127}, -5.464237297370464 \times 10^{19127}, 0., 0.\}
        \sqrt{q} = 9.41281756206668 \times 10^{19127}
       n= 18,q= \{2.888535597280191 \times 10^{38255}, -8.376035600060306 \times 10^{38255}, 0., 0.\}
        ,\q\= 8.860113445675085 \times 10^{38255}
       n= 19,q= \{-6.181433447672278 \times 10^{76511}, -4.838895398972067 \times 10^{76511}, 0., 0.\}
        \sqrt{q} = 7.850161027023243 \times 10^{76511}
       n= 20,q= \{1.479521078580855 \times 10^{153023}, 5.982261973798685 \times 10^{153023}, 0., 0.\}
        ,\q\= 6.162502815019462 \times 10^{153023}
ln[ \circ ] := P = \{-2.4, -1.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.4, -1.2, 0, 0}
```

```
n= 1, q= \{-2.4, -1.2, 0, 0\}, q= 2.68328
       n= 2,q= \{1.92, 4.56, 0., 0.\}, q= 4.94773
       n= 3,q= \{-19.5072, 16.3104, 0., 0.\}, q= 25.4275
       n= 4,q= \{112.102, -637.54, 0., 0.\}, \q = 647.321
       n=5,q=\{-393893., -142940., 0., 0.\}, q=419027.
       n= 6,q= \{1.3472 \times 10^{11}, 1.12606 \times 10^{11}, 0., 0.\}, \q\= 1.75584 \times 10^{11}
       n= 7,q= \{5.46938 \times 10^{21}, 3.03407 \times 10^{22}, 0., 0.\}, \q\= 3.08297 \times 10^{22}
       n= 8,q= \{-8.90642 \times 10^{44}, 3.31889 \times 10^{44}, 0., 0.\}, \q\= 9.5047×10<sup>44</sup>
       n= 9,q= \{6.83093 \times 10^{89}, -5.91189 \times 10^{89}, 0., 0.\},\q\= 9.03394×10<sup>89</sup>
       n= 10,q= \{1.17111 \times 10^{179}, -8.07675 \times 10^{179}, 0., 0.\}, \q\= 8.16121 \times 10^{179}
       n= 11,q= \{-6.386230150888532 \times 10^{359}, -1.891758440540132 \times 10^{359}, 0., 0.\}
        |q| = 6.660531926015563 \times 10^{359}
       n= 12,q= \{3.720518554276293\times10^{719}, 2.416240958235052\times10^{719}, 0., 0.\}
        \sqrt{q} = 4.436268553747259 \times 10^{719}
       n= 13,q= \{8.004037944461517 \times 10^{1438}, 1.797933863343168 \times 10^{1439}, 0., 0.\}
        ,\q\= 1.968047868096680 \times 10^{1439}
       n= 14,q= \{-2.591919942792292 \times 10^{2878}, 2.878146172766201 \times 10^{2878}, 0., 0.\}
        \sqrt{q} = 3.873212411119887 \times 10^{2878}
       n= 15,q= \{-1.565676401964332 \times 10^{5756}, -1.491984892692805 \times 10^{5757}, 0., 0.\}
        \q = 1.500177438165313 \times 10^{5757}
       n= 16,q= \{-2.201505494066881 \times 10^{11514}, 4.671931077152820 \times 10^{11513}, 0., 0.\}
       , q = 2.250532345980240 \times 10^{11514}
       n= 17,q= \{4.628357040509997 \times 10^{23028}, -2.057056386850747 \times 10^{23028}, 0., 0.\}
        \sqrt{q} = 5.064895840303323 \times 10^{23028}
       n= 18,q= \{1.719020791575501 \times 10^{46057}, -1.904158282161342 \times 10^{46057}, 0., 0.\}
        ,\q\= 2.565316987312191 \times 10^{46057}
       n= 19,q= \{-6.707862816547707 \times 10^{92113}, -6.546575354972072 \times 10^{92114}, 0., 0.\}
        \sqrt{q} = 6.580851245392495 \times 10^{92114}
       n= 20,q=\{-4.240769464267148\times10^{184229}, 8.782705879868954\times10^{184228}, 0., 0.\}
        \sqrt{q} = 4.330760311398395 \times 10^{184229}
ln[-]:= P = \{-2.4, -1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Outfol= \{-2.4, -1.4, 0, 0\}
```

```
n= 1, q= \{-2.4, -1.4, 0, 0\}, q= 2.77849
       n= 2, q= \{1.4, 5.32, 0., 0.\}, q= 5.50113
       n= 3,q= \{-28.7424, 13.496, 0., 0.\}, \q = 31.7532
       n=4,q=\{641.584, -777.215, 0., 0.\}, q=1007.82
       n= 5, q= \{-192436., -997298., 0., 0.\}, q= 1.01569 \times 10^6
       n = 6,q = \{-9.57572 \times 10^{11}, 3.83832 \times 10^{11}, 0., 0.\}, \ | = 1.03163 \times 10^{12}\}
       n= 7,q= \{7.69616 \times 10^{23}, -7.35093 \times 10^{23}, 0., 0.\},\q\= 1.06427 \times 10^{24}
       n=8,q=\{5.19478\times10^{46}, -1.13148\times10^{48}, 0., 0.\}, q=1.13267\times10^{48}\}
       n= 9,q= \left\{-1.27755\times10^{96}, -1.17556\times10^{95}, 0., 0.\right\},\q\= 1.28294\times10^{96}
       n= 10,q= \{1.61831 \times 10^{192}, 3.00366 \times 10^{191}, 0., 0.\},\q\= 1.64594 \times 10^{192}
       n= 11,q= \{2.528694624649801 \times 10^{384}, 9.72168510308598 \times 10^{383}, 0., 0.\}
        q = 2.709134200656815 \times 10^{384}
       n= 12,q= \{5.449184892297159 \times 10^{768}, 4.916634572542315 \times 10^{768}, 0., 0.\}
        \sqrt{q} = 7.339408117168438 \times 10^{768}
       n= 13,q= \{5.520320470521257 \times 10^{1536}, 5.358330166728697 \times 10^{1537}, 0., 0.\}
        \q = 5.386691151035795 \times 10^{1537}
       n= 14,q= \{-2.840696279470222 \times 10^{3075}, 5.915939941440801 \times 10^{3074}, 0., 0.\}
        \sqrt{q} = 2.901644155664734 \times 10^{3075}
       n= 15,q= \left\{7.719571898288617\times10^{6150}\text{, }-3.361077716244033}\times10^{6150}\text{, }0.\text{, }0.\right\}
        \q = 8.419538806103309 \times 10^{6150}
       n= 16,q= \{4.829494687821511 \times 10^{12301}, -5.189216217256304 \times 10^{12301}, 0., 0.\}
       ,\q\ = 7.088863370747953 \times 10^{12301}
       n= 17,q= \{-3.603946009739632 \times 10^{24602}, -5.012258431039312 \times 10^{24603}, 0., 0.\}
        \sqrt{q} = 5.025198388913203 \times 10^{24603}
       n= 18,q=\{-2.499285031111348\times10^{49207}, 3.612781754465592\times10^{49206}, 0., 0.\}
        ,\q\= 2.525261884793585 \times 10^{49207}
       n= 19,q= \{6.115903746683258 \times 10^{98414}, -1.805874271921610 \times 10^{98414}, 0., 0.\}
        \sqrt{q} = 6.376947586791248 \times 10^{98414}
       n= 20,q= \{3.414309675270592 \times 10^{196829}, -2.208910645136855 \times 10^{196829}, 0., 0.\}
        \sqrt{q} = 4.066546052468272 \times 10^{196829}
ln[-]:= P = \{-2.2, 1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, 1.4, 0, 0}
```

```
n= 1,q= \{-2.2, 1.4, 0, 0\}, q= 2.60768
       n= 2,q= \{0.68, -4.76, 0., 0.\}, q= 4.80833
       n= 3,q= \{-24.3952, -5.0736, 0., 0.\}, \q = 24.9172
       n= 4,q= \{567.184, 248.943, 0., 0.\}, \q = 619.412
       n=5,q=\{259723.,282395.,0.,0.\},\q=383670.
       n= 6,q= \{-1.22905 \times 10^{10}, 1.46689 \times 10^{11}, 0., 0.\}, q = 1.47203 \times 10^{11}
       n= 7,q= \{-2.13666 \times 10^{22}, -3.60575 \times 10^{21}, 0., 0.\},\q\= 2.16687×10<sup>22</sup>
       n= 8,q= \{4.43529 \times 10^{44}, 1.54085 \times 10^{44}, 0., 0.\}, \q\= 4.69532 \times 10^{44}
       n= 9,q= \{1.72976 \times 10^{89}, 1.36682 \times 10^{89}, 0., 0.\},\q\= 2.2046 \times 10^{89}
       n= 10,q= \{1.12385 \times 10^{178}, 4.72855 \times 10^{178}, 0., 0.\},\q\= 4.86027 \times 10^{178}
       n= 11,q= \{-2.109610344132128 \times 10^{357}, 1.062835565311691 \times 10^{357}, 0., 0.\}
        q = 2.362218288592462 \times 10^{357}
       n= 12,q= \{3.320836365177853 \times 10^{714}, -4.484337805386124 \times 10^{714}, 0., 0.\}
        \sqrt{q} = 5.580075242960698 \times 10^{714}
       n= 13,q= \{-9.08133138852758 \times 10^{1428}, -2.978350411573617 \times 10^{1429}, 0., 0.\}
        ,\q\ =\ 3.113723971710289\times 10^{1429}
       n= 14,q= \{-8.045865376238171 \times 10^{2858}, 5.409477415731507 \times 10^{2858}, 0., 0.\}
        \sqrt{q} = 9.69527697200330 \times 10^{2858}
       n= 15,q= \{3.547350374123897 \times 10^{5717}, -8.704785408555294 \times 10^{5717}, 0., 0.\}
        \q = 9.39983955638574 \times 10^{5717}
       n= 16,q= \{-6.318959433220020 \times 10^{11435}, -6.175784755141373 \times 10^{11435}, 0., 0.\}
       , q = 8.835698368579411 \times 10^{11435}
       n= 17,q= \{1.788930976843686 \times 10^{22870}, 7.804906667207393 \times 10^{22871}, 0., 0.\}
        \sqrt{q} = 7.806956566051686 \times 10^{22871}
       n= 18,q= \{-6.088456534341931 \times 10^{45743}, 2.792487861668224 \times 10^{45742}, 0., 0.\}
        ,\q\= 6.094857082421753 \times 10^{45743}
       n= 19,q= \{3.699132308599532 \times 10^{91487}, -3.400388193688885 \times 10^{91486}, 0., 0.\}
        \sqrt{q} = 3.714728285514660 \times 10^{91487}
       n= 20,q=\{1.356795343784711\times10^{182975}, -2.515697165810991\times10^{182974}, 0., 0.\}
        ,\q\= 1.379920623520269 \times 10^{182975}
ln[-]:= P = \{-2.2, 1.2, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, 1.2, 0, 0}
```

 $n= 1,q= \{-2.2, 1.2, 0, 0\}, q= 2.50599$

```
n= 2,q= \{1.2, -4.08, 0., 0.\}, q= 4.25281
       n= 3,q= \{-17.4064, -8.592, 0., 0.\}, \q = 19.4115
       n= 4, q= \{226.96, 300.312, 0., 0.\}, q= 376.428
       n=5,q=\{-38678.3, 136319., 0., 0.\}, q= 141700.
       n= 6,q= \{-1.70868 \times 10^{10}, -1.05452 \times 10^{10}, 0., 0.\}, \q\= 2.00788 \times 10^{10}
       n= 7,q= \{1.80759 \times 10^{20}, 3.60366 \times 10^{20}, 0., 0.\}, \q\= 4.03159 \times 10^{20}
       n=8,q=\{-9.71899\times10^{40}, 1.30279\times10^{41}, 0., 0.\}, q=1.62537\times10^{41}\}
       n = 9, q = \{-7.52666 \times 10^{81}, -2.53235 \times 10^{82}, 0., 0.\}, q = 2.64184 \times 10^{82}
       n= 10,q= \{-5.84631 \times 10^{164}, 3.81203 \times 10^{164}, 0., 0.\}, \q\= 6.97932 \times 10<sup>164</sup>
       n= 11,q= \{1.964769367876027 \times 10^{329}, -4.457265130109520 \times 10^{329}, 0., 0.\}
        |q| = 4.871091367346151 \times 10^{329}
       n= 12,q= \{-1.600689377114627 \times 10^{659}, -1.751499598428228 \times 10^{659}, 0., 0.\}
        \sqrt{q} = 2.372753110903420 \times 10^{659}
       n= 13,q= \{-5.055443612866299 \times 10^{1317}, 5.607213602449199 \times 10^{1318}, 0., 0.\}
        \q = 5.629957325301856 \times 10^{1318}
       n= 14,q= \{-3.118526928226262 \times 10^{2637}, -5.669390438495767 \times 10^{2636}, 0., 0.\}
        \sqrt{q} = 3.169641948472003 \times 10^{2637}
       n= 15,q= \{9.40379032263125 \times 10^{5274}, 3.536029349815509 \times 10^{5274}, 0., 0.\}
        \sqrt{q} = 1.004663008151340 \times 10^{5275}
       n= 16,q= \{7.592776886925651 \times 10^{10549}, 6.650415716067033 \times 10^{10549}, 0., 0.\}
       ,\q\= 1.009347759947699×10<sup>10550</sup>
       n= 17,q= \{1.342223165812101 \times 10^{21099}, 1.009902454748017 \times 10^{21100}, 0., 0.\}
        ,\q\= 1.018782900511438 \times 10^{21100}
       n= 18,q=\{-1.001887337837645\times10^{42200}, 2.711028939946591\times10^{42199}, 0., 0.\}
        ,\q\= 1.037918598374498 \times 10^{42200}
       n= 19,q= \{9.30281458587124 \times 10^{84399}, -5.432291134887805 \times 10^{84399}, 0., 0.\}
        \sqrt{q} = 1.077275016851682 \times 10^{84400}
       n= 20,q= \{5.703257224491797 \times 10^{168799}, -1.010711944086666 \times 10^{168800}, 0., 0.\}
        ,\q\ =\ 1.160521461932792 \times 10^{168\,800}
ln[-]:= P = \{-2.2, 0.7, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, 0.7, 0, 0}
```

```
n= 2,q= \{2.15, -2.38, 0., 0.\}, q= 3.20732
       n= 3,q= \{-3.2419, -9.534, 0., 0.\}, \q = 10.0701
       n= 4, q= \{-82.5872, 62.5165, 0., 0.\}, q= 103.581
       n=5,q=\{2910.13, -10325.4, 0., 0.\}, q=10727.7
       n=6,q=\{-9.81458\times10^7, -6.00968\times10^7, 0., 0.\}, q=1.15084\times10^8
       n= 7,q= \{6.02097 \times 10^{15}, 1.17965 \times 10^{16}, 0., 0.\}, \q\= 1.32442 \times 10^{16}
       n= 8,q= \{-1.02905 \times 10^{32}, 1.42053 \times 10^{32}, 0., 0.\}, q = 1.7541 \times 10^{32}
       n = 9,q = \{-9.58951 \times 10^{63}, -2.9236 \times 10^{64}, 0., 0.\}, q = 3.07685 \times 10^{64}\}
       n= 10,q= \{-7.62783 \times 10^{128}, 5.60717 \times 10^{128}, 0., 0.\},\q\= 9.467 \times 10^{128}
       n= 11,q= \{2.67434 \times 10^{257}, -8.55411 \times 10^{257}, 0., 0.\},\q\= 8.96242 \times 10^{257}
       n= 12,q= \{-6.602067593304522 \times 10^{515}, -4.575324516462220 \times 10^{515}, 0., 0.\}
        \sqrt{q} = 8.032489709766331 \times 10^{515}
       n= 13,q= \{2.265370207562151 \times 10^{1031}, 6.041320343797380 \times 10^{1031}, 0., 0.\}
        ,\q\ =\ 6.452089093750200 \times 10^{1031}
       n= 14,q= \{-3.136564931906991 \times 10^{2063}, 2.737165424235544 \times 10^{2063}, 0., 0.\}
        \sqrt{q} = 4.162945367369028 \times 10^{2063}
       n= 15,q= \{2.345965012438163 \times 10^{4126}, -1.717059416497106 \times 10^{4127}, 0., 0.\}
        ,\q\= 1.733011413169925 \times 10^{4127}
       n= 16,q= \{-2.893257521385542 \times 10^{8254}, -8.056322630759397 \times 10^{8253}, 0., 0.\}
        \q = 3.003328558177222 \times 10^{8254}
       n= 17,q= \{7.721895741745150 \times 10^{16508}, 4.661803209230636 \times 10^{16508}, 0., 0.\}
        \sqrt{q} = 9.01998242836287 \times 10^{16508}
       n= 18,q= \{3.789526468478882 \times 10^{33017}, 7.199591670042385 \times 10^{33017}, 0., 0.\}
        ,\q\= 8.136008300797494 \times 10^{33017}
       n= 19,q= \{-3.747360936004167 \times 10^{66035}, 5.456608639173139 \times 10^{66035}, 0., 0.\}
        \q = 6.619463107064573 \times 10^{66035}
       n= 20,q= \left\{-1.573186385640891\times10^{132071}, -4.089576411500056\times10^{132071}, 0., 0.\right\}
        ,\q\= 4.381729182578896 \times 10^{132071}
ln[ \circ ] := P = \{-2.2, 0.5, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, 0.5, 0, 0}
```

 $n= 1,q= \{-2.2, 0.7, 0, 0\}, q= 2.30868$

```
n= 1,q= \{-2.2, 0.5, 0, 0\}, q= 2.2561
       n= 2,q= \{2.39, -1.7, 0., 0.\}, q= 2.93293
       n= 3,q= \{0.6221, -7.626, 0., 0.\}, \q = 7.65133
       n=4,q=\{-59.9689, -8.98827, 0., 0.\}, q=60.6387
       n=5,q={3513.28, 1078.53, 0., 0.},\q\ 3675.1
       n = 6, q = \{1.11799 \times 10^7, 7.57837 \times 10^6, 0., 0.\}, q = 1.35063 \times 10^7\}
       n= 7,q= \{6.75579 \times 10^{13}, 1.6945 \times 10^{14}, 0., 0.\},\q\= 1.82421 \times 10^{14}
       n= 8,q= \{-2.41494 \times 10^{28}, 2.28954 \times 10^{28}, 0., 0.\}, \q\= 3.32775 \times 10^{28}
       n= 9,q= \{5.89901 \times 10^{55}, -1.10582 \times 10^{57}, 0., 0.\}, q= 1.10739 \times 10^{57}
       n= 10,q= \{-1.21936 \times 10^{114}, -1.30465 \times 10^{113}, 0., 0.\}, \q\= 1.22632 \times 10^{114}
       n= 11,q= \{1.46981 \times 10^{228}, 3.18167 \times 10^{227}, 0., 0.\},\q\= 1.50385 \times 10^{228}
       n = 12, q = \{2.059118015286201 \times 10^{456}, 9.35290092289640 \times 10^{455}, 0., 0.\}
        \sqrt{q} = 2.261577891121893 \times 10^{456}
       n= 13,q= \{3.365199444141022 \times 10^{912}, 3.851745357104583 \times 10^{912}, 0., 0.\}
        n= 14,q= \{-3.511374997129671 \times 10^{1824}, 2.592378266940221 \times 10^{1825}, 0., 0.\}
        \sqrt{q} = 2.616050959482376 \times 10^{1825}
       n= 15,q= \{-6.597127535199311 \times 10^{3650}, -1.820562445927248 \times 10^{3650}, 0., 0.\}
        ,\q\ =\ 6.843722622608659 \times 10^{3650}
       n= 16,q= \{4.020764409616434 \times 10^{7301}, 2.402096528315291 \times 10^{7301}, 0., 0.\}
        , q = 4.683653933520555 \times 10^{7301}
       n= 17,q= \{1.039647870629381 \times 10^{14603}, 1.931652845902663 \times 10^{14603}, 0., 0.\}
        \sqrt{q} = 2.193661416898256 \times 10^{14603}
       n= 18,q= \{-2.650415022179652 \times 10^{29206}, 4.016477536075778 \times 10^{29206}, 0., 0.\}
        ,\q\= 4.812150411988066 \times 10^{29206}
       n= 19,q= \{-9.10739200800579 \times 10^{58412}, -2.129066479572471 \times 10^{58413}, 0., 0.\}
        \sqrt{q} = 2.315679158759691 \times 10^{58413}
       n= 20,q= \{-3.703478182564238 \times 10^{116826}, 3.878048608114267 \times 10^{116826}, 0., 0.\}
        ,\q\= 5.362369966313992 \times 10^{116826}
ln[ \circ ] := P = \{-2.2, 0.4, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, 0.4, 0, 0}
```

```
n= 2,q= \{2.48, -1.36, 0., 0.\}, q= 2.82843
       n= 3,q= \{2.1008, -6.3456, 0., 0.\}, \q = 6.68431
       n= 4, q= \{-38.0533, -26.2617, 0., 0.\}, q= 46.2356
       n=5,q=\{756.177, 1999.09, 0., 0.\}, q=2137.32
       n=6,q=\{-3.42454\times10^6, 3.02332\times10^6, 0., 0.\}, q=4.56815\times10^6
       n= 7,q= \{2.587 \times 10^{12}, -2.0707 \times 10^{13}, 0., 0.\}, \q\= 2.0868 \times 10^{13}
       n= 8,q= \{-4.22087 \times 10^{26}, -1.07138 \times 10^{26}, 0., 0.\},\q\= 4.35472×10<sup>26</sup>
       n= 9,q= \{1.66679 \times 10^{53}, 9.04433 \times 10^{52}, 0., 0.\},\q\= 1.89636 \times 10^{53}
       n= 10,q= \{1.96019 \times 10^{106}, 3.015 \times 10^{106}, 0., 0.\},\q\= 3.59619 \times 10^{106}
       n= 11,q= \{-5.24787 \times 10^{212}, 1.18199 \times 10^{213}, 0., 0.\}, \q\= 1.29326 \times 10^{213}
       n= 12,q= \{-1.121709751939305 \times 10^{426}, -1.240591418501088 \times 10^{426}, 0., 0.\}
        \sqrt{q} = 1.672513029920628 \times 10^{426}
       n= 13,q= \{-2.808343000628042 \times 10^{851}, 2.783166984609771 \times 10^{852}, 0., 0.\}
        \q = 2.797299835254278 \times 10^{852}
       n= 14,q= \{-7.667150560130081 \times 10^{1704}, -1.563217504161581 \times 10^{1704}, 0., 0.\}
        \q = 7.824886368313612 \times 10^{1704}
       n= 15,q= \{5.634154874638586 \times 10^{3409}, 2.397084792527523 \times 10^{3409}, 0., 0.\}
        ,\q\ =\ 6.122884667702018\times 10^{3409}
       n= 16,q= \{2.599768564884702 \times 10^{6819}, 2.701109393748193 \times 10^{6819}, 0., 0.\}
        \q = 3.748971665398045 \times 10^{6819}
       n= 17,q= \{-5.371953660320695 \times 10^{13637}, 1.404451858436265 \times 10^{13639}, 0., 0.\}
        \sqrt{q} = 1.405478854795739 \times 10^{13639}
       n= 18,q= \{-1.969599234052217 \times 10^{27278}, -1.508930060334180 \times 10^{27277}, 0., 0.\}
        ,\q\= 1.975370811277943 \times 10^{27278}
       n= 19,q= \{3.856552443509277 \times 10^{54556}, 5.943974982145131 \times 10^{54555}, 0., 0.\}
        \sqrt{q} = 3.902089842048879 \times 10^{54556}
       n= 20,q= \{1.451968836365371 \times 10^{109113}, 4.584650248309963 \times 10^{109112}, 0., 0.\}
        ,\q\= 1.522630513542105 \times 10^{109113}
ln[ \circ ] := P = \{-2.2, 0.2, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, 0.2, 0, 0}
```

 $n= 1,q= \{-2.2, 0.4, 0, 0\}, q= 2.23607$

 $n= 1,q= \{-2.2, 0.2, 0, 0\}, q= 2.20907$

```
n= 2,q= \{2.6, -0.68, 0., 0.\}, q= 2.68745
       n= 3,q= \{4.0976, -3.336, 0., 0.\}, q= 5.28386
       n=4,q=\{3.46143, -27.1392, 0., 0.\}, q=27.359
       n=5,q=\{-726.754, -187.681, 0., 0.\}, q=750.597
       n=6,q=\{492945., 272796., 0., 0.\},\q=563394.
       n= 7,q= \{1.68577 \times 10^{11}, 2.68947 \times 10^{11}, 0., 0.\}, \q\= 3.17412 \times 10^{11}
       n= 8,q= \{-4.3914 \times 10^{22}, 9.06766 \times 10^{22}, 0., 0.\}, q= 1.00751 \times 10^{23}
       n = 9,q = \{-6.29381 \times 10^{45}, -7.96394 \times 10^{45}, 0., 0.\}, q = 1.01507 \times 10^{46}\}
       n= 10,q= \{-2.38122 \times 10^{91}, 1.00247×10<sup>92</sup>, 0., 0.\},\q\= 1.03036×10<sup>92</sup>
       n= 11,q= \{-9.48245 \times 10^{183}, -4.77421 \times 10^{183}, 0., 0.\},\q\= 1.06165 \times 10^{184}
       n= 12,q= \{6.712382991058453 \times 10^{367}, 9.05424202148467 \times 10^{367}, 0., 0.\}
       \sqrt{q} = 1.127099747148714 \times 10^{368}
      n= 13,q= \{-3.692321316496795 \times 10^{735}, 1.215510802838808 \times 10^{736}, 0., 0.\}
        n= 14,q= \{-1.341134144775277 \times 10^{1472}, -8.976112895507726 \times 10^{1471}, 0., 0.\}
        n= 15,q= \{9.92934767153112\times10^{2943}, 2.407634298304617\times10^{2944}, 0., 0.\}
        \sqrt{q} = 2.604346821411114 \times 10^{2944}
      n= 16,q= \{-4.810783462551363 \times 10^{5888}, 4.781247602753883 \times 10^{5888}, 0., 0.\}
       ,\q\= 6.782622366194172×10<sup>5888</sup>
       n= 17,q= \{2.833088847179252 \times 10^{11775}, -4.600309379538345 \times 10^{11777}, 0., 0.\}
        \sqrt{q} = 4.600396616239743 \times 10^{11777}
      n= 18,q= \{-2.116204374822688 \times 10^{23555}, -2.606617039348838 \times 10^{23553}, 0., 0.\}
        ,\q\= 2.116364902671008 \times 10^{23555}
       n= 19,q= \{4.477641510779700 \times 10^{47110}, 1.103226876431475 \times 10^{47109}, 0., 0.\}
       \sqrt{q} = 4.479000401257664 \times 10^{47110}
       n= 20,q= \{2.003710240364870 \times 10^{94221}, 9.87970891543480 \times 10^{94219}, 0., 0.\}
        ,\q\= 2.006144459446632 \times 10^{94221}
ln[ \circ ] := P = \{-2.2, -0.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-2.2, -0.2, 0, 0}
```

```
n= 1, q= \{-2.2, -0.2, 0, 0\}, q= 2.20907
       n= 2,q= \{2.6, 0.68, 0., 0.\}, q= 2.68745
       n= 3,q= \{4.0976, 3.336, 0., 0.\}, q= 5.28386
       n= 4, q= \{3.46143, 27.1392, 0., 0.\}, q= 27.359
       n=5,q=\{-726.754, 187.681, 0., 0.\}, q=750.597
       n=6,q=\{492945., -272796., 0., 0.\}, q=563394.
       n= 7,q= \{1.68577 \times 10^{11}, -2.68947 \times 10^{11}, 0., 0.\},\q\= 3.17412 \times 10^{11}
       n= 8,q= \{-4.3914 \times 10^{22}, -9.06766 \times 10^{22}, 0., 0.\}, \q\= 1.00751 \times 10^{23}
       n= 9,q= \{-6.29381 \times 10^{45}, 7.96394 \times 10^{45}, 0., 0.\}, \q\= 1.01507 \times 10<sup>46</sup>
       n= 10,q= \{-2.38122 \times 10^{91}, -1.00247 \times 10^{92}, 0., 0.\},\q\= 1.03036\times 10^{92}
       n= 11,q= \{-9.48245 \times 10^{183}, 4.77421 \times 10^{183}, 0., 0.\},\q\= 1.06165 \times 10^{184}
       n= 12,q= \{6.712382991058453\times10^{367}, -9.05424202148467\times10^{367}, 0., 0.\}
       \sqrt{q} = 1.127099747148714 \times 10^{368}
       n= 13,q= \{-3.692321316496795 \times 10^{735}, -1.215510802838808 \times 10^{736}, 0., 0.\}
        \sqrt{q} = 1.270353840022696 \times 10^{736}
       n= 14,q= \{-1.341134144775277 \times 10^{1472}, 8.976112895507726 \times 10^{1471}, 0., 0.\}
        n= 15,q= \{9.92934767153112 \times 10^{2943}, -2.407634298304617 \times 10^{2944}, 0., 0.\}
        \sqrt{q} = 2.604346821411114 \times 10^{2944}
      n= 16,q= \{-4.810783462551363 \times 10^{5888}, -4.781247602753883 \times 10^{5888}, 0., 0.\}
        \q = 6.782622366194172 \times 10^{5888}
       n= 17,q= \{2.833088847179252 \times 10^{11775}, 4.600309379538345 \times 10^{11777}, 0., 0.\}
        \sqrt{q} = 4.600396616239743 \times 10^{11777}
       n= 18,q= \{-2.116204374822688 \times 10^{23555}, 2.606617039348838 \times 10^{23553}, 0., 0.\}
        ,\q\= 2.116364902671008 \times 10^{23555}
       n= 19,q= \{4.477641510779700 \times 10^{47110}, -1.103226876431475 \times 10^{47109}, 0., 0.\}
       \sqrt{q} = 4.479000401257664 \times 10^{47110}
       n= 20,q= \{2.003710240364870 \times 10^{94221}, -9.87970891543480 \times 10^{94219}, 0., 0.\}
        \sqrt{q} = 2.006144459446632 \times 10^{94221}
ln[ \circ ] := P = \{-2.2, -0.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, -0.4, 0, 0}
```

```
n= 1,q= \{-2.2, -0.4, 0, 0\}, q= 2.23607
       n= 2,q= \{2.48, 1.36, 0., 0.\}, q= 2.82843
       n= 3,q= \{2.1008, 6.3456, 0., 0.\}, \q = 6.68431
       n= 4, q= \{-38.0533, 26.2617, 0., 0.\}, q= 46.2356
       n=5,q=\{756.177, -1999.09, 0., 0.\}, q=2137.32
       n = 6,q = \{-3.42454 \times 10^6, -3.02332 \times 10^6, 0., 0.\}, q = 4.56815 \times 10^6\}
       n= 7,q= \{2.587 \times 10^{12}, 2.0707 \times 10^{13}, 0., 0.\}, \q\= 2.0868 \times 10^{13}
       n= 8,q= \{-4.22087 \times 10^{26}, 1.07138 \times 10^{26}, 0., 0.\}, \q\= 4.35472 \times 10^{26}
       n = 9,q = \{1.66679 \times 10^{53}, -9.04433 \times 10^{52}, 0., 0.\}, q = 1.89636 \times 10^{53}\}
       n= 10,q= \{1.96019 \times 10^{106}, -3.015 \times 10^{106}, 0., 0.\},\q\= 3.59619 \times 10^{106}
       n= 11,q= \{-5.24787 \times 10^{212}, -1.18199 \times 10^{213}, 0., 0.\},\q\= 1.29326 \times 10^{213}
       n= 12,q= \{-1.121709751939305 \times 10^{426}, 1.240591418501088 \times 10^{426}, 0., 0.\}
        \sqrt{q} = 1.672513029920628 \times 10^{426}
       n= 13,q= \{-2.808343000628042 \times 10^{851}, -2.783166984609771 \times 10^{852}, 0., 0.\}
        \sqrt{q} = 2.797299835254278 \times 10^{852}
       n= 14,q= \{-7.667150560130081 \times 10^{1704}, 1.563217504161581 \times 10^{1704}, 0., 0.\}
        \q = 7.824886368313612 \times 10^{1704}
       n= 15,q= \{5.634154874638586 \times 10^{3409}, -2.397084792527523 \times 10^{3409}, 0., 0.\}
        \sqrt{q} = 6.122884667702018 \times 10^{3409}
       n= 16,q= \{2.599768564884702 \times 10^{6819}, -2.701109393748193 \times 10^{6819}, 0., 0.\}
        \q = 3.748971665398045 \times 10^{6819}
       n= 17,q= \{-5.371953660320695 \times 10^{13637}, -1.404451858436265 \times 10^{13639}, 0., 0.\}
        ,\q\= 1.405478854795739 \times 10^{13639}
       n= 18,q= \{-1.969599234052217 \times 10^{27278}, 1.508930060334180 \times 10^{27277}, 0., 0.\}
        ,\q\= 1.975370811277943 \times 10^{27278}
       n= 19,q= \{3.856552443509277 \times 10^{54556}, -5.943974982145131 \times 10^{54555}, 0., 0.\}
        \sqrt{q} = 3.902089842048879 \times 10^{54556}
       n= 20,q= \{1.451968836365371 \times 10^{109113}, -4.584650248309963 \times 10^{109112}, 0., 0.\}
        ,\q\= 1.522630513542105 \times 10^{109113}
ln[ \circ ] := P = \{-2.2, -0.5, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, -0.5, 0, 0}
```

0}]

Out[\circ]= {-2.2, -0.7, 0, 0}

 $n= 1, q= \{-2.2, -0.7, 0, 0\}, q= 2.30868$

```
n= 2,q= \{2.15, 2.38, 0., 0.\}, q= 3.20732
       n= 3,q= \{-3.2419, 9.534, 0., 0.\}, \q = 10.0701
       n=4,q=\{-82.5872, -62.5165, 0., 0.\}, q=103.581
       n= 5, q= \{2910.13, 10325.4, 0., 0.\}, q= 10727.7
       n=6,q=\{-9.81458\times10^7, 6.00968\times10^7, 0., 0.\}, q=1.15084\times10^8
       n= 7,q= \{6.02097 \times 10^{15}, -1.17965 \times 10^{16}, 0., 0.\}, \q\= 1.32442 \times 10^{16}
       n= 8,q= \{-1.02905 \times 10^{32}, -1.42053 \times 10^{32}, 0., 0.\}, \q\= 1.7541×10<sup>32</sup>
       n= 9,q= \{-9.58951 \times 10^{63}, 2.9236 \times 10^{64}, 0., 0.\}, q= 3.07685 \times 10^{64}\}
       n= 10,q= \{-7.62783 \times 10^{128}, -5.60717 \times 10^{128}, 0., 0.\},\q\= 9.467 \times 10^{128}
       n= 11,q= \{2.67434 \times 10^{257}, 8.55411 \times 10^{257}, 0., 0.\},\q\= 8.96242 \times 10^{257}
       n= 12,q= \{-6.602067593304522 \times 10^{515}, 4.575324516462220 \times 10^{515}, 0., 0.\}
        \sqrt{q} = 8.032489709766331 \times 10^{515}
       n= 13,q= \{2.265370207562151\times10^{1031}, -6.041320343797380\times10^{1031}, 0., 0.\}
        \sqrt{q} = 6.452089093750200 \times 10^{1031}
       n= 14,q= \{-3.136564931906991 \times 10^{2063}, -2.737165424235544 \times 10^{2063}, 0., 0.\}
        \sqrt{q} = 4.162945367369028 \times 10^{2063}
       n= 15,q= \{2.345965012438163 \times 10^{4126}, 1.717059416497106 \times 10^{4127}, 0., 0.\}
        \sqrt{q} = 1.733011413169925 \times 10^{4127}
       n= 16,q= \{-2.893257521385542 \times 10^{8254}, 8.056322630759397 \times 10^{8253}, 0., 0.\}
        ,\q\= 3.003328558177222×10<sup>8254</sup>
       n= 17,q= \{7.721895741745150 \times 10^{16508}, -4.661803209230636 \times 10^{16508}, 0., 0.\}
        \sqrt{q} = 9.01998242836287 \times 10^{16508}
       n= 18,q= \{3.789526468478882 \times 10^{33017}, -7.199591670042385 \times 10^{33017}, 0., 0.\}
        ,\q\= 8.136008300797494 \times 10^{33017}
       n= 19,q= \{-3.747360936004167 \times 10^{66035}, -5.456608639173139 \times 10^{66035}, 0., 0.\}
        \q = 6.619463107064573 \times 10^{66035}
       n= 20,q= \{-1.573186385640891 \times 10^{132071}, 4.089576411500056 \times 10^{132071}, 0., 0.\}
        ,\q\= 4.381729182578896 \times 10^{132071}
ln[ \circ ] := P = \{-2.2, -1.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-2.2, -1.2, 0, 0}
```

```
n= 1,q= \{-2.2, -1.2, 0, 0\}, q= 2.50599
       n= 2,q= \{1.2, 4.08, 0., 0.\}, q= 4.25281
       n= 3,q= \{-17.4064, 8.592, 0., 0.\}, q= 19.4115
       n=4,q=\{226.96, -300.312, 0., 0.\},\q=376.428
       n=5,q=\{-38678.3, -136319., 0., 0.\}, q=141700.
       n= 6,q= \{-1.70868 \times 10^{10}, 1.05452 \times 10^{10}, 0., 0.\}, \q\= 2.00788 \times 10^{10}
       n= 7,q= \{1.80759 \times 10^{20}, -3.60366 \times 10^{20}, 0., 0.\}, \q\= 4.03159 \times 10^{20}
       n=8,q=\{-9.71899\times10^{40}, -1.30279\times10^{41}, 0., 0.\}, q=1.62537\times10^{41}\}
       n= 9,q= \{-7.52666 \times 10^{81}, 2.53235 \times 10^{82}, 0., 0.\}, \q\= 2.64184 \times 10^{82}
       n= 10,q= \{-5.84631 \times 10^{164}, -3.81203 \times 10^{164}, 0., 0.\}, \q\= 6.97932 \times 10<sup>164</sup>
       n= 11,q= \{1.964769367876027 \times 10^{329}, 4.457265130109520 \times 10^{329}, 0., 0.\}
        |q| = 4.871091367346151 \times 10^{329}
       n= 12,q= \{-1.600689377114627 \times 10^{659}, 1.751499598428228 \times 10^{659}, 0., 0.\}
        \sqrt{q} = 2.372753110903420 \times 10^{659}
       n= 13,q= \{-5.055443612866299 \times 10^{1317}, -5.607213602449199 \times 10^{1318}, 0., 0.\}
        ,\q\ =\ 5.629957325301856\times 10^{1318}
       n= 14,q= \{-3.118526928226262 \times 10^{2637}, 5.669390438495767 \times 10^{2636}, 0., 0.\}
        \sqrt{q} = 3.169641948472003 \times 10^{2637}
       n= 15,q= \{9.40379032263125 \times 10^{5274}, -3.536029349815509 \times 10^{5274}, 0., 0.\}
        \sqrt{q} = 1.004663008151340 \times 10^{5275}
       n= 16,q= \{7.592776886925651 \times 10^{10549}, -6.650415716067033 \times 10^{10549}, 0., 0.\}
       ,\q\= 1.009347759947699×10<sup>10550</sup>
       n= 17,q= \{1.342223165812101 \times 10^{21099}, -1.009902454748017 \times 10^{21100}, 0., 0.\}
        ,\q\= 1.018782900511438 \times 10^{21100}
       n= 18,q= \{-1.001887337837645 \times 10^{42200}, -2.711028939946591 \times 10^{42199}, 0., 0.\}
        ,\q\= 1.037918598374498 \times 10^{42200}
       n= 19,q= \{9.30281458587124 \times 10^{84399}, 5.432291134887805 \times 10^{84399}, 0., 0.\}
        \sqrt{q} = 1.077275016851682 \times 10^{84400}
       n= 20,q= \{5.703257224491797 \times 10^{168799}, 1.010711944086666 \times 10^{168800}, 0., 0.\}
        ,\q\ =\ 1.160521461932792 \times 10^{168\,800}
ln[-]:= P = \{-2.2, -1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Outfol= \{-2.2, -1.4, 0, 0\}
```

```
n= 1, q= \{-2.2, -1.4, 0, 0\}, q= 2.60768
       n= 2,q= \{0.68, 4.76, 0., 0.\}, q= 4.80833
       n= 3,q= \{-24.3952, 5.0736, 0., 0.\}, \q = 24.9172
       n= 4, q= \{567.184, -248.943, 0., 0.\}, q= 619.412
       n=5,q=\{259723., -282395., 0., 0.\}, q=383670.
       n = 6, q = \{-1.22905 \times 10^{10}, -1.46689 \times 10^{11}, 0., 0.\}, q = 1.47203 \times 10^{11}
       n= 7,q= \{-2.13666 \times 10^{22}, 3.60575 \times 10^{21}, 0., 0.\}, \q\= 2.16687 \times 10^{22}
       n= 8,q= \{4.43529 \times 10^{44}, -1.54085 \times 10^{44}, 0., 0.\}, \q\= 4.69532 \times 10^{44}
       n = 9,q = \{1.72976 \times 10^{89}, -1.36682 \times 10^{89}, 0., 0.\}, q = 2.2046 \times 10^{89}\}
       n= 10,q= \{1.12385 \times 10^{178}, -4.72855 \times 10^{178}, 0., 0.\}, \q\= 4.86027 \times 10^{178}
       n= 11,q= \{-2.109610344132128 \times 10^{357}, -1.062835565311691 \times 10^{357}, 0., 0.\}
        q = 2.362218288592462 \times 10^{357}
       n= 12,q= \{3.320836365177853\times10^{714}, 4.484337805386124\times10^{714}, 0., 0.\}
        \sqrt{q} = 5.580075242960698 \times 10^{714}
       n= 13,q= \{-9.08133138852758 \times 10^{1428}, 2.978350411573617 \times 10^{1429}, 0., 0.\}
        \q = 3.113723971710289 \times 10^{1429}
       n= 14,q= \{-8.045865376238171 \times 10^{2858}, -5.409477415731507 \times 10^{2858}, 0., 0.\}
        \sqrt{q} = 9.69527697200330 \times 10^{2858}
       n= 15,q= \{3.547350374123897 \times 10^{5717}, 8.704785408555294 \times 10^{5717}, 0., 0.\}
        \q = 9.39983955638574 \times 10^{5717}
       n= 16,q= \{-6.318959433220020 \times 10^{11435}, 6.175784755141373 \times 10^{11435}, 0., 0.\}
       ,\q\= 8.835698368579411×10<sup>11435</sup>
       n= 17,q= \{1.788930976843686 \times 10^{22870}, -7.804906667207393 \times 10^{22871}, 0., 0.\}
        \sqrt{q} = 7.806956566051686 \times 10^{22871}
       n= 18,q= \{-6.088456534341931 \times 10^{45743}, -2.792487861668224 \times 10^{45742}, 0., 0.\}
        ,\q\= 6.094857082421753 \times 10^{45743}
       n= 19,q= \{3.699132308599532 \times 10^{91487}, 3.400388193688885 \times 10^{91486}, 0., 0.\}
        \sqrt{q} = 3.714728285514660 \times 10^{91487}
       n= 20,q= \{1.356795343784711 \times 10^{182975}, 2.515697165810991 \times 10^{182974}, 0., 0.\}
        ,\q\= 1.379920623520269 \times 10^{182975}
ln[-]:= P = \{-1.7, 1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.7, 1.4, 0, 0}
```

```
n= 1, q= \{-1.7, 1.4, 0, 0\}, q= 2.20227
       n= 2, q= \{-0.77, -3.36, 0., 0.\}, \q = 3.4471
       n= 3,q= \{-12.3967, 6.5744, 0., 0.\}, q= 14.0321
       n=4,q=\{108.755, -161.602, 0., 0.\}, q=194.789
       n=5,q=\{-14289.1, -35148.7, 0., 0.\}, q=37942.2
       n = 6,q = \{-1.03126 \times 10^9, 1.00449 \times 10^9, 0., 0.\}, q = 1.43961 \times 10^9\}
       n= 7,q= \{5.4497 \times 10^{16}, -2.07176 \times 10^{18}, 0., 0.\}, \q\= 2.07248 \times 10^{18}
       n= 8,q= \{-4.28923 \times 10^{36}, -2.2581 \times 10^{35}, 0., 0.\}, \q\= 4.29517 \times 10<sup>36</sup>
       n= 9,q= \{1.83465 \times 10^{73}, 1.9371 \times 10^{72}, 0., 0.\},\q\= 1.84485 \times 10^{73}
       n= 10,q= \{3.32843 \times 10^{146}, 7.10782 \times 10^{145}, 0., 0.\},\q\= 3.40348 \times 10^{146}
       n= 11,q= \{1.05732 \times 10^{293}, 4.73157 \times 10^{292}, 0., 0.\},\q\= 1.15837 \times 10^{293}
       n= 12,q= \{8.940544236564429 \times 10^{585}, 1.000560736774964 \times 10^{586}, 0., 0.\}
        \sqrt{q} = 1.341810381699036 \times 10^{586}
       n= 13,q= \{-2.017884755160050 \times 10^{1171}, 1.789111505701213 \times 10^{1172}, 0., 0.\}
        n= 14,q= \{-3.160201390981388 \times 10^{2344}, -7.220441665271841 \times 10^{2343}, 0., 0.\}
        \q = 3.241638568683535 \times 10^{2344}
       n= 15,q= \{9.46552505314476 \times 10^{4688}, 4.563609958818408 \times 10^{4688}, 0., 0.\}
        \sqrt{q} = 1.050822060997663 \times 10^{4689}
       n= 16,q= \{6.876962867548461 \times 10^{9377}, 8.639392879595317 \times 10^{9377}, 0., 0.\}
        \q = 1.104227003879377 \times 10^{9378}
       n= 17,q= \{-2.734649104636191 \times 10^{18755}, 1.188255680622791 \times 10^{18756}, 0., 0.\}
        |q| = 1.219317276096426 \times 10^{18756}
       n= 18,q= \{-1.337168505277458 \times 10^{37512}, -6.498924666187968 \times 10^{37511}, 0., 0.\}
        ,\q\= 1.486734619787208 \times 10^{37512}
       n= 19,q= \{1.365659393338087 \times 10^{75024}, 1.738031476359474 \times 10^{75024}, 0., 0.\}
        \q = 2.210379829673815 \times 10^{75024}
       n= 20,q= \left\{-1.155727834203739 \times 10^{150048}, 4.747118023215157 \times 10^{150048}, 0., 0.\right\}
        ,\q\= 4.885778991428844 \times 10^{150048}
ln[ \circ ] := P = \{-1.7, 1.2, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.7, 1.2, 0, 0}
```

 $n= 1, q= \{-1.7, 1.2, 0, 0\}, q= 2.08087$

```
n= 2, q= \{-0.25, -2.88, 0., 0.\}, q= 2.89083
       n= 3,q= \{-9.9319, 2.64, 0., 0.\}, \q = 10.2768
       n= 4,q= \{89.973, -51.2404, 0., 0.\}, q= 103.541
       n=5,q=\{5467.87, -9219.31, 0., 0.\}, q=10718.8
       n=6,q=\{-5.50982\times10^7, -1.0082\times10^8, 0., 0.\}, q=1.14893\times10^8
       n= 7,q= \{-7.12885 \times 10^{15}, 1.111 \times 10^{16}, 0., 0.\},\q\= 1.32005 \times 10^{16}
       n= 8,q= \{-7.26115 \times 10^{31}, -1.58403 \times 10^{32}, 0., 0.\}, q= 1.74253 \times 10^{32}
       n= 9,q= \{-1.98191 \times 10^{64}, 2.30038 \times 10^{64}, 0., 0.\}, \q\= 3.03639 \times 10^{64}
       n= 10,q= \{-1.36378 \times 10^{128}, -9.11827 \times 10^{128}, 0., 0.\}, \q\= 9.21969 \times 10^{128}
       n= 11,q= \{-8.12829 \times 10^{257}, 2.48706 \times 10^{257}, 0., 0.\},\q\= 8.50027 \times 10^{257}
       n= 12,q= \{5.988355632813150 \times 10^{515}, -4.043111978051559 \times 10^{515}, 0., 0.\}
        \sqrt{q} = 7.225452072507918 \times 10^{515}
      n= 13,q= \{1.951364871798100 \times 10^{1031}, -4.842318477571874 \times 10^{1031}, 0., 0.\}
        \sqrt{q} = 5.220715765210897 \times 10^{1031}
       n= 14,q= \{-1.964022337534638 \times 10^{2063}, -1.889826035038522 \times 10^{2063}, 0., 0.\}
        n= 15,q= \{2.859412996256020 \times 10^{4125}, 7.423321093740348 \times 10^{4126}, 0., 0.\}
        \sqrt{q} = 7.428826185044443 \times 10^{4126}
      n= 16,q= \{-5.502393363393881 \times 10^{8253}, 4.245268162164521 \times 10^{8252}, 0., 0.\}
        \q = 5.518745848760197 \times 10^{8253}
       n= 17,q= \{3.009610970783415 \times 10^{16507}, -4.671827072264280 \times 10^{16506}, 0., 0.\}
        \sqrt{q} = 3.045655574320791 \times 10^{16507}
      n= 18,q= \{8.839498513528476 \times 10^{33014}, -2.812076402057908 \times 10^{33014}, 0., 0.\}
        ,\q\= 9.27601787739130 \times 10^{33014}
       n= 19,q= \{7.022896027966118 \times 10^{66029}, -4.971469035183876 \times 10^{66029}, 0., 0.\}
        \sqrt{q} = 8.604450766168308 \times 10^{66029}
       n= 20,q= \{2.460556425183018 \times 10^{132059}, -6.982822028069879 \times 10^{132059}, 0., 0.\}
        ,\q\= 7.403657298741438 \times 10^{132059}
ln[ \circ ] := P = \{-1.7, 0.7, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.7, 0.7, 0, 0}
```

```
n= 2,q= \{0.7, -1.68, 0., 0.\}, q= 1.82
       n= 3,q= \{-4.0324, -1.652, 0., 0.\}, q= 4.35768
       n= 4, q= \{11.8311, 14.023, 0., 0.\}, q= 18.3473
       n=5,q=\{-58.3699, 332.517, 0., 0.\}, q=337.602
       n=\ 6, q=\ \{-107\,163.,\ -38\,817.3,\ 0.,\ 0.\}\,, \ \ |\ 113\,976.
       n= 7,q= \{9.97702 \times 10^9, 8.31953 \times 10^9, 0., 0.\},\q\= 1.29906 \times 10^{10}
       n= 8,q= \{3.03265 \times 10^{19}, 1.66008 \times 10^{20}, 0., 0.\}, \q\= 1.68756 \times 10^{20}
       n= 9,q= \{-2.6639 \times 10^{40}, 1.00689 \times 10^{40}, 0., 0.\}, \q\= 2.84784 \times 10^{40}
       n= 10,q= \{6.08256\times10^{80}, -5.36451\times10^{80}, 0., 0.\},\q\= 8.11021\times10^{80}
       n= 11,q= \{8.21959 \times 10^{160}, -6.52599 \times 10^{161}, 0., 0.\},\q\= 6.57755 \times 10^{161}
       n= 12,q= \{-4.191290585161919 \times 10^{323}, -1.072818517595919 \times 10^{323}, 0., 0.\}
        \sqrt{q} = 4.326413796779458 \times 10^{323}
       n= 13,q= \{1.641597719757023 \times 10^{647}, 8.992988304774280 \times 10^{646}, 0., 0.\}
        n= 14,q= \{1.886104687013389 \times 10^{1294}, 2.952573818983807 \times 10^{1294}, 0., 0.\}
        \q = 3.503581460009528 \times 10^{1294}
       n= 15,q= \{-5.160301266174752 \times 10^{2588}, 1.113772663747676 \times 10^{2589}, 0., 0.\}
        \sqrt{q} = 1.227508304692250 \times 10^{2589}
       n= 16,q= \{-9.74202454934747 \times 10^{5177}, -1.149480497393592 \times 10^{5178}, 0., 0.\}
        \q = 1.506776638088441 \times 10^{5178}
       n= 17,q= \{-3.722349906873325 \times 10^{10355}, 2.239653444920901 \times 10^{10356}, 0., 0.\}
        \sqrt{q} = 2.270375837089106 \times 10^{10356}
       n= 18,q= \{-4.877488665054063 \times 10^{20712}, -1.667354758425968 \times 10^{20712}, 0., 0.\}
        ,\q\= 5.154606441638060 \times 10^{20712}
       n= 19,q= \{2.100982378728515 \times 10^{41425}, 1.626500786969322 \times 10^{41425}, 0., 0.\}
        \sqrt{q} = 2.656996756817658 \times 10^{41425}
       n= 20,q= \{1.768622145715902 \times 10^{82850}, 6.834498984821216 \times 10^{82850}, 0., 0.\}
        ,\q\= 7.059631765739552 \times 10^{82850}
ln[ \circ ] := P = \{-1.7, 0.5, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.7, 0.5, 0, 0}
```

 $n= 1,q= \{-1.7, 0.7, 0, 0\}, q= 1.83848$

```
n= 1, q= \{-1.7, 0.5, 0, 0\}, q= 1.772
       n= 2, q= \{0.94, -1.2, 0., 0.\}, q= 1.52434
       n= 3,q= \{-2.2564, -1.756, 0., 0.\}, \q = 2.85917
       n=4,q=\{0.307805, 8.42448, 0., 0.\},\q\= 8.4301
       n=5,q=\{-72.5771, 5.68619, 0., 0.\}, q=72.7995
       n=6,q=\{5233.4, -824.874, 0., 0.\},\q=5298.01
       n= 7,q= \{2.6708\times10^7, -8.63379\times10^6, 0., 0.\},\q\= 2.80689\times10^7
       n= 8,q= \{6.38777 \times 10^{14}, -4.61183 \times 10^{14}, 0., 0.\}, \q\= 7.87861 \times 10^{14}
       n = 9,q = \{1.95346 \times 10^{29}, -5.89186 \times 10^{29}, 0., 0.\}, q = 6.20725 \times 10^{29}\}
       n= 10,q= \{-3.0898 \times 10^{59}, -2.3019 \times 10^{59}, 0., 0.\},\q\= 3.853 \times 10^{59}
       n= 11,q= \{4.24812 \times 10^{118}, 1.42248 \times 10^{119}, 0., 0.\},\q\= 1.48456 \times 10^{119}
       n= 12,q= \{-1.84299 \times 10^{238}, 1.20857 \times 10^{238}, 0., 0.\}, \q\= 2.20392 \times 10^{238}
       n= 13,q= \{1.935962613380683 \times 10^{476}, -4.454785477860046 \times 10^{476}, 0., 0.\}
        ,\q\= 4.857269283677870 \times 10^{476}
       n= 14,q= \{-1.609716241334500 \times 10^{953}, -1.724859627153650 \times 10^{953}, 0., 0.\}
        \q = 2.359306489416053 \times 10^{953}
       n= 15,q= \{-3.839543557685598 \times 10^{1905}, 5.553069111702801 \times 10^{1906}, 0., 0.\}
        \q = 5.566327111000699 \times 10^{1906}
       n= 16,q= \{-3.068915561203409 \times 10^{3813}, -4.264250146644275 \times 10^{3812}, 0., 0.\}
        ,\q\ =\ 3.098399750666139 \times 10^{3813}
       n= 17,q= \{9.23640442866488 \times 10^{7626}, 2.617324726380107 \times 10^{7626}, 0., 0.\}
        \sqrt{q} = 9.60008101492799 \times 10^{7626}
       n= 18,q= \{7.846077804653943 \times 10^{15253}, 4.834933938798261 \times 10^{15253}, 0., 0.\}
        \sqrt{q} = 9.21615554931808 \times 10^{15253}
       n= 19,q= \{3.818435072413998 \times 10^{30507}, 7.587053572834620 \times 10^{30507}, 0., 0.\}
        ,\q\= 8.493752310922651 \times 10^{30507}
       n= 20,q=\{-4.298293551482128\times10^{61015}, 5.794134291759129\times10^{61015}, 0., 0.\}
        \sqrt{q} = 7.214382831930387 \times 10^{61015}
ln[ \circ ] := P = \{-1.7, 0.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\bullet]= \{-1.7, 0.4, 0, 0\}
```

```
n= 1, q= \{-1.7, 0.4, 0, 0\}, q= 1.74642
       n= 2,q= \{1.03, -0.96, 0., 0.\}, q= 1.40801
       n= 3,q= \{-1.5607, -1.5776, 0., 0.\}, q= 2.21915
       n=4,q=\{-1.75304, 5.32432, 0., 0.\}, q=5.60549
       n=5,q=\{-26.9753, -18.2675, 0., 0.\}, q=32.5786
       n=6,q={392.264, 985.939, 0., 0.},\q=1061.11
       n= 7, q= \{-818206., 773497., 0., 0.\}, q= 1.12595 \times 10^6
       n= 8,q= \{7.11642 \times 10^{10}, -1.26576 \times 10^{12}, 0., 0.\}, q= 1.26776 \times 10^{12}
       n = 9,q = \{-1.59708 \times 10^{24}, -1.80154 \times 10^{23}, 0., 0.\}, q = 1.60721 \times 10^{24}\}
       n= 10,q= \{2.51822 \times 10^{48}, 5.75441 \times 10^{47}, 0., 0.\}, \q\= 2.58313 \times 10^{48}
       n= 11,q= \{6.01031 \times 10^{96}, 2.89817×10<sup>96</sup>, 0., 0.\},\q\= 6.67257×10<sup>96</sup>
       n= 12,q= \{2.77244 \times 10^{193}, 3.48378 \times 10^{193}, 0., 0.\},\q\= 4.45232 \times 10^{193}
       n= 13,q= \{-4.450324293558656 \times 10^{386}, 1.931712070691921 \times 10^{387}, 0., 0.\}
        n= 14,q= \{-3.533457660878485 \times 10^{774}, -1.719349031272150 \times 10^{774}, 0., 0.\}
        \q = 3.929565387235252 \times 10^{774}
       n= 15,q= \{9.52916194988437 \times 10^{1548}, 1.215049401254516 \times 10^{1549}, 0., 0.\}
       ,\q\= 1.544148413255733×10<sup>1549</sup>
       n= 16,q= \{-5.682957728177167 \times 10^{3097}, 2.315680504332865 \times 10^{3098}, 0., 0.\}
        \sqrt{q} = 2.384394322160199 \times 10^{3098}
       n= 17,q= \{-5.039416112744825 \times 10^{6196}, -2.631982883617531 \times 10^{6196}, 0., 0.\}
        ,\q\ =\ 5.685336283549797 \times 10^{6196}
       n= 18,q= \{1.846838085773651 \times 10^{12393}, 2.652731390434154 \times 10^{12393}, 0., 0.\}
        \sqrt{q} = 3.232304865704782 \times 10^{12393}
       n= 19,q= \{-3.626172914730638 \times 10^{24786}, 9.79833072636218 \times 10^{24786}, 0., 0.\}
        \sqrt{q} = 1.044779474485881 \times 10^{24787}
      n= 20,q= \{-8.285815501564710 \times 10^{49573}, -7.106088297901502 \times 10^{49573}, 0., 0.\}
        \sqrt{q} = 1.091564150306993 \times 10^{49574}
ln[ \circ ] := P = \{-1.7, 0.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-1.7, 0.2, 0, 0}
```

```
n= 1, q= \{-1.7, 0.2, 0, 0\}, q= 1.71172
       n= 2,q= \{1.15, -0.48, 0., 0.\}, q= 1.24615
       n= 3,q= \{-0.6079, -0.904, 0., 0.\}, \q = 1.08938
       n= 4, q= \{-2.14767, 1.29908, 0., 0.\}, q= 2.51
       n=5,q=\{1.22488, -5.38001, 0., 0.\}, q=5.51769
       n=6,q=\{-29.1442, -12.9798, 0., 0.\}, q=31.9039
       n= 7, q= \{679.209, 756.771, 0., 0.\}, q= 1016.87
       n= 8,q= \{-111379., 1.02801 \times 10^6, 0., 0.\}, q= 1.03403 \times 10^6
       n= 9,q= \{-1.0444 \times 10^{12}, -2.28998 \times 10^{11}, 0., 0.\}, \q\= 1.06922 \times 10^{12}
       n= 10,q= \{1.03834 \times 10^{24}, 4.78334 \times 10^{23}, 0., 0.\}, \q\= 1.14322 \times 10^{24}
       n= 11,q= \{8.4935 \times 10^{47}, 9.93347 \times 10^{47}, 0., 0.\},\q\= 1.30696\times 10^{48}
       n= 12,q= \left\{-2.65344\times10^{95},\,1.6874\times10^{96},\,0.,\,0.\right\},\q\= 1.70813\times10^{96}
       n= 13,q= \{-2.7769 \times 10^{192}, -8.95481 \times 10^{191}, 0., 0.\}, \q\= 2.91772 \times 10^{192}
       n= 14,q= \{6.909315014496898 \times 10^{384}, 4.973330293805434 \times 10^{384}, 0., 0.\}
        ,\q\= 8.513086877322180 \times 10^{384}
       n= 15,q= \{2.300461975826942 \times 10^{769}, 6.872461134208431 \times 10^{769}, 0., 0.\}
       , q = 7.247264818083511 \times 10^{769}
      n= 16,q= \{-4.193859673897983 \times 10^{1539}, 3.161967103918998 \times 10^{1539}, 0., 0.\}
        \q = 5.252284734343103 \times 10^{1539}
      n = 17, q = \{7.590422998081801 \times 10^{3078}, -2.652169265463576 \times 10^{3079}, 0., 0.\}
        \sqrt{q} = 2.758649493061360 \times 10^{3079}
      n= 18,q= \{-6.457856599771514 \times 10^{6158}, -4.026217317476089 \times 10^{6158}, 0., 0.\}
        \q = 7.610147025567696 \times 10^{6158}
      n= 19,q= \{2.549348597566814 \times 10^{12317}, 5.200146815155465 \times 10^{12317}, 0., 0.\}
        ,\q\= 5.791433775075686 \times 10^{12317}
       n= 20,q= \{-2.054234862725564 \times 10^{24635}, 2.651397398071624 \times 10^{24635}, 0., 0.\}
        \sqrt{q} = 3.354070517108741 \times 10^{24635}
ln[-]:= P = \{-1.7, -0.2, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module[\{q = t, n = 1\}, While[n \le 20, q = quaternion[q] + P;
            Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]]; 
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.7, -0.2, 0, 0}
```

```
n= 1, q= \{-1.7, -0.2, 0, 0\}, q= 1.71172
       n= 2,q= \{1.15, 0.48, 0., 0.\}, q= 1.24615
       n= 3,q= \{-0.6079, 0.904, 0., 0.\}, \q = 1.08938
       n=4,q=\{-2.14767, -1.29908, 0., 0.\}, q=2.51
       n=5,q=\{1.22488,5.38001,0.,0.\},\q=5.51769
       n=6,q=\{-29.1442, 12.9798, 0., 0.\}, \q = 31.9039
       n= 7, q= \{679.209, -756.771, 0., 0.\}, q= 1016.87
       n= 8,q= \{-111379., -1.02801 \times 10^6, 0., 0.\}, q= 1.03403 \times 10^6\}
       n = 9,q = \{-1.0444 \times 10^{12}, 2.28998 \times 10^{11}, 0., 0.\}, q = 1.06922 \times 10^{12}\}
       n= 10,q= \{1.03834 \times 10^{24}, -4.78334 \times 10^{23}, 0., 0.\}, \q\= 1.14322 \times 10^{24}
       n= 11,q= \{8.4935 \times 10^{47}, -9.93347 \times 10^{47}, 0., 0.\},\q\= 1.30696\times 10^{48}
       n= 12,q= \{-2.65344 \times 10^{95}, -1.6874 \times 10^{96}, 0., 0.\}, \q\= 1.70813×10<sup>96</sup>
       n= 13,q= \{-2.7769 \times 10^{192}, 8.95481 \times 10^{191}, 0., 0.\},\q\= 2.91772×10<sup>192</sup>
       n= 14,q= \{6.909315014496898 \times 10^{384}, -4.973330293805434 \times 10^{384}, 0., 0.\}
        ,\q\= 8.513086877322180 \times 10^{384}
       n= 15,q= \{2.300461975826942 \times 10^{769}, -6.872461134208431 \times 10^{769}, 0., 0.\}
       , q = 7.247264818083511 \times 10^{769}
      n= 16,q= \{-4.193859673897983 \times 10^{1539}, -3.161967103918998 \times 10^{1539}, 0., 0.\}
        \q = 5.252284734343103 \times 10^{1539}
      n= 17,q= \{7.590422998081801 \times 10^{3078}, 2.652169265463576 \times 10^{3079}, 0., 0.\}
        \sqrt{q} = 2.758649493061360 \times 10^{3079}
       n= 18,q= \{-6.457856599771514 \times 10^{6158}, 4.026217317476089 \times 10^{6158}, 0., 0.\}
       \q = 7.610147025567696 \times 10^{6158}
      n= 19,q= \{2.549348597566814 \times 10^{12317}, -5.200146815155465 \times 10^{12317}, 0., 0.\}
        ,\q\= 5.791433775075686 \times 10^{12317}
       n= 20,q= \{-2.054234862725564 \times 10^{24635}, -2.651397398071624 \times 10^{24635}, 0., 0.\}
        \sqrt{q} = 3.354070517108741 \times 10^{24635}
ln[-]:= P = \{-1.7, -0.4, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module[\{q = t, n = 1\}, While[n \le 20, q = quaternion[q] + P;
            Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]]; 
           n++]]
       iteration[
        {0,
          0,
          0,
          0}]
Out[\circ]= {-1.7, -0.4, 0, 0}
```

```
n= 1, q= \{-1.7, -0.4, 0, 0\}, q= 1.74642
       n= 2,q= \{1.03, 0.96, 0., 0.\}, q= 1.40801
       n= 3,q= \{-1.5607, 1.5776, 0., 0.\}, q= 2.21915
       n=4,q=\{-1.75304, -5.32432, 0., 0.\}, q=5.60549
       n=5,q=\{-26.9753, 18.2675, 0., 0.\}, q=32.5786
       n=6,q={392.264, -985.939, 0., 0.},\q\ 1061.11
       n= 7, q= \{-818206., -773497., 0., 0.\}, q= 1.12595 \times 10^6
       n= 8,q= \{7.11642 \times 10^{10}, 1.26576 \times 10^{12}, 0., 0.\}, q= 1.26776 \times 10^{12}
       n= 9,q= \{-1.59708 \times 10^{24}, 1.80154 \times 10^{23}, 0., 0.\}, \q\= 1.60721 \times 10^{24}
       n= 10,q= \{2.51822 \times 10^{48}, -5.75441 \times 10^{47}, 0., 0.\}, \q\= 2.58313 \times 10^{48}
       n= 11,q= \{6.01031 \times 10^{96}, -2.89817×10<sup>96</sup>, 0., 0.\},\q\= 6.67257×10<sup>96</sup>
       n= 12,q= \{2.77244 \times 10^{193}, -3.48378 \times 10^{193}, 0., 0.\}, \q\= 4.45232 \times 10^{193}
       n= 13,q= \{-4.450324293558656 \times 10^{386}, -1.931712070691921 \times 10^{387}, 0., 0.\}
        \q = 1.982313140559597 \times 10^{387}
       n= 14,q= \left\{-3.533457660878485\times10^{774}\text{, }1.719349031272150\times10^{774}\text{, }0.\text{, }0.\right\}
        \q = 3.929565387235252 \times 10^{774}
       n= 15,q= \{9.52916194988437 \times 10^{1548}, -1.215049401254516 \times 10^{1549}, 0., 0.\}
        ,\q\= 1.544148413255733×10<sup>1549</sup>
       n= 16,q= \{-5.682957728177167 \times 10^{3097}, -2.315680504332865 \times 10^{3098}, 0., 0.\}
        \sqrt{q} = 2.384394322160199 \times 10^{3098}
       n= 17,q= \{-5.039416112744825 \times 10^{6196}, 2.631982883617531 \times 10^{6196}, 0., 0.\}
        ,\q\ =\ 5.685336283549797 \times 10^{6196}
       n= 18,q= \{1.846838085773651 \times 10^{12393}, -2.652731390434154 \times 10^{12393}, 0., 0.\}
        \sqrt{q} = 3.232304865704782 \times 10^{12393}
       n= 19,q= \{-3.626172914730638 \times 10^{24786}, -9.79833072636218 \times 10^{24786}, 0., 0.\}
        \sqrt{q} = 1.044779474485881 \times 10^{24787}
       n= 20,q= \{-8.285815501564710 \times 10^{49573}, 7.106088297901502 \times 10^{49573}, 0., 0.\}
        \sqrt{q} = 1.091564150306993 \times 10^{49574}
ln[\circ]:= P = \{-1.7, -0.5, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\bullet]= {-1.7, -0.5, 0, 0}
```

```
n= 1, q= \{-1.7, -0.5, 0, 0\}, q= 1.772
               n= 2,q= \{0.94, 1.2, 0., 0.\}, q= 1.52434
               n= 3,q= \{-2.2564, 1.756, 0., 0.\}, \q = 2.85917
               n=4,q=\{0.307805, -8.42448, 0., 0.\}, q=8.4301
               n=5,q=\{-72.5771, -5.68619, 0., 0.\}, q=72.7995
               n=6,q=\{5233.4,824.874,0.,0.\},\q=5298.01
               n= 7,q= \{2.6708 \times 10^7, 8.63379 \times 10^6, 0., 0.\},\q\= 2.80689 \times 10^7
               n= 8,q= \{6.38777 \times 10^{14}, 4.61183 \times 10^{14}, 0., 0.\}, \q\= 7.87861 \times 10^{14}
               n= 9,q= \{1.95346 \times 10^{29}, 5.89186 \times 10^{29}, 0., 0.\}, \q\= 6.20725 \times 10^{29}
               n= 10,q= \{-3.0898 \times 10^{59}, 2.3019 \times 10^{59}, 0., 0.\},\q\= 3.853 \times 10^{59}
               n= 11,q= \{4.24812 \times 10^{118}, -1.42248 \times 10^{119}, 0., 0.\}, \q\= 1.48456 \times 10^{119}
                \text{n= 12,q= } \left\{ -1.84299 \times 10^{238} \text{, } -1.20857 \times 10^{238} \text{, 0., 0.} \right\} \text{,} \\ \text{q} = 2.20392 \times 10^{238} \text{, } -1.20857 \times 10^{238} \text{, } -1.
               n= 13,q= \{1.935962613380683 \times 10^{476}, 4.454785477860046 \times 10^{476}, 0., 0.\}
                  ,\q\= 4.857269283677870 \times 10^{476}
               n= 14,q= \{-1.609716241334500 \times 10^{953}, 1.724859627153650 \times 10^{953}, 0., 0.\}
                  \q = 2.359306489416053 \times 10^{953}
               n= 15,q= \{-3.839543557685598 \times 10^{1905}, -5.553069111702801 \times 10^{1906}, 0., 0.\}
                 \q = 5.566327111000699 \times 10^{1906}
               n= 16,q= \{-3.068915561203409 \times 10^{3813}, 4.264250146644275 \times 10^{3812}, 0., 0.\}
                  ,\q\ =\ 3.098399750666139 \times 10^{3813}
               n= 17,q= \{9.23640442866488 \times 10^{7626}, -2.617324726380107 \times 10^{7626}, 0., 0.\}
                  \sqrt{q} = 9.60008101492799 \times 10^{7626}
               n= 18,q= \{7.846077804653943 \times 10^{15253}, -4.834933938798261 \times 10^{15253}, 0., 0.\}
                  \sqrt{q} = 9.21615554931808 \times 10^{15253}
               n= 19,q= \{3.818435072413998 \times 10^{30507}, -7.587053572834620 \times 10^{30507}, 0., 0.\}
                  \sqrt{q} = 8.493752310922651 \times 10^{30507}
               n= 20,q= \{-4.298293551482128 \times 10^{61015}, -5.794134291759129 \times 10^{61015}, 0., 0.\}
                  \sqrt{q} = 7.214382831930387 \times 10^{61015}
 ln[\circ]:= P = \{-1.7, -0.7, 0, 0\}
               quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
               iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
                         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
                         n++]]
               iteration[
                   {0,
                      0,
                      0,
                      0}]
Out[\phi]= {-1.7, -0.7, 0, 0}
```

 $n= 1, q= \{-1.7, -0.7, 0, 0\}, q= 1.83848$

```
n= 2,q= \{0.7, 1.68, 0., 0.\}, q= 1.82
       n= 3,q= \{-4.0324, 1.652, 0., 0.\}, \q = 4.35768
       n= 4, q= \{11.8311, -14.023, 0., 0.\}, q= 18.3473
       n=5,q=\{-58.3699, -332.517, 0., 0.\}, q=337.602
       n=6,q=\{-107163., 38817.3, 0., 0.\}, q=113976.
       n= 7,q= \{9.97702 \times 10^9, -8.31953 \times 10^9, 0., 0.\}, \q\= 1.29906 \times 10<sup>10</sup>
       n= 8,q= \{3.03265 \times 10^{19}, -1.66008 \times 10^{20}, 0., 0.\}, q = 1.68756 \times 10^{20}\}
       n= 9,q= \{-2.6639 \times 10^{40}, -1.00689 \times 10^{40}, 0., 0.\}, \q\= 2.84784 \times 10^{40}
       n= 10,q= \{6.08256 \times 10^{80}, 5.36451 \times 10^{80}, 0., 0.\}, \q\= 8.11021 \times 10^{80}
       n= 11,q= \{8.21959 \times 10^{160}, 6.52599 \times 10^{161}, 0., 0.\},\q\= 6.57755 \times 10^{161}
       n= 12,q= \{-4.191290585161919 \times 10^{323}, 1.072818517595919 \times 10^{323}, 0., 0.\}
        \sqrt{q} = 4.326413796779458 \times 10^{323}
       n= 13,q= \{1.641597719757023 \times 10^{647}, -8.992988304774280 \times 10^{646}, 0., 0.\}
        \sqrt{q} = 1.871785634096364 \times 10^{647}
       n= 14,q= \{1.886104687013389 \times 10^{1294}, -2.952573818983807 \times 10^{1294}, 0., 0.\}
        \sqrt{q} = 3.503581460009528 \times 10^{1294}
       n= 15,q= \{-5.160301266174752 \times 10^{2588}, -1.113772663747676 \times 10^{2589}, 0., 0.\}
        \sqrt{q} = 1.227508304692250 \times 10^{2589}
       n= 16,q= \{-9.74202454934747 \times 10^{5177}, 1.149480497393592 \times 10^{5178}, 0., 0.\}
        \q = 1.506776638088441 \times 10^{5178}
       n= 17,q= \{-3.722349906873325 \times 10^{10355}, -2.239653444920901 \times 10^{10356}, 0., 0.\}
        \sqrt{q} = 2.270375837089106 \times 10^{10356}
       n= 18,q= \{-4.877488665054063 \times 10^{20712}, 1.667354758425968 \times 10^{20712}, 0., 0.\}
        ,\q\= 5.154606441638060 \times 10^{20712}
       n= 19,q= \{2.100982378728515 \times 10^{41425}, -1.626500786969322 \times 10^{41425}, 0., 0.\}
        \sqrt{q} = 2.656996756817658 \times 10^{41425}
       n= 20,q= \{1.768622145715902 \times 10^{82850}, -6.834498984821216 \times 10^{82850}, 0., 0.\}
        \sqrt{q} = 7.059631765739552 \times 10^{82850}
ln[ \circ ] := P = \{-1.7, -1.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.7, -1.2, 0, 0}
```

```
n= 1, q= \{-1.7, -1.2, 0, 0\}, q= 2.08087
       n= 2,q= \{-0.25, 2.88, 0., 0.\}, q= 2.89083
       n= 3,q= \{-9.9319, -2.64, 0., 0.\}, \q = 10.2768
       n=4,q=\{89.973,51.2404,0.,0.\},\q=103.541
       n=5,q=\{5467.87,9219.31,0.,0.\},\q\=10718.8
       n=6,q=\{-5.50982\times10^7, 1.0082\times10^8, 0., 0.\}, q=1.14893\times10^8
       n= 7,q= \{-7.12885 \times 10^{15}, -1.111 \times 10^{16}, 0., 0.\}, \q\= 1.32005 \times 10<sup>16</sup>
       n= 8,q= \{-7.26115 \times 10^{31}, 1.58403 \times 10^{32}, 0., 0.\}, q= 1.74253 \times 10^{32}
       n = 9,q = \{-1.98191 \times 10^{64}, -2.30038 \times 10^{64}, 0., 0.\}, q = 3.03639 \times 10^{64}\}
       n= 10,q= \{-1.36378 \times 10^{128}, 9.11827 \times 10^{128}, 0., 0.\}, \q\= 9.21969 \times 10^{128}
       n= 11,q= \{-8.12829 \times 10^{257}, -2.48706 \times 10^{257}, 0., 0.\},\q\= 8.50027 \times 10^{257}
       n= 12,q= \{5.988355632813150 \times 10^{515}, 4.043111978051559 \times 10^{515}, 0., 0.\}
        \sqrt{q} = 7.225452072507918 \times 10^{515}
       n= 13,q= \{1.951364871798100 \times 10^{1031}, 4.842318477571874 \times 10^{1031}, 0., 0.\}
        \sqrt{q} = 5.220715765210897 \times 10^{1031}
       n= 14,q= \{-1.964022337534638 \times 10^{2063}, 1.889826035038522 \times 10^{2063}, 0., 0.\}
        n= 15,q= \{2.859412996256020 \times 10^{4125}, -7.423321093740348 \times 10^{4126}, 0., 0.\}
        \sqrt{q} = 7.428826185044443 \times 10^{4126}
       n= 16,q= \{-5.502393363393881 \times 10^{8253}, -4.245268162164521 \times 10^{8252}, 0., 0.\}
        \q = 5.518745848760197 \times 10^{8253}
       n= 17,q= \{3.009610970783415 \times 10^{16507}, 4.671827072264280 \times 10^{16506}, 0., 0.\}
        \sqrt{q} = 3.045655574320791 \times 10^{16507}
       n= 18,q= \{8.839498513528476 \times 10^{33014}, 2.812076402057908 \times 10^{33014}, 0., 0.\}
        ,\q\= 9.27601787739130 \times 10^{33014}
       n= 19,q= \{7.022896027966118 \times 10^{66029}, 4.971469035183876 \times 10^{66029}, 0., 0.\}
        \sqrt{q} = 8.604450766168308 \times 10^{66029}
       n= 20,q= \{2.460556425183018 \times 10^{132059}, 6.982822028069879 \times 10^{132059}, 0., 0.\}
        ,\q\= 7.403657298741438 \times 10^{132059}
ln[ \circ ] := P = \{-1.7, -1.4, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.7, -1.4, 0, 0}
```

```
n= 1,q= \{-1.7, -1.4, 0, 0\}, q= 2.20227
       n= 2, q= \{-0.77, 3.36, 0., 0.\}, q= 3.4471
       n= 3,q= \{-12.3967, -6.5744, 0., 0.\}, \q = 14.0321
       n= 4, q= \{108.755, 161.602, 0., 0.\}, q= 194.789
       n=5,q=\{-14289.1, 35148.7, 0., 0.\}, q=37942.2
       n = 6,q = \{-1.03126 \times 10^9, -1.00449 \times 10^9, 0., 0.\}, q = 1.43961 \times 10^9\}
       n= 7,q= \{5.4497 \times 10^{16}, 2.07176 \times 10^{18}, 0., 0.\},\q\= 2.07248 \times 10^{18}
       n= 8,q= \{-4.28923 \times 10^{36}, 2.2581 \times 10^{35}, 0., 0.\}, \q\= 4.29517×10<sup>36</sup>
       n= 9,q= \{1.83465 \times 10^{73}, -1.9371 \times 10^{72}, 0., 0.\},\q\= 1.84485 \times 10^{73}
       n= 10,q= \{3.32843 \times 10^{146}, -7.10782 \times 10^{145}, 0., 0.\}, \q\= 3.40348 \times 10^{146}
       n= 11,q= \{1.05732 \times 10^{293}, -4.73157 \times 10^{292}, 0., 0.\}, \q\= 1.15837 \times 10^{293}
       n= 12,q= \{8.940544236564429 \times 10^{585}, -1.000560736774964 \times 10^{586}, 0., 0.\}
        \sqrt{q} = 1.341810381699036 \times 10^{586}
       n= 13,q= \{-2.017884755160050 \times 10^{1171}, -1.789111505701213 \times 10^{1172}, 0., 0.\}
        n= 14,q= \{-3.160201390981388 \times 10^{2344}, 7.220441665271841 \times 10^{2343}, 0., 0.\}
        \q = 3.241638568683535 \times 10^{2344}
       n= 15,q= \{9.46552505314476 \times 10^{4688}, -4.563609958818408 \times 10^{4688}, 0., 0.\}
        \sqrt{q} = 1.050822060997663 \times 10^{4689}
       n= 16,q= \{6.876962867548461 \times 10^{9377}, -8.639392879595317 \times 10^{9377}, 0., 0.\}
        \q = 1.104227003879377 \times 10^{9378}
       n= 17,q= \{-2.734649104636191 \times 10^{18755}, -1.188255680622791 \times 10^{18756}, 0., 0.\}
        |q| = 1.219317276096426 \times 10^{18756}
       n= 18,q= \{-1.337168505277458 \times 10^{37512}, 6.498924666187968 \times 10^{37511}, 0., 0.\}
        ,\q\= 1.486734619787208 \times 10^{37512}
       n= 19,q= \{1.365659393338087 \times 10^{75024}, -1.738031476359474 \times 10^{75024}, 0., 0.\}
        \sqrt{q} = 2.210379829673815 \times 10^{75024}
       n= 20,q= \left\{-1.155727834203739\times10^{150048}, -4.747118023215157\times10^{150048}, 0., 0.\right\}
        ,\q\= 4.885778991428844 \times 10^{150048}
ln[ \circ ] := P = \{-1.5, 1.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.5, 1.4, 0, 0}
```

```
n= 1,q= \{-1.5, 1.4, 0, 0\}, q= 2.05183
       n= 2,q= \{-1.21, -2.8, 0., 0.\}, q= 3.05026
       n= 3,q= \{-7.8759, 8.176, 0., 0.\}, q= 11.3524
       n= 4, q= \{-6.31718, -127.387, 0., 0.\}, q= 127.543
       n= 5,q= \{-16189., 1610.85, 0., 0.\}, \q = 16268.9
       n=6,q=\{2.59488\times10^8, -5.21559\times10^7, 0., 0.\}, q=2.64678\times10^8
       n= 7,q= \{6.46137 \times 10^{16}, -2.70677 \times 10^{16}, 0., 0.\}, \q\= 7.00542 \times 10^{16}
       n= 8,q= \{3.44227 \times 10^{33}, -3.49789 \times 10^{33}, 0., 0.\}, \q\= 4.90759 \times 10<sup>33</sup>
       n= 9,q= \{-3.85959 \times 10^{65}, -2.40814 \times 10^{67}, 0., 0.\},\q\= 2.40845 \times 10^{67}
       n= 10,q= \{-5.79763 \times 10^{134}, 1.85888 \times 10^{133}, 0., 0.\}, \q\= 5.80061 \times 10^{134}
       n= 11,q= \{3.3578 \times 10^{269}, -2.15542 \times 10^{268}, 0., 0.\},\q\= 3.36471 \times 10^{269}
       n= 12,q= \{1.122833072883896 \times 10^{539}, -1.447494313562347 \times 10^{538}, 0., 0.\}
        \sqrt{q} = 1.132124775561354 \times 10^{539}
       n= 13,q= \{1.239801711683940 \times 10^{1078}, -3.250588976158354 \times 10^{1077}, 0., 0.\}
        ,\q\ =\ 1.281706507439847\times 10^{1078}
       n= 14,q= \{1.431444997375206 \times 10^{2156}, -8.060171553244148 \times 10^{2155}, 0., 0.\}
        n= 15,q= \{1.399371125833242 \times 10^{4312}, -2.307538449575456 \times 10^{4312}, 0., 0.\}
        \sqrt{q} = 2.698698435187765 \times 10^{4312}
       n= 16,q= \{-3.366494148453304 \times 10^{8624}, -6.458205356171798 \times 10^{8624}, 0., 0.\}
        \sqrt{q} = 7.282973244084894 \times 10^{8624}
       n= 17,q= \{-3.037513357091577 \times 10^{17249}, 4.348302108212429 \times 10^{17249}, 0., 0.\}
        \sqrt{q} = 5.304169927405644 \times 10^{17249}
       n= 18,q= \{-9.68124382977492 \times 10^{34498}, -2.641605146872943 \times 10^{34499}, 0., 0.\}
        ,\q\ =\ 2.813421861879440\times 10^{34499}
       n= 19,q= \{-6.040812931070075 \times 10^{68998}, 5.114804705773068 \times 10^{68998}, 0., 0.\}
        \q = 7.915342572901173 \times 10^{68998}
       n= 20,q= \{1.033019368998511 \times 10^{137997}, -6.179515681306405 \times 10^{137997}, 0., 0.\}
        ,\q\= 6.265264804638176 \times 10^{137997}
ln[ \circ ] := P = \{-1.5, 1.2, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.5, 1.2, 0, 0}
```

```
n= 1, q= \{-1.5, 1.2, 0, 0\}, q= 1.92094
       n= 2, q= \{-0.69, -2.4, 0., 0.\}, \q = 2.49722
       n= 3,q= \{-6.7839, 4.512, 0., 0.\}, q= 8.14736
       n=4,q=\{24.1632, -60.0179, 0., 0.\}, q=64.6994
       n=5,q=\{-3019.79, -2899.24, 0., 0.\}, q=4186.26
       n=6,q=\{713524., 1.75102\times10^7, 0., 0.\}, q=1.75248\times10^7
       n= 7,q= \{-3.06099 \times 10^{14}, 2.49879 \times 10^{13}, 0., 0.\}, \q\= 3.07117 \times 10^{14}
       n=8,q=\{9.30722\times10^{28}, -1.52976\times10^{28}, 0., 0.\}, q=9.4321\times10^{28}\}
       n=9,q=\{8.42842\times10^{57}, -2.84756\times10^{57}, 0., 0.\}, q=8.89646\times10^{57}\}
       n= 10,q= \{6.29298 \times 10^{115}, -4.80008 \times 10^{115}, 0., 0.\}, \q\= 7.91469 \times 10^{115}
       n= 11,q= \{1.65607 \times 10^{231}, -6.04136 \times 10^{231}, 0., 0.\}, \q\= 6.26423 \times 10^{231}
       n= 12,q= \{-3.375546788832662 \times 10^{463}, -2.000987394327528 \times 10^{463}, 0., 0.\}
        \sqrt{q} = 3.924062521909681 \times 10^{463}
       n= 13,q= \{7.390365571340825 \times 10^{926}, 1.350885314683385 \times 10^{927}, 0., 0.\}
        \q = 1.539826667585617 \times 10^{927}
       n= 14,q= \{-1.278716100646629 \times 10^{1854}, 1.996707264093201 \times 10^{1854}, 0., 0.\}
        \sqrt{q} = 2.371066166207825 \times 10^{1854}
       n= 15,q= \{-2.351725032429633 \times 10^{3708}, -5.106443453748114 \times 10^{3708}, 0., 0.\}
        ,\q\ =\ 5.621954764535475\times 10^{3708}
       n= 16,q= \{-2.054515411817080 \times 10^{7417}, 2.401790179373174 \times 10^{7417}, 0., 0.\}
        \q = 3.160637537448312 \times 10^{7417}
       n= 17,q= \{-1.547562488339518 \times 10^{14834}, -9.86902987894619 \times 10^{14834}, 0., 0.\}
        \sqrt{q} = 9.98962964312733 \times 10^{14834}
       n= 18,q= \{-9.50028010962171 \times 10^{29669}, 3.054588087391804 \times 10^{29669}, 0., 0.\}
        ,\q\= 9.97927004068483 \times 10^{29669}
       n= 19,q= \{8.092481377763799 \times 10^{59339}, -5.803888489947155 \times 10^{59339}, 0., 0.\}
        \sqrt{q} = 9.95858305449098 \times 10^{59339}
       n= 20,q= \{3.180313324571281 \times 10^{118679}, -9.39357190470300 \times 10^{118679}, 0., 0.\}
        ,\q\= 9.91733764531949 \times 10^{118679}
ln[ \circ ] := P = \{-1.5, 0.7, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.5, 0.7, 0, 0}
```

Out[\circ]= {-1.5, 0.5, 0, 0}

0, 0, 0}]

```
n= 1,q= \{-1.5, 0.5, 0, 0\}, q= 1.58114
       n= 2,q= \{0.5, -1., 0., 0.\}, q= 1.11803
       n= 3, q= \{-2.25, -0.5, 0., 0.\}, q = 2.30489
       n= 4, q= \{3.3125, 2.75, 0., 0.\}, q= 4.30525
       n= 5, q= \{1.91016, 18.7188, 0., 0.\}, q= 18.816
       n=6,q=\{-348.243,72.0115,0.,0.\},\q\=355.61
       n= 7, q= \{116086., -50154.5, 0., 0.\}, q= 126457.
       n= 8,q= \{1.09605 \times 10^{10}, -1.16445 \times 10^{10}, 0., 0.\}, \q\= 1.59914×10<sup>10</sup>
       n = 9,q = \{-1.54613 \times 10^{19}, -2.55258 \times 10^{20}, 0., 0.\}, q = 2.55726 \times 10^{20}\}
       n= 10,q= \{-6.49175 \times 10^{40}, 7.89324 \times 10^{39}, 0., 0.\}, \q\= 6.53956 \times 10^{40}
       n= 11,q= \{4.15198 \times 10^{81}, -1.02482 \times 10^{81}, 0., 0.\}, \q\= 4.27658 \times 10^{81}
       n= 12,q= \{1.61887 \times 10^{163}, -8.51004 \times 10^{162}, 0., 0.\},\q\= 1.82892 \times 10^{163}
       n= 13,q= \{1.896516045749107 \times 10^{326}, -2.755323121004310 \times 10^{326}, 0., 0.\}
        \q = 3.344933274809045 \times 10^{326}
      n = 14, q = \{-3.995032389357101 \times 10^{652}, -1.045102902041637 \times 10^{653}, 0., 0.\}
        |q| = 1.118857861292476 \times 10^{653}
       n= 15,q= \{-9.32637237935728 \times 10^{1305}, 8.350439887734880 \times 10^{1305}, 0., 0.\}
       \q = 1.251842913775974 \times 10^{1306}
       n= 16,q=\{1.725137543976461\times10^{2611}, -1.557586238489077\times10^{2612}, 0., 0.\}
        ,\q\= 1.567110680771121 \times 10^{2612}
       n= 17,q= \{-2.396313894874182 \times 10^{5224}, -5.374100995997164 \times 10^{5223}, 0., 0.\}
        ,\q\ =\ 2.455835885786925 \times 10^{5224}
       n= 18,q= \{5.453510667615295 \times 10^{10448}, 2.575606577833037 \times 10^{10448}, 0., 0.\}
        \sqrt{q} = 6.031129897918849 \times 10^{10448}
       n= 19,q= \{2.310702935801701 \times 10^{20897}, 2.809219589558518 \times 10^{20897}, 0., 0.\}
        n= 20,q= \{-2.552366644836728 \times 10^{41794}, 1.298254390580904 \times 10^{41795}, 0., 0.\}
        \sqrt{q} = 1.323106275988193 \times 10^{41795}
ln[ \circ ] := P = \{-1.5, 0.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-1.5, 0.4, 0, 0}
```

```
n= 1, q= \{-1.5, 0.4, 0, 0\}, q= 1.55242
       n= 2, q= \{0.59, -0.8, 0., 0.\}, q= 0.994032
       n= 3,q= \{-1.7919, -0.544, 0., 0.\}, \q = 1.87266
       n= 4,q= \{1.41497, 2.34959, 0., 0.\}, q= 2.74275
       n=5,q=\{-5.01842, 7.04919, 0., 0.\}, q=8.65307
       n=6,q=\{-26.0065, -70.3516, 0., 0.\}, q=75.0046
       n= 7, q= \{-4274.51, 3659.6, 0., 0.\}, q= 5627.09
       n=8,q=\{4.87875\times10^6, -3.1286\times10^7, 0., 0.\}, q=3.16641\times10^7
       n = 9,q = \{-9.5501 \times 10^{14}, -3.05273 \times 10^{14}, 0., 0.\}, q = 1.00261 \times 10^{15}\}
       n= 10,q= \{8.18853 \times 10^{29}, 5.83077 \times 10^{29}, 0., 0.\}, \q\= 1.00524\times10<sup>30</sup>
       n= 11,q= \{3.30541 \times 10^{59}, 9.54909 \times 10^{59}, 0., 0.\},\q\= 1.0105\times 10^{60}
       n= 12,q= \{-8.02595 \times 10^{119}, 6.31274 \times 10^{119}, 0., 0.\}, \q\= 1.02111 \times 10^{120}
       n= 13,q= \{2.45652 \times 10^{239}, -1.01331 \times 10^{240}, 0., 0.\}, \q\= 1.04266 \times 10^{240}
       n= 14,q= \{-9.66459426368118 \times 10^{479}, -4.978441367050818 \times 10^{479}, 0., 0.\}
        \sqrt{q} = 1.087148843198308 \times 10^{480}
       n= 15,q= \{6.861950383641632 \times 10^{959}, 9.62292317561448 \times 10^{959}, 0., 0.\}
       ,\q\= 1.181892607267420×10<sup>960</sup>
      n= 16,q= \{-4.551428737621881 \times 10^{1919}, 1.320640427533235 \times 10^{1920}, 0., 0.\}
        n = 17, q = \{-1.536936103298664 \times 10^{3840}, -1.202160158788003 \times 10^{3840}, 0., 0.\}
        \sqrt{q} = 1.951246174371670 \times 10^{3840}
       n= 18,q= \{9.16983538245684 \times 10^{7679}, 3.695286699977072 \times 10^{7680}, 0., 0.\}
       \q = 3.807361633000077 \times 10^{7680}
       n= 19,q= \{-1.281428498561386 \times 10^{15361}, 6.777034145954383 \times 10^{15360}, 0., 0.\}
        \sqrt{q} = 1.449600260444101 \times 10^{15361}
       n= 20,q= \{1.182777078770973 \times 10^{30722}, -1.736856938069914 \times 10^{30722}, 0., 0.\}
        \sqrt{q} = 2.101340915079606 \times 10^{30722}
ln[-]:= P = \{-1.5, 0.2, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ] = \{-1.5, 0.2, 0, 0\}
```

```
n= 1,q= \{-1.5, 0.2, 0, 0\}, q= 1.51327
       n= 2, q= \{0.71, -0.4, 0., 0.\}, q= 0.814923
       n= 3,q= \{-1.1559, -0.368, 0., 0.\}, \q = 1.21307
       n=4,q=\{-0.299319, 1.05074, 0., 0.\}, \q = 1.09254
       n=5,q=\{-2.51447, -0.429015, 0., 0.\}, q=2.5508
       n=6,q=\{4.63849, 2.35749, 0., 0.\}, q=5.20321
       n= 7, q= \{14.4579, 22.0704, 0., 0.\}, q= 26.3843
       n= 8, q= \{-279.571, 638.382, 0., 0.\}, q= 696.916
       n=9,q=\{-329373., -356946., 0., 0.\},\q=485692.
       n= 10,q= \{-1.89245 \times 10^{10}, 2.35137 \times 10^{11}, 0., 0.\}, \q\= 2.35897 \times 10<sup>11</sup>
       n= 11,q= \{-5.49311 \times 10^{22}, -8.89967 \times 10^{21}, 0., 0.\}, \q\= 5.56474 \times 10^{22}
       n= 12,q= \{2.93822 \times 10^{45}, 9.77738 \times 10^{44}, 0., 0.\}, \q\= 3.09663 \times 10^{45}
       n= 13,q= \{7.67719 \times 10^{90}, 5.74562 \times 10^{90}, 0., 0.\}, \q\= 9.58913 \times 10^{90}
       n= 14,q= \{2.5927 \times 10^{181}, 8.82205 \times 10^{181}, 0., 0.\}, \q\= 9.19514 \times 10^{181}
       n= 15,q= \{-7.110641858204215 \times 10^{363}, 4.574587149368102 \times 10^{363}, 0., 0.\}
        \q = 8.455062106383943 \times 10^{363}
       n= 16,q= \{2.963438004848211 \times 10^{727}, -6.505650173659985 \times 10^{727}, 0., 0.\}
        \sqrt{q} = 7.148807522280967 \times 10^{727}
       n= 17,q= \{-3.354151937346345 \times 10^{1455}, -3.855818194174273 \times 10^{1455}, 0., 0.\}
        \sqrt{q} = 5.110544899062094 \times 10^{1455}
       n= 18,q=\{-3.616998727721109\times10^{2910}, 2.586600013208984\times10^{2911}, 0., 0.\}
        \sqrt{q} = 2.611766916532959 \times 10^{2911}
       n= 19,q= \{-6.559672830369358 \times 10^{5822}, -1.871145791380060 \times 10^{5822}, 0., 0.\}
        \sqrt{q} = 6.821326426296080 \times 10^{5822}
      n= 20,q= \{3.952812106888663 \times 10^{11645}, 2.454820841875150 \times 10^{11645}, 0., 0.\}
        \sqrt{q} = 4.653049421408525 \times 10^{11645}
ln[\circ]:= P = \{-1.5, -0.4, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}1
Out[\bullet]= {-1.5, -0.4, 0, 0}
```

```
n= 1, q= \{-1.5, -0.4, 0, 0\}, q= 1.55242
       n= 2,q= \{0.59, 0.8, 0., 0.\}, q= 0.994032
       n= 3,q= \{-1.7919, 0.544, 0., 0.\}, q= 1.87266
       n= 4, q= \{1.41497, -2.34959, 0., 0.\}, q= 2.74275
       n=5,q=\{-5.01842, -7.04919, 0., 0.\}, q=8.65307
       n=6,q=\{-26.0065, 70.3516, 0., 0.\}, \q = 75.0046
       n= 7, q= \{-4274.51, -3659.6, 0., 0.\}, q= 5627.09
       n= 8,q= \{4.87875 \times 10^6, 3.1286 \times 10^7, 0., 0.\}, q= 3.16641 \times 10^7
       n= 9,q= \{-9.5501 \times 10^{14}, 3.05273 \times 10^{14}, 0., 0.\}, q= 1.00261 \times 10^{15}
       n= 10,q= \{8.18853 \times 10^{29}, -5.83077 \times 10^{29}, 0., 0.\}, \q\= 1.00524 \times 10<sup>30</sup>
       n= 11,q= \{3.30541 \times 10^{59}, -9.54909 \times 10^{59}, 0., 0.\},\q\= 1.0105\times 10^{60}
       n= 12,q= \{-8.02595 \times 10^{119}, -6.31274 \times 10^{119}, 0., 0.\}, \q\= 1.02111 \times 10^{120}
       n= 13,q= \{2.45652 \times 10^{239}, 1.01331 \times 10^{240}, 0., 0.\},\q\= 1.04266 \times 10^{240}
       n= 14,q= \{-9.66459426368118 \times 10^{479}, 4.978441367050818 \times 10^{479}, 0., 0.\}
        \sqrt{q} = 1.087148843198308 \times 10^{480}
       n= 15,q= \{6.861950383641632 \times 10^{959}, -9.62292317561448 \times 10^{959}, 0., 0.\}
       ,\q\= 1.181892607267420×10<sup>960</sup>
      n= 16,q= \{-4.551428737621881 \times 10^{1919}, -1.320640427533235 \times 10^{1920}, 0., 0.\}
        n = 17, q = \{-1.536936103298664 \times 10^{3840}, 1.202160158788003 \times 10^{3840}, 0., 0.\}
        \sqrt{q} = 1.951246174371670 \times 10^{3840}
       n= 18,q= \{9.16983538245684 \times 10^{7679}, -3.695286699977072 \times 10^{7680}, 0., 0.\}
       ,\q\= 3.807361633000077×10<sup>7680</sup>
      n= 19,q= \{-1.281428498561386 \times 10^{15361}, -6.777034145954383 \times 10^{15360}, 0., 0.\}
        \sqrt{q} = 1.449600260444101 \times 10^{15361}
       n= 20,q= \{1.182777078770973 \times 10^{30722}, 1.736856938069914 \times 10^{30722}, 0., 0.\}
        \sqrt{q} = 2.101340915079606 \times 10^{30722}
ln[-]:= P = \{-1.5, -0.7, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
          0,
          0,
          0}]
Out[\circ]= {-1.5, -0.7, 0, 0}
```

```
n= 1,q= \{-1.5, -0.7, 0, 0\}, q= 1.65529
      n= 2,q= \{0.26, 1.4, 0., 0.\}, q= 1.42394
      n= 3,q= \{-3.3924, 0.028, 0., 0.\}, \q = 3.39252
      n=4,q=\{10.0076, -0.889974, 0., 0.\}, q=10.0471
      n=5,q=\{97.8599, -18.513, 0., 0.\}, q=99.5956
      n=6,q=\{9232.32, -3624.06, 0., 0.\}, q=9918.15
      n= 7,q= \{7.2102 \times 10^7, -6.6917 \times 10^7, 0., 0.\},\q\= 9.83696 \times 10^7
      n= 8,q= \{7.20812 \times 10^{14}, -9.6497 \times 10^{15}, 0., 0.\}, q= 9.67658 \times 10^{15}
      n = 9, q = \{-9.25971 \times 10^{31}, -1.39112 \times 10^{31}, 0., 0.\}, q = 9.36363 \times 10^{31}\}
      n= 10,q= \{8.38071 \times 10^{63}, 2.57628 \times 10^{63}, 0., 0.\}, \q\= 8.76775 \times 10^{63}
      n= 11,q= \{6.3599 \times 10^{127}, 4.31821 \times 10^{127}, 0., 0.\}, \q\= 7.68735 \times 10^{127}
      n= 12,q= \{2.18014 \times 10^{255}, 5.49268 \times 10^{255}, 0., 0.\}, \q\= 5.90954 \times 10^{255}
      n= 13,q= \{-2.541655728910854 \times 10^{511}, 2.394967644262645 \times 10^{511}, 0., 0.\}
       n = 14, q = \{7.241438272404024 \times 10^{1021}, -1.217436646719256 \times 10^{1023}, 0., 0.\}
        n= 15,q= \{-1.476908145949724 \times 10^{2046}, -1.763198465556008 \times 10^{2045}, 0., 0.\}
       \q = 1.487395831600332 \times 10^{2046}
      n= 16,q= \{2.150168983283261 \times 10^{4092}, 5.208164353411445 \times 10^{4091}, 0., 0.\}
        \sqrt{q} = 2.212346359862042 \times 10^{4092}
      n= 17,q= \{4.351976897351914 \times 10^{8184}, 2.239686690509361 \times 10^{8184}, 0., 0.\}
        ,\q\= 4.894476415994827 \times 10^{8184}
      n= 18,q= \{1.392350644344002 \times 10^{16369}, 1.949412946880661 \times 10^{16369}, 0., 0.\}
        ,\q\ =\ 2.395589938672957 \times 10^{16369}
      n= 19,q= \{-1.861570520660788 \times 10^{32738}, 5.428532745363655 \times 10^{32738}, 0., 0.\}
       \q = 5.738851154271100 \times 10^{32738}
      n= 20,q= \{-2.600352296409219 \times 10^{65477}, -2.021119305842151 \times 10^{65477}, 0., 0.\}
        q = 3.293441257087874 \times 10^{65477}
ln[ \circ ] := P = \{-1.5, -1.2, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-1.5, -1.2, 0, 0}
```

```
n= 1,q= \{-1.5, -1.2, 0, 0\}, q= 1.92094
       n= 2,q= \{-0.69, 2.4, 0., 0.\}, q= 2.49722
       n= 3,q= \{-6.7839, -4.512, 0., 0.\}, \q = 8.14736
       n= 4,q= \{24.1632, 60.0179, 0., 0.\}, \q = 64.6994
       n=5,q=\{-3019.79,2899.24,0.,0.\}, q=4186.26
       n = 6, q = \{713524., -1.75102 \times 10^7, 0., 0.\}, q = 1.75248 \times 10^7\}
       n= 7,q= \{-3.06099 \times 10^{14}, -2.49879 \times 10^{13}, 0., 0.\},\q\= 3.07117×10<sup>14</sup>
       n= 8,q= \{9.30722 \times 10^{28}, 1.52976 \times 10^{28}, 0., 0.\}, q= 9.4321 \times 10^{28}
       n= 9,q= \{8.42842 \times 10^{57}, 2.84756 \times 10^{57}, 0., 0.\}, \q\= 8.89646 \times 10^{57}
       n= 10,q= \{6.29298 \times 10^{115}, 4.80008 \times 10^{115}, 0., 0.\},\q\= 7.91469 \times 10^{115}
       n= 11,q= \{1.65607 \times 10^{231}, 6.04136 \times 10^{231}, 0., 0.\},\q\= 6.26423 \times 10^{231}
       n= 12,q= \{-3.375546788832662 \times 10^{463}, 2.000987394327528 \times 10^{463}, 0., 0.\}
        \sqrt{q} = 3.924062521909681 \times 10^{463}
       n= 13,q= \{7.390365571340825 \times 10^{926}, -1.350885314683385 \times 10^{927}, 0., 0.\}
        \q = 1.539826667585617 \times 10^{927}
       n= 14,q= \{-1.278716100646629 \times 10^{1854}, -1.996707264093201 \times 10^{1854}, 0., 0.\}
        \sqrt{q} = 2.371066166207825 \times 10^{1854}
       n= 15,q= \{-2.351725032429633 \times 10^{3708}, 5.106443453748114 \times 10^{3708}, 0., 0.\}
        \sqrt{q} = 5.621954764535475 \times 10^{3708}
       n= 16,q= \{-2.054515411817080 \times 10^{7417}, -2.401790179373174 \times 10^{7417}, 0., 0.\}
        \q = 3.160637537448312 \times 10^{7417}
       n= 17,q= \{-1.547562488339518 \times 10^{14834}, 9.86902987894619 \times 10^{14834}, 0., 0.\}
        \sqrt{q} = 9.98962964312733 \times 10^{14834}
       n= 18,q= \{-9.50028010962171 \times 10^{29669}, -3.054588087391804 \times 10^{29669}, 0., 0.\}
        ,\q\= 9.97927004068483 \times 10^{29669}
       n= 19,q= \{8.092481377763799 \times 10^{59339}, 5.803888489947155 \times 10^{59339}, 0., 0.\}
        \protect\ \q\= 9.95858305449098\times10<sup>59339</sup>
       n= 20,q= \{3.180313324571281 \times 10^{118679}, 9.39357190470300 \times 10^{118679}, 0., 0.\}
        ,\q\= 9.91733764531949 \times 10^{118679}
ln[ \circ ] := P = \{-1.5, -1.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.5, -1.4, 0, 0}
```

 $n= 1, q= \{-1.5, -1.4, 0, 0\}, q= 2.05183$

```
n= 2,q= \{-1.21, 2.8, 0., 0.\}, q= 3.05026
       n= 3,q= \{-7.8759, -8.176, 0., 0.\}, \q = 11.3524
       n= 4, q= \{-6.31718, 127.387, 0., 0.\}, q= 127.543
       n=5,q=\{-16189.,-1610.85,0.,0.\},\q=16268.9
       n=6,q=\{2.59488\times10^8,5.21559\times10^7,0.,0.\},\q=2.64678\times10^8
       n= 7,q= \{6.46137 \times 10^{16}, 2.70677 \times 10^{16}, 0., 0.\}, \q\= 7.00542 \times 10^{16}
       n= 8,q= \{3.44227 \times 10^{33}, 3.49789 \times 10^{33}, 0., 0.\}, \q\= 4.90759 \times 10<sup>33</sup>
       n= 9,q= \{-3.85959 \times 10^{65}, 2.40814 \times 10^{67}, 0., 0.\},\q\= 2.40845 \times 10^{67}
       n= 10,q= \{-5.79763 \times 10^{134}, -1.85888 \times 10^{133}, 0., 0.\}, \q\= 5.80061 \times 10^{134}
       n= 11,q= \{3.3578 \times 10^{269}, 2.15542 \times 10^{268}, 0., 0.\},\q\= 3.36471 \times 10^{269}
       n= 12,q= \left\{1.122833072883896\times10^{539}\text{, }1.447494313562347\times10^{538}\text{, 0., 0.}\right\}
        \sqrt{q} = 1.132124775561354 \times 10^{539}
       n= 13,q= \{1.239801711683940 \times 10^{1078}, 3.250588976158354 \times 10^{1077}, 0., 0.\}
        ,\q\ =\ 1.281706507439847\times 10^{1078}
       n= 14,q= \{1.431444997375206 \times 10^{2156}, 8.060171553244148 \times 10^{2155}, 0., 0.\}
        n= 15,q= \{1.399371125833242 \times 10^{4312}, 2.307538449575456 \times 10^{4312}, 0., 0.\}
        \sqrt{q} = 2.698698435187765 \times 10^{4312}
       n= 16,q= \{-3.366494148453304 \times 10^{8624}, 6.458205356171798 \times 10^{8624}, 0., 0.\}
        \sqrt{q} = 7.282973244084894 \times 10^{8624}
       n= 17,q= \{-3.037513357091577 \times 10^{17249}, -4.348302108212429 \times 10^{17249}, 0., 0.\}
        \sqrt{q} = 5.304169927405644 \times 10^{17249}
       n = 18, q = \{-9.68124382977492 \times 10^{34498}, 2.641605146872943 \times 10^{34499}, 0., 0.\}
        ,\q\ =\ 2.813421861879440\times 10^{34499}
       n= 19,q= \{-6.040812931070075 \times 10^{68998}, -5.114804705773068 \times 10^{68998}, 0., 0.\}
        \q = 7.915342572901173 \times 10^{68998}
       n= 20,q= \{1.033019368998511 \times 10^{137997}, 6.179515681306405 \times 10^{137997}, 0., 0.\}
        ,\q\= 6.265264804638176 \times 10^{137997}
ln[ \circ ] := P = \{-1.4, 1.4, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.4, 1.4, 0, 0}
```

Out[\circ]= {-1.4, 1.2, 0, 0}

0, 0, 0}]

```
n= 1,q= \{-1.4, 1.2, 0, 0\}, q= 1.84391
       n= 2, q= \{-0.88, -2.16, 0., 0.\}, q= 2.33238
       n= 3,q= \{-5.2912, 5.0016, 0., 0.\}, q= 7.28099
       n= 4,q= \{1.58079, -51.7289, 0., 0.\}, \q = 51.7531
       n=5,q=\{-2674.78, -162.346, 0., 0.\}, q=2679.71
       n=6,q=\{7.12811\times10^6,868480.,0.,0.\},\q=7.18082\times10^6
       n= 7,q= \{5.00557 \times 10^{13}, 1.23812 \times 10^{13}, 0., 0.\}, \q\= 5.15642 \times 10^{13}
       n= 8,q= \{2.35228 \times 10^{27}, 1.2395 \times 10^{27}, 0., 0.\}, q= 2.65887 \times 10^{27}
       n= 9,q= \{3.99683 \times 10^{54}, 5.83131 \times 10^{54}, 0., 0.\}, \q\= 7.06957 \times 10<sup>54</sup>
       n= 10,q= \{-1.80295 \times 10^{109}, 4.66135 \times 10^{109}, 0., 0.\}, \q\= 4.99788 \times 10^{109}
       n= 11,q= \{-1.84776 \times 10^{219}, -1.68083 \times 10^{219}, 0., 0.\}, \q\= 2.49788 \times 10^{219}
       n= 12,q= \{5.890204334801011 \times 10^{437}, 6.211558628922021 \times 10^{438}, 0., 0.\}
        \sqrt{q} = 6.239423504747270 \times 10^{438}
       n= 13,q= \{-3.823651552947854 \times 10^{877}, 7.317469912389422 \times 10^{876}, 0., 0.\}
        \sqrt{q} = 3.893040567159271 \times 10^{877}
       n= 14,q= \{1.408485753917329 \times 10^{1755}, -5.595891038831402 \times 10^{1754}, 0., 0.\}
        \q = 1.515576485754778 \times 10^{1755}
       n= 15,q= \{1.670692153803330 \times 10^{3510}, -1.576346561733534 \times 10^{3510}, 0., 0.\}
        \sqrt{q} = 2.296972084172802 \times 10^{3510}
       n= 16,q= \{3.063437900908750 \times 10^{7019}, -5.267179664726144 \times 10^{7020}, 0., 0.\}
        \sqrt{q} = 5.276080755469145 \times 10^{7020}
       n= 17,q= \{-2.764933510277738 \times 10^{14041}, -3.227135563163583 \times 10^{14040}, 0., 0.\}
        \sqrt{q} = 2.783702813823186 \times 10^{14041}
       n= 18,q= \{7.540713276826421 \times 10^{28082}, 1.784563052160002 \times 10^{28082}, 0., 0.\}
        ,\q\= 7.749001355687124 \times 10^{28082}
       n= 19,q= \{5.367769143617164 \times 10^{56165}, 2.691375660151362 \times 10^{56165}, 0., 0.\}
        \sqrt{q} = 6.004702201044089 \times 10^{56165}
       n= 20,q= \{2.156944263511337 \times 10^{112331}, 2.889336644488551 \times 10^{112331}, 0., 0.\}
        ,\q\= 3.605644852322372 \times 10^{112331}
ln[ \circ ] := P = \{-1.4, 0.7, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.4, 0.7, 0, 0}
```

```
n= 1,q= \{-1.4, 0.7, 0, 0\}, q= 1.56525
       n= 2,q= \{0.07, -1.26, 0., 0.\}, q= 1.26194
       n= 3,q= \{-2.9827, 0.5236, 0., 0.\}, \q = 3.02831
       n=4,q=\{7.22234, -2.42348, 0., 0.\}, q=7.6181
       n=5,q=\{44.889, -34.3065, 0., 0.\}, q=56.4974
       n=6,q=\{836.686, -3079.26, 0., 0.\}, q=3190.91
       n= 7,q= \{-8.78181 \times 10^6, -5.15275 \times 10^6, 0., 0.\},\q\= 1.01819\times 10^7
       n= 8,q= \{5.05694 \times 10^{13}, 9.05009 \times 10^{13}, 0., 0.\}, \q\= 1.03671×10<sup>14</sup>
       n= 9,q= \{-5.63315 \times 10^{27}, 9.15316 \times 10^{27}, 0., 0.\}, \q\= 1.07477 \times 10^{28}
       n= 10,q= \{-5.20479 \times 10^{55}, -1.03122 \times 10^{56}, 0., 0.\},\q\= 1.15513\times 10^{56}
       n= 11,q= \{-7.92521 \times 10^{111}, 1.07346 \times 10^{112}, 0., 0.\},\q\= 1.33432\times 10^{112}
       n= 12,q= \{-5.24225 \times 10^{223}, -1.70148 \times 10^{224}, 0., 0.\}, \q\= 1.78041 \times 10^{224}
       n= 13,q= \{-2.620219935058118 \times 10^{448}, 1.783914851960451 \times 10^{448}, 0., 0.\}
        \q = 3.169843009854123 \times 10^{448}
       n= 14,q= \{3.683200309030891 \times 10^{896}, -9.34849851510605 \times 10^{896}, 0., 0.\}
        \sqrt{q} = 1.004790470712105 \times 10^{897}
       n= 15,q= \{-7.382845997049475 \times 10^{1793}, -6.886478523962686 \times 10^{1793}, 0., 0.\}
       \sqrt{q} = 1.009603890033853 \times 10^{1794}
       n= 16,q= \{7.082828555150167 \times 10^{3586}, 1.016836208088102 \times 10^{3588}, 0., 0.\}
        ,\q\ =\ 1.019300014771488 \times 10^{3588}
       n= 17,q= \{-1.028939228044825 \times 10^{7176}, 1.440415306111405 \times 10^{7175}, 0., 0.\}
        ,\q\= 1.038972520113155 \times 10^{7176}
       n= 18,q= \{1.037967972468679 \times 10^{14352}, -2.964199626268438 \times 10^{14351}, 0., 0.\}
        \sqrt{q} = 1.079463897550280 \times 10^{14352}
       n= 19,q= \{9.89512717627042 \times 10^{28703}, -6.153488552140536 \times 10^{28703}, 0., 0.\}
        ,\q\ =\ 1.165242306114441\times 10^{28704}
       n= 20,q= \left\{6.004812047324077\times10^{57407}, -1.217791036023094\times10^{57408}, 0., 0.\right\}
        \sqrt{q} = 1.357789631958900 \times 10^{57408}
ln[ \circ ] := P = \{-1.4, 0.5, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-1.4, 0.5, 0, 0}
```

```
n= 1,q= \{-1.4, 0.5, 0, 0\}, q= 1.48661
       n= 2,q= \{0.31, -0.9, 0., 0.\}, q= 0.951893
       n= 3,q= \{-2.1139, -0.058, 0., 0.\}, q= 2.1147
       n= 4, q= \{3.06521, 0.745212, 0., 0.\}, q= 3.1545
       n=5,q=\{7.44017,5.06846,0.,0.\},\q=9.00252
       n=6,q=\{28.2667, 75.9204, 0., 0.\},\q=81.0118
       n= 7, q= \{-4966.3, 4292.55, 0., 0.\}, q= 6564.31
       n= 8,q= \{6.2382 \times 10^6, -4.26362 \times 10^7, 0., 0.\}, q= 4.30901 \times 10^7
       n= 9,q= \{-1.77893 \times 10^{15}, -5.31946 \times 10^{14}, 0., 0.\},\q\= 1.85676×10<sup>15</sup>
       n= 10,q= \{2.88162 \times 10^{30}, 1.89259 \times 10^{30}, 0., 0.\}, \q\= 3.44755 \times 10^{30}
       n= 11,q= \{4.72183 \times 10^{60}, 1.09074×10<sup>61</sup>, 0., 0.\},\q\= 1.18856×10<sup>61</sup>
       n= 12,q= \{-9.66759 \times 10^{121}, 1.03006 \times 10^{122}, 0., 0.\}, \q\= 1.41267 \times 10^{122}
       n= 13,q= \{-1.26398 \times 10^{243}, -1.99164 \times 10^{244}, 0., 0.\},\q\= 1.99564 \times 10^{244}
       n= 14,q= \{-3.950643493356471 \times 10^{488}, 5.034781154769981 \times 10^{487}, 0., 0.\}
        |q| = 3.982596417459857 \times 10^{488}
       n= 15,q= \{1.535409379883555 \times 10^{977}, -3.978125081913161 \times 10^{976}, 0., 0.\}
       ,\q\= 1.586107422436409×10<sup>977</sup>
      n= 16,q= \{2.199227172160938 \times 10^{1954}, -1.221610113023901 \times 10^{1954}, 0., 0.\}
        \q = 2.515736755507870 \times 10^{1954}
      n = 17, q = \{3.344268886528727 \times 10^{3908}, -5.373196308697514 \times 10^{3908}, 0., 0.\}
        \sqrt{q} = 6.328931423013263 \times 10^{3908}
       n= 18,q= \{-1.768710418639650 \times 10^{7817}, -3.593882647277620 \times 10^{7817}, 0., 0.\}
        \q = 4.005537295720468 \times 10^{7817}
      n= 19,q= \{-9.78765593739875 \times 10^{15634}, 1.271307536321635 \times 10^{15635}, 0., 0.\}
        \sqrt{q} = 1.604432902740764 \times 10^{15635}
       n= 20,q= \{-6.582407644192146 \times 10^{31269}, -2.488624151227645 \times 10^{31270}, 0., 0.\}
        \sqrt{q} = 2.574204939397155 \times 10^{31270}
ln[-]:= P = \{-1.4, 0.4, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module[\{q = t, n = 1\}, While[n \le 20, q = quaternion[q] + P;
            Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]]; 
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ] = \{-1.4, 0.4, 0, 0\}
```

```
n= 1, q= \{-1.4, 0.4, 0, 0\}, q= 1.45602
       n= 2,q= \{0.4, -0.72, 0., 0.\}, q= 0.82365
       n= 3,q= \{-1.7584, -0.176, 0., 0.\}, \q = 1.76719
       n= 4,q= \{1.66099, 1.01896, 0., 0.\}, q= 1.94863
       n=5,q=\{0.32063,3.78496,0.,0.\},\q=3.79852
       n=6,q=\{-15.6231, 2.82715, 0., 0.\}, q=15.8769
       n= 7, q= \{234.69, -87.9378, 0., 0.\}, q= 250.624
       n= 8, q= \{47344.9, -41275.8, 0., 0.\}, q= 62811.1
       n = 9, q = \{5.37845 \times 10^8, -3.9084 \times 10^9, 0., 0.\}, q = 3.94523 \times 10^9
       n= 10,q= \{-1.49863 \times 10^{19}, -4.20422 \times 10^{18}, 0., 0.\}, \q\= 1.55649 \times 10^{19}
       n= 11,q= \{2.06914 \times 10^{38}, 1.26011 \times 10^{38}, 0., 0.\}, \q\= 2.42265 \times 10<sup>38</sup>
       n= 12,q= \{2.69344 \times 10^{76}, 5.2147 \times 10^{76}, 0., 0.\}, \q\= 5.86922 \times 10^{76}
       n= 13,q= \{-1.99385 \times 10^{153}, 2.8091 \times 10^{153}, 0., 0.\},\q\= 3.44478×10<sup>153</sup>
       n= 14,q= \{-3.91561 \times 10^{306}, -1.12018 \times 10^{307}, 0., 0.\},\q\= 1.18665 \times 10^{307}
       n= 15,q= \{-1.101494188285400 \times 10^{614}, 8.772414221892157 \times 10^{613}, 0., 0.\}
        \q = 1.408134212222354 \times 10^{614}
       n= 16,q= \{4.437369340219544 \times 10^{1227}, -1.932552656529280 \times 10^{1228}, 0., 0.\}
        ,\q\= 1.982841959631070 \times 10^{1228}
       n= 17,q= \{-3.537857303643172 \times 10^{2456}, -1.715089981288571 \times 10^{2456}, 0., 0.\}
        ,\q\ =\ 3.931662236873581\times 10^{2456}
       n= 18,q= \{9.57490065702491 \times 10^{4912}, 1.213548723341401 \times 10^{4913}, 0., 0.\}
        , q = 1.545796794485777 \times 10^{4913}
       n= 19,q= \{-5.559132780045843 \times 10^{9825}, 2.323921693690663 \times 10^{9826}, 0., 0.\}
        |q| = 2.389487729842504 \times 10^{9826}
       n= 20,q= \{-5.091572465744278 \times 10^{19652}, -2.583797853131084 \times 10^{19652}, 0., 0.\}
        \sqrt{q} = 5.709651611067882 \times 10^{19652}
ln[\circ]:= P = \{-1.4, 0.2, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}1
Out[\bullet]= {-1.4, 0.2, 0, 0}
```

```
n= 1,q= \{-1.4, 0.2, 0, 0\}, q= 1.41421
      n= 2, q= \{0.52, -0.36, 0., 0.\}, q= 0.632456
      n= 3,q= \{-1.2592, -0.1744, 0., 0.\}, q= 1.27122
      n= 4,q= \{0.155169, 0.639209, 0., 0.\}, q= 0.657773
      n=5,q=\{-1.78451, 0.398371, 0., 0.\}, q=1.82844
      n=6,q=\{1.62578, -1.2218, 0., 0.\}, q=2.0337
      n=7,q=\{-0.249628, -3.77274, 0., 0.\}, q=3.78099
      n= 8, q= \{-15.5712, 2.08356, 0., 0.\}, q= 15.71
      n= 9, q= \{236.722, -64.6872, 0., 0.\}, q= 245.401
      n=10,q=\{51851.4, -30625.6, 0., 0.\},\q=60220.4
      n= 11,q= \{1.75064 \times 10^9, -3.17596 \times 10^9, 0., 0.\},\q\= 3.62649 \times 10^9
      n= 12,q= \{-7.02195 \times 10^{18}, -1.11199 \times 10^{19}, 0., 0.\},\q\= 1.31515\times 10^{19}
      n= 13,q= \{-7.43454 \times 10^{37}, 1.56167 \times 10^{38}, 0., 0.\}, \q\= 1.72961 \times 10<sup>38</sup>
      n= 14,q= \{-1.8861 \times 10^{76}, -2.32207 \times 10^{76}, 0., 0.\},\q\= 2.99155 \times 10^{76}
      n= 15,q= \{-1.83462 \times 10^{152}, 8.75931 \times 10^{152}, 0., 0.\},\q\= 8.94937 \times 10^{152}
      n= 16,q= \{-7.33597 \times 10^{305}, -3.214 \times 10^{305}, 0., 0.\},\q\= 8.00913×10<sup>305</sup>
      n= 17,q= \{4.348662672976242 \times 10^{611}, 4.715552369916906 \times 10^{611}, 0., 0.\}
       , q = 6.414616215859358 \times 10^{611}
      n= 18,q= \{-3.325567110092071 \times 10^{1222}, 4.101269314704461 \times 10^{1223}, 0., 0.\}
       \sqrt{q} = 4.114730119676582 \times 10^{1223}
      n= 19,q= \{-1.670981602569914 \times 10^{2447}, -2.727809268522201 \times 10^{2446}, 0., 0.\}
        ,\q\= 1.693100395777366 \times 10^{2447}
      n= 20,q= \{2.717770082072762 \times 10^{4894}, 9.11623820604058 \times 10^{4893}, 0., 0.\}
       \sqrt{q} = 2.866588950181474 \times 10^{4894}
ln[-]:= P = \{-1.4, -0.2, 0, 0\}
      quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out 0 = \{-1.4, -0.2, 0, 0\}
```

```
n= 1, q= \{-1.4, -0.2, 0, 0\}, q= 1.41421
      n= 2,q= \{0.52, 0.36, 0., 0.\}, q= 0.632456
      n= 3,q= \{-1.2592, 0.1744, 0., 0.\}, \q = 1.27122
      n= 4, q= \{0.155169, -0.639209, 0., 0.\}, q= 0.657773
      n=5,q=\{-1.78451, -0.398371, 0., 0.\}, q=1.82844
      n=6,q=\{1.62578, 1.2218, 0., 0.\}, q=2.0337
      n=7,q=\{-0.249628, 3.77274, 0., 0.\}, q=3.78099
      n= 8, q= \{-15.5712, -2.08356, 0., 0.\}, q= 15.71
      n= 9, q= \{236.722, 64.6872, 0., 0.\}, q= 245.401
      n= 10, q= \{51851.4, 30625.6, 0., 0.\}, q= 60220.4
      n= 11,q= \{1.75064 \times 10^9, 3.17596 \times 10^9, 0., 0.\}, \q\= 3.62649 \times 10^9
      n= 12,q= \{-7.02195 \times 10^{18}, 1.11199 \times 10^{19}, 0., 0.\}, \q\= 1.31515 \times 10^{19}
      n= 13,q= \{-7.43454 \times 10^{37}, -1.56167 \times 10^{38}, 0., 0.\},\q\= 1.72961\times10<sup>38</sup>
      n= 14,q= \{-1.8861 \times 10^{76}, 2.32207 \times 10^{76}, 0., 0.\}, \q\= 2.99155 \times 10^{76}
      n= 15,q= \{-1.83462 \times 10^{152}, -8.75931 \times 10^{152}, 0., 0.\}, \q\= 8.94937 \times 10^{152}
      n= 16,q= \{-7.33597 \times 10^{305}, 3.214 \times 10^{305}, 0., 0.\}, q= 8.00913 \times 10^{305}
      n= 17,q= \{4.348662672976242 \times 10^{611}, -4.715552369916906 \times 10^{611}, 0., 0.\}
       ,\q\= 6.414616215859358 \times 10^{611}
      n= 18,q= \{-3.325567110092071 \times 10^{1222}, -4.101269314704461 \times 10^{1223}, 0., 0.\}
       \sqrt{q} = 4.114730119676582 \times 10^{1223}
      n= 19,q= \{-1.670981602569914 \times 10^{2447}, 2.727809268522201 \times 10^{2446}, 0., 0.\}
        ,\q\= 1.693100395777366 \times 10^{2447}
      n= 20,q= {2.717770082072762\times10<sup>4894</sup>, -9.11623820604058\times10<sup>4893</sup>, 0., 0.}
       q = 2.866588950181474 \times 10^{4894}
ln[-]:= P = \{-1.4, -0.4, 0, 0\}
      quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out 0 = \{-1.4, -0.4, 0, 0\}
```

```
n= 1, q= \{-1.4, -0.4, 0, 0\}, q= 1.45602
       n= 2,q= \{0.4, 0.72, 0., 0.\}, q= 0.82365
       n= 3,q= \{-1.7584, 0.176, 0., 0.\}, q= 1.76719
       n= 4, q= \{1.66099, -1.01896, 0., 0.\}, q= 1.94863
       n=5,q=\{0.32063, -3.78496, 0., 0.\}, q=3.79852
       n=6,q=\{-15.6231, -2.82715, 0., 0.\}, q=15.8769
       n= 7, q= \{234.69, 87.9378, 0., 0.\}, q= 250.624
       n= 8, q= \{47344.9, 41275.8, 0., 0.\}, q= 62811.1
       n = 9, q = \{5.37845 \times 10^8, 3.9084 \times 10^9, 0., 0.\}, q = 3.94523 \times 10^9
       n= 10,q= \{-1.49863 \times 10^{19}, 4.20422 \times 10^{18}, 0., 0.\}, \q\= 1.55649 \times 10<sup>19</sup>
       n= 11,q= \{2.06914 \times 10^{38}, -1.26011 \times 10^{38}, 0., 0.\}, \q\= 2.42265 \times 10^{38}
       n= 12,q= \{2.69344 \times 10^{76}, -5.2147 \times 10^{76}, 0., 0.\}, \q\= 5.86922 \times 10^{76}
       n= 13,q= \{-1.99385 \times 10^{153}, -2.8091 \times 10^{153}, 0., 0.\}, \q\= 3.44478 \times 10<sup>153</sup>
       n= 14,q= \{-3.91561 \times 10^{306}, 1.12018 \times 10^{307}, 0., 0.\}, \q\= 1.18665 \times 10^{307}
       n= 15,q= \left\{-1.101494188285400\times10^{614}\text{, }-8.772414221892157\times10^{613}\text{, 0., 0.}\right\}
        \q = 1.408134212222354 \times 10^{614}
       n= 16,q= \{4.437369340219544 \times 10^{1227}, 1.932552656529280 \times 10^{1228}, 0., 0.\}
        ,\q\= 1.982841959631070 \times 10^{1228}
       n= 17,q= \left\{-3.537857303643172\times10^{2456}, 1.715089981288571\times10^{2456}, 0., 0.\right\}
        ,\q\ =\ 3.931662236873581\times 10^{2456}
       n= 18,q= \{9.57490065702491 \times 10^{4912}, -1.213548723341401 \times 10^{4913}, 0., 0.\}
        ,\q\ =\ 1.545796794485777\times 10^{4913}
       n= 19,q= \{-5.559132780045843 \times 10^{9825}, -2.323921693690663 \times 10^{9826}, 0., 0.\}
        |q| = 2.389487729842504 \times 10^{9826}
       n= 20,q= \{-5.091572465744278 \times 10^{19652}, 2.583797853131084 \times 10^{19652}, 0., 0.\}
        \sqrt{q} = 5.709651611067882 \times 10^{19652}
Inf = I = \{-1.4, -0.5, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}1
Out[\bullet]= {-1.4, -0.5, 0, 0}
```

```
n= 1, q= \{-1.4, -0.5, 0, 0\}, q= 1.48661
       n= 2,q= \{0.31, 0.9, 0., 0.\}, q= 0.951893
       n= 3,q= \{-2.1139, 0.058, 0., 0.\}, q= 2.1147
       n= 4,q= \{3.06521, -0.745212, 0., 0.\}, \q = 3.1545
       n=5,q=\{7.44017, -5.06846, 0., 0.\}, q=9.00252
       n=6,q=\{28.2667, -75.9204, 0., 0.\}, q=81.0118
       n= 7, q= \{-4966.3, -4292.55, 0., 0.\}, q= 6564.31
       n= 8,q= \{6.2382 \times 10^6, 4.26362 \times 10^7, 0., 0.\}, q= 4.30901 \times 10^7
       n= 9,q= \{-1.77893 \times 10^{15}, 5.31946 \times 10^{14}, 0., 0.\}, \q\= 1.85676 \times 10^{15}
       n= 10,q= \{2.88162 \times 10^{30}, -1.89259 \times 10^{30}, 0., 0.\}, \q\= 3.44755 \times 10^{30}
       n= 11,q= \{4.72183 \times 10^{60}, -1.09074 \times 10^{61}, 0., 0.\},\q\= 1.18856\times 10^{61}
       n= 12,q= \{-9.66759 \times 10^{121}, -1.03006 \times 10^{122}, 0., 0.\}, \q\= 1.41267 \times 10^{122}
       n= 13,q= \{-1.26398 \times 10^{243}, 1.99164 \times 10^{244}, 0., 0.\}, \q\= 1.99564 \times 10^{244}
       n= 14,q= \{-3.950643493356471 \times 10^{488}, -5.034781154769981 \times 10^{487}, 0., 0.\}
        \sqrt{q} = 3.982596417459857 \times 10^{488}
       n= 15,q= \{1.535409379883555 \times 10^{977}, 3.978125081913161 \times 10^{976}, 0., 0.\}
        ,\q\= 1.586107422436409×10<sup>977</sup>
       n= 16,q= \{2.199227172160938 \times 10^{1954}, 1.221610113023901 \times 10^{1954}, 0., 0.\}
        \q = 2.515736755507870 \times 10^{1954}
       n = 17, q = \{3.344268886528727 \times 10^{3908}, 5.373196308697514 \times 10^{3908}, 0., 0.\}
        \sqrt{q} = 6.328931423013263 \times 10^{3908}
       n= 18,q=\{-1.768710418639650\times10^{7817}, 3.593882647277620\times10^{7817}, 0., 0.\}
        \q = 4.005537295720468 \times 10^{7817}
       n= 19,q= \{-9.78765593739875 \times 10^{15634}, -1.271307536321635 \times 10^{15635}, 0., 0.\}
        \sqrt{q} = 1.604432902740764 \times 10^{15635}
       n= 20,q= \{-6.582407644192146 \times 10^{31269}, 2.488624151227645 \times 10^{31270}, 0., 0.\}
        \sqrt{q} = 2.574204939397155 \times 10^{31270}
ln[-]:= P = \{-1.4, -0.7, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module[\{q = t, n = 1\}, While[n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.4, -0.7, 0, 0}
```

```
n= 1,q= \{-1.4, -0.7, 0, 0\}, q= 1.56525
       n= 2,q= \{0.07, 1.26, 0., 0.\}, q= 1.26194
       n= 3,q= \{-2.9827, -0.5236, 0., 0.\}, \q = 3.02831
       n=4,q=\{7.22234, 2.42348, 0., 0.\}, q=7.6181
       n=5,q=\{44.889,34.3065,0.,0.\}, q=56.4974
       n=6,q=\{836.686,3079.26,0.,0.\},\q=3190.91
       n= 7,q= \{-8.78181 \times 10^6, 5.15275 \times 10^6, 0., 0.\}, q= 1.01819 \times 10^7
       n= 8,q= \{5.05694 \times 10^{13}, -9.05009 \times 10^{13}, 0., 0.\}, q= 1.03671 \times 10^{14}\}
       n= 9,q= \{-5.63315 \times 10^{27}, -9.15316 \times 10^{27}, 0., 0.\},\q\= 1.07477×10<sup>28</sup>
       n= 10,q= \{-5.20479 \times 10^{55}, 1.03122 \times 10^{56}, 0., 0.\}, \q\= 1.15513 \times 10<sup>56</sup>
       n= 11,q= \{-7.92521 \times 10^{111}, -1.07346 \times 10^{112}, 0., 0.\},\q\= 1.33432×10<sup>112</sup>
       n= 12,q= \{-5.24225 \times 10^{223}, 1.70148 \times 10^{224}, 0., 0.\},\q\= 1.78041 \times 10^{224}
       n= 13,q= \{-2.620219935058118 \times 10^{448}, -1.783914851960451 \times 10^{448}, 0., 0.\}
        \q = 3.169843009854123 \times 10^{448}
       n = 14, q = \{3.683200309030891 \times 10^{896}, 9.34849851510605 \times 10^{896}, 0., 0.\}
        \sqrt{q} = 1.004790470712105 \times 10^{897}
       n= 15,q= \{-7.382845997049475 \times 10^{1793}, 6.886478523962686 \times 10^{1793}, 0., 0.\}
        \sqrt{q} = 1.009603890033853 \times 10^{1794}
       n= 16,q= \{7.082828555150167 \times 10^{3586}, -1.016836208088102 \times 10^{3588}, 0., 0.\}
        ,\q\ =\ 1.019300014771488 \times 10^{3588}
       n= 17,q= \{-1.028939228044825 \times 10^{7176}, -1.440415306111405 \times 10^{7175}, 0., 0.\}
        ,\q\= 1.038972520113155 \times 10^{7176}
       n= 18,q= \{1.037967972468679 \times 10^{14352}, 2.964199626268438 \times 10^{14351}, 0., 0.\}
        \sqrt{q} = 1.079463897550280 \times 10^{14352}
       n= 19,q= \{9.89512717627042 \times 10^{28703}, 6.153488552140536 \times 10^{28703}, 0., 0.\}
        \q = 1.165242306114441 \times 10^{28704}
       n= 20,q= \{6.004812047324077 \times 10^{57407}, 1.217791036023094 \times 10^{57408}, 0., 0.\}
        \sqrt{q} = 1.357789631958900 \times 10^{57408}
ln[ \circ ] := P = \{-1.4, -1.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.4, -1.2, 0, 0}
```

```
n= 1, q= \{-1.4, -1.2, 0, 0\}, q= 1.84391
       n= 2,q= \{-0.88, 2.16, 0., 0.\}, \q = 2.33238
       n= 3,q= \{-5.2912, -5.0016, 0., 0.\}, \q = 7.28099
       n= 4,q= \{1.58079, 51.7289, 0., 0.\}, q= 51.7531
       n=5,q=\{-2674.78, 162.346, 0., 0.\}, q=2679.71
       n = 6, q = \{7.12811 \times 10^6, -868480., 0., 0.\}, q = 7.18082 \times 10^6\}
       n= 7,q= \{5.00557 \times 10^{13}, -1.23812 \times 10^{13}, 0., 0.\}, \q\= 5.15642 \times 10^{13}
       n= 8,q= \{2.35228 \times 10^{27}, -1.2395 \times 10^{27}, 0., 0.\}, q= 2.65887 \times 10^{27}
       n= 9,q= {3.99683 \times 10^{54}}, -5.83131 \times 10^{54}, 0., 0.},\q\= 7.06957 \times 10^{54}
       n= 10,q= \{-1.80295 \times 10^{109}, -4.66135 \times 10^{109}, 0., 0.\}, \q\= 4.99788 \times 10^{109}
       n= 11,q= \{-1.84776 \times 10^{219}, 1.68083 \times 10^{219}, 0., 0.\}, \q\= 2.49788 \times 10^{219}
       n= 12,q= \{5.890204334801011 \times 10^{437}, -6.211558628922021 \times 10^{438}, 0., 0.\}
        \sqrt{q} = 6.239423504747270 \times 10^{438}
       n= 13,q= \{-3.823651552947854 \times 10^{877}, -7.317469912389422 \times 10^{876}, 0., 0.\}
        \q = 3.893040567159271 \times 10^{877}
       n= 14,q= \{1.408485753917329 \times 10^{1755}, 5.595891038831402 \times 10^{1754}, 0., 0.\}
        \q = 1.515576485754778 \times 10^{1755}
       n= 15,q= \{1.670692153803330 \times 10^{3510}, 1.576346561733534 \times 10^{3510}, 0., 0.\}
        \sqrt{q} = 2.296972084172802 \times 10^{3510}
       n= 16,q= \{3.063437900908750 \times 10^{7019}, 5.267179664726144 \times 10^{7020}, 0., 0.\}
        \q = 5.276080755469145 \times 10^{7020}
       n= 17,q= \{-2.764933510277738 \times 10^{14041}, 3.227135563163583 \times 10^{14040}, 0., 0.\}
        \sqrt{q} = 2.783702813823186 \times 10^{14041}
       n= 18,q= \{7.540713276826421 \times 10^{28082}, -1.784563052160002 \times 10^{28082}, 0., 0.\}
        ,\q\= 7.749001355687124 \times 10^{28082}
       n= 19,q= \{5.367769143617164 \times 10^{56165}, -2.691375660151362 \times 10^{56165}, 0., 0.\}
        \sqrt{q} = 6.004702201044089 \times 10^{56165}
       n= 20,q= \{2.156944263511337 \times 10^{112331}, -2.889336644488551 \times 10^{112331}, 0., 0.\}
        ,\q\= 3.605644852322372 \times 10^{112331}
ln[ \circ ] := P = \{-1.4, -1.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.4, -1.4, 0, 0}
```

```
n= 1,q= \{-1.4, -1.4, 0, 0\}, q= 1.9799
       n= 2,q= \{-1.4, 2.52, 0., 0.\}, q= 2.88278
       n= 3,q= \{-5.7904, -8.456, 0., 0.\}, \q = 10.2485
       n=4,q=\{-39.3752, 96.5272, 0., 0.\}, q=104.249
       n=5,q=\{-7768.5, -7602.96, 0., 0.\}, q=10.869.9
       n=6,q=\{2.54463\times10^6, 1.18127\times10^8, 0., 0.\}, q=1.18155\times10^8
       n= 7,q= \{-1.39476 \times 10^{16}, 6.0118 \times 10^{14}, 0., 0.\}, \q\= 1.39605 \times 10^{16}
       n= 8,q= \{1.94173 \times 10^{32}, -1.677 \times 10^{31}, 0., 0.\}, \q\= 1.94896 \times 10^{32}
       n= 9,q= \{3.7422 \times 10^{64}, -6.51256 \times 10^{63}, 0., 0.\}, q= 3.79844 \times 10^{64}\}
       n= 10,q= \{1.35799 \times 10^{129}, -4.87426 \times 10^{128}, 0., 0.\}, \q\= 1.44282 \times 10^{129}
       n= 11,q= \{1.60656 \times 10^{258}, -1.32384 \times 10^{258}, 0., 0.\}, \q\= 2.08172 \times 10^{258}
       n= 12,q= \{8.284704992926566 \times 10^{515}, -4.253649742555875 \times 10^{516}, 0., 0.\}
        \sqrt{q} = 4.333578140583586 \times 10^{516}
       n= 13,q= \{-1.740717276414744 \times 10^{1033}, -7.048046652062692 \times 10^{1032}, 0., 0.\}
        ,\q\= 1.877989950054389 \times 10^{1033}
       n= 14,q= \{2.533347020312244 \times 10^{2066}, 2.453731314444525 \times 10^{2066}, 0., 0.\}
        \sqrt{q} = 3.526846252505286 \times 10^{2066}
       n= 15,q= \{3.970497618392657 \times 10^{4131}, 1.243230582818976 \times 10^{4133}, 0., 0.\}
        ,\q\= 1.243864448881058 \times 10^{4133}
       n= 16,q= \left\{-1.544045796922646\times10^{8266}\text{, }9.87248813639132\times10^{8264}\text{, }0.\text{, }0.\right\}
        \q = 1.547198767190178 \times 10^{8266}
       n= 17,q= \{2.374330820794169 \times 10^{16532}, -3.048714762432741 \times 10^{16531}, 0., 0.\}
        \sqrt{q} = 2.393824025194807 \times 10^{16532}
       n= 18,q= \{5.544500229546361 \times 10^{33064}, -1.447731484850846 \times 10^{33064}, 0., 0.\}
        \sqrt{q} = 5.730393463599868 \times 10^{33064}
       n= 19,q= \{2.864555634321122 \times 10^{66129}, -1.605389510015402 \times 10^{66129}, 0., 0.\}
        \sqrt{q} = 3.283740924766809 \times 10^{66129}
       n= 20,q= \{5.628403503253391 \times 10^{132258}, -9.19745513238929 \times 10^{132258}, 0., 0.\}
        ,\q\= 1.078295446098838 \times 10^{132259}
ln[ \circ ] := P = \{-1.2, 1.4, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.2, 1.4, 0, 0}
```

```
n= 2,q= \{-1.72, -1.96, 0., 0.\}, q= 2.60768
                      n= 3,q= \{-2.0832, 8.1424, 0., 0.\}, q= 8.40467
                      n= 4, q= \{-63.159, -32.5245, 0., 0.\}, q= 71.0415
                      n=5,q=\{2930.01,4109.83,0.,0.\},\q=5047.34
                      n=6,q=\{-8.30571\times10^6, 2.40837\times10^7, 0., 0.\}, q=2.54756\times10^7
                      n= 7,q= \{-5.11038 \times 10^{14}, -4.00064 \times 10^{14}, 0., 0.\},\q\= 6.49008×10<sup>14</sup>
                      n=8,q=\{1.01109\times10^{29},4.08896\times10^{29},0.,0.\},\q=4.21212\times10^{29}\}
                      n=9,q=\{-1.56973\times10^{59}, 8.26863\times10^{58}, 0., 0.\}, q=1.77419\times10^{59}\}
                      n= 10,q= \{1.78035 \times 10^{118}, -2.5959 \times 10^{118}, 0., 0.\},\q\= 3.14776 \times 10^{118}
                       \texttt{n= 11,q= } \left\{ -3.56906 \times 10^{236} \text{, } -9.24324 \times 10^{236} \text{, 0., 0.} \right\} \text{,} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, } \\  \texttt{10,q= } \left\{ -3.56906 \times 10^{236} \text{, } -9.24324 \times 10^{236} \text{, 0., 0.} \right\} \text{,} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, } \\  \texttt{10,q= } \left\{ -3.56906 \times 10^{236} \text{, } -9.24324 \times 10^{236} \text{, 0., 0.} \right\} \text{,} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, } \\  \texttt{10,q= } \left\{ -3.56906 \times 10^{236} \text{, } -9.24324 \times 10^{236} \text{, 0., 0.} \right\} \text{,} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9.90837 \times 10^{236} \text{, 0., 0.} \\  \texttt{q} = 9
                      n= 12,q= \{-7.269930800602665 \times 10^{473}, 6.597940593002617 \times 10^{473}, 0., 0.\}
                         \sqrt{q} = 9.81757169132689 \times 10^{473}
                      n = \text{ 13,q} = \left\{9.31907377675958 \times 10^{946}, -9.59331430752327 \times 10^{947}, \text{ 0., 0.}\right\}, \\ |q| = 9.63847139143430 \times 10^{947}, \\ |q| = 10.63847139143430 \times 10^{947}, \\ |q| = 10.638471391430 \times 10^{947}, \\ |q| = 10.63847139140 \times 10^{947}, \\ |q| = 10.6384713910 \times 10^{947}, \\ |q| = 10.6384713910 \times 1
                      n= 14, q= \{-9.11632280423638 \times 10^{1895}, -1.788016075909052 \times 10^{1895}, 0., 0.\}
                          \q = 9.29001307634975 \times 10^{1895}
                      n= 15,q= \{7.991033998333100 \times 10^{3791}, 3.260026345430187 \times 10^{3791}, 0., 0.\}
                         \q = 8.630434295874941 \times 10^{3791}
                      n= 16,q= \{5.322885258961659 \times 10^{7583}, 5.210196272358846 \times 10^{7583}, 0., 0.\}
                          \sqrt{q} = 7.448439613541439 \times 10^{7583}
                      n= 17,q= \{1.186962283569322 \times 10^{15166}, 5.546655386887177 \times 10^{15167}, 0., 0.\}
                          ,\q\ = 5.547925267657334 \times 10^{15167}
                      n= 18,q= \{-3.075129718625838 \times 10^{30335}, 1.316734148838337 \times 10^{30334}, 0., 0.\}
                           \sqrt{q} = 3.077947477551070 \times 10^{30335}
                      n= 19,q= \{9.43908489818866 \times 10^{60670}, -8.098256625244533 \times 10^{60669}, 0., 0.\}
                          \sqrt{q} = 9.47376067456300 \times 10^{60670}
                      n= 20,q= {8.844050611152998×10^{121341}, -1.528802636260038×10^{121341}, 0., 0.}
                          \sqrt{q} = 8.975214131889632 \times 10^{121341}
 ln[ \circ ] := P = \{-1.2, 1.2, 0, 0\}
                                                                                                                                                                                                                                                                                                                                                                                                                 +
                      quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
                      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
                                     Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
                                     n++]]
                      iteration[
                            {0,
                               0,
                               0,
                               0}]
Out[*]= {-1.2, 1.2, 0, 0}
```

 $n= 1, q= \{-1.2, 1.4, 0, 0\}, q= 1.84391$

 $n= 1, q= \{-1.2, 1.2, 0, 0\}, q= 1.69706$

```
n= 2,q= \{-1.2, -1.68, 0., 0.\}, q= 2.06456
       n= 3,q= \{-2.5824, 5.232, 0., 0.\}, q= 5.8346
       n= 4, q= \{-21.905, -25.8222, 0., 0.\}, q= 33.8618
       n=5,q=\{-188.157, 1132.47, 0., 0.\}, q=1148.
       n=6,q=\{-1.2471\times10^6, -426165., 0., 0.\}, q=1.3179\times10^6
       n= 7,q= \{1.37363 \times 10^{12}, 1.06294 \times 10^{12}, 0., 0.\}, \q\= 1.73686 \times 10^{12}
       n= 8,q= \{7.57023 \times 10^{23}, 2.92016 \times 10^{24}, 0., 0.\}, \q\= 3.01669 \times 10^{24}
       n= 9,q= \{-7.95426 \times 10^{48}, 4.42126 \times 10^{48}, 0., 0.\}, \q\= 9.10043 \times 10^{48}
       n= 10,q= \{4.37227 \times 10^{97}, -7.03357 \times 10^{97}, 0., 0.\}, \q\= 8.28178 \times 10^{97}
       n= 11,q= \{-3.03544 \times 10^{195}, -6.15054 \times 10^{195}, 0., 0.\},\q\= 6.85879 \times 10^{195}
       n= 12,q= \{-2.861525537922634 \times 10^{391}, 3.733915561824339 \times 10^{391}, 0., 0.\}
        \sqrt{q} = 4.704301630105938 \times 10^{391}
       n= 13,q= \{-5.753797018650549 \times 10^{782}, -2.136938947321417 \times 10^{783}, 0., 0.\}
        \sqrt{q} = 2.213045382701738 \times 10^{783}
       n= 14,q= \{-4.235446263260845 \times 10^{1566}, 2.459102588827242 \times 10^{1566}, 0., 0.\}
        n= 15,q= \{1.189181950659341 \times 10^{3133}, -2.083079374164682 \times 10^{3133}, 0., 0.\}
        \sqrt{q} = 2.398619059134709 \times 10^{3133}
       n= 16,q= \left\{-2.925065967296369 \times 10^{6266}, -4.954320787094791 \times 10^{6266}, 0., 0.\right\}
        \q = 5.753373390844279 \times 10^{6266}
       n= 17,q= \{-1.598928354840411 \times 10^{12533}, 2.898343025079986 \times 10^{12533}, 0., 0.\}
        \sqrt{q} = 3.310130537447499 \times 10^{12533}
       n= 18,q= \{-5.843820407117144 \times 10^{25066}, -9.26848568970865 \times 10^{25066}, 0., 0.\}
        ,\q\= 1.095696417494247 \times 10^{25067}
       n= 19,q= \{-5.175459002969517 \times 10^{50133}, 1.083267316331852 \times 10^{50134}, 0., 0.\}
        \sqrt{q} = 1.200550639309727 \times 10^{50134}
       n= 20,q= \{-9.05614319718630 \times 10^{100267}, -1.121281116986462 \times 10^{100268}, 0., 0.\}
        ,\q\= 1.441321837546995 \times 10^{100268}
ln[ \circ ] := P = \{-1.2, 0.7, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.2, 0.7, 0, 0}
```

```
n= 1, q= \{-1.2, 0.7, 0, 0\}, q= 1.38924
       n= 2,q= \{-0.25, -0.98, 0., 0.\}, q= 1.01139
       n= 3,q= \{-2.0979, 1.19, 0., 0.\}, q= 2.4119
       n= 4, q= \{1.78508, -4.293, 0., 0.\}, q= 4.64934
       n= 5, q= \{-16.4433, -14.6267, 0., 0.\}, q= 22.0074
       n=6,q=\{55.2418,481.725,0.,0.\},\q=484.882
       n=7,q=\{-229008., 53223.5, 0., 0.\},\q=235112.
       n= 8,q= \{4.96122 \times 10^{10}, -2.43772 \times 10^{10}, 0., 0.\}, \q\= 5.52776 \times 10^{10}
       n = 9,q = \{1.86712 \times 10^{21}, -2.41882 \times 10^{21}, 0., 0.\}, q = 3.05562 \times 10^{21}\}
       n= 10,q= \{-2.36455 \times 10^{42}, -9.03242 \times 10^{42}, 0., 0.\},\q\= 9.33679\times 10^{42}
       n= 11,q= \{-7.59934 \times 10^{85}, 4.27151 \times 10^{85}, 0., 0.\},\q\= 8.71756 \times 10^{85}
       n= 12,q= \{3.95042 \times 10^{171}, -6.49214 \times 10^{171}, 0., 0.\}, \q\= 7.59959 \times 10^{171}
       n= 13,q= \{-2.654206309011530 \times 10^{343}, -5.129336444277694 \times 10^{343}, 0., 0.\}
        \sqrt{q} = 5.775370420102242 \times 10^{343}
      n= 14,q= \left\{-1.926528122779873\times10^{687}, 2.722863430288925\times10^{687}, 0., 0.\right\}
        \q = 3.335490348939195 \times 10^{687}
       n= 15,q= \{-3.702474652143029 \times 10^{1374}, -1.049134594588097 \times 10^{1375}, 0., 0.\}
       \q = 1.112549586786651 \times 10^{1375}
       n= 16,q= \{-9.63600212063915 \times 10^{2749}, 7.768788486297568 \times 10^{2749}, 0., 0.\}
        ,\q\= 1.237766583059148 \times 10^{2750}
       n= 17,q= \{3.249846232413256 \times 10^{5499}, -1.497201246575208 \times 10^{5500}, 0., 0.\}
        ,\q\= 1.532066114137919 \times 10^{5500}
       n= 18,q= \{-2.135996567403049 \times 10^{11000}, -9.73134766069374 \times 10^{10999}, 0., 0.\}
        \sqrt{q} = 2.347226578089662 \times 10^{11000}
       n= 19,q= \{3.615490063024715 \times 10^{22000}, 4.157225039889503 \times 10^{22000}, 0., 0.\}
        \sqrt{q} = 5.509472608890504 \times 10^{22000}
      n= 20,q= \left\{-4.210751636453822\times10^{44000}, 3.006081164295604\times10^{44001}, 0., 0.\right\}
        ,\q\ =\ 3.035428842811473\times 10^{44001}
ln[ \circ ] := P = \{-1.2, 0.5, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-1.2, 0.5, 0, 0}
```

```
n= 1,q= \{-1.2, 0.5, 0, 0\}, q = 1.3
      n= 2, q= \{-0.01, -0.7, 0., 0.\}, q= 0.700071
      n= 3,q= \{-1.6899, 0.514, 0., 0.\}, q= 1.76634
      n= 4,q= \{1.39157, -1.23722, 0., 0.\}, q= 1.86203
      n=5,q=\{-0.79425, -2.94334, 0., 0.\}, q=3.04862
      n=6,q=\{-9.23241, 5.1755, 0., 0.\}, q=10.5841
      n= 7, q= \{57.2516, -95.0646, 0., 0.\}, \q = 110.973
      n= 8, q= \{-5760.73, -10884.7, 0., 0.\}, q= 12315.1
      n= 9,q= \{-8.52908 \times 10^7, 1.25408 \times 10^8, 0., 0.\}, q= 1.51663 \times 10^8
      n= 10,q= \{-8.45257 \times 10^{15}, -2.13923 \times 10^{16}, 0., 0.\},\q\= 2.30016\times 10^{16}
      n= 11,q= \{-3.86182 \times 10^{32}, 3.61639 \times 10^{32}, 0., 0.\}, \q\= 5.29074 \times 10^{32}
      n= 12,q= \{1.83541 \times 10^{64}, -2.79317 \times 10^{65}, 0., 0.\}, \q\= 2.7992 \times 10<sup>65</sup>
      n= 13,q= \{-7.76813 \times 10^{130}, -1.02532 \times 10^{130}, 0., 0.\}, \q\= 7.8355 \times 10^{130}
      n= 14,q= \{5.92925 \times 10^{261}, 1.59297 \times 10^{261}, 0., 0.\},\q\= 6.13951 \times 10^{261}
      n= 15,q= \{3.261845244992378 \times 10^{523}, 1.889019464754236 \times 10^{523}, 0., 0.\}
        n= 16,q= \{7.071239864059008 \times 10^{1046}, 1.232337831761330 \times 10^{1047}, 0., 0.\}
        \sqrt{q} = 1.420802894049977 \times 10^{1047}
      n= 17,q= \left\{-1.018632199439644\times10^{2094}, 1.742831280387752\times10^{2094}, 0., 0.\right\}
        ,\q\= 2.018680863740789 \times 10^{2094}
      n= 18,q= \{-1.999849314162765 \times 10^{4188}, -3.550608120787173 \times 10^{4188}, 0., 0.\}
       \q = 4.075072429633257 \times 10^{4188}
      n= 19,q= \{-8.607420748042535 \times 10^{8376}, 1.420136243043394 \times 10^{8377}, 0., 0.\}
        n= 20,q= \{-1.275910029467076 \times 10^{16754}, -2.444742032683778 \times 10^{16754}, 0., 0.\}
        \sqrt{q} = 2.757663868143738 \times 10^{16754}
ln[\circ]:= P = \{-1.2, 0.4, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}1
Out[\bullet]= {-1.2, 0.4, 0, 0}
```

| The proof of th

```
n= 1, q= \{-1.3, -0.4, 0, 0\}, q= 1.36015
      n= 2, q= \{0.23, 0.64, 0., 0.\}, q= 0.680074
      n= 3,q= \{-1.6567, -0.1056, 0., 0.\}, q= 1.66006
      n= 4, q= \{1.4335, -0.050105, 0., 0.\}, q= 1.43438
      n=5,q=\{0.752422, -0.543651, 0., 0.\}, q=0.928275
      n=6,q=\{-1.02942, -1.21811, 0., 0.\}, q=1.59483
      n= 7, q= \{-1.72409, 2.10789, 0., 0.\}, q= 2.72318
      n= 8, q= \{-2.77071, -7.66839, 0., 0.\}, q= 8.15359
      n= 9, q= \{-52.4273, 42.0937, 0., 0.\}, q = 67.2347
      n= 10,q= \{975.444, -4414.12, 0., 0.\}, \q = 4520.61
      n= 11,q= \{-1.8533 \times 10^7, -8.61145 \times 10^6, 0., 0.\},\q\= 2.0436×10<sup>7</sup>
      n= 12,q= \{2.69314 \times 10^{14}, 3.19192 \times 10^{14}, 0., 0.\}, \q\= 4.17628 \times 10^{14}
      n= 13,q= \{-2.93533\times10^{28}, 1.71925\times10^{29}, 0., 0.\},\q\= 1.74413×10<sup>29</sup>
      n= 14,q= \{-2.86967 \times 10^{58}, -1.00931 \times 10^{58}, 0., 0.\},\q\= 3.042 \times 10^{58}
      n= 15,q= \{7.21631\times10^{116}, 5.79281\times10^{116}, 0., 0.\},\q\= 9.25375\times10^{116}
      n= 16,q= \{1.85186 \times 10^{233}, 8.36055 \times 10^{233}, 0., 0.\},\q\= 8.56318 \times 10^{233}
      n= 17,q= \{-6.646933788163123 \times 10^{467}, 3.096507333232949 \times 10^{467}, 0., 0.\}
       , q = 7.332808905800695 \times 10^{467}
      n= 18,q= \{3.459337111945913 \times 10^{935}, -4.116455843712195 \times 10^{935}, 0., 0.\}
       \sqrt{q} = 5.377008644898999 \times 10^{935}
      n= 19,q= \{-4.978195459145986 \times 10^{1870}, -2.848041693968044 \times 10^{1871}, 0., 0.\}
        ,\q\= 2.891222196731857 \times 10^{1871}
      n= 20,q=\{-7.863517190285748\times10^{3742}, 2.835621645674031\times10^{3742}, 0., 0.\}
       |q| = 8.359165790874982 \times 10^{3742}
ln[-]:= P = \{-1.2, -0.5, 0, 0\}
      quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
          n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out 0 = \{-1.2, -0.5, 0, 0\}
```

```
n= 1,q= \{-1.2, -0.5, 0, 0\}, q= 1.3
      n= 2,q= \{-0.01, 0.7, 0., 0.\}, q= 0.700071
      n= 3,q= \{-1.6899, -0.514, 0., 0.\}, \q = 1.76634
      n= 4,q= \{1.39157, 1.23722, 0., 0.\}, q= 1.86203
      n=5,q=\{-0.79425, 2.94334, 0., 0.\}, q=3.04862
      n=6,q=\{-9.23241, -5.1755, 0., 0.\}, q=10.5841
      n= 7, q= \{57.2516, 95.0646, 0., 0.\}, q= 110.973
      n= 8, q= \{-5760.73, 10884.7, 0., 0.\}, q= 12315.1
      n = 9,q = \{-8.52908 \times 10^7, -1.25408 \times 10^8, 0., 0.\}, q = 1.51663 \times 10^8
      n= 10,q= \{-8.45257 \times 10^{15}, 2.13923 \times 10^{16}, 0., 0.\}, \q\= 2.30016 \times 10<sup>16</sup>
      n= 11,q= \{-3.86182 \times 10^{32}, -3.61639 \times 10^{32}, 0., 0.\},\q\= 5.29074×10<sup>32</sup>
      n= 12,q= \{1.83541 \times 10^{64}, 2.79317 \times 10^{65}, 0., 0.\}, \q\= 2.7992×10<sup>65</sup>
      n= 13,q= \{-7.76813 \times 10^{130}, 1.02532 \times 10^{130}, 0., 0.\}, \q\= 7.8355 \times 10^{130}
      n= 14,q= \{5.92925 \times 10^{261}, -1.59297 \times 10^{261}, 0., 0.\},\q\= 6.13951×10<sup>261</sup>
      n= 15,q= \{3.261845244992378 \times 10^{523}, -1.889019464754236 \times 10^{523}, 0., 0.\}
        n= 16,q=\{7.071239864059008\times10^{1046}, -1.232337831761330\times10^{1047}, 0., 0.\}
        , q = 1.420802894049977 \times 10^{1047}
      n= 17,q= \{-1.018632199439644 \times 10^{2094}, -1.742831280387752 \times 10^{2094}, 0., 0.\}
        \sqrt{q} = 2.018680863740789 \times 10^{2094}
      n= 18,q= \left\{-1.999849314162765\times10^{4188}, 3.550608120787173\times10^{4188}, 0., 0.\right\}
        \q = 4.075072429633257 \times 10^{4188}
      n= 19,q= \{-8.607420748042535 \times 10^{8376}, -1.420136243043394 \times 10^{8377}, 0., 0.\}
        n= 20,q= \{-1.275910029467076 \times 10^{16754}, 2.444742032683778 \times 10^{16754}, 0., 0.\}
        \sqrt{q} = 2.757663868143738 \times 10^{16754}
ln[\circ]:= P = \{-1.2, -0.7, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}1
Out[\bullet]= {-1.2, -0.7, 0, 0}
```

```
n= 1, q= \{-1.2, -0.7, 0, 0\}, q= 1.38924
       n= 2,q= \{-0.25, 0.98, 0., 0.\}, q= 1.01139
       n= 3,q= \{-2.0979, -1.19, 0., 0.\}, q= 2.4119
       n= 4, q= \{1.78508, 4.293, 0., 0.\}, \q = 4.64934
       n=5,q=\{-16.4433, 14.6267, 0., 0.\}, q=22.0074
       n=6,q=\{55.2418,-481.725,0.,0.\},\q=484.882
       n= 7, q= \{-229008., -53223.5, 0., 0.\}, q= 235112.
       n= 8,q= \{4.96122 \times 10^{10}, 2.43772 \times 10^{10}, 0., 0.\}, \q\= 5.52776 \times 10^{10}
       n= 9,q= \{1.86712 \times 10^{21}, 2.41882 \times 10^{21}, 0., 0.\}, \q\= 3.05562×10<sup>21</sup>
       n= 10,q= \{-2.36455 \times 10^{42}, 9.03242 \times 10^{42}, 0., 0.\}, \q\= 9.33679 \times 10^{42}
       n= 11,q= \{-7.59934 \times 10^{85}, -4.27151 \times 10^{85}, 0., 0.\},\q\= 8.71756 \times 10^{85}
       n= 12,q= \{3.95042 \times 10^{171}, 6.49214 \times 10^{171}, 0., 0.\}, \q\= 7.59959 \times 10^{171}
       n= 13,q= \{-2.654206309011530 \times 10^{343}, 5.129336444277694 \times 10^{343}, 0., 0.\}
        \q = 5.775370420102242 \times 10^{343}
      n= 14,q= \{-1.926528122779873 \times 10^{687}, -2.722863430288925 \times 10^{687}, 0., 0.\}
        \q = 3.335490348939195 \times 10^{687}
       n= 15,q= \{-3.702474652143029 \times 10^{1374}, 1.049134594588097 \times 10^{1375}, 0., 0.\}
       \q = 1.112549586786651 \times 10^{1375}
       n= 16,q= \{-9.63600212063915 \times 10^{2749}, -7.768788486297568 \times 10^{2749}, 0., 0.\}
        ,\q\ =\ 1.237766583059148\times 10^{2750}
      n=17,q=\{3.249846232413256\times10^{5499}, 1.497201246575208\times10^{5500}, 0., 0.\}
        ,\q\ =\ 1.532066114137919\times 10^{5500}
       n= 18,q= \{-2.135996567403049 \times 10^{11000}, 9.73134766069374 \times 10^{10999}, 0., 0.\}
        \sqrt{q} = 2.347226578089662 \times 10^{11000}
       n= 19,q= \{3.615490063024715 \times 10^{22000}, -4.157225039889503 \times 10^{22000}, 0., 0.\}
        \sqrt{q} = 5.509472608890504 \times 10^{22000}
      n= 20,q= \left\{-4.210751636453822\times10^{44000}, -3.006081164295604\times10^{44001}, 0., 0.\right\}
        ,\q\ =\ 3.035428842811473\times 10^{44001}
ln[*]:= P = \{-1.2, -1.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\bullet]= {-1.2, -1.2, 0, 0}
```

```
n= 2,q= \{-1.2, 1.68, 0., 0.\}, q= 2.06456
       n= 3,q= \{-2.5824, -5.232, 0., 0.\}, q= 5.8346
       n= 4,q= \{-21.905, 25.8222, 0., 0.\}, q= 33.8618
       n= 5, q= \{-188.157, -1132.47, 0., 0.\}, q= 1148.
       n= 6,q= \{-1.2471\times10^6, 426165., 0., 0.\},\q\= 1.3179\times10^6
       n= 7,q= \{1.37363 \times 10^{12}, -1.06294 \times 10^{12}, 0., 0.\},\q\= 1.73686 \times 10^{12}
       n= 8,q= \{7.57023 \times 10^{23}, -2.92016 \times 10^{24}, 0., 0.\}, q= 3.01669 \times 10^{24}\}
       n = 9, q = \{-7.95426 \times 10^{48}, -4.42126 \times 10^{48}, 0., 0.\}, q = 9.10043 \times 10^{48}\}
       n= 10,q= \{4.37227 \times 10^{97}, 7.03357 \times 10^{97}, 0., 0.\}, \q\= 8.28178 \times 10^{97}
       n= 11,q= \{-3.03544 \times 10^{195}, 6.15054 \times 10^{195}, 0., 0.\},\q\= 6.85879 \times 10^{195}
       n= 12,q= \{-2.861525537922634 \times 10^{391}, -3.733915561824339 \times 10^{391}, 0., 0.\}
        \sqrt{q} = 4.704301630105938 \times 10^{391}
       n= 13,q= \{-5.753797018650549 \times 10^{782}, 2.136938947321417 \times 10^{783}, 0., 0.\}
        \sqrt{q} = 2.213045382701738 \times 10^{783}
       n= 14,q= \{-4.235446263260845 \times 10^{1566}, -2.459102588827242 \times 10^{1566}, 0., 0.\}
        n= 15,q= \{1.189181950659341 \times 10^{3133}, 2.083079374164682 \times 10^{3133}, 0., 0.\}
        \sqrt{q} = 2.398619059134709 \times 10^{3133}
       n= 16,q= \left\{-2.925065967296369\times10^{6266}\text{, }4.954320787094791\times10^{6266}\text{, }0.\text{, }0.\right\}
        \q = 5.753373390844279 \times 10^{6266}
       n= 17,q= \{-1.598928354840411 \times 10^{12533}, -2.898343025079986 \times 10^{12533}, 0., 0.\}
        \sqrt{q} = 3.310130537447499 \times 10^{12533}
       n= 18,q= \{-5.843820407117144 \times 10^{25066}, 9.26848568970865 \times 10^{25066}, 0., 0.\}
        ,\q\= 1.095696417494247 \times 10^{25067}
       n= 19,q= \{-5.175459002969517 \times 10^{50133}, -1.083267316331852 \times 10^{50134}, 0., 0.\}
        \sqrt{q} = 1.200550639309727 \times 10^{50134}
       n= 20,q= \{-9.05614319718630 \times 10^{100267}, 1.121281116986462 \times 10^{100268}, 0., 0.\}
        ,\q\= 1.441321837546995 \times 10^{100268}
ln[ \circ ] := P = \{-1.2, -1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-1.2, -1.4, 0, 0}
```

 $n= 1, q= \{-1.2, -1.2, 0, 0\}, q= 1.69706$

```
n= 1, q= \{-1.2, -1.4, 0, 0\}, q= 1.84391
          n= 2,q= \{-1.72, 1.96, 0., 0.\}, q= 2.60768
          n= 3,q= \{-2.0832, -8.1424, 0., 0.\}, \q = 8.40467
          n= 4,q= \{-63.159, 32.5245, 0., 0.\}, \q = 71.0415
          n=5,q=\{2930.01, -4109.83, 0., 0.\}, q=5047.34
          n=6,q=\{-8.30571\times10^6, -2.40837\times10^7, 0., 0.\}, q=2.54756\times10^7\}
          n= 7,q= \{-5.11038 \times 10^{14}, 4.00064 \times 10^{14}, 0., 0.\}, \q\= 6.49008 \times 10^{14}
          n= 8,q= \{1.01109 \times 10^{29}, -4.08896 \times 10^{29}, 0., 0.\}, \q\= 4.21212×10<sup>29</sup>
          n= 9,q= \{-1.56973 \times 10^{59}, -8.26863 \times 10^{58}, 0., 0.\},\q\= 1.77419×10<sup>59</sup>
          n= 10,q= \{1.78035 \times 10^{118}, 2.5959 \times 10^{118}, 0., 0.\},\q\= 3.14776\times 10^{118}
          n= 11,q= \{-3.56906 \times 10^{236}, 9.24324 \times 10^{236}, 0., 0.\},\q\= 9.90837 \times 10^{236}
          n= 12,q= \{-7.269930800602665 \times 10^{473}, -6.597940593002617 \times 10^{473}, 0., 0.\}
           \sqrt{q} = 9.81757169132689 \times 10^{473}
         n=\ 13, q=\ \left\{9.31907377675958\times 10^{946},\ 9.59331430752327\times 10^{947},\ 0.,\ 0.\right\}, \\ |\ q>=\ 9.63847139143430\times 10^{947},\ 0.,\ 0.
          n= 14,q= \{-9.11632280423638 \times 10^{1895}, 1.788016075909052 \times 10^{1895}, 0., 0.\}
           \q = 9.29001307634975 \times 10^{1895}
          n= 15,q= \{7.991033998333100 \times 10^{3791}, -3.260026345430187 \times 10^{3791}, 0., 0.\}
           \sqrt{q} = 8.630434295874941 \times 10^{3791}
          n= 16,q= \{5.322885258961659 \times 10^{7583}, -5.210196272358846 \times 10^{7583}, 0., 0.\}
           \sqrt{q} = 7.448439613541439 \times 10^{7583}
          n= 17,q= \{1.186962283569322 \times 10^{15166}, -5.546655386887177 \times 10^{15167}, 0., 0.\}
           \sqrt{q} = 5.547925267657334 \times 10^{15167}
          n= 18,q= \{-3.075129718625838 \times 10^{30335}, -1.316734148838337 \times 10^{30334}, 0., 0.\}
            \sqrt{q} = 3.077947477551070 \times 10^{30335}
          n= 19,q= \{9.43908489818866 \times 10^{60670}, 8.098256625244533 \times 10^{60669}, 0., 0.\}
           \sqrt{q} = 9.47376067456300 \times 10^{60670}
         n= 20,q= {8.844050611152998×10^{121341}, 1.528802636260038×10^{121341}, 0., 0.}
           \sqrt{q} = 8.975214131889632 \times 10^{121341}
ln[ \circ ] := P = \{-0.76, 1.4, 0, 0\}
                                                                                                                                                                                    +
          quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
          iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
                Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
                n++]]
          iteration[
            {0,
              0,
              0,
              0}]
Out[\bullet]= {-0.76, 1.4, 0, 0}
```

```
n= 1,q= \{-0.76, 1.4, 0, 0\}, q= 1.59298
       n= 2, q= \{-2.1424, -0.728, 0., 0.\}, q= 2.26271
       n= 3,q= \{3.29989, 4.51933, 0., 0.\}, q= 5.59586
       n= 4, q= \{-10.2951, 31.2266, 0., 0.\}, q= 32.88
       n=5,q=\{-869.875, -641.562, 0., 0.\}, q=1080.87
       n=6,q={345080., 1.11616\times10^6, 0., 0.},\q=1.16828\times10^6
       n= 7,q= \{-1.12673 \times 10^{12}, 7.70327 \times 10^{11}, 0., 0.\}, \q\= 1.36489 \times 10^{12}
       n= 8,q= \{6.76116 \times 10^{23}, -1.7359 \times 10^{24}, 0., 0.\}, q= 1.86292 \times 10^{24}
       n= 9,q= \{-2.55622 \times 10^{48}, -2.34734 \times 10^{48}, 0., 0.\}, \q\= 3.47048 \times 10^{48}
       n= 10,q= \{1.02423 \times 10^{96}, 1.20006 \times 10^{97}, 0., 0.\}, \q\= 1.20443 \times 10^{97}
       n= 11,q= \{-1.42966 \times 10^{194}, 2.45828 \times 10^{193}, 0., 0.\}, \q\= 1.45064 \times 10^{194}
       n= 12,q= \{1.983493845087599 \times 10^{388}, -7.028993412131522 \times 10^{387}, 0., 0.\}
        \sqrt{q} = 2.104356271494509 \times 10^{388}
       n= 13,q= \{3.440180349622503 \times 10^{776}, -2.788393034024830 \times 10^{776}, 0., 0.\}
        \q = 4.428315317378270 \times 10^{776}
       n= 14,q= \{4.059705125730606 \times 10^{1552}, -1.918514984535299 \times 10^{1553}, 0., 0.\}
        \sqrt{q} = 1.960997655012701 \times 10^{1553}
       n= 15,q= \{-3.515887688807643 \times 10^{3106}, -1.557721023301785 \times 10^{3106}, 0., 0.\}
        \sqrt{q} = 3.845511802965310 \times 10^{3106}
       n= 16,q= \{9.93497145387279 \times 10^{6212}, 1.095354433684718 \times 10^{6213}, 0., 0.\}
        \q = 1.478796102674551 \times 10^{6213}
       n= 17,q= \{-2.127647575000976 \times 10^{12425}, 2.176463006106134 \times 10^{12426}, 0., 0.\}
        \sqrt{q} = 2.186837913285442 \times 10^{12426}
       n= 18,q= \{-4.691722374914475 \times 10^{24852}, -9.26149247404210 \times 10^{24851}, 0., 0.\}
        ,\q\= 4.782260058982626 \times 10^{24852}
       n= 19,q= \{2.115450641480574\times10^{49705}, 8.690470293113068\times10^{49704}, 0., 0.\}
        \sqrt{q} = 2.287001127174051 \times 10^{49705}
       n= 20,q= \{3.719888677385763 \times 10^{99410}, 3.676852191266782 \times 10^{99410}, 0., 0.\}
        ,\q\= 5.230374155695378 \times 10^{99410}
ln[ \circ ] := P = \{-0.76, 1.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-0.76, 1.2, 0, 0}
```

```
n= 1,q= \{-0.76, 1.2, 0, 0\}, q= 1.42042
       n= 2, q= \{-1.6224, -0.624, 0., 0.\}, q= 1.73826
       n= 3,q= \{1.48281, 3.22476, 0., 0.\}, q= 3.54933
       n=4,q=\{-8.96033, 10.7634, 0., 0.\},\q=14.0049
       n=5,q=\{-36.3226, -191.687, 0., 0.\}, q=195.098
       n=6,q=\{-35425.3, 13926.3, 0., 0.\},\q=38064.3
       n = 7,q = \{1.06101 \times 10^9, -9.86687 \times 10^8, 0., 0.\}, q = 1.44889 \times 10^9\}
       n= 8,q= \{1.52183 \times 10^{17}, -2.09376 \times 10^{18}, 0., 0.\}, \q\= 2.09928×10<sup>18</sup>
       n= 9,q= \{-4.36068 \times 10^{36}, -6.3727 \times 10^{35}, 0., 0.\}, \q\= 4.407 \times 10^{36}
       n= 10,q= \{1.86094 \times 10^{73}, 5.55786 \times 10^{72}, 0., 0.\}, \q\= 1.94216 \times 10^{73}
       n= 11,q= \{3.1542 \times 10^{146}, 2.06857 \times 10^{146}, 0., 0.\}, \q\= 3.77199 \times 10<sup>146</sup>
       n= 12,q= \{5.66999 \times 10^{292}, 1.30493 \times 10^{293}, 0., 0.\}, \q\= 1.42279 \times 10^{293}
       n= 13,q= \{-1.381367456221049 \times 10^{586}, 1.479793520389373 \times 10^{586}, 0., 0.\}
        \q = 2.024343081617587 \times 10^{586}
       n = 14, q = \{-2.816128138797619 \times 10^{1171}, -4.088277221985320 \times 10^{1172}, 0., 0.\}
        \sqrt{q} = 4.097964912092987 \times 10^{1172}
       n= 15,q= \{-1.663470486686272 \times 10^{2345}, 2.302622504807644 \times 10^{2344}, 0., 0.\}
        \sqrt{q} = 1.679331642074528 \times 10^{2345}
       n= 16,q=\{2.714113356079798\times10^{4690}, -7.660689157454269\times10^{4689}, 0., 0.\}
        \sqrt{q} = 2.820154764072730 \times 10^{4690}
       n= 17,q= \{6.779549725979369 \times 10^{9380}, -4.158395751804465 \times 10^{9380}, 0., 0.\}
        ,\q\=7.953272893322117\times10^{9380}
       n= 18,q= \{2.867003925840152 \times 10^{18761}, -5.638410155931947 \times 10^{18761}, 0., 0.\}
        , q = 6.325454971565236 \times 10^{18761}
       n= 19,q= \{-2.357195757573368 \times 10^{37523}, -3.233068810510775 \times 10^{37523}, 0., 0.\}
        \sqrt{q} = 4.001138059729936 \times 10^{37523}
       n= 20,q= \{-4.896362093975675 \times 10^{75046}, 1.524195216815755 \times 10^{75047}, 0., 0.\}
        \sqrt{q} = 1.600910577301944 \times 10^{75047}
ln[ \circ ] := P = \{-0.76, 0.7, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-0.76, 0.7, 0, 0}
```

```
n= 1,q= \{-0.76, 0.7, 0, 0\}, q= 1.03325
       n= 2,q= \{-0.6724, -0.364, 0., 0.\}, \q = 0.764603
       n= 3,q= \{-0.440374, 1.18951, 0., 0.\}, \q = 1.26841
       n=4,q=\{-1.981, -0.347657, 0., 0.\}, q=2.01127
       n=5,q=\{3.04349, 2.07741, 0., 0.\}, q=3.6849
       n=6,q=\{4.18717, 13.3452, 0., 0.\}, q=13.9866
       n= 7, q= \{-161.321, 112.457, 0., 0.\}, q= 196.65
       n= 8, q= \{13377.2, -36282.7, 0., 0.\}, q= 38670.2
       n = 9, q = \{-1.13748 \times 10^9, -9.70721 \times 10^8, 0., 0.\}, q = 1.49538 \times 10^9\}
       n= 10,q= \{3.51564 \times 10^{17}, 2.20835 \times 10^{18}, 0., 0.\},\q\= 2.23616 \times 10^{18}
       n= 11,q= \{-4.75323 \times 10^{36}, 1.55276\times 10^{36}, 0., 0.\},\q\= 5.00042\times 10^{36}
       n= 12,q= \left\{2.01821\times10^{73}, -1.47612\times10^{73}, 0., 0.\right\},\q\= 2.50042\times10^{73}
       n= 13,q= \{1.89425 \times 10^{146}, -5.95826 \times 10^{146}, 0., 0.\}, \q\= 6.25212×10<sup>146</sup>
       n= 14,q= \{-3.19127 \times 10^{293}, -2.25729 \times 10^{293}, 0., 0.\}, \q\= 3.9089 \times 10^{293}
       n= 15,q= \{5.088852345853266 \times 10^{586}, 1.440720366790744 \times 10^{587}, 0., 0.\}
        \q = 1.527952668528911 \times 10^{587}
       n= 16,q= \{-1.816710993306693 \times 10^{1174}, 1.466322643652331 \times 10^{1174}, 0., 0.\}
        ,\q\= 2.334639357264619 \times 10^{1174}
       n= 17,q= \{1.150336737913830 \times 10^{2348}, -5.327768932915444 \times 10^{2348}, 0., 0.\}
        \sqrt{q} = 5.450540928488952 \times 10^{2348}
       n= 18,q= \left\{-2.706184719194474 \times 10^{4697}, -1.225745666929720 \times 10^{4697}, 0., 0.\right\}
        \sqrt{q} = 2.970839641313320 \times 10^{4697}
       n= 19,q= \{5.820983294404689 \times 10^{9394}, 6.634188386928096 \times 10^{9394}, 0., 0.\}
        \q = 8.825888174398658 \times 10^{9394}
       n= 20,q=\{-1.012860903951314\times10^{18789}, 7.723499954448408\times10^{18789}, 0., 0.\}
        \sqrt{q} = 7.789630206699008 \times 10^{18789}
ln[\circ]:= P = \{-0.76, 0.5, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}1
Out[\bullet]= {-0.76, 0.5, 0, 0}
```

```
n= 1,q= \{-0.76, 0.5, 0, 0\}, q= 0.909725
       n= 2, q= \{-0.4324, -0.26, 0., 0.\}, q= 0.504549
       n= 3,q= \{-0.64063, 0.724848, 0., 0.\}, \q = 0.967374
       n=4,q=\{-0.874998, -0.428719, 0., 0.\}, q=0.974382
       n=5,q=\{-0.178179, 1.25026, 0., 0.\}, q=1.26289
       n=6,q=\{-2.29139, 0.0544602, 0., 0.\}, q=2.29204
       n= 7, q= \{4.48752, 0.250421, 0., 0.\}, q= 4.4945
       n= 8,q= \{19.3151, 2.74753, 0., 0.\}, q= 19.5095
       n= 9, q= \{364.764, 106.638, 0., 0.\}, q= 380.032
       n= 10,q= \{121680., 77795.6, 0., 0.\}, q= 144424.
       n= 11,q= \{8.75389 \times 10^9, 1.89324 \times 10^{10}, 0., 0.\},\q\= 2.08582\times 10^{10}
       n= 12,q= \{-2.81803 \times 10^{20}, 3.31463 \times 10^{20}, 0., 0.\},\q\= 4.35064×10<sup>20</sup>
       n= 13,q= \{-3.04548 \times 10^{40}, -1.86815 \times 10^{41}, 0., 0.\},\q\= 1.89281\times 10^{41}
       n= 14,q= \{-3.39723\times10^{82}, 1.13788\times10^{82}, 0., 0.\},\q\= 3.58273×10<sup>82</sup>
       n= 15,q= \{1.02464 \times 10^{165}, -7.73131 \times 10^{164}, 0., 0.\}, \q\= 1.2836 \times 10^{165}
       n= 16,q= \{4.521601872800172 \times 10^{329}, -1.584364420914123 \times 10^{330}, 0., 0.\}
        ,\q\ =\ 1.647622363656079 \times 10^{330}
      n= 17,q= \left\{-2.305761783297446\times10^{660}, -1.432773026560652\times10^{660}, 0., 0.\right\}
        \sqrt{q} = 2.714659453219646 \times 10^{660}
      n= 18,q= \{3.263698855675246 \times 10^{1320}, 6.607266577565935 \times 10^{1320}, 0., 0.\}
        ,\q\ =\ 7.369375946954788 \times 10^{1320}
       n= 19,q= \left\{-3.300424140648396\times10^{2641}, 4.312825673668648\times10^{2641}, 0., 0.\right\}
        q = 5.430770184755578 \times 10^{2641}
       n= 20,q= \{-7.707665783280729 \times 10^{5282}, -2.846830793556837 \times 10^{5283}, 0., 0.\}
        ,\q\= 2.949326479963013 \times 10^{5283}
ln[ \circ ] := P = \{-0.76, 0.4, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-0.76, 0.4, 0, 0}
```

,\q\= $1.837130145489553 \times 10^{1849}$ $ln[\circ] := P = \{-0.76, 0.2, 0, 0\}$ quaternion[$\{x_{y_{z}}, y_{z}, x_{y}\}$] := $\{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}$ iteration[t_] := Module [q = t, n = 1], While $[n \le 20, q = quaternion[q] + P$; Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]]; iteration[**{0,** 0, 0, 0}] Out[\circ]= {-0.76, 0.2, 0, 0}

```
n= 1, q= \{-0.76, 0.2, 0, 0\}, q= 0.785875
     n= 2, q= \{-0.2224, -0.104, 0., 0.\}, q= 0.245515
     n= 3,q= \{-0.721354, 0.246259, 0., 0.\}, \q = 0.762231
     n=4,q=\{-0.300292, -0.15528, 0., 0.\}, q=0.338064
     n=5,q=\{-0.693937, 0.293259, 0., 0.\}, q=0.753359
     n = 6, q = \{-0.364452, -0.207006, 0., 0.\}, \\ | q = 0.419138
     n=7,q=\{-0.670026, 0.350888, 0., 0.\},\q=0.756345
     n=8,q=\{-0.434187, -0.270208, 0., 0.\}, q=0.511401
     n=10,q=\{-0.533541, -0.360247, 0., 0.\}, q=0.643774
     n= 11,q= {-0.605112, 0.584414, 0., 0.},\q\= 0.841249
     n = \ 12, q = \ \{-0.735379, \ -0.507271, \ 0., \ 0.\} \ , \ \ | \ -0.893368
     n= 13, q= \{-0.476542, 0.946073, 0., 0.\}, \q = 1.05931
     n= 14, q= \{-1.42796, -0.701688, 0., 0.\}, q= 1.59105
     n= 15,q= {0.78671, 2.20397, 0., 0.},q = 2.34017
     n= 16, q= \{-4.99856, 3.66777, 0., 0.\}, \q = 6.19985
     n= 17, q= \{10.7731, -36.4671, 0., 0.\}, q= 38.0252
     n= 18, q= \{-1214.55, -785.529, 0., 0.\}, q= 1446.44
     n= 19,q= \{858081., 1.90813 \times 10^6, 0., 0.\}, \q\= 2.09219 \times 10^6
     n= 20,q= \{-2.90467 \times 10^{12}, 3.27466 \times 10^{12}, 0., 0.\}, \q\= 4.37727 \times 10^{12}
ln[\circ]:= P = \{-0.76, -0.2, 0, 0\}
     quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.76, -0.2, 0, 0\}
```

```
n= 1,q= \{-0.76, -0.2, 0, 0\}, q= 0.785875
      n= 2, q= \{-0.2224, 0.104, 0., 0.\}, \q = 0.245515
      n= 3, q= \{-0.721354, -0.246259, 0., 0.\}, \q = 0.762231
      n=4,q=\{-0.300292, 0.15528, 0., 0.\}, q=0.338064
      n=5,q=\{-0.693937, -0.293259, 0., 0.\}, q=0.753359
      n=6,q=\{-0.364452, 0.207006, 0., 0.\}, q=0.419138
      n=7,q=\{-0.670026, -0.350888, 0., 0.\}, q=0.756345
      n= 8, q= \{-0.434187, 0.270208, 0., 0.\}, q= 0.511401
      n=9,q=\{-0.644494, -0.434642, 0., 0.\}, q=0.777358
      n= 10,q= {-0.533541, 0.360247, 0., 0.},\q\= 0.643774
      n= 11,q= {-0.605112, -0.584414, 0., 0.},\q\= 0.841249
      n= 12,q= \{-0.735379, 0.507271, 0., 0.\}, q= 0.893368
      n= 13, q= \{-0.476542, -0.946073, 0., 0.\}, q= 1.05931
      n= 14, q= \{-1.42796, 0.701688, 0., 0.\}, q= 1.59105
      n= 15, q= \{0.78671, -2.20397, 0., 0.\}, q= 2.34017
      n= 16, q= \{-4.99856, -3.66777, 0., 0.\}, q= 6.19985
      n= 17, q= \{10.7731, 36.4671, 0., 0.\}, q= 38.0252
      n= 18, q= \{-1214.55, 785.529, 0., 0.\}, \q = 1446.44
      n=19,q=\{858081., -1.90813\times10^6, 0., 0.\}, q=2.09219\times10^6
      n= 20,q= \{-2.90467 \times 10^{12}, -3.27466 \times 10^{12}, 0., 0.\}, \q\= 4.37727 \times 10^{12}
ln[*] = P = \{-0.76, -0.4, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
      iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.76, -0.4, 0, 0\}
```

```
n= 1,q= \{-0.76, -0.4, 0, 0\}, q= 0.858836
        n= 2, q= \{-0.3424, 0.208, 0., 0.\}, q= 0.400627
        n= 3, q= \{-0.686026, -0.542438, 0., 0.\}, \q = 0.874569
        n=4,q=\{-0.583607, 0.344254, 0., 0.\}, \q = 0.677575
        n=5,q=\{-0.537913, -0.801818, 0., 0.\}, q=0.965538
        n=6,q=\{-1.11356,0.462617,0.,0.\},\q=1.20583
        n= 7, q= \{0.266006, -1.43031, 0., 0.\}, q= 1.45483
        n= 8, q= \{-2.73502, -1.16094, 0., 0.\}, q= 2.97121
        n= 9, q= \{5.37253, 5.95037, 0., 0.\}, q= 8.01693
        n= 10,q= \{-7.30283, 63.5372, 0., 0.\}, q= 63.9555
        n= 11,q= \{-3984.4, -928.402, 0., 0.\}, q= 4091.14
        n= 12,q= \{1.50135 \times 10^7, 7.39826 \times 10^6, 0., 0.\}, q= 1.67374 \times 10^7 \}
        n= 13,q= \{1.70672 \times 10^{14}, 2.22148 \times 10^{14}, 0., 0.\}, \q\= 2.80141×10<sup>14</sup>
        n= 14,q= \{-2.02208 \times 10^{28}, 7.5829 \times 10^{28}, 0., 0.\}, \q\= 7.84788 \times 10^{28}
        n= 15,q= \{-5.34116\times10^{57}, -3.06665\times10^{57}, 0., 0.\},\q\= 6.15892×10<sup>57</sup>
        n= 16,q= \{1.91236 \times 10^{115}, 3.27589 \times 10^{115}, 0., 0.\},\q\= 3.79323 \times 10^{115}
        n= 17,q= \{-7.0743\times10^{230}, 1.25294\times10^{231}, 0., 0.\},\q\= 1.43886\times10^{231}
        n= 18,q= \{-1.069392274423501 \times 10^{462}, -1.772731165321300 \times 10^{462}, 0., 0.\}
         |q| = 2.070308098109574 \times 10^{462}
        n= 19,q= \{-1.998975947904746 \times 10^{924}, 3.791490025648736 \times 10^{924}, 0., 0.\}
         ,\q\= 4.286175621098083 \times 10^{924}
        n= 20,q= \{-1.037949177429218 \times 10^{1849}, -1.515819473598514 \times 10^{1849}, 0., 0.\}
         ,\q\= 1.837130145489553 \times 10^{1849}
ln[185]:= P = \{-0.76, -0.7, 0, 0\}
        quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
        iteration[
         {0,
          0,
          0,
          0}]
Out[185]= \{-0.76, -0.7, 0, 0\}
```

```
n= 1,q= \{-0.76, -0.7, 0, 0\}, q= 1.03325
      n= 2, q= \{-0.6724, 0.364, 0., 0.\}, q= 0.764603
      n= 3,q= \{-0.440374, -1.18951, 0., 0.\}, q= 1.26841
      n= 4, q= \{-1.981, 0.347657, 0., 0.\}, q= 2.01127
      n=5,q=\{3.04349,-2.07741,0.,0.\},\q=3.6849
      n=6,q=\{4.18717, -13.3452, 0., 0.\}, q=13.9866
      n= 7, q= \{-161.321, -112.457, 0., 0.\}, q= 196.65
      n= 8, q= \{13377.2, 36282.7, 0., 0.\}, q= 38670.2
      n = 9,q = \{-1.13748 \times 10^9, 9.70721 \times 10^8, 0., 0.\}, q = 1.49538 \times 10^9\}
      n= 10,q= \{3.51564 \times 10^{17}, -2.20835 \times 10^{18}, 0., 0.\}, \q\= 2.23616 \times 10^{18}
      n= 11,q= \{-4.75323\times10^{36}, -1.55276\times10^{36}, 0., 0.\},\q\= 5.00042\times10^{36}
      n= 12,q= \{2.01821\times10^{73}, 1.47612\times10^{73}, 0., 0.\},\q\= 2.50042\times10^{73}
      n= 13,q= \{1.89425 \times 10^{146}, 5.95826 \times 10^{146}, 0., 0.\},\q\= 6.25212 \times 10^{146}
      n= 14,q= \{-3.19127 \times 10^{293}, 2.25729 \times 10^{293}, 0., 0.\},\q\= 3.9089 \times 10^{293}
      n= 15,q= \{5.088852345853266 \times 10^{586}, -1.440720366790744 \times 10^{587}, 0., 0.\}
       |q| = 1.527952668528911 \times 10^{587}
      n= 16,q= \{-1.816710993306693 \times 10^{1174}, -1.466322643652331 \times 10^{1174}, 0., 0.\}
       ,\q\= 2.334639357264619 \times 10^{1174}
      n= 17,q= \{1.150336737913830 \times 10^{2348}, 5.327768932915444 \times 10^{2348}, 0., 0.\}
       \sqrt{q} = 5.450540928488952 \times 10^{2348}
      n= 18,q= \left\{-2.706184719194474\times10^{4697}, 1.225745666929720\times10^{4697}, 0., 0.\right\}
       \sqrt{q} = 2.970839641313320 \times 10^{4697}
      n= 19,q= \{5.820983294404689 \times 10^{9394}, -6.634188386928096 \times 10^{9394}, 0., 0.\}
       \q = 8.825888174398658 \times 10^{9394}
      n= 20,q= \{-1.012860903951314 \times 10^{18789}, -7.723499954448408 \times 10^{18789}, 0., 0.\}
       \sqrt{q} = 7.789630206699008 \times 10^{18789}
ln[-]:= P = \{-0.76, -1.2, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
          n++]]
      iteration[
        {0,
         0,
         0,
         0}1
```

Out[\bullet]= {-0.76, -1.2, 0, 0}

```
n= 1, q= \{-0.76, -1.2, 0, 0\}, q= 1.42042
       n= 2, q= \{-1.6224, 0.624, 0., 0.\}, q= 1.73826
       n= 3,q= \{1.48281, -3.22476, 0., 0.\}, q= 3.54933
       n=4,q=\{-8.96033, -10.7634, 0., 0.\}, q=14.0049
       n=5,q=\{-36.3226, 191.687, 0., 0.\}, q=195.098
       n=6,q=\{-35425.3, -13926.3, 0., 0.\}, q=38064.3
       n= 7,q= \{1.06101 \times 10^9, 9.86687 \times 10^8, 0., 0.\},\q\= 1.44889 \times 10^9
       n= 8,q= \{1.52183 \times 10^{17}, 2.09376 \times 10^{18}, 0., 0.\}, q= 2.09928 \times 10^{18}
       n= 9,q= \{-4.36068 \times 10^{36}, 6.3727 \times 10^{35}, 0., 0.\}, \q\= 4.407 \times 10^{36}
       n= 10,q= \{1.86094 \times 10^{73}, -5.55786 \times 10^{72}, 0., 0.\}, \q\= 1.94216 \times 10<sup>73</sup>
       n= 11,q= \{3.1542 \times 10^{146}, -2.06857 \times 10^{146}, 0., 0.\},\q\= 3.77199 \times 10^{146}
       n= 12,q= \{5.66999 \times 10^{292}, -1.30493 \times 10^{293}, 0., 0.\}, \q\= 1.42279 \times 10^{293}
       n= 13,q= \{-1.381367456221049 \times 10^{586}, -1.479793520389373 \times 10^{586}, 0., 0.\}
        \q = 2.024343081617587 \times 10^{586}
       n = 14, q = \{-2.816128138797619 \times 10^{1171}, 4.088277221985320 \times 10^{1172}, 0., 0.\}
        \sqrt{q} = 4.097964912092987 \times 10^{1172}
       n= 15,q= \{-1.663470486686272 \times 10^{2345}, -2.302622504807644 \times 10^{2344}, 0., 0.\}
        \sqrt{q} = 1.679331642074528 \times 10^{2345}
       n= 16,q= \{2.714113356079798 \times 10^{4690}, 7.660689157454269 \times 10^{4689}, 0., 0.\}
        \sqrt{q} = 2.820154764072730 \times 10^{4690}
       n= 17,q= \{6.779549725979369 \times 10^{9380}, 4.158395751804465 \times 10^{9380}, 0., 0.\}
        ,\q\=7.953272893322117\times10^{9380}
       n= 18,q= \{2.867003925840152 \times 10^{18761}, 5.638410155931947 \times 10^{18761}, 0., 0.\}
        , q = 6.325454971565236 \times 10^{18761}
       n= 19,q= \{-2.357195757573368 \times 10^{37523}, 3.233068810510775 \times 10^{37523}, 0., 0.\}
        \sqrt{q} = 4.001138059729936 \times 10^{37523}
       n= 20,q= \{-4.896362093975675 \times 10^{75046}, -1.524195216815755 \times 10^{75047}, 0., 0.\}
        \sqrt{q} = 1.600910577301944 \times 10^{75047}
ln[\circ]:= P = \{-0.76, -0.7, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-0.76, -0.7, 0, 0}
```

```
n= 1,q= \{-0.76, -0.7, 0, 0\}, q= 1.03325
       n= 2, q= \{-0.6724, 0.364, 0., 0.\}, q= 0.764603
       n= 3,q= \{-0.440374, -1.18951, 0., 0.\}, q= 1.26841
       n= 4, q= \{-1.981, 0.347657, 0., 0.\}, q= 2.01127
       n=5,q=\{3.04349,-2.07741,0.,0.\},\q=3.6849
       n=6,q=\{4.18717, -13.3452, 0., 0.\}, q=13.9866
       n= 7, q= \{-161.321, -112.457, 0., 0.\}, q= 196.65
       n= 8, q= \{13377.2, 36282.7, 0., 0.\}, q= 38670.2
       n = 9,q = \{-1.13748 \times 10^9, 9.70721 \times 10^8, 0., 0.\}, q = 1.49538 \times 10^9\}
       n= 10,q= \{3.51564 \times 10^{17}, -2.20835 \times 10^{18}, 0., 0.\}, \q\= 2.23616 \times 10^{18}
       n= 11,q= \{-4.75323\times10^{36}, -1.55276\times10^{36}, 0., 0.\},\q\= 5.00042\times10^{36}
       n= 12,q= \{2.01821\times10^{73}, 1.47612\times10^{73}, 0., 0.\},\q\= 2.50042\times10^{73}
       n= 13,q= \{1.89425 \times 10^{146}, 5.95826 \times 10^{146}, 0., 0.\},\q\= 6.25212 \times 10^{146}
       n= 14,q= \{-3.19127 \times 10^{293}, 2.25729 \times 10^{293}, 0., 0.\},\q\= 3.9089 \times 10^{293}
       n= 15,q= \{5.088852345853266 \times 10^{586}, -1.440720366790744 \times 10^{587}, 0., 0.\}
        |q| = 1.527952668528911 \times 10^{587}
       n= 16,q= \{-1.816710993306693 \times 10^{1174}, -1.466322643652331 \times 10^{1174}, 0., 0.\}
        ,\q\= 2.334639357264619 \times 10^{1174}
       n= 17,q= \{1.150336737913830 \times 10^{2348}, 5.327768932915444 \times 10^{2348}, 0., 0.\}
        \sqrt{q} = 5.450540928488952 \times 10^{2348}
       n= 18,q= \left\{-2.706184719194474\times10^{4697}, 1.225745666929720\times10^{4697}, 0., 0.\right\}
        \sqrt{q} = 2.970839641313320 \times 10^{4697}
       n= 19,q= \{5.820983294404689 \times 10^{9394}, -6.634188386928096 \times 10^{9394}, 0., 0.\}
        \q = 8.825888174398658 \times 10^{9394}
       n= 20,q= \{-1.012860903951314 \times 10^{18789}, -7.723499954448408 \times 10^{18789}, 0., 0.\}
        \sqrt{q} = 7.789630206699008 \times 10^{18789}
ln[-]:= P = \{-0.76, -1.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}1
Out[\bullet]= {-0.76, -1.4, 0, 0}
```

```
n= 1,q= \{-0.76, -1.4, 0, 0\}, q= 1.59298
       n= 2, q= \{-2.1424, 0.728, 0., 0.\}, q= 2.26271
       n= 3,q= \{3.29989, -4.51933, 0., 0.\}, q= 5.59586
       n= 4,q= \{-10.2951, -31.2266, 0., 0.\}, q= 32.88
       n=5,q=\{-869.875,641.562,0.,0.\}, q=1080.87
       n=6,q={345080., -1.11616\times10^6, 0., 0.}, q=1.16828\times10^6
       n= 7,q= \{-1.12673\times10^{12}, -7.70327\times10^{11}, 0., 0.\},\q\= 1.36489\times10^{12}
       n= 8,q= \{6.76116 \times 10^{23}, 1.7359 \times 10^{24}, 0., 0.\}, q= 1.86292 \times 10^{24}
       n= 9,q= \{-2.55622 \times 10^{48}, 2.34734 \times 10^{48}, 0., 0.\}, \q\= 3.47048 \times 10^{48}
       n= 10,q= \{1.02423 \times 10^{96}, -1.20006 \times 10^{97}, 0., 0.\}, \q\= 1.20443 \times 10<sup>97</sup>
       n= 11,q= \{-1.42966 \times 10^{194}, -2.45828 \times 10^{193}, 0., 0.\}, \q\= 1.45064 \times 10^{194}
       n= 12,q= \{1.983493845087599 \times 10^{388}, 7.028993412131522 \times 10^{387}, 0., 0.\}
       \sqrt{q} = 2.104356271494509 \times 10^{388}
       n= 13,q= \{3.440180349622503 \times 10^{776}, 2.788393034024830 \times 10^{776}, 0., 0.\}
        \q = 4.428315317378270 \times 10^{776}
       n= 14,q= \{4.059705125730606 \times 10^{1552}, 1.918514984535299 \times 10^{1553}, 0., 0.\}
        n= 15,q= \{-3.515887688807643 \times 10^{3106}, 1.557721023301785 \times 10^{3106}, 0., 0.\}
        \sqrt{q} = 3.845511802965310 \times 10^{3106}
      n= 16,q= \{9.93497145387279 \times 10^{6212}, -1.095354433684718 \times 10^{6213}, 0., 0.\}
       ,\q\= 1.478796102674551×10<sup>6213</sup>
       n= 17,q= \{-2.127647575000976 \times 10^{12425}, -2.176463006106134 \times 10^{12426}, 0., 0.\}
        \sqrt{q} = 2.186837913285442 \times 10^{12426}
       n= 18,q= \{-4.691722374914475 \times 10^{24852}, 9.26149247404210 \times 10^{24851}, 0., 0.\}
        ,\q\= 4.782260058982626 \times 10^{24852}
       n= 19,q= \{2.115450641480574 \times 10^{49705}, -8.690470293113068 \times 10^{49704}, 0., 0.\}
       \sqrt{q} = 2.287001127174051 \times 10^{49705}
       n= 20,q= \{3.719888677385763 \times 10^{99410}, -3.676852191266782 \times 10^{99410}, 0., 0.\}
        ,\q\= 5.230374155695378 \times 10^{99410}
ln[ \circ ] := P = \{-0.6, 1.4, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
        {0,
          0,
          0,
          0}]
Out[\circ]= {-0.6, 1.4, 0, 0}
```

```
n= 2,q= \{-2.2, -0.28, 0., 0.\}, q= 2.21775
       n= 3,q= \{4.1616, 2.632, 0., 0.\}, q= 4.92406
       n= 4,q= \{9.79149, 23.3067, 0., 0.\}, \q \ge 25.2799
       n=5,q=\{-447.927, 457.814, 0., 0.\}, q=640.494
       n=6,q=\{-8955.4, -410133., 0., 0.\},\q=410231.
       n= 7,q= \{-1.68129 \times 10^{11}, 7.34581 \times 10^{9}, 0., 0.\}, q= 1.68289 \times 10^{11}\}
       n= 8,q= \{2.82134 \times 10^{22}, -2.47009 \times 10^{21}, 0., 0.\}, q= 2.83213 \times 10^{22}
       n = 9,q = \{7.89896 \times 10^{44}, -1.39379 \times 10^{44}, 0., 0.\}, \ | = 8.02099 \times 10^{44} \}
       n= 10,q= \{6.04509 \times 10^{89}, -2.2019 \times 10^{89}, 0., 0.\}, \q\= 6.43362 \times 10^{89}
       n= 11,q= \{3.16948 \times 10^{179}, -2.66214 \times 10^{179}, 0., 0.\}, \q\= 4.13915 \times 10^{179}
       n= 12,q= \{2.958576594875668 \times 10^{358}, -1.687519702024897 \times 10^{359}, 0., 0.\}
       \sqrt{q} = 1.713258445010460 \times 10^{359}
       n= 13,q= \{-2.760190990044736 \times 10^{718}, -9.98531258760484 \times 10^{717}, 0., 0.\}
        \sqrt{q} = 2.935254499399658 \times 10^{718}
       n= 14,q= \{6.621589626802344 \times 10^{1436}, 5.512273967417436 \times 10^{1436}, 0., 0.\}
        n= 15,q= \{1.346028489390845 \times 10^{2873}, 7.300003224548780 \times 10^{2873}, 0., 0.\}
        \sqrt{q} = 7.423061347764437 \times 10^{2873}
      n= 16,q= \{-5.147825438417078 \times 10^{5747}, 1.965202462577538 \times 10^{5747}, 0., 0.\}
        \q = 5.510183977267438 \times 10^{5747}
       n= 17,q= \{2.263808602549316 \times 10^{11495}, -2.023303845699307 \times 10^{11495}, 0., 0.\}
        \sqrt{q} = 3.036212746333480 \times 10^{11495}
       n= 18,q= \{1.031070936954682 \times 10^{22990}, -9.16074530293041 \times 10^{22990}, 0., 0.\}
        ,\q\= 9.21858784099790 \times 10^{22990}
       n= 19,q= \{-8.285614722812904 \times 10^{45981}, -1.889075648539133 \times 10^{45981}, 0., 0.\}
       \q = 8.498236178219425 \times 10^{45981}
       n= 20,q= \{6.508280452899040 \times 10^{91963}, 3.130430601208634 \times 10^{91963}, 0., 0.\}
        ,\q\= 7.222001814079749 \times 10^{91963}
ln[ \circ ] := P = \{-0.6, 1.2, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-0.6, 1.2, 0, 0}
```

 $n= 1,q= \{-0.6, 1.4, 0, 0\}, q= 1.52315$

```
n= 1,q= \{-0.6, 1.2, 0, 0\}, q= 1.34164
       n= 2,q= \{-1.68, -0.24, 0., 0.\}, q= 1.69706
       n= 3,q= \{2.1648, 2.0064, 0., 0.\}, q= 2.95161
       n=4,q=\{0.0607181, 9.88691, 0., 0.\}, q=9.8871
       n=5,q=\{-98.3473, 2.40063, 0., 0.\}, q=98.3766
       n=6,q=\{9665.83, -470.991, 0., 0.\}, q=9677.3
       n= 7,q= \{9.32064 \times 10^7, -9.10503 \times 10^6, 0., 0.\},\q\= 9.365 \times 10^7
       n= 8,q= \{8.60453 \times 10^{15}, -1.69729 \times 10^{15}, 0., 0.\}, q= 8.77033 \times 10^{15}
       n= 9,q= \{7.11571 \times 10^{31}, -2.92088 \times 10^{31}, 0., 0.\}, \q\= 7.69187 \times 10^{31}
       n= 10,q= \{4.21018 \times 10^{63}, -4.15683 \times 10^{63}, 0., 0.\}, \q\= 5.91648 \times 10<sup>63</sup>
       n= 11,q= \{4.46373\times10^{125}, -3.50019\times10^{127}, 0., 0.\},\q\= 3.50048\times10^{127}
       n= 12,q= \{-1.22494 \times 10^{255}, -3.12479 \times 10^{253}, 0., 0.\}, \q\= 1.22533 \times 10^{255}
       n= 13,q= \{1.499491845522758 \times 10^{510}, 7.655324534377576 \times 10^{508}, 0., 0.\}
        ,\q\= 1.501444702332360 \times 10^{510}
       n=14,q=\{2.242615395416582\times10^{1020}, 2.295819342825895\times10^{1019}, 0., 0.\}
        \q = 2.254336194161910 \times 10^{1020}
       n= 15,q= \{4.976615947210536 \times 10^{2040}, 1.029727960663307 \times 10^{2040}, 0., 0.\}
        \sqrt{q} = 5.082031676308407 \times 10^{2040}
       n= 16,q= \{2.370636661305841 \times 10^{4081}, 1.024912118065119 \times 10^{4081}, 0., 0.\}
        \sqrt{q} = 2.582704595900204 \times 10^{4081}
       n= 17,q= \left\{4.569473330170578\times10^{8162},\,4.859388483403584\times10^{8162},\,0.,\,0.\right\}
        \sqrt{q} = 6.670363029684035 \times 10^{8162}
       n= 18,q= \{-2.733569917495197 \times 10^{16324}, 4.440969215170145 \times 10^{16325}, 0., 0.\}
        ,\q\= 4.449374294777557 \times 10^{16325}
       n= 19,q= \{-1.964748352515059 \times 10^{32651}, -2.427939970222273 \times 10^{32650}, 0., 0.\}
        \sqrt{q} = 1.979693161502728 \times 10^{32651}
       n= 20,q= \{3.801287163720610 \times 10^{65302}, 9.54058211299935 \times 10^{65301}, 0., 0.\}
        \sqrt{q} = 3.919185013700668 \times 10^{65302}
ln[ \circ ] := P = \{-0.6, 0.7, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-0.6, 0.7, 0, 0}
```

```
n= 1, q= \{-0.6, 0.7, 0, 0\}, q= 0.921954
      n= 2, q= \{-0.73, -0.14, 0., 0.\}, q= 0.743303
      n= 3,q= \{-0.0867, 0.9044, 0., 0.\}, q= 0.908546
      n= 4,q= {-1.41042, 0.543177, 0., 0.},\q\= 1.5114
      n= 5, q= \{1.09425, -0.832218, 0., 0.\}, q= 1.37476
      n=6,q=\{-0.0952035, -1.12131, 0., 0.\}, q=1.12534
      n= 7, q= \{-1.84827, 0.913505, 0., 0.\}, q= 2.0617
      n= 8, q= \{1.98162, -2.67681, 0., 0.\}, q= 3.33049
      n= 9, q= \{-3.83852, -9.90885, 0., 0.\}, q= 10.6264
      n= 10,q= {-84.051, 76.7705, 0., 0.},\q\= 113.834
      n= 11,q= \{1170.25, -12904.6, 0., 0.\}, q= 12957.5
      n= 12,q= \left\{-1.65159\times10^{8}, -3.02032\times10^{7}, 0., 0.\right\},\q\= 1.67898\times10^{8}
      n= 13,q= \{2.63652 \times 10^{16}, 9.97666 \times 10^{15}, 0., 0.\}, \q\= 2.81896 \times 10^{16}
      n= 14,q= \{5.95589 \times 10^{32}, 5.26073 \times 10^{32}, 0., 0.\}, \q\= 7.94656 \times 10^{32}
      n= 15,q= \{7.79736 \times 10^{64}, 6.26646 \times 10^{65}, 0., 0.\}, q= 6.31479 \times 10^{65}
      n= 16,q= \{-3.86606 \times 10^{131}, 9.77238 \times 10^{130}, 0., 0.\}, \q\= 3.98765 \times 10<sup>131</sup>
      n= 17,q= \{1.39914 \times 10^{263}, -7.55611 \times 10^{262}, 0., 0.\}, \q\= 1.59014 \times 10^{263}
      n= 18,q= \{1.386642056834385 \times 10^{526}, -2.114409520858638 \times 10^{526}, 0., 0.\}
        q = 2.528537881005454 \times 10^{526}
      n= 19,q= \{-2.547951428115762 \times 10^{1052}, -5.863858333987257 \times 10^{1052}, 0., 0.\}
        ,\q\= 6.393503815679550 \times 10^{1052}
      n= 20,q=\{-2.789277808103465\times10^{2105}, 2.988165243270269\times10^{2105}, 0., 0.\}
        \sqrt{q} = 4.087689104110896 \times 10^{2105}
ln[ \circ ] := P = \{-0.6, 0.5, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
      iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-0.6, 0.5, 0, 0}
```

```
n= 1,q= \{-0.6, 0.5, 0, 0\}, q= 0.781025
      n= 2,q= \{-0.49, -0.1, 0., 0.\}, q= 0.5001
      n= 3,q= \{-0.3699, 0.598, 0., 0.\}, \q = 0.703157
      n=4,q=\{-0.820778,0.0575996,0.,0.\}, \q = 0.822797
      n=5,q=\{0.0703588,0.405447,0.,0.\},\q=0.411507
      n=7,q=\{-0.333564, -0.346094, 0., 0.\}, q=0.480673
      n=8,q=\{-0.608516, 0.730889, 0., 0.\}, q=0.951047
      n=9,q=\{-0.763907, -0.389516, 0., 0.\}, q=0.857483
      n=10,q=\{-0.168168, 1.09511, 0., 0.\}, q=1.10794
      n= 11,q= \{-1.77098, 0.131676, 0., 0.\}, q= 1.77587
      n= 12,q= \{2.51903, 0.0336103, 0., 0.\}, q= 2.51926
      n= 13, q= \{5.74439, 0.669331, 0., 0.\}, q= 5.78325
      n= 14, q= \{31.95, 8.18979, 0., 0.\}, q= 32.9829
      n= 15, q= \{953.128, 523.827, 0., 0.\}, q= 1087.59
      n= 16, q= \{634058., 998550., 0., 0.\}, q= 1.18285 \times 10^6
      n= 17,q= \{-5.95072 \times 10^{11}, 1.26628 \times 10^{12}, 0., 0.\}, \q\= 1.39913 \times 10^{12}
      n= 18,q= \{-1.24935 \times 10^{24}, -1.50705 \times 10^{24}, 0., 0.\}, \q\= 1.95757 \times 10^{24}
      n= 19,q= \{-7.10343\times10^{47}, 3.76566\times10^{48}, 0., 0.\}, q= 3.83208\times10^{48}
      n= 20,q= \{-1.36756 \times 10^{97}, -5.34982 \times 10^{96}, 0., 0.\},\q\= 1.46848 \times 10^{97}
ln[\circ]:= P = \{-0.6, 0.4, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
      iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.6, 0.4, 0, 0\}
```

```
n= 1, q= \{-0.6, 0.4, 0, 0\}, \q = 0.72111
     n= 2, q= \{-0.4, -0.08, 0., 0.\}, q= 0.407922
     n= 3, q= \{-0.4464, 0.464, 0., 0.\}, \q = 0.64387
     n=4,q=\{-0.616023, -0.0142592, 0., 0.\}, q=0.616188
     n=5,q=\{-0.220719,0.417568,0.,0.\},\q=0.472313
     n = 6, q = \{-0.725646, 0.21567, 0., 0.\}, \\ | q = 0.757018
     n= 7, q= \{-0.119951, 0.0870003, 0., 0.\}, q= 0.14818
     n=8,q=\{-0.593181, 0.379128, 0., 0.\}, q=0.70399
     n=9,q=\{-0.391875, -0.0497834, 0., 0.\}, q=0.395024
     n= 10,q= {-0.448912, 0.439018, 0., 0.},\q\= 0.6279
     n= 11,q= {-0.591214, 0.00583894, 0., 0.},\q\= 0.591243
     n= 12, q= \{-0.2505, 0.393096, 0., 0.\}, q= 0.466127
     n= 13, q= \{-0.691774, 0.203059, 0., 0.\}, \q = 0.720961
     n= 14,q= {-0.162681, 0.119058, 0., 0.},q= 0.201594
     n=15,q=\{-0.58771, 0.361263, 0., 0.\},\q=0.689865
     n= 16, q= \{-0.385108, -0.0246354, 0., 0.\}, \q = 0.385896
     n= 17,q= {-0.452298, 0.418975, 0., 0.},q = 0.616533
     n= 18, q= \{-0.570966, 0.0209969, 0., 0.\}, q= 0.571352
     n= 19, q= \{-0.274439, 0.376023, 0., 0.\}, q= 0.465521
     n=20,q=\{-0.666077, 0.193609, 0., 0.\}, q=0.693644
ln[-]:= P = \{-0.6, 0.2, 0, 0\}
     quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.6, 0.2, 0, 0\}
```

```
n= 1, q= \{-0.6, 0.2, 0, 0\}, \q = 0.632456
     n= 2,q= \{-0.28, -0.04, 0., 0.\}, \q = 0.282843
     n= 3,q= \{-0.5232, 0.2224, 0., 0.\}, q= 0.568507
     n=4,q=\{-0.375724, -0.0327194, 0., 0.\}, q=0.377145
     n=5,q=\{-0.459902, 0.224587, 0., 0.\}, q=0.51181
     n= 7, q= \{-0.407385, 0.205773, 0., 0.\}, q= 0.456404
     n= 8, q= \{-0.47638, 0.0323426, 0., 0.\}, q= 0.477477
     n= 10, q= \{-0.488667, 0.073413, 0., 0.\}, \q = 0.494151
     n= 11,q= {-0.366594, 0.128251, 0., 0.},q= 0.388381
     n= 12, q= \{-0.482057, 0.105968, 0., 0.\}, q= 0.493567
     n= 13, q= \{-0.37885, 0.0978349, 0., 0.\}, \q = 0.391279
     n= 14,q= {-0.466044, 0.12587, 0., 0.},\q\= 0.482743
     n= 15,q= {-0.398646, 0.0826776, 0., 0.},\q\= 0.407129
     n= 16, q= \{-0.447917, 0.134082, 0., 0.\}, q= 0.467555
     n= 17, q= \{-0.417348, 0.079885, 0., 0.\}, q= 0.424925
     n= 18, q= \{-0.432202, 0.13332, 0., 0.\}, q= 0.452297
     n=19,q=\{-0.430976,0.0847575,0.,0.\},\q\=0.439231
     n=20,q=\{-0.421444,0.126943,0.,0.\},\q=0.440147
ln[-]:= P = \{-0.6, -0.2, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
        Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
        n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.6, -0.2, 0, 0\}
```

```
n= 1,q= \{-0.6, -0.2, 0, 0\}, q= 0.632456
     n= 2, q= \{-0.28, 0.04, 0., 0.\}, q = 0.282843
     n= 3, q= \{-0.5232, -0.2224, 0., 0.\}, \q = 0.568507
     n=4,q=\{-0.375724,0.0327194,0.,0.\},\q=0.377145
     n=5,q=\{-0.459902, -0.224587, 0., 0.\}, q=0.51181
     n=6,q=\{-0.438929, 0.00657607, 0., 0.\}, \q = 0.438978
     n= 7, q= \{-0.407385, -0.205773, 0., 0.\}, q = 0.456404
     n=8,q=\{-0.47638, -0.0323426, 0., 0.\}, q=0.477477
     n=9,q=\{-0.374108, -0.169185, 0., 0.\}, q=0.410585
     n= 11,q= {-0.366594, -0.128251, 0., 0.},q= 0.388381
     n= 12, q= \{-0.482057, -0.105968, 0., 0.\}, \q = 0.493567
     n= 13, q= \{-0.37885, -0.0978349, 0., 0.\}, \q = 0.391279
     n= 14, q= \{-0.466044, -0.12587, 0., 0.\}, q = 0.482743
     n=15,q=\{-0.398646, -0.0826776, 0., 0.\}, \q = 0.407129
     n= 16, q= \{-0.447917, -0.134082, 0., 0.\}, \q = 0.467555
     n=17,q=\{-0.417348, -0.079885, 0., 0.\}, q=0.424925
     n= 18, q= \{-0.432202, -0.13332, 0., 0.\}, q= 0.452297
     n=19,q=\{-0.430976, -0.0847575, 0., 0.\}, \q = 0.439231
     n=20,q=\{-0.421444, -0.126943, 0., 0.\}, q=0.440147
ln[-]:= P = \{-0.6, -0.4, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.6, -0.4, 0, 0\}
```

```
n= 1, q= \{-0.6, -0.4, 0, 0\}, q= 0.72111
     n= 2, q= \{-0.4, 0.08, 0., 0.\}, q= 0.407922
     n= 3,q= \{-0.4464, -0.464, 0., 0.\}, \q = 0.64387
     n=4,q=\{-0.616023, 0.0142592, 0., 0.\}, q=0.616188
     n=5,q=\{-0.220719, -0.417568, 0., 0.\}, q=0.472313
     n=6,q=\{-0.725646,-0.21567,0.,0.\},\q=0.757018
     n= 7, q= \{-0.119951, -0.0870003, 0., 0.\}, q= 0.14818
     n= 8, q= \{-0.593181, -0.379128, 0., 0.\}, \q = 0.70399
     n=10,q=\{-0.448912, -0.439018, 0., 0.\}, q=0.6279
     n=11,q=\{-0.591214, -0.00583894, 0., 0.\}, q=0.591243
     n= 12, q= \{-0.2505, -0.393096, 0., 0.\}, q= 0.466127
     n= 13, q= \{-0.691774, -0.203059, 0., 0.\}, q= 0.720961
     n= 14, q= \{-0.162681, -0.119058, 0., 0.\}, q= 0.201594
     n=15,q=\{-0.58771, -0.361263, 0., 0.\}, q=0.689865
     n= 16, q= \{-0.385108, 0.0246354, 0., 0.\}, q= 0.385896
     n= 17, q= \{-0.452298, -0.418975, 0., 0.\}, q= 0.616533
     n= 18, q= \{-0.570966, -0.0209969, 0., 0.\}, \q = 0.571352
     n=19,q=\{-0.274439, -0.376023, 0., 0.\}, q=0.465521
     n=20,q=\{-0.666077, -0.193609, 0., 0.\}, q=0.693644
ln[-]:= P = \{-0.6, -0.5, 0, 0\}
     quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.6, -0.5, 0, 0\}
```

```
n= 1, q= \{-0.6, -0.5, 0, 0\}, q= 0.781025
      n= 2,q= \{-0.49, 0.1, 0., 0.\}, q= 0.5001
      n= 3,q= \{-0.3699, -0.598, 0., 0.\}, q= 0.703157
      n=4,q=\{-0.820778, -0.0575996, 0., 0.\}, \q = 0.822797
      n=5,q=\{0.0703588,-0.405447,0.,0.\},\q=0.411507
      n=6,q=\{-0.759437, -0.557054, 0., 0.\}, q=0.941835
      n= 7, q= \{-0.333564, 0.346094, 0., 0.\}, q= 0.480673
      n=8,q=\{-0.608516, -0.730889, 0., 0.\}, q=0.951047
      n=9,q=\{-0.763907, 0.389516, 0., 0.\},\q=0.857483
      n=10,q=\{-0.168168, -1.09511, 0., 0.\}, q=1.10794
      n= 11,q= {-1.77098, -0.131676, 0., 0.},q = 1.77587
      n= 12, q= \{2.51903, -0.0336103, 0., 0.\}, q= 2.51926
      n= 13, q= \{5.74439, -0.669331, 0., 0.\}, q= 5.78325
      n= 14,q= \{31.95, -8.18979, 0., 0.\}, q= 32.9829
      n= 15,q= {953.128, -523.827, 0., 0.},q = 1087.59
      n= 16, q= \{634058., -998550., 0., 0.\}, q= 1.18285 \times 10^6
      n= 17,q= \{-5.95072 \times 10^{11}, -1.26628 \times 10^{12}, 0., 0.\}, \q\= 1.39913 \times 10^{12}
      n= 18,q= \{-1.24935 \times 10^{24}, 1.50705 \times 10^{24}, 0., 0.\}, q= 1.95757 \times 10^{24}\}
      n= 19,q= \{-7.10343\times10^{47}, -3.76566\times10^{48}, 0., 0.\},\q\= 3.83208×10<sup>48</sup>
      n= 20,q= \{-1.36756 \times 10^{97}, 5.34982 \times 10^{96}, 0., 0.\}, \q\= 1.46848 \times 10^{97}
ln[-]:= P = \{-0.6, -0.7, 0, 0\}
      quaternion[\{x_{y_{y_{z}}}, y_{z_{y_{z}}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
          n++]]
      iteration[
        {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.6, -0.7, 0, 0\}
```

```
n= 1,q= \{-0.6, -0.7, 0, 0\}, q= 0.921954
      n= 2, q= \{-0.73, 0.14, 0., 0.\}, \q = 0.743303
      n= 3,q= \{-0.0867, -0.9044, 0., 0.\}, \q = 0.908546
      n=4,q=\{-1.41042, -0.543177, 0., 0.\}, q=1.5114
      n= 5, q= \{1.09425, 0.832218, 0., 0.\}, q= 1.37476
      n=6,q=\{-0.0952035, 1.12131, 0., 0.\}, q=1.12534
      n= 7, q= \{-1.84827, -0.913505, 0., 0.\}, q= 2.0617
      n= 8, q= \{1.98162, 2.67681, 0., 0.\}, q= 3.33049
      n=9,q=\{-3.83852, 9.90885, 0., 0.\}, q=10.6264
      n= 10,q= {-84.051, -76.7705, 0., 0.},\q\= 113.834
      n= 11,q= {1170.25, 12904.6, 0., 0.},q= 12957.5
      n= 12,q= \left\{-1.65159\times10^{8},\ 3.02032\times10^{7},\ 0.,\ 0.\right\},\q\= 1.67898\times10^{8}
      n= 13,q= \{2.63652 \times 10^{16}, -9.97666 \times 10^{15}, 0., 0.\}, \q\= 2.81896 \times 10^{16}
      n= 14,q= \{5.95589 \times 10^{32}, -5.26073 \times 10^{32}, 0., 0.\}, q= 7.94656 \times 10^{32}\}
      n= 15,q= \{7.79736 \times 10^{64}, -6.26646 \times 10^{65}, 0., 0.\}, q= 6.31479 \times 10^{65}
      n= 16,q= \{-3.86606 \times 10^{131}, -9.77238 \times 10^{130}, 0., 0.\}, \q\= 3.98765 \times 10^{131}
      n= 17,q= \{1.39914 \times 10^{263}, 7.55611 \times 10^{262}, 0., 0.\},\q\= 1.59014\times 10^{263}
      n= 18,q= \{1.386642056834385 \times 10^{526}, 2.114409520858638 \times 10^{526}, 0., 0.\}
        q = 2.528537881005454 \times 10^{526}
      n= 19,q= \{-2.547951428115762 \times 10^{1052}, 5.863858333987257 \times 10^{1052}, 0., 0.\}
        ,\q\= 6.393503815679550 \times 10^{1052}
      n= 20,q= \{-2.789277808103465 \times 10^{2105}, -2.988165243270269 \times 10^{2105}, 0., 0.\}
        \sqrt{q} = 4.087689104110896 \times 10^{2105}
ln[ \circ ] := P = \{-0.6, -1.2, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
      iteration[
        {0,
         0,
         0,
         0}]
Outfol= \{-0.6, -1.2, 0, 0\}
```

```
n= 1, q= \{-0.6, -1.2, 0, 0\}, q= 1.34164
       n= 2,q= \{-1.68, 0.24, 0., 0.\}, q= 1.69706
       n= 3,q= \{2.1648, -2.0064, 0., 0.\}, q= 2.95161
       n=4,q=\{0.0607181, -9.88691, 0., 0.\}, q=9.8871
       n=5,q=\{-98.3473, -2.40063, 0., 0.\}, q=98.3766
       n= 6,q= \{9665.83, 470.991, 0., 0.\}, q= 9677.3
       n= 7,q= \{9.32064 \times 10^7, 9.10503 \times 10^6, 0., 0.\}, q= 9.365 \times 10^7
       n= 8,q= \{8.60453 \times 10^{15}, 1.69729 \times 10^{15}, 0., 0.\}, q= 8.77033 \times 10^{15}
       n= 9,q= \{7.11571\times10^{31}, 2.92088\times10^{31}, 0., 0.\}, q= 7.69187\times10^{31}
       n= 10,q= \{4.21018\times10^{63}, 4.15683\times10^{63}, 0., 0.\},\q\= 5.91648\times10^{63}
       n= 11,q= \{4.46373 \times 10^{125}, 3.50019 \times 10^{127}, 0., 0.\},\q\= 3.50048 \times 10^{127}
       n= 12,q= \left\{-1.22494\times10^{255}, 3.12479\times10^{253}, 0., 0.\right\},\q\= 1.22533\times10^{255}
       n= 13,q= \{1.499491845522758 \times 10^{510}, -7.655324534377576 \times 10^{508}, 0., 0.\}
        \sqrt{q} = 1.501444702332360 \times 10^{510}
      n = 14, q = \{2.242615395416582 \times 10^{1020}, -2.295819342825895 \times 10^{1019}, 0., 0.\}
        \q = 2.254336194161910 \times 10^{1020}
       n= 15,q= \{4.976615947210536 \times 10^{2040}, -1.029727960663307 \times 10^{2040}, 0., 0.\}
       \sqrt{q} = 5.082031676308407 \times 10^{2040}
       n= 16,q=\{2.370636661305841\times10^{4081}, -1.024912118065119\times10^{4081}, 0., 0.\}
        \sqrt{q} = 2.582704595900204 \times 10^{4081}
       n= 17,q= \{4.569473330170578\times10^{8162}, -4.859388483403584\times10^{8162}, 0., 0.\}
        \sqrt{q} = 6.670363029684035 \times 10^{8162}
       n= 18,q= \{-2.733569917495197 \times 10^{16324}, -4.440969215170145 \times 10^{16325}, 0., 0.\}
        \sqrt{q} = 4.449374294777557 \times 10^{16325}
       n= 19,q= \{-1.964748352515059 \times 10^{32651}, 2.427939970222273 \times 10^{32650}, 0., 0.\}
        \sqrt{q} = 1.979693161502728 \times 10^{32651}
       n= 20,q= \{3.801287163720610 \times 10^{65302}, -9.54058211299935 \times 10^{65301}, 0., 0.\}
        \sqrt{q} = 3.919185013700668 \times 10^{65302}
ln[ \circ ] := P = \{-0.6, -1.4, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\bullet]= {-0.6, -1.4, 0, 0}
```

```
n= 1, q= \{-0.6, -1.4, 0, 0\}, q= 1.52315
       n= 2,q= \{-2.2, 0.28, 0., 0.\}, q= 2.21775
       n= 3,q= \{4.1616, -2.632, 0., 0.\}, q= 4.92406
       n=4,q=\{9.79149, -23.3067, 0., 0.\}, q=25.2799
       n=5,q=\{-447.927, -457.814, 0., 0.\}, q=640.494
       n=6,q=\{-8955.4,410133.,0.,0.\},\q=410231.
       n= 7,q= \{-1.68129 \times 10^{11}, -7.34581 \times 10^{9}, 0., 0.\},\q\= 1.68289 \times 10^{11}
       n= 8,q= \{2.82134 \times 10^{22}, 2.47009 \times 10^{21}, 0., 0.\}, \q\= 2.83213 \times 10^{22}
       n= 9,q= \{7.89896 \times 10^{44}, 1.39379 \times 10^{44}, 0., 0.\}, q= 8.02099 \times 10^{44}\}
       n= 10,q= \{6.04509 \times 10^{89}, 2.2019 \times 10^{89}, 0., 0.\}, \q\= 6.43362 \times 10^{89}
       n= 11,q= \{3.16948 \times 10^{179}, 2.66214 \times 10^{179}, 0., 0.\},\q\= 4.13915 \times 10^{179}
       n= 12,q= \{2.958576594875668 \times 10^{358}, 1.687519702024897 \times 10^{359}, 0., 0.\}
       \sqrt{q} = 1.713258445010460 \times 10^{359}
      n= 13,q= \{-2.760190990044736 \times 10^{718}, 9.98531258760484 \times 10^{717}, 0., 0.\}
        \sqrt{q} = 2.935254499399658 \times 10^{718}
       n= 14,q= \{6.621589626802344 \times 10^{1436}, -5.512273967417436 \times 10^{1436}, 0., 0.\}
        n= 15,q= \{1.346028489390845 \times 10^{2873}, -7.300003224548780 \times 10^{2873}, 0., 0.\}
        \sqrt{q} = 7.423061347764437 \times 10^{2873}
      n= 16,q= \{-5.147825438417078 \times 10^{5747}, -1.965202462577538 \times 10^{5747}, 0., 0.\}
       \q = 5.510183977267438 \times 10^{5747}
       n= 17,q= \{2.263808602549316 \times 10^{11495}, 2.023303845699307 \times 10^{11495}, 0., 0.\}
        \sqrt{q} = 3.036212746333480 \times 10^{11495}
       n= 18,q= \{1.031070936954682 \times 10^{22990}, 9.16074530293041 \times 10^{22990}, 0., 0.\}
        ,\q\= 9.21858784099790 \times 10^{22990}
       n= 19,q= \{-8.285614722812904 \times 10^{45981}, 1.889075648539133 \times 10^{45981}, 0., 0.\}
       ,\q\= 8.498236178219425 \times 10^{45981}
       n= 20,q= \{6.508280452899040 \times 10^{91963}, -3.130430601208634 \times 10^{91963}, 0., 0.\}
        ,\q\= 7.222001814079749 \times 10^{91963}
ln[ \circ ] := P = \{-0.4, 1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {-0.4, 1.4, 0, 0}
```

```
n= 1, q= \{-0.4, 1.4, 0, 0\}, q= 1.45602
       n= 2,q= \{-2.2, 0.28, 0., 0.\}, q= 2.21775
       n= 3,q= \{4.3616, 0.168, 0., 0.\}, q= 4.36483
       n= 4, q= \{18.5953, 2.8655, 0., 0.\}, q= 18.8148
       n=5,q={337.175, 107.97, 0., 0.},\q\ 354.04
       n=6,q=\{102029., 72810.9, 0., 0.\},\q=125345.
       n= 7,q= \{5.10855 \times 10^9, 1.48577 \times 10^{10}, 0., 0.\}, \q\= 1.57114×10<sup>10</sup>
       n=8,q=\{-1.94653\times10^{20}, 1.51802\times10^{20}, 0., 0.\}, q=2.46848\times10^{20}\}
       n= 9,q= \{1.48459 \times 10^{40}, -5.90977 \times 10^{40}, 0., 0.\}, \q\= 6.09339 \times 10^{40}
       n= 10,q= \{-3.27214 \times 10^{81}, -1.75472 \times 10^{81}, 0., 0.\},\q\= 3.71294 \times 10^{81}
       n= 11,q= \{7.62784 \times 10^{162}, 1.14833 \times 10^{163}, 0., 0.\},\q\= 1.37859 \times 10^{163}
       n= 12,q= \{-7.368332879060586 \times 10^{325}, 1.751863389089434 \times 10^{326}, 0., 0.\}
        \sqrt{q} = 1.900512727712806 \times 10^{326}
       n= 13,q= \{-2.526102039865464 \times 10^{652}, -2.581662521890037 \times 10^{652}, 0., 0.\}
        \q = 3.611948628198369 \times 10^{652}
       n= 14,q= \{-2.837898611191654 \times 10^{1303}, 1.304308592558128 \times 10^{1305}, 0., 0.\}
        \sqrt{q} = 1.304617289274408 \times 10^{1305}
       n= 15,q= \{-1.700415537768224 \times 10^{2610}, -7.402991086772104 \times 10^{2608}, 0., 0.\}
        ,\q\= 1.702026271473705\times10^{2610}
       n= 16,q= \{2.885932573380517 \times 10^{5220}, 2.517632213981392 \times 10^{5219}, 0., 0.\}
        \q = 2.896893428786683 \times 10^{5220}
       n= 17,q= \{8.265222098449927 \times 10^{10440}, 1.453143362824201 \times 10^{10440}, 0., 0.\}
        \sqrt{q} = 8.391991537747464 \times 10^{10440}
       n= 18,q= \{6.620227070378498 \times 10^{20881}, 2.402110526926086 \times 10^{20881}, 0., 0.\}
        ,\q\= 7.042552196962504 \times 10^{20881}
       n= 19,q= \{3.805727147980315 \times 10^{41763}, 3.180503427279446 \times 10^{41763}, 0., 0.\}
        \sqrt{q} = 4.959754144694139 \times 10^{41763}
       n= 20,q= \{4.367957073938079 \times 10^{83526}, 2.420825647488365 \times 10^{83527}, 0., 0.\}
        ,\q\= 2.459916117581069 \times 10^{83527}
ln[ \circ ] := P = \{-0.4, 1.2, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-0.4, 1.2, 0, 0}
```

```
n= 1,q= \{-0.4, 1.2, 0, 0\}, q= 1.26491
       n= 2, q= \{-1.68, 0.24, 0., 0.\}, q= 1.69706
       n= 3,q= \{2.3648, 0.3936, 0., 0.\}, q= 2.39733
       n= 4, q= \{5.03736, 3.06157, 0., 0.\}, q= 5.89476
       n=5,q=\{15.6018, 32.0445, 0., 0.\}, q=35.6407
       n=6,q=\{-783.832, 1001.1, 0., 0.\}, q=1271.45
       n = 7,q = \{-387809., -1.56939 \times 10^6, 0., 0.\}, q = 1.61659 \times 10^6\}
       n= 8,q= \{-2.31258\times10^{12}, 1.21724\times10^{12}, 0., 0.\}, q= 2.61337\times10^{12}
       n = 9,q = \{3.86635 \times 10^{24}, -5.62995 \times 10^{24}, 0., 0.\}, q = 6.82971 \times 10^{24}\}
       n= 10,q= \{-1.67477 \times 10^{49}, -4.35347 \times 10^{49}, 0., 0.\}, \q\= 4.6645 \times 10^{49}
       n= 11,q= \{-1.61478 \times 10^{99}, 1.45821 \times 10^{99}, 0., 0.\}, \q\= 2.17575 \times 10^{99}
       n= 12,q= \{4.8114 \times 10^{197}, -4.70939 \times 10^{198}, 0., 0.\},\q\= 4.7339×10<sup>198</sup>
       n= 13,q= \{-2.194686007948114 \times 10^{397}, -4.531754507338190 \times 10^{396}, 0., 0.\}
        \q = 2.240985199110210 \times 10^{397}
       n = 14, q = \{4.611278684335427 \times 10^{794}, 1.989155641742184 \times 10^{794}, 0., 0.\}
        \sqrt{q} = 5.022014662631027 \times 10^{794}
       n= 15,q= \{1.730715093753151 \times 10^{1589}, 1.834510202118258 \times 10^{1589}, 0., 0.\}
        \q = 2.522063127168103 \times 10^{1589}
       n= 16,q= \{-3.700529459309967 \times 10^{3177}, 6.350028992900426 \times 10^{3178}, 0., 0.\}
        \sqrt{q} = 6.360802417420950 \times 10^{3178}
       n= 17,q= \{-4.018592902788379 \times 10^{6357}, -4.699693871140086 \times 10^{6356}, 0., 0.\}
        \sqrt{q} = 4.045980739346820 \times 10^{6357}
       n= 18,q= \{1.592821769351681 \times 10^{12715}, 3.777231287168318 \times 10^{12714}, 0., 0.\}
        \sqrt{q} = 1.636996014316544 \times 10^{12715}
       n= 19,q= \{2.394406426952987 \times 10^{25430}, 1.203291244415593 \times 10^{25430}, 0., 0.\}
        \sqrt{q} = 2.679755950888252 \times 10^{25430}
       n= 20,q= \{4.285272318546543 \times 10^{50860}, 5.762336578249909 \times 10^{50860}, 0., 0.\}
        \sqrt{q} = 7.181091956320998 \times 10^{50860}
ln[ \circ ] := P = \{-0.4, 0.7, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-0.4, 0.7, 0, 0}
```

```
n= 1, q= \{-0.4, 0.7, 0, 0\}, q = 0.806226
      n= 2, q= \{-0.73, 0.14, 0., 0.\}, \q = 0.743303
      n= 3,q= \{0.1133, 0.4956, 0., 0.\}, \q = 0.508386
      n=4,q=\{-0.632782,0.812303,0.,0.\},\q=1.02968
      n=5,q=\{-0.659422, -0.328022, 0., 0.\}, q=0.736503
      n=6,q=\{-0.0727606, 1.13261, 0., 0.\}, q=1.13495
      n= 7, q= \{-1.67751, 0.535181, 0., 0.\}, q= 1.76081
      n= 8, q= \{2.12763, -1.09555, 0., 0.\}, q= 2.39312
      n= 9, q= \{2.92658, -3.96183, 0., 0.\}, q= 4.92554
      n=10,q=\{-7.53122, -22.4892, 0., 0.\}, q=23.7167
      n= 11,q= \{-449.445, 339.442, 0., 0.\}, q= 563.225
      n= 12, q= \{86779.4, -305121., 0., 0.\}, q= 317221.
      n= 13,q= \{-8.55679 \times 10^{10}, -5.29564 \times 10^{10}, 0., 0.\}, \q\= 1.00629 \times 10<sup>11</sup>
      n= 14,q= \{4.5175 \times 10^{21}, 9.06273 \times 10^{21}, 0., 0.\}, \q\= 1.01262 \times 10^{22}
      n= 15,q= \{-6.17253 \times 10^{43}, 8.18817 \times 10^{43}, 0., 0.\},\q\= 1.02541×10<sup>44</sup>
      n= 16,q= \{-2.8946 \times 10^{87}, -1.01083 \times 10^{88}, 0., 0.\}, \q\= 1.05146 \times 10^{88}
      n= 17,q= \{-9.38 \times 10^{175}, 5.85192 \times 10^{175}, 0., 0.\},\q\= 1.10557\times 10^{176}
      n= 18,q= \{5.373953760047012 \times 10^{351}, -1.097820054054608 \times 10^{352}, 0., 0.\}
       |q| = 1.222294015871671 \times 10^{352}
      n= 19,q= \{-9.16415080933229 \times 10^{703}, -1.179926841468355 \times 10^{704}, 0., 0.\}
       ,\q\ =\ 1.494002661235697 \times 10^{704}
      n= 20,q= \{-5.524107506556323 \times 10^{1407}, 2.162605503839024 \times 10^{1408}, 0., 0.\}
       ,\q\ =\ 2.232043951779346\times 10^{1408}
ln[ \circ ] := P = \{-0.4, 0.5, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
      iteration[
        {0,
         0,
         0,
         0}]
Out[0]= {-0.4, 0.5, 0, 0}
```

```
n= 1, q= \{-0.4, 0.5, 0, 0\}, \q = 0.640312
     n= 2,q= \{-0.49, 0.1, 0., 0.\}, q= 0.5001
     n= 3,q= \{-0.1699, 0.402, 0., 0.\}, \q = 0.436429
     n=4,q=\{-0.532738,0.3634,0.,0.\},\q\=0.64488
     n=5,q=\{-0.24825, 0.112806, 0., 0.\}, q=0.272678
     n=6,q=\{-0.351097, 0.443992, 0., 0.\}, q=0.566037
     n=7,q=\{-0.47386, 0.188231, 0., 0.\}, q=0.509877
     n=8,q=\{-0.210888,0.321609,0.,0.\},\q=0.384586
     n=9,q=\{-0.458959, 0.364353, 0., 0.\}, q=0.586
     n=10,q=\{-0.32211, 0.165554, 0., 0.\},\q=0.362164
     n= 11,q= {-0.323653, 0.393347, 0., 0.},q= 0.509385
     n= 12, q= \{-0.44997, 0.245384, 0., 0.\}, q= 0.512529
     n= 13, q= \{-0.25774, 0.279169, 0., 0.\}, q= 0.379954
     n= 14,q= {-0.411505, 0.356094, 0., 0.},q = 0.544187
     n= 16, q= \{-0.315038, 0.352058, 0., 0.\}, q= 0.472434
     n= 17, q= \{-0.424696, 0.278176, 0., 0.\}, q= 0.50769
     n= 18, q= \{-0.297015, 0.263719, 0., 0.\}, q= 0.397198
     n= 19, q= \{-0.38133, 0.343343, 0., 0.\}, q= 0.513124
     n=20,q=\{-0.372472,0.238146,0.,0.\},\q\=0.442096
ln[\circ]:= P = \{-0.4, 0.4, 0, 0\}
     quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
        Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
        n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.4, 0.4, 0, 0\}
```

```
n= 1, q= \{-0.4, 0.4, 0, 0\}, q= 0.565685
     n= 2, q= \{-0.4, 0.08, 0., 0.\}, q= 0.407922
     n = \ 3, q = \ \{-0.2464, \ 0.336, \ 0., \ 0.\} \ , \ \ | \ q = \ 0.416664
     n=4,q=\{-0.452183,0.234419,0.,0.\},\q=0.509335
     n=5,q=\{-0.250483, 0.187999, 0., 0.\}, q=0.313186
     n=6,q=\{-0.372602,0.305819,0.,0.\},\q=0.482035
     n= 7, q= \{-0.354693, 0.172103, 0., 0.\}, q= 0.394241
     n= 8, q= \{-0.303812, 0.277913, 0., 0.\}, q= 0.411749
     n= 10,q= {-0.305249, 0.222058, 0., 0.},q= 0.377474
     n= 11, q= \{-0.356133, 0.264434, 0., 0.\}, q= 0.443572
     n= 12, q= \{-0.343095, 0.211653, 0., 0.\}, q= 0.403126
     n= 13, q= \{-0.327083, 0.254766, 0., 0.\}, q= 0.414595
     n= 14, q= \{-0.357923, 0.233341, 0., 0.\}, q= 0.427266
     n= 15,q= {-0.326339, 0.232964, 0., 0.},\q\= 0.400961
     n= 16, q= \{-0.347775, 0.247949, 0., 0.\}, q= 0.427114
     n= 17, q= \{-0.340531, 0.227539, 0., 0.\}, q= 0.409555
     n= 18, q= \{-0.335812, 0.245032, 0., 0.\}, q= 0.415705
     n= 19, q= \{-0.347271, 0.235431, 0., 0.\}, q= 0.419553
     n=20,q=\{-0.334831,0.236484,0.,0.\},\q\=0.409922
ln[-]:= P = \{-0.4, 0.2, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.4, 0.2, 0, 0\}
```

```
n= 1, q= \{-0.4, 0.2, 0, 0\}, \q = 0.447214
     n= 2, q= \{-0.28, 0.04, 0., 0.\}, q = 0.282843
     n= 3,q= \{-0.3232, 0.1776, 0., 0.\}, q= 0.368782
     n=4,q=\{-0.327084,0.0851994,0.,0.\},\q=0.337998
     n=5,q=\{-0.300275,0.144265,0.,0.\},\q\=0.333133
     n= 7, q= \{-0.303523, 0.125035, 0., 0.\}, q= 0.328268
     n= 8, q= \{-0.323507, 0.124098, 0., 0.\}, q= 0.346493
     n= 10, q= \{-0.317768, 0.125604, 0., 0.\}, \q = 0.341691
     n= 11,q= {-0.3148, 0.120174, 0., 0.},q= 0.336958
     n= 12, q= \{-0.315343, 0.124338, 0., 0.\}, q= 0.338971
     n= 13, q= \{-0.316019, 0.121581, 0., 0.\}, q= 0.3386
     n= 14,q= {-0.314914, 0.123156, 0., 0.},q = 0.338139
     n=15,q=\{-0.315996, 0.122433, 0., 0.\}, q=0.338886
     n= 16, q= \{-0.315136, 0.122623, 0., 0.\}, q= 0.338153
     n= 17,q= {-0.315726, 0.122714, 0., 0.},\q\= 0.338735
     n= 18, q= \{-0.315376, 0.122512, 0., 0.\}, q= 0.338336
     n= 19, q= \{-0.315547, 0.122725, 0., 0.\}, q= 0.338573
     n=20,q=\{-0.315491, 0.122549, 0., 0.\}, q=0.338457
ln[-]:= P = \{-0.4, -0.2, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
        Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
        n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.4, -0.2, 0, 0\}
```

```
n= 1, q= \{-0.4, -0.2, 0, 0\}, q= 0.447214
     n= 2,q= \{-0.28, -0.04, 0., 0.\}, \q = 0.282843
     n= 3,q= \{-0.3232, -0.1776, 0., 0.\}, q= 0.368782
     n=4,q=\{-0.327084, -0.0851994, 0., 0.\}, q=0.337998
     n=5,q=\{-0.300275, -0.144265, 0., 0.\}, q=0.333133
     n=7,q=\{-0.303523, -0.125035, 0., 0.\},\q=0.328268
     n=8,q=\{-0.323507, -0.124098, 0., 0.\}, q=0.346493
     n= 10,q= {-0.317768, -0.125604, 0., 0.},\q\= 0.341691
     n= 11,q= {-0.3148, -0.120174, 0., 0.},\q\= 0.336958
     n= 13, q= \{-0.316019, -0.121581, 0., 0.\}, \q = 0.3386
     n= 14, q= \{-0.314914, -0.123156, 0., 0.\}, q= 0.338139
     n= 15, q= \{-0.315996, -0.122433, 0., 0.\}, q= 0.338886
     n= 16, q= \{-0.315136, -0.122623, 0., 0.\}, q= 0.338153
     n= 17, q= \{-0.315726, -0.122714, 0., 0.\}, q= 0.338735
     n= 18, q= \{-0.315376, -0.122512, 0., 0.\}, q= 0.338336
     n= 19, q= \{-0.315547, -0.122725, 0., 0.\}, \q = 0.338573
     n=20,q=\{-0.315491, -0.122549, 0., 0.\}, q=0.338457
ln[-]:= P = \{-0.4, -0.4, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
        Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
        n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.4, -0.4, 0, 0\}
```

```
n= 1, q= \{-0.4, -0.4, 0, 0\}, q= 0.565685
     n = 2, q = \{-0.4, -0.08, 0., 0.\}, q = 0.407922
     n= 3,q= \{-0.2464, -0.336, 0., 0.\}, q= 0.416664
     n=4,q=\{-0.452183,-0.234419,0.,0.\},\q=0.509335
     n=5,q=\{-0.250483, -0.187999, 0., 0.\}, q=0.313186
     n=6,q=\{-0.372602, -0.305819, 0., 0.\},\q=0.482035
     n= 7, q= \{-0.354693, -0.172103, 0., 0.\}, q= 0.394241
     n= 8, q= \{-0.303812, -0.277913, 0., 0.\}, q= 0.411749
     n=\ 9, q=\ \{-0.384934,\ -0.231133,\ 0.,\ 0.\}\,, \ \ |\ q =\ 0.448995
     n= 10,q= {-0.305249, -0.222058, 0., 0.},\q\= 0.377474
     n= 11, q= \{-0.356133, -0.264434, 0., 0.\}, \q = 0.443572
     n= 12,q= \{-0.343095, -0.211653, 0., 0.\}, q= 0.403126
     n= 13, q= \{-0.327083, -0.254766, 0., 0.\}, q= 0.414595
     n= 14,q= {-0.357923, -0.233341, 0., 0.},\q\= 0.427266
     n = 15, q = \{-0.326339, -0.232964, 0., 0.\}, \\ | q = 0.400961
     n= 16, q= \{-0.347775, -0.247949, 0., 0.\}, q= 0.427114
     n= 17, q= \{-0.340531, -0.227539, 0., 0.\}, q= 0.409555
     n= 18, q= \{-0.335812, -0.245032, 0., 0.\}, \q = 0.415705
     n= 19, q= \{-0.347271, -0.235431, 0., 0.\}, q= 0.419553
     n=20,q=\{-0.334831, -0.236484, 0., 0.\}, q=0.409922
ln[-]:= P = \{-0.4, -0.5, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.4, -0.5, 0, 0\}
```

```
n= 1, q= \{-0.4, -0.5, 0, 0\}, q= 0.640312
     n= 2,q= \{-0.49, -0.1, 0., 0.\}, \q = 0.5001
     n= 3, q= \{-0.1699, -0.402, 0., 0.\}, \q = 0.436429
     n=4,q=\{-0.532738, -0.3634, 0., 0.\}, q=0.64488
     n=5,q=\{-0.24825, -0.112806, 0., 0.\}, q=0.272678
     n=6,q=\{-0.351097, -0.443992, 0., 0.\},\q=0.566037
     n= 7, q= \{-0.47386, -0.188231, 0., 0.\}, \q = 0.509877
     n=8,q=\{-0.210888,-0.321609,0.,0.\},\q=0.384586
     n= 9, q= \{-0.458959, -0.364353, 0., 0.\}, \q = 0.586
     n=10,q=\{-0.32211, -0.165554, 0., 0.\}, q=0.362164
     n= 11,q= {-0.323653, -0.393347, 0., 0.},\q\= 0.509385
     n= 12, q= \{-0.44997, -0.245384, 0., 0.\}, q= 0.512529
     n= 13, q= \{-0.25774, -0.279169, 0., 0.\}, q= 0.379954
     n= 14, q= \{-0.411505, -0.356094, 0., 0.\}, \q = 0.544187
     n=15,q=\{-0.357466, -0.206931, 0., 0.\}, q=0.41304
     n= 16, q= \{-0.315038, -0.352058, 0., 0.\}, q= 0.472434
     n= 17, q= \{-0.424696, -0.278176, 0., 0.\}, q= 0.50769
     n=18,q=\{-0.297015, -0.263719, 0., 0.\}, q=0.397198
     n= 19, q= \{-0.38133, -0.343343, 0., 0.\}, q= 0.513124
     n=20,q=\{-0.372472, -0.238146, 0., 0.\}, q=0.442096
ln[-]:= P = \{-0.4, -0.7, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.4, -0.7, 0, 0\}
```

```
n= 1,q= \{-0.4, -0.7, 0, 0\}, q= 0.806226
      n= 2, q= \{-0.73, -0.14, 0., 0.\}, q= 0.743303
      n= 3,q= \{0.1133, -0.4956, 0., 0.\}, q= 0.508386
      n=4,q=\{-0.632782, -0.812303, 0., 0.\}, q=1.02968
      n=5,q=\{-0.659422, 0.328022, 0., 0.\}, q=0.736503
      n=6,q=\{-0.0727606,-1.13261,0.,0.\},\q\=1.13495
      n= 7, q= \{-1.67751, -0.535181, 0., 0.\}, \q = 1.76081
      n= 8, q= \{2.12763, 1.09555, 0., 0.\}, q= 2.39312
      n= 9, q= \{2.92658, 3.96183, 0., 0.\}, q= 4.92554
      n= 10,q= {-7.53122, 22.4892, 0., 0.},\q\= 23.7167
      n= 11,q= \{-449.445, -339.442, 0., 0.\}, q= 563.225
      n= 12, q= \{86779.4, 305121., 0., 0.\}, q= 317221.
      n= 13,q= \{-8.55679 \times 10^{10}, 5.29564 \times 10^{10}, 0., 0.\}, \q\= 1.00629 \times 10<sup>11</sup>
      n= 14,q= \{4.5175 \times 10^{21}, -9.06273 \times 10^{21}, 0., 0.\}, \q\= 1.01262 \times 10^{22}
      n= 15,q= \{-6.17253\times10^{43}, -8.18817\times10^{43}, 0., 0.\},\q\= 1.02541×10<sup>44</sup>
      n= 16,q= \{-2.8946\times10^{87}, 1.01083\times10^{88}, 0., 0.\},\q\= 1.05146\times10^{88}
      n= 17,q= \{-9.38 \times 10^{175}, -5.85192 \times 10^{175}, 0., 0.\},\q\= 1.10557\times 10^{176}
      n= 18,q= \{5.373953760047012 \times 10^{351}, 1.097820054054608 \times 10^{352}, 0., 0.\}
       |q| = 1.222294015871671 \times 10^{352}
      n= 19,q= \{-9.16415080933229 \times 10^{703}, 1.179926841468355 \times 10^{704}, 0., 0.\}
       ,\q\ =\ 1.494002661235697 \times 10^{704}
      n= 20,q= \{-5.524107506556323 \times 10^{1407}, -2.162605503839024 \times 10^{1408}, 0., 0.\}
       ,\q\ =\ 2.232043951779346\times 10^{1408}
ln[ \circ ] := P = \{-0.4, -1.2, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
      iteration[
        {0,
         0,
         0,
         0}]
Outfol= \{-0.4, -1.2, 0, 0\}
```

```
n= 1, q= \{-0.4, -1.2, 0, 0\}, q= 1.26491
       n= 2,q= \{-1.68, -0.24, 0., 0.\}, q= 1.69706
       n= 3,q= \{2.3648, -0.3936, 0., 0.\}, \q = 2.39733
       n= 4, q= \{5.03736, -3.06157, 0., 0.\}, q= 5.89476
       n=5,q=\{15.6018, -32.0445, 0., 0.\}, q=35.6407
       n=6,q=\{-783.832, -1001.1, 0., 0.\}, q=1271.45
       n= 7,q= \{-387809., 1.56939 \times 10^6, 0., 0.\}, \q\= 1.61659 \times 10^6
       n= 8,q= \{-2.31258 \times 10^{12}, -1.21724 \times 10^{12}, 0., 0.\}, q= 2.61337 \times 10^{12}\}
       n= 9,q= \{3.86635 \times 10^{24}, 5.62995 \times 10^{24}, 0., 0.\}, q= 6.82971 \times 10^{24}\}
       n= 10,q= \{-1.67477 \times 10^{49}, 4.35347 \times 10^{49}, 0., 0.\}, \q\= 4.6645 \times 10^{49}
       n= 11,q= \{-1.61478 \times 10^{99}, -1.45821 \times 10^{99}, 0., 0.\}, \q\= 2.17575 \times 10^{99}
       n= 12,q= \{4.8114 \times 10^{197}, 4.70939 \times 10^{198}, 0., 0.\}, \q\= 4.7339 \times 10^{198}
       n= 13,q= \{-2.194686007948114 \times 10^{397}, 4.531754507338190 \times 10^{396}, 0., 0.\}
        \sqrt{q} = 2.240985199110210 \times 10^{397}
       n=14,q=\{4.611278684335427\times10^{794}, -1.989155641742184\times10^{794}, 0., 0.\}
        \sqrt{q} = 5.022014662631027 \times 10^{794}
       n= 15,q= \{1.730715093753151 \times 10^{1589}, -1.834510202118258 \times 10^{1589}, 0., 0.\}
       \sqrt{q} = 2.522063127168103 \times 10^{1589}
       n= 16,q= \{-3.700529459309967 \times 10^{3177}, -6.350028992900426 \times 10^{3178}, 0., 0.\}
        \sqrt{q} = 6.360802417420950 \times 10^{3178}
       n= 17,q= \{-4.018592902788379 \times 10^{6357}, 4.699693871140086 \times 10^{6356}, 0., 0.\}
        \sqrt{q} = 4.045980739346820 \times 10^{6357}
       n= 18,q= \{1.592821769351681 \times 10^{12715}, -3.777231287168318 \times 10^{12714}, 0., 0.\}
        \sqrt{q} = 1.636996014316544 \times 10^{12715}
       n= 19,q= \{2.394406426952987 \times 10^{25430}, -1.203291244415593 \times 10^{25430}, 0., 0.\}
        \sqrt{q} = 2.679755950888252 \times 10^{25430}
       n= 20,q= \{4.285272318546543 \times 10^{50860}, -5.762336578249909 \times 10^{50860}, 0., 0.\}
        \sqrt{q} = 7.181091956320998 \times 10^{50860}
ln[ \circ ] := P = \{-0.4, -1.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
          0,
          0,
          0}]
Out[\bullet]= {-0.4, -1.4, 0, 0}
```

```
n= 1, q= \{-0.4, -1.4, 0, 0\}, q= 1.45602
       n= 2,q= \{-2.2, -0.28, 0., 0.\}, q= 2.21775
       n= 3,q= \{4.3616, -0.168, 0., 0.\}, \q = 4.36483
       n= 4,q= \{18.5953, -2.8655, 0., 0.\}, q= 18.8148
       n=5,q={337.175, -107.97, 0., 0.}, q=354.04
       n=6,q=\{102029., -72810.9, 0., 0.\}, q=125345.
       n= 7,q= \{5.10855 \times 10^9, -1.48577 \times 10^{10}, 0., 0.\}, \q\= 1.57114 \times 10^{10}
       n=8,q=\{-1.94653\times10^{20}, -1.51802\times10^{20}, 0., 0.\}, q=2.46848\times10^{20}\}
       n= 9,q= \{1.48459 \times 10^{40}, 5.90977 \times 10^{40}, 0., 0.\}, \q\= 6.09339 \times 10^{40}
       n= 10,q= \{-3.27214 \times 10^{81}, 1.75472 \times 10^{81}, 0., 0.\}, \q\= 3.71294 \times 10^{81}
       n= 11,q= \{7.62784 \times 10^{162}, -1.14833 \times 10^{163}, 0., 0.\},\q\= 1.37859\times 10^{163}
       n= 12,q= \{-7.368332879060586 \times 10^{325}, -1.751863389089434 \times 10^{326}, 0., 0.\}
        \sqrt{q} = 1.900512727712806 \times 10^{326}
      n= 13,q= \{-2.526102039865464 \times 10^{652}, 2.581662521890037 \times 10^{652}, 0., 0.\}
        \q = 3.611948628198369 \times 10^{652}
       n= 14,q= \{-2.837898611191654 \times 10^{1303}, -1.304308592558128 \times 10^{1305}, 0., 0.\}
        \sqrt{q} = 1.304617289274408 \times 10^{1305}
       n= 15,q= \{-1.700415537768224 \times 10^{2610}, 7.402991086772104 \times 10^{2608}, 0., 0.\}
        ,\q\= 1.702026271473705\times10^{2610}
      n= 16,q= \{2.885932573380517 \times 10^{5220}, -2.517632213981392 \times 10^{5219}, 0., 0.\}
        \sqrt{q} = 2.896893428786683 \times 10^{5220}
       n= 17,q= \{8.265222098449927 \times 10^{10440}, -1.453143362824201 \times 10^{10440}, 0., 0.\}
        \sqrt{q} = 8.391991537747464 \times 10^{10440}
       n= 18,q= \{6.620227070378498 \times 10^{20881}, -2.402110526926086 \times 10^{20881}, 0., 0.\}
        ,\q\= 7.042552196962504 \times 10^{20881}
       n= 19,q= \{3.805727147980315 \times 10^{41763}, -3.180503427279446 \times 10^{41763}, 0., 0.\}
        \sqrt{q} = 4.959754144694139 \times 10^{41763}
       n= 20,q= \{4.367957073938079 \times 10^{83526}, -2.420825647488365 \times 10^{83527}, 0., 0.\}
        ,\q\= 2.459916117581069 \times 10^{83527}
ln[ \circ ] := P = \{-0.2, 1.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {-0.2, 1.4, 0, 0}
```

```
n= 1, q= \{-0.2, 1.4, 0, 0\}, q= 1.41421
     n= 2,q= \{-2.12, 0.84, 0., 0.\}, q= 2.28035
     n= 3,q= \{3.5888, -2.1616, 0., 0.\}, \q = 4.18951
     n=4,q=\{8.00697, -14.1151, 0., 0.\}, q=16.228
     n=5,q=\{-135.324, -224.638, 0., 0.\}, q=262.25
     n=6,q=\{-32149.9,60799.5,0.,0.\},\q=68776.5
     n= 8,q= \{-8.19199 \times 10^{18}, 2.08212 \times 10^{19}, 0., 0.\}, q= 2.23748 \times 10^{19}
     n= 9,q= \{-3.66414 \times 10^{38}, -3.41134 \times 10^{38}, 0., 0.\}, \q\= 5.00631 \times 10^{38}
     n= 10,q= \{1.78867 \times 10^{76}, 2.49993 \times 10^{77}, 0., 0.\}, \q\= 2.50632 \times 10^{77}
     n= 11,q= \{-6.21764 \times 10^{154}, 8.94307 \times 10^{153}, 0., 0.\},\q\= 6.28163 \times 10^{154}
     n= 12,q= \{3.785928632868169 \times 10^{309}, -1.112095870419438 \times 10^{309}, 0., 0.\}
      \sqrt{q} = 3.945885558170056 \times 10^{309}
     n= 13,q= \{1.309649838816708 \times 10^{619}, -8.420631196630800 \times 10^{618}, 0., 0.\}
       n= 14,q= \{1.006112402815910 \times 10^{1238}, -2.205615657880493 \times 10^{1238}, 0., 0.\}
       |q| = 2.424252997809347 \times 10^{1238}
     n= 15,q= \{-3.852478263187599 \times 10^{2476}, -4.438194538477073 \times 10^{2476}, 0., 0.\}
       \sqrt{q} = 5.877002597387605 \times 10^{2476}
     n = 16, q = \{-4.855981993034777 \times 10^{4952}, 3.419609597456168 \times 10^{4953}, 0., 0.\}
      \q = 3.453915952970065 \times 10^{4953}
     n= 17,q= \left\{-1.145792418784755 \times 10^{9907}, -3.321112525691211 \times 10^{9906}, 0., 0.\right\}
       n= 18,q= \{1.202542382861590 \times 10^{19814}, 7.610611107736162 \times 10^{19813}, 0., 0.\}
       ,\q\= 1.423138151027651 \times 10^{19814}
     n= 19,q= \{8.668941682464599 \times 10^{39627}, 1.830416483305985 \times 10^{39628}, 0., 0.\}
      \sqrt{q} = 2.025322196910401 \times 10^{39628}
     n= 20,q= \{-2.598919003418529 \times 10^{79256}, 3.173554749680305 \times 10^{79256}, 0., 0.\}
       ,\q\= 4.101930001297973 \times 10^{79256}
ln[ \circ ] := P = \{-0.2, 1.2, 0, 0\}
     quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
          n++]]
     iteration[
       {0,
        0,
        0,
        0}]
```

Out[\circ]= {-0.2, 1.2, 0, 0}

```
n= 1,q= \{-0.2, 1.2, 0, 0\}, q= 1.21655
       n= 2,q= \{-1.6, 0.72, 0., 0.\}, q= 1.75454
       n= 3,q= \{1.8416, -1.104, 0., 0.\}, q= 2.14716
       n=4,q=\{1.97267, -2.86625, 0., 0.\}, q=3.47949
       n=5,q=\{-4.52396, -10.1084, 0., 0.\}, q=11.0745
       n=6,q=\{-81.9129, 92.6597, 0., 0.\}, q=123.675
       n= 7, q= \{-1876.3, -15178.8, 0., 0.\}, q= 15294.4
       n=8,q=\{-2.26877\times10^8, 5.69602\times10^7, 0., 0.\}, q=2.33918\times10^8
       n=9,q=\{4.82287\times10^{16}, -2.58459\times10^{16}, 0., 0.\}, q=5.47176\times10^{16}\}
       n= 10,q= \{1.658 \times 10^{33}, -2.49303 \times 10^{33}, 0., 0.\}, \q\= 2.99402 \times 10<sup>33</sup>
       n= 11,q= \{-3.46623 \times 10^{66}, -8.26686 \times 10^{66}, 0., 0.\},\q\= 8.96413 \times 10^{66}
       n= 12,q= \{-5.63261 \times 10^{133}, 5.73097 \times 10^{133}, 0., 0.\},\q\= 8.03557×10<sup>133</sup>
       n= 13,q= \{-1.1177 \times 10^{266}, -6.45607 \times 10^{267}, 0., 0.\}, \q\= 6.45704 \times 10^{267}
       n= 14,q= \{-4.166834615242078 \times 10^{535}, 1.443195247254052 \times 10^{534}, 0., 0.\}
        \q = 4.169333140443032 \times 10^{535}
       n= 15,q= \{1.734168258556263 \times 10^{1071}, -1.202711182562207 \times 10^{1070}, 0., 0.\}
       ,\q\= 1.738333883599656 \times 10^{1071}
      n= 16,q= \{2.992874407097459 \times 10^{2142}, -4.171407114020091 \times 10^{2141}, 0., 0.\}
        \sqrt{q} = 3.021804690870662 \times 10^{2142}
      n = 17, q = \{8.783290843549990 \times 10^{4284}, -2.496899518627000 \times 10^{4284}, 0., 0.\}
        \sqrt{q} = 9.13130358976794 \times 10^{4284}
      n= 18,q= \{7.091169083626936 \times 10^{8569}, -4.386198935844181 \times 10^{8569}, 0., 0.\}
       n= 19,q= \{3.104593786778585 \times 10^{17139}, -6.220655657699125 \times 10^{17139}, 0., 0.\}
        \sqrt{q} = 6.952342007738710 \times 10^{17139}
       n= 20,q= \{-2.905805423075994 \times 10^{34279}, -3.862521780916351 \times 10^{34279}, 0., 0.\}
        \sqrt{q} = 4.833505939256832 \times 10^{34279}
ln[-]:= P = \{-0.2, 0.7, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module[\{q = t, n = 1\}, While[n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ] = \{-0.2, 0.7, 0, 0\}
```

```
n= 1, q= \{-0.2, 0.7, 0, 0\}, q= 0.728011
     n= 2, q= \{-0.65, 0.42, 0., 0.\}, q= 0.773886
     n= 3,q= \{0.0461, 0.154, 0., 0.\}, \q = 0.160752
     n=4,q=\{-0.221591, 0.714199, 0., 0.\}, q=0.747785
     n=5,q=\{-0.660977, 0.38348, 0., 0.\}, q=0.764165
     n=6,q=\{0.0898341, 0.193056, 0., 0.\}, q=0.212934
     n= 7, q= \{-0.229201, 0.734686, 0., 0.\}, q= 0.769608
     n= 8, q= \{-0.687231, 0.363219, 0., 0.\}, q= 0.777312
     n = 9, q = \{0.140358, 0.200769, 0., 0.\}, q = 0.244967
     n= 10, q= \{-0.220608, 0.756359, 0., 0.\}, q= 0.787875
     n= 11,q= {-0.723411, 0.366282, 0., 0.},q= 0.810856
     n= 12,q= \{0.189161, 0.170055, 0., 0.\}, q= 0.254363
     n= 13, q= \{-0.193137, 0.764336, 0., 0.\}, q= 0.788359
     n= 14, q= \{-0.746907, 0.404758, 0., 0.\}, \q = 0.849529
     n= 15,q= {0.194041, 0.0953671, 0., 0.},q= 0.21621
     n= 16, q= \{-0.171443, 0.73701, 0., 0.\}, q= 0.756688
     n= 17, q= \{-0.713792, 0.44729, 0., 0.\}, q= 0.842358
     n= 18, q= \{0.10943, 0.0614568, 0., 0.\}, q= 0.125507
     n= 19, q= \{-0.191802, 0.71345, 0., 0.\}, q= 0.738782
     n=20,q=\{-0.672224,0.426318,0.,0.\},\q\=0.79601
ln[-]:= P = \{-0.2, 0.5, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.2, 0.5, 0, 0\}
```

```
n= 1, q= \{-0.2, 0.5, 0, 0\}, q= 0.538516
     n= 2,q= \{-0.41, 0.3, 0., 0.\}, q= 0.508035
     n = \ 3, q = \ \{-0.1219,\ 0.254,\ 0.,\ 0.\} \ , \ \ | \ q = \ 0.281737
     n= 4, q= \{-0.249656, 0.438075, 0., 0.\}, q= 0.50422
     n=5,q=\{-0.329581, 0.281264, 0., 0.\}, q=0.433282
     n=6,q=\{-0.170485,0.314602,0.,0.\},\q=0.357826
     n= 7, q= \{-0.269909, 0.39273, 0., 0.\}, q= 0.476537
     n= 8, q= \{-0.281386, 0.287997, 0., 0.\}, q= 0.402642
     n = 9, q = \{-0.203764, 0.337923, 0., 0.\}, \\ | q = 0.394604
     n= 10, q= \{-0.272672, 0.362287, 0., 0.\}, q= 0.453433
     n= 11, q= \{-0.256902, 0.302429, 0., 0.\}, q= 0.396815
     n= 12, q= \{-0.225465, 0.344611, 0., 0.\}, q= 0.411815
     n= 13, q= \{-0.267922, 0.344605, 0., 0.\}, q= 0.436503
     n= 14,q= {-0.24697, 0.315345, 0., 0.},\q\= 0.400546
     n=15,q=\{-0.238449, 0.344238, 0., 0.\}, q=0.418757
     n= 16, q= \{-0.261642, 0.335834, 0., 0.\}, q= 0.425724
     n= 17, q= \{-0.244328, 0.324263, 0., 0.\}, q= 0.406008
     n= 18, q= \{-0.245451, 0.341547, 0., 0.\}, q= 0.420595
     n= 19, q= \{-0.256408, 0.332334, 0., 0.\}, q= 0.419751
     n=20,q=\{-0.244701,0.329574,0.,0.\},\q\=0.410484
ln[-]:= P = \{-0.2, 0.4, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{-0.2, 0.4, 0, 0\}
```

```
n= 1, q= \{-0.2, 0.4, 0, 0\}, q= 0.447214
     n= 2,q= \{-0.32, 0.24, 0., 0.\}, q= 0.4
     n= 3, q= \{-0.1552, 0.2464, 0., 0.\}, q= 0.291204
     n= 4, q= \{-0.236626, 0.323517, 0., 0.\}, q= 0.400818
     n=5,q=\{-0.248672, 0.246895, 0., 0.\}, q=0.350421
     n= 7, q= \{-0.237196, 0.289605, 0., 0.\}, q= 0.374343
     n= 8, q= \{-0.227609, 0.262614, 0., 0.\}, q= 0.347522
     n=9,q=\{-0.21716,0.280453,0.,0.\},\q\=0.354701
     n= 10, q= \{-0.231496, 0.278193, 0., 0.\}, \q = 0.361914
     n= 11, q= \{-0.223801, 0.271199, 0., 0.\}, q= 0.351619
     n= 12, q= \{-0.223462, 0.278611, 0., 0.\}, q= 0.357154
     n= 13, q= \{-0.227689, 0.275482, 0., 0.\}, q= 0.357397
     n= 14, q= \{-0.224048, 0.274552, 0., 0.\}, q= 0.354367
     n=15,q=\{-0.225181, 0.276974, 0., 0.\}, q=0.356961
     n= 16, q= \{-0.226008, 0.275261, 0., 0.\}, q= 0.356158
     n= 17, q= \{-0.224689, 0.275577, 0., 0.\}, \q = 0.355567
     n= 18, q= \{-0.225458, 0.276162, 0., 0.\}, q= 0.356506
     n= 19, q= \{-0.225434, 0.275474, 0., 0.\}, q= 0.355959
     n=20,q=\{-0.225066,0.275797,0.,0.\},\q\=0.355976
ln[-]:= P = \{-0.2, 0.2, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.2, 0.2, 0, 0\}
```

```
n= 1, q= \{-0.2, 0.2, 0, 0\}, q= 0.282843
     n= 2,q= \{-0.2, 0.12, 0., 0.\}, q= 0.233238
     n= 3,q= \{-0.1744, 0.152, 0., 0.\}, \q = 0.231343
     n=4,q=\{-0.192689, 0.146982, 0., 0.\}, q=0.242348
     n=5,q=\{-0.184475, 0.143356, 0., 0.\}, q=0.233628
     n = 6, q = \{-0.18652, 0.147109, 0., 0.\}, \\ | q = 0.237551
     n= 7, q= \{-0.186851, 0.145123, 0., 0.\}, q= 0.236588
     n= 8, q= \{-0.186147, 0.145767, 0., 0.\}, q= 0.236429
     n= 10,q= {-0.186419, 0.145614, 0., 0.},q= 0.236549
     n= 11,q= {-0.186451, 0.14571, 0., 0.},\q\= 0.236633
     n= 12,q= \{-0.186467, 0.145665, 0., 0.\}, q= 0.236618
     n= 13, q= \{-0.186448, 0.145677, 0., 0.\}, q= 0.236611
     n= 14, q= \{-0.186459, 0.145678, 0., 0.\}, q= 0.23662
     n= 15,q= {-0.186455, 0.145674, 0., 0.},\q\= 0.236615
     n= 16, q= \{-0.186455, 0.145677, 0., 0.\}, q= 0.236616
     n= 17, q= \{-0.186456, 0.145676, 0., 0.\}, q= 0.236616
     n= 18, q= \{-0.186456, 0.145676, 0., 0.\}, q= 0.236616
     n= 19, q= \{-0.186456, 0.145676, 0., 0.\}, q= 0.236616
     n=20,q=\{-0.186456, 0.145676, 0., 0.\}, q=0.236616
ln[-]:= P = \{-0.2, -0.2, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.2, -0.2, 0, 0\}
```

```
n= 1, q= \{-0.2, -0.2, 0, 0\}, q= 0.282843
     n= 2, q= \{-0.2, -0.12, 0., 0.\}, q= 0.233238
     n= 3,q= \{-0.1744, -0.152, 0., 0.\}, q= 0.231343
     n=4,q=\{-0.192689, -0.146982, 0., 0.\}, q=0.242348
     n=5,q=\{-0.184475, -0.143356, 0., 0.\}, q=0.233628
     n=6,q=\{-0.18652, -0.147109, 0., 0.\}, \q = 0.237551
     n=7,q=\{-0.186851, -0.145123, 0., 0.\}, q=0.236588
     n=8,q=\{-0.186147, -0.145767, 0., 0.\}, q=0.236429
     n = 9, q = \{-0.186597, -0.145732, 0., 0.\}, \\ \  \  \, | q = 0.236762
     n=10,q=\{-0.186419, -0.145614, 0., 0.\}, \q = 0.236549
     n= 11,q= \{-0.186451, -0.14571, 0., 0.\}, q = 0.236633
     n= 13, q= \{-0.186448, -0.145677, 0., 0.\}, q= 0.236611
     n= 14, q= \{-0.186459, -0.145678, 0., 0.\}, q= 0.23662
     n = 15, q = \{-0.186455, -0.145674, 0., 0.\}, \\ | q = 0.236615|
     n= 16, q= \{-0.186455, -0.145677, 0., 0.\}, q= 0.236616
     n= 17, q= \{-0.186456, -0.145676, 0., 0.\}, q= 0.236616
     n= 18, q= \{-0.186456, -0.145676, 0., 0.\}, q= 0.236616
     n= 19, q= \{-0.186456, -0.145676, 0., 0.\}, q= 0.236616
     n=20,q=\{-0.186456, -0.145676, 0., 0.\}, q=0.236616
ln[-]:= P = \{-0.2, -0.4, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.2, -0.4, 0, 0\}
```

```
n= 1, q= \{-0.2, -0.4, 0, 0\}, q= 0.447214
     n= 2,q= \{-0.32, -0.24, 0., 0.\}, q= 0.4
     n= 3, q= \{-0.1552, -0.2464, 0., 0.\}, q= 0.291204
     n=4,q=\{-0.236626,-0.323517,0.,0.\},\q=0.400818
     n=5,q=\{-0.248672, -0.246895, 0., 0.\}, q=0.350421
     n = 6, q = \{-0.199119, -0.277209, 0., 0.\}, \\ | q = 0.341311
     n= 7, q= \{-0.237196, -0.289605, 0., 0.\}, q= 0.374343
     n= 8, q= \{-0.227609, -0.262614, 0., 0.\}, q= 0.347522
     n = 9, q = \{-0.21716, -0.280453, 0., 0.\}, \\ | q = 0.354701
     n=10,q=\{-0.231496, -0.278193, 0., 0.\}, q=0.361914
     n= 11, q= \{-0.223801, -0.271199, 0., 0.\}, q= 0.351619\}
     n= 13, q= \{-0.227689, -0.275482, 0., 0.\}, \q = 0.357397
     n= 14, q= \{-0.224048, -0.274552, 0., 0.\}, q= 0.354367
     n=15,q=\{-0.225181, -0.276974, 0., 0.\}, q=0.356961
     n= 16, q= \{-0.226008, -0.275261, 0., 0.\}, q= 0.356158
     n= 17,q= {-0.224689, -0.275577, 0., 0.},\q\= 0.355567
     n= 18, q= \{-0.225458, -0.276162, 0., 0.\}, \q = 0.356506
     n= 19, q= \{-0.225434, -0.275474, 0., 0.\}, \q = 0.355959
     n=20,q=\{-0.225066, -0.275797, 0., 0.\}, q=0.355976
ln[-]:= P = \{-0.2, -0.5, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.2, -0.5, 0, 0\}
```

```
n= 1, q= \{-0.2, -0.5, 0, 0\}, q= 0.538516
     n= 2, q= \{-0.41, -0.3, 0., 0.\}, q= 0.508035
     n= 3,q= \{-0.1219, -0.254, 0., 0.\}, q= 0.281737
     n=4,q=\{-0.249656, -0.438075, 0., 0.\}, q=0.50422
     n=5,q=\{-0.329581, -0.281264, 0., 0.\}, q=0.433282
     n= 7, q= \{-0.269909, -0.39273, 0., 0.\}, q= 0.476537
     n=8,q=\{-0.281386,-0.287997,0.,0.\}, q=0.402642
     n=9,q=\{-0.203764, -0.337923, 0., 0.\}, \q = 0.394604
     n=10,q=\{-0.272672, -0.362287, 0., 0.\}, q=0.453433
     n= 11,q= \{-0.256902, -0.302429, 0., 0.\}, \q = 0.396815
     n= 13, q= \{-0.267922, -0.344605, 0., 0.\}, \q = 0.436503
     n= 14, q= \{-0.24697, -0.315345, 0., 0.\}, q = 0.400546
     n= 15, q= \{-0.238449, -0.344238, 0., 0.\}, q= 0.418757
     n= 16, q= \{-0.261642, -0.335834, 0., 0.\}, q= 0.425724
     n=17,q=\{-0.244328, -0.324263, 0., 0.\}, q=0.406008
     n= 18, q= \{-0.245451, -0.341547, 0., 0.\}, \q = 0.420595
     n= 19, q= \{-0.256408, -0.332334, 0., 0.\}, q= 0.419751
     n=20,q=\{-0.244701, -0.329574, 0., 0.\}, q=0.410484
ln[-]:= P = \{-0.2, -0.7, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
        Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
        n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{-0.2, -0.7, 0, 0\}
```

```
n= 1, q= \{-0.2, -0.7, 0, 0\}, q= 0.728011
     n= 2,q= \{-0.65, -0.42, 0., 0.\}, \q = 0.773886
     n= 3, q= \{0.0461, -0.154, 0., 0.\}, q= 0.160752
     n=4,q=\{-0.221591, -0.714199, 0., 0.\}, q=0.747785
     n=5,q=\{-0.660977, -0.38348, 0., 0.\}, q=0.764165
     n = 6, q = \{0.0898341, -0.193056, 0., 0.\}, \\ | q = 0.212934
     n=7,q=\{-0.229201, -0.734686, 0., 0.\}, q=0.769608
     n=8,q=\{-0.687231, -0.363219, 0., 0.\}, q=0.777312
     n= 10,q= {-0.220608, -0.756359, 0., 0.},\q\= 0.787875
     n= 11,q= {-0.723411, -0.366282, 0., 0.},q= 0.810856
     n= 12,q= \{0.189161, -0.170055, 0., 0.\}, q= 0.254363
     n= 13, q= \{-0.193137, -0.764336, 0., 0.\}, \q = 0.788359
     n= 14, q= \{-0.746907, -0.404758, 0., 0.\}, \q = 0.849529
     n= 16, q= \{-0.171443, -0.73701, 0., 0.\}, q= 0.756688
     n= 17,q= {-0.713792, -0.44729, 0., 0.},\q\= 0.842358
     n= 18, q= \{0.10943, -0.0614568, 0., 0.\}, q= 0.125507
     n= 19, q= \{-0.191802, -0.71345, 0., 0.\}, q= 0.738782
     n=20,q=\{-0.672224,-0.426318,0.,0.\},\q=0.79601
ln[-]:= P = \{-0.2, -1.2, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
        Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
        n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet]= {-0.2, -1.2, 0, 0}
```

```
n= 1,q= \{-0.2, -1.2, 0, 0\}, q= 1.21655
       n= 2,q= \{-1.6, -0.72, 0., 0.\}, q= 1.75454
       n= 3,q= \{1.8416, 1.104, 0., 0.\}, q= 2.14716
       n= 4,q= \{1.97267, 2.86625, 0., 0.\}, q= 3.47949
       n=5,q=\{-4.52396, 10.1084, 0., 0.\}, q=11.0745
       n=6,q=\{-81.9129, -92.6597, 0., 0.\}, q=123.675
       n= 7, q= \{-1876.3, 15178.8, 0., 0.\}, q= 15294.4
       n= 8,q= \{-2.26877 \times 10^8, -5.69602 \times 10^7, 0., 0.\}, q= 2.33918 \times 10^8\}
       n= 9,q= \{4.82287 \times 10^{16}, 2.58459 \times 10^{16}, 0., 0.\}, \q\= 5.47176 \times 10^{16}
       n= 10,q= \{1.658 \times 10^{33}, 2.49303 \times 10^{33}, 0., 0.\},\q\= 2.99402 \times 10^{33}
       n= 11,q= \{-3.46623 \times 10^{66}, 8.26686 \times 10^{66}, 0., 0.\},\q\= 8.96413 \times 10^{66}
       n= 12,q= \{-5.63261 \times 10^{133}, -5.73097 \times 10^{133}, 0., 0.\}, \q\= 8.03557 \times 10^{133}
       n= 13,q= \{-1.1177 \times 10^{266}, 6.45607 \times 10^{267}, 0., 0.\},\q\= 6.45704 \times 10^{267}
       n= 14,q= \{-4.166834615242078 \times 10^{535}, -1.443195247254052 \times 10^{534}, 0., 0.\}
        |q| = 4.169333140443032 \times 10^{535}
       n= 15,q= \{1.734168258556263 \times 10^{1071}, 1.202711182562207 \times 10^{1070}, 0., 0.\}
        \q = 1.738333883599656 \times 10^{1071}
      n= 16,q= \{2.992874407097459 \times 10^{2142}, 4.171407114020091 \times 10^{2141}, 0., 0.\}
        \sqrt{q} = 3.021804690870662 \times 10^{2142}
      n = 17, q = \{8.783290843549990 \times 10^{4284}, 2.496899518627000 \times 10^{4284}, 0., 0.\}
        \sqrt{q} = 9.13130358976794 \times 10^{4284}
       n= 18,q= \{7.091169083626936 \times 10^{8569}, 4.386198935844181 \times 10^{8569}, 0., 0.\}
        \q = 8.338070524850885 \times 10^{8569}
       n= 19,q= \{3.104593786778585 \times 10^{17139}, 6.220655657699125 \times 10^{17139}, 0., 0.\}
        ,\q\= 6.952342007738710 \times 10^{17139}
       n= 20,q= \{-2.905805423075994 \times 10^{34279}, 3.862521780916351 \times 10^{34279}, 0., 0.\}
        \sqrt{q} = 4.833505939256832 \times 10^{34279}
ln[-]:= P = \{-0.2, -1.4, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]]; 
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ] = \{-0.2, -1.4, 0, 0\}
```

```
n= 1, q= \{-0.2, -1.4, 0, 0\}, q= 1.41421
      n= 2, q= \{-2.12, -0.84, 0., 0.\}, q= 2.28035
      n= 3,q= \{3.5888, 2.1616, 0., 0.\}, \q = 4.18951
      n= 4,q= \{8.00697, 14.1151, 0., 0.\}, q= 16.228
      n=5,q=\{-135.324, 224.638, 0., 0.\}, q=262.25
      n=6,q=\{-32149.9, -60799.5, 0., 0.\}, \q = 68776.5
      n= 7,q= \{-2.66297 \times 10^9, 3.9094 \times 10^9, 0., 0.\},\q\= 4.7302 \times 10^9
      n= 8,q= \{-8.19199 \times 10^{18}, -2.08212 \times 10^{19}, 0., 0.\},\q\= 2.23748×10<sup>19</sup>
      n=9,q=\{-3.66414\times10^{38}, 3.41134\times10^{38}, 0., 0.\}, q=5.00631\times10^{38}\}
      n= 10,q= \{1.78867 \times 10^{76}, -2.49993 \times 10^{77}, 0., 0.\}, \q\= 2.50632 \times 10<sup>77</sup>
      n= 11,q= \{-6.21764 \times 10^{154}, -8.94307 \times 10^{153}, 0., 0.\},\q\= 6.28163 \times 10^{154}
      n= 12,q= \{3.785928632868169 \times 10^{309}, 1.112095870419438 \times 10^{309}, 0., 0.\}
       \sqrt{q} = 3.945885558170056 \times 10^{309}
      n= 13,q= \{1.309649838816708 \times 10^{619}, 8.420631196630800 \times 10^{618}, 0., 0.\}
        n= 14,q= \{1.006112402815910 \times 10^{1238}, 2.205615657880493 \times 10^{1238}, 0., 0.\}
        \q = 2.424252997809347 \times 10^{1238}
      n= 15,q= \{-3.852478263187599 \times 10^{2476}, 4.438194538477073 \times 10^{2476}, 0., 0.\}
        \sqrt{q} = 5.877002597387605 \times 10^{2476}
      n= 16,q= \left\{-4.855981993034777\times10^{4952}, -3.419609597456168\times10^{4953}, 0., 0.\right\}
       \q = 3.453915952970065 \times 10^{4953}
      n= 17,q= \{-1.145792418784755 \times 10^{9907}, 3.321112525691211 \times 10^{9906}, 0., 0.\}
        n= 18,q= \{1.202542382861590 \times 10^{19814}, -7.610611107736162 \times 10^{19813}, 0., 0.\}
        ,\q\= 1.423138151027651 \times 10^{19814}
      n= 19,q= \{8.668941682464599 \times 10^{39627}, -1.830416483305985 \times 10^{39628}, 0., 0.\}
       n= 20,q= \{-2.598919003418529 \times 10^{79256}, -3.173554749680305 \times 10^{79256}, 0., 0.\}
        \sqrt{q} = 4.101930001297973 \times 10^{79256}
ln[ \circ ] := P = \{0.2, 1.4, 0, 0\}
      quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {0.2, 1.4, 0, 0}
```

```
n= 1, q= \{0.2, 1.4, 0, 0\}, q= 1.41421
       n= 2, q= \{-1.72, 1.96, 0., 0.\}, q= 2.60768
       n= 3,q= \{-0.6832, -5.3424, 0., 0.\}, \q = 5.38591
       n=4,q=\{-27.8745, 8.69986, 0., 0.\}, q=29.2006
       n=5,q=\{701.499, -483.608, 0., 0.\}, q=852.043
       n=6,q=\{258224., -678499., 0., 0.\}, q=725976.
       n= 7,q= \{-3.93681 \times 10^{11}, -3.5041 \times 10^{11}, 0., 0.\}, \q\= 5.27041 \times 10^{11}
       n= 8,q= \{3.21978\times10^{22}, 2.759\times10^{23}, 0., 0.\}, q= 2.77772\times10^{23}
       n= 9,q= \{-7.50841 \times 10^{46}, 1.77668 \times 10^{46}, 0., 0.\}, \q\= 7.71575 \times 10^{46}
       n= 10,q= \{5.32196 \times 10^{93}, -2.668 \times 10^{93}, 0., 0.\}, \q\= 5.95328 \times 10^{93}
       n= 11,q= \{2.1205 \times 10^{187}, -2.8398 \times 10^{187}, 0., 0.\},\q\= 3.54415 \times 10^{187}
       n= 12,q= \{-3.567922784821928 \times 10^{374}, -1.204361909176134 \times 10^{375}, 0., 0.\}
       \sqrt{q} = 1.256100449111812 \times 10^{375}
       n= 13,q= \{-1.323186878289867 \times 10^{750}, 8.594140593842332 \times 10^{749}, 0., 0.\}
        n= 14,q= \{1.012230989411198 \times 10^{1500}, -2.274330812790092 \times 10^{1500}, 0., 0.\}
        \sqrt{q} = 2.489416040345771 \times 10^{1500}
       n= 15,q= \{-4.147969070082069 \times 10^{3000}, -4.604296257757779 \times 10^{3000}, 0., 0.\}
        \sqrt{q} = 6.197192221930815 \times 10^{3000}
      n= 16,q= \{-3.993896622844785 \times 10^{6000}, 3.819695693334777 \times 10^{6001}, 0., 0.\}
        \q = 3.840519143555980 \times 10^{6001}
       n= 17,q= \left\{-1.443056308734054 \times 10^{12003}, -3.051093945980907 \times 10^{12002}, 0., 0.\right\}
        \sqrt{q} = 1.474958729201996 \times 10^{12003}
       n= 18,q= \{1.989319767505139 \times 10^{24006}, 8.805800734576052 \times 10^{24005}, 0., 0.\}
        ,\q\= 2.175503252849166 \times 10^{24006}
       n= 19,q= \{3.181971871616098 \times 10^{48012}, 3.503510694000682 \times 10^{48012}, 0., 0.\}
       \sqrt{q} = 4.732814403157301 \times 10^{48012}
       n= 20,q= \left\{-2.149642191221089 \times 10^{96024}, 2.229614496043273 \times 10^{96025}, 0., 0.\right\}
        ,\q\= 2.239953217473320 \times 10^{96025}
ln[ \circ ] := P = \{0.2, 1.2, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.2, 1.2, 0, 0}
```

```
n= 1,q= \{0.2, 1.2, 0, 0\}, q= 1.21655
       n= 2,q= \{-1.2, 1.68, 0., 0.\}, q= 2.06456
       n= 3,q= \{-1.1824, -2.832, 0., 0.\}, q= 3.06892
       n=4,q=\{-6.42215, 7.89711, 0., 0.\}, q=10.1788
       n= 5, q= \{-20.9203, -100.233, 0., 0.\}, q= 102.393
       n=6,q=\{-9608.79, 4195.01, 0., 0.\}, q=10484.6
       n = 7, q = \{7.47306 \times 10^7, -8.0618 \times 10^7, 0., 0.\}, q = 1.09927 \times 10^8
       n= 8,q= \{-9.14596 \times 10^{14}, -1.20493 \times 10^{16}, 0., 0.\}, \q\= 1.20839 \times 10^{16}
       n= 9,q= \{-1.44348 \times 10^{32}, 2.20404 \times 10^{31}, 0., 0.\}, \q\= 1.46021 \times 10^{32}
       n= 10,q= \{2.03507 \times 10^{64}, -6.363 \times 10^{63}, 0., 0.\}, \q\= 2.13222 \times 10^{64}
       n= 11,q= \{3.73662 \times 10^{128}, -2.58983 \times 10^{128}, 0., 0.\}, \q\= 4.54638 \times 10^{128}
       n= 12,q= \{7.25513\times10^{256}, -1.93544\times10^{257}, 0., 0.\}, q= 2.06695\times10^{257}
       n= 13,q= \{-3.219556164276026 \times 10^{514}, -2.808371994655656 \times 10^{514}, 0., 0.\}
        \q = 4.272293898515638 \times 10^{514}
       n= 14,q= \{2.478588634561570 \times 10^{1028}, 1.808342273394755 \times 10^{1029}, 0., 0.\}
        \q = 1.825249515529395 \times 10^{1029}
       n= 15,q= \{-3.208667761552735 \times 10^{2058}, 8.964273212466944 \times 10^{2057}, 0., 0.\}
        \sqrt{q} = 3.331535793940290 \times 10^{2058}
       n= 16,q= \{9.49196686175031 \times 10^{4116}, -5.752674892518690 \times 10^{4116}, 0., 0.\}
        ,\q\= 1.109913074630536 \times 10^{4117}
       n= 17,q= \{5.700416648555115 \times 10^{8233}, -1.092083988924209 \times 10^{8234}, 0., 0.\}
        ,\q\= 1.231907033235810 \times 10^{8234}
       n= 18,q= \{-8.676999391933682 \times 10^{16467}, -1.245066750416808 \times 10^{16468}, 0., 0.\}
        ,\q\= 1.517594938535855 \times 10^{16468}
       n= 19,q= \{-7.972880285172952 \times 10^{32935}, 2.160688687256698 \times 10^{32936}, 0., 0.\}
        \sqrt{q} = 2.303094397469645 \times 10^{32936}
       n= 20,q= \{-4.032907402822076 \times 10^{65872}, -3.445382447405030 \times 10^{65872}, 0., 0.\}
        \sqrt{q} = 5.304243803656067 \times 10^{65872}
ln[ \circ ] := P = \{0.2, 0.7, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.2, 0.7, 0, 0}
```

```
n= 1,q= \{0.2, 0.7, 0, 0\}, q= 0.728011
      n= 2,q= \{-0.25, 0.98, 0., 0.\}, q= 1.01139
      n= 3,q= \{-0.6979, 0.21, 0., 0.\}, q= 0.72881
      n=4,q=\{0.642964,0.406882,0.,0.\},\q=0.760892
      n= 5, q= \{0.44785, 1.22322, 0., 0.\}, q= 1.30263
      n=6,q=\{-1.0957, 1.79564, 0., 0.\}, q=2.10354
      n= 7, q= \{-1.82376, -3.23497, 0., 0.\}, q= 3.71364
      n= 8, q= \{-6.9389, 12.4996, 0., 0.\}, q= 14.2965
      n= 9, q= \{-107.892, -172.767, 0., 0.\}, q= 203.689
      n= 10, q= \{-18207.6, 37281.3, 0., 0.\}, q= 41489.9
      n= 11,q= \{-1.05838 \times 10^9, -1.3576 \times 10^9, 0., 0.\},\q\= 1.72141\times 10^9
      n= 12,q= \{-7.22925 \times 10^{17}, 2.87371 \times 10^{18}, 0., 0.\}, \q\= 2.96325 \times 10^{18}
      n= 13,q= \{-7.7356 \times 10^{36}, -4.15495 \times 10^{36}, 0., 0.\}, \q\= 8.78084 \times 10<sup>36</sup>
      n= 14,q= \{4.25759 \times 10^{73}, 6.42821 \times 10^{73}, 0., 0.\}, \q\= 7.71032 \times 10<sup>73</sup>
      n= 15,q= \{-2.31949 \times 10^{147}, 5.47374 \times 10^{147}, 0., 0.\}, \q\= 5.9449 \times 10^{147}
      n= 16,q= \{-2.45818 \times 10^{295}, -2.53925 \times 10^{295}, 0., 0.\},\q\= 3.53418×10<sup>295</sup>
      n= 17,q= \{-4.051753532351323 \times 10^{589}, 1.248385573817264 \times 10^{591}, 0., 0.\}
       , q = 1.249042918231296 \times 10^{591}
      n= 18,q= \{-1.556824870246367 \times 10^{1182}, -1.011630131690107 \times 10^{1181}, 0., 0.\}
       \q = 1.560108211583752 \times 10^{1182}
      n= 19,q= \{2.413469721384185 \times 10^{2364}, 3.149861897011531 \times 10^{2363}, 0., 0.\}
        ,\q\= 2.433937631851052 \times 10^{2364}
      n= 20,q=\{5.725619796335804\times10^{4728}, 1.520419262995816\times10^{4728}, 0., 0.\}
       \sqrt{q} = 5.924052395740706 \times 10^{4728}
ln[-]:= P = \{0.2, 0.5, 0, 0\}
      quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out [0] = \{0.2, 0.5, 0, 0\}
```

```
n= 1,q= \{0.2, 0.5, 0, 0\}, q= 0.538516
     n= 2, q= \{-0.01, 0.7, 0., 0.\}, q = 0.700071
     n= 3,q= \{-0.2899, 0.486, 0., 0.\}, \q = 0.565896
     n=4,q=\{0.047846,0.218217,0.,0.\},\q=0.223401
     n=5,q=\{0.15467, 0.520882, 0., 0.\}, q=0.543361
     n=6,q=\{-0.0473947, 0.66113, 0., 0.\},\q=0.662827
     n=7,q=\{-0.234847, 0.437332, 0., 0.\}, q= 0.496399
     n=8,q=\{0.0638938,0.294588,0.,0.\},\q=0.301438
     n=9,q=\{0.1173,0.537645,0.,0.\},\q=0.550292
     n=10,q=\{-0.0753025, 0.626132, 0., 0.\},\q=0.630644
     n= 11,q= {-0.18637, 0.405701, 0., 0.},\q\= 0.446461
     n= 12,q= \{0.0701403, 0.348778, 0., 0.\}, q= 0.355761
     n= 13, q= \{0.0832732, 0.548927, 0., 0.\}, q= 0.555207
     n= 14, q= \{-0.0943863, 0.591422, 0., 0.\}, q= 0.598906
     n=15,q=\{-0.140871, 0.388356, 0., 0.\}, q=0.413116
     n= 16, q= \{0.0690244, 0.390584, 0., 0.\}, q= 0.396636
     n= 17, q= \{0.0522086, 0.55392, 0., 0.\}, q= 0.556375
     n= 18, q= \{-0.104101, 0.557839, 0., 0.\}, q= 0.567469
     n= 19, q= \{-0.100347, 0.383857, 0., 0.\}, \q = 0.396756
     n=20,q=\{0.0627236,0.422962,0.,0.\},\q=0.427588
ln[-]:= P = \{0.2, 0.4, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{0.2, 0.4, 0, 0\}
```

```
n= 1, q= \{0.2, 0.4, 0, 0\}, q= 0.447214
     n= 2,q= \{0.08, 0.56, 0., 0.\}, q= 0.565685
     n= 3,q= \{-0.1072, 0.4896, 0., 0.\}, q= 0.501199
     n=4,q=\{-0.0282163, 0.29503, 0., 0.\}, q=0.296376
     n=5,q=\{0.113754,0.383351,0.,0.\},\q=0.399872
     n=6,q=\{0.0659821,0.487215,0.,0.\}, q=0.491663
     n=7,q=\{-0.0330249, 0.464295, 0., 0.\},\q=0.465468
     n= 8, q= \{-0.0144792, 0.369333, 0., 0.\}, q= 0.369617
     n= 10,q= {0.0525126, 0.449677, 0., 0.},q= 0.452733
     n= 11,q= {0.000547999, 0.447227, 0., 0.},\q\= 0.447228
     n=12,q=\{-0.0000120764, 0.40049, 0., 0.\}, \q = 0.40049
     n= 13, q= \{0.0396076, 0.39999, 0., 0.\}, q= 0.401947
     n= 14, q= \{0.0415765, 0.431685, 0., 0.\}, q= 0.433683
     n= 15, q= \{0.0153764, 0.435896, 0., 0.\}, q= 0.436167
     n= 16, q= \{0.0102312, 0.413405, 0., 0.\}, q= 0.413532
     n= 17, q= \{0.029201, 0.408459, 0., 0.\}, q= 0.409502
     n= 18, q= \{0.0340138, 0.423855, 0., 0.\}, q= 0.425217
     n= 19, q= \{0.021504, 0.428834, 0., 0.\}, q= 0.429373
     n=20,q=\{0.016564,0.418443,0.,0.\},\q=0.418771
ln[-]:= P = \{0.2, 0.2, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{0.2, 0.2, 0, 0\}
```

```
n= 1, q= \{0.2, 0.2, 0, 0\}, q= 0.282843
     n= 2,q= \{0.2, 0.28, 0., 0.\}, q= 0.344093
     n= 3,q= \{0.1616, 0.312, 0., 0.\}, q= 0.351367
     n= 4, q= \{0.128771, 0.300838, 0., 0.\}, q= 0.327239
     n=5,q=\{0.126078,0.277478,0.,0.\}, q=0.304778
     n=6,q=\{0.138902, 0.269968, 0., 0.\}, q=0.303605
     n= 7, q= \{0.146411, 0.274998, 0., 0.\}, q= 0.311545
     n= 8, q= \{0.145812, 0.280525, 0., 0.\}, q= 0.316158
     n= 9,q= {0.142567, 0.281808, 0., 0.},\q\= 0.315818
     n= 10,q= {0.140909, 0.280353, 0., 0.},\q\= 0.313773
     n= 11,q= \{0.141258, 0.279009, 0., 0.\}, q= 0.312729
     n= 12, q= \{0.142108, 0.278824, 0., 0.\}, q= 0.31295
     n= 13, q= \{0.142452, 0.279246, 0., 0.\}, q= 0.313482
     n= 14, q= \{0.142314, 0.279558, 0., 0.\}, q= 0.313697
     n = 15, q = \{0.142101, 0.27957, 0., 0.\}, q = 0.313611
     n= 16, q= \{0.142033, 0.279454, 0., 0.\}, q= 0.313477
     n= 17, q= \{0.142079, 0.279383, 0., 0.\}, q= 0.313435
     n= 18, q= \{0.142131, 0.279389, 0., 0.\}, q= 0.313464
     n= 19, q= \{0.142143, 0.27942, 0., 0.\}, q= 0.313497
     n=20,q=\{0.142129,0.279435,0.,0.\},\q=0.313504
ln[-]:= P = \{0.2, -0.2, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{0.2, -0.2, 0, 0\}
```

```
n= 1, q= \{0.2, -0.2, 0, 0\}, q= 0.282843
     n= 2,q= \{0.2, -0.28, 0., 0.\}, q= 0.344093
     n= 3,q= \{0.1616, -0.312, 0., 0.\}, q= 0.351367
     n=4,q=\{0.128771, -0.300838, 0., 0.\}, q=0.327239
     n=5,q=\{0.126078,-0.277478,0.,0.\},\q=0.304778
     n=6,q=\{0.138902, -0.269968, 0., 0.\}, q=0.303605
     n= 7, q= \{0.146411, -0.274998, 0., 0.\}, \q = 0.311545
     n= 8, q= \{0.145812, -0.280525, 0., 0.\}, q= 0.316158
     n= 9, q= \{0.142567, -0.281808, 0., 0.\}, q= 0.315818
     n= 10, q= \{0.140909, -0.280353, 0., 0.\}, q= 0.313773
     n= 11,q= \{0.141258, -0.279009, 0., 0.\}, q= 0.312729
     n= 12,q= \{0.142108, -0.278824, 0., 0.\}, q= 0.31295
     n= 13, q= \{0.142452, -0.279246, 0., 0.\}, q= 0.313482
     n= 14, q= \{0.142314, -0.279558, 0., 0.\}, q= 0.313697
     n = 15, q = \{0.142101, -0.27957, 0., 0.\}, \\ q = 0.313611
     n= 16, q= \{0.142033, -0.279454, 0., 0.\}, q= 0.313477
     n= 17, q= \{0.142079, -0.279383, 0., 0.\}, q= 0.313435
     n= 18, q= \{0.142131, -0.279389, 0., 0.\}, q= 0.313464
     n= 19, q= \{0.142143, -0.27942, 0., 0.\}, q= 0.313497
     n=20,q=\{0.142129,-0.279435,0.,0.\},\q=0.313504
ln[-]:= P = \{0.2, -0.4, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\bullet] = \{0.2, -0.4, 0, 0\}
```

```
n= 1, q= \{0.2, -0.4, 0, 0\}, q= 0.447214
     n= 2,q= \{0.08, -0.56, 0., 0.\}, q= 0.565685
     n= 3,q= \{-0.1072, -0.4896, 0., 0.\}, \q = 0.501199
     n=4,q=\{-0.0282163, -0.29503, 0., 0.\},\q=0.296376
     n=5,q=\{0.113754,-0.383351,0.,0.\},\q=0.399872
     n=7,q=\{-0.0330249, -0.464295, 0., 0.\}, q=0.465468
     n= 8, q= \{-0.0144792, -0.369333, 0., 0.\}, q= 0.369617
     n=10,q=\{0.0525126,-0.449677,0.,0.\},\q\=0.452733
     n=11,q=\{0.000547999, -0.447227, 0., 0.\}, q=0.447228
     n= 12, q= \{-0.0000120764, -0.40049, 0., 0.\}, \q = 0.40049
     n= 13, q= \{0.0396076, -0.39999, 0., 0.\}, q= 0.401947
     n= 14, q= \{0.0415765, -0.431685, 0., 0.\}, q= 0.433683
     n= 15, q= \{0.0153764, -0.435896, 0., 0.\}, q = 0.436167
     n= 16, q= \{0.0102312, -0.413405, 0., 0.\}, q= 0.413532
     n= 17, q= \{0.029201, -0.408459, 0., 0.\}, q= 0.409502
     n= 18, q= \{0.0340138, -0.423855, 0., 0.\}, \q = 0.425217
     n= 19, q= \{0.021504, -0.428834, 0., 0.\}, q= 0.429373
     n=20,q=\{0.016564,-0.418443,0.,0.\},\q=0.418771
ln[-]:= P = \{0.2, -0.5, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
        Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
        n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{0.2, -0.5, 0, 0\}
```

```
n= 1, q= \{0.2, -0.5, 0, 0\}, q= 0.538516
     n= 2, q= \{-0.01, -0.7, 0., 0.\}, q = 0.700071
     n= 3,q= \{-0.2899, -0.486, 0., 0.\}, q= 0.565896
     n=4,q=\{0.047846, -0.218217, 0., 0.\}, q=0.223401
     n=5,q=\{0.15467, -0.520882, 0., 0.\}, q=0.543361
     n=6,q=\{-0.0473947, -0.66113, 0., 0.\},\q=0.662827
     n=7,q=\{-0.234847, -0.437332, 0., 0.\}, q=0.496399
     n=8,q=\{0.0638938,-0.294588,0.,0.\},\q=0.301438
     n=9,q=\{0.1173,-0.537645,0.,0.\},\q=0.550292
     n=10,q=\{-0.0753025, -0.626132, 0., 0.\},\q=0.630644
     n= 11,q= {-0.18637, -0.405701, 0., 0.},\q\= 0.446461
     n= 12, q= \{0.0701403, -0.348778, 0., 0.\}, q= 0.355761
     n= 13, q= \{0.0832732, -0.548927, 0., 0.\}, q = 0.555207
     n= 14, q= \{-0.0943863, -0.591422, 0., 0.\}, \q = 0.598906
     n=15,q=\{-0.140871, -0.388356, 0., 0.\}, q=0.413116
     n= 16, q= \{0.0690244, -0.390584, 0., 0.\}, q= 0.396636
     n= 17, q= \{0.0522086, -0.55392, 0., 0.\}, q= 0.556375
     n=18,q=\{-0.104101, -0.557839, 0., 0.\}, q=0.567469
     n= 19, q= \{-0.100347, -0.383857, 0., 0.\}, \q = 0.396756
     n=20,q=\{0.0627236,-0.422962,0.,0.\},\q=0.427588
ln[-]:= P = \{0.2, -0.7, 0, 0\}
     quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
     iteration[
       {0,
        0,
        0,
        0}1
Out[\circ]= {0.2, -0.7, 0, 0}
```

```
n= 1, q= \{0.2, -0.7, 0, 0\}, q= 0.728011
      n= 2, q= \{-0.25, -0.98, 0., 0.\}, q= 1.01139
      n= 3,q= \{-0.6979, -0.21, 0., 0.\}, q= 0.72881
      n=4,q=\{0.642964, -0.406882, 0., 0.\}, q=0.760892
      n= 5, q= \{0.44785, -1.22322, 0., 0.\}, q= 1.30263
      n=6,q=\{-1.0957, -1.79564, 0., 0.\}, q=2.10354
      n= 7, q= \{-1.82376, 3.23497, 0., 0.\}, q= 3.71364
      n=8,q=\{-6.9389, -12.4996, 0., 0.\}, q=14.2965
      n=9,q=\{-107.892, 172.767, 0., 0.\}, q=203.689
      n=10,q=\{-18207.6, -37281.3, 0., 0.\}, q=41489.9
      n= 11,q= \{-1.05838 \times 10^9, 1.3576 \times 10^9, 0., 0.\}, \q\= 1.72141 \times 10^9
      n= 12,q= \{-7.22925 \times 10^{17}, -2.87371 \times 10^{18}, 0., 0.\},\q\= 2.96325\times 10^{18}
      n= 13,q= \{-7.7356 \times 10^{36}, 4.15495 \times 10^{36}, 0., 0.\},\q\= 8.78084×10<sup>36</sup>
      n= 14,q= \{4.25759 \times 10^{73}, -6.42821 \times 10^{73}, 0., 0.\}, \q\= 7.71032×10<sup>73</sup>
      n= 15,q= \{-2.31949 \times 10^{147}, -5.47374 \times 10^{147}, 0., 0.\}, \q\= 5.9449 \times 10^{147}
      n= 16,q= \{-2.45818 \times 10^{295}, 2.53925 \times 10^{295}, 0., 0.\}, \q\= 3.53418 \times 10^{295}
      n= 17,q= \{-4.051753532351323 \times 10^{589}, -1.248385573817264 \times 10^{591}, 0., 0.\}
       ,\q\= 1.249042918231296×10<sup>591</sup>
      n= 18,q= \{-1.556824870246367 \times 10^{1182}, 1.011630131690107 \times 10^{1181}, 0., 0.\}
       ,\q\= 1.560108211583752 \times 10^{1182}
      n= 19,q= \{2.413469721384185 \times 10^{2364}, -3.149861897011531 \times 10^{2363}, 0., 0.\}
        ,\q\= 2.433937631851052 \times 10^{2364}
      n= 20,q= \{5.725619796335804 \times 10^{4728}, -1.520419262995816 \times 10^{4728}, 0., 0.\}
       \sqrt{q} = 5.924052395740706 \times 10^{4728}
ln[-]:= P = \{0.2, -1.2, 0, 0\}
      quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {0.2, -1.2, 0, 0}
```

```
n= 2,q= \{-1.2, -1.68, 0., 0.\}, q= 2.06456
       n= 3,q= \{-1.1824, 2.832, 0., 0.\}, q= 3.06892
       n=4,q=\{-6.42215, -7.89711, 0., 0.\}, q=10.1788
       n=5,q=\{-20.9203, 100.233, 0., 0.\}, q=102.393
       n=6,q=\{-9608.79, -4195.01, 0., 0.\},\q=10484.6
       n= 7,q= \{7.47306 \times 10^7, 8.0618 \times 10^7, 0., 0.\},\q\= 1.09927×10<sup>8</sup>
       n= 8,q= \{-9.14596 \times 10^{14}, 1.20493 \times 10^{16}, 0., 0.\}, q= 1.20839 \times 10^{16}
       n= 9,q= \{-1.44348 \times 10^{32}, -2.20404 \times 10^{31}, 0., 0.\},\q\= 1.46021×10<sup>32</sup>
       n= 10,q= \{2.03507 \times 10^{64}, 6.363 \times 10^{63}, 0., 0.\},\q\= 2.13222 \times 10^{64}
       n= 11,q= \{3.73662 \times 10^{128}, 2.58983 \times 10^{128}, 0., 0.\},\q\= 4.54638 \times 10^{128}
       n= 12,q= \{7.25513 \times 10^{256}, 1.93544 \times 10^{257}, 0., 0.\},\q\= 2.06695 \times 10^{257}
       n= 13,q= \{-3.219556164276026 \times 10^{514}, 2.808371994655656 \times 10^{514}, 0., 0.\}
        \sqrt{q} = 4.272293898515638 \times 10^{514}
       n= 14,q= \{2.478588634561570 \times 10^{1028}, -1.808342273394755 \times 10^{1029}, 0., 0.\}
        \q = 1.825249515529395 \times 10^{1029}
       n= 15,q= \{-3.208667761552735 \times 10^{2058}, -8.964273212466944 \times 10^{2057}, 0., 0.\}
       \sqrt{q} = 3.331535793940290 \times 10^{2058}
       n= 16,q= \{9.49196686175031 \times 10^{4116}, 5.752674892518690 \times 10^{4116}, 0., 0.\}
        ,\q\= 1.109913074630536 \times 10^{4117}
       n= 17,q= \{5.700416648555115 \times 10^{8233}, 1.092083988924209 \times 10^{8234}, 0., 0.\}
        ,\q\= 1.231907033235810 \times 10^{8234}
       n= 18,q= \{-8.676999391933682 \times 10^{16467}, 1.245066750416808 \times 10^{16468}, 0., 0.\}
        ,\q\= 1.517594938535855 \times 10^{16468}
       n= 19,q= \{-7.972880285172952 \times 10^{32935}, -2.160688687256698 \times 10^{32936}, 0., 0.\}
        \sqrt{q} = 2.303094397469645 \times 10^{32936}
       n= 20,q=\{-4.032907402822076\times10^{65872}, 3.445382447405030\times10^{65872}, 0., 0.\}
        \sqrt{q} = 5.304243803656067 \times 10^{65872}
ln[ \circ ] := P = \{0.2, -1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {0.2, -1.4, 0, 0}
```

 $n= 1,q= \{0.2, -1.2, 0, 0\}, q= 1.21655$

```
n= 1,q= \{0.2, -1.4, 0, 0\}, q= 1.41421
       n= 2, q= \{-1.72, -1.96, 0., 0.\}, q= 2.60768
       n= 3,q= \{-0.6832, 5.3424, 0., 0.\}, q= 5.38591
       n=4,q=\{-27.8745, -8.69986, 0., 0.\}, q=29.2006
       n= 5, q= \{701.499, 483.608, 0., 0.\}, q= 852.043
       n=6,q=\{258224.,678499.,0.,0.\},\q=725976.
       n= 7,q= \{-3.93681 \times 10^{11}, 3.5041 \times 10^{11}, 0., 0.\}, \q\= 5.27041 \times 10^{11}
       n= 8,q= \{3.21978 \times 10^{22}, -2.759 \times 10^{23}, 0., 0.\}, \q\= 2.77772 \times 10^{23}
       n = 9,q = \{-7.50841 \times 10^{46}, -1.77668 \times 10^{46}, 0., 0.\}, q = 7.71575 \times 10^{46}\}
       n= 10,q= \{5.32196 \times 10^{93}, 2.668 \times 10^{93}, 0., 0.\},\q\= 5.95328 \times 10^{93}
       n= 11,q= \{2.1205 \times 10^{187}, 2.8398 \times 10^{187}, 0., 0.\},\q\= 3.54415 \times 10^{187}
       n= 12,q= \{-3.567922784821928 \times 10^{374}, 1.204361909176134 \times 10^{375}, 0., 0.\}
        \sqrt{q} = 1.256100449111812 \times 10^{375}
      n= 13,q= \{-1.323186878289867 \times 10^{750}, -8.594140593842332 \times 10^{749}, 0., 0.\}
        n= 14,q= \{1.012230989411198 \times 10^{1500}, 2.274330812790092 \times 10^{1500}, 0., 0.\}
        \q = 2.489416040345771 \times 10^{1500}
       n= 15,q= \{-4.147969070082069 \times 10^{3000}, 4.604296257757779 \times 10^{3000}, 0., 0.\}
        \sqrt{q} = 6.197192221930815 \times 10^{3000}
      n= 16,q= \{-3.993896622844785 \times 10^{6000}, -3.819695693334777 \times 10^{6001}, 0., 0.\}
        \sqrt{q} = 3.840519143555980 \times 10^{6001}
       n= 17,q= \{-1.443056308734054 \times 10^{12003}, 3.051093945980907 \times 10^{12002}, 0., 0.\}
        \sqrt{q} = 1.474958729201996 \times 10^{12003}
       n= 18,q= \{1.989319767505139 \times 10^{24006}, -8.805800734576052 \times 10^{24005}, 0., 0.\}
        ,\q\= 2.175503252849166 \times 10^{24006}
       n= 19,q= \{3.181971871616098 \times 10^{48012}, -3.503510694000682 \times 10^{48012}, 0., 0.\}
        \sqrt{q} = 4.732814403157301 \times 10^{48012}
       n= 20,q= \{-2.149642191221089 \times 10^{96024}, -2.229614496043273 \times 10^{96025}, 0., 0.\}
        ,\q\= 2.239953217473320 \times 10^{96025}
ln[ \circ ] := P = \{0.4, 1.4, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.4, 1.4, 0, 0}
```

```
n= 1, q= \{0.4, 1.4, 0, 0\}, q= 1.45602
       n= 2,q= \{-1.4, 2.52, 0., 0.\}, q= 2.88278
       n= 3,q= \{-3.9904, -5.656, 0., 0.\}, \q = 6.92197
       n=4,q=\{-15.667, 46.5394, 0., 0.\},\q=49.1057
       n=5,q=\{-1920.06, -1456.87, 0., 0.\}, q=2410.21
       n=6,q=\{1.56416\times10^6, 5.59456\times10^6, 0., 0.\}, q=5.8091\times10^6
       n= 7,q= \{-2.88525 \times 10^{13}, 1.75016 \times 10^{13}, 0., 0.\}, \q\= 3.37457\times 10^{13}
       n= 8,q= \{5.26159 \times 10^{26}, -1.00993 \times 10^{27}, 0., 0.\}, q= 1.13877 \times 10^{27}\}
       n=9,q=\{-7.43108\times10^{53}, -1.06276\times10^{54}, 0., 0.\}, q=1.2968\times10^{54}\}
       n= 10,q= \{-5.77259 \times 10^{107}, 1.5795 \times 10^{108}, 0., 0.\},\q\= 1.68168 \times 10^{108}
       n= 11,q= \{-2.16159 \times 10^{216}, -1.82356 \times 10^{216}, 0., 0.\},\q\= 2.82804 \times 10^{216}
       n= 12,q= \{1.347102556624021\times10^{432}, 7.883568009646126\times10^{432}, 0., 0.\}
        \sqrt{q} = 7.997832822757598 \times 10^{432}
       n= 13,q= \{-6.033595926465281 \times 10^{865}, 2.123994924222728 \times 10^{865}, 0., 0.\}
        \sqrt{q} = 6.396532986077876 \times 10^{865}
       n= 14,q= \{3.189292536573453 \times 10^{1731}, -2.563065424524637 \times 10^{1731}, 0., 0.\}
        n= 15,q= \{3.602282513449470 \times 10^{3462}, -1.634873085837179 \times 10^{3463}, 0., 0.\}
        \sqrt{q} = 1.674089125423679 \times 10^{3463}
       n= 16,q= \left\{-2.543045613727740\times10^{6926}\text{, }-1.177854945764089\times10^{6926}\text{, 0., 0.}\right\}
        \sqrt{q} = 2.802574399861817 \times 10^{6926}
       n= 17,q= \{5.079738720238976 \times 10^{13852}, 5.990677706865782 \times 10^{13852}, 0., 0.\}
        \sqrt{q} = 7.854423266760824 \times 10^{13852}
       n= 18,q= \{-1.008447392164355 \times 10^{27705}, 6.086215501607711 \times 10^{27705}, 0., 0.\}
        ,\q\= 6.169196485343378 \times 10^{27705}
       n= 19,q= \{-3.602505298924691 \times 10^{55411}, -1.227525630149313 \times 10^{55411}, 0., 0.\}
        \sqrt{q} = 3.805898527477308 \times 10^{55411}
       n= 20,q= \{1.147122525610701 \times 10^{110823}, 8.844335174357543 \times 10^{110822}, 0., 0.\}
        ,\q\= 1.448486360145394 \times 10^{110823}
ln[ \circ ] := P = \{0.4, 1.2, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.4, 1.2, 0, 0}
```

```
n= 1, q= \{0.4, 1.2, 0, 0\}, q= 1.26491
       n= 2,q= \{-0.88, 2.16, 0., 0.\}, q= 2.33238
       n= 3,q= \{-3.4912, -2.6016, 0., 0.\}, \q = 4.35394
       n= 4,q= \{5.82015, 19.3654, 0., 0.\}, q= 20.2211
       n=5,q=\{-340.745, 226.619, 0., 0.\}, q=409.223
       n=6,q=\{64751.2, -154438., 0., 0.\}, q=167463.
       n= 7,q= \{-1.96583 \times 10^{10}, -2. \times 10^{10}, 0., 0.\}, \q\= 2.80437 \times 10^{10}
       n= 8,q= \{-1.35541\times10^{19}, 7.86332\times10^{20}, 0., 0.\}, q= 7.86449\times10^{20}
       n= 9,q= \{-6.18135 \times 10^{41}, -2.13161 \times 10^{40}, 0., 0.\}, \q\= 6.18502 \times 10^{41}
       n= 10,q= \{3.81636 \times 10^{83}, 2.63525 \times 10^{82}, 0., 0.\},\q\= 3.82545 \times 10^{83}
       n= 11,q= \{1.44952 \times 10^{167}, 2.01141 \times 10^{166}, 0., 0.\},\q\= 1.46341 \times 10^{167}
       n= 12,q= \{2.060643558121792 \times 10^{334}, 5.831156125852581 \times 10^{333}, 0., 0.\}
       \sqrt{q} = 2.141559172955424 \times 10^{334}
       n= 13,q= \{3.906228055988158 \times 10^{668}, 2.403186861428109 \times 10^{668}, 0., 0.\}
        \q = 4.586275691269520 \times 10^{668}
       n= 14,q= \{9.48331053444834 \times 10^{1336}, 1.877479188378521 \times 10^{1337}, 0., 0.\}
        n= 15,q= \{-2.625596315866682 \times 10^{2674}, 3.560943633071510 \times 10^{2674}, 0., 0.\}
        \sqrt{q} = 4.424259889722260 \times 10^{2674}
      n= 16,q= \{-5.786563544019832 \times 10^{5348}, -1.869920096800295 \times 10^{5349}, 0., 0.\}
       ,\q\= 1.957407557180522×10<sup>5349</sup>
       n= 17,q= \{-3.161757991927832 \times 10^{10698}, 2.164082292474925 \times 10^{10698}, 0., 0.\}
        \sqrt{q} = 3.831444344907420 \times 10^{10698}
       n= 18,q= \{5.313461430915991 \times 10^{21396}, -1.368460896684420 \times 10^{21397}, 0., 0.\}
        ,\q\= 1.467996576812304 \times 10^{21397}
       n= 19,q= \{-1.590356501976008 \times 10^{42794}, -1.454252838849876 \times 10^{42794}, 0., 0.\}
       \q = 2.155013949532644 \times 10^{42794}
       n= 20,q= \{4.143824840744420 \times 10^{85587}, 4.625560915563934 \times 10^{85588}, 0., 0.\}
        ,\q\= 4.644085122680286 \times 10^{85588}
ln[ \circ ] := P = \{0.4, 0.7, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\circ] = \{0.4, 0.7, 0, 0\}
```

```
n= 1,q= \{0.4, 0.7, 0, 0\}, q= 0.806226
       n= 2,q= \{0.07, 1.26, 0., 0.\}, q= 1.26194
       n= 3,q= \{-1.1827, 0.8764, 0., 0.\}, \q = 1.47202
       n= 4,q= \{1.0307, -1.37304, 0., 0.\}, q= 1.71685
       n=5,q=\{-0.422882,-2.13038,0.,0.\},\q=2.17195
       n=6,q=\{-3.95971, 2.5018, 0., 0.\}, q=4.68383
       n= 7, q= \{9.82026, -19.1128, 0., 0.\}, q= 21.4881
       n= 8, q= \{-268.462, -374.685, 0., 0.\}, q= 460.935
       n= 9, q= \{-68317., 201178., 0., 0.\}, q= 212462.
       n= 10,q= \{-3.58055 \times 10^{10}, -2.74878 \times 10^{10}, 0., 0.\},\q\= 4.51399\times 10^{10}
       n= 11,q= \{5.26453 \times 10^{20}, 1.96843 \times 10^{21}, 0., 0.\},\q\= 2.03761\times 10^{21}
       n= 12,q= \{-3.59755 \times 10^{42}, 2.07257 \times 10^{42}, 0., 0.\},\q\= 4.15186×10<sup>42</sup>
       n= 13,q= \{8.64685 \times 10^{84}, -1.49124 \times 10^{85}, 0., 0.\}, \q\= 1.72379 \times 10<sup>85</sup>
       n= 14,q= \{-1.4761 \times 10^{170}, -2.5789 \times 10^{170}, 0., 0.\},\q\= 2.97146\times 10^{170}
       n= 15,q= \left\{-4.471842015190418\times10^{340}, 7.613437991289128\times10^{340}, 0., 0.\right\}
        \q = 8.829598465164026 \times 10^{340}
       n= 16,q= \{-3.796706703838234 \times 10^{681}, -6.809218377898732 \times 10^{681}, 0., 0.\}
        ,\q\ =\ 7.796180905602693 \times 10^{681}
       n= 17,q= \{-3.195047312294365 \times 10^{1363}, 5.170521012653324 \times 10^{1363}, 0., 0.\}
        ,\q\ = 6.078043671288403 \times 10^{1363}
       n= 18,q=\{-1.652596021449011\times10^{2727}, -3.304011852927908\times10^{2727}, 0., 0.\}
        \sqrt{q} = 3.694261487008900 \times 10^{2727}
       n= 19,q= \{-8.185420714179011 \times 10^{5454}, 1.092039368593807 \times 10^{5455}, 0., 0.\}
        \q = 1.364756793439721 \times 10^{5455}
       n= 20,q= \{-5.225388598776531 \times 10^{10909}, -1.787760333677344 \times 10^{10910}, 0., 0.\}
        \sqrt{q} = 1.862561105239870 \times 10^{10910}
ln[\cdot]:= P = \{0.4, 0.5, 0, 0\}
       quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}1
Out[\bullet] = \{0.4, 0.5, 0, 0\}
```

```
n= 1, q= \{0.4, 0.5, 0, 0\}, q= 0.640312
      n= 2,q= \{0.31, 0.9, 0., 0.\}, q= 0.951893
      n= 3,q= \{-0.3139, 1.058, 0., 0.\}, \q = 1.10358
      n=4,q=\{-0.620831, -0.164212, 0., 0.\}, \q = 0.642181
      n= 5, q= \{0.758465, 0.703896, 0., 0.\}, q= 1.03477
      n=6,q=\{0.479799, 1.56776, 0., 0.\},\q=1.63954
      n= 7, q= \{-1.82767, 2.00442, 0., 0.\}, q= 2.71258
      n=8,q=\{-0.277336,-6.82684,0.,0.\},\q=6.83247
      n= 9, q= \{-46.1288, 4.28666, 0., 0.\}, q= 46.3276
      n= 10,q= \{2109.89, -394.977, 0., 0.\}, q= 2146.55
      n= 11,q= \{4.29564 \times 10^6, -1.66672 \times 10^6, 0., 0.\}, \q\= 4.60766 \times 10^6
      n= 12,q= \{1.56746 \times 10^{13}, -1.43193 \times 10^{13}, 0., 0.\},\q\= 2.12305×10<sup>13</sup>
      n= 13,q= \{4.06515 \times 10^{25}, -4.48897 \times 10^{26}, 0., 0.\}, \q\= 4.50734×10<sup>26</sup>
      n= 14,q= \{-1.99856 \times 10^{53}, -3.64967 \times 10^{52}, 0., 0.\},\q\= 2.03161\times 10^{53}
      n= 15,q= \{3.86105 \times 10^{106}, 1.45882 \times 10^{106}, 0., 0.\}, \q\= 4.12745 \times 10^{106}
      n= 16,q= \{1.27796 \times 10^{213}, 1.12651 \times 10^{213}, 0., 0.\},\q\= 1.70359×10<sup>213</sup>
      n= 17,q= \{3.641396820235645 \times 10^{425}, 2.879269401595104 \times 10^{426}, 0., 0.\}
       ,\q\= 2.902204333775631×10<sup>426</sup>
      n= 18,q= \{-8.157594578937608 \times 10^{852}, 2.096912488714040 \times 10^{852}, 0., 0.\}
        q = 8.422789994986053 \times 10^{852}
      n= 19,q= \left\{6.214930732898733\times10^{1705}\text{, }-3.421152390088045\times10^{1705}\text{, }0.\text{, }0.\right\}
        \sqrt{q} = 7.094339129963715 \times 10^{1705}
      n= 20,q=\{2.692108033852405\times10^{3411}, -4.252445026217629\times10^{3411}, 0., 0.\}
        \sqrt{q} = 5.032964769093433 \times 10^{3411}
Inf = P = \{0.4, 0.4, 0, 0\}
      quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out [0] = \{0.4, 0.4, 0, 0\}
```

```
n= 1,q= \{0.4, 0.4, 0, 0\}, q= 0.565685
      n= 2,q= \{0.4, 0.72, 0., 0.\}, q= 0.82365
      n= 3,q= \{0.0416, 0.976, 0., 0.\}, q= 0.976886
      n=4,q=\{-0.550845, 0.481203, 0., 0.\}, \q = 0.731428
      n=5,q=\{0.471874, -0.130137, 0., 0.\}, q=0.48949
      n=6,q=\{0.60573,0.277183,0.,0.\},\q=0.666137
      n= 7, q= \{0.690078, 0.735796, 0., 0.\}, q= 1.00876
      n= 8, q= \{0.334811, 1.41551, 0., 0.\}, q= 1.45457
      n= 9, q= \{-1.49158, 1.34786, 0., 0.\}, q= 2.01036
      n=10,q=\{0.808082,-3.62088,0.,0.\},\q=3.70995
      n= 11,q= {-12.0578, -5.45193, 0., 0.},q = 13.233
      n= 12, q= \{116.066, 131.876, 0., 0.\}, q= 175.678
      n= 13, q= \{-3919.62, 30613.1, 0., 0.\}, q= 30863.
      n = 14, q = \{-9.21795 \times 10^8, -2.39983 \times 10^8, 0., 0.\}, q = 9.52522 \times 10^8
      n= 15,q= \{7.92115 \times 10^{17}, 4.42431 \times 10^{17}, 0., 0.\}, q= 9.07299 \times 10^{17}
      n= 16,q= \{4.31701\times10^{35}, 7.00912\times10^{35}, 0., 0.\},\q\= 8.23191\times10^{35}
      n= 17,q= \{-3.04912\times10^{71}, 6.05169\times10^{71}, 0., 0.\},\q\= 6.77644\times10^{71}
      n=~18, q=~\left\{-2.73258\times10^{143}\text{, }-3.69047\times10^{143}\text{, }0.\text{, }0.\right\}\text{,} \\ q\,=~4.59201\times10^{143}
      n= 19,q= \{-6.15259 \times 10^{286}, 2.0169 \times 10^{287}, 0., 0.\},\q\= 2.10866 \times 10^{287}
      n= 20,q= \{-3.689346603113686 \times 10^{574}, -2.481832496743607 \times 10^{574}, 0., 0.\}
        \q = 4.446433503359642 \times 10^{574}
ln[-]:= P = \{0.4, 0.2, 0, 0\}
      quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \left\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\right\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n=",n,",","q=",q,",","\q\= ", N[Sqrt[Plus@@(q^2)]]]; 
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[\circ] = \{0.4, 0.2, 0, 0\}
```

```
n= 1, q= \{0.4, 0.2, 0, 0\}, q= 0.447214
     n= 2,q= \{0.52, 0.36, 0., 0.\}, q= 0.632456
     n= 3,q= \{0.5408, 0.5744, 0., 0.\}, q= 0.788923
     n=4,q=\{0.362529,0.821271,0.,0.\},\q=0.897727
     n=5,q=\{-0.143059, 0.79547, 0., 0.\}, q=0.808231
     n = 6, q = \{-0.212306, -0.0275976, 0., 0.\}, \\ | q = 0.214092
     n= 7, q= \{0.444312, 0.211718, 0., 0.\}, q= 0.492177
     n= 8, q= \{0.552589, 0.388138, 0., 0.\}, q= 0.675282
     n= 10,q= {0.312103, 0.897774, 0., 0.},\q\= 0.950477
     n= 11,q= {-0.308589, 0.760396, 0., 0.},q= 0.820628
     n= 12, q= \{-0.0829747, -0.2693, 0., 0.\}, q= 0.281793
     n= 13, q= \{0.334362, 0.24469, 0., 0.\}, q= 0.414332
     n= 14, q= \{0.451925, 0.36363, 0., 0.\}, q= 0.580054
     n= 15, q= \{0.472009, 0.528667, 0., 0.\}, q= 0.708718
     n= 16, q= \{0.343304, 0.699071, 0., 0.\}, q= 0.778818
     n= 17, q= \{0.029157, 0.679987, 0., 0.\}, q= 0.680612
     n= 18, q= \{-0.0615327, 0.239653, 0., 0.\}, q= 0.247426
     n= 19, q= \{0.346353, 0.170507, 0., 0.\}, q= 0.386048
     n=20,q=\{0.490888,0.318111,0.,0.\},\q=0.584949
ln[\circ]:= P = \{0.4, -0.2, 0, 0\}
     quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{0.4, -0.2, 0, 0\}
```

```
n= 1, q= \{0.4, -0.2, 0, 0\}, q= 0.447214
     n= 2,q= \{0.52, -0.36, 0., 0.\}, q= 0.632456
     n= 3,q= \{0.5408, -0.5744, 0., 0.\}, q= 0.788923
     n=4,q=\{0.362529, -0.821271, 0., 0.\}, q=0.897727
     n=5,q=\{-0.143059, -0.79547, 0., 0.\}, q=0.808231
     n=6,q=\{-0.212306,0.0275976,0.,0.\},\q=0.214092
     n= 7, q= \{0.444312, -0.211718, 0., 0.\}, q= 0.492177
     n= 8, q= \{0.552589, -0.388138, 0., 0.\}, q= 0.675282
     n= 9, q= \{0.554703, -0.628961, 0., 0.\}, q= 0.838623
     n= 10, q= \{0.312103, -0.897774, 0., 0.\}, q= 0.950477
     n= 11,q= {-0.308589, -0.760396, 0., 0.},\q\= 0.820628
     n= 12, q= \{-0.0829747, 0.2693, 0., 0.\}, q= 0.281793
     n= 13, q= \{0.334362, -0.24469, 0., 0.\}, q= 0.414332
     n= 14, q= \{0.451925, -0.36363, 0., 0.\}, q= 0.580054
     n= 16, q= \{0.343304, -0.699071, 0., 0.\}, q= 0.778818
     n= 17, q= \{0.029157, -0.679987, 0., 0.\}, q= 0.680612
     n= 18, q= \{-0.0615327, -0.239653, 0., 0.\}, \q = 0.247426
     n= 19, q= \{0.346353, -0.170507, 0., 0.\}, q= 0.386048
     n=20,q=\{0.490888,-0.318111,0.,0.\},\q=0.584949
ln[\circ]:= P = \{0.4, -0.4, 0, 0\}
     quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
     iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
         n++]]
     iteration[
      {0,
       0,
       0,
       0}1
Out[\bullet] = \{0.4, -0.4, 0, 0\}
```

```
n= 1, q= \{0.4, -0.4, 0, 0\}, q= 0.565685
      n= 2,q= \{0.4, -0.72, 0., 0.\}, q= 0.82365
      n= 3,q= \{0.0416, -0.976, 0., 0.\}, \q = 0.976886
      n=4,q=\{-0.550845, -0.481203, 0., 0.\}, \q = 0.731428
      n= 5, q= \{0.471874, 0.130137, 0., 0.\}, q= 0.48949
      n=6,q=\{0.60573, -0.277183, 0., 0.\}, q=0.666137
      n= 7, q= \{0.690078, -0.735796, 0., 0.\}, q= 1.00876
      n= 8, q= \{0.334811, -1.41551, 0., 0.\}, q= 1.45457
      n= 9, q= \{-1.49158, -1.34786, 0., 0.\}, q= 2.01036
      n= 10, q= \{0.808082, 3.62088, 0., 0.\}, q= 3.70995
      n= 11,q= \{-12.0578, 5.45193, 0., 0.\}, q= 13.233
      n= 12, q= \{116.066, -131.876, 0., 0.\}, q= 175.678
      n= 13, q= \{-3919.62, -30613.1, 0., 0.\}, \q = 30863.
      n = 14, q = \{-9.21795 \times 10^8, 2.39983 \times 10^8, 0., 0.\}, q = 9.52522 \times 10^8
      n= 15,q= \{7.92115 \times 10^{17}, -4.42431 \times 10^{17}, 0., 0.\},\q\= 9.07299 \times 10^{17}
      n= 16,q= \{4.31701 \times 10^{35}, -7.00912 \times 10^{35}, 0., 0.\}, \q\= 8.23191×10<sup>35</sup>
      n= 17,q= \{-3.04912 \times 10^{71}, -6.05169 \times 10^{71}, 0., 0.\}, \q\= 6.77644 \times 10^{71}
      n= 18,q= \{-2.73258 \times 10^{143}, 3.69047 \times 10^{143}, 0., 0.\}, \q\= 4.59201 \times 10<sup>143</sup>
      n= 19,q= \left\{-6.15259\times10^{286}, -2.0169\times10^{287}, 0., 0.\right\},\q\= 2.10866\times10^{287}
      n= 20,q= \{-3.689346603113686 \times 10^{574}, 2.481832496743607 \times 10^{574}, 0., 0.\}
        \q = 4.446433503359642 \times 10^{574}
ln[-]:= P = \{0.4, -1.2, 0, 0\}
      quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n=",n,",","q=",q,",","\q\= ", N[Sqrt[Plus@@(q^2)]]]; 
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[\circ]= {0.4, -1.2, 0, 0}
```

```
n= 1,q= \{0.4, -1.2, 0, 0\}, q= 1.26491
       n= 2, q= \{-0.88, -2.16, 0., 0.\}, q= 2.33238
       n= 3,q= \{-3.4912, 2.6016, 0., 0.\}, \q = 4.35394
       n = 4, q = \{5.82015, -19.3654, 0., 0.\}, \\ q = 20.2211
       n=5,q=\{-340.745, -226.619, 0., 0.\}, q=409.223
       n=6,q=\{64751.2, 154438., 0., 0.\},\q=167463.
       n= 7,q= \{-1.96583 \times 10^{10}, 2.\times 10^{10}, 0., 0.\},\q\= 2.80437 \times 10^{10}
       n= 8,q= \{-1.35541 \times 10^{19}, -7.86332 \times 10^{20}, 0., 0.\}, q= 7.86449 \times 10^{20}\}
       n= 9,q= \{-6.18135 \times 10^{41}, 2.13161 \times 10^{40}, 0., 0.\}, \q\= 6.18502 \times 10^{41}
       n= 10,q= \{3.81636 \times 10^{83}, -2.63525 \times 10^{82}, 0., 0.\}, \q\= 3.82545 \times 10^{83}
       n= 11,q= \{1.44952 \times 10^{167}, -2.01141 \times 10^{166}, 0., 0.\},\q\= 1.46341\times 10^{167}
       n= 12,q= \{2.060643558121792 \times 10^{334}, -5.831156125852581 \times 10^{333}, 0., 0.\}
        \sqrt{q} = 2.141559172955424 \times 10^{334}
       n= 13,q= \{3.906228055988158 \times 10^{668}, -2.403186861428109 \times 10^{668}, 0., 0.\}
        \q = 4.586275691269520 \times 10^{668}
       n= 14,q= \{9.48331053444834 \times 10^{1336}, -1.877479188378521 \times 10^{1337}, 0., 0.\}
        n= 15,q= \{-2.625596315866682 \times 10^{2674}, -3.560943633071510 \times 10^{2674}, 0., 0.\}
        \sqrt{q} = 4.424259889722260 \times 10^{2674}
       n= 16,q= \{-5.786563544019832 \times 10^{5348}, 1.869920096800295 \times 10^{5349}, 0., 0.\}
        \q = 1.957407557180522 \times 10^{5349}
       n= 17,q= \left\{-3.161757991927832\times10^{10698}, -2.164082292474925\times10^{10698}, 0., 0.\right\}
        \sqrt{q} = 3.831444344907420 \times 10^{10698}
       n= 18,q= \{5.313461430915991 \times 10^{21396}, 1.368460896684420 \times 10^{21397}, 0., 0.\}
        ,\q\= 1.467996576812304 \times 10^{21397}
       n= 19,q= \{-1.590356501976008 \times 10^{42794}, 1.454252838849876 \times 10^{42794}, 0., 0.\}
        \q = 2.155013949532644 \times 10^{42794}
       n= 20,q= \{4.143824840744420 \times 10^{85587}, -4.625560915563934 \times 10^{85588}, 0., 0.\}
        \sqrt{q} = 4.644085122680286 \times 10^{85588}
ln[ \circ ] := P = \{0.4, -0.5, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.4, -0.5, 0, 0}
```

```
n= 1, q= \{0.4, -0.5, 0, 0\}, q= 0.640312
      n= 2,q= \{0.31, -0.9, 0., 0.\}, q= 0.951893
      n= 3,q= \{-0.3139, -1.058, 0., 0.\}, \q = 1.10358
      n=4,q=\{-0.620831, 0.164212, 0., 0.\}, q=0.642181
      n=5,q=\{0.758465, -0.703896, 0., 0.\}, q=1.03477
      n=6,q=\{0.479799, -1.56776, 0., 0.\},\q=1.63954
      n=7,q=\{-1.82767, -2.00442, 0., 0.\}, q=2.71258
      n= 8, q= \{-0.277336, 6.82684, 0., 0.\}, \q = 6.83247
      n= 9, q= \{-46.1288, -4.28666, 0., 0.\}, q= 46.3276
      n= 10,q= \{2109.89, 394.977, 0., 0.\}, q= 2146.55
      n= 11,q= \{4.29564 \times 10^6, 1.66672 \times 10^6, 0., 0.\}, \q\= 4.60766 \times 10^6
      n= 12,q= \{1.56746 \times 10^{13}, 1.43193 \times 10^{13}, 0., 0.\}, \q\= 2.12305 \times 10<sup>13</sup>
      n= 13,q= \{4.06515 \times 10^{25}, 4.48897 \times 10^{26}, 0., 0.\}, \q\= 4.50734 \times 10^{26}
      n= 14,q= \{-1.99856 \times 10^{53}, 3.64967 \times 10^{52}, 0., 0.\}, \q\= 2.03161 \times 10<sup>53</sup>
      n= 15,q= \{3.86105 \times 10^{106}, -1.45882 \times 10^{106}, 0., 0.\},\q\= 4.12745 \times 10^{106}
      n= 16,q= \{1.27796 \times 10^{213}, -1.12651 \times 10^{213}, 0., 0.\}, \q\= 1.70359 \times 10^{213}
      n= 17,q= \{3.641396820235645 \times 10^{425}, -2.879269401595104 \times 10^{426}, 0., 0.\}
       ,\q\= 2.902204333775631×10<sup>426</sup>
      n= 18,q= \{-8.157594578937608 \times 10^{852}, -2.096912488714040 \times 10^{852}, 0., 0.\}
       q = 8.422789994986053 \times 10^{852}
      n= 19,q= \{6.214930732898733 \times 10^{1705}, 3.421152390088045 \times 10^{1705}, 0., 0.\}
        \sqrt{q} = 7.094339129963715 \times 10^{1705}
      n= 20,q=\{2.692108033852405\times10^{3411}, 4.252445026217629\times10^{3411}, 0., 0.\}
       \sqrt{q} = 5.032964769093433 \times 10^{3411}
ln[-]:= P = \{0.4, -0.7, 0, 0\}
      quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out 0 = \{0.4, -0.7, 0, 0\}
```

```
n= 1, q= \{0.4, -0.7, 0, 0\}, q= 0.806226
       n= 2,q= \{0.07, -1.26, 0., 0.\}, q= 1.26194
       n= 3,q= \{-1.1827, -0.8764, 0., 0.\}, \q = 1.47202
       n= 4, q= \{1.0307, 1.37304, 0., 0.\}, q= 1.71685
       n=5,q=\{-0.422882, 2.13038, 0., 0.\}, q=2.17195
       n=6,q=\{-3.95971, -2.5018, 0., 0.\}, q=4.68383
       n= 7, q= \{9.82026, 19.1128, 0., 0.\}, q= 21.4881
       n= 8, q= \{-268.462, 374.685, 0., 0.\}, q= 460.935
       n= 9, q= \{-68317., -201178., 0., 0.\}, q= 212462.
       n= 10,q= \{-3.58055 \times 10^{10}, 2.74878 \times 10^{10}, 0., 0.\}, \q\= 4.51399 \times 10^{10}
       n= 11,q= \{5.26453 \times 10^{20}, -1.96843 \times 10^{21}, 0., 0.\}, \q\= 2.03761 \times 10^{21}
       n= 12,q= \{-3.59755 \times 10^{42}, -2.07257 \times 10^{42}, 0., 0.\},\q\= 4.15186×10<sup>42</sup>
       n= 13,q= \{8.64685 \times 10^{84}, 1.49124 \times 10^{85}, 0., 0.\}, q= 1.72379 \times 10^{85}
       n= 14,q= \{-1.4761 \times 10^{170}, 2.5789 \times 10^{170}, 0., 0.\}, \q\= 2.97146 \times 10^{170}
       n= 15,q= \{-4.471842015190418 \times 10^{340}, -7.613437991289128 \times 10^{340}, 0., 0.\}
        \q = 8.829598465164026 \times 10^{340}
       n= 16,q= \{-3.796706703838234 \times 10^{681}, 6.809218377898732 \times 10^{681}, 0., 0.\}
        ,\q\ =\ 7.796180905602693 \times 10^{681}
       n= 17,q= \{-3.195047312294365 \times 10^{1363}, -5.170521012653324 \times 10^{1363}, 0., 0.\}
        ,\q\ =\ 6.078043671288403\times 10^{1363}
       n= 18,q= \{-1.652596021449011 \times 10^{2727}, 3.304011852927908 \times 10^{2727}, 0., 0.\}
        \sqrt{q} = 3.694261487008900 \times 10^{2727}
       n= 19,q= \left\{-8.185420714179011\times10^{5454}, -1.092039368593807\times10^{5455}, 0., 0.\right\}
        \sqrt{q} = 1.364756793439721 \times 10^{5455}
       n= 20,q= \{-5.225388598776531 \times 10^{10909}, 1.787760333677344 \times 10^{10910}, 0., 0.\}
        ,\q\= 1.862561105239870 \times 10^{10910}
ln[\cdot]:= P = \{0.4, -1.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}1
Out[\bullet]= {0.4, -1.4, 0, 0}
```

```
n= 1,q= \{0.4, -1.4, 0, 0\}, q= 1.45602
       n= 2,q= \{-1.4, -2.52, 0., 0.\}, q= 2.88278
       n= 3,q= \{-3.9904, 5.656, 0., 0.\}, \q = 6.92197
       n=4,q=\{-15.667, -46.5394, 0., 0.\}, q=49.1057
       n=5,q=\{-1920.06, 1456.87, 0., 0.\}, q=2410.21
       n=6,q=\{1.56416\times10^6, -5.59456\times10^6, 0., 0.\}, q=5.8091\times10^6
       n= 7,q= \{-2.88525 \times 10^{13}, -1.75016 \times 10^{13}, 0., 0.\}, \q\= 3.37457 \times 10<sup>13</sup>
       n= 8,q= \{5.26159 \times 10^{26}, 1.00993 \times 10^{27}, 0., 0.\}, \q\= 1.13877 \times 10^{27}
       n=9,q=\{-7.43108\times10^{53}, 1.06276\times10^{54}, 0., 0.\}, q=1.2968\times10^{54}\}
       n= 10,q= \{-5.77259 \times 10^{107}, -1.5795 \times 10^{108}, 0., 0.\}, \q\= 1.68168 \times 10^{108}
       n= 11,q= \{-2.16159 \times 10^{216}, 1.82356 \times 10^{216}, 0., 0.\},\q\= 2.82804 \times 10^{216}
       n= 12,q= \{1.347102556624021 \times 10^{432}, -7.883568009646126 \times 10^{432}, 0., 0.\}
        \sqrt{q} = 7.997832822757598 \times 10^{432}
       n= 13,q= \{-6.033595926465281 \times 10^{865}, -2.123994924222728 \times 10^{865}, 0., 0.\}
        \sqrt{q} = 6.396532986077876 \times 10^{865}
       n= 14,q= \{3.189292536573453 \times 10^{1731}, 2.563065424524637 \times 10^{1731}, 0., 0.\}
        n= 15,q= \{3.602282513449470 \times 10^{3462}, 1.634873085837179 \times 10^{3463}, 0., 0.\}
        \sqrt{q} = 1.674089125423679 \times 10^{3463}
       n= 16,q= \left\{-2.543045613727740\times10^{6926}\text{, }1.177854945764089\times10^{6926}\text{, }0.\text{, }0.\right\}
        \q = 2.802574399861817 \times 10^{6926}
       n= 17,q= \{5.079738720238976 \times 10^{13852}, -5.990677706865782 \times 10^{13852}, 0., 0.\}
        \sqrt{q} = 7.854423266760824 \times 10^{13852}
       n= 18,q= \{-1.008447392164355 \times 10^{27705}, -6.086215501607711 \times 10^{27705}, 0., 0.\}
        ,\q\= 6.169196485343378 \times 10^{27705}
       n= 19,q= \{-3.602505298924691 \times 10^{55411}, 1.227525630149313 \times 10^{55411}, 0., 0.\}
        \sqrt{q} = 3.805898527477308 \times 10^{55411}
       n= 20,q= \{1.147122525610701 \times 10^{110823}, -8.844335174357543 \times 10^{110822}, 0., 0.\}
        ,\q\= 1.448486360145394 \times 10^{110823}
ln[ \circ ] := P = \{0.7, 1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.7, 1.4, 0, 0}
```

```
n= 1,q= \{0.7, 1.4, 0, 0\}, q= 1.56525
       n= 2, q= \{-0.77, 3.36, 0., 0.\}, q= 3.4471
       n= 3,q= \{-9.9967, -3.7744, 0., 0.\}, \q = 10.6855
       n= 4,q= \{86.3879, 76.8631, 0., 0.\}, q= 115.632
       n= 5,q= \{1555.64, 13281.5, 0., 0.\}, q= 13372.3
       n=6,q=\{-1.73978\times10^8, 4.13224\times10^7, 0., 0.\}, q=1.78818\times10^8
       n= 7,q= \{2.85607 \times 10^{16}, -1.43783 \times 10^{16}, 0., 0.\},\q\= 3.19758 \times 10^{16}
       n= 8,q= \{6.08979 \times 10^{32}, -8.21312 \times 10^{32}, 0., 0.\}, \q\= 1.02245 \times 10<sup>33</sup>
       n = 9,q = \{-3.03698 \times 10^{65}, -1.00032 \times 10^{66}, 0., 0.\}, q = 1.04541 \times 10^{66}\}
       n= 10,q= \{-9.08416 \times 10^{131}, 6.07594 \times 10^{131}, 0., 0.\}, \q\= 1.09288 \times 10^{132}
       n= 11,q= \{4.5605 \times 10^{263}, -1.1039 \times 10^{264}, 0., 0.\}, \q\= 1.19439 \times 10^{264}
       n= 12,q= \{-1.010604083742845 \times 10^{528}, -1.006862097503111 \times 10^{528}, 0., 0.\}
        \sqrt{q} = 1.426566471450272 \times 10^{528}
       n= 13,q= \{7.549330689352586 \times 10^{1053}, 2.035077895005061 \times 10^{1056}, 0., 0.\}
        \sqrt{q} = 2.035091897466079 \times 10^{1056}
       n= 14,q= \{-4.141485046344373 \times 10^{2112}, 3.072695201596953 \times 10^{2110}, 0., 0.\}
        \sqrt{q} = 4.141599031132087 \times 10^{2112}
       n= 15,q= \{1.715095424351386 \times 10^{4225}, -2.545104245877578 \times 10^{4223}, 0., 0.\}
        ,\q\ =\ 1.715284253467424 \times 10^{4225}
       n= 16,q= \{2.940904559068822 \times 10^{8450}, -8.730193293203838 \times 10^{8448}, 0., 0.\}
        \sqrt{q} = 2.942200070193299 \times 10^{8450}
       n= 17,q= \{8.641297998058115 \times 10^{16900}, -5.134933051507045 \times 10^{16899}, 0., 0.\}
        \sqrt{q} = 8.656541253045455 \times 10^{16900}
       n= 18,q= \{7.440835571680859 \times 10^{33801}, -8.874497339630054 \times 10^{33800}, 0., 0.\}
        ,\q\= 7.493570646567778 \times 10^{33801}
       n= 19,q= \{5.457846697448020 \times 10^{67603}, -1.320673509710129 \times 10^{67603}, 0., 0.\}
        \sqrt{q} = 5.615360103510222 \times 10^{67603}
       n= 20,q= \{2.804391205359419 \times 10^{135207}, -1.441606710675703 \times 10^{135207}, 0., 0.\}
        ,\q\= 3.153226909209433 \times 10^{135207}
ln[ \circ ] := P = \{0.7, 1.2, 0, 0\}
       quaternion[\{x\_,\,y\_,\,z\_,\,w\_\}] := \big\{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\big\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.7, 1.2, 0, 0}
```

```
n= 1, q= \{0.7, 1.2, 0, 0\}, q= 1.38924
       n= 2,q= \{-0.25, 2.88, 0., 0.\}, q= 2.89083
       n= 3,q= \{-7.5319, -0.24, 0., 0.\}, q= 7.53572
       n= 4,q= \{57.3719, 4.81531, 0., 0.\}, q= 57.5736
       n=5,q={3269.05,553.727,0.,0.},q=3315.61
       n=6,q=\{1.03801\times10^7, 3.62033\times10^6, 0., 0.\}, q=1.09933\times10^7
       n= 7,q= \{9.46391 \times 10^{13}, 7.51585 \times 10^{13}, 0., 0.\}, \q\= 1.20853×10<sup>14</sup>
       n= 8,q= \{3.30777 \times 10^{27}, 1.42259 \times 10^{28}, 0., 0.\}, q= 1.46054 \times 10^{28}
       n=9,q=\{-1.91434\times10^{56}, 9.41118\times10^{55}, 0., 0.\}, q=2.13317\times10^{56}\}
       n= 10,q= \{2.779 \times 10^{112}, -3.60324 \times 10^{112}, 0., 0.\},\q\= 4.5504 \times 10^{112}
       n= 11,q= \{-5.26053 \times 10^{224}, -2.00268 \times 10^{225}, 0., 0.\},\q\= 2.07062×10<sup>225</sup>
       n= 12,q= \{-3.733992082336978 \times 10^{450}, 2.107030575602885 \times 10^{450}, 0., 0.\}
        \sqrt{q} = 4.287455506180871 \times 10^{450}
       n= 13,q= \{9.50311902442982 \times 10^{900}, -1.573527097308619 \times 10^{901}, 0., 0.\}
        n= 14,q= \{-1.572894814039690 \times 10^{1802}, -2.990683058777874 \times 10^{1802}, 0., 0.\}
        \sqrt{q} = 3.379080237889289 \times 10^{1802}
       n= 15,q= \{-6.470187062028030 \times 10^{3604}, 9.40805974717615 \times 10^{3604}, 0., 0.\}
        \sqrt{q} = 1.141818325409393 \times 10^{3605}
       n= 16,q= \left\{-4.664826758880124 \times 10^{7209}, -1.217438129099317 \times 10^{7210}, 0., 0.\right\}
        \sqrt{q} = 1.303749088240711 \times 10^{7210}
       n= 17,q= \{-1.264549511281204 \times 10^{14420}, 1.135827592380689 \times 10^{14420}, 0., 0.\}
        \sqrt{q} = 1.699761685088485 \times 10^{14420}
       n= 18,q= \{3.089811468682173 \times 10^{28839}, -2.872620453689414 \times 10^{28840}, 0., 0.\}
        ,\q\= 2.889189786094845 \times 10^{28840}
       n= 19,q= \left\{-8.156478921834776\times10^{57680}, -1.775171124596108\times10^{57680}, 0., 0.\right\}
        \sqrt{q} = 8.347417620074774 \times 10^{57680}
       n= 20,q=\{6.337691588073519\times10^{115361}, 2.895829172083578\times10^{115361}, 0., 0.\}
        ,\q\= 6.967938092393481 \times 10^{115361}
ln[ \circ ] := P = \{0.7, 0.7, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ] = \{0.7, 0.7, 0, 0\}
```

```
n= 2,q= \{0.7, 1.68, 0., 0.\}, q= 1.82
               n= 3,q= \{-1.6324, 3.052, 0., 0.\}, \q = 3.46113
               n=4,q=\{-5.94997, -9.26417, 0., 0.\}, q=11.0103
               n=5,q=\{-49.7226, 110.943, 0., 0.\}, q=121.576
               n=6,q=\{-9835.34, -11032.1, 0., 0.\}, q=14779.7
               n = 7,q = \{-2.49727 \times 10^7, 2.17008 \times 10^8, 0., 0.\}, q = 2.18441 \times 10^8\}
               n= 8,q= \{-4.6469 \times 10^{16}, -1.08386 \times 10^{16}, 0., 0.\}, \q\= 4.77163 \times 10^{16}
               n= 9,q= \{2.04189 \times 10^{33}, 1.00732 \times 10^{33}, 0., 0.\}, \q\= 2.27684 \times 10^{33}
               n= 10,q= \{3.15464 \times 10^{66}, 4.11366 \times 10^{66}, 0., 0.\},\q\= 5.18401 \times 10^{66}
               n= 11,q= \{-6.97047 \times 10^{132}, 2.59542 \times 10^{133}, 0., 0.\}, \q\= 2.68739 \times 10^{133}
                \text{n= 12,q= } \left\{ -6.25034 \times 10^{266} \text{, } -3.61826 \times 10^{266} \text{, 0., 0.} \right\} \text{,} \\ \text{q} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{,} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0.} \\ \text{= } 7.22209 \times 10^{266} \text{, o., 0
               n= 13,q= \{2.597491455811477 \times 10^{533}, 4.523075060020172 \times 10^{533}, 0., 0.\}
                  \sqrt{q} = 5.215857538467678 \times 10^{533}
               n = 14, q = \{-1.371124613556286 \times 10^{1067}, 2.349729764479276 \times 10^{1067}, 0., 0.\}
                  \q = 2.720516986159010 \times 10^{1067}
               n= 15,q= \{-3.641247260179959 \times 10^{2134}, -6.443544630566697 \times 10^{2134}, 0., 0.\}
                 \q = 7.401212671979705 \times 10^{2134}
               n= 16,q= \{-2.826058579633686 \times 10^{4269}, 4.692507846379654 \times 10^{4269}, 0., 0.\}
                  ,\q = 5.477794901587297 \times 10^{4269}
               n= 17,q= \{-1.403302279281345 \times 10^{8539}, -2.652260411851922 \times 10^{8539}, 0., 0.\}
                  \sqrt{q} = 3.000623698385578 \times 10^{8539}
               n= 18,q= \{-5.065228005240709 \times 10^{17078}, 7.443846162398963 \times 10^{17078}, 0., 0.\}
                  \sqrt{q} = 9.003742579313146 \times 10^{17078}
               n= 19,q= \{-2.975431094438700 \times 10^{34157}, -7.540955609697362 \times 10^{34157}, 0., 0.\}
                  , q = 8.106738043453655 \times 10^{34157}
               n= 20,q= \{-4.801282130967343 \times 10^{68315}, 4.487518760575095 \times 10^{68315}, 0., 0.\}
                  ,\q\= 6.571920170517879 \times 10^{68315}
 ln[ \circ ] := P = \{0.7, 0.5, 0, 0\}
               quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
               iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
                         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
                         n++]]
               iteration[
                   {0,
                     0,
                     0,
                     0}]
Out[*]= {0.7, 0.5, 0, 0}
```

 $n= 1, q= \{0.7, 0.7, 0, 0\}, q= 0.989949$

```
n= 1,q= \{0.7, 0.5, 0, 0\}, q= 0.860233
       n= 2,q= \{0.94, 1.2, 0., 0.\}, q= 1.52434
       n= 3,q= \{0.1436, 2.756, 0., 0.\}, q= 2.75974
       n= 4, q= \{-6.87492, 1.29152, 0., 0.\}, q= 6.99518
       n=5,q=\{46.2964, -17.2582, 0., 0.\}, q=49.4086
       n= 6,q= \{1846.21, -1597.49, 0., 0.\}, \q = 2441.41
       n = 7,q = \{856533., -5.89861 \times 10^6, 0., 0.\}, q = 5.96047 \times 10^6\}
       n= 8,q= \{-3.40599 \times 10^{13}, -1.01047 \times 10^{13}, 0., 0.\},\q\= 3.55272×10<sup>13</sup>
       n= 9,q= \{1.05797 \times 10^{27}, 6.8833 \times 10^{26}, 0., 0.\}, \q\= 1.26218 \times 10^{27}
       n= 10,q= \{6.45505 \times 10^{53}, 1.45647 \times 10^{54}, 0., 0.\}, \q\= 1.5931\times10<sup>54</sup>
       n= 11,q= \{-1.70462 \times 10^{108}, 1.88031 \times 10^{108}, 0., 0.\}, \q\= 2.53797 \times 10^{108}
       n= 12,q= \{-6.29849 \times 10^{215}, -6.41044 \times 10^{216}, 0., 0.\}, \q\= 6.44131 \times 10^{216}
       n= 13,q= \{-4.069705546952813 \times 10^{433}, 8.075214753160699 \times 10^{432}, 0., 0.\}
        \sqrt{q} = 4.149047381266589 \times 10^{433}
       n= 14,q= \{1.591041230580186 \times 10^{867}, -6.572749254754658 \times 10^{866}, 0., 0.\}
        \q = 1.721459417199514 \times 10^{867}
       n= 15,q= \{2.099401869747332 \times 10^{1734}, -2.091503012515970 \times 10^{1734}, 0., 0.\}
        \q = 2.963422525064890 \times 10^{1734}
       n= 16,q= \{3.310335933521645 \times 10^{3466}, -8.781810670116408 \times 10^{3468}, 0., 0.\}
        ,\q\ =\ 8.781873062061970\times 10^{3468}
       n= 17,q= \{-7.711910281337113 \times 10^{6937}, -5.814148684534029 \times 10^{6935}, 0., 0.\}
        \sqrt{q} = 7.712129447816968 \times 10^{6937}
       n= 18,q= \{5.947017975490048 \times 10^{13875}, 8.967638603496125 \times 10^{13873}, 0., 0.\}
        \sqrt{q} = 5.947694061988565 \times 10^{13875}
       n= 19,q= \{3.535898094658945 \times 10^{27751}, 1.066614159453798 \times 10^{27750}, 0., 0.\}
        \q = 3.537506465501404 \times 10^{27751}
       n= 20,q= \{1.249119867816129 \times 10^{55503}, 7.542877948297877 \times 10^{55501}, 0., 0.\}
        ,\q\= 1.251395199346423 \times 10^{55503}
ln[ \circ ] := P = \{0.7, 0.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[*]= {0.7, 0.4, 0, 0}
```

```
n= 1, q= \{0.7, 0.4, 0, 0\}, q= 0.806226
       n= 2,q= \{1.03, 0.96, 0., 0.\}, q= 1.40801
       n= 3,q= \{0.8393, 2.3776, 0., 0.\}, q= 2.52139
       n=4,q=\{-4.24856, 4.39104, 0., 0.\},\q=6.10995
       n=5,q=\{-0.530988, -36.9112, 0., 0.\}, q=36.915
       n=6,q=\{-1361.45, 39.5988, 0., 0.\}, q=1362.03
       n = 7,q = \{1.85198 \times 10^6, -107823., 0., 0.\}, q = 1.85512 \times 10^6\}
       n= 8,q= \{3.41822 \times 10^{12}, -3.99374 \times 10^{11}, 0., 0.\}, \q\= 3.44147 \times 10^{12}
       n= 9,q= \{1.15247 \times 10^{25}, -2.7303 \times 10^{24}, 0., 0.\}, \q\= 1.18437 \times 10^{25}
       n= 10,q= \{1.25365 \times 10^{50}, -6.29319 \times 10^{49}, 0., 0.\}, \q\= 1.40274 \times 10^{50}
       n= 11,q= \{1.1756 \times 10^{100}, -1.57789 \times 10^{100}, 0., 0.\},\q\= 1.96768 \times 10^{100}
       n= 12,q= \{-1.10771 \times 10^{200}, -3.70992 \times 10^{200}, 0., 0.\}, \q\= 3.87176 \times 10^{200}
       n= 13,q= \{-1.253648225199897 \times 10^{401}, 8.219051945099823 \times 10^{400}, 0., 0.\}
        \sqrt{q} = 1.499053708613986 \times 10^{401}
       n= 14,q= \{8.961057237843609 \times 10^{801}, -2.060759976760032 \times 10^{802}, 0., 0.\}
        \sqrt{q} = 2.247162021309345 \times 10^{802}
       n= 15,q= \{-3.443726213616913 \times 10^{1604}, -3.693317621040782 \times 10^{1604}, 0., 0.\}
       \q = 5.049737150015099 \times 10^{1604}
       n= 16,q= \{-1.781344815538059 \times 10^{3208}, 2.543754941358279 \times 10^{3209}, 0., 0.\}
        \sqrt{q} = 2.549984528424262 \times 10^{3209}
       n= 17,q= \{-6.438957308166219 \times 10^{6418}, -9.06260935357578 \times 10^{6417}, 0., 0.\}
        ,\q\= 6.502421095203105 \times 10^{6418}
       n= 18,q= \{4.063886233343197 \times 10^{12837}, 1.167075094565246 \times 10^{12837}, 0., 0.\}
        \sqrt{q} = 4.228148009934235 \times 10^{12837}
       n= 19,q= \{1.515310704120188 \times 10^{25675}, 9.48572082016282 \times 10^{25674}, 0., 0.\}
        ,\q\= 1.787723559391083 \times 10^{25675}
      n= 20,q= \{1.396377535240515 \times 10^{51350}, 2.874762859017690 \times 10^{51350}, 0., 0.\}
        ,\q\ =\ 3.195955524801924\times 10^{51350}
ln[\circ]:= P = \{0.7, 0.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[*]= {0.7, 0.2, 0, 0}
```

```
n= 1,q= \{0.7, 0.2, 0, 0\}, q= 0.728011
       n= 2,q= \{1.15, 0.48, 0., 0.\}, q= 1.24615
       n= 3,q= \{1.7921, 1.304, 0., 0.\}, q= 2.21631
       n= 4, q= \{2.21121, 4.8738, 0., 0.\}, q= 5.35195
       n=5,q=\{-18.1645, 21.7539, 0., 0.\}, q=28.3405
       n=6,q=\{-142.586, -790.097, 0., 0.\}, q=802.86
       n=7,q=\{-603922., 225314., 0., 0.\},\q=644584.
       n= 8,q= \{3.13955 \times 10^{11}, -2.72145 \times 10^{11}, 0., 0.\}, q= 4.15488 \times 10^{11}
       n= 9,q= \{2.45053 \times 10^{22}, -1.70883 \times 10^{23}, 0., 0.\}, \q\= 1.72631 \times 10^{23}
       n= 10,q= \{-2.86003 \times 10^{46}, -8.37506 \times 10^{45}, 0., 0.\},\q\= 2.98013\times 10^{46}
       n= 11,q= \{7.47837 \times 10^{92}, 4.79059 \times 10^{92}, 0., 0.\},\q\= 8.8812 \times 10^{92}
       n= 12,q= \{3.29763 \times 10^{185}, 7.16516 \times 10^{185}, 0., 0.\}, \q\= 7.88758 \times 10^{185}
       n= 13,q= \{-4.046518720106294 \times 10^{371}, 4.725607152058041 \times 10^{371}, 0., 0.\}
        \q = 6.221388647862532 \times 10^{371}
      n= 14,q= \{-5.957049203411436 \times 10^{742}, -3.824451560934211 \times 10^{743}, 0., 0.\}
        |q| = 3.870567670775279 \times 10^{743}
       n= 15,q= \{-1.427156538981347 \times 10^{1487}, 4.556489224909752 \times 10^{1486}, 0., 0.\}
       \sqrt{q} = 1.498129409405077 \times 10^{1487}
       n= 16,q=\{1.829159846190031\times10^{2974}, -1.300564678425601\times10^{2974}, 0., 0.\}
        \sqrt{q} = 2.244391727324404 \times 10^{2974}
       n= 17,q= \{1.654357260145651 \times 10^{5948}, -4.757881374298317 \times 10^{5948}, 0., 0.\}
        ,\q\ =\ 5.037294225682223 \times 10^{5948}
       n= 18,q= \{-1.990053722769822 \times 10^{11897}, -1.574247118896438 \times 10^{11897}, 0., 0.\}
        , q = 2.537433311609147 \times 10^{11897}
      n= 19,q= \{1.482059828156293 \times 10^{23794}, 6.265672679039045 \times 10^{23794}, 0., 0.\}
        \q = 6.438567810863763 \times 10^{23794}
      n= 20,q= \{-3.706215278662166 \times 10^{47589}, 1.857220354796037 \times 10^{47589}, 0., 0.\}
        \sqrt{q} = 4.145515545509098 \times 10^{47589}
ln[\circ]:= P = \{0.7, -0.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
          0,
          0,
          0}]
Out[\circ]= {0.7, -0.2, 0, 0}
```

```
n= 1, q= \{0.7, -0.2, 0, 0\}, q= 0.728011
       n= 2,q= \{1.15, -0.48, 0., 0.\}, q= 1.24615
       n= 3,q= \{1.7921, -1.304, 0., 0.\}, q= 2.21631
       n= 4,q= \{2.21121, -4.8738, 0., 0.\}, \q = 5.35195
       n=5,q=\{-18.1645, -21.7539, 0., 0.\}, q=28.3405
       n=6,q=\{-142.586,790.097,0.,0.\},\q\=802.86
       n=7,q=\{-603922., -225314., 0., 0.\}, q=644584.
       n= 8,q= \{3.13955 \times 10^{11}, 2.72145 \times 10^{11}, 0., 0.\}, \q\= 4.15488 \times 10^{11}
       n= 9,q= \{2.45053 \times 10^{22}, 1.70883 \times 10^{23}, 0., 0.\}, q= 1.72631 \times 10^{23}
       n= 10,q= \{-2.86003 \times 10^{46}, 8.37506 \times 10^{45}, 0., 0.\}, \q\= 2.98013 \times 10<sup>46</sup>
       n= 11,q= \{7.47837 \times 10^{92}, -4.79059 \times 10^{92}, 0., 0.\},\q\= 8.8812 \times 10^{92}
       n= 12,q= \{3.29763\times10^{185}, -7.16516\times10^{185}, 0., 0.\},\q\= 7.88758\times10^{185}
       n= 13,q= \{-4.046518720106294 \times 10^{371}, -4.725607152058041 \times 10^{371}, 0., 0.\}
        ,\q\= 6.221388647862532 \times 10^{371}
       n = 14, q = \{-5.957049203411436 \times 10^{742}, 3.824451560934211 \times 10^{743}, 0., 0.\}
        |q| = 3.870567670775279 \times 10^{743}
       n= 15,q= \{-1.427156538981347 \times 10^{1487}, -4.556489224909752 \times 10^{1486}, 0., 0.\}
        \sqrt{q} = 1.498129409405077 \times 10^{1487}
       n= 16,q= \{1.829159846190031 \times 10^{2974}, 1.300564678425601 \times 10^{2974}, 0., 0.\}
        \sqrt{q} = 2.244391727324404 \times 10^{2974}
       n= 17,q= \{1.654357260145651 \times 10^{5948}, 4.757881374298317 \times 10^{5948}, 0., 0.\}
        ,\q\ =\ 5.037294225682223 \times 10^{5948}
       n= 18,q= \{-1.990053722769822 \times 10^{11897}, 1.574247118896438 \times 10^{11897}, 0., 0.\}
        , q = 2.537433311609147 \times 10^{11897}
       n= 19,q= \{1.482059828156293 \times 10^{23794}, -6.265672679039045 \times 10^{23794}, 0., 0.\}
        \sqrt{q} = 6.438567810863763 \times 10^{23794}
       n= 20,q= \left\{-3.706215278662166 \times 10^{47589}, -1.857220354796037 \times 10^{47589}, 0., 0.\right\}
        \sqrt{q} = 4.145515545509098 \times 10^{47589}
ln[\circ]:= P = \{0.7, -0.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.7, -0.4, 0, 0}
```

```
n= 1,q= \{0.7, -0.4, 0, 0\}, q= 0.806226
       n= 2,q= \{1.03, -0.96, 0., 0.\}, q= 1.40801
       n= 3,q= \{0.8393, -2.3776, 0., 0.\}, \q = 2.52139
       n=4,q=\{-4.24856, -4.39104, 0., 0.\}, q=6.10995
       n=5,q=\{-0.530988, 36.9112, 0., 0.\}, q=36.915
       n=6,q=\{-1361.45, -39.5988, 0., 0.\}, q=1362.03
       n = 7, q = \{1.85198 \times 10^6, 107823., 0., 0.\}, q = 1.85512 \times 10^6
       n= 8,q= \{3.41822 \times 10^{12}, 3.99374 \times 10^{11}, 0., 0.\}, \q\= 3.44147 \times 10^{12}
       n= 9,q= \{1.15247 \times 10^{25}, 2.7303 \times 10^{24}, 0., 0.\}, \q\= 1.18437 \times 10^{25}
       n= 10,q= \{1.25365 \times 10^{50}, 6.29319 \times 10^{49}, 0., 0.\}, \q\= 1.40274\times 10^{50}
       n= 11,q= \{1.1756 \times 10^{100}, 1.57789 \times 10^{100}, 0., 0.\},\q\= 1.96768 \times 10^{100}
       n= 12,q= \{-1.10771 \times 10^{200}, 3.70992 \times 10^{200}, 0., 0.\}, \q\= 3.87176 \times 10^{200}
       n= 13,q= \{-1.253648225199897 \times 10^{401}, -8.219051945099823 \times 10^{400}, 0., 0.\}
        ,\q\= 1.499053708613986 \times 10^{401}
       n = 14, q = \{8.961057237843609 \times 10^{801}, 2.060759976760032 \times 10^{802}, 0., 0.\}
        \sqrt{q} = 2.247162021309345 \times 10^{802}
       n= 15,q= \{-3.443726213616913 \times 10^{1604}, 3.693317621040782 \times 10^{1604}, 0., 0.\}
        \sqrt{q} = 5.049737150015099 \times 10^{1604}
       n= 16,q= \{-1.781344815538059 \times 10^{3208}, -2.543754941358279 \times 10^{3209}, 0., 0.\}
        \sqrt{q} = 2.549984528424262 \times 10^{3209}
       n= 17,q= \{-6.438957308166219 \times 10^{6418}, 9.06260935357578 \times 10^{6417}, 0., 0.\}
        \sqrt{q} = 6.502421095203105 \times 10^{6418}
       n= 18,q= \{4.063886233343197 \times 10^{12837}, -1.167075094565246 \times 10^{12837}, 0., 0.\}
        \sqrt{q} = 4.228148009934235 \times 10^{12837}
       n= 19,q= \{1.515310704120188 \times 10^{25675}, -9.48572082016282 \times 10^{25674}, 0., 0.\}
        \q = 1.787723559391083 \times 10^{25675}
       n= 20,q= \{1.396377535240515 \times 10^{51350}, -2.874762859017690 \times 10^{51350}, 0., 0.\}
        ,\q\ =\ 3.195955524801924\times 10^{51350}
ln[\circ]:= P = \{0.7, -0.5, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.7, -0.5, 0, 0}
```

```
n= 1, q= \{0.7, -0.5, 0, 0\}, q= 0.860233
       n= 2,q= \{0.94, -1.2, 0., 0.\}, q= 1.52434
       n= 3,q= \{0.1436, -2.756, 0., 0.\}, \q = 2.75974
       n= 4, q= \{-6.87492, -1.29152, 0., 0.\}, q= 6.99518
       n=5,q=\{46.2964, 17.2582, 0., 0.\}, q=49.4086
       n=6,q=\{1846.21, 1597.49, 0., 0.\}, q=2441.41
       n= 7, q= \{856533., 5.89861 \times 10^6, 0., 0.\}, q= 5.96047 \times 10^6
       n= 8,q= \{-3.40599 \times 10^{13}, 1.01047 \times 10^{13}, 0., 0.\}, \q\= 3.55272 \times 10<sup>13</sup>
       n= 9,q= \{1.05797 \times 10^{27}, -6.8833 \times 10^{26}, 0., 0.\}, q= 1.26218 \times 10^{27}\}
       n= 10,q= \{6.45505 \times 10^{53}, -1.45647 \times 10^{54}, 0., 0.\},\q\= 1.5931\times10<sup>54</sup>
       n= 11,q= \{-1.70462 \times 10^{108}, -1.88031 \times 10^{108}, 0., 0.\}, \q\= 2.53797 \times 10<sup>108</sup>
       n= 12,q= \{-6.29849 \times 10^{215}, 6.41044 \times 10^{216}, 0., 0.\},\q\= 6.44131 \times 10^{216}
       n= 13,q= \{-4.069705546952813 \times 10^{433}, -8.075214753160699 \times 10^{432}, 0., 0.\}
        \sqrt{q} = 4.149047381266589 \times 10^{433}
       n= 14,q= \{1.591041230580186 \times 10^{867}, 6.572749254754658 \times 10^{866}, 0., 0.\}
        n= 15,q= \{2.099401869747332 \times 10^{1734}, 2.091503012515970 \times 10^{1734}, 0., 0.\}
       \q = 2.963422525064890 \times 10^{1734}
       n= 16,q= \{3.310335933521645 \times 10^{3466}, 8.781810670116408 \times 10^{3468}, 0., 0.\}
        ,\q\ =\ 8.781873062061970\times 10^{3468}
       n= 17,q= \{-7.711910281337113 \times 10^{6937}, 5.814148684534029 \times 10^{6935}, 0., 0.\}
        \sqrt{q} = 7.712129447816968 \times 10^{6937}
       n= 18,q= \{5.947017975490048 \times 10^{13875}, -8.967638603496125 \times 10^{13873}, 0., 0.\}
        \sqrt{q} = 5.947694061988565 \times 10^{13875}
       n= 19,q= \{3.535898094658945 \times 10^{27751}, -1.066614159453798 \times 10^{27750}, 0., 0.\}
        \sqrt{q} = 3.537506465501404 \times 10^{27751}
       n= 20,q= \{1.249119867816129 \times 10^{55503}, -7.542877948297877 \times 10^{55501}, 0., 0.\}
        ,\q\= 1.251395199346423 \times 10^{55503}
ln[\circ]:= P = \{0.7, -0.7, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
        {0,
         0,
         0,
         0}]
Out[\bullet]= {0.7, -0.7, 0, 0}
```

```
n= 1,q= \{0.7, -0.7, 0, 0\}, q= 0.989949
       n= 2,q= \{0.7, -1.68, 0., 0.\}, q= 1.82
       n= 3,q= \{-1.6324, -3.052, 0., 0.\}, q= 3.46113
       n=4,q=\{-5.94997, 9.26417, 0., 0.\}, q=11.0103
       n= 5, q= \{-49.7226, -110.943, 0., 0.\}, q= 121.576
       n=6,q=\{-9835.34, 11032.1, 0., 0.\}, q=14779.7
       n = 7, q = \{-2.49727 \times 10^7, -2.17008 \times 10^8, 0., 0.\}, q = 2.18441 \times 10^8\}
       n= 8,q= \{-4.6469 \times 10^{16}, 1.08386 \times 10^{16}, 0., 0.\}, \q\= 4.77163 \times 10<sup>16</sup>
       n= 9,q= \{2.04189 \times 10^{33}, -1.00732 \times 10^{33}, 0., 0.\}, \q\= 2.27684 \times 10^{33}
       n= 10,q= \{3.15464 \times 10^{66}, -4.11366 \times 10^{66}, 0., 0.\}, \q\= 5.18401 \times 10^{66}
       n= 11,q= \{-6.97047 \times 10^{132}, -2.59542 \times 10^{133}, 0., 0.\}, \q\= 2.68739 \times 10^{133}
       n= 12,q= \{-6.25034 \times 10^{266}, 3.61826 \times 10^{266}, 0., 0.\},\q\= 7.22209 \times 10^{266}
       n= 13,q= \{2.597491455811477 \times 10^{533}, -4.523075060020172 \times 10^{533}, 0., 0.\}
        \q = 5.215857538467678 \times 10^{533}
       n= 14,q= \{-1.371124613556286 \times 10^{1067}, -2.349729764479276 \times 10^{1067}, 0., 0.\}
        \q = 2.720516986159010 \times 10^{1067}
       n= 15,q= \{-3.641247260179959 \times 10^{2134}, 6.443544630566697 \times 10^{2134}, 0., 0.\}
        \sqrt{q} = 7.401212671979705 \times 10^{2134}
       n= 16,q= \{-2.826058579633686 \times 10^{4269}, -4.692507846379654 \times 10^{4269}, 0., 0.\}
        ,\q = 5.477794901587297 \times 10^{4269}
       n= 17,q= \{-1.403302279281345 \times 10^{8539}, 2.652260411851922 \times 10^{8539}, 0., 0.\}
        \sqrt{q} = 3.000623698385578 \times 10^{8539}
       n= 18,q= \{-5.065228005240709 \times 10^{17078}, -7.443846162398963 \times 10^{17078}, 0., 0.\}
        \sqrt{q} = 9.003742579313146 \times 10^{17078}
       n= 19,q= \{-2.975431094438700 \times 10^{34157}, 7.540955609697362 \times 10^{34157}, 0., 0.\}
        \q = 8.106738043453655 \times 10^{34157}
       n= 20,q= \{-4.801282130967343 \times 10^{68315}, -4.487518760575095 \times 10^{68315}, 0., 0.\}
        ,\q\= 6.571920170517879 \times 10^{68315}
ln[\circ]:= P = \{0.7, -1.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.7, -1.2, 0, 0}
```

```
n= 1, q= \{0.7, -1.2, 0, 0\}, q= 1.38924
       n= 2, q= \{-0.25, -2.88, 0., 0.\}, q= 2.89083
       n= 3,q= \{-7.5319, 0.24, 0., 0.\}, \q = 7.53572
       n= 4,q= \{57.3719, -4.81531, 0., 0.\}, \q = 57.5736
       n=5,q={3269.05, -553.727, 0., 0.},\q=3315.61
       n = 6,q = \{1.03801 \times 10^7, -3.62033 \times 10^6, 0., 0.\}, q = 1.09933 \times 10^7
       n= 7,q= \{9.46391 \times 10^{13}, -7.51585 \times 10^{13}, 0., 0.\},\q\= 1.20853 \times 10^{14}
       n= 8,q= \{3.30777 \times 10^{27}, -1.42259 \times 10^{28}, 0., 0.\}, q= 1.46054 \times 10^{28}
       n= 9,q= \left\{-1.91434\times10^{56}, -9.41118\times10^{55}, 0., 0.\right\},\q\= 2.13317\times10^{56}
       n= 10,q= \{2.779 \times 10^{112}, 3.60324 \times 10^{112}, 0., 0.\}, \q\= 4.5504 \times 10^{112}
       n= 11,q= \{-5.26053 \times 10^{224}, 2.00268 \times 10^{225}, 0., 0.\}, \q\= 2.07062 \times 10^{225}
       n= 12,q= \{-3.733992082336978 \times 10^{450}, -2.107030575602885 \times 10^{450}, 0., 0.\}
        \sqrt{q} = 4.287455506180871 \times 10^{450}
       n= 13,q= \{9.50311902442982 \times 10^{900}, 1.573527097308619 \times 10^{901}, 0., 0.\}
        n= 14,q= \{-1.572894814039690 \times 10^{1802}, 2.990683058777874 \times 10^{1802}, 0., 0.\}
        \sqrt{q} = 3.379080237889289 \times 10^{1802}
       n= 15,q= \{-6.470187062028030 \times 10^{3604}, -9.40805974717615 \times 10^{3604}, 0., 0.\}
        ,\q\= 1.141818325409393 \times 10^{3605}
       n= 16,q= \{-4.664826758880124 \times 10^{7209}, 1.217438129099317 \times 10^{7210}, 0., 0.\}
        \q = 1.303749088240711 \times 10^{7210}
       n= 17,q= \{-1.264549511281204 \times 10^{14420}, -1.135827592380689 \times 10^{14420}, 0., 0.\}
        \sqrt{q} = 1.699761685088485 \times 10^{14420}
       n= 18,q= \{3.089811468682173\times10^{28839}, 2.872620453689414\times10^{28840}, 0., 0.\}
        ,\q\= 2.889189786094845 \times 10^{28840}
       n= 19,q= \left\{-8.156478921834776\times10^{57\,680}, 1.775171124596108\times10^{57\,680}, 0., 0.\right\}
        \q = 8.347417620074774 \times 10^{57680}
       n= 20,q= \{6.337691588073519 \times 10^{115361}, -2.895829172083578 \times 10^{115361}, 0., 0.\}
        ,\q\= 6.967938092393481 \times 10^{115361}
ln[ \circ ] := P = \{0.7, -1.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[\circ]= {0.7, -1.4, 0, 0}
```

```
n= 1,q= \{0.7, -1.4, 0, 0\}, q= 1.56525
      n= 2, q= \{-0.77, -3.36, 0., 0.\}, \q = 3.4471
      n= 3,q= \{-9.9967, 3.7744, 0., 0.\}, q= 10.6855
      n= 4,q= \{86.3879, -76.8631, 0., 0.\}, q= 115.632
      n=5,q=\{1555.64, -13281.5, 0., 0.\}, q=13372.3
      n = 6,q = \{-1.73978 \times 10^8, -4.13224 \times 10^7, 0., 0.\}, q = 1.78818 \times 10^8
      n= 7,q= \{2.85607 \times 10^{16}, 1.43783 \times 10^{16}, 0., 0.\}, \q\= 3.19758 \times 10^{16}
      n= 8,q= \{6.08979 \times 10^{32}, 8.21312 \times 10^{32}, 0., 0.\}, \q\= 1.02245 \times 10^{33}
      n=9,q=\{-3.03698\times10^{65}, 1.00032\times10^{66}, 0., 0.\}, q=1.04541\times10^{66}\}
      n= 10,q= \{-9.08416 \times 10^{131}, -6.07594 \times 10^{131}, 0., 0.\}, \q\= 1.09288 \times 10^{132}
      n= 11,q= \{4.5605 \times 10^{263}, 1.1039 \times 10^{264}, 0., 0.\},\q\= 1.19439 \times 10^{264}
      n= 12,q= \{-1.010604083742845 \times 10^{528}, 1.006862097503111 \times 10^{528}, 0., 0.\}
       \sqrt{q} = 1.426566471450272 \times 10^{528}
      n= 13,q= \{7.549330689352586 \times 10^{1053}, -2.035077895005061 \times 10^{1056}, 0., 0.\}
        \sqrt{q} = 2.035091897466079 \times 10^{1056}
      n= 14,q= \{-4.141485046344373 \times 10^{2112}, -3.072695201596953 \times 10^{2110}, 0., 0.\}
        \sqrt{q} = 4.141599031132087 \times 10^{2112}
      n= 15,q= \{1.715095424351386 \times 10^{4225}, 2.545104245877578 \times 10^{4223}, 0., 0.\}
        ,\q\= 1.715284253467424 \times 10^{4225}
      n= 16,q= \{2.940904559068822 \times 10^{8450}, 8.730193293203838 \times 10^{8448}, 0., 0.\}
       \sqrt{q} = 2.942200070193299 \times 10^{8450}
      n= 17,q= \{8.641297998058115 \times 10^{16900}, 5.134933051507045 \times 10^{16899}, 0., 0.\}
        \sqrt{q} = 8.656541253045455 \times 10^{16900}
      n= 18,q= \{7.440835571680859 \times 10^{33801}, 8.874497339630054 \times 10^{33800}, 0., 0.\}
        ,\q\= 7.493570646567778 \times 10^{33801}
      n= 19,q= \{5.457846697448020 \times 10^{67603}, 1.320673509710129 \times 10^{67603}, 0., 0.\}
       \q = 5.615360103510222 \times 10^{67603}
      n= 20,q= \{2.804391205359419 \times 10^{135207}, 1.441606710675703 \times 10^{135207}, 0., 0.\}
        ,\q\= 3.153226909209433 \times 10^{135207}
 ln[1]:= P = \{0.9, 1.4, 0, 0\}
      quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[1]= \{0.9, 1.4, 0, 0\}
```

```
n= 1,q= \{0.9, 1.4, 0, 0\}, q= 1.66433
      n= 2,q= \{-0.25, 3.92, 0., 0.\}, q= 3.92796
      n= 3,q= \{-14.4039, -0.56, 0., 0.\}, \q = 14.4148
      n= 4,q= \{208.059, 17.5324, 0., 0.\}, q= 208.796
      n=5,q=\{42982.,7296.92,0.,0.\},\q=43596.9
      n= 6,q= \{1.7942 \times 10^9, 6.27272 \times 10^8, 0., 0.\}, q= 1.90069 \times 10^9
      n= 7,q= \{2.82569 \times 10^{18}, 2.25091 \times 10^{18}, 0., 0.\}, \q\= 3.61264 \times 10^{18}
      n= 8,q= \{2.91797 \times 10^{36}, 1.27208 \times 10^{37}, 0., 0.\}, q= 1.30511 \times 10^{37}
      n= 9,q= \{-1.53303 \times 10^{74}, 7.42375 \times 10^{73}, 0., 0.\},\q\= 1.70332 \times 10^{74}
      n= 10,q= \{1.79906 \times 10^{148}, -2.27617 \times 10^{148}, 0., 0.\}, \q\= 2.9013 \times 10^{148}
      n= 11,q= \{-1.94431 \times 10^{296}, -8.18993 \times 10^{296}, 0., 0.\},\q\= 8.41756 \times 10^{296}
      n= 12,q= \{-6.329467547249228 \times 10^{593}, 3.184752059507657 \times 10^{593}, 0., 0.\}
       \sqrt{q} = 7.085534920683082 \times 10^{593}
      n= 13,q= \{2.991951375114289 \times 10^{1187}, -4.031556961337771 \times 10^{1187}, 0., 0.\}
        \q = 5.020480511221941 \times 10^{1187}
      n= 14,q= \{-7.301678501462757 \times 10^{2374}, -2.412444478865226 \times 10^{2375}, 0., 0.\}
        \sqrt{q} = 2.520522456355933 \times 10^{2375}
      n= 15,q= \{-5.286743274220077 \times 10^{4750}, 3.522978797460548 \times 10^{4750}, 0., 0.\}
        \sqrt{q} = 6.353033452994545 \times 10^{4750}
      n= 16,q= \{1.553827484015464 \times 10^{9501}, -3.725016892538898 \times 10^{9501}, 0., 0.\}
       \q = 4.036103405486779 \times 10^{9501}
      n= 17,q= \left\{-1.146137099961832\times10^{19003}, -1.157606725209764\times10^{19003}, 0., 0.\right\}
        \sqrt{q} = 1.629013069978197 \times 10^{19003}
      n= 18,q= \{-2.642307834195556 \times 10^{38004}, 2.653552029856463 \times 10^{38006}, 0., 0.\}
        ,\q\ =\ 2.653683582159791\times 10^{38\,006}
      n= 19,q= \{-7.040640196086290 \times 10^{76012}, -1.402300263387050 \times 10^{76011}, 0., 0.\}
       \sqrt{q} = 7.042036554224420 \times 10^{76012}
      n= 20,q= \{4.955094991045904 \times 10^{152025}, 1.974618320277052 \times 10^{152024}, 0., 0.\}
        ,\q\= 4.959027883103295 \times 10^{152025}
 ln[5]:= P = \{0.9, 1.2, 0, 0\}
      quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[5]= \{0.9, 1.2, 0, 0\}
```

```
n= 1,q= \{0.9, 1.2, 0, 0\}, q= 1.5
      n= 2,q= \{0.27, 3.36, 0., 0.\}, q= 3.37083
      n= 3,q= \{-10.3167, 3.0144, 0., 0.\}, \q = 10.7481
      n= 4,q= \{98.2477, -60.9973, 0., 0.\}, \q = 115.643
      n=5,q=\{5932.84, -11984.5, 0., 0.\}, q=13372.6
      n=6,q=\{-1.0843\times10^8, -1.42204\times10^8, 0., 0.\}, q=1.78827\times10^8
      n= 7,q= \{-8.46503\times10^{15}, 3.08382×10<sup>16</sup>, 0., 0.\},\q\= 3.19789×10<sup>16</sup>
      n= 8,q= \{-8.7934 \times 10^{32}, -5.22093 \times 10^{32}, 0., 0.\}, \q\= 1.02265 \times 10<sup>33</sup>
      n= 9,q= \{5.00657 \times 10^{65}, 9.18194 \times 10^{65}, 0., 0.\}, \q\= 1.04582×10<sup>66</sup>
      n= 10,q= \{-5.92424 \times 10^{131}, 9.194 \times 10^{131}, 0., 0.\}, \q\= 1.09374 \times 10^{132}
      n= 11,q= \{-4.94331 \times 10^{263}, -1.08935 \times 10^{264}, 0., 0.\},\q\= 1.19626 \times 10^{264}
      n= 12,q= \{-9.42318178220839 \times 10^{527}, 1.076999165134003 \times 10^{528}, 0., 0.\}
       \sqrt{q} = 1.431045334957904 \times 10^{528}
      n= 13,q= \{-2.719636526938977 \times 10^{1055}, -2.029751782468877 \times 10^{1056}, 0., 0.\}
        \sqrt{q} = 2.047890750704780 \times 10^{1056}
      n= 14,q= \{-4.045928070048975 \times 10^{2112}, 1.104037417644371 \times 10^{2112}, 0., 0.\}
        \sqrt{q} = 4.193856526822189 \times 10^{2112}
      n= 15,q= \{1.515063532845137 \times 10^{4225}, -8.933711956863486 \times 10^{4224}, 0., 0.\}
        ,\q\ =\ 1.758843256756907 \times 10^{4225}
      n= 16,q= \{1.497305415275132 \times 10^{8450}, -2.707028239757287 \times 10^{8450}, 0., 0.\}
       ,\q\= 3.093529601839244×10<sup>8450</sup>
      n= 17,q= \{-5.086078384231203 \times 10^{16\,900}, -8.106496085382589 \times 10^{16\,900}, 0., 0.\}
        \sqrt{q} = 9.56992539745567 \times 10^{16900}
      n= 18,q= \{-3.984708545177935 \times 10^{33801}, 8.246054902343849 \times 10^{33801}, 0., 0.\}
        ,\q\= 9.15834721128671 \times 10^{33801}
      n= 19,q= \{-5.211951926245497 \times 10^{67603}, -6.571625086675188 \times 10^{67603}, 0., 0.\}
       \q = 8.387532364248308 \times 10^{67603}
      n= 20,q= \{-1.602181339832452 \times 10^{135207}, 6.850198805811995 \times 10^{135207}, 0., 0.\}
        ,\q\= 7.035069916131281 \times 10^{135207}
 ln[9]:= P = \{0.9, 0.7, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
Out[9]= \{0.9, 0.7, 0, 0\}
```

```
n= 1, q= \{0.9, 0.7, 0, 0\}, q= 1.14018
       n= 2,q= \{1.22, 1.96, 0., 0.\}, q= 2.30868
       n= 3,q= \{-1.4532, 5.4824, 0., 0.\}, q= 5.67173
       n=4,q=\{-27.0449, -15.234, 0., 0.\}, q=31.0404
       n=5,q=\{500.251,824.707,0.,0.\},\q=964.569
       n=6,q=\{-429889.,825123.,0.,0.\},\q=930394.
       n= 7,q= \{-4.96022 \times 10^{11}, -7.09423 \times 10^{11}, 0., 0.\},\q\= 8.65632 \times 10^{11}
       n= 8,q= \{-2.57243 \times 10^{23}, 7.0378 \times 10^{23}, 0., 0.\}, q= 7.49319 \times 10^{23}
       n= 9,q= \{-4.29132 \times 10^{47}, -3.62085 \times 10^{47}, 0., 0.\},\q\= 5.6148 \times 10^{47}
       n= 10,q= \{5.30488 \times 10^{94}, 3.10764 \times 10^{95}, 0., 0.\}, q= 3.15259 \times 10^{95}
       n= 11,q= \{-9.376 \times 10^{190}, 3.29713 \times 10^{190}, 0., 0.\}, \q\= 9.93884 \times 10^{190}
       n= 12,q= \{7.703838218690188 \times 10^{381}, -6.182785256280149 \times 10^{381}, 0., 0.\}
        \sqrt{q} = 9.87805430360791 \times 10^{381}
       n= 13,q= \{2.112228977447643\times10^{763}, -9.52623547105704\times10^{763}, 0., 0.\}
        \sqrt{q} = 9.75759568250268 \times 10^{763}
       n= 14,q= \{-8.628765099685591 \times 10^{1527}, -4.024318121591257 \times 10^{1527}, 0., 0.\}
        \sqrt{q} = 9.52106735031949 \times 10^{1527}
       n= 15,q= \{5.826045080178430 \times 10^{3055}, 6.944979151523782 \times 10^{3055}, 0., 0.\}
        \sqrt{q} = 9.06507234893199 \times 10^{3055}
       n= 16,q= \{-1.428993413882870 \times 10^{6111}, 8.092352323535379 \times 10^{6111}, 0., 0.\}
        n= 17,q= \left\{-6.344414395130783\times10^{12223}, -2.312783634630360\times10^{12223}, 0., 0.\right\}
        \sqrt{q} = 6.752818830514907 \times 10^{12223}
       n= 18,q= \{3.490262587652868 \times 10^{24447}, 2.934651556874350 \times 10^{24447}, 0., 0.\}
        ,\q\= 4.560056215775671 \times 10^{24447}
       n= 19,q= \{3.569753170504246 \times 10^{48894}, 2.048540907351157 \times 10^{48895}, 0., 0.\}
        \sqrt{q} = 2.079411269103434 \times 10^{48895}
       n= 20,q=\{-4.069088472107850\times10^{97790}, 1.462557079784887\times10^{97790}, 0., 0.\}
        \sqrt{q} = 4.323951226074352 \times 10^{97790}
ln[13]:= P = \{0.9, 0.5, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[13]= \{0.9, 0.5, 0, 0\}
```

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168 | PROVAAA.nb
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```
n= 1,q= \{0.9, 0.5, 0, 0\}, q= 1.02956
       n= 2,q= \{1.46, 1.4, 0., 0.\}, q= 2.02277
       n= 3,q= \{1.0716, 4.588, 0., 0.\}, q= 4.71148
       n=4,q=\{-19.0014, 10.333, 0., 0.\},\q=21.6293
       n=5,q=\{255.183, -392.183, 0., 0.\}, q=467.895
       n=6,q=\{-88688.5, -200157., 0., 0.\}, q=218925.
       n= 7,q= \{-3.2197 \times 10^{10}, 3.55032 \times 10^{10}, 0., 0.\}, \q\= 4.79283 \times 10^{10}
       n= 8,q= \{-2.23831 \times 10^{20}, -2.28619 \times 10^{21}, 0., 0.\}, q= 2.29712 \times 10^{21}
       n= 9,q= \{-5.17656 \times 10^{42}, 1.02344 \times 10^{42}, 0., 0.\}, \q\= 5.27676 \times 10^{42}
       n= 10,q= \{2.57494 \times 10^{85}, -1.05958 \times 10^{85}, 0., 0.\}, \q\= 2.78442 \times 10^{85}
       n= 11,q= \{5.50759 \times 10^{170}, -5.4567 \times 10^{170}, 0., 0.\},\q\= 7.75301 \times 10^{170}
       n= 12,q= \{5.578808944410104 \times 10^{339}, -6.010653992644118 \times 10^{341}, 0., 0.\}
        \sqrt{q} = 6.010912886593225 \times 10^{341}
       n= 13,q= \{-3.612484910836485 \times 10^{683}, -6.706458051183462 \times 10^{681}, 0., 0.\}
        n= 14,q= \{1.304554957306206 \times 10^{1367}, 4.845395703011623 \times 10^{1365}, 0., 0.\}
        \q = 1.305454488898052 \times 10^{1367}
       n= 15,q= \{1.699515850680320 \times 10^{2734}, 1.264216996894800 \times 10^{2733}, 0., 0.\}
        \sqrt{q} = 1.704211422584073 \times 10^{2734}
       n= 16,q= \{2.872371680561276 \times 10^{5468}, 4.297113649844373 \times 10^{5467}, 0., 0.\}
        \sqrt{q} = 2.904336572866030 \times 10^{5468}
       n= 17,q= \{8.065867214093620 \times 10^{10936}, 2.468581511193256 \times 10^{10936}, 0., 0.\}
        \sqrt{q} = 8.435170928487197 \times 10^{10936}
       n= 18,q= \{5.896431923798520 \times 10^{21873}, 3.982250135290274 \times 10^{21873}, 0., 0.\}
        ,\q\= 7.115210859279556 \times 10^{21873}
       n= 19,q= \{1.890959329197092 \times 10^{43747}, 4.696213365255309 \times 10^{43747}, 0., 0.\}
        \sqrt{q} = 5.062622557200972 \times 10^{43747}
       n= 20,q= \{-1.847869278732508 \times 10^{87495}, 1.776069694985919 \times 10^{87495}, 0., 0.\}
         ,\q\= 2.563014715668011 \times 10^{87495}
ln[17] = P = \{0.9, 0.4, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[17]= \{0.9, 0.4, 0, 0\}
```

```
n= 2,q= \{1.55, 1.12, 0., 0.\}, q= 1.9123
       n= 3,q= \{2.0481, 3.872, 0., 0.\}, q= 4.38031
       n= 4,q= \{-9.89767, 16.2605, 0., 0.\}, \q = 19.0359
       n=5,q=\{-165.54, -321.482, 0., 0.\}, q=361.599
       n=6,q=\{-75946.4, 106436., 0., 0.\},\q=130754.
       n= 7,q= \{-5.56084 \times 10^9, -1.61669 \times 10^{10}, 0., 0.\}, \q\= 1.70965 \times 10^{10}
       n=8,q=\{-2.30446\times10^{20}, 1.79803\times10^{20}, 0., 0.\}, q=2.92292\times10^{20}\}
       n= 9,q= \{2.07761 \times 10^{40}, -8.28697 \times 10^{40}, 0., 0.\}, \q\= 8.54344 \times 10^{40}
       n= 10,q= \{-6.43575 \times 10^{81}, -3.44341 \times 10^{81}, 0., 0.\},\q\= 7.29904\times 10^{81}
       n= 11,q= \{2.95618 \times 10^{163}, 4.43219 \times 10^{163}, 0., 0.\},\q\= 5.3276 \times 10^{163}
       n= 12,q= \{-1.090530531291127 \times 10^{327}, 2.620466381232606 \times 10^{327}, 0., 0.\}
        \sqrt{q} = 2.838327129639643 \times 10^{327}
       n= 13,q= \{-5.677587215492202 \times 10^{654}, -5.715397189912260 \times 10^{654}, 0., 0.\}
        \q = 8.056100894848416 \times 10^{654}
       n= 14,q= \{-4.307684489364568 \times 10^{1307}, 6.489933203381180 \times 10^{1309}, 0., 0.\}
        n= 15,q= \{-4.211737736978352 \times 10^{2619}, -5.591316919443444 \times 10^{2617}, 0., 0.\}
        \sqrt{q} = 4.212108859891550 \times 10^{2619}
       n= 16,q= \{1.773560848259816 \times 10^{5239}, 4.709832093805100 \times 10^{5237}, 0., 0.\}
        \q = 1.774186104757689 \times 10^{5239}
       n= 17,q= \{3.143299830644895 \times 10^{10478}, 1.670634760690056 \times 10^{10477}, 0., 0.\}
        \sqrt{q} = 3.147736334315263 \times 10^{10478}
       n= 18,q= \{9.85242362029597 \times 10^{20956}, 1.050261192069306 \times 10^{20956}, 0., 0.\}
        ,\q\= 9.90824403036849 \times 10^{20956}
       n= 19,q= \{9.59672026221991 \times 10^{41913}, 2.069523635244766 \times 10^{41913}, 0., 0.\}
        \sqrt{q} = 9.81732997653328 \times 10^{41913}
       n= 20,q= \{8.781411171446546 \times 10^{83827}, 3.972127880699289 \times 10^{83827}, 0., 0.\}
        ,\q\= 9.63799678681389 \times 10^{83827}
ln[21]:= P = \{0.9, 0.2, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[21]= \{0.9, 0.2, 0, 0\}
```

 $n= 1,q= \{0.9, 0.4, 0, 0\}, q= 0.984886$

 $n= 1,q= \{0.9, 0.2, 0, 0\}, q= 0.921954$

```
n= 2,q= \{1.67, 0.56, 0., 0.\}, q= 1.76139
       n= 3,q= \{3.3753, 2.0704, 0., 0.\}, q= 3.9597
       n= 4, q= \{8.00609, 14.1764, 0., 0.\}, q= 16.2809
       n=5,q=\{-135.974, 227.196, 0., 0.\}, q=264.777
       n=6,q=\{-33128.1, -61785.2, 0., 0.\}, q=70106.3
       n = 7,q = \{-2.71994 \times 10^9, 4.09366 \times 10^9, 0., 0.\}, q = 4.91489 \times 10^9\}
       n= 8,q= \{-9.35996 \times 10^{18}, -2.2269 \times 10^{19}, 0., 0.\}, q = 2.41561 \times 10^{19}
       n= 9,q= \{-4.08302 \times 10^{38}, 4.16875 \times 10^{38}, 0., 0.\}, \q\= 5.83519 \times 10^{38}
       n= 10,q= \{-7.0744 \times 10^{75}, -3.40421 \times 10^{77}, 0., 0.\}, \q\= 3.40495 \times 10<sup>77</sup>
       n= 11,q= \{-1.15837 \times 10^{155}, 4.81655 \times 10^{153}, 0., 0.\},\q\= 1.15937 \times 10^{155}
       n= 12,q= \{1.339492474009539 \times 10^{310}, -1.115866656092269 \times 10^{309}, 0., 0.\}
        \sqrt{q} = 1.344132311891199 \times 10^{310}
       n= 13,q= \{1.781788503986410 \times 10^{620}, -2.989389975667570 \times 10^{619}, 0., 0.\}
        n= 14,q= \{3.085405748671911 \times 10^{1240}, -1.065292138515338 \times 10^{1240}, 0., 0.\}
        \q = 3.264134797204346 \times 10^{1240}
       n= 15,q= \{8.384881293555095 \times 10^{2480}, -6.573716976380435 \times 10^{2480}, 0., 0.\}
        ,\q\ =\ 1.065457597432026 \times 10^{2481}
       n= 16,q= \{2.709247942145783 \times 10^{4961}, -1.102396730087557 \times 10^{4962}, 0., 0.\}
        \q = 1.135199891925625 \times 10^{4962}
       n= 17,q= \{-1.141878306387527 \times 10^{9924}, -5.973332144835911 \times 10^{9923}, 0., 0.\}
        \sqrt{q} = 1.288678794627951 \times 10^{9924}
       n= 18,q= \{9.47079097473148 \times 10^{19\,847}, 1.364163678607081 \times 10^{19\,848}, 0., 0.\}
        ,\q\= 1.660693035723748 \times 10^{19848}
       n= 19,q= \{-9.63983725160251 \times 10^{39695}, 2.583941811081688 \times 10^{39696}, 0., 0.\}
        \sqrt{q} = 2.757901358901356 \times 10^{39696}
       n= 20,q= \{-5.747490660682278 \times 10^{79392}, -4.981755705287704 \times 10^{79392}, 0., 0.\}
         \sqrt{q} = 7.606019905429948 \times 10^{79392}
ln[25]:= P = \{0.9, -0.2, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[25]= \{0.9, -0.2, 0, 0\}
```

```
n= 1,q= \{0.9, -0.2, 0, 0\}, q= 0.921954
       n= 2,q= \{1.67, -0.56, 0., 0.\}, q= 1.76139
       n= 3,q= \{3.3753, -2.0704, 0., 0.\}, q= 3.9597
       n= 4,q= \{8.00609, -14.1764, 0., 0.\}, q= 16.2809
       n=5,q=\{-135.974, -227.196, 0., 0.\}, q=264.777
       n=6,q=\{-33128.1,61785.2,0.,0.\},\q=70106.3
       n = 7, q = \{-2.71994 \times 10^9, -4.09366 \times 10^9, 0., 0.\}, q = 4.91489 \times 10^9\}
       n= 8,q= \{-9.35996 \times 10^{18}, 2.2269 \times 10^{19}, 0., 0.\}, q= 2.41561 \times 10^{19}
       n= 9,q= \{-4.08302 \times 10^{38}, -4.16875 \times 10^{38}, 0., 0.\},\q\= 5.83519 \times 10^{38}
       n= 10,q= \{-7.0744 \times 10^{75}, 3.40421 \times 10^{77}, 0., 0.\}, \q\= 3.40495 \times 10^{77}
       n= 11,q= \{-1.15837 \times 10^{155}, -4.81655 \times 10^{153}, 0., 0.\},\q\= 1.15937\times 10^{155}
       n= 12,q= \{1.339492474009539 \times 10^{310}, 1.115866656092269 \times 10^{309}, 0., 0.\}
        \sqrt{q} = 1.344132311891199 \times 10^{310}
       n= 13,q= \{1.781788503986410 \times 10^{620}, 2.989389975667570 \times 10^{619}, 0., 0.\}
        n= 14,q= \{3.085405748671911 \times 10^{1240}, 1.065292138515338 \times 10^{1240}, 0., 0.\}
        \q = 3.264134797204346 \times 10^{1240}
       n= 15,q= \{8.384881293555095 \times 10^{2480}, 6.573716976380435 \times 10^{2480}, 0., 0.\}
        \sqrt{q} = 1.065457597432026 \times 10^{2481}
       n= 16,q= \{2.709247942145783 \times 10^{4961}, 1.102396730087557 \times 10^{4962}, 0., 0.\}
        n= 17,q= \{-1.141878306387527 \times 10^{9924}, 5.973332144835911 \times 10^{9923}, 0., 0.\}
        ,\q\= 1.288678794627951 \times 10^{9924}
       n= 18,q= \{9.47079097473148 \times 10^{19847}, -1.364163678607081 \times 10^{19848}, 0., 0.\}
        ,\q\= 1.660693035723748 \times 10^{19848}
       n= 19,q= \{-9.63983725160251 \times 10^{39695}, -2.583941811081688 \times 10^{39696}, 0., 0.\}
        \sqrt{q} = 2.757901358901356 \times 10^{39696}
       n= 20,q= \{-5.747490660682278 \times 10^{79392}, 4.981755705287704 \times 10^{79392}, 0., 0.\}
        \sqrt{q} = 7.606019905429948 \times 10^{79392}
ln[29]:= P = \{0.9, -0.4, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[29]= \{0.9, -0.4, 0, 0\}
```

```
n= 1, q= \{0.9, -0.4, 0, 0\}, q= 0.984886
       n= 2,q= \{1.55, -1.12, 0., 0.\}, q= 1.9123
       n= 3,q= \{2.0481, -3.872, 0., 0.\}, \q = 4.38031
       n=4,q=\{-9.89767, -16.2605, 0., 0.\}, q=19.0359
       n=5,q=\{-165.54, 321.482, 0., 0.\}, q=361.599
       n=6,q=\{-75946.4, -106436., 0., 0.\}, q=130754.
       n= 7,q= \{-5.56084 \times 10^9, 1.61669 \times 10^{10}, 0., 0.\},\q\= 1.70965 \times 10^{10}
       n=8,q=\{-2.30446\times10^{20}, -1.79803\times10^{20}, 0., 0.\}, q=2.92292\times10^{20}\}
       n= 9,q= \{2.07761\times10^{40}, 8.28697\times10^{40}, 0., 0.\},\q\= 8.54344\times10^{40}
       n= 10,q= \{-6.43575 \times 10^{81}, 3.44341 \times 10^{81}, 0., 0.\}, \q\= 7.29904 \times 10<sup>81</sup>
       n= 11,q= \{2.95618 \times 10^{163}, -4.43219 \times 10^{163}, 0., 0.\}, \q\= 5.3276 \times 10^{163}
       n= 12,q= \{-1.090530531291127 \times 10^{327}, -2.620466381232606 \times 10^{327}, 0., 0.\}
        \sqrt{q} = 2.838327129639643 \times 10^{327}
       n= 13,q= \{-5.677587215492202 \times 10^{654}, 5.715397189912260 \times 10^{654}, 0., 0.\}
        \sqrt{q} = 8.056100894848416 \times 10^{654}
       n= 14,q= \{-4.307684489364568 \times 10^{1307}, -6.489933203381180 \times 10^{1309}, 0., 0.\}
        n= 15,q= \{-4.211737736978352 \times 10^{2619}, 5.591316919443444 \times 10^{2617}, 0., 0.\}
        \sqrt{q} = 4.212108859891550 \times 10^{2619}
       n= 16,q= \{1.773560848259816 \times 10^{5239}, -4.709832093805100 \times 10^{5237}, 0., 0.\}
        \q = 1.774186104757689 \times 10^{5239}
       n= 17,q= \{3.143299830644895 \times 10^{10478}, -1.670634760690056 \times 10^{10477}, 0., 0.\}
        \sqrt{q} = 3.147736334315263 \times 10^{10478}
       n= 18,q= \{9.85242362029597 \times 10^{20956}, -1.050261192069306 \times 10^{20956}, 0., 0.\}
        ,\q\= 9.90824403036849 \times 10^{20956}
       n= 19,q= \{9.59672026221991 \times 10^{41913}, -2.069523635244766 \times 10^{41913}, 0., 0.\}
        \sqrt{q} = 9.81732997653328 \times 10^{41913}
       n= 20,q= \{8.781411171446546 \times 10^{83827}, -3.972127880699289 \times 10^{83827}, 0., 0.\}
        ,\q\= 9.63799678681389 \times 10^{83827}
ln[33]:= P = \{0.9, -0.5, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[33]= \{0.9, -0.5, 0, 0\}
```

```
n= 1,q= \{0.9, -0.5, 0, 0\}, q= 1.02956
       n= 2, q= \{1.46, -1.4, 0., 0.\}, q= 2.02277
       n= 3,q= \{1.0716, -4.588, 0., 0.\}, q= 4.71148
       n= 4, q= \{-19.0014, -10.333, 0., 0.\}, q= 21.6293
       n=5,q=\{255.183,392.183,0.,0.\}, q=467.895
       n=6,q=\{-88688.5, 200157., 0., 0.\},\q=218925.
       n= 7,q= \{-3.2197 \times 10^{10}, -3.55032 \times 10^{10}, 0., 0.\},\q\= 4.79283 \times 10^{10}
       n= 8,q= \{-2.23831\times10^{20}, 2.28619\times10^{21}, 0., 0.\}, q= 2.29712\times10^{21}
       n= 9,q= \{-5.17656\times10^{42}, -1.02344\times10^{42}, 0., 0.\},\q\= 5.27676\times10^{42}
       n= 10,q= \{2.57494 \times 10^{85}, 1.05958 \times 10^{85}, 0., 0.\}, \q\= 2.78442 \times 10^{85}
       n= 11,q= \{5.50759 \times 10^{170}, 5.4567 \times 10^{170}, 0., 0.\}, \q\= 7.75301 \times 10^{170}
       n= 12,q= \{5.578808944410104 \times 10^{339}, 6.010653992644118 \times 10^{341}, 0., 0.\}
        \sqrt{q} = 6.010912886593225 \times 10^{341}
       n= 13,q= \{-3.612484910836485 \times 10^{683}, 6.706458051183462 \times 10^{681}, 0., 0.\}
        n= 14,q= \{1.304554957306206 \times 10^{1367}, -4.845395703011623 \times 10^{1365}, 0., 0.\}
        \q = 1.305454488898052 \times 10^{1367}
       n= 15,q= \{1.699515850680320 \times 10^{2734}, -1.264216996894800 \times 10^{2733}, 0., 0.\}
        \sqrt{q} = 1.704211422584073 \times 10^{2734}
       n= 16,q= \{2.872371680561276 \times 10^{5468}, -4.297113649844373 \times 10^{5467}, 0., 0.\}
        \sqrt{q} = 2.904336572866030 \times 10^{5468}
       n= 17,q= \{8.065867214093620 \times 10^{10936}, -2.468581511193256 \times 10^{10936}, 0., 0.\}
        \sqrt{q} = 8.435170928487197 \times 10^{10936}
       n= 18,q= \{5.896431923798520 \times 10^{21873}, -3.982250135290274 \times 10^{21873}, 0., 0.\}
        ,\q\ =\ 7.115210859279556 \times 10^{21873}
       n= 19,q= \{1.890959329197092 \times 10^{43747}, -4.696213365255309 \times 10^{43747}, 0., 0.\}
        \sqrt{q} = 5.062622557200972 \times 10^{43747}
       n= 20,q= \{-1.847869278732508 \times 10^{87495}, -1.776069694985919 \times 10^{87495}, 0., 0.\}
        \sqrt{q} = 2.563014715668011 \times 10^{87495}
ln[37]:= P = \{0.9, -0.7, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[37]= \{0.9, -0.7, 0, 0\}
```

```
n= 1,q= \{0.9, -0.7, 0, 0\}, q= 1.14018
       n= 2,q= \{1.22, -1.96, 0., 0.\}, q= 2.30868
       n= 3,q= \{-1.4532, -5.4824, 0., 0.\}, \q = 5.67173
       n=4,q=\{-27.0449, 15.234, 0., 0.\}, q=31.0404
       n=5,q=\{500.251, -824.707, 0., 0.\}, q=964.569
       n=6,q=\{-429889., -825123., 0., 0.\}, q=930394.
       n= 7,q= \{-4.96022 \times 10^{11}, 7.09423 \times 10^{11}, 0., 0.\}, \q\= 8.65632 \times 10^{11}
       n= 8,q= \{-2.57243 \times 10^{23}, -7.0378 \times 10^{23}, 0., 0.\}, \q\= 7.49319 \times 10^{23}
       n=9,q=\{-4.29132\times10^{47}, 3.62085\times10^{47}, 0., 0.\}, q=5.6148\times10^{47}\}
       n= 10,q= \{5.30488 \times 10^{94}, -3.10764 \times 10^{95}, 0., 0.\}, \q\= 3.15259 \times 10<sup>95</sup>
       n= 11,q= \{-9.376 \times 10^{190}, -3.29713 \times 10^{190}, 0., 0.\},\q\= 9.93884 \times 10^{190}
       n= 12,q= \{7.703838218690188 \times 10^{381}, 6.182785256280149 \times 10^{381}, 0., 0.\}
        \sqrt{q} = 9.87805430360791 \times 10^{381}
       n= 14,q= \{-8.628765099685591 \times 10^{1527}, 4.024318121591257 \times 10^{1527}, 0., 0.\}
        \sqrt{q} = 9.52106735031949 \times 10^{1527}
       n= 15,q= \{5.826045080178430 \times 10^{3055}, -6.944979151523782 \times 10^{3055}, 0., 0.\}
        \sqrt{q} = 9.06507234893199 \times 10^{3055}
       n= 16,q= \{-1.428993413882870 \times 10^{6111}, -8.092352323535379 \times 10^{6111}, 0., 0.\}
        ,\q\= 8.217553669137128 \times 10^{6111}
       n= 17,q= \left\{-6.344414395130783\times10^{12223}, 2.312783634630360\times10^{12223}, 0., 0.\right\}
        \sqrt{q} = 6.752818830514907 \times 10^{12223}
       n= 18,q= \{3.490262587652868 \times 10^{24447}, -2.934651556874350 \times 10^{24447}, 0., 0.\}
        \sqrt{q} = 4.560056215775671 \times 10^{24447}
       n= 19,q= \{3.569753170504246 \times 10^{48894}, -2.048540907351157 \times 10^{48895}, 0., 0.\}
        \sqrt{q} = 2.079411269103434 \times 10^{48895}
       n= 20,q= \left\{-4.069088472107850\times10^{97790}, -1.462557079784887\times10^{97790}, 0., 0.\right\}
        |q| = 4.323951226074352 \times 10^{97790}
ln[41]:= P = \{0.9, -1.2, 0, 0\}
                                                                                                                       +
       quaternion[\{x_{y_{z}}, y_{z}, x_{y}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[41]= \{0.9, -1.2, 0, 0\}
```

```
n= 1, q= \{0.9, -1.2, 0, 0\}, q= 1.5
      n= 2,q= \{0.27, -3.36, 0., 0.\}, q= 3.37083
      n= 3,q= \{-10.3167, -3.0144, 0., 0.\}, q= 10.7481
      n= 4,q= \{98.2477, 60.9973, 0., 0.\}, q= 115.643
      n=5,q=\{5932.84, 11984.5, 0., 0.\}, q=13372.6
      n=6,q=\{-1.0843\times10^8, 1.42204\times10^8, 0., 0.\}, q=1.78827\times10^8
      n= 7,q= \{-8.46503 \times 10^{15}, -3.08382 \times 10^{16}, 0., 0.\},\q\= 3.19789\times 10^{16}
      n= 8,q= \{-8.7934 \times 10^{32}, 5.22093 \times 10^{32}, 0., 0.\}, \q\= 1.02265 \times 10<sup>33</sup>
      n= 9,q= \{5.00657 \times 10^{65}, -9.18194 \times 10^{65}, 0., 0.\},\q\= 1.04582 \times 10^{66}
      n= 10,q= \left\{-5.92424\times10^{131}, -9.194\times10^{131}, 0., 0.\right\},\q\= 1.09374\times10^{132}
      n= 11,q= \{-4.94331 \times 10^{263}, 1.08935 \times 10^{264}, 0., 0.\},\q\= 1.19626\times10<sup>264</sup>
      n= 12,q= \{-9.42318178220839 \times 10^{527}, -1.076999165134003 \times 10^{528}, 0., 0.\}
       \sqrt{q} = 1.431045334957904 \times 10^{528}
      n= 13,q= \{-2.719636526938977 \times 10^{1055}, 2.029751782468877 \times 10^{1056}, 0., 0.\}
        \sqrt{q} = 2.047890750704780 \times 10^{1056}
      n= 14,q= \{-4.045928070048975 \times 10^{2112}, -1.104037417644371 \times 10^{2112}, 0., 0.\}
        \sqrt{q} = 4.193856526822189 \times 10^{2112}
      n= 15,q= \{1.515063532845137 \times 10^{4225}, 8.933711956863486 \times 10^{4224}, 0., 0.\}
        ,\q\ =\ 1.758843256756907 \times 10^{4225}
      n= 16,q= \{1.497305415275132 \times 10^{8450}, 2.707028239757287 \times 10^{8450}, 0., 0.\}
        \q = 3.093529601839244 \times 10^{8450}
      n= 17,q= \{-5.086078384231203 \times 10^{16900}, 8.106496085382589 \times 10^{16900}, 0., 0.\}
        \sqrt{q} = 9.56992539745567 \times 10^{16900}
      n= 18,q= \{-3.984708545177935 \times 10^{33801}, -8.246054902343849 \times 10^{33801}, 0., 0.\}
        ,\q\= 9.15834721128671 \times 10^{33801}
      n= 19,q= \{-5.211951926245497 \times 10^{67603}, 6.571625086675188 \times 10^{67603}, 0., 0.\}
       \q = 8.387532364248308 \times 10^{67603}
      n= 20,q= \left\{-1.602181339832452\times10^{135\,207}, -6.850198805811995\times10^{135\,207}, 0., 0.\right\}
        ,\q\= 7.035069916131281 \times 10^{135207}
ln[45]:= P = \{0.9, -1.4, 0, 0\}
      quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
           n++]]
      iteration[
        {0,
         0,
         0,
         0}]
```

Out[45]= $\{0.9, -1.4, 0, 0\}$

```
n= 1,q= \{0.9, -1.4, 0, 0\}, q= 1.66433
       n= 2, q= \{-0.25, -3.92, 0., 0.\}, q= 3.92796
       n= 3,q= \{-14.4039, 0.56, 0., 0.\}, \q = 14.4148
       n=4,q=\{208.059, -17.5324, 0., 0.\}, q=208.796
       n=5,q=\{42982., -7296.92, 0., 0.\}, q=43596.9
       n = 6,q = \{1.7942 \times 10^9, -6.27272 \times 10^8, 0., 0.\}, q = 1.90069 \times 10^9\}
       n= 7,q= \{2.82569 \times 10^{18}, -2.25091 \times 10^{18}, 0., 0.\},\q\= 3.61264 \times 10^{18}
       n= 8,q= \{2.91797 \times 10^{36}, -1.27208 \times 10^{37}, 0., 0.\}, q= 1.30511 \times 10^{37}
       n= 9,q= \{-1.53303 \times 10^{74}, -7.42375 \times 10^{73}, 0., 0.\},\q\= 1.70332×10<sup>74</sup>
       n= 10,q= \{1.79906 \times 10^{148}, 2.27617 \times 10^{148}, 0., 0.\}, \q\= 2.9013 \times 10^{148}
       n= 11,q= \{-1.94431 \times 10^{296}, 8.18993 \times 10^{296}, 0., 0.\},\q\= 8.41756 \times 10^{296}
       n= 12,q= \{-6.329467547249228 \times 10^{593}, -3.184752059507657 \times 10^{593}, 0., 0.\}
        \sqrt{q} = 7.085534920683082 \times 10^{593}
       n= 13,q= \{2.991951375114289 \times 10^{1187}, 4.031556961337771 \times 10^{1187}, 0., 0.\}
         \q = 5.020480511221941 \times 10^{1187}
       n= 14,q= \{-7.301678501462757 \times 10^{2374}, 2.412444478865226 \times 10^{2375}, 0., 0.\}
         \sqrt{q} = 2.520522456355933 \times 10^{2375}
       n= 15,q= \{-5.286743274220077 \times 10^{4750}, -3.522978797460548 \times 10^{4750}, 0., 0.\}
         \sqrt{q} = 6.353033452994545 \times 10^{4750}
       n= 16,q= \{1.553827484015464 \times 10^{9501}, 3.725016892538898 \times 10^{9501}, 0., 0.\}
        , q = 4.036103405486779 \times 10^{9501}
       n= 17,q= \{-1.146137099961832 \times 10^{19003}, 1.157606725209764 \times 10^{19003}, 0., 0.\}
         \sqrt{q} = 1.629013069978197 \times 10^{19003}
       n= 18,q= \{-2.642307834195556 \times 10^{38004}, -2.653552029856463 \times 10^{38006}, 0., 0.\}
         ,\q\= 2.653683582159791 \times 10^{38006}
       n= 19,q= \{-7.040640196086290 \times 10^{76012}, 1.402300263387050 \times 10^{76011}, 0., 0.\}
        \sqrt{q} = 7.042036554224420 \times 10^{76012}
       n= 20,q= \{4.955094991045904 \times 10^{152025}, -1.974618320277052 \times 10^{152024}, 0., 0.\}
         ,\q\= 4.959027883103295 \times 10^{152025}
ln[49]:= P = \{1.2, 1.4, 0, 0\}
       quaternion[\{x_{y_{z}}, y_{z_{z}}, w_{z_{z}}\}] := \{x^{2} - y^{2} - z^{2} - w^{2}, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[49]= \{1.2, 1.4, 0, 0\}
```

```
n= 1, q= \{1.2, 1.4, 0, 0\}, q= 1.84391
       n= 2,q= \{0.68, 4.76, 0., 0.\}, q= 4.80833
       n= 3,q= \{-20.9952, 7.8736, 0., 0.\}, q= 22.423
       n= 4, q= \{380.005, -329.216, 0., 0.\}, q= 502.779
       n=5,q=\{36022., -250206., 0., 0.\}, q=252785.
       n= 6,q= \{-6.13053 \times 10^{10}, -1.80258 \times 10^{10}, 0., 0.\}, \q\= 6.39005 \times 10^{10}
       n= 7,q= \{3.43341 \times 10^{21}, 2.21015 \times 10^{21}, 0., 0.\}, \q\= 4.08327 \times 10^{21}
       n= 8,q= \{6.90352 \times 10^{42}, 1.51767 \times 10^{43}, 0., 0.\}, q= 1.66731 \times 10^{43}
       n= 9,q= \left\{-1.82674\times10^{86}, 2.09546\times10^{86}, 0., 0.\right\},\q\= 2.77991\times10^{86}
       n= 10,q= \{-1.05395 \times 10^{172}, -7.65572 \times 10^{172}, 0., 0.\}, \q\= 7.72793 \times 10^{172}
       n= 11,q= \{-5.749923850784894 \times 10^{345}, 1.613741499892428 \times 10^{345}, 0., 0.\}
        |q| = 5.972083884064259 \times 10^{345}
       n= 12,q= \{3.045746266134992 \times 10^{691}, -1.855778147846572 \times 10^{691}, 0., 0.\}
         \sqrt{q} = 3.566578591830005 \times 10^{691}
       n= 13,q= \{5.832657783650395 \times 10^{1382}, -1.130445872915722 \times 10^{1383}, 0., 0.\}
        ,\q\= 1.272048285170010 \times 10^{1383}
       n= 14,q= \{-9.37708903380412 \times 10^{2765}, -1.318700783931470 \times 10^{2766}, 0., 0.\}
        \sqrt{q} = 1.618106839803963 \times 10^{2766}
       n= 15,q= \{-8.596737700625773 \times 10^{5531}, 2.473114931974537 \times 10^{5532}, 0., 0.\}
        \sqrt{q} = 2.618269745020368 \times 10^{5532}
       n= 16,q= \{-5.377258475821815 \times 10^{11064}, -4.252144074737210 \times 10^{11064}, 0., 0.\}
        , q = 6.855336457689025 \times 10^{11064}
       n= 17,q= \{1.083417948347479 \times 10^{22129}, 4.572975553259234 \times 10^{22129}, 0., 0.\}
        \sqrt{q} = 4.699563794812031 \times 10^{22129}
       n= 18,q= \{-1.973831095990514 \times 10^{44259}, 9.90888758351059 \times 10^{44258}, 0., 0.\}
         ,\q\= 2.208589986150806 \times 10^{44259}
       n= 19,q= \{2.914148664072610 \times 10^{88518}, -3.911694087801502 \times 10^{88518}, 0., 0.\}
         \sqrt{q} = 4.877869726925617 \times 10^{88518}
       n= 20,q=\{-6.809088200225046\times10^{177036}, -2.279851620045494\times10^{177037}, 0., 0.\}
        \sqrt{q} = 2.379361307285740 \times 10^{177037}
ln[53] := P = \{1.2, 1.2, 0, 0\}
       quaternion[\{x_{y}, y_{z}, z_{w}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
            Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[53]= \{1.2, 1.2, 0, 0\}
```

 $n= 1,q= \{1.2, 1.2, 0, 0\}, q= 1.69706$

```
n= 2,q= \{1.2, 4.08, 0., 0.\}, q= 4.25281
       n= 3,q= \{-14.0064, 10.992, 0., 0.\}, \q = 17.8046
       n= 4,q= \{76.5552, -306.717, 0., 0.\}, \q = 316.126
       n=5,q=\{-88213.2,-46960.3,0.,0.\},\q=99934.2
       n=6,q=\{5.57631\times10^9, 8.28504\times10^9, 0., 0.\}, q=9.98685\times10^9
       n= 7,q= \{-3.75467 \times 10^{19}, 9.23998 \times 10^{19}, 0., 0.\}, \q\= 9.97371 \times 10^{19}
       n= 8,q= \{-7.12797 \times 10^{39}, -6.93862 \times 10^{39}, 0., 0.\}, q= 9.94748 \times 10^{39}\}
       n= 9,q= \{2.66354 \times 10^{78}, 9.89166 \times 10^{79}, 0., 0.\}, \q\= 9.89525 \times 10^{79}
       n= 10,q= \{-9.7774 \times 10^{159}, 5.26937 \times 10^{158}, 0., 0.\},\q\= 9.79159 \times 10^{159}
       n= 11,q= \{9.53198651360759 \times 10^{319}, -1.030414830696073 \times 10^{319}, 0., 0.\}
        q = 9.58751905442254 \times 10^{319}
       n= 12,q= \{8.979701217227850 \times 10^{639}, -1.964380053923242 \times 10^{639}, 0., 0.\}
        \q = 9.19205216189153 \times 10^{639}
       n= 13,q= \{7.677624495443186 \times 10^{1279}, -3.527909192262530 \times 10^{1279}, 0., 0.\}
        \q = 8.449382294693482 \times 10^{1279}
       n= 14,q= \{4.649977462417878 \times 10^{2559}, -5.417192406442796 \times 10^{2559}, 0., 0.\}
        \sqrt{q} = 7.139206116187969 \times 10^{2559}
       n= 15,q= \{-7.723683167427284 \times 10^{5118}, -5.037964519908054 \times 10^{5119}, 0., 0.\}
        \q = 5.096826396941570 \times 10^{5119}
       n= 16,q= \{-2.478453368714440 \times 10^{10239}, 7.782328352101944 \times 10^{10238}, 0., 0.\}
        \q = 2.597763932056039 \times 10^{10239}
       n= 17,q= \{5.537084755092657 \times 10^{20478}, -3.857627584141791 \times 10^{20478}, 0., 0.\}
        \sqrt{q} = 6.748377446691252 \times 10^{20478}
       n= 18,q=\{1.577801700714788\times10^{40957}, -4.272002177395286\times10^{40957}, 0., 0.\}
        \sqrt{q} = 4.554059816301114 \times 10^{40957}
       n= 19,q= \{-1.576054439689159 \times 10^{81915}, -1.348074460190312 \times 10^{81915}, 0., 0.\}
        \sqrt{q} = 2.073946081044854 \times 10^{81915}
       n= 20,q= \{6.666428466465081 \times 10^{163829}, 4.249277476029013 \times 10^{163830}, 0., 0.\}
        \sqrt{q} = 4.301252347081308 \times 10^{163830}
ln[57] = P = \{1.2, 0.7, 0, 0\}
       quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
            Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[57]= \{1.2, 0.7, 0, 0\}
```

```
n= 1,q= \{1.2, 0.7, 0, 0\}, q= 1.38924
                 n= 2,q= \{2.15, 2.38, 0., 0.\}, q= 3.20732
                 n= 3,q= \{0.1581, 10.934, 0., 0.\}, \q = 10.9351
                 n= 4,q= \{-118.327, 4.15733, 0., 0.\}, q= 118.4
                 n=5,q=\{13985.3, -983.152, 0., 0.\}, q= 14019.8
                 n=6,q=\{1.94621\times10^8, -2.74993\times10^7, 0., 0.\}, q=1.96555\times10^8
                 n= 7,q= \{3.71213 \times 10^{16}, -1.07039 \times 10^{16}, 0., 0.\},\q\= 3.86337 \times 10^{16}
                 n= 8,q= \{1.26342 \times 10^{33}, -7.94687 \times 10^{32}, 0., 0.\}, \q\= 1.49257 \times 10<sup>33</sup>
                 n= 9,q= \{9.64698 \times 10^{65}, -2.00804 \times 10^{66}, 0., 0.\}, \q\= 2.22775 \times 10^{66}
                  \text{n= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{q} = 4.96288 \times 10^{132} \text{, } \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, 0., 0.} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{,} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{,} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{,} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{, } -3.87431 \times 10^{132}\text{,} \right\} \text{,} \\ \text{m= 10,q= } \left\{ -3.10159 \times 10^{132} \times
                 n= 11,q= \{-5.39041 \times 10^{264}, 2.40331 \times 10^{265}, 0., 0.\}, \q\= 2.46302 \times 10^{265}
                 n= 12,q= \{-5.485329218685683 \times 10^{530}, -2.590962267205347 \times 10^{530}, 0., 0.\}
                  \sqrt{q} = 6.066458778187548 \times 10^{530}
                 n= 13,q= \{2.337575116728501 \times 10^{1061}, 2.842456205762718 \times 10^{1061}, 0., 0.\}
                   ,\q\ =\ 3.680192210744875 \times 10^{1061}
                 n= 14,q= \{-2.615299855330720 \times 10^{2122}, 1.328890979396288 \times 10^{2123}, 0., 0.\}
                   \q = 1.354381470802725 \times 10^{2123}
                 n= 15,q= \{-1.697553301787895 \times 10^{4246}, -6.950896772330818 \times 10^{4245}, 0., 0.\}
                   ,\q\ =\ 1.834349168453753\times 10^{4246}
                 n= 16,q= \{2.398537553014996 \times 10^{8492}, 2.359903553251401 \times 10^{8492}, 0., 0.\}
                   \q = 3.364836871806975 \times 10^{8492}
                 n= 17,q= \{1.838376125745748 \times 10^{16983}, 1.132063458793402 \times 10^{16985}, 0., 0.\}
                   |q| = 1.132212717387175 \times 10^{16985}
                 n= 18,q= \{-1.281229712057309 \times 10^{33\,970}, 4.162316870949891 \times 10^{33\,968}, 0., 0.\}
                   ,\q\= 1.281905637413252 \times 10^{33970}
                 n= 19,q= \{1.639817086885036 \times 10^{67940}, -1.066576809211682 \times 10^{67939}, 0., 0.\}
                  \q = 1.643282063231875 \times 10^{67940}
                 n= 20,q= \{2.677624217540644 \times 10^{135880}, -3.497981752441274 \times 10^{135879}, 0., 0.\}
                    \sqrt{q} = 2.700375939339607 \times 10^{135880}
 ln[61]:= P = \{1.2, 0.5, 0, 0\}
                 quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
                 iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
                           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
                           n++]]
                 iteration[
                     {0,
                       0,
                       0,
                       0}]
Out[61]= \{1.2, 0.5, 0, 0\}
```

 $n= 1, q= \{1.2, 0.5, 0, 0\}, q= 1.3$

```
n= 2,q= \{2.39, 1.7, 0., 0.\}, q= 2.93293
       n= 3,q= \{4.0221, 8.626, 0., 0.\}, q= 9.51762
       n = 4, q = \{-57.0306, 69.8893, 0., 0.\}, \\ q = 90.2053
       n=5,q=\{-1630.82, -7971.15, 0., 0.\}, q=8136.27
       n=6,q=\{-6.08797\times10^7, 2.59991\times10^7, 0., 0.\}, q=6.61988\times10^7
       n= 7,q= \{3.03038 \times 10^{15}, -3.16563 \times 10^{15}, 0., 0.\},\q\= 4.38229 \times 10^{15}
       n= 8,q= \{-8.37977 \times 10^{29}, -1.91862 \times 10^{31}, 0., 0.\},\q\= 1.92044×10<sup>31</sup>
       n= 9,q= \{-3.67406 \times 10^{62}, 3.21551 \times 10^{61}, 0., 0.\}, \q\= 3.68811 \times 10^{62}
       n= 10,q= \{1.33953 \times 10^{125}, -2.3628 \times 10^{124}, 0., 0.\},\q\= 1.36021 \times 10^{125}
       n= 11,q= \{1.73852 \times 10^{250}, -6.33009 \times 10^{249}, 0., 0.\},\q\= 1.85018\times10<sup>250</sup>
       n= 12,q= \{2.621758059145861 \times 10^{500}, -2.201000113042697 \times 10^{500}, 0., 0.\}
        \sqrt{q} = 3.423158894692187 \times 10^{500}
       n= 13,q= \{2.029213823082307 \times 10^{1000}, -1.154097956910128 \times 10^{1001}, 0., 0.\}
        ,\q\= 1.171801681831023 \times 10^{1001}
       n= 14,q= \{-1.290765006746249 \times 10^{2002}, -4.683823054706161 \times 10^{2001}, 0., 0.\}
        n= 15,q= \{1.446692318562675 \times 10^{4004}, 1.209142979361207 \times 10^{4004}, 0., 0.\}
        , q = 1.885456286718614 \times 10^{4004}
       n= 16,q= \{6.308919200497515 \times 10^{8007}, 3.498515720571689 \times 10^{8008}, 0., 0.\}
        \q = 3.554945409126743 \times 10^{8008}
       n= 17,q= \{-1.184158763230318 \times 10^{16017}, 4.414370600551425 \times 10^{16016}, 0., 0.\}
        \sqrt{q} = 1.263763686187130 \times 10^{16017}
       n= 18,q= \{1.207365298545029 \times 10^{32034}, -1.045463126157850 \times 10^{32034}, 0., 0.\}
        ,\q\= 1.597098654525284 \times 10^{32034}
       n= 19,q= \{3.647378159749816 \times 10^{64067}, -2.524511798862785 \times 10^{64068}, 0., 0.\}
        \q = 2.550724112286472 \times 10^{64068}
       n= 20,q= \{-6.240126148195213 \times 10^{128136}, -1.841569839840568 \times 10^{128136}, 0., 0.\}
         ,\q\= 6.506193496999612 \times 10^{128136}
ln[65]:= P = \{1.2, 0.4, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q²)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[65]= \{1.2, 0.4, 0, 0\}
```

```
n= 2,q= \{2.48, 1.36, 0., 0.\}, q= 2.82843
       n= 3,q= \{5.5008, 7.1456, 0., 0.\}, q= 9.01767
       n=4,q=\{-19.6008,79.013,0.,0.\},\q=81.4079
       n=5,q=\{-5857.67, -3097.04, 0., 0.\}, q=6626.
       n= 6,q= \{2.47206 \times 10^7, 3.62828 \times 10^7, 0., 0.\}, \q\= 4.39039 \times 10^7
       n= 7,q= \{-7.05334 \times 10^{14}, 1.79387 \times 10^{15}, 0., 0.\}, \q\= 1.92755 \times 10<sup>15</sup>
       n= 8,q= \{-2.72047 \times 10^{30}, -2.53055 \times 10^{30}, 0., 0.\},\q\= 3.71546×10<sup>30</sup>
       n= 9,q= \{9.97263 \times 10^{59}, 1.37686 \times 10^{61}, 0., 0.\}, \q\= 1.38047 \times 10^{61}
       n= 10,q= \{-1.8858 \times 10^{122}, 2.74618 \times 10^{121}, 0., 0.\},\q\= 1.90569\times 10^{122}
       n= 11,q= \left\{3.48082\times10^{244}, -1.03575\times10^{244}, 0., 0.\right\},\q\= 3.63165\times10^{244}
       n= 12,q= \{1.104332903387162 \times 10^{489}, -7.210511519241572 \times 10^{488}, 0., 0.\}
        \sqrt{q} = 1.318888139758134 \times 10^{489}
       n= 13,q= \{6.996363978123637 \times 10^{977}, -1.592561024190124 \times 10^{978}, 0., 0.\}
        n= 14,q= \{-2.046759526625636 \times 10^{1956}, -2.228427316521494 \times 10^{1956}, 0., 0.\}
        \q = 3.025741704913356 \times 10^{1956}
       n= 15,q= \{-7.766637451863888 \times 10^{3911}, 9.12210967896633 \times 10^{3912}, 0., 0.\}
        n= 16,q= \{-8.260967842200434 \times 10^{7825}, -1.416962373453400 \times 10^{7825}, 0., 0.\}
        \q = 8.381609156817824 \times 10^{7825}
       n= 17,q= \{6.623580732208701 \times 10^{15651}, 2.341096120141308 \times 10^{15651}, 0., 0.\}
        \sqrt{q} = 7.025137205765239 \times 10^{15651}
       n= 18,q= \{3.839109067234566 \times 10^{31303}, 3.101287830723303 \times 10^{31303}, 0., 0.\}
        ,\q\= 4.935255275982703 \times 10^{31303}
       n= 19,q= \{5.120772221130212 \times 10^{62606}, 2.381236446206810 \times 10^{62607}, 0., 0.\}
        n= 20,q= \left\{-5.408063931336649 \times 10^{125214}, 2.438753889135731 \times 10^{125214}, 0., 0.\right\}
        ,\q\= 5.932510094150626 \times 10^{125214}
ln[69]:= P = \{1.2, 0.2, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[69]= \{1.2, 0.2, 0, 0\}
```

 $n= 1, q= \{1.2, 0.4, 0, 0\}, q= 1.26491$

```
n= 1,q= \{1.2, 0.2, 0, 0\}, q= 1.21655
       n= 2,q= \{2.6, 0.68, 0., 0.\}, q= 2.68745
       n= 3,q= \{7.4976, 3.736, 0., 0.\}, q= 8.37686
       n= 4, q= \{43.4563, 56.2221, 0., 0.\}, q= 71.0589
       n=5,q=\{-1271.27, 4886.61, 0., 0.\}, q=5049.26
       n = 6,q = \{-2.22628 \times 10^7, -1.24244 \times 10^7, 0., 0.\}, q = 2.54951 \times 10^7\}
       n= 7,q= \{3.41267 \times 10^{14}, 5.53204 \times 10^{14}, 0., 0.\}, \q\= 6.49998 \times 10^{14}
       n= 8,q= \{-1.89571 \times 10^{29}, 3.7758 \times 10^{29}, 0., 0.\}, q= 4.22497 \times 10^{29}\}
       n = 9,q = \{-1.06629 \times 10^{59}, -1.43157 \times 10^{59}, 0., 0.\}, q = 1.78504 \times 10^{59}\}
       n= 10,q= \{-9.12399 \times 10^{117}, 3.05294 \times 10^{118}, 0., 0.\}, \q\= 3.18636 \times 10^{118}
       n= 11,q= \{-8.48797 \times 10^{236}, -5.571 \times 10^{236}, 0., 0.\},\q\= 1.01529 \times 10^{237}
       n= 12,q= \{4.100964033503680 \times 10^{473}, 9.45729667623086 \times 10^{473}, 0., 0.\}
        \sqrt{q} = 1.030816988734363 \times 10^{474}
       n= 13,q= \{-7.262255441815651 \times 10^{947}, 7.756806704679331 \times 10^{947}, 0., 0.\}
        \sqrt{q} = 1.062583664263381 \times 10^{948}
       n= 14,q= \{-7.427696151577181 \times 10^{1894}, -1.126638234043392 \times 10^{1896}, 0., 0.\}
        n= 15,q= \{-1.263796643396398 \times 10^{3792}, 1.673665295044763 \times 10^{3791}, 0., 0.\}
        ,\q\ =\ 1.274830777420429 \times 10^{3792}
       n= 16,q= \{1.569170400661629 \times 10^{7584}, -4.230345164093226 \times 10^{7583}, 0., 0.\}
        \q = 1.625193511058374 \times 10^{7584}
       n= 17,q= \{2.283337544238907 \times 10^{15168}, -1.327626483215430 \times 10^{15168}, 0., 0.\}
        \sqrt{q} = 2.641253948386246 \times 10^{15168}
       n= 18,q= \{3.451038261995992 \times 10^{30336}, -6.062838787703313 \times 10^{30336}, 0., 0.\}
        ,\q\ = 6.976222419865932 \times 10^{30336}
       n= 19,q= \{-2.484834907991946 \times 10^{60673}, -4.184617726535505 \times 10^{60673}, 0., 0.\}
        \q = 4.866767925144009 \times 10^{60673}
       n= 20,q= \{-1.133662099725984 \times 10^{121347}, 2.079616840699463 \times 10^{121347}, 0., 0.\}
         ,\q\= 2.368543003721052 \times 10^{121347}
ln[73]:= P = \{1.2, -0.2, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[73]= \{1.2, -0.2, 0, 0\}
```

```
n= 1,q= \{1.2, -0.2, 0, 0\}, q= 1.21655
       n= 2,q= \{2.6, -0.68, 0., 0.\}, q= 2.68745
       n= 3,q= \{7.4976, -3.736, 0., 0.\}, q= 8.37686
       n=4,q=\{43.4563, -56.2221, 0., 0.\}, q=71.0589
       n=5,q=\{-1271.27, -4886.61, 0., 0.\}, q=5049.26
       n=6,q=\{-2.22628\times10^7, 1.24244\times10^7, 0., 0.\}, q=2.54951\times10^7\}
       n= 7,q= \{3.41267 \times 10^{14}, -5.53204 \times 10^{14}, 0., 0.\}, q = 6.49998 \times 10^{14}\}
       n=8,q=\{-1.89571\times10^{29}, -3.7758\times10^{29}, 0., 0.\}, q=4.22497\times10^{29}\}
       n= 9,q= \{-1.06629 \times 10^{59}, 1.43157 \times 10^{59}, 0., 0.\}, \q\= 1.78504 \times 10<sup>59</sup>
       n= 10,q= \{-9.12399 \times 10^{117}, -3.05294 \times 10^{118}, 0., 0.\},\q\= 3.18636 \times 10^{118}
       n= 11,q= \{-8.48797 \times 10^{236}, 5.571 \times 10^{236}, 0., 0.\}, \q\= 1.01529 \times 10^{237}
       n= 12,q= \{4.100964033503680 \times 10^{473}, -9.45729667623086 \times 10^{473}, 0., 0.\}
        \sqrt{q} = 1.030816988734363 \times 10^{474}
       n= 13,q= \{-7.262255441815651 \times 10^{947}, -7.756806704679331 \times 10^{947}, 0., 0.\}
        \sqrt{q} = 1.062583664263381 \times 10^{948}
       n= 14,q= \{-7.427696151577181 \times 10^{1894}, 1.126638234043392 \times 10^{1896}, 0., 0.\}
        n= 15,q= \{-1.263796643396398 \times 10^{3792}, -1.673665295044763 \times 10^{3791}, 0., 0.\}
        ,\q\ =\ 1.274830777420429 \times 10^{3792}
       n= 16,q= \{1.569170400661629 \times 10^{7584}, 4.230345164093226 \times 10^{7583}, 0., 0.\}
        \q = 1.625193511058374 \times 10^{7584}
       n= 17,q= \{2.283337544238907 \times 10^{15168}, 1.327626483215430 \times 10^{15168}, 0., 0.\}
        \sqrt{q} = 2.641253948386246 \times 10^{15168}
       n= 18,q= \{3.451038261995992 \times 10^{30336}, 6.062838787703313 \times 10^{30336}, 0., 0.\}
        ,\q\ = 6.976222419865932 \times 10^{30336}
       n= 19,q= \{-2.484834907991946 \times 10^{60673}, 4.184617726535505 \times 10^{60673}, 0., 0.\}
        \sqrt{q} = 4.866767925144009 \times 10^{60673}
       n= 20,q= \left\{-1.133662099725984 \times 10^{121347}, -2.079616840699463 \times 10^{121347}, 0., 0.\right\}
         ,\q\= 2.368543003721052 \times 10^{121347}
ln[77]:= P = \{1.2, -0.4, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[77]= \{1.2, -0.4, 0, 0\}
```

```
n= 1,q= \{1.2, -0.4, 0, 0\}, q= 1.26491
       n= 2,q= \{2.48, -1.36, 0., 0.\}, q= 2.82843
       n= 3,q= \{5.5008, -7.1456, 0., 0.\}, q= 9.01767
       n=4,q=\{-19.6008,-79.013,0.,0.\},\q=81.4079
       n=5,q=\{-5857.67,3097.04,0.,0.\},\q=6626.
       n = 6,q = \{2.47206 \times 10^7, -3.62828 \times 10^7, 0., 0.\}, q = 4.39039 \times 10^7\}
       n= 7,q= \{-7.05334 \times 10^{14}, -1.79387 \times 10^{15}, 0., 0.\}, \q\= 1.92755 \times 10<sup>15</sup>
       n= 8,q= \{-2.72047 \times 10^{30}, 2.53055 \times 10^{30}, 0., 0.\}, q= 3.71546 \times 10^{30}\}
       n= 9,q= \{9.97263 \times 10^{59}, -1.37686 \times 10^{61}, 0., 0.\}, \q\= 1.38047 \times 10<sup>61</sup>
       n= 10,q= \{-1.8858 \times 10^{122}, -2.74618 \times 10^{121}, 0., 0.\}, \q\= 1.90569 \times 10^{122}
       n= 11,q= \{3.48082 \times 10^{244}, 1.03575 \times 10^{244}, 0., 0.\},\q\= 3.63165 \times 10^{244}
       n= 12,q= \{1.104332903387162 \times 10^{489}, 7.210511519241572 \times 10^{488}, 0., 0.\}
        \sqrt{q} = 1.318888139758134 \times 10^{489}
       n= 13,q= \{6.996363978123637 \times 10^{977}, 1.592561024190124 \times 10^{978}, 0., 0.\}
        n= 14,q= \{-2.046759526625636 \times 10^{1956}, 2.228427316521494 \times 10^{1956}, 0., 0.\}
        \q = 3.025741704913356 \times 10^{1956}
       n= 15,q= \{-7.766637451863888 \times 10^{3911}, -9.12210967896633 \times 10^{3912}, 0., 0.\}
        n= 16,q= \{-8.260967842200434 \times 10^{7825}, 1.416962373453400 \times 10^{7825}, 0., 0.\}
        \q = 8.381609156817824 \times 10^{7825}
       n= 17,q= \{6.623580732208701 \times 10^{15651}, -2.341096120141308 \times 10^{15651}, 0., 0.\}
        \sqrt{q} = 7.025137205765239 \times 10^{15651}
       n= 18,q= \{3.839109067234566 \times 10^{31303}, -3.101287830723303 \times 10^{31303}, 0., 0.\}
        ,\q\= 4.935255275982703 \times 10^{31303}
       n= 19,q= \{5.120772221130212 \times 10^{62606}, -2.381236446206810 \times 10^{62607}, 0., 0.\}
        \q = 2.435674463911511 \times 10^{62607}
       n= 20,q= \{-5.408063931336649 \times 10^{125214}, -2.438753889135731 \times 10^{125214}, 0., 0.\}
        ,\q\= 5.932510094150626 \times 10^{125214}
ln[81]:= P = \{1.2, -0.5, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[81]= \{1.2, -0.5, 0, 0\}
```

```
n= 1, q= \{1.2, -0.5, 0, 0\}, q= 1.3
        n= 2,q= \{2.39, -1.7, 0., 0.\}, q= 2.93293
        n= 3,q= \{4.0221, -8.626, 0., 0.\}, \q = 9.51762
        n=4,q=\{-57.0306, -69.8893, 0., 0.\}, q=90.2053
        n=5,q=\{-1630.82,7971.15,0.,0.\}, q=8136.27
        n = 6,q = \{-6.08797 \times 10^7, -2.59991 \times 10^7, 0., 0.\}, \q = 6.61988 \times 10^7
        n= 7,q= \{3.03038 \times 10^{15}, 3.16563 \times 10^{15}, 0., 0.\}, \q\= 4.38229 \times 10^{15}
        n= 8,q= \{-8.37977 \times 10^{29}, 1.91862 \times 10^{31}, 0., 0.\}, q= 1.92044 \times 10^{31}
         \text{n= 9,q= } \left\{ \text{-3.67406} \times \text{10}^{62} \text{, -3.21551} \times \text{10}^{61} \text{, 0., 0.} \right\} \text{,} \text{,} \text{q} \text{= 3.68811} \times \text{10}^{62} 
        n= 10,q= \{1.33953 \times 10^{125}, 2.3628 \times 10^{124}, 0., 0.\},\q\= 1.36021 \times 10^{125}
        n= 11,q= \{1.73852 \times 10^{250}, 6.33009 \times 10^{249}, 0., 0.\},\q\= 1.85018 \times 10^{250}
        n= 12,q= \{2.621758059145861 \times 10^{500}, 2.201000113042697 \times 10^{500}, 0., 0.\}
        \sqrt{q} = 3.423158894692187 \times 10^{500}
        n= 13,q= \{2.029213823082307 \times 10^{1000}, 1.154097956910128 \times 10^{1001}, 0., 0.\}
         ,\q\= 1.171801681831023\times10^{1001}
        n= 14,q= \{-1.290765006746249 \times 10^{2002}, 4.683823054706161 \times 10^{2001}, 0., 0.\}
         \sqrt{q} = 1.373119181542015 \times 10^{2002}
        n= 15,q= \{1.446692318562675 \times 10^{4004}, -1.209142979361207 \times 10^{4004}, 0., 0.\}
         ,\q\= 1.885456286718614 \times 10^{4004}
        n= 16,q= \{6.308919200497515 \times 10^{8007}, -3.498515720571689 \times 10^{8008}, 0., 0.\}
         \q = 3.554945409126743 \times 10^{8008}
        n= 17,q= \{-1.184158763230318 \times 10^{16017}, -4.414370600551425 \times 10^{16016}, 0., 0.\}
         ,\q\ =\ 1.263763686187130\times 10^{16017}
        n= 18,q= \{1.207365298545029 \times 10^{32034}, 1.045463126157850 \times 10^{32034}, 0., 0.\}
         ,\q\ =\ 1.597098654525284 \times 10^{32034}
        n= 19,q= \{3.647378159749816 \times 10^{64067}, 2.524511798862785 \times 10^{64068}, 0., 0.\}
        \q = 2.550724112286472 \times 10^{64068}
        n= 20,q= \left\{-6.240126148195213\times10^{128136}, 1.841569839840568\times10^{128136}, 0., 0.\right\}
         ,\q\ = 6.506193496999612 \times 10^{128136}
ln[85]:= P = \{1.2, -0.5, 0, 0\}
        quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
        iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[85]= \{1.2, -0.5, 0, 0\}
```

 $n= 1,q= \{1.2, -0.5, 0, 0\}, q= 1.3$

```
n= 2,q= \{2.39, -1.7, 0., 0.\}, q= 2.93293
       n= 3,q= \{4.0221, -8.626, 0., 0.\}, \q = 9.51762
       n=4,q=\{-57.0306, -69.8893, 0., 0.\}, q=90.2053
       n=5,q=\{-1630.82,7971.15,0.,0.\}, q=8136.27
       n = 6,q = \{-6.08797 \times 10^7, -2.59991 \times 10^7, 0., 0.\}, q = 6.61988 \times 10^7\}
       n= 7,q= \{3.03038 \times 10^{15}, 3.16563 \times 10^{15}, 0., 0.\}, \q\= 4.38229 \times 10^{15}
       n= 8,q= \{-8.37977 \times 10^{29}, 1.91862 \times 10^{31}, 0., 0.\}, q = 1.92044 \times 10^{31}
       n= 9,q= \{-3.67406 \times 10^{62}, -3.21551 \times 10^{61}, 0., 0.\},\q\= 3.68811×10<sup>62</sup>
       n= 10,q= \{1.33953 \times 10^{125}, 2.3628 \times 10^{124}, 0., 0.\},\q\= 1.36021 \times 10^{125}
       n= 11,q= \{1.73852 \times 10^{250}, 6.33009 \times 10^{249}, 0., 0.\},\q\= 1.85018 \times 10^{250}
       n= 12,q= \{2.621758059145861 \times 10^{500}, 2.201000113042697 \times 10^{500}, 0., 0.\}
        \sqrt{q} = 3.423158894692187 \times 10^{500}
       n= 13,q= \{2.029213823082307 \times 10^{1000}, 1.154097956910128 \times 10^{1001}, 0., 0.\}
        ,\q\= 1.171801681831023\times10^{1001}
       n= 14,q= \{-1.290765006746249 \times 10^{2002}, 4.683823054706161 \times 10^{2001}, 0., 0.\}
        n= 15,q= \{1.446692318562675 \times 10^{4004}, -1.209142979361207 \times 10^{4004}, 0., 0.\}
        , q = 1.885456286718614 \times 10^{4004}
       n= 16,q= \{6.308919200497515 \times 10^{8007}, -3.498515720571689 \times 10^{8008}, 0., 0.\}
        \q = 3.554945409126743 \times 10^{8008}
       n= 17,q= \{-1.184158763230318 \times 10^{16017}, -4.414370600551425 \times 10^{16016}, 0., 0.\}
        \sqrt{q} = 1.263763686187130 \times 10^{16017}
       n= 18,q= \{1.207365298545029 \times 10^{32034}, 1.045463126157850 \times 10^{32034}, 0., 0.\}
        ,\q\= 1.597098654525284\times10^{32034}
       n= 19,q= \{3.647378159749816 \times 10^{64067}, 2.524511798862785 \times 10^{64068}, 0., 0.\}
        \q = 2.550724112286472 \times 10^{64068}
       n= 20,q= \{-6.240126148195213 \times 10^{128136}, 1.841569839840568 \times 10^{128136}, 0., 0.\}
         ,\q\= 6.506193496999612 \times 10^{128136}
ln[89]:= P = \{1.2, -0.7, 0, 0\}
       quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[89]= \{1.2, -0.7, 0, 0\}
```

```
n= 1,q= \{1.2, -0.7, 0, 0\}, q= 1.38924
       n= 2,q= \{2.15, -2.38, 0., 0.\}, q= 3.20732
       n= 3,q= \{0.1581, -10.934, 0., 0.\}, \q = 10.9351
       n= 4, q= \{-118.327, -4.15733, 0., 0.\}, q= 118.4
       n= 5, q= \{13985.3, 983.152, 0., 0.\}, q= 14019.8
       n=6,q=\{1.94621\times10^8, 2.74993\times10^7, 0., 0.\}, q=1.96555\times10^8
       n= 7,q= \{3.71213 \times 10^{16}, 1.07039 \times 10^{16}, 0., 0.\}, \q\= 3.86337 \times 10^{16}
       n= 8,q= \{1.26342 \times 10^{33}, 7.94687 \times 10^{32}, 0., 0.\}, \q\= 1.49257 \times 10^{33}
       n=9,q=\{9.64698\times10^{65}, 2.00804\times10^{66}, 0., 0.\}, q=2.22775\times10^{66}\}
       n= 10,q= \left\{-3.10159\times10^{132}, 3.87431\times10^{132}, 0., 0.\right\},\q\= 4.96288\times10^{132}
       n= 11,q= \{-5.39041 \times 10^{264}, -2.40331 \times 10^{265}, 0., 0.\},\q\= 2.46302×10<sup>265</sup>
       n= 12,q= \{-5.485329218685683 \times 10^{530}, 2.590962267205347 \times 10^{530}, 0., 0.\}
        \sqrt{q} = 6.066458778187548 \times 10^{530}
       n= 13,q= \{2.337575116728501 \times 10^{1061}, -2.842456205762718 \times 10^{1061}, 0., 0.\}
        \sqrt{q} = 3.680192210744875 \times 10^{1061}
       n= 14,q= \{-2.615299855330720 \times 10^{2122}, -1.328890979396288 \times 10^{2123}, 0., 0.\}
        \sqrt{q} = 1.354381470802725 \times 10^{2123}
       n= 15,q= \{-1.697553301787895 \times 10^{4246}, 6.950896772330818 \times 10^{4245}, 0., 0.\}
        ,\q\ =\ 1.834349168453753\times 10^{4246}
       n= 16,q= \{2.398537553014996 \times 10^{8492}, -2.359903553251401 \times 10^{8492}, 0., 0.\}
        \q = 3.364836871806975 \times 10^{8492}
       n= 17,q= \{1.838376125745748 \times 10^{16983}, -1.132063458793402 \times 10^{16985}, 0., 0.\}
        |q| = 1.132212717387175 \times 10^{16985}
       n= 18,q= \{-1.281229712057309 \times 10^{33970}, -4.162316870949891 \times 10^{33968}, 0., 0.\}
        ,\q\= 1.281905637413252 \times 10^{33970}
       n= 19,q= \{1.639817086885036 \times 10^{67940}, 1.066576809211682 \times 10^{67939}, 0., 0.\}
        \q = 1.643282063231875 \times 10^{67940}
       n= 20,q= \{2.677624217540644 \times 10^{135880}, 3.497981752441274 \times 10^{135879}, 0., 0.\}
         ,\q\= 2.700375939339607 \times 10^{135880}
ln[93]:= P = \{1.2, -1.2, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus@@ (q<sup>2</sup>)]]];
            n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[93]= \{1.2, -1.2, 0, 0\}
```

 $n= 1,q= \{1.2, -1.2, 0, 0\}, q= 1.69706$

```
n= 2,q= \{1.2, -4.08, 0., 0.\}, q= 4.25281
       n= 3,q= \{-14.0064, -10.992, 0., 0.\}, \q = 17.8046
       n= 4, q= \{76.5552, 306.717, 0., 0.\}, q= 316.126
       n=5,q=\{-88213.2,46960.3,0.,0.\},\q\=99934.2
       n=6,q=\{5.57631\times10^9, -8.28504\times10^9, 0., 0.\}, q=9.98685\times10^9
       n= 7,q= \{-3.75467 \times 10^{19}, -9.23998 \times 10^{19}, 0., 0.\}, \q\= 9.97371 \times 10^{19}
       n= 8,q= \{-7.12797 \times 10^{39}, 6.93862 \times 10^{39}, 0., 0.\}, q = 9.94748 \times 10^{39}
       n = 9,q = \{2.66354 \times 10^{78}, -9.89166 \times 10^{79}, 0., 0.\}, q = 9.89525 \times 10^{79}\}
       n= 10,q= \{-9.7774 \times 10^{159}, -5.26937 \times 10^{158}, 0., 0.\}, \q\= 9.79159 \times 10<sup>159</sup>
       n= 12,q= \{8.979701217227850 \times 10^{639}, 1.964380053923242 \times 10^{639}, 0., 0.\}
        ,\q\ = 9.19205216189153 \times 10^{639}
       n= 13,q= \{7.677624495443186 \times 10^{1279}, 3.527909192262530 \times 10^{1279}, 0., 0.\}
        \q = 8.449382294693482 \times 10^{1279}
       n= 14,q= \{4.649977462417878 \times 10^{2559}, 5.417192406442796 \times 10^{2559}, 0., 0.\}
        \q = 7.139206116187969 \times 10^{2559}
       n= 15,q= \{-7.723683167427284 \times 10^{5118}, 5.037964519908054 \times 10^{5119}, 0., 0.\}
        \sqrt{q} = 5.096826396941570 \times 10^{5119}
       n= 16,q= \{-2.478453368714440 \times 10^{10239}, -7.782328352101944 \times 10^{10238}, 0., 0.\}
        \sqrt{q} = 2.597763932056039 \times 10^{10239}
       n= 17,q= \{5.537084755092657 \times 10^{20478}, 3.857627584141791 \times 10^{20478}, 0., 0.\}
        \sqrt{q} = 6.748377446691252 \times 10^{20478}
       n= 18,q= \{1.577801700714788 \times 10^{40957}, 4.272002177395286 \times 10^{40957}, 0., 0.\}
        ,\q\= 4.554059816301114 \times 10^{40957}
       n= 19,q= \{-1.576054439689159 \times 10^{81915}, 1.348074460190312 \times 10^{81915}, 0., 0.\}
        \q = 2.073946081044854 \times 10^{81915}
       n= 20,q= \{6.666428466465081 \times 10^{163829}, -4.249277476029013 \times 10^{163830}, 0., 0.\}
        ,\q\= 4.301252347081308 \times 10^{163830}
ln[97]:= P = \{1.2, -1.4, 0, 0\}
       quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
       iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
           Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
           n++]]
       iteration[
         {0,
          0,
          0,
          0}]
Out[97]= \{1.2, -1.4, 0, 0\}
```

```
n= 1,q= \{1.2, -1.4, 0, 0\}, q= 1.84391
n= 2,q= \{0.68, -4.76, 0., 0.\}, q= 4.80833
n= 3,q= \{-20.9952, -7.8736, 0., 0.\}, \q \ge 22.423
n= 4,q= \{380.005, 329.216, 0., 0.\}, \q = 502.779
n=5,q=\{36022., 250206., 0., 0.\}, q=252785.
n= 6,q= \{-6.13053 \times 10^{10}, 1.80258 \times 10^{10}, 0., 0.\}, \q\= 6.39005 \times 10^{10}
n= 7,q= \{3.43341 \times 10^{21}, -2.21015 \times 10^{21}, 0., 0.\},\q\= 4.08327 \times 10^{21}
n= 8,q= \{6.90352 \times 10^{42}, -1.51767 \times 10^{43}, 0., 0.\}, \q\= 1.66731 \times 10^{43}
n= 9,q= \left\{-1.82674\times10^{86}, -2.09546\times10^{86}, 0., 0.\right\},\q\= 2.77991\times10^{86}
n= 10,q= \{-1.05395 \times 10^{172}, 7.65572 \times 10^{172}, 0., 0.\}, \q\= 7.72793 \times 10^{172}
n= 11,q= \{-5.749923850784894 \times 10^{345}, -1.613741499892428 \times 10^{345}, 0., 0.\}
 |q| = 5.972083884064259 \times 10^{345}
n= 12,q= \{3.045746266134992 \times 10^{691}, 1.855778147846572 \times 10^{691}, 0., 0.\}
 \sqrt{q} = 3.566578591830005 \times 10^{691}
n= 13,q= \{5.832657783650395 \times 10^{1382}, 1.130445872915722 \times 10^{1383}, 0., 0.\}
 ,\q\= 1.272048285170010 \times 10^{1383}
n= 14,q= \{-9.37708903380412 \times 10^{2765}, 1.318700783931470 \times 10^{2766}, 0., 0.\}
 \sqrt{q} = 1.618106839803963 \times 10^{2766}
n= 15,q= \{-8.596737700625773\times10^{5531}, -2.473114931974537\times10^{5532}, 0., 0.\}
 \sqrt{q} = 2.618269745020368 \times 10^{5532}
n= 16,q= \{-5.377258475821815 \times 10^{11064}, 4.252144074737210 \times 10^{11064}, 0., 0.\}
\q = 6.855336457689025 \times 10^{11064}
n= 17,q= \{1.083417948347479 \times 10^{22129}, -4.572975553259234 \times 10^{22129}, 0., 0.\}
 \sqrt{q} = 4.699563794812031 \times 10^{22129}
n= 18,q= \{-1.973831095990514 \times 10^{44259}, -9.90888758351059 \times 10^{44258}, 0., 0.\}
 ,\q\= 2.208589986150806 \times 10^{44259}
n= 19,q= \{2.914148664072610 \times 10^{88518}, 3.911694087801502 \times 10^{88518}, 0., 0.\}
 \sqrt{q} = 4.877869726925617 \times 10^{88518}
n= 20,q=\{-6.809088200225046\times10^{177036}, 2.279851620045494\times10^{177037}, 0., 0.\}
 \sqrt{q} = 2.379361307285740 \times 10^{177037}
quaternion[\{x_{y}, y_{z}, z_{w}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
```

 $n= 1,q= \{1.5, 1.4, 0, 0\}, q= 2.05183$

```
n= 2,q= \{1.79, 5.6, 0., 0.\}, q= 5.87912
        n= 3,q= \{-26.6559, 21.448, 0., 0.\}, \q = 34.2134
        n= 4,q= \{252.02, -1142.03, 0., 0.\}, q= 1169.51
        n = 5,q = \{-1.24072 \times 10^6, -575629., 0., 0.\}, q = 1.36775 \times 10^6\}
        n= 6,q= \{1.20804 \times 10^{12}, 1.42839 \times 10^{12}, 0., 0.\}, q= 1.87074 \times 10^{12}
        n= 7,q= \{-5.80938 \times 10^{23}, 3.4511 \times 10^{24}, 0., 0.\}, \q\= 3.49965 \times 10^{24}
        n= 8,q= \{-1.15726 \times 10^{49}, -4.00975 \times 10^{48}, 0., 0.\}, q= 1.22475 \times 10^{49}\}
        n = 9,q = \{1.17846 \times 10^{98}, 9.28062 \times 10^{97}, 0., 0.\}, q = 1.50002 \times 10^{98}\}
        n= 10,q= \{5.27478 \times 10^{195}, 2.18737 \times 10^{196}, 0., 0.\},\q\= 2.25007 \times 10^{196}
        n= 11,q= \{-4.506368803642956 \times 10^{392}, 2.307580477805702 \times 10^{392}, 0., 0.\}
         |q| = 5.062833935257648 \times 10^{392}
        n= 12,q= \{1.498243213289646 \times 10^{785}, -2.079761735415824 \times 10^{785}, 0., 0.\}
         \sqrt{q} = 2.563228745599644 \times 10^{785}
        n= 13,q= \{-2.080676149931359 \times 10^{1570}, -6.231977810692509 \times 10^{1570}, 0., 0.\}
         \q = 6.570141602268325 \times 10^{1570}
        n= 14,q= \{-3.450833419207062 \times 10^{3141}, 2.593345519521870 \times 10^{3141}, 0., 0.\}
         \sqrt{q} = 4.316676067385699 \times 10^{3141}
        n= 15,q= \{5.182810303492141\times10^{6282}, -1.789840677263394\times10^{6283}, 0., 0.\}
         q = 1.863369227074046 \times 10^{6283}
        n= 16,q= \{-2.934914423566842 \times 10^{12566}, -1.855280940746014 \times 10^{12566}, 0., 0.\}
         , q = 3.472144876406528 \times 10^{12566}
        n= 17,q= \{5.171655304565275 \times 10^{25132}, 1.089018158552827 \times 10^{25133}, 0., 0.\}
         ,\q\= 1.205579004275611 \times 10^{25133}
        n= 18,q= \{-9.18500363765410 \times 10^{50265}, 1.126405307289527 \times 10^{50266}, 0., 0.\}
         ,\q\= 1.453420735550172 \times 10^{50266}
        n= 19,q= \{-4.251459980528252 \times 10^{100531}, -2.069207368985438 \times 10^{100532}, 0., 0.\}
         ,\q\= 2.112431834527204 \times 10^{100532}
        n= 20,q= \{-4.100870016203305 \times 10^{201064}, 1.759430464131149 \times 10^{201064}, 0., 0.\}
         \sqrt{q} = 4.462368255523970 \times 10^{201064}
In[105]:= P = \{1.5, 1.2, 0, 0\}
        quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
             Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[105]= \{1.5, 1.2, 0, 0\}
```

```
n= 2,q= \{2.31, 4.8, 0., 0.\}, q= 5.32692
        n= 3,q= \{-16.2039, 23.376, 0., 0.\}, q= 28.443
        n= 4, q= \{-282.371, -756.365, 0., 0.\}, q= 807.354
        n=5,q=\{-492353., 427152., 0., 0.\}, q=651821.
        n= 6,q= \{5.99523 \times 10^{10}, -4.20619 \times 10^{11}, 0., 0.\}, \q\= 4.2487 \times 10^{11}
        n= 7,q= \{-1.73326 \times 10^{23}, -5.04341 \times 10^{22}, 0., 0.\}, \q\= 1.80515 \times 10^{23}
        n=8,q=\{2.74983\times10^{46}, 1.74831\times10^{46}, 0., 0.\}, q=3.25855\times10^{46}\}
        n=9,q=\{4.505\times10^{92}, 9.61512\times10^{92}, 0., 0.\}, q=1.06182\times10^{93}
        n= 10,q= \{-7.21556 \times 10^{185}, 8.66322 \times 10^{185}, 0., 0.\}, \q\= 1.12746 \times 10^{186}
        n= 11,q= \{-2.298705891478494 \times 10^{371}, -1.250199632295119 \times 10^{372}, 0., 0.\}
         |q| = 1.271156799276167 \times 10^{372}
        n= 12,q= \{-1.510158632835671 \times 10^{744}, 5.747682520562074 \times 10^{743}, 0., 0.\}
         \sqrt{q} = 1.615839608346030 \times 10^{744}
        n= 13,q= \{1.950220552756356 \times 10^{1488}, -1.735982475445102 \times 10^{1488}, 0., 0.\}
         \sqrt{q} = 2.610937639899852 \times 10^{1488}
        n= 14,q= \{7.897250493408041 \times 10^{2975}, -6.771097405675786 \times 10^{2976}, 0., 0.\}
         \sqrt{q} = 6.816995359445809 \times 10^{2976}
        n= 15,q= \{-4.522409442359303 \times 10^{5953}, -1.069461046557740 \times 10^{5953}, 0., 0.\}
         \sqrt{q} = 4.647142573070570 \times 10^{5953}
        n= 16,q= \{1.930844023423620 \times 10^{11907}, 9.67308147037637 \times 10^{11906}, 0., 0.\}
         , q = 2.159593409444496 \times 10^{11907}
        n= 17,q= \{2.792473591465327 \times 10^{23814}, 3.735442309033197 \times 10^{23814}, 0., 0.\}
         \sqrt{q} = 4.663843694116102 \times 10^{23814}
        n= 18,q= \{-6.155620485084001 \times 10^{47628}, 2.086224800083493 \times 10^{47629}, 0., 0.\}
         ,\q\= 2.175143800314653 \times 10^{47629}
        n= 19,q= \{-3.973417280919554 \times 10^{95258}, -2.568401623176845 \times 10^{95258}, 0., 0.\}
         \sqrt{q} = 4.731250552047270 \times 10^{95258}
        n= 20,q= \{9.19135799037269 \times 10^{190516}, 2.041066278774542 \times 10^{190517}, 0., 0.\}
         ,\q\ =\ 2.238473178624759 \times 10^{190517}
In[109]:= P = \{1.5, 0.7, 0, 0\}
        quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
             Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[109]= \{1.5, 0.7, 0, 0\}
```

 $n= 1,q= \{1.5, 1.2, 0, 0\}, q= 1.92094$

 $n= 1,q= \{1.5, 0.7, 0, 0\}, q= 1.65529$

```
n= 2,q= \{3.26, 2.8, 0., 0.\}, q= 4.29739
        n= 3,q= \{4.2876, 18.956, 0., 0.\}, \q = 19.4349
        n= 4, q= \{-339.446, 163.251, 0., 0.\}, q= 376.663
        n=5,q=\{88574.3, -110830., 0., 0.\}, q= 141875.
        n= 6,q= \{-4.43778\times10^9, -1.96333\times10^{10}, 0., 0.\},\q\= 2.01286\times10^{10}
        n= 7,q= \{-3.65773\times10^{20}, 1.74257\times10^{20}, 0., 0.\},\q\= 4.05161\times10^{20}
        n= 8,q= \{1.03424 \times 10^{41}, -1.27477 \times 10^{41}, 0., 0.\}, \q\= 1.64155 \times 10^{41}
        n = 9,q = \{-5.55372 \times 10^{81}, -2.63684 \times 10^{82}, 0., 0.\}, q = 2.69469 \times 10^{82}\}
        n= 10,q= \{-6.6445 \times 10^{164}, 2.92885 \times 10^{164}, 0., 0.\},\q\= 7.26137 \times 10^{164}
        n= 11,q= \{3.557113583294676 \times 10^{329}, -3.892150961232142 \times 10^{329}, 0., 0.\}
         |q| = 5.272750340143156 \times 10^{329}
        n= 12,q= \{-2.495782060560797 \times 10^{658}, -2.768964610486456 \times 10^{659}, 0., 0.\}
          |q| = 2.780189614947977 \times 10^{659}
        n= 13,q= \{-7.604875733188239 \times 10^{1318}, 1.382146440235962 \times 10^{1318}, 0., 0.\}
         \sqrt{q} = 7.729454295064580 \times 10^{1318}
        n= 14,q= \{5.592380613497841 \times 10^{2637}, -2.102210384612595 \times 10^{2637}, 0., 0.\}
         \sqrt{q} = 5.974446369949229 \times 10^{2637}
        n= 15,q= \{2.685543242505345 \times 10^{5275}, -2.351272120080263 \times 10^{5275}, 0., 0.\}
         \sqrt{q} = 3.569400942739952 \times 10^{5275}
        n= 16,q= \{1.683661924699387 \times 10^{10550}, -1.262888590674553 \times 10^{10551}, 0., 0.\}
         , q = 1.274062309003286 \times 10^{10551}
        n= 17,q= \{-1.566540417689135 \times 10^{21102}, -4.252554870512029 \times 10^{21101}, 0., 0.\}
         \sqrt{q} = 1.623234767222784 \times 10^{21102}
        n= 18,q= \{2.273206650986495 \times 10^{42204}, 1.332359816619576 \times 10^{42204}, 0., 0.\}
         \sqrt{q} = 2.634891109520807 \times 10^{42204}
        n= 19,q= \{3.392285797146686 \times 10^{84408}, 6.057458393293535 \times 10^{84408}, 0., 0.\}
         \sqrt{q} = 6.942651159031788 \times 10^{84408}
        n= 20,q=\{-2.518519925695918\times10^{168817}, 4.109726014875328\times10^{168817}, 0., 0.\}
         \sqrt{q} = 4.820040511600542 \times 10^{168817}
ln[113]:= P = \{1.5, 0.5, 0, 0\}
        quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
             Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[113]= \{1.5, 0.5, 0, 0\}
```

```
n= 1, q= \{1.5, 0.5, 0, 0\}, q= 1.58114
        n= 2,q= \{3.5, 2., 0., 0.\}, q= 4.03113
        n= 3,q= \{9.75, 14.5, 0., 0.\}, q= 17.4732
        n= 4,q= \{-113.688, 283.25, 0., 0.\}, q= 305.214
        n=5,q=\{-67304.2, -64403.5, 0., 0.\}, q=93154.
        n=6,q=\{3.82051\times10^8, 8.66925\times10^9, 0., 0.\}, q=8.67766\times10^9
        n= 7,q= \{-7.50099 \times 10^{19}, 6.62418 \times 10^{18}, 0., 0.\}, q= 7.53019 \times 10^{19}
        n= 8,q= \{5.58261 \times 10^{39}, -9.93759 \times 10^{38}, 0., 0.\}, q= 5.67037 \times 10^{39}\}
        n= 9,q= {3.0178\times10^{79}}, -1.10955\times10^{79}, 0., 0.},\q\= 3.21531\times10^{79}
        n= 10,q= \{7.87599 \times 10^{158}, -6.69682 \times 10^{158}, 0., 0.\}, \q\= 1.03382 \times 10^{159}
        n= 11,q= \{1.718388547301242 \times 10^{317}, -1.054881491231032 \times 10^{318}, 0., 0.\}
         |q| = 1.068786018123724 \times 10^{318}
        n= 12,q= \{-1.083246368546844 \times 10^{636}, -3.625392546582919 \times 10^{635}, 0., 0.\}
         ,\q\= 1.142303552536766 \times 10^{636}
        n= 13,q= \{1.041987983801735 \times 10^{1272}, 7.854386621285486 \times 10^{1271}, 0., 0.\}
         ,\q\= 1.304857406138115 \times 10^{1272}
        n= 14,q= \{4.688250664209213 \times 10^{2543}, 1.636835295902518 \times 10^{2544}, 0., 0.\}
         \sqrt{q} = 1.702652850353490 \times 10^{2544}
        n= 15,q= \{-2.459432843007701 \times 10^{5088}, 1.534778832643212 \times 10^{5088}, 0., 0.\}
         \sqrt{q} = 2.899026728816863 \times 10^{5088}
        n= 16,q= \{3.693263844135281 \times 10^{10176}, -7.549370935511472 \times 10^{10176}, 0., 0.\}
         , q = 8.404355974394604 \times 10^{10176}
        n= 17,q= \{-4.335280369954845 \times 10^{20353}, -5.576363744418052 \times 10^{20353}, 0., 0.\}
         \sqrt{q} = 7.063319934434227 \times 10^{20353}
        n= 18,q=\{-1.230117672394431\times10^{40707}, 4.835020055380695\times10^{40707}, 0., 0.\}
         \sqrt{q} = 4.989048849617594 \times 10^{40707}
        n= 19,q= \{-2.186422944799645 \times 10^{81415}, -1.189528723301058 \times 10^{81415}, 0., 0.\}
         \sqrt{q} = 2.489060842387063 \times 10^{81415}
        n= 20,q= \{3.365466709988106 \times 10^{162830}, 5.201625788247325 \times 10^{162830}, 0., 0.\}
         ,\q\= 6.195423877104598 \times 10^{162830}
ln[117] = P = \{1.5, 0.4, 0, 0\}
        quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
            Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
            n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[117]= \{1.5, 0.4, 0, 0\}
```

```
n= 1,q= \{1.5, 0.4, 0, 0\}, q= 1.55242
        n= 2,q= \{3.59, 1.6, 0., 0.\}, q= 3.93041
        n= 3,q= \{11.8281, 11.888, 0., 0.\}, q= 16.7699
        n= 4,q= \{0.0794056, 281.625, 0., 0.\}, q= 281.625
        n=5,q=\{-79311.1, 45.1252, 0., 0.\}, q=79311.1
        n = 6,q = \{6.29025 \times 10^9, -7.15786 \times 10^6, 0., 0.\}, q = 6.29025 \times 10^9
        n= 7,q= \{3.95671 \times 10^{19}, -9.00493 \times 10^{16}, 0., 0.\},\q\= 3.95672 \times 10^{19}
        n= 8,q= \{1.56555 \times 10^{39}, -7.12599 \times 10^{36}, 0., 0.\}, \q\= 1.56557 \times 10^{39}
        n= 9,q= \{2.4509 \times 10^{78}, -2.23122 \times 10^{76}, 0., 0.\}, \q\= 2.451 \times 10^{78}
        n= 10,q= \{6.0064 \times 10^{156}, -1.0937 \times 10^{155}, 0., 0.\}, \q\= 6.00739 \times 10^{156}
        n= 11,q= \{3.606485557065646 \times 10^{313}, -1.313836526660478 \times 10^{312}, 0., 0.\}
         \sqrt{q} = 3.608877905597663 \times 10^{313}
        n= 12,q= \{1.298947640913523\times10^{627}, -9.47666491549261\times10^{625}, 0., 0.\}
         \sqrt{q} = 1.302399973751098 \times 10^{627}
        n= 13,q= \{1.678284256042754 \times 10^{1254}, -2.461938307141416 \times 10^{1253}, 0., 0.\}
         \q = 1.696245691626860 \times 10^{1254}
        n= 14,q= \{2.756026641799277 \times 10^{2508}, -8.263664600447978 \times 10^{2507}, 0., 0.\}
         \sqrt{q} = 2.877249446362684 \times 10^{2508}
        n= 15,q= \{6.912801324020428 \times 10^{5016}, -4.554975959545641 \times 10^{5016}, 0., 0.\}
         |q| = 8.278564376594369 \times 10^{5016}
        n= 16,q= \{2.703901615333985 \times 10^{10033}, -6.297528768805665 \times 10^{10033}, 0., 0.\}
         , q = 6.853462813741731 \times 10^{10033}
        n= 17,q= \{-3.234778464852926 \times 10^{20067}, -3.405579642117176 \times 10^{20067}, 0., 0.\}
         \sqrt{q} = 4.696995253934073 \times 10^{20067}
        n= 18,q=\{-1.134180982126702\times10^{40134}, 2.203259137332435\times10^{40135}, 0., 0.\}
         \sqrt{q} = 2.206176441547921 \times 10^{40135}
        n= 19,q= \{-4.841487161236687 \times 10^{80270}, -4.997789224518663 \times 10^{80269}, 0., 0.\}
         \sqrt{q} = 4.867214491241045 \times 10^{80270}
        n= 20,q= \{2.319021896109253 \times 10^{160541}, 4.839346473014833 \times 10^{160540}, 0., 0.\}
         \sqrt{q} = 2.368977690374683 \times 10^{160541}
ln[121]:= P = \{1.5, 0.2, 0, 0\}
        quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
             Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[121]= \{1.5, 0.2, 0, 0\}
```

```
n= 2,q= \{3.71, 0.8, 0., 0.\}, q= 3.79527
        n= 3,q= \{14.6241, 6.136, 0., 0.\}, q= 15.8592
        n= 4,q= \{177.714, 179.667, 0., 0.\}, q= 252.71
        n=5,q=\{-696.518,63858.8,0.,0.\},\q\=63862.6
        n = 6,q = \{-4.07746 \times 10^9, -8.89576 \times 10^7, 0., 0.\}, \q = 4.07843 \times 10^9
        n= 7,q= \{1.66178 \times 10^{19}, 7.25443 \times 10^{17}, 0., 0.\}, \q\= 1.66336 \times 10^{19}
        n= 8,q= \{2.75624 \times 10^{38}, 2.41105 \times 10^{37}, 0., 0.\}, q= 2.76677 \times 10^{38}
        n= 9,q= \{7.53873 \times 10^{76}, 1.32909 \times 10^{76}, 0., 0.\},\q\= 7.655 \times 10^{76}
        n= 10,q= \{5.5066 \times 10^{153}, 2.00392 \times 10^{153}, 0., 0.\}, \q\= 5.8599 \times 10^{153}
        n= 11,q= \{2.6307 \times 10^{307}, 2.20696 \times 10^{307}, 0., 0.\}, \q\= 3.43384 \times 10^{307}
        n= 12,q= \left\{\text{2.049871379662801}\times\text{10}^{614}\text{, 1.161169809912458}\times\text{10}^{615}\text{, 0., 0.}\right\}
         , q = 1.179124698318096 \times 10^{615}
        n= 13,q= \{-1.306295600720527 \times 10^{1230}, 4.760497520536086 \times 10^{1229}, 0., 0.\}
         n= 14,q= \{1.479784830031501\times10^{2460}, -1.243723393663454\times10^{2460}, 0., 0.\}
         \sqrt{q} = 1.933031562892106 \times 10^{2460}
        n= 15,q= \{6.429152632456209 \times 10^{4919}, -3.680886021396951 \times 10^{4920}, 0., 0.\}
         \sqrt{q} = 3.736611023137097 \times 10^{4920}
        n= 16,q= \{-1.313558186680129 \times 10^{9841}, -4.732995610847094 \times 10^{9840}, 0., 0.\}
         n= 17,q= \{1.501422635271410 \times 10^{19682}, 1.243413026429863 \times 10^{19682}, 0., 0.\}
         \sqrt{q} = 1.949447584317367 \times 10^{19682}
        n= 18,q= \{7.081939754098726 \times 10^{39363}, 3.733776925746249 \times 10^{39364}, 0., 0.\}
         \sqrt{q} = 3.800345884000817 \times 10^{39364}
        n= 19,q= \{-1.343955142442827 \times 10^{78729}, 5.288476648675778 \times 10^{78728}, 0., 0.\}
         \q = 1.444262883804195 \times 10^{78729}
        n= 20,q= \{1.526535572262631 \times 10^{157458}, -1.421495077535325 \times 10^{157458}, 0., 0.\}
         ,\q\ =\ 2.085895277534411\times 10^{157458}
ln[125]:= P = \{1.5, -0.2, 0, 0\}
        quaternion[\{x_{y_{y_{z}}}, y_{y_{z}}, x_{y_{z}}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
            Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
            n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[125]= \{1.5, -0.2, 0, 0\}
```

 $n= 1,q= \{1.5, 0.2, 0, 0\}, q= 1.51327$

 $n= 1,q= \{1.5, -0.2, 0, 0\}, q= 1.51327$

```
n= 2,q= \{3.71, -0.8, 0., 0.\}, q= 3.79527
        n= 3,q= \{14.6241, -6.136, 0., 0.\}, \q = 15.8592
        n=4,q=\{177.714, -179.667, 0., 0.\}, q=252.71
        n=5,q=\{-696.518, -63858.8, 0., 0.\}, q=63862.6
        n = 6,q = \{-4.07746 \times 10^9, 8.89576 \times 10^7, 0., 0.\}, q = 4.07843 \times 10^9\}
        n= 7,q= \{1.66178 \times 10^{19}, -7.25443 \times 10^{17}, 0., 0.\},\q\= 1.66336 \times 10^{19}
        n= 8,q= \{2.75624 \times 10^{38}, -2.41105 \times 10^{37}, 0., 0.\}, q= 2.76677 \times 10^{38}
        n = 9,q = \{7.53873 \times 10^{76}, -1.32909 \times 10^{76}, 0., 0.\}, q = 7.655 \times 10^{76}\}
        n= 10,q= \{5.5066 \times 10^{153}, -2.00392 \times 10^{153}, 0., 0.\}, \q\= 5.8599 \times 10^{153}
        n= 11,q= \{2.6307 \times 10^{307}, -2.20696 \times 10^{307}, 0., 0.\},\q\= 3.43384\times 10^{307}
        n= 12,q= \{2.049871379662801 \times 10^{614}, -1.161169809912458 \times 10^{615}, 0., 0.\}
         , q = 1.179124698318096 \times 10^{615}
        n= 13,q= \{-1.306295600720527 \times 10^{1230}, -4.760497520536086 \times 10^{1229}, 0., 0.\}
         n= 14,q= \{1.479784830031501 \times 10^{2460}, 1.243723393663454 \times 10^{2460}, 0., 0.\}
         \sqrt{q} = 1.933031562892106 \times 10^{2460}
        n= 15,q= \{6.429152632456209 \times 10^{4919}, 3.680886021396951 \times 10^{4920}, 0., 0.\}
         \sqrt{q} = 3.736611023137097 \times 10^{4920}
        n= 16,q= \left\{-1.313558186680129\times10^{9841}\text{, 4.732995610847094}\times10^{9840}\text{, 0., 0.}\right\}
         \q = 1.396226193822966 \times 10^{9841}
        n= 17,q= \{1.501422635271410 \times 10^{19682}, -1.243413026429863 \times 10^{19682}, 0., 0.\}
         \sqrt{q} = 1.949447584317367 \times 10^{19682}
        n= 18,q= \{7.081939754098726 \times 10^{39363}, -3.733776925746249 \times 10^{39364}, 0., 0.\}
         ,\q\= 3.800345884000817 \times 10^{39364}
        n= 19,q= \{-1.343955142442827 \times 10^{78729}, -5.288476648675778 \times 10^{78728}, 0., 0.\}
         \q = 1.444262883804195 \times 10^{78729}
        n= 20,q= \{1.526535572262631 \times 10^{157458}, 1.421495077535325 \times 10^{157458}, 0., 0.\}
         ,\q\= 2.085895277534411 \times 10^{157458}
In[129]:= P = \{1.5, -0.4, 0, 0\}
        quaternion[\{x_{\_},\,y_{\_},\,z_{\_},\,w_{\_}\}] := \{x^2-y^2-z^2-w^2,\,2\,x\,y,\,2\,x\,z,\,2\,x\,w\}
        iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
             Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[129]= \{1.5, -0.4, 0, 0\}
```

```
n= 2,q= \{3.59, -1.6, 0., 0.\}, q= 3.93041
        n= 3,q= \{11.8281, -11.888, 0., 0.\}, \q = 16.7699
        n=4,q=\{0.0794056, -281.625, 0., 0.\}, q=281.625
        n=5,q=\{-79311.1, -45.1252, 0., 0.\}, q=79311.1
        n=6,q=\{6.29025\times10^9, 7.15786\times10^6, 0., 0.\},\q=6.29025\times10^9
        n= 7,q= \{3.95671\times10^{19}, 9.00493\times10^{16}, 0., 0.\},\q\= 3.95672\times10^{19}
        n= 8,q= \{1.56555 \times 10^{39}, 7.12599 \times 10^{36}, 0., 0.\}, q= 1.56557 \times 10^{39}\}
        n= 9,q= \{2.4509 \times 10^{78}, 2.23122 \times 10^{76}, 0., 0.\},\q\= 2.451 \times 10^{78}
        n= 10,q= \{6.0064 \times 10^{156}, 1.0937 \times 10^{155}, 0., 0.\},\q\= 6.00739 \times 10^{156}
        n= 11,q= \{3.606485557065646 \times 10^{313}, 1.313836526660478 \times 10^{312}, 0., 0.\}
         \sqrt{q} = 3.608877905597663 \times 10^{313}
        n= 12,q= \{1.298947640913523 \times 10^{627}, 9.47666491549261 \times 10^{625}, 0., 0.\}
         \sqrt{q} = 1.302399973751098 \times 10^{627}
        n= 13,q= \{1.678284256042754 \times 10^{1254}, 2.461938307141416 \times 10^{1253}, 0., 0.\}
         \q = 1.696245691626860 \times 10^{1254}
        n= 14,q= \{2.756026641799277 \times 10^{2508}, 8.263664600447978 \times 10^{2507}, 0., 0.\}
         \sqrt{q} = 2.877249446362684 \times 10^{2508}
        n= 15,q= \{6.912801324020428 \times 10^{5016}, 4.554975959545641 \times 10^{5016}, 0., 0.\}
         |q| = 8.278564376594369 \times 10^{5016}
        n= 16,q= \{2.703901615333985 \times 10^{10033}, 6.297528768805665 \times 10^{10033}, 0., 0.\}
         , q = 6.853462813741731 \times 10^{10033}
        n= 17,q= \{-3.234778464852926 \times 10^{20067}, 3.405579642117176 \times 10^{20067}, 0., 0.\}
         \sqrt{q} = 4.696995253934073 \times 10^{20067}
        n= 18,q= \{-1.134180982126702 \times 10^{40134}, -2.203259137332435 \times 10^{40135}, 0., 0.\}
         \sqrt{q} = 2.206176441547921 \times 10^{40135}
        n= 19,q= \{-4.841487161236687 \times 10^{80270}, 4.997789224518663 \times 10^{80269}, 0., 0.\}
         ,\q\= 4.867214491241045 \times 10^{80270}
        n= 20,q= \{2.319021896109253 \times 10^{160541}, -4.839346473014833 \times 10^{160540}, 0., 0.\}
         \sqrt{q} = 2.368977690374683 \times 10^{160541}
In[133]:= P = \{1.5, -0.5, 0, 0\}
        quaternion[\{x_{y}, y_{z}, z_{w}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
             Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[133]= \{1.5, -0.5, 0, 0\}
```

 $n= 1,q= \{1.5, -0.4, 0, 0\}, q= 1.55242$

 $n= 1,q= \{1.5, -0.5, 0, 0\}, q= 1.58114$

```
n= 2,q= \{3.5, -2., 0., 0.\}, q= 4.03113
        n= 3, q= \{9.75, -14.5, 0., 0.\}, q = 17.4732
        n= 4, q= \{-113.688, -283.25, 0., 0.\}, q= 305.214
        n=5,q=\{-67304.2,64403.5,0.,0.\},\q=93154.
        n=6,q=\{3.82051\times10^8, -8.66925\times10^9, 0., 0.\}, q=8.67766\times10^9
        n= 7,q= \{-7.50099 \times 10^{19}, -6.62418 \times 10^{18}, 0., 0.\},\q\= 7.53019×10<sup>19</sup>
        n= 8,q= \{5.58261 \times 10^{39}, 9.93759 \times 10^{38}, 0., 0.\}, q= 5.67037 \times 10^{39}
        n = 9, q = \{3.0178 \times 10^{79}, 1.10955 \times 10^{79}, 0., 0.\}, q = 3.21531 \times 10^{79}
        n= 10,q= \{7.87599 \times 10^{158}, 6.69682 \times 10^{158}, 0., 0.\}, q= 1.03382 \times 10^{159}\}
        n= 11,q= \{1.718388547301242 \times 10^{317}, 1.054881491231032 \times 10^{318}, 0., 0.\}
         \sqrt{q} = 1.068786018123724 \times 10^{318}
        n= 12,q= \{-1.083246368546844 \times 10^{636}, 3.625392546582919 \times 10^{635}, 0., 0.\}
         n= 13,q= \{1.041987983801735 \times 10^{1272}, -7.854386621285486 \times 10^{1271}, 0., 0.\}
         \q = 1.304857406138115 \times 10^{1272}
        n= 14,q= \{4.688250664209213 \times 10^{2543}, -1.636835295902518 \times 10^{2544}, 0., 0.\}
         \sqrt{q} = 1.702652850353490 \times 10^{2544}
        n= 15,q= \{-2.459432843007701 \times 10^{5088}, -1.534778832643212 \times 10^{5088}, 0., 0.\}
         ,\q\= 2.899026728816863 \times 10^{5088}
        n= 16,q= \{3.693263844135281 \times 10^{10176}, 7.549370935511472 \times 10^{10176}, 0., 0.\}
        \sqrt{q} = 8.404355974394604 \times 10^{10176}
        n= 17,q= \{-4.335280369954845 \times 10^{20353}, 5.576363744418052 \times 10^{20353}, 0., 0.\}
         \sqrt{q} = 7.063319934434227 \times 10^{20353}
        n= 18,q= \{-1.230117672394431 \times 10^{40707}, -4.835020055380695 \times 10^{40707}, 0., 0.\}
         \sqrt{q} = 4.989048849617594 \times 10^{40707}
        n= 19,q= \{-2.186422944799645 \times 10^{81415}, 1.189528723301058 \times 10^{81415}, 0., 0.\}
         \sqrt{q} = 2.489060842387063 \times 10^{81415}
        n= 20,q= \{3.365466709988106 \times 10^{162830}, -5.201625788247325 \times 10^{162830}, 0., 0.\}
         ,\q\= 6.195423877104598 \times 10^{162830}
In[137]:= P = \{1.5, -0.7, 0, 0\}
        quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
            Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
            n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[137]= \{1.5, -0.7, 0, 0\}
```

```
n= 1,q= \{1.5, -0.7, 0, 0\}, q= 1.65529
        n= 2,q= \{3.26, -2.8, 0., 0.\}, q= 4.29739
        n= 3,q= \{4.2876, -18.956, 0., 0.\}, \q = 19.4349
        n= 4,q= \{-339.446, -163.251, 0., 0.\}, q= 376.663
        n= 5, q= \{88574.3, 110830., 0., 0.\}, q= 141875.
        n = 6,q = \{-4.43778 \times 10^9, 1.96333 \times 10^{10}, 0., 0.\}, q = 2.01286 \times 10^{10}
        n= 7,q= \{-3.65773\times10^{20}, -1.74257\times10^{20}, 0., 0.\},\q\= 4.05161×10<sup>20</sup>
        n= 8,q= \{1.03424 \times 10^{41}, 1.27477 \times 10^{41}, 0., 0.\}, q= 1.64155 \times 10^{41}
        n= 9,q= \{-5.55372 \times 10^{81}, 2.63684 \times 10^{82}, 0., 0.\}, \q\= 2.69469 \times 10^{82}
        n= 10,q= \{-6.6445 \times 10^{164}, -2.92885 \times 10^{164}, 0., 0.\}, \q\= 7.26137 \times 10<sup>164</sup>
        n= 11,q= \{3.557113583294676 \times 10^{329}, 3.892150961232142 \times 10^{329}, 0., 0.\}
         |q| = 5.272750340143156 \times 10^{329}
        n= 12,q= \{-2.495782060560797 \times 10^{658}, 2.768964610486456 \times 10^{659}, 0., 0.\}
          |q| = 2.780189614947977 \times 10^{659}
        n= 13,q= \{-7.604875733188239 \times 10^{1318}, -1.382146440235962 \times 10^{1318}, 0., 0.\}
         \sqrt{q} = 7.729454295064580 \times 10^{1318}
        n= 14,q= \{5.592380613497841 \times 10^{2637}, 2.102210384612595 \times 10^{2637}, 0., 0.\}
         \sqrt{q} = 5.974446369949229 \times 10^{2637}
        n= 15,q= \{2.685543242505345 \times 10^{5275}, 2.351272120080263 \times 10^{5275}, 0., 0.\}
         \sqrt{q} = 3.569400942739952 \times 10^{5275}
        n= 16,q= \{1.683661924699387 \times 10^{10550}, 1.262888590674553 \times 10^{10551}, 0., 0.\}
         , q = 1.274062309003286 \times 10^{10551}
        n= 17,q= \left\{-1.566540417689135\times10^{21102},\,4.252554870512029\times10^{21101},\,0.,\,0.\right\}
         \sqrt{q} = 1.623234767222784 \times 10^{21102}
        n= 18,q= \{2.273206650986495 \times 10^{42204}, -1.332359816619576 \times 10^{42204}, 0., 0.\}
          ,\q\= 2.634891109520807 \times 10^{42204}
        n= 19,q= \{3.392285797146686 \times 10^{84408}, -6.057458393293535 \times 10^{84408}, 0., 0.\}
         \sqrt{q} = 6.942651159031788 \times 10^{84408}
        n= 20,q= \{-2.518519925695918 \times 10^{168817}, -4.109726014875328 \times 10^{168817}, 0., 0.\}
         \sqrt{q} = 4.820040511600542 \times 10^{168817}
ln[141]:= P = \{1.5, -1.2, 0, 0\}
        quaternion[\{x_{y}, y_{z}, z_{w}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
             Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[141]= \{1.5, -1.2, 0, 0\}
```

 $n= 1,q= \{1.5, -1.2, 0, 0\}, q= 1.92094$

```
n= 2,q= \{2.31, -4.8, 0., 0.\}, q= 5.32692
        n= 3,q= \{-16.2039, -23.376, 0., 0.\}, q= 28.443
        n=4,q=\{-282.371,756.365,0.,0.\},\q\=807.354
        n=5,q=\{-492353., -427152., 0., 0.\}, q=651821.
        n= 6,q= \{5.99523 \times 10^{10}, 4.20619 \times 10^{11}, 0., 0.\}, q= 4.2487 \times 10^{11}
        n= 7,q= \{-1.73326 \times 10^{23}, 5.04341 \times 10^{22}, 0., 0.\}, \q\= 1.80515 \times 10^{23}
        n= 8,q= \{2.74983 \times 10^{46}, -1.74831 \times 10^{46}, 0., 0.\}, \q\= 3.25855 \times 10^{46}
        n= 9,q= \{4.505 \times 10^{92}, -9.61512 \times 10^{92}, 0., 0.\}, \q\= 1.06182 \times 10<sup>93</sup>
        n= 10,q= \{-7.21556 \times 10^{185}, -8.66322 \times 10^{185}, 0., 0.\}, \q\= 1.12746 \times 10^{186}
        n= 11,q= \{-2.298705891478494 \times 10^{371}, 1.250199632295119 \times 10^{372}, 0., 0.\}
         |q| = 1.271156799276167 \times 10^{372}
        n= 12,q= \{-1.510158632835671 \times 10^{744}, -5.747682520562074 \times 10^{743}, 0., 0.\}
          \sqrt{q} = 1.615839608346030 \times 10^{744}
        n= 13,q= \{1.950220552756356 \times 10^{1488}, 1.735982475445102 \times 10^{1488}, 0., 0.\}
         \q = 2.610937639899852 \times 10^{1488}
        n= 14,q= \{7.897250493408041 \times 10^{2975}, 6.771097405675786 \times 10^{2976}, 0., 0.\}
         \sqrt{q} = 6.816995359445809 \times 10^{2976}
        n= 15,q= \{-4.522409442359303 \times 10^{5953}, 1.069461046557740 \times 10^{5953}, 0., 0.\}
         \sqrt{q} = 4.647142573070570 \times 10^{5953}
        n= 16,q= \{1.930844023423620 \times 10^{11907}, -9.67308147037637 \times 10^{11906}, 0., 0.\}
         \sqrt{q} = 2.159593409444496 \times 10^{11907}
        n= 17,q= \{2.792473591465327 \times 10^{23814}, -3.735442309033197 \times 10^{23814}, 0., 0.\}
         \sqrt{q} = 4.663843694116102 \times 10^{23814}
        n= 18,q= \{-6.155620485084001 \times 10^{47628}, -2.086224800083493 \times 10^{47629}, 0., 0.\}
          ,\q\= 2.175143800314653 \times 10^{47629}
        n= 19,q= \{-3.973417280919554 \times 10^{95258}, 2.568401623176845 \times 10^{95258}, 0., 0.\}
         \sqrt{q} = 4.731250552047270 \times 10^{95258}
        n= 20,q= \{9.19135799037269 \times 10^{190516}, -2.041066278774542 \times 10^{190517}, 0., 0.\}
         ,\q\ =\ 2.238473178624759 \times 10^{190517}
ln[145]:= P = \{1.5, -1.4, 0, 0\}
        quaternion[\{x_{,}, y_{,}, z_{,}, w_{,}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
             Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[145]= \{1.5, -1.4, 0, 0\}
```

```
n= 2,q= \{1.79, -5.6, 0., 0.\}, q= 5.87912
        n= 3,q= \{-26.6559, -21.448, 0., 0.\}, \q = 34.2134
        n= 4, q= \{252.02, 1142.03, 0., 0.\}, q= 1169.51
        n = 5, q = \{-1.24072 \times 10^6, 575629., 0., 0.\}, q = 1.36775 \times 10^6\}
        n= 6,q= \{1.20804 \times 10^{12}, -1.42839 \times 10^{12}, 0., 0.\}, \q\= 1.87074 \times 10^{12}
        n= 7,q= \{-5.80938 \times 10^{23}, -3.4511 \times 10^{24}, 0., 0.\}, \q\= 3.49965 \times 10^{24}
        n= 8,q= \{-1.15726 \times 10^{49}, 4.00975 \times 10^{48}, 0., 0.\}, q= 1.22475 \times 10^{49}\}
        n= 9,q= \{1.17846 \times 10^{98}, -9.28062 \times 10^{97}, 0., 0.\}, \q\= 1.50002 \times 10^{98}
        n= 10,q= \{5.27478 \times 10^{195}, -2.18737 \times 10^{196}, 0., 0.\}, \q\= 2.25007 \times 10^{196}
        n= 11,q= \{-4.506368803642956 \times 10^{392}, -2.307580477805702 \times 10^{392}, 0., 0.\}
         |q| = 5.062833935257648 \times 10^{392}
        n= 12,q= \{1.498243213289646\times10^{785}, 2.079761735415824\times10^{785}, 0., 0.\}
          \sqrt{q} = 2.563228745599644 \times 10^{785}
        n= 13,q= \{-2.080676149931359 \times 10^{1570}, 6.231977810692509 \times 10^{1570}, 0., 0.\}
         ,\q\ =\ 6.570141602268325 \times 10^{1570}
        n= 14,q= \{-3.450833419207062 \times 10^{3141}, -2.593345519521870 \times 10^{3141}, 0., 0.\}
         \sqrt{q} = 4.316676067385699 \times 10^{3141}
        n= 15,q= \left\{5.182810303492141\times10^{6282}\text{, }1.789840677263394\times10^{6283}\text{, 0., 0.}\right\}
         |q| = 1.863369227074046 \times 10^{6283}
        n= 16,q= \{-2.934914423566842 \times 10^{12566}, 1.855280940746014 \times 10^{12566}, 0., 0.\}
         \sqrt{q} = 3.472144876406528 \times 10^{12566}
        n= 17,q= \{5.171655304565275 \times 10^{25132}, -1.089018158552827 \times 10^{25133}, 0., 0.\}
         ,\q\= 1.205579004275611 \times 10^{25133}
        n= 18,q= \{-9.18500363765410 \times 10^{50265}, -1.126405307289527 \times 10^{50266}, 0., 0.\}
          \sqrt{q} = 1.453420735550172 \times 10^{50266}
        n= 19,q= \{-4.251459980528252 \times 10^{100531}, 2.069207368985438 \times 10^{100532}, 0., 0.\}
          ,\q\= 2.112431834527204 \times 10^{100532}
        n= 20,q=\{-4.100870016203305\times10^{201064}, -1.759430464131149\times10^{201064}, 0., 0.\}
         \sqrt{q} = 4.462368255523970 \times 10^{201064}
ln[149]:= P = {0.124, 0.624, 0, 0}
        quaternion[\{x_{y}, y_{z}, z_{w}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
        iteration[t_] := Module[{q = t, n = 1}, While[n \le 20, q = quaternion[q] + P;}
             Print["n=", n, ",", "q=", q, ",", "\q\= ", N[Sqrt[Plus@@(q^2)]]];
             n++]]
        iteration[
          {0,
           0,
           0,
           0}]
Out[149]= \{0.124, 0.624, 0, 0\}
```

 $n= 1,q= \{1.5, -1.4, 0, 0\}, q= 2.05183$

```
n= 1,q= \{0.124, 0.624, 0, 0\}, q= 0.636201
      n= 2, q= \{-0.25, 0.778752, 0., 0.\}, \q = 0.817896
      n= 3, q= \{-0.419955, 0.234624, 0., 0.\}, q= 0.481051
      n= 4,q= \{0.245314, 0.426937, 0., 0.\}, \q = 0.492396
      n=5,q=\{0.00190342,0.833467,0.,0.\},\q=0.833469
      n = 6, q = \{-0.570663, 0.627173, 0., 0.\}, \\ q = 0.84794
      n= 7, q= \{0.0563109, -0.0918092, 0., 0.\}, q= 0.107703
      n= 8, q= \{0.118742, 0.61366, 0., 0.\}, q= 0.625043
      n=10,q=\{-0.411619, 0.256869, 0., 0.\}, q=0.485192
      n= 11, q= \{0.227449, 0.412536, 0., 0.\}, q = 0.471083
      n= 12,q= \{0.00554678, 0.811662, 0., 0.\}, \q = 0.81168
      n= 13, q= \{-0.534764, 0.633004, 0., 0.\}, q= 0.828653
      n= 14, q= \{0.00927782, -0.0530153, 0., 0.\}, q= 0.053821
      n= 15, q= \{0.121275, 0.623016, 0., 0.\}, q= 0.63471
      n= 16, q= \{-0.249442, 0.775113, 0., 0.\}, q= 0.814261
      n= 17,q= {-0.414579, 0.237309, 0., 0.},q= 0.477694
      n= 18, q= \{0.23956, 0.427233, 0., 0.\}, q= 0.489813
      n=19,q=\{-0.0011389, 0.828696, 0., 0.\}, q=0.828697
      n=20,q=\{-0.562736,0.622112,0.,0.\},\q\=0.838866
ln[157] = P = \{0.124, 0.67, 0, 0\}
      quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q²)]]];
          n++]]
      iteration[
       {0,
        0,
        0,
        0}1
Out[157]= \{0.124, 0.67, 0, 0\}
```

```
n= 1,q= \{0.124, 0.67, 0, 0\}, q= 0.681378
      n= 2,q= \{-0.309524, 0.83616, 0., 0.\}, \q = 0.89161
      n= 3, q= \{-0.479358, 0.152377, 0., 0.\}, q= 0.502994
      n= 4,q= \{0.330566, 0.523914, 0., 0.\}, \q = 0.619483
      n= 5, q= \{-0.0412119, 1.01638, 0., 0.\}, q= 1.01721
      n=6,q=\{-0.907322, 0.586226, 0., 0.\},\q=1.08023
      n= 7, q= \{0.603571, -0.393792, 0., 0.\}, q= 0.720673
      n= 8, q= \{0.333226, 0.194637, 0., 0.\}, q= 0.385906
      n=9,q=\{0.197156,0.799716,0.,0.\},\q=0.82366
      n=10,q=\{-0.476676, 0.985338, 0., 0.\}, q=1.09458
      n= 11,q= {-0.619671, -0.269373, 0., 0.},q= 0.675687
      n= 12, q= \{0.43543, 1.00384, 0., 0.\}, q= 1.09421
      n= 13, q= \{-0.694105, 1.54421, 0., 0.\}, q= 1.69303
      n= 14, q= \{-1.7788, -1.47369, 0., 0.\}, q= 2.30995
      n= 15, q= \{1.11637, 5.91278, 0., 0.\}, q = 6.01725
      n= 16, q= \{-33.5907, 13.8717, 0., 0.\}, q= 36.3423
      n= 17, q= \{936.036, -931.252, 0., 0.\}, q= 1320.38
      n= 18,q= \{8932.64, -1.74337 \times 10^6, 0., 0.\}, \q\= 1.74339 \times 10^6
      n= 19,q= \{-3.03926 \times 10^{12}, -3.11458 \times 10^{10}, 0., 0.\},\q\= 3.03942\times 10^{12}
      n= 20,q= \{9.23613 \times 10^{24}, 1.8932 \times 10^{23}, 0., 0.\}, \q\= 9.23807 \times 10^{24}
ln[161] = P = \{0.14, 0.624, 0, 0\}
      quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
          Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
          n++]]
      iteration[
        {0,
         0,
         0,
         0}1
```

Out[161]= $\{0.14, 0.624, 0, 0\}$

```
n= 1,q= \{0.14, 0.624, 0, 0\}, q= 0.639512
      n= 2,q= \{-0.229776, 0.79872, 0., 0.\}, \q = 0.831114
      n= 3, q= \{-0.445157, 0.256947, 0., 0.\}, q= 0.51399
      n=4,q=\{0.272143,0.395237,0.,0.\},\q=0.479869
      n=5,q=\{0.0578494,0.839122,0.,0.\},\q=0.841114
      n=6,q=\{-0.560779,0.721085,0.,0.\},\q=0.913475
      n=7,q=\{-0.0654912, -0.184739, 0., 0.\}, q=0.196004
      n= 8, q= \{0.110161, 0.648198, 0., 0.\}, q= 0.657492
      n= 10,q= {-0.376163, 0.212951, 0., 0.},q= 0.432258
      n= 11,q= {0.23615, 0.463791, 0., 0.},q= 0.520451
      n= 12, q= \{-0.0193356, 0.843049, 0., 0.\}, q= 0.843271
      n= 13, q= \{-0.570358, 0.591398, 0., 0.\}, \q = 0.82162
      n= 14, q= \{0.115556, -0.0506173, 0., 0.\}, q= 0.126156
      n= 16, q= \{-0.212175, 0.808659, 0., 0.\}, q= 0.836031
      n= 17,q= {-0.468911, 0.280845, 0., 0.},q= 0.546582
      n= 18, q= \{0.281004, 0.360617, 0., 0.\}, q= 0.457174
      n= 19, q= \{0.0889184, 0.82667, 0., 0.\}, q= 0.831438
      n=20,q=\{-0.535477, 0.771012, 0., 0.\},\q=0.93872
ln[165]:= P = \{0.14, 0.67, 0, 0\}
      quaternion[\{x_{y}, y_{z}, w_{z}\}] := \{x^2 - y^2 - z^2 - w^2, 2xy, 2xz, 2xw\}
      iteration[t_] := Module [q = t, n = 1], While [n \le 20, q = quaternion[q] + P;
         Print["n= ", n, ",", "q= ", q, ",", "\q\= ", N[Sqrt[Plus @@ (q<sup>2</sup>)]]];
         n++]]
      iteration[
       {0,
        0,
        0,
        0}1
Out[165]= \{0.14, 0.67, 0, 0\}
```

- $n= 1, q= \{0.14, 0.67, 0, 0\}, q= 0.684471$
- $n= 2, q= \{-0.2893, 0.8576, 0., 0.\}, \q = 0.905081$
- $n= 3, q= \{-0.511783, 0.173793, 0., 0.\}, q= 0.540487$
- $n = 5, q = \{0.0360006, 1.03585, 0., 0.\}, q = 1.03648$
- $n=6,q=\{-0.931697, 0.744583, 0., 0.\},\q\=1.19267$
- $n= 7, q= \{0.453656, -0.717451, 0., 0.\}, \q = 0.848846$
- $n= 8, q= \{-0.168932, 0.0190484, 0., 0.\}, \q = 0.170002$
- $n= 9, q= \{0.168175, 0.663564, 0., 0.\}, q= 0.684544$
- $n= 10, q= \{-0.272035, 0.89319, 0., 0.\}, q= 0.933698$
- $n= 11, q= \{-0.583786, 0.184043, 0., 0.\}, q= 0.612109$
- n= 12,q= {0.446934, 0.455117, 0., 0.}, q= 0.637872
- n= 13,q= $\{0.132618, 1.07681, 0., 0.\}$, \q\= 1.08495
- $n= 14, q= \{-1.00194, 0.955611, 0., 0.\}, q= 1.38459$
- $n= 15, q= \{0.230695, -1.24493, 0., 0.\}, q= 1.26613$
- $n= 16, q= \{-1.35663, 0.0956004, 0., 0.\}, q = 1.36$
- $n = 17, q = \{1.97132, 0.41061, 0., 0.\}, q = 2.01363$
- $n= 18, q= \{3.8575, 2.28889, 0., 0.\}, \q = 4.48545$
- $n= 19, q= \{9.78128, 18.3287, 0., 0.\}, q= 20.7754$
- $n=20,q=\{-240.13,359.227,0.,0.\},\q\=432.095$