COMMODITY PRICE SHOCKS, FACTOR UTILIZATION, AND PRODUCTIVITY DYNAMICS*

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Abstract

I estimate the responses of Chilean manufacturing firms to copper price variations to investigate the effects of international price shocks on aggregate productivity and output. I exploit sectoral variation in product tradability and reliance on copper in production processes. I find that, when the price of copper increases 1% in a year, establishments in non-tradable sectors with average reliance on copper to produce display 0.39% higher RTFP, 0.66% higher output, and 0.17% higher employment responses relative to similar establishments in tradable sectors. At the same time, establishments for which copper is a more important input feature lower relative RTFP and output responses, but no employment differentials. When the same analysis is done at a 4-years horizon, results are suggestive of larger employment responses. To interpret and aggregate these results, I develop a multi-sector small open economy framework featuring variable factor utilization. I find that variable factor utilization can generate a positive and strong association between commodity price shocks and measured aggregate TFP, as is observed in Chilean data.

JEL Codes: E23, E32, F41, F44, F47

Keywords: Factor utilization, commodity prices, aggregate productivity

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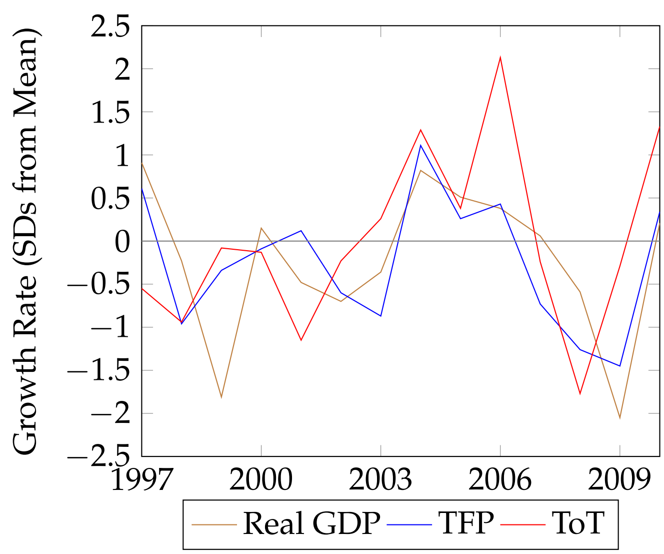
1 Introduction

A very persistent feature of business cycles in open economies is that terms of trade shocks are procyclical (Mendoza (1995), Kose (2002), Fernández et al. (2018), Drechsel and Tenreyro (2018), among others). At the same time, it has been extensively documented that TFP positively and strongly comoves with output (Basu and Fernald (2001), Basu et al. (2006)). Figure 1 plots measures of detrended real GDP, TFP, and terms of trade for Chile, a commodity exporting economy.[1] The procyclicality of TFP and the terms of trade is confirmed in this case. More notably, the correlation coefficient between TFP deviations from trend and terms of trade shocks is 0.59. Similar magnitudes have been observed in other countries.[2]

FIGURE 1 - CHILE: REAL GDP, TFP, AND TERMS OF TRADE INDEX



Note: Real GDP, TFP, and ToT growth rates are expressed in terms of std. dev. from their respective sample means. Real GDP is measured with the chain-weighting method (2013 is the reference year). TFP is the Solow residual obtained from this real GDP measure and capital and labor inputs as calculated by the Chilean National Commission of Productivity (CNP, in Spanish). Terms of trade are an index defined as the ratio between the exports price index and the imports price index, with base year 2003.

[1]Chile's main export is copper, a commodity. Copper represents around 51% of Chile's total exports and 9.7% of its GDP.

[2]For example, according to Kehoe and Ruhl (2008), the correlation coefficient for Mexico in the 1980-2005 period is 0.71, while for the United States in the 1970-2007 period is 0.42.

In spite of the pervasiveness of these patterns, a basic open-economy real business cycle model is not able to generate any correlation between exogenous price shocks and aggregate efficiency, as productivity is invariant, up to a first-order, to price movements (Kehoe and Ruhl (2008)). In this paper, I fill this gap in the literature by proposing variable factor utilization as an explanation. If firms need to change their output levels, but face short run constraints to alter the capital stock or the total amount of manhours they have hired, they can use machines at a different power or require workers to exert a different level of effort per hour worked. As these adjustments are normally unobserved by the econometrician, whenever terms of trade shocks create variations in the product demand or the input costs firms face, firms' measured productivity should change as a consequence.

I start by documenting some stylized facts from the Chilean manufacturing sector in the period 1996-2007. I focus on the effects that yearly fluctuations in the price of copper, Chile's biggest export, have on manufacturing establishments.[3] I exploit two sources of variation across industries: i) the international tradability of the sectoral product, and ii) the reliance of the industry's production processes on copper. My goal is to characterize the supply and demand perturbations that changes in copper prices induce on manufacturing firms. I find that, when the price of copper increases 1% in a year, establishments in non-tradable sectors that have an average reliance on copper to produce display 0.66% higher output, 0.39% higher revenue-based total factor productivity (RTFP), and 0.17% higher employment, relative to comparable firms in tradable sectors. At the same time, conditional on sectoral tradability, establishments that have one standard-deviation additional reliance on copper to produce feature -0.04% lower output, -0.02% lower RTFP, but no relative employment responses. When the price of copper increases by the same amount in the lapse of 4 years, similar responses on both margins are observed for output and TFP. In the case of employment, however, larger differentials are found for non-tradable firms relative to tradable ones, which hints at reallocation costs taking place at the very short run.

Guided by these empirical facts, I write a small open economy, multi-sector model featuring a full array of inter-industry linkages, hiring costs, and the possibility of variable factor utilization by firms. In this model, if firms want to adjust

[3]Shocks to the price of this commodity are tightly associated with variations in Chile's terms of trade. The correlation coefficient between the average yearly LME copper price and the yearly terms of trade index is 0.97 for the 1996-2010 period.

their scale of production, they have two options to do so. They can either hire more input units, such as more machines or more manhours, or they can use each input unit more intensively. If manhours are used more intensively, workers have to be compensated for their effort with a higher wage per hour worked, while if capital is used more intensively, it gets depreciated faster. Firms have incentives to vary the rate at which they use their inputs over time. In particular, firms have to make their input hiring/firing decisions before knowing the current period's perturbations, which implies that they cannot respond to unexpected supply or demand shocks by adjusting on this margin. For this reason, they will resort to change how intensively they use their factors. As changes in utilization are usually not observed by the statistician, any conventional TFP measure will capture swings in this dimension as variations in TFP.

I then study a restricted version of this model, which abstracts from capital accumulation, considers only three sectors, takes into account only downstream effects from the commodity producing industry, and imposes balanced trade. Within this setting, I show that for a higher commodity price to increase national income and constitute and effective resource windfall, we need the output ratio between the commodity producing sector and the other tradable sector to be larger than a certain threshold level. This threshold level is increasing in: i) the importance of the commodity in the production processes of other sectors, and ii) in the factor price elasticity of the intensity of use of factors. This means that whenever there is a commodity price shock, the economy will trade-off the benefit of larger revenues for the commodity producing sector with the damage of higher costs for all other industries that use the commodity. If the effect of larger revenues is bigger than that of higher costs, then the economy will enjoy an increase in domestic income. Put in another way, the economy needs to have a comparative advantage in commodity production that is large enough to make it a net exporter of the good. When this condition holds, ceteris paribus, households demand more of all final consumption goods, tradables and non-tradables.[4] Non-tradable industries,

[4] In the model it is assumed that each identical household possesses an equal share of all firms in the economy. In a more general context one could think of commodity-producing firms being possessed by a subset of low-MPC households. Of course, this feature would weaken the aggregate demand effects of commodity price shocks. However, commodity producing industries are usually heavily taxed or governments own large shares of them. In Chile, for example, private copper mines are subject to a special royalty tax and CODELCO, the state-owned copper mining company,

therefore, try to raise production but they cannot attract inputs due to short-run hiring costs, so they resort to use more intensively the inputs they have at their disposal instead. This higher factor utilization is measured as an increase in sectoral TFP. On the other hand, if an industry uses the commodity to produce, it faces higher costs. Higher costs lead to sectoral output reduction which, given the frictions to input adjustment, results in less factor utilization and lower measured sectoral TFP, just as in the data. Relatedly, the combination of demand increases for firms in non-tradable sectors and cost increases for tradable firms that use the commodity in their production processes, induce larger TFP and output responses of non-tradable sectors relative to tradable ones when commodity prices increase.

Numerical simulations are conducted on a calibrated version of the full model to Chilean data for the period 1996-2009. After feeding the model with actual data on copper price shocks, I find that the model is able to qualitatively match the positive correlation between commodity price shocks and aggregate TFP, and quantitatively generate an accurate volatility of real GDP. Importantly, these results are achieved under a reasonable parameterization and without losing the ability to generate sensible patterns for the trade balance, a problem in other papers that had used this mechanism to explain TFP dynamics for other type of events.[5] However, the model predicts a 55% higher volatility of TFP, an 81% lower volatility of aggregate hours, and a 56% lower volatility in aggregate consumption relative to what is observed in the data. These numerical results point to the existence of other shocks that occur simultaneously and in opposite directions with copper price perturbations (such as oil price shocks) and to an excessive consumption smoothing behavior within the model.

Related Literature My paper contributes to a large literature that investigates the importance of terms of trade shocks on business cycle fluctuations. Mendoza (1995), Kose (2002), Fernández et al. (2018), and Drechsel and Tenreyro (2018), us-

is the largest copper mining company in the world (9% share of world output). As long as governments can increase spending, reduce other taxes, increase transfers to households, or payoff outstanding debt (which could in turn give way to lower future taxes or higher future spending), there should be a positive income effect on all households over the wealth and firm-ownership distribution.

[5] Kehoe and Ruhl (2009), for example, evaluate the ability of variable factor utilization to explain the observed fall of TFP in Mexico during the 1994-95 financial crisis.

ing variants of open-economy real business cycle models, find that terms of trade explain over 40% of the output variance of developing economies. Relatedly, Kohn et al. (2020), highlight how differences in the composition of trade between countries can explain the larger output response to commodity price shocks of developing economies relative to developed ones. Schmitt-Grohé and Uribe (2018) have recently pointed to an apparent disconnect between the predictions of quantitative models and the results obtained via time-series analysis, which they show assign a much lower relevance for terms of trade shocks in explaining real GDP volatility. Fernández et al. (2020) argue that terms of trade shocks do explain a large fraction of output variance, but mostly if these shocks occur at high frequencies. My paper will differentiate from this literature in two respects. First, by focusing on explaining the relation between aggregate TFP and terms of trade shocks, a business cycle feature that has not been explicitly addressed but for Kehoe and Ruhl (2008). Importantly, Kehoe and Ruhl (2008) note that terms of trade shocks display a positive correlation with TFP deviations from trend in many economies, but that this relation does not naturally arise in standard real business cycle models. And second, by incorporating variable factor utilization, a mechanism that has not been considered in these papers and that helps to explain the positive correlation between productivity and terms of trade observed in the data.

I am not the first to consider factor utilization as a potential explanation for business cycle phenomena. Greenwood et al. (1988), Burnside et al. (1993, 1995), Burnside et al. (1996), and Bils and Cho (1994) use real business cycle models with any of variable capital or labor utilization to show how small technological perturbations can create comparatively large fluctuations in aggregate variables. Bachmann (2012) shows how labor hoarding can explain the differential behavior of aggregate employment in the recoveries following the 1991 and 2001 recessions, compared to previous ones. In open economy contexts, Meza and Quintin (2007) and Kehoe and Ruhl (2009) use RBC models endowed with the possibility of factor hoarding to account for the observed fall in measured productivity during the 1994-1995 Mexican financial crisis. I view my work as complementary to this literature, by also including labor hoarding and variable capital utilization, but distinct in several ways. First, it directly and quantitatively assesses the effect of terms of trade shocks on productivity through the input utilization channel, an exercise not performed before. Second, I allow the model to include many sectors linked by input-output linkages, a feature not present in previous models and that should play a role in amplifying the transmission of international price shocks across the economy. Third, I provide micro-evidence that strongly suggests that variable factor utilization is an important driver of firm-level responses to terms-of-trade shocks, which provides a stronger foundation for structural modeling assumptions.

A very close literature to that of factor hoarding is the one that centers in identifying technological shocks. Basu and Fernald (1997), Basu and Kimball (1997), Basu et al. (2006), and Fernald (2014) stress the role of variable factor utilization, sectoral heterogeneity, and aggregation to appropriately account for "pure" technological change. They find that technology shocks are usually permanent, so temporary movements in TFP ought to be induced by varying input utilization. In a very recent paper, Huo et al. (2020) apply Basu et al. (2006) methodology over a multi-country sector-level panel database. They obtain pure TFP series for many economies and then study the international business cycle co-movement generated by technology shocks. My paper has a similar flavor to this line of research, as I also use commodity price shocks as an exogenous shifter that creates sectoral responses, but it is different in many important dimensions. First, I focus on commodity price shocks in themselves, and on how they impact productivity. I do not use them as an instrument to estimate sectoral production functions. Second, I use an establishment-level database, not sector-level data, which allows me to track what is happening at the most disaggregated level. And third, I study the heterogeneity in output, TFP, and employment responses to copper price shocks according to two dimensions, tradability of the industry product and technological reliance on copper of the industry to which the firm belongs. Based on the results I obtain from this exercise I build my TFP theory, not the other way around as in the papers inspired by Basu et al. (2006).

This paper will also contribute to the Dutch disease literature. van Wijnbergen (1984a,b) uses variants of multi-sector models with increasing returns to scale to point at the inefficient equilibria that natural resource booms can create in an economy. Krugman (1987), using a similar framework, argues that natural resource booms can be harmful in the long run and that currency overvaluation may cause a permanent loss of competitiveness in some sectors. More recently, using regional data for the U.S., Allcott and Keniston (2018) find no evidence of a crowding out of the overall local manufacturing sector nor foregone local learning-by-doing effects when there is an oil and gas boom. I use the main insights from this literature when choosing the sources of variation over which firm-level responses are evaluated, i.e. tradability of the industry product and reliance on copper to produce. Nonetheless, I distinguish myself theoretically and empirically. Theoretically, I choose an efficient model, as I am concerned with short-run responses to terms of trade shocks, not long-run effects, from which I abstract. Empirically, I complement what is found in Allcott and Keniston (2018) by exploiting cross-industry variation instead of cross-regional, and separate from them when choosing the unit of analysis, which is the establishment. My results do find evidence of a Dutch disease type of situation occurring across industries. Non-tradable industries get benefited by copper price shocks relative to tradable ones, whereas tradable ones that rely more on copper to produce get crowded out, which is consistent with the within-region split of the manufacturing sector that Allcott and Keniston (2018) do in their paper.

Finally, this paper will also be inserted in the literature about shocks in models with production networks. In particular, the basic setup will be a variant of Long and Plosser (1983), which has been used in various papers such as Horvath (1998), Carvalho (2010), Foerster et al. (2011), Jones (2013), DiGiovanni et al. (2020), Caliendo and Parro (2015), Acemoglu et al. (2016), Bigio and La'O (2020), Atalay (2017), Baqaee and Farhi (2019), and Liu (2019). My paper will extend on them by adding the factor utilization channel and inserting the basic set up in a small open economy context. In addition, the empirical section of this article will be heavily based on Acemoglu et al. (2015), who study how different types of shocks propagate across sectors via input-output linkages. They perform their analysis at the industry-level, while here I will do it at the establishment-level. In addition, I will look at price shocks and distinguish firms by trade orientation, two aspects they do not consider in their article.

The remainder of the paper is organized as follows. Section 2 presents the data and the obtained stylized facts. Section 3 describes the model and provides intuition by deriving analytical results for a simplified version of it. Section 4 uses stochastic simulations to show the quantitative relevance of the proposed mechanism at the aggregate level. Section 5, finally, concludes.

2 Commodity Price Shocks and Chilean Firms' Behavior

The goal of this section is to document the impact of commodity price shocks on Chilean firms' output, input choices, and RTFP. In section 2.1. I describe the data to be used. In section 2.2. I describe and justify the two sources of variation considered and the empirical specification that is utilized. In section 2.3. I display the results from the exercise and summarize the main messages to be taken away.

2.1 Data

2.1.1 Manufacturing Sector Data (ENIA)

My main source of microdata is the annual Chilean manufacturing census (Encuesta Nacional Industrial Anual, ENIA in Spanish) for the 1995-2007 period.[6] The ENIA provides annual establishment level data and covers all manufacturing firms with more than 10 employees.[7] It contains information on establishments' gross output, value added, employment, wage bill, stock of capital, materials, and industry affiliation. Employment corresponds to the number of workers hired for production in a given year. Capital stock is measured as the sum of the reported book value of machinery, equipment, and vehicles. Plant's industry affiliation is at the 4-digit International Standard Industrial Classification (ISIC) level. To obtain real values, I deflate gross output, capital, and intermediates with their own industry-specific price deflators.[8] I employ these variables to construct measures of average real wages, capital intensity (capital per worker), materials intensity (materials per worker), and revenue-based total factor productivity (RTFP). The RTFP measure is computed as the residual of a sector-specific constant returns to scale Cobb-Douglas production function in capital, labor, and materials. The out-

[6]This survey has been used in several studies before such as Bergoeing et al. (2010), Corbo et al. (1991), Hsieh and Parker (2007), Levinsohn and Petrin (2003, 2012), Liu (1993), Liu and Tybout (1996), Pavcnik (2002), among others

[7]A unit of observation is a plant, not a firm, so the existence of multi-plant firms within the sample is possible. Pavcnik (2002) reported that approximately 90% of the plants are single-plant establishments for the 1979-1986 version of the survey. Unfortunately, for this issue of the survey there is no publicly available information on this matter.

[8]Provided by the National Institute of Statistics, INE in Spanish

put elasticities are obtained by calculating the sectoral labor and materials shares of output per what is observed in Chilean national accounts. For robustness, I also also estimate these parameters using the Olley and Pakes (1996) methodology. [9]

The establishment-level analysis focuses on an unbalanced panel of 9,575 private establishments for which there is enough information to compute the RTFP measure. This unbalanced panel accounts for around 64% of value added and 76% of employment of the full survey. For the sake of robustness, I also provide results for a balanced panel of continuing firms during the whole 1995-2007 period.

2.1.2 Input-Output Tables

To characterize sectors I use the 1996 Chilean Input-Output tables. I distinguish sectors in terms of how tradable their products are and in terms of how relevant is copper in their production processes. Similar to Pavcnik (2002), I determine the tradability of the industry's product according to the share that the industry's trade flows represent of the industry's output, i.e.:

$$\text{Tradability}_i = \frac{\text{Exports}_i + \text{Imports}_i}{\text{Output}_i}$$

where i indexes industries. Hence, an industry's product will be non-tradable (NT) if $\text{Tradability}_i < 30\%$, and tradable ($T$) otherwise.[10]

To measure the sectoral cost exposure to the copper industry, I follow Acemoglu et al. (2015) and Acemoglu et al. (2016), and use the downstream coefficients associated to the Leontief inverse matrix. In the Chilean Input-Output tables these correspond to the terms found in the Matrix of Direct and Indirect Requirements, which are at the 2- and 3-digit ISIC level. [11]

To be more clear, let $\mathbf{H}$ be the matrix that summarizes the shares that intermediate demand for each sector's output represent of the total value of production of each sector, i.e.:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & \\ h_{21} & h_{22} & & \\ & & \ddots & \\ & & & h_{JJ} \end{bmatrix}$$

[9] More details about TFP estimation in this case can be found in Appendix A.1

[10] All results are robust to cutoff points between 20% and 45%.

[11] Level of aggregation is determined by that used in Chilean national accounts when reporting Input-Output tables

where:

$$h_{ij} = \frac{\text{Sales}_{i \to j}}{\text{Sales}_j}$$

thus we can write:

$$\mathbf{S} = \mathbf{HS} + \mathbf{F}$$

where $\mathbf{S}$ is the vector of total sectoral sales, Sales_i, and $\mathbf{F}$ the vector of sectoral final demands, f_i. The Leontief inverse matrix, $\mathbf{A}$, is thus given by:

$$\mathbf{A} \equiv (\mathbf{I} - \mathbf{H})^{-1}$$

with typical entry a_{ij}. In this sense, a_{ij} represents the sector i's production that is required to satisfy an additional unit of sector j's final demand, f_j. Thus, I take the terms $a_{i,Cu}$ (upstream) and $a_{Cu,i}$ (downstream) associated to each manufacturing sector i in the tables relative to the copper industry (Cu). The downstream coefficients, $a_{Cu,i}$, will be the ones used to measure the importance of copper for firms' production processes.

2.1.3 Copper Prices

Finally, to measure commodity price shocks, I use a yearly time series for the London Metal Exchange (LME) price of copper, in dollars per pound of copper, constructed from the monthly series published by the Central Bank of Chile. I calculate the May-to-May percent variation in the commodity price during the 1996-2006 period. The reason for choosing May as reference month is to make the yearly copper price change to coincide with the 12-month period that the ENIA covers. For robustness, I also provide results in which this copper price series is expressed in Chilean pesos and deflated by the domestic consumer price index.

2.2 Empirical Approach

I consider two main sources of variation for firms, both defined at the sectoral level: i) the tradability of the industry's product, and ii) the sector's cost exposure to the copper industry. These variables are intended to capture two channels of transmission of international price shocks on domestic firms. First, the change in aggregate demand that comes from the effect that the price shock has on households' budgets. And second, the propagation of the price shock through the production network.

In principle, if the commodity price shock represents an increase in the disposable income of households, households will demand more of all goods, which should disproportionately benefit firms in industries that sell a less tradable good, as domestic demand is a more important part of their customer base. On the other hand, firms in industries with a technology that uses directly or indirectly the commodity, should see themselves negatively affected in the case of a positive price shock.

I consider the following specification, based on Pavcnik (2002) and Acemoglu et al. (2015):

$$\begin{aligned}\Delta y_{fit} =& \alpha_0 + \alpha_1 \Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i} + \alpha_2 \Delta \ln(PCu_t) \cdot \mathbb{1}\{\text{NT}\}_i \\ & + \chi' \mathbf{Z}_{fit} + \rho_i + \kappa_t + \epsilon_{fit}\end{aligned} \tag{1}$$

where f indexes firms, i industries, and t time; Δy_{fit} is the yearly log change in a firm outcome; $\Delta \ln(PCu_t)$ is the yearly log change in the international price of copper; $\mathbb{1}\{NT\}_i$ is an indicator variable that takes the value of one if a firm belongs to an industry that sells a non-tradable product; $\tilde{a}_{Cu,i}$ is a normalized downstream Leontief inverse term with respect to the copper industry;[12] $\mathbf{Z}_{fit}$ is a vector of controls; ρ_i is a 4-digit ISIC-level industry fixed effect; and κ_t a time fixed effect. Controls include the interaction of the upstream Leontief inverse term with respect to the copper industry with the yearly log change in the price of copper, the interaction of this last interaction with the non-tradable dummy,[13], the interaction of the non-tradable dummy with the yearly log change in the price of copper and the downstream Leontief inverse term,[14] and lagged firm's employment and RTFP. Errors are clustered at the 4-digit ISIC industry level.

There are two identifying assumptions that need to be made in order for (1) to provide unbiased estimates of α_1 and α_2. The first one is that copper price

[12]Normalization is made in order to have comparable regression coefficients between the upstream and downstream directions, and it is performed considering just the terms in a given network direction. More details in Appendix A

[13]Results for the regressors containing the upstream terms are not reported in the main body of the paper, but in Appendix B. The reason is that most of these estimates are not statistically different from zero. This is consistent with results from Acemoglu et al. (2015) in which price shocks are akin to productivity shocks.

[14]These triple interactions are included to account for possible heterogeneous effects of demand shocks across sectors. For example, among non-tradable manufacturing sectors, sectors that are more cost exposed to the copper sector are also more important in households' consumption baskets. Examples include but are not limited to dairy, bakery products, and grain mill products.

shocks are driven by demand/supply shocks occurring in markets different from the Chilean manufacturing sector. This assumption is a plausible one, since Chile comprises a very small share of world output and is usually considered a small open economy. Furthermore, even though Chile is the biggest player in the world copper market (representing 28% of world output in 2019, according to the Chilean Copper Commission), it is very hard that developments in the Chilean manufacturing sector will create sensible movements in the world supply or demand for the metal. In the particular period under consideration, the most drastic movements in the price of the metal are related to the entry of China to international markets, an arguably exogenous event to Chilean developments.

The second assumption is that there are not any confounders that have the same trend of the copper price series and that differentially affect non-tradable and copper intensive industries, as defined in 1996. This is more difficult to sustain. First of all, sectoral characterization, even though it was fixed at practically the beginning of the sample, it is persistent over time. Non-tradable firms, in particular, should differentially respond to any perturbation that affects households' budgets. Of particular concern are movements in nominal exchange rates and prices of other goods or inputs. Even though I have included the interaction of the log change in copper prices with the non-tradable dummy within the set of controls, which should partially correct for this problem, I have run this same specification but using a measure for the price of copper that is expressed in Chilean pesos and is deflated by the domestic consumer price index.[15] As it will be clear in the next section, overall results are not affected by this change in regressors.

2.3 Results

2.3.1 Output, Inputs, and RTFP

The main regression results are presented in Table 1. I run specification (1) for 1-year differences and 4-year differences. The motivation is to inspect if there are any variations in terms of the magnitudes of the responses as the time horizon lengthens. We can observe in Panel A that, when the price of copper increases 1%

[15] I could have used other deflators, such as an index of import prices, as in Drechsel and Tenreyro (2018), for example. I chose the Chilean CPI as it provides a closer counterpart with the units in which variables are denoted in the model to be laid out later in the paper. In any case, Fernández et al. (2018) also use a CPI measure to express real price shocks, in this case being the U.S. CPI.

in a year, conditional on an average cost exposure to the copper industry, the average firm in a non-tradable industry increases its output in approximately 0.66% relative to a similar firm in a tradable one. At the same time, firms with downstream Leontief inverse terms that are one standard deviation over the average decrease their output in 0.04%. Relative employment responses for non-tradable firms are weaker than in the case of output. For a 1% copper price shock, non-tradable firms with an average reliance on copper to produce feature 0.17% higher employment than similar tradable firms, whereas firms with one standard deviation larger Leontief inverse term do not display statistically significant responses. Jointly, these results point to a positive relative response to copper price shocks of labor productivity, measured as real output per worker, for non-tradable firms with an average reliance on copper, and a negative relative response of the same variable for firms in industries that rely more on copper to produce. These patterns are supported by what is found for RTFP. When the price of copper increases by 1%, non-tradable establishments with average cost exposure to the copper sector have a 0.39% higher RTFP than similar tradable ones, and establishments in industries with a one standard deviation larger downstream Leontief inverse term feature 0.02% lower RTFP.

These results may be interpreted as that productivity, measured either as RTFP or real output per worker, increases for firms that are facing a bigger copper price-induced demand shock, and decreases for those that face a larger cost shock, such as tradable firms that rely more on copper as an input. On the other hand, the weaker results on employment relative to those on output suggest the existence of frictions to the adjustment of workers on the extensive margin, as firms that face bigger demand increases elevate their production levels without hiring proportionately more workers, and firms that face bigger cost shocks reduce their output without firing proportionately less workers. This conjecture is reinforced by the relative responses found for materials per worker. For a 1% copper price shock, non-tradable firms with an average reliance on copper display a 0.5% higher materials per worker ratio relative to tradable ones, while firms with a one standard deviation larger downstream term feature a 0.04% lower coefficient. Capital per worker, on the other hand, is actually 0.52% lower for non-tradable firms relative to tradable ones and does not show any significant reaction for firms that rely more on copper to produce. This points to materials as the most important observable

TABLE 1 - EFFECTS OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL GROSS OUTPUT, INPUTS, AND RTFP

	Output	Employment	Capital/ worker	Materials/ worker	RTFP
Panel A. 1-Year Differences					
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.04***	0.01	-0.01	-0.04***	-0.02**
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.66***	0.17**	-0.52***	0.50*	0.39***
	(0.12)	(0.07)	(0.21)	(0.27)	(0.13)
Observations	31,504	31,504	31,504	31,504	31,504
Panel B. 4-Year Differences					
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.05**	0.01	0.01	-0.06***	-0.02**
	(0.01)	(0.01)	(0.04)	(0.01)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.74***	0.29*	-0.42	0.32	0.44**
	(0.19)	(0.16)	(0.40)	(0.30)	(0.20)
Observations	17,148	17,148	16,624	17,148	16,624
Industry and Time FE	Y	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

input that is adjusted in the short run when faced with copper price induced perturbations, regardless of the exact transmission channel considered.

When I consider the same specification with four-year differences in Panel B, similar patterns for output, employment, and RTFP are observed. However, point estimates are larger on the NT/T margin. Notably, when the price of copper increases in 1%, conditional on an average reliance on copper to produce, employment in non-tradable firms is now 0.29% higher than employment in tradable ones. This is 0.12 percentage points higher than the estimate found in Panel A of Table 1. Additionally, estimates for capital per worker and materials per worker are no longer significant and they are smaller in magnitude for the NT/T margin (-0.42 vs. -0.52 and 0.32 vs. 0.50, respectively).

These longer-horizon responses reinforce the hypothesis of the existence of short-run costs to the adjustment of inputs on the extensive margin. As it is shown in Panel B, firms react to copper price induced demand shocks by hiring more workers and capital goods relative to the short-run scenario. In the case of cost shocks, firms seem to rely even more on substituting away the materials that are intensive in copper than in the short-run case. This is expressed in the coefficient associated to the cost-exposure to copper in the materials per worker column, -0.06, which is significant and larger in magnitude than the analog one found in Panel A. The positive point estimate for employment found in the corresponding row in both tables supports this intuition.

2.3.2 Measures of Intensity of Input Usage

In Table 2 I present results related to some measures of intensity of input usage. In particular, I display the responses of the firm's average real wage, the number of workdays, and its real electricity and fuel consumption. Average real wage and workdays are meant to capture the intensity with which labor is used, while electricity and fuel consumption the intensity with which capital is utilized (as in Costello (1993), Burnside et al. (1995), and Burnside et al. (1996)). The average real wage is calculated as the total wage bill divided by the total number of workers and deflated by the industry-specific gross output deflator. The real electricity and fuel consumption correspond to the establishment's total electricity and fuel expenditures divided by the corresponding intermediate input deflator.

We can observe that average real wages track the movements in RTFP reported

TABLE 2 - EFFECT OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL MEASURES OF INPUT USAGE INTENSITY

	Avg. Real Wage	Workdays	Electricity Consumption	Fuel Consumption
Panel A. 1-Year Differences				
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.09***	0.00	-0.06***	-0.08***
	(0.01)	(0.00)	(0.01)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.34*	0.11***	0.24	0.52
	(0.18)	(0.04)	(0.25)	(0.33)
Observations	31,504	31,482	31,269	27,443
Panel B. 4-Year Differences				
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.08***	-0.00	-0.10***	-0.11***
	(0.01)	(0.00)	(0.02)	(0.04)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.44*	-0.01	0.07	-0.35
	(0.18)	(0.07)	(0.25)	(0.06)
Observations	17,148	17,135	17,025	14,403
Industry and Time FE	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

in the previous section. In Panel A, when the price of copper goes up by 1%, the average real wage increases in 0.34% for non-tradable firms with an average cost exposure to the copper sector, relative to similar tradable ones. At the same time, firms with a one standard deviation larger downstream term show 0.09% lower average real wages. The total number of workdays increases 0.11% for non-tradable firms relative to tradable ones, but does not sensibly react when firms have a bigger cost exposure to the copper sector. This is suggestive that when there is a copper price induced demand increase, non-tradable firms make their workers work harder relative to similar tradable firms, as the total number of days an average worker works in a year for a non-tradable firm is greater than for tradable ones. However, when there is a copper induced cost shock, there does not seem to be any impact on the fraction of the year a worker spends working.

Electricity and fuel consumption do not feature sensible differences between non-tradable and tradable firms, conditional on an average reliance on copper to produce, although point estimates are positive in both cases. On the cost exposure margin, however, both indicators show statistically significant negative responses to copper price shocks. This is suggestive of a lower utilization of capital for firms that rely more on copper to produce when the price of copper goes up.[16] Lower capital utilization could explain the lower average real wage rate for firms more reliant on copper to produce that is observed in the first column of the table. If labor and capital are complements, a lower amount of capital services should lead to a lower marginal productivity of labor, and a lower average real wage.

In terms of responses over a longer time horizon, we can appreciate in Panel B that the average real wage preserves the qualitative features found in Panel A, and closely tracks the responses of output and RTFP in Table 1. Also similar to what was found for yearly differences, outcomes for electricity and fuel consumption display no statistically significant responses on the NT/T margin, but significantly negative responses on the cost-exposure dimension. In this case, point estimates are even more negative than in Panel A (-0.10 vs -0.06 in the case of electricity, -0.11 vs. -0.08 in the case of fuel), which suggests that capital utilization is even lower as time horizons get longer. The most remarkable result here is that workdays now display no heterogeneous responses across firms based on the two dimen-

[16]Actually, in the 1996 Input-Output tables, the five sectors most reliant on copper to produce are Basic Precious and Non-Ferrous Metals, Electrical Machinery and Apparatus, Fabricated Metal Products, Basic Chemicals, and Basic Iron and Steel. All strongly capital-intensive industries.

sions considered. Since employment reactions on the T/NT margin were larger on a 4-year horizon than in a 1-year one, this can be indicative of firms reducing the use of additional workdays to satisfy demand increases in order to rely more on new workforce members. In this sense, the observed movements in average real wage could reflect variations in work effort on a per workday basis.

2.3.3 Robustness Tests and Summary of Results

In Appendix B, I present more detailed results and a full set of robustness tests. In Appendix B.1 I display the estimates for those controls that are omitted in the tables within the main body of the paper. We can appreciate in Tables B.1.1 and B.1.2 that copper price shocks tend to not propagate upstream, a result that it is in line with the theoretical and empirical findings of Acemoglu et al. (2015), who state that supply-side shocks only propagate downstream in a closed-economy context. In the case under consideration in this paper, an exogenous increase in the price of copper corresponds to a supply shock for those manufacturing firms downstream the copper industry. For those local firms that are upstream the copper industry two things may be happening. One of them is that they sell tradable products and the world prices for them has not changed, so even though the copper industry may be demanding more inputs, the received price does not incentivize these firms to supply more of them, so the copper industry brings them from abroad. The other possibility is that the copper industry, given the nature of the extraction process, does not respond to one-year changes in the price of the metal because the time it takes to extract the mineral is too long to take advantage of the opportunity.[17] This last conjecture would be coherent with the tenuous positive effects found for some variables such as materials per worker, employment, and electricity and fuel consumption for the regressions with 4-year differences.

In Appendix B.2.1 I run the same regressions but after estimating RTFP via the Olley and Pakes (1996) method. I consider a Cobb-Douglas production function in capital, materials, production workers, and non-production workers. Results do not change much in qualitative and quantitative terms and, if anything, reinforce the interpretation of frictions to the adjustment of employment in the short-run. In Appendix B.2.2, to account for the possible joint movement of the

[17] Actually, copper, just like many other commodities such as oil, is commonly traded in future contracts where the transaction price is set long before extraction occurs.

international price of copper with other variables that may disproportionately affect non-tradable industries relative to tradable ones, such as exchange rates and prices of other goods and inputs, I use a modified copper price series. I express the LME price of copper in Chilean pesos using the average annual nominal exchange rate, as reported by the Central Bank of Chile, and deflate this modified series by the Chilean consumer price index. The idea is to obtain a real price of copper, net of variations on other prices, that can account more accurately for the effect of changes in the price of the metal on firms producing non-tradables relative to those producing tradables. Results are robust to this modification. Notably, patterns related to output per worker and RTFP are preserved, and even though employment responses are somewhat stronger in the short run than in the baseline specification, the point estimate in the short run on the non-tradable/tradable margin is still smaller than in the long-run. To evaluate the magnitude of the possible biases that using an unbalanced panel might entail for my results, in Appendix B.2.3 I provide estimates for a balanced panel of 1,319 firms that were present during the entire 1995-2007 period. Once again, patterns do not change much and, if anything, reinforce the conjecture about short-run frictions in the reallocation of labor. Finally, in Appendix B.2.4. I evaluate how important is the choice of an empirical specification in differences for the results obtained. I estimate the same regression but in levels and adding firm fixed effects, once again finding similar results overall.

I will summarize the main findings of this empirical section as follows:

1. The labor productivity and RTFP of firms in non-tradable industries, relative to those of similar firms in tradable industries, are positively correlated with copper price shocks

2. The relative labor productivity and RTFP of firms that rely more on copper to produce is negatively associated with copper price shocks

3. Conditional on a copper price shock of the same size, the 4-year employment responses of firms in non-tradable industries, relative to firms in tradable ones, are larger than the 1-year responses

4. Measures of factor utilization tend to track the patterns observed for RTFP and labor productivity

3 Theoretical Framework

In this section, I move to a theoretical analysis of the interaction between frictions to the reallocation of factors, variable factor utilization, and sectoral heterogeneity to interpret my empirical results. My goal is to clarify how the degree of comparative advantage in the production of the good that is facing an exogenous price shock (in this case, copper) and the size of the non-tradable sector affects the transmission of price shocks to aggregate TFP through the factor utilization channel. Booms and busts in commodity prices have heterogeneous effects over the tradable and non-tradable margin and over firms with different reliance on the commodity to produce. Importantly, the model features Dutch disease type of responses by non-commodity tradable firms, but an association between price shocks and aggregate TFP that not necessarily goes in the same direction.

The starting point is a standard networks model based on Long and Plosser (1983), which I embed in a small open economy framework. I add quasi-fixity of primary factors and the possibility for varying factor utilization by firms. Price shocks are the only source of perturbations in this economy.

3.1 Set Up

This is a small open economy consisting of J sectors, of which J^T produce internationally traded goods while J^{NT} produce non-tradable goods ($J^T + J^{NT} = J$). Firms in each sector produce by combining labor, capital services, and intermediates coming from all J sectors of the economy. There is also an infinitely lived representative household that consumes a composite, C_t, made of goods produced in every sector of the economy, and that has access to an internationally traded risk-free bond, B_t, to smooth consumption over time. The economy is subject to stochastic shocks to the price of internationally traded goods. Price shocks are the only source of randomness in this environment

3.2 Households

Households derive utility from consuming a consumption composite, C_t, disutility from supplying hours to each sector-specific labor market, $\{N_{jt}\}_{j=1}^{J}$, and disutility from exerting effort per hour worked in each sector-specific labor market, $\{e_{jt}\}_{j=1}^{J}$.

Household's lifetime expected utility will be given by:

$$U_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{u\left(C_t, \{N_{jt}\}_{j=1}^{J}, \{e_{jt}\}_{j=1}^{J}\right)^{1-\sigma}}{1-\sigma} \right] \tag{2}$$

where $\sigma > 0$ and:

$$u\left(C_t, \{N_{jt}\}_{j=1}^{J}, \{e_{jt}\}_{j=1}^{J}\right) = C_t - \sum_{j=1}^{J} \frac{N_{jt}^{1+\psi_n}}{1+\psi_n} - \chi \sum_{j=1}^{J} N_{jt} \frac{e_{jt}^{1+\psi_e}}{1+\psi_e} \tag{3}$$

with χ, ψ_n, and $\psi_e > 0$. This utility function specification is a variation of the Greenwood et al. (1988), GHH from now on, type of preferences, which eliminates the wealth effect on labor supply. I extend their specification by adding an effort disutility component. This component is convex in its argument, e_{jt}, has convexity parameter ψ_e, and is weighted by the relative disutility factor χ and the current amount of sector-specific hours supplied, N_{jt}. Similar to the standard GHH preferences, wealth effects on any of hours or effort per hour supplied to each industry are eliminated.

The consumption composite, C_t, is a Cobb-Douglas aggregator of consumption of each sector's good:

$$C_t = \prod_{j=1}^{J} C_{jt}^{\alpha_j}, \qquad \alpha_j \in [0, 1] \tag{4}$$

The consumption composite will be the numeraire of this economy. Hence, the solution to the static problem of getting one unit of the consumption composite at the minimum cost will imply:

$$1 = \prod_{j=1}^{J} \left(\frac{P_{jt}}{\alpha_j} \right)^{\alpha_j} \tag{5}$$

It can be verified that $\sum_{j=1}^{J} P_{jt} C_{jt} = C_t$. Households have access to an internationally-traded risk-free bond, B_t, denominated in domestic consumption composite units. The budget constraint of the household is as follows:

$$B_{t+1} + \sum_{j=1}^{J} P_{jt} C_{jt} = (1 + r_t) B_t + \sum_{j=1}^{J} w_{jt} e_{jt} N_{jt} + \sum_{j=1}^{J} \pi_{jt} \tag{6}$$

where r_t is the interest-rate international bonds bear, w_{jt} is the wage per labor service unit paid in sector j, and π_{jt} are the profits generated by sector j firms

To provide a meaningful role to labor effort, it will be assumed, as in Huo et al. (2020), that households choose the amount of hours to supply to each sector before shocks are realized. This is to say that there is quasi-fixity of hours employed in each sector within a period, so that the only margin over which the labor input can be adjusted is via labor effort.[18] This quasi-fixity can reflect hiring/firing costs or the time it takes to build a fully functional workforce.

The problem of the household can then be summarized as follows:

$$
\begin{aligned}
V\left(B_t, \{N_{jt}\}_{j=1}^J\right) = \max_{\mathbf{v}_t} & \frac{u\left(C_t, \{N_{jt}\}_{j=1}^J, \{e_{jt}\}_{j=1}^J\right)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V\left(B_{t+1}, \{N_{jt+1}\}_{j=1}^J\right)\right] \\
\text{s.t.} \quad & B_{t+1} + \sum_{j=1}^J P_{jt} C_{jt} = (1+r_t) B_t + \sum_{j=1}^J w_{jt} e_{jt} N_{jt} + \sum_{j=1}^J \pi_{jt} \\
& C_t = \prod_{j=1}^J C_{jt}^{\alpha_j}
\end{aligned}
\tag{7}
$$

where $\mathbf{v}_t = \left\{B_{t+1}, \{C_{jt}\}_{j=1}^J, \{e_{jt}\}_{j=1}^J, \{N_{jt+1}\}_{j=1}^J\right\}$.

3.3 Firms

Firms are classified as belonging either to the group of sectors that produce tradable goods, $\mathcal{J}^T$, which has cardinality J^T, or to the group of sectors that produce non-tradable goods, $\mathcal{J}^{NT}$, that has cardinality J^{NT}.

Firms produce with sector-specific, constant returns to scale, Cobb-Douglas technologies that use labor, capital services, and intermediates from all sectors. Thus, a sector j firm features the following production function:

$$
Y_{jt} = Z_j (u_{jt} K_{jt})^{\gamma_{jk}} (e_{jt} N_{jt})^{\gamma_{jn}} \prod_{i=1}^J M_{ij,t}^{\varphi_{ij}}, \qquad \gamma_{jk}, \gamma_{jn}, \varphi_{ij} \in [0,1] \tag{8}
$$

with $\sum_{i=1}^J \varphi_{ij} = 1 - \gamma_{jk} - \gamma_{jn}$, and where u_{jt} is capital utilization, K_{jt} is capital stock, e_{jt} is effort per hour worked, N_{jt} is hours employed, and $M_{ij,t}$ is the amount of materials sector j purchases from sector i.

[18] An alternative specification would have been to place the effort choice on firms. In this case, firms would directly decide how much effort to extract from households and would internally adjust compensation to that end. The results in this model are isomorphic to the specification considered in this paper.

The firm demands raw labor input, which in equilibrium is equal to the product of total hours employed in a sector and the effort per employed hour, $L_{jt} = e_{jt}N_{jt}$. As it was said in section 3.2, hours worked, N_{jt}, are assigned to each sector before shocks are realized. However, it is assumed firms are not aware of this. In this sense, the firm is indifferent with respect to the exact hours-effort per hour mix (e_{jt}, N_{jt}), and offers a sector-specific wage, w_{jt}, in order to lure in just a certain amount of labor input, L_{jt}, without looking at how that labor input is being generated[19] It is important to note that the firm manager has the ability to induce households to provide the desired level of effort per hour if needed. This is not equivalent to say that the manager observes the households' effort, but rather that it can design a contract in which households are incentivized to provide more or less effort, and that that effort can be implicitly monitored through an observable variable such as output. In this margin, the firm managers operate just as the econometrician of section 2, just observing output and the amount of hours used in the process.

In summary, if firms need to respond to a certain demand increase, they will increase wages per hour worked, waiting for households to provide more labor input. As households will not react but on the hours margin but with a lag, they will work harder every hour that they are currently working in the firm. A higher output is rewarded with a subsequent higher wage per hour. The effort response depends on the wage elasticity of effort, $1/\psi_e$. The only agents that observe effort are households themselves.

Capital stock is accumulated by firms via the reinvestment of operational profits. To invest, firms have to purchase a sector-specific investment good, I_{jt}. This investment good is produced with the sector's own output in a one-to-one fashion. Investment choices have to be made one period in advance, so the capital stock is fixed within a period. As in the case of labor effort, when faced with a shock, firms can only change the amount of capital services they have by adjusting the utilization rate of their current capital stock, u_{jt}. Materials and labor service units are fully flexible inputs from the firm's perspective.[20]

[19]One can think of this assumption as if the firm were offering a piece rate compensation scheme. A higher output per worker can be obtained if the worker either works for longer hours or works harder per unit of time. If there are no other associated direct costs with each option, the employer is indifferent with respect to which method the worker used. She only cares about her total output.

[20]This would align with the assumptions needed to implement the Olley and Pakes (1996)

Following Greenwood et al. (1988), it will be assumed that a higher capital utilization rate implies a higher depreciation rate. Let δ_j denote the sector-specific depreciation rate. Hence, δ_j and u_{jt} are related according to the following convex function:

$$\delta_j = \bar{\delta}\frac{u_{jt}^{1+\psi_u}}{1+\psi_u} \tag{9}$$

where $\bar{\delta}$, and ψ_u are positive. In this case, and in opposition to the situation of labor effort, firm managers do observe how much they utilize capital. As it is clear from (9), the strength of the capital utilization response to a demand/supply shock will depend on the convexity term, ψ_e. Households, on the other hand, do not directly observe the capital utilization rate of firms. They only observe the wage per hour they earn. Finally, the econometrician, just like with labor effort, only observes the capital stock.

It is a typical feature of small open economy models that investment rates are too volatile because of the weak response of interest rates to changes in this variable. For this reason, I will assume that firms face quadratic capital adjustment costs in terms of the sectoral good and, thus, smooth investment rates over time. The functional form for the capital adjustment costs is given by:

$$\frac{\tau}{2}\left(K_{jt+1} - K_{jt}\right)^2$$

where $\tau > 0$.

Therefore, the problem of a sector j firm can be stated as follows:

$$\begin{aligned}
\mathcal{V}_{jt}(K_{jt}) = &\max_{\mathbf{x}_{jt}} \pi_{jt} + \frac{1}{1+r_{t+1}}\mathbb{E}_t\left[\mathcal{V}_{jt+1}(K_{jt+1})\right] \\
\text{s.t.} \quad & \pi_{jt} = P_{jt}Y_{jt} - w_{jt}L_{jt} - \sum_{i=1}^{J} P_{it}M_{ij,t} - P_{jt}\left(I_{jt} + \frac{\tau}{2}\left(K_{jt+1} - K_{jt}\right)^2\right) \\
& Y_{jt} = Z_j(u_{jt}K_{jt})^{\gamma_{jk}} L_{jt}^{\gamma_{jn}} \prod_{i=1}^{J} M_{i,j,t}^{\varphi_{ij}} \\
& K_{jt+1} = I_{jt} + (1-\delta_{jt})K_{jt} \\
& \delta_{jt} = \bar{\delta}\frac{u_{jt}^{1+\psi_u}}{1+\psi_u}
\end{aligned} \tag{10}$$

where $\mathbf{x}_{jt} = \left\{L_{jt}, u_{jt}, \{M_{ij,t}\}_{i=1}^{J}, I_{jt}\right\}$

methodology. However, in equilibrium firms will do face a less than fully flexible labor input, which would invalidate the procedure.

3.4 Endogenous Interest Rate Premium and Copper Price Shocks

3.4.1 Interest Rate Premium

As the steady-state in stochastic small-open economy models depends on initial conditions,[21] I will follow Schmitt-Grohé and Uribe (2003) and assume that the interest rate on internationally-traded bonds faced by domestic households is composed of the international interest rate, r^*, and a premium that is a function of the amount of bonds, B_t, the household holds:

$$1 + r_t = 1 + r^* + \phi(\exp\{\bar{B} - B_t\} - 1) \tag{11}$$

where $\phi > 0$ and $\bar{B}$ is the steady-state bond holdings level.

3.4.2 Copper Price Shocks

Copper prices will be the only source of perturbations in this economy. They will display a stochastic behavior corresponding to an AR(1) process in logs:

$$\log(P_{Cu,t}) = \rho_{Cu} \log(P_{Cu,t-1}) + \varepsilon_{Cu,t} \tag{12}$$

where $\varepsilon_{Cu,t} \sim \mathcal{N}(0, \sigma^2_{Cu})$

3.5 Equilibrium

An equilibrium in this economy is a sequence of allocations:

$$\left\{ B_{t+1}, \left\{ C_{jt}, e_{jt}, N_{jt+1}, u_{jt}, L_{jt}, \{M_{ij,t}\}_{i=1}^{J}, I_{jt} \right\}_{j=1}^{J} \right\}_{t=0}^{\infty}$$

and prices: $\{\{P_{jt}\}_{j\in\mathcal{J}^{NT}}, r_t\}_{t=0}^{\infty}$ such that, given international prices for non-copper tradable sectors, $\{P_{jt}\}_{j\in\mathcal{J}^{T}|j\neq Cu}$, a stochastic process for the international copper price, $\{P_{Cu,t}\}$, dictated by (12), an international interest rate, r^*, and initial values $\{N_{j,0}, K_{j,0}\}_{j=1}^{J}$ and B_0:

1. Households solve (7)

2. Firms in each sector solve (10)

[21] This implies that transitory shocks have permanent effects. For this reason, equilibrium dynamics display a random walk behavior, so the unconditional variance of variables such as assets and consumption tends to infinity.

3. Labor markets clear:

$$L_{jt} = e_{jt} N_{jt}, \qquad \forall j = 1, \ldots, J \tag{13}$$

4. Non-tradable markets clear:

$$C_{jt} + I_{jt} + \frac{\tau}{2} \left(K_{jt+1} - K_{jt} \right)^2 + \sum_{i=1}^{J} M_{j \to i,t} = Y_{jt}, \qquad j \in \mathcal{J}^{NT} \tag{14}$$

5. The balance of payments condition holds:

$$B_{t+1} + \sum_{j=1}^{J} P_{jt} \left(C_{jt} + I_{jt} + \frac{\tau}{2} \left(K_{jt+1} - K_{jt} \right)^2 + \sum_{i=1}^{J} M_{i \to j,t} \right) = (1 + r_t) B_t + \sum_{j=1}^{J} P_{jt} Y_{jt} \tag{15}$$

3.6 Factor Utilization in a Restricted Model

In this section, I use a restricted version of the model just described and focus on an arbitrary period. The main goals here are to convey in the simplest possible way the main insights behind the full model laid out in Section 3.5, and transparently illustrate how variable factor utilization can rationalize the patterns observed in the empirical section of the paper. There three sectors. Two of them, called T and NT, produce final consumption goods. The remaining sector is a commodity producing industry, Cu. Goods Cu and T are tradable and perfectly substitutable with foreign varieties, while good NT is non-tradable. Trade will be balanced each period.

There is no capital, $\gamma_{jk} = 0 \; \forall j$. Labor is the only primary production input,[22] while the commodity is an intermediate input for both sectors.

The household's period budget constraint is given by:

$$C_T + P_{NT} C_{NT} = w_T e_T N_T + w_{NT} e_{NT} N_{NT} + \sum_{j=\{T,NT,Cu\}} \Pi_j = E \tag{16}$$

where good T will be the numeraire, so $P_T = 1$

[22] I could have also included capital with its own utilization rate, but given that I assume a Cobb-Douglas production function, utilization rates of both inputs would have positively co-moved when the firm is faced with demand shocks or shocks to the commodity price. So unless the shock to the price of the commodity affects the relative cost of capital and labor, which is not the case in this environment, adding capital would have complicated the exposition without changing the main results. Plus, the main results from the empirical section do not talk about capital.

The demand functions for T and NT goods are:

$$C_T = \alpha E \tag{17}$$

$$C_{NT} = (1-\alpha)\frac{E}{P_{NT}} \tag{18}$$

The optimal effort supply choice per sector will be given by:

$$e_j = w_j^{\frac{1}{\psi_e}} \tag{19}$$

where $1/\psi_e$ will be the wage elasticity of labor effort.

In the Cu sector it will be assumed that firms, in order to produce, only use a sector-specific commodity endowment $\bar{Cu}$ (no alternative use in other sectors) according to a linear production function, $Y_{Cu} = \bar{Cu}$.[23] Total industry profits are therefore going to be $\Pi_{Cu} = P_{Cu}\bar{Cu}$, where P_{Cu} is the international price of Cu.

The solution to the firm's problem will generate the following materials and labor demand functions:

$$M_j = \frac{(1-\gamma_j)P_jY_j}{PCu} \tag{20}$$

$$L_j = \frac{\gamma_j P_j Y_j}{w_j} \tag{21}$$

If we combine the labor demand function (21) with the household's effort supply function (19), we obtain the following relation between labor utilization and sectoral revenues, P_jY_j:

$$e_j = \left[\gamma_j \frac{P_jY_j}{N_j}\right]^{\frac{1}{1+\psi_e}} \tag{22}$$

We can appreciate that labor effort is an increasing function of revenues, P_jY_j and a negative one of the current number of hours hired, N_j. Furthermore, the response of effort to each one of these variables depends positively on the wage elasticity of effort, $1/\psi_e$. It is important to remind that effort per hours, e_j, is not directly observed by neither firms nor the econometrician. Revenues, P_jY_j, and hours

[23] Of course, the commodity producing sector could also use labor and intermediates coming from sectors T and NT. However, considering a more complete production function would not add much to the analysis and would unnecessarily complicate it. The crucial thing that is needed is a positive relation between profits in the commodity industry and commodity prices, and this is the easiest way of getting it. Plus, papers such as Fernández et al. (2018) have used a similar modeling strategy.

worked, N_j, are. Hence, according to this restricted model, whenever we observe an increase in revenues per hour worked, we should infer that effort is increasing as well.

Similarly, as in the real world we only observe the number of workers/hours worked, N_j, and the intermediate consumption, M_j, per the TFP measurement procedure performed in Section 2, we have:

Definition 1. *Measured TFP, TFP_j, is given by:*

$$TFP_j = \frac{Y_j}{N_j^{\gamma_j} M_j^{1-\gamma_j}} = Z_j e_j^{\gamma_j} \tag{23}$$

where γ_j is both the output elasticity of labor and the labor cost share of output.

We can appreciate that any variation in input use gets loaded onto measured total factor productivity. It is important to note that this result is robust to allowing for production functions that display returns to scale that are not constant. As long as factor utilization cannot be totally corrected for with observable variables, some variation of it will be reflected in measured TFP. It is important to note that here I talk of TFP and not RTFP, as it was the case in the empirical section. The reason is that as I am abstracting from any within sector heterogeneity, the real world counterpart of a sector j firm's revenues can be any of sector j's average revenues or sector j's aggregate revenues. In both situations, and as long as firm-level price variations are washed out at the sectoral level, I can eliminate sectoral price variations by making use of the industry-specific price deflator, thus obtaining a TFP measure and not a RTFP one.

3.6.1 Results

Let me consider a change in PCu. I will start characterizing the responses of tradable firms that rely more on copper for their production processes. [24]

[24]Of course, the parallel is not perfect. The term $\tilde{a}_{Cu,i}$ that was used in the empirical section is a Leontief inverse coefficient, that comprises direct and indirect effects that propagate through a whole array of input-output linkages. In this model, instead, there is just one downstream linkage from Cu to each other sector. Nonetheless, adding more upstream and downstream linkages would not change the main message that I am trying to convey here, which is that the downstream propagation of commodity price shocks has harmful effects on tradable firms, and which are larger the more important is the commodity for them to produce.

Proposition 1. *In sector T, firms' output (Y_T), TFP_T, and factor utilization (e_T) will be smaller the higher is PCu, with this response being stronger the larger is the materials output elasticity $1 - \gamma_T$.*[25]

Proof. See Appendix C. ∎

Thus, if there is a considerable friction for adjusting the workforce on the extensive margin, but there is the possibility of adjusting it on the intensive margin via varying labor effort, firms will do it. As a consequence, when there is a commodity price shock, firms in tradable sectors that rely on the commodity to produce will reduce output by cutting on wages to workers, who in turn will reduce their labor effort. The lower productivity per hour worked will show itself as a lower measured TFP. At the same time, firms that produce with more commodity intensive technologies will be disproportionately negatively affected by an increase in the price of that commodity. Therefore, they will display a disproportionately bigger fall in TFP relative to comparable tradable firms. This is consistent with listed results 2 and 4 in Section 2.3.4, where firms in tradable sectors that relied more on copper to produce displayed disproportionately lower RTFP and output when the price of copper increased, but no response in terms of employment.

Non-tradable firms' behavior, on the other hand, depends crucially on the behavior of the economy's aggregate demand, C. A higher aggregate demand is translated into a higher consumption of tradable, T, and non-tradable, NT, goods. Domestic tradable production depends on the international price of the good, which is exogenously determined, so changes in domestic demand do not directly affect the production decisions of firms in this sector. This is not the case with the NT sector, whose only customers are domestic residents. Hence, any change in domestic demand will fully impact firms there. For this reason, it is necessary to establish the circumstances under which an increase in the price of the commodity, PCu, induces a correspondingly higher aggregate demand. As sectors other than Cu use Cu in their production processes, it is possible that an increase in the commodity price will make production in these other sectors so expensive, that

[25] Note that the derivative does not consider the effect of increasing the output elasticity of materials on the output elasticity of labor, even though in this simple setup they should both add up to one. The reason for doing this is that it does not affect the main result and that in the general quantitative setting there are no restrictions on the degree of returns to scale that manufacturing firms have.

the income windfall in the Cu sector is more than offset by the loss in production happening in the rest of the economy. At the same time, the cost pressure that non-tradable firms that use the commodity face when PCu goes up could make them reduce their production. The following proposition establishes when an increase in aggregate demand expresses itself as a higher expenditure on non-tradables, and when as a higher non-tradable output.

Lemma 1. *Let ϕ_j, $j = Cu, T, NT$ be the GDP share of sector j. If:*

$$\frac{\phi_{Cu}}{\phi_T} > \frac{(1-\gamma_T)}{\gamma_T}\frac{(1+\psi_e)}{\psi_e}$$

then $dE/dPCu$, $de_{NT}/dPCu$, and $dTFP_{NT}/dPCu > 0$, while if

$$\frac{\phi_{Cu}}{\phi_T} > \frac{(1-\gamma_T+\psi(1-\gamma_{NT}))}{\gamma_T\gamma_{NT}}\frac{(1+\psi_e)}{\psi_e}$$

then $dY_{NT}/dPCu > 0$

Proof. See Appendix C. ■

It is clear that what is needed for generating a positive aggregate demand effect is that the relative importance of the commodity producing sector to that of the non-commodity tradable sector be larger than a certain threshold. This threshold is determined by the importance of the commodity for firms in sector T, $1-\gamma_T$, and by the wage effort elasticity, $1/\psi_e$. The larger $1-\gamma_T$, the bigger is the cost pressure induced by a higher commodity price on non-commodity tradable firms, so the larger the production reduction and the loss of income coming from this sector. On the other hand, a larger $1/\psi_e$ implies that a lower wage will induce a sharper decline in effort by households, so production and productivity in non-commodity tradable sectors should fall more strongly. In summary, the country must have a strong comparative advantage in commodity production if it stands to benefit from higher commodity prices.

We can also observe that the condition for having an actual non-tradable output increase is more stringent than that for having an increase in non-tradable revenues and TFP. The reason for this is that the more important is the commodity for the production processes of non-tradable firms, the more expensive it gets to produce when the price of the commodity goes up. In this way, a bigger demand increase is needed in order to compensate for this cost pressure.

In Section 2 it was also highlighted the positive output and RTFP responses to copper price shocks of non-tradable firms relative to that of similar tradable firms, at the same time that there was a weaker relative employment response. In this restricted model, the instantaneous employment response is totally muted, but the intuition carries on if we allowed for a less stark assumption. In this way, conditional on cost structure, the impact of commodity-price induced demand shocks on non-tradable versus tradable firms is as follows:

Proposition 2. *If $\gamma_T = \gamma_{NT}$, and*

$$\frac{\phi_{Cu}}{\phi_T} > \frac{(1-\gamma_T+\psi(1-\gamma_{NT}))}{\gamma_T\gamma_{NT}}\frac{(1+\psi_e)}{\psi_e}$$

sector NT firms' output, TFP, and factor utilization responses to changes in PCu are going to be larger than those of sector T firms.

Proof. See Appendix C. ■

We can appreciate that this theoretical result is consistent with what was found in the listed empirical results 1 and 3 of section 2.3.4. Output per worker and RTFP disproportionately increase for non-tradable firms relative to similar tradable ones.

The motivating fact for this paper is the positive co-movement between aggregate TFP and commodity prices. The model can also speak to this aggregate pattern and establish conditions under which one should see it happening. An important and convenient simplifying assumption made here is that sector-specific hours, which is the only primary production input, are fixed. Thus, any change in real GDP, which here it corresponds to a change in E when keeping prices fixed, will be equivalent to a change in aggregate TFP if we measure aggregate TFP as the ratio between total value added and hours to the power of a macro output elasticity.

Proposition 3. *If:*

$$\frac{\phi_{Cu}}{\phi_T} > \frac{(1-\gamma_T+\psi_e(1-\gamma_{NT}))}{\gamma_T\gamma_{NT}}\frac{(1+\psi_e)}{\psi_e}$$

and:

$$\phi_{NT} > \frac{(1-\gamma_T)(1+\psi_e)^2}{\gamma_T\psi\left[\frac{\phi_{Cu}}{\phi_T}\gamma_{NT}-(1+\psi_e)\left(1-\frac{\gamma_{NT}}{\gamma_T}\right)\right]}$$

then $dE^{real}/dPCu > 0$, where E^{real} is real GDP.

Proof. See Appendix C. ∎

Thus, in addition to the condition from Proposition 2 for having an increase in non-tradables output, we need that the GDP share of non-tradables be larger than a certain value. This threshold value is more demanding the larger the relative reliance on commodities between the non-commodity tradable sector and the non-tradable sector, the smaller the effort wage elasticity, and the smaller the importance of the commodity industry in the whole tradable bundle. If sector T relies more on the commodity to produce, firms there will reduce production and productivity more strongly when commodity prices go up. At the same time, if it is harder to extract effort from households, productivity in sector NT will not go up enough to compensate for any eventual loss of output due to higher costs. Finally, if ϕ_{Cu}/ϕ_T is smaller, then any increase in PCu will not generate a large income effect on households, so it will be harder to generate the necessary increase in aggregate demand to make non-tradables produce more and get more productive. In summary, to match the micro- and macro-patterns we need that the tradable sector is concentrated in the commodity producing industry and that the non-tradable sector has an important participation in GDP.

In the next section I will calibrate the full model to incorporate more realistic features that will allow me to gauge the quantitative relevance of the factor utilization mechanism in a stochastic business cycle setup.

4 Quantitative Analysis

In this section I will evaluate the predictions of the model with respect to key aggregate variables when the model is fed with realistic copper price shocks. Special attention will be placed on the behavior of aggregate TFP and real GDP, which are the two variables highlighted in Figure 1 in the Introduction.

4.1 Calibration

I set $\sigma = 2$, which is a standard value in the open-economy literature that features GHH preferences (see Drechsel and Tenreyro (2018), for example). I will normalize the relative distaste for hours worked relative to effort per hour, χ, to 1, as it does not affect the dynamics of the model. For the inter-temporal discount factor

I choose $\beta = 0.96$, in order to match a steady-state international interest rate, r^*, of 4%. The depreciation rate parameter, $\bar{\delta}$, will be set to generate a steady state capital utilization rate equal to 1. The total number of sectors, J, will be equal to 43, which corresponds to a slightly coarser classification of sectors than the one used in the 1996 Chilean Input-Output Tables. Of these 43 sectors, and following the definition of non-tradability used in Section 2, 29 industries will be tradable, and 14 will be non-tradable. More details about the classification procedure can be found in Appendix D.[26] The steady-state level of internationally traded bonds, $\bar{B}$, will be set to match the 1996 trade balance share of GDP, which is equal to -1.6%. The parameter that rules the elasticity of the domestic interest rate with respect to international asset holdings, ϕ, will target the volatility of the trade balance to GDP ratio in the 1996-2009 period, which is equal to 5.37%. Similarly, the capital adjustment costs parameter, τ, will be set to match the volatility of investment in the 1996-2009 period, which is equal to 9.77%. The parameter that rules the convexity of the depreciation rate with respect to the capital utilization rate, ψ_u, will match a steady state depreciation rate equal to 4%. This is a value consistent with the shares of GDP that are observed in Chilean national accounts for the net stock of capital and the consumption of fixed capital. The parameter ψ_n will be set to 1.33, in order to match a wage elasticity of aggregate hours worked of 0.75, as suggested by Chetty et al. (2011). Following the approach in Huo et al. (2020) I will set ψ_e to this same value.

The parameters α_j, γ_{jk}, γ_{jn}, and φ_{ij} are chosen to match the model-predicted cost shares to those observed in 1996 I-O tables. Final consumption shares, α_j, are chosen so that the model's steady-state consumption choices are proportional to the amount that the industry sells to consumers or the government. For simplicity, they are restricted to sum to 1. The output elasticities of labor, γ_{jn}, are set to match the sectoral wage bill/gross output ratio. Similarly, intermediate purchase shares, φ_{ij}, will be set to match the sector j's output share of the corresponding intermediate purchases of sector i by sector j firms. Capital output elasticities, γ_{jk}, will just

[26] The reason for a coarser classification is that I only need detail in terms of sector subdivisions for the manufacturing industry. The rest of sectors are grouped according to two criteria: i) tradability of the industry; ii) characteristics of their activities. This means that, for example, Agriculture groups non-mining extractive activities such as Agriculture, Forestry, Hunting, and Fishing, which are all tradable also. On the other hand, Construction, even though is a non-tradable industry like Services, it is a very different activity from it.

be equal to $1 - \gamma_{jn} - \sum_{i=1}^{J} \varphi_{ij}$.

Steady-state final good prices, P_j, will be all normalized to one. TFP parameters, Z_j, on the other hand, will be set to match the value-added based GDP shares of each sector in the 1996 Input-Output tables and a steady-state value for GDP of 1.

Finally, the parameters governing the stochastic process for the international price of copper, ρ_{Cu} and σ_{Cu}, will be estimated by fitting an AR(1) process to a yearly series for the mining GDP deflator/CPI ratio. A summary of some of the parameter values and targets can be found in Table 4. Output elasticities, consumption weights, and GDP shares can be found in Tables 1 and 2 of Appendix D.

TABLE 4 - CALIBRATION

Parameter	Description	Value	Target/Source
Panel A. Externally assigned			
σ	Relative risk aversion coefficient	2	Literature
χ	Effort distaste parameter	1	Normalization
$\{J^T, J^{NT}\}$	Number of T/NT sectors	$\{29, 14\}$	I-O tables
ψ_n	Inverse of hours wage elasticity	1.33	Chetty et al. (2011)
ψ_e	Inverse of effort wage elsaticity	1.33	Arbitrary
ρ_{Cu}	Copper price persistence	0.38	Estimation
σ_{Cu}	S.D. copper price shocks	0.25	Estimation
Panel B. Internally calibrated parameters			
ψ_u	Convexity depreciation rate function	1.04	SS capital utilization rate = 1
β	Inter=temporal discount factor	0.96	SS $r^* = 4\%$
$\bar{\delta}$	Depreciation rate level parameter	0.08	SS depreciation rate = 4%
$\bar{B}$	SS bond holdings level	0.39	TB/GDP = -1.6%
ϕ	Elasticity of domestic interest rate function	0.0001	$\sigma_{TB/GDP} = 5.37\%$
τ	Capital adjustment costs function parameter	0.06	$\sigma_I = 9.77\%$

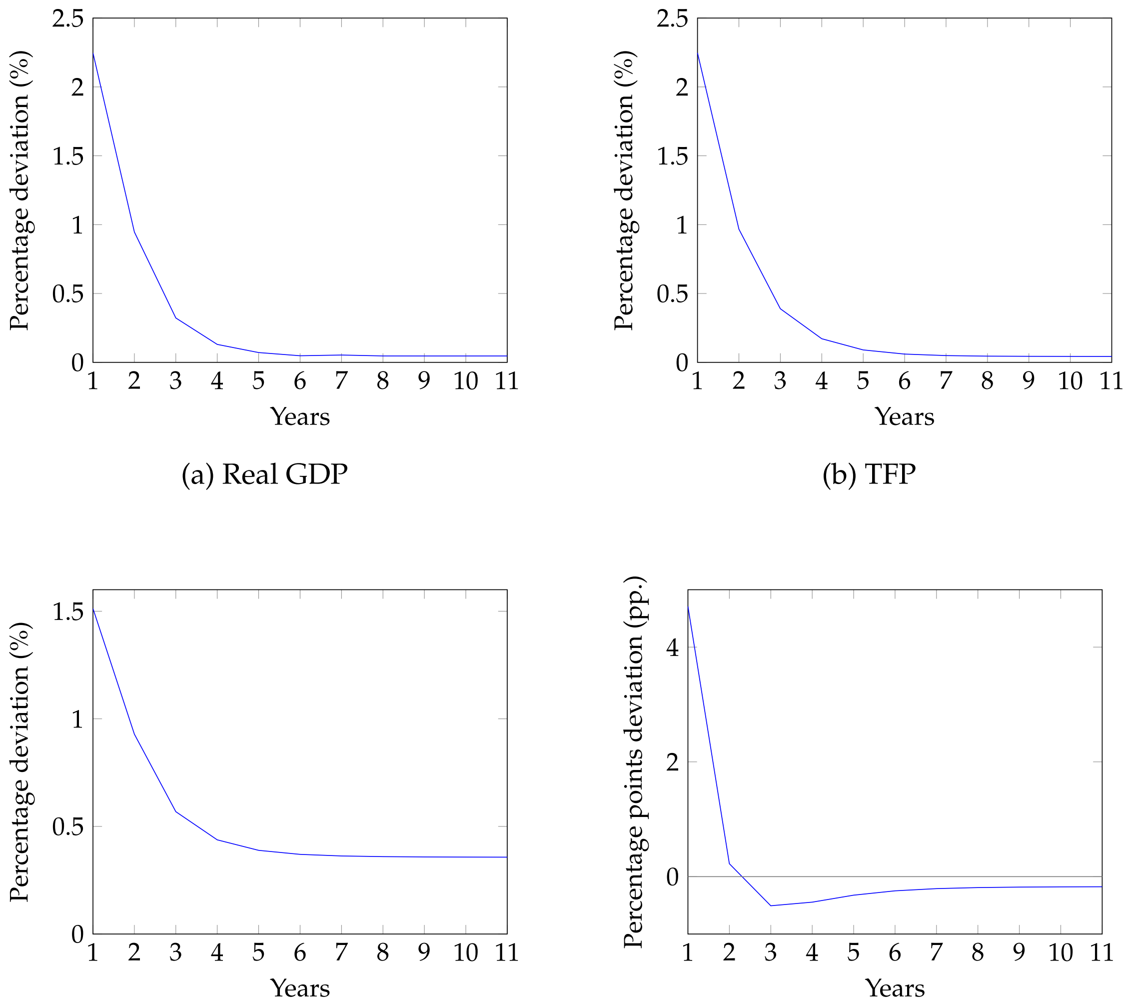
4.2 Results

4.2.1 Impulse Response Functions to Copper Price Shocks

Figure 4 displays the impulse response functions to a one-standard deviation copper price shock, which is equivalent to a 25% deviation of the price of copper from its steady-state value, using the calibration described above. The figure shows that the responses are in line with what the theory predicted in Section 3. We can observe that TFP and real GDP both co-move in response to the copper price shock, and display an impact elasticity to the shock of around 10%. This increase in TFP and real GDP, as expected, is accompanied by an increase in aggregate consumption. As the shock is transitory and its persistence limited, the response of aggregate consumption is lower than that of real GDP (its impact elasticity being around 5.6%), which rationalizes the increase in the trade balance to GDP ratio that occurs when the shock arrives. However, as consumption is smoothed over time, its level remains above its steady state for several periods after the shock hit and unfolded. In consequence, even though on impact the trade balance to GDP ratio behaves in a procyclical way, negative values are observed shortly after the initial positive response to the copper price shock.

Positive copper price shocks have three effects on the economy. First, they raise revenues in the copper sector. These higher revenues will induce the copper sector to increase production but, as managers do not have the ability to hire more inputs in the very short run, they increase the utilization of the two factors of production, labor and capital, displaying a higher sectoral TFP. Second, firms that use copper in their production processes will face higher production costs. Higher costs induce firms to cut back on production and hire less inputs. As they cannot fire inputs in the very short run, they use them less intensively, featuring a lower TFP. And third, if the positive effect on the copper sector is larger than the negative effect on other sectors that are downstream the copper industry, households enjoy higher wealth and increase consumption of all final goods. Sectors producing final goods that are non-tradable thus face a higher demand and higher revenues, which makes them want a larger scale, so they use their inputs more intensively and display a larger TFP. As it is clear from the plots, given the size of the Chilean copper sector (around 6% of GDP), the first effect dominates the second, which creates the positive general equilibrium effect on non-tradables and the ensuing positive response of TFP and real GDP.

FIGURE 2 - IMPULSE RESPONSES TO 1 S.D. COPPER PRICE SHOCK



(a) Real GDP

(b) TFP

(c) Consumption

(d) Trade Balance/GDP

4.2.2 Factor Utilization and Productivity Dynamics During the 1997-2009 Period

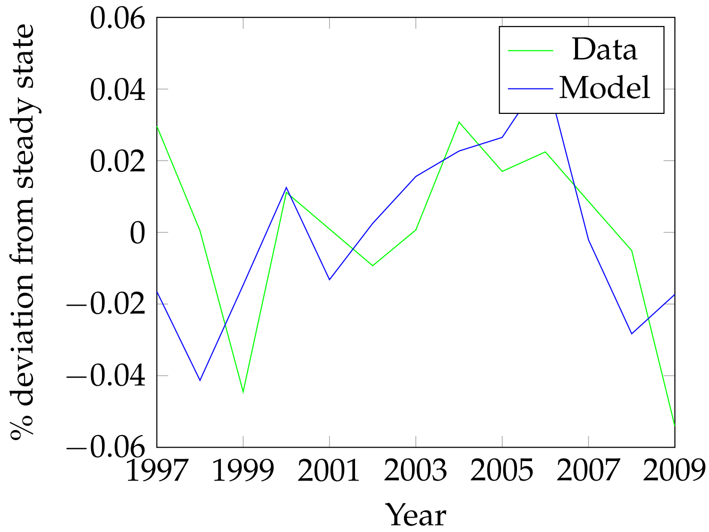
The period 1997-2009 was unique for the Chilean economy, as it featured important changes in the value of copper in international markets. These fluctuations were driven to a big extent by the entry of China to international markets after its WTO accession in 2001. Jointly with this major event, Chile faced two international financial crises in this period, the 1997-98 Asian crisis and the Great Recession of 2008-09. It is thus of special interest to know to what degree fluctuations in TFP and real GDP can be explained by the variability in factor utilization that was induced by terms of trade shocks, and to what degree other factors might have influenced in their evolution.

To make the analysis it will be assumed that the Chilean economy was at its non-stochastic steady state in 1996. The model will then be fed with the copper price shocks implied by the actual copper price series for the period 1996-2009. I will compare the model predicted evolution of aggregate TFP and real GDP with the actual detrended series for both variables. After that, I will evaluate the performance of the model in terms of the differences of predicted second moments for several variables with respect to their data counterparts.

We can appreciate in Figures 3 and 4 that the model is quite successful in describing the overall evolution of real GDP and aggregate TFP during the period of analysis. Some differences have to be noted though. Even though the fit with the data in the case of real GDP is quite high, this occurs to a lesser extent with TFP. In particular, the model predicted series for aggregate TFP is more volatile than that of the data. This tends to happen in periods in which the copper price perturbation is large, which in this case correspond to the Asian crisis and the build up of the Great Recession. A possible explanation for this higher than actual predicted volatility is that I am only considering copper price shocks as a source of fluctuations for terms of trade shocks. In particular, it was usually the case in this period that increases in the price of copper were accompanied by increases in the price of oil, and viceversa. Chile is a net importer of oil, so increases in the price of crude could have played a countervailing force on factor utilization and TFP. At the same time, although not included in the model, it might well be the case that the central government has had a more counter-cyclical behavior than what is described by the household sector. It is important to note that since 2001 Chile adopted a fiscal rule in which government spending would be set in order to keep a structural balance in public finances. This means that the government spends according to long-run revenues, which could have allowed it to have a larger fiscal deficit when the price of copper was low and a higher surplus when it was high.

In Table 5 I display the fit of the model with respect to second moments of several aggregate variables. Unsurprisingly, the volatility of aggregate investment and the trade balance to real GDP are close to their data counterparts, as these moments were used in the calibration section to assign values to the parameter of the capital adjustment cost function (τ) and the elasticity of the domestic interest

FIGURE 3 - REAL GDP, MODEL VS. DATA, 1997-2009



rate to changes in accumulated international assets (ϕ). The most remarkable result is the one of real GDP, where predicted volatility is quite close to that observed in the data, even though it was not explicitly targeted in the calibration procedure.

The model underpredicts the volatility of aggregate consumption and hours worked. This could be due to two reasons, both of them related to the complementarity that GHH preferences induce over consumption and labor. One of them is that the parameter for the inverse of the wage elasticity of hours worked, ψ_n, was set at a value that was too high for the cyclical characteristics of the Chilean labor supply. If hours vary little, then consumption will vary little as well. The other is that Chilean households could be better described as financially constrained, so variations in income create larger fluctuations in their desired consumption than what would be predicted by a model with a non-financially constrained household such as this. If consumption varies less than in the data then, hours will vary less too.

The model predicts an overall negative correlation of the aggregate investment rate and the trade balance to GDP ratio with copper prices. The reason for this apparent contradictory result is that, on impact, trade balance co-moves positively with a positive copper price shock but goes below its steady state level shortly after. As copper price shocks are persistent, the correlation coefficient tends to capture the negative correlation that is observed after the shock hits. On the other hand, on impact, as consumption rates are relatively stable, the increase in the

FIGURE 4 - TFP, MODEL VS. DATA, 1997-2009

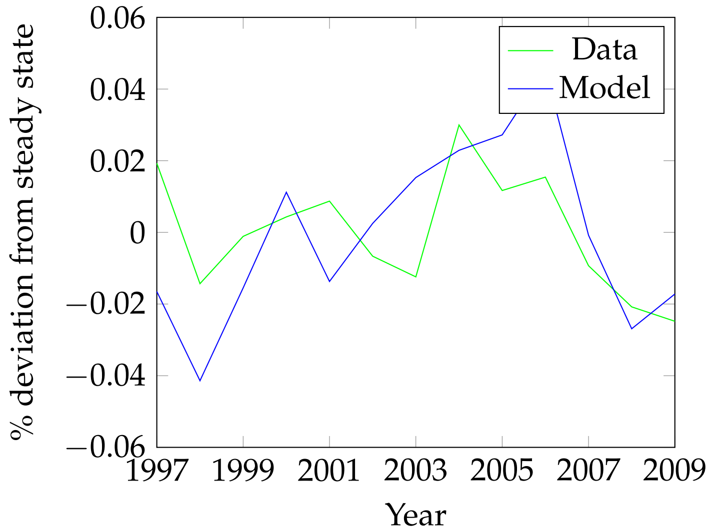


TABLE 5 - EMPIRICAL AND THEORETICAL SECOND MOMENTS, CHILE, 1996-2009

	Model			**Data**		
Variable (x)	σ_x	$\text{Corr}(x_t, x_{t-1})$	$\text{Corr}(x_t, PCu_t)$	σ_x	$\text{Corr}(x_t, x_{t-1})$	$\text{Corr}(x_t, PCu_t)$
Real GDP	2.33	0.52	1.00	2.45	0.14	0.46
TFP	2.35	0.53	1.00	1.58	0.19	0.56
C	1.59	0.59	0.96	3.50	0.12	0.47
I	9.78	0.22	-0.84	9.77	-0.09	0.32
N	0.38	0.45	0.50	2.57	-0.33	0.22
TB/GDP	4.53	0.32	-0.93	5.37	0.62	0.70

trade balance is partly achieved with lower investment rates. As investment rates are smoothed over time due to the presence of capital adjustment costs, they tend to be below their steady state levels in later periods too. This behavior creates the negative correlation with copper prices.

Finally, with the exception of the trade balance to GDP ratio, all aggregate variables considered tend to be more persistent in the model than in the data. This could be an outcome of the choice of just having one shock variable to explain Chile's business cycle in this period. Absent any other perturbations, when faced with a positive copper price shock, the model predicts a more protracted boom than what is observed in the data.

5 Conclusion

The relation between terms of trade shocks and the productivity of open economies is an open question in international macroeconomics. Focusing in the Chilean case, this paper has contributed to fill this gap by providing evidence suggestive of the existence of variable factor utilization as a short-run response to this type of shocks at the firm-level. In particular, when faced with positive copper price shocks, non-tradable firms tend to increase output and TFP relative to comparable tradable firms, but display no significant employment differences. Additionally, firms that rely more on copper to produce reduce output and TFP, with no sensible responses of employment either.

Building over these findings I elaborate a general equilibrium multi-sector model aiming at understanding under which contexts terms of trade shocks should co-move positively with aggregate TFP and when otherwise. The basic trade-off that the model incorporates is the one between the increase in revenues for the commodity industry and the increase in costs for the non-commodity tradable sectors. If the effect on commodity sector revenues overcome the negative effect on other tradable sectors, then households will enjoy a wealth increase and will demand more from every final good. The increase in factor utilization rates by firms in the commodity and non-tradable sectors will be expressed as a larger observed sectoral TFP, which in turn will lead to a higher aggregate TFP.

I calibrate the model to Chilean data for the period 1996-2009 in order to evaluate its ability to generate realistic patterns for aggregate TFP and other aggregate variables. The framework is successful in yielding a positive co-movement between copper price shocks and aggregate TFP, a pattern for TFP that closely resembles the one observed in the data for the period, and TFP deviations from trend that are in the orders of magnitude of those observed in reality. However, the volatility in TFP induced by copper price shocks is higher than that observed in the data for the period, suggesting that other shocks creating forces in the opposite direction, such as oil price shocks, are occurring simultaneously. Relatedly, the model predicts that, absent any other shocks, an increase in copper prices should create a more protracted boom in the economy than what is observed in the data.

In summary, this paper has assigned terms of trade shocks an important role in generating business cycle fluctuations by expanding the set of possible effects on domestic agents. This is confirmed by the results coming from the quantitative analysis. If taken at face value, these results leave little room for other productivity enhancing mechanisms such as R&D to play an important role on productivity dynamics at business cycle frequencies. My findings suggest that other types of shocks or productivity detrimental policies might be happening jointly, and in the opposite direction, with variations in commodity prices. The extension of the model to incorporate these elements and the improvement of its estimation by making use of the establishment-level results shown in Section 2 are tasks left for future work.

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Appendix A: Construction of Variables

Appendix A.1: TFP Estimation

Plant TFP will be measured using the Olley-Pakes (1996) method with investment as proxy variable.

The logged production function considered is:

$$y_{fit} = \beta_0 + \beta_p^i p_{fit} + \beta_{np}^i np_{fit} + \beta_m^i m_{fit} + \beta_k^i k_{fit} + \omega_{fit} + \mu_{fit}$$

where y_{fit} is real gross output, p_{fit} is number of blue-collar workers, np_{fit} is number of white-collar workers, m_{fit} are real materials purchases, and k_{fit} is the plant's stock of capital.

Hence, the log of TFP will correspond to the projected residual after applying the method industry-by-industry.

Appendix A.2: Construction of Normalized Leontief Inverse Coefficients

I take the Leontief inverse coefficients for the manufacturing sector that appear on the row and column associated with the cross with the copper industry, as reported by the Central Bank of Chile. There are 37 manufacturing sub-sectors included in the Chilean input-output matrix. In this matrix, rows correspond to the downstream direction while columns to the upstream direction. I compute the average and the standard deviation of both the row and the column coefficients. I finish by expressing each industry coefficient as a standardized deviation from the corresponding direction average.

More specifically, let $\mathcal{J}$ be the set of all manufacturing industries that appear in the input-output matrix, whose cardinality is denoted by J. A normalized upstream Leontief inverse coefficient for manufacturing sector i, $\tilde{a}_{i \to Cu}$, is given by:

$$\tilde{a}_{i \to Cu} = \frac{a_{i \to Cu} - \bar{a}_{j \to Cu, j \in \mathcal{J}}}{\sigma_{j \to Cu, j \in \mathcal{J}}}$$

where $a_{i \to Cu}$ is the raw upstream Leontief inverse term for manufacturing sector i, $\bar{a}_{j \to Cu, j \in \mathcal{J}}$ is the average of all upstream Leontief inverse coefficients that correspond to manufacturing industries, and $\sigma_{j \to Cu, j \in \mathcal{J}}$ is the standard deviation of those same coefficients, i.e.:

$$\bar{a}_{j\to Cu,j\in\mathcal{J}} = \frac{1}{J}\sum_{k=1}^{J} a_{k\to Cu}$$

$$\sigma_{j\to Cu,j\in\mathcal{J}} = \sqrt{\frac{1}{J}\sum_{k=1}^{J}\left(a_{k\to Cu} - \bar{a}_{j\to Cu,j\in\mathcal{J}}\right)^2}$$

Similarly, a normalized downstream Leontief inverse coefficient for manufacturing sector i, $\tilde{a}_{Cu\to i}$, is given by:

$$\tilde{a}_{Cu\to i} = \frac{a_{Cu\to i} - \bar{a}_{Cu\to j,j\in\mathcal{J}}}{\sigma_{Cu\to j,j\in\mathcal{J}}}$$

where $a_{Cu\to i}$ is the raw downstream Leontief inverse term for manufacturing sector i, $\bar{a}_{Cu\to j,j\in\mathcal{J}}$ is the average of all downstream Leontief inverse coefficients that correspond to manufacturing industries, and $\sigma_{Cu\to j,j\in\mathcal{J}}$ is the standard deviation of those same coefficients, i.e.:

$$\bar{a}_{Cu\to j,j\in\mathcal{J}} = \frac{1}{J}\sum_{k=1}^{J} a_{Cu\to k}$$

$$\sigma_{Cu\to j,j\in\mathcal{J}} = \sqrt{\frac{1}{J}\sum_{k=1}^{J}\left(a_{Cu\to k} - \bar{a}_{Cu\to j,j\in\mathcal{J}}\right)^2}$$

Appendix B: Additional Results

Appendix B.1: Results for Omitted Controls in Baseline Specification

TABLE 1 - EFFECTS OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL GROSS OUTPUT, INPUTS, AND RTFP

	Output	Employment	Capital/ worker	Materials/ worker	RTFP
$\Delta \ln(PCu_t) \cdot \tilde{a}_{i,Cu}$	-0.01	0.03**	-0.03	0.02	-0.05
	(0.03)	(0.01)	(0.02)	(0.02)	(0.04)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{i,Cu} \cdot \mathbb{1}\{NT\}$	0.01	-0.00	0.04	0.01	-0.00
	(0.04)	(0.01)	(0.04)	(0.03)	(0.06)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i} \cdot \mathbb{1}\{NT\}$	2.67***	0.70**	-2.22***	1.96*	1.59***
	(0.46)	(0.27)	(0.83)	(1.04)	(0.51)
$\log(\text{RTFP})_{t-1}$	-0.04***	0.03***	0.20***	0.12***	-0.19***
	(0.01)	(0.00)	(0.01)	(0.02)	(0.01)
$\log(\text{Employment})_{t-1}$	0.01***	-0.04***	0.06***	0.06***	0.00
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
$\log(\text{Age})_{t-1}$	-0.00***	-0.00	-0.02	-0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Observations	31,504	31,504	31,504	31,504	31,504
Industry and Time FE	Y	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the yearly change in the natural log of the listed outcome. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

TABLE 2 - EFFECT OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL MEASURES OF INPUT USAGE INTENSITY

	Avg. Real Wage	Workdays	Electricity Consumption	Fuel Consumption
$\Delta \ln(PCu_t) \cdot \tilde{a}_{i,Cu}$	-0.06*	0.00	0.02	0.03
	(0.03)	(0.00)	(0.03)	(0.03)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{i,Cu} \cdot \mathbb{1}\{NT\}$	0.00	0.01	0.07	-0.01
	(0.06)	(0.00)	(0.05)	(0.05)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i} \cdot \mathbb{1}\{NT\}$	1.35*	0.44***	0.88	1.94
	(0.73)	(0.15)	(0.98)	(1.39)
$\log(\text{RTFP})_{t-1}$	-0.02***	-0.00	0.02**	0.02
	(0.01)	(0.00)	(0.01)	(0.01)
$\log(\text{Employment})_{t-1}$	0.04***	0.00	0.02***	0.03***
	(0.00)	(0.00)	(0.00)	(0.01)
$\log(\text{Age})_{t-1}$	-0.00***	-0.00*	-0.00**	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Observations	31,504	31,482	31,269	27,443
Industry and Time FE	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the yearly change in the natural log of the listed outcome. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

TABLE 3 - EFFECTS OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL GROSS OUTPUT, INPUTS, AND RTFP (4-YEAR DIFFERENCES)

	Output	Employment	Capital/ worker	Materials/ worker	RTFP
$\Delta \ln(PCu_t) \cdot \tilde{a}_{i,Cu}$	0.03	0.05***	-0.05	0.06***	-0.05
	(0.03)	(0.02)	(0.04)	(0.02)	(0.03)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{i,Cu} \cdot \mathbb{1}\{NT\}$	0.06	-0.06*	0.11	0.14***	0.03
	(0.06)	(0.03)	(0.07)	(0.04)	(0.07)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i} \cdot \mathbb{1}\{NT\}$	3.05***	1.25*	-2.22	1.07	1.94**
	(0.77)	(0.64)	(1.59)	(1.15)	(0.84)
$\log(\text{RTFP})_{t-1}$	-0.09***	0.05***	0.62***	0.12***	-0.39***
	(0.01)	(0.01)	(0.04)	(0.03)	(0.02)
$\log(\text{Employment})_{t-1}$	0.03***	-0.07***	0.06**	0.11***	0.02*
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)
$\log(\text{Age})_{t-1}$	-0.02***	-0.01	-0.05***	-0.00	0.00
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)
Observations	17,148	17,148	16,624	17,148	16,624
Industry and Time FE	Y	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the 4-year change in the natural log of the listed outcome. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

TABLE 4 - EFFECT OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL MEASURES OF INPUT USAGE INTENSITY (4-YEAR DIFFERENCES)

	Avg. Real Wage	Workdays	Electricity Consumption	Fuel Consumption
$\Delta \ln(PCu_t) \cdot \tilde{a}_{i,Cu}$	0.44*	-0.01	0.07	-0.35
	(0.26)	(0.07)	(0.25)	(0.79)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{i,Cu} \cdot \mathbb{1}\{NT\}$	0.14**	0.00	0.20***	-0.00
	(0.06)	(0.00)	(0.04)	(0.11)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i} \cdot \mathbb{1}\{NT\}$	1.82*	-0.05	0.32	-2.32
	(1.02)	(0.26)	(0.96)	(3.30)
$\log(\text{RTFP})_{t-1}$	-0.03**	-0.00	0.02	0.07
	(0.01)	(0.00)	(0.02)	(0.06)
$\log(\text{Employment})_{t-1}$	0.05***	0.00	0.07***	0.18***
	(0.01)	(0.00)	(0.01)	(0.02)
$\log(\text{Age})_{t-1}$	-0.00	-0.00	-0.01	-0.01
	(0.00)	(0.00)	(0.01)	(0.02)
Observations	17,148	17,135	17,025	14,403
Industry and Time FE	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the 4-year change in the natural log of the listed outcome. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Appendix B.2: Results for Robustness

B.2.1. Results with RTFP calculated with Olley-Pakes (1996) methodology

TABLE 1 - EFFECTS OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL GROSS OUTPUT, INPUTS, AND RTFP

	Output	Employment	Capital/ worker	Materials/ worker	RTFP
Panel A. 1-Year Differences					
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.03***	0.00	-0.02	-0.04***	-0.02**
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.59***	0.09	-0.23	0.44*	0.39***
	(0.13)	(0.14)	(0.41)	(0.24)	(0.11)
Observations	27,865	27,865	31,504	31,504	26,787
Panel B. 4-Year Differences					
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.05***	-0.00	0.02	-0.06***	-0.03***
	(0.02)	(0.01)	(0.03)	(0.01)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.76***	0.31**	-0.33	0.32	0.50***
	(0.19)	(0.13)	(0.45)	(0.30)	(0.18)
Observations	15,606	15,606	15,131	15,606	13,771
Industry and Time FE	Y	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

TABLE 2 - EFFECT OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL MEASURES OF INPUT USAGE INTENSITY

	Avg. Real Wage	Workdays	Electricity Consumption	Fuel Consumption
Panel A. 1-Year Differences				
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.09***	0.00*	-0.05***	-0.06***
	(0.01)	(0.00)	(0.01)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.31	0.07**	0.30	0.66*
	(0.22)	(0.03)	(0.23)	(0.33)
Observations	27,865	27,849	27,652	24,312
Panel B. 4-Year Differences				
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.08***	-0.00	-0.12***	-0.11***
	(0.01)	(0.00)	(0.02)	(0.03)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.36	-0.00	0.13	-0.08
	(0.27)	(0.07)	(0.25)	(0.73)
Observations	15,606	15,597	15,499	13,185
Industry and Time FE	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

B.2.2. Results with copper price series in Chilean pesos and deflated by Chilean CPI

TABLE 1 - EFFECTS OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL GROSS OUTPUT, INPUTS, AND RTFP

	Output	Employment	Capital/ worker	Materials/ worker	RTFP
Panel A. 1-Year Differences					
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.04***	0.01**	-0.03	-0.05***	-0.02**
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i} \cdot \mathbb{1}\{NT\}$	0.70***	0.33***	-0.64***	0.49	0.31*
	(0.15)	(0.11)	(0.41)	(0.30)	(0.19)
Observations	31,504	31,504	31,504	31,504	31,504
Panel B. 4-Year Differences					
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.06**	0.01	0.00	-0.08***	-0.03**
	(0.03)	(0.01)	(0.05)	(0.02)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.81***	0.48***	-0.63	0.32	0.41*
	(0.24)	(0.16)	(0.73)	(0.42)	(0.24)
Observations	17,148	17,148	16,624	17,148	16,624
Industry and Time FE	Y	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

TABLE 2 - EFFECT OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL MEASURES OF INPUT USAGE INTENSITY

	Avg. Real Wage	Workdays	Electricity Consumption	Fuel Consumption
Panel A. 1-Year Differences				
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.10***	0.00*	-0.05***	-0.09***
	(0.01)	(0.00)	(0.02)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.29*	0.13***	0.51*	0.61*
	(0.17)	(0.04)	(0.26)	(0.32)
Observations	31,504	31,482	27,652	27,443
Panel B. 4-Year Differences				
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.10***	0.00	-0.12***	-0.16***
	(0.02)	(0.00)	(0.03)	(0.05)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.49	-0.00	0.29	-0.69
	(0.33)	(0.09)	(0.31)	(0.92)
Observations	17,148	17,135	17,025	14,403
Industry and Time FE	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

B.2.3. Results with balanced panel, 1319 firms

TABLE 1 - EFFECTS OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL GROSS OUTPUT, INPUTS, AND RTFP

	Output	Employment	Capital/ worker	Materials/ worker	RTFP
Panel A. 1-Year Differences					
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.02	0.02***	0.02	-0.02	-0.04**
	(0.02)	(0.01)	(0.03)	(0.01)	(0.02)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.50***	0.05	-0.35	0.21	0.44**
	(0.14)	(0.06)	(0.40)	(0.40)	(0.20)
Observations	13,190	13,190	13,190	13,190	13,190
Panel B. 4-Year Differences					
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.04	0.01	0.00	-0.07***	-0.01
	(0.02)	(0.01)	(0.03)	(0.01)	(0.01)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.92***	0.37**	-0.21	0.31	0.45*
	(0.20)	(0.15)	(0.25)	(0.19)	(0.22)
Observations	9,233	9,233	9,233	9,233	9,233
Industry and Time FE	Y	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

TABLE 2 - EFFECT OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL MEASURES OF INPUT USAGE INTENSITY

	Avg. Real Wage	Workdays	Electricity Consumption	Fuel Consumption
Panel A. 1-Year Differences				
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.09***	-0.00	-0.05***	-0.07***
	(0.02)	(0.00)	(0.02)	(0.02)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.67***	0.04	-0.15	0.23
	(0.15)	(0.04)	(0.29)	(0.42)
Observations	13,190	13,187	13,138	11,821
Panel B. 4-Year Differences				
$\Delta \ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.09***	0.00	-0.11***	-0.12***
	(0.02)	(0.00)	(0.02)	(0.03)
$\Delta \ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.47	0.03	0.21	-0.09
	(0.32)	(0.11)	(0.23)	(0.79)
Observations	9,233	9,229	9,195	7,975
Industry and Time FE	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

B.2.4. Results with balanced panel, levels fixed effects, 1319 firms

TABLE 1 - EFFECTS OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL GROSS OUTPUT, INPUTS, AND RTFP

	Output	Employment	Capital/ worker	Materials/ worker	RTFP
$\ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.07**	0.01	-0.03	-0.07***	-0.03**
	(0.03)	(0.01)	(0.02)	(0.02)	(0.01)
$\ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.77***	0.18**	0.16	-0.17	0.62***
	(0.25)	(0.08)	(0.40)	(0.40)	(0.20)
Observations	13,190	13,190	13,190	13,190	13,190
Firm, Industry, and Time FE	Y	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

TABLE 2 - EFFECT OF COPPER PRICE SHOCKS ON ESTABLISHMENT-LEVEL MEASURES OF INPUT USAGE INTENSITY

	Avg. Real Wage	Workdays	Electricity Consumption	Fuel Consumption
$\ln(PCu_t) \cdot \tilde{a}_{Cu,i}$	-0.12***	0.00	-0.14***	-0.17***
	(0.03)	(0.00)	(0.02)	(0.02)
$\ln(PCu_t) \cdot \mathbb{1}\{NT\}$	0.10	-0.00	-0.51*	-0.33
	(0.24)	(0.07)	(0.27)	(0.67)
Observations	13,190	13,188	13,162	12,263
Industry and Time FE	Y	Y	Y	Y
Firm-Level Controls	Y	Y	Y	Y

Note: This table presents estimates of equation (1) for the 1995-2007 period. The dependent variable is the change in the natural log of the listed outcome. Firm-level controls are lagged age, employment, and RTFP, interactions of all the regressors included in the table, and analogous interactions for upstream coefficients. Robust standard errors in parentheses, clustered at the 4-digit industry-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Appendix C: Proofs Lemmas, Propositions, and Corollaries

C.1. Proof Proposition 1

We have that:

$$d \log Y_T = \gamma_T d \log e_T + (1 - \gamma_T) d \log M_T$$

From the input utilization function:

$$d \log e_T = d \log Y_T - d \log w_T$$

From the labor-effort function we get $d \log e_T = (1/\psi) d \log w_T$. Using the labor market clearing condition we obtain:

$$d \log w_T = \frac{\psi}{1+\psi} d \log Y_T$$

Substituting in the effort function we get:

$$d \log e_T = \frac{1}{1+\psi} d \log Y_T$$

From the demand for commodities we get:

$$d \log M_T = d \log Y_T - d \log PCu$$

Substituting for e_T and M_T into the log-differentiated production function expression of above and solving for $d \log Y_T$ we get:

$$d \log Y_T = -\frac{(1-\gamma_T)}{\gamma_T} \frac{(1+\psi)}{\psi} d \log PCu < 0$$

As $d \log Y_T = d \log TFP_T$, TFP_T is also a negative function of PCu. As e_T depends positively on Y_T, labor utilization rate in sector T is a negative function of PCu as well.

Differentiating $d \log Y_T / d \log PCu$ with respect to $1 - \gamma_T$ we obtain:

$$\frac{d^2 \log Y_T}{d \log(PCu) d(1-\gamma_T)} = -\frac{(1+\psi)}{\psi \gamma_T} < 0$$

The sign of this cross-derivative carries on to TFP_T and e_T.

C.2. Proof Lemma 1

The NT sector production function in log-differences is given by:

$$d\log Y_{NT} = \gamma_{NT} d\log e_{NT} + (1-\gamma_{NT}) d\log M_{NT}$$

The labor utilization function in log-differences is in turn given by:

$$d\log e_{NT} = d\log P_{NT} + d\log Y_{NT} - d\log w_{NT}$$

From the effort supply function we obtain:

$$d\log e_{NT} = \frac{1}{\psi} d\log w_{NT}$$

Combining effort supply and demand to solve for w_{NT} in log-differences:

$$\frac{1}{\psi} d\log w_{NT} = d\log P_{NT} + d\log Y_{NT} - d\log w_{NT}$$

$$\Rightarrow d\log w_{NT} = \frac{\psi}{1+\psi}\left(d\log P_{NT} + d\log Y_{NT}\right)$$

Substituting in the effort labor supply function:

$$d\log e_{NT} = \frac{1}{1+\psi}\left(d\log P_{NT} + d\log Y_{NT}\right)$$

On the other hand, the materials demand function in log differences is given by:

$$d\log M_{NT} = d\log P_{NT} + d\log Y_{NT} - d\log P_{Cu}$$

Substituting in the production function:

$$d\log Y_{NT} = \frac{\gamma_{NT}}{1+\psi}\left(d\log P_{NT} + d\log Y_{NT}\right) + (1-\gamma_{NT})\left(d\log P_{NT} + d\log Y_{NT} - d\log P_{Cu}\right)$$

Solving for $d\log Y_{NT}$ one gets:

$$d\log Y_{NT} = \frac{(1+\psi(1-\gamma_{NT}))}{\gamma_{NT}\psi} d\log P_{NT} - \frac{(1+\psi)(1-\gamma_{NT})}{\gamma_{NT}\psi} d\log P_{Cu}$$

To solve for the demand for NT goods we need to solve for $d\log E$ first. GDP is given by:

$$E = \gamma_T P_T Y_T + \gamma_{NT} P_{NT} Y_{NT} + P_{Cu} \bar{Cu}$$

In log-differences:

$$d\log E = \phi_T d\log Y_T + (1-\phi_T-\phi_{Cu})\left(d\log P_{NT} + d\log Y_{NT}\right) + \phi_{Cu} d\log P_{Cu}$$

From the NT demand schedule and the NT market clearing condition we know:

$$d \log E = d \log P_{NT} + d \log Y_{NT}$$

We can then substitute for $d \log P_{NT} + d \log Y_{NT}$ in the expression for $d \log E$ above and solve for $d \log E$. We get:

$$d \log E = \frac{\phi_T}{\phi_T + \phi_{Cu}} d \log Y_T + \frac{\phi_{Cu}}{\phi_T + \phi_{Cu}} d \log P_{Cu}$$

Substituting for $d \log Y_T$ from Proposition 1's proof:

$$d \log E = -\frac{\phi_T}{\phi_T + \phi_{Cu}} \frac{(1 - \gamma_T)}{\gamma_T} \frac{(1 + \psi)}{\psi} d \log P_{Cu} + \frac{\phi_{Cu}}{\phi_T + \phi_{Cu}} d \log P_{Cu}$$
$$\Rightarrow d \log E = \left(\frac{\phi_{Cu} \gamma_T \psi - \phi_T (1 - \gamma_T)(1 + \psi)}{\gamma_T \psi (\phi_T + \phi_{Cu})} \right) d \log P_{Cu}$$

If $d \log E / d \log P_{Cu} > 0$ then:

$$\phi_{Cu} \gamma_T \psi > \phi_T (1 - \gamma_T)(1 + \psi) \iff \frac{\phi_{Cu}}{\phi_T} > \frac{(1 - \gamma_T)}{\gamma_T} \frac{(1 + \psi)}{\psi}$$

Given that $d \log E = d \log P_{NT} + d \log Y_{NT}$, we have:

$$d \log Y_{NT} = \left(\frac{\phi_{Cu} \gamma_T \psi - \phi_T (1 - \gamma_T)(1 + \psi)}{\gamma_T \psi (\phi_T + \phi_{Cu})} \right) d \log P_{Cu} - d \log P_{NT}$$

Combining NT supply and demand:

$$\frac{(1 + \psi(1 - \gamma_{NT}))}{\gamma_{NT} \psi} d \log P_{NT} - \frac{(1 + \psi)(1 - \gamma_{NT})}{\gamma_{NT} \psi} d \log P_{Cu} =$$
$$\left(\frac{\phi_{Cu} \gamma_T \psi - \phi_T (1 - \gamma_T)(1 + \psi)}{\gamma_T \psi (\phi_T + \phi_{Cu})} \right) d \log P_{Cu} - d \log P_{NT}$$

We can then solve for $d \log P_{NT}$ to get:

$$d \log P_{NT} = \frac{(\gamma_T \phi_{Cu} (1 + \psi - \gamma_{NT}) + (1 + \psi) \phi_T (\gamma_T - \gamma_{NT}))}{(1 + \psi) \gamma_T (\phi_T + \phi_{Cu})} d \log P_{Cu}$$

Then I substitute for $d \log P_{NT}$ in the expression for $d \log Y_{NT}$ of above and solve to get:

$$d \log Y_{NT} = \frac{(\psi \phi_{Cu} \gamma_T \gamma_{NT} - (1 + \psi) \phi_T (1 + \psi(1 - \gamma_{NT}) - \gamma_T))}{\psi (1 + \psi) \gamma_T (\phi_T + \phi_{Cu})} d \log P_{Cu}$$

Hence, for $d \log Y_{NT} / d \log P_{Cu} > 0$ we need:

$$\psi \phi_{Cu} \gamma_T \gamma_{NT} - (1 + \psi) \phi_T (1 + \psi(1 - \gamma_{NT}) - \gamma_T) > 0$$
$$\iff \frac{\phi_{Cu}}{\phi_T} > \frac{(1 + \psi)}{\psi} \frac{(1 + \psi(1 - \gamma_{NT}) - \gamma_T)}{\gamma_T \gamma_{NT}}$$

C.3. Proof Proposition 2

We have that:

$$d\log(e_{NT}/e_T) = \frac{1}{1+\psi_e}\left(d\log P_{NT} + d\log Y_{NT} - d\log Y_T\right)$$

Differentiating both sides by $\log PCu$:

$$\frac{d\log(e_{NT}/e_T)}{d\log PCu} = \frac{1}{1+\psi_e}\left(\frac{d\log P_{NT} + d\log Y_{NT}}{d\log PCu} - \frac{d\log Y_T}{d\log PCu}\right)$$

From the non-tradables demand schedule we know that $d\log E = d\log P_{NT} + d\log Y_{NT}$. Hence:

$$\frac{d\log(e_{NT}/e_T)}{d\log PCu} = \frac{1}{1+\psi_e}\left(\frac{d\log E}{d\log PCu} - \frac{d\log Y_T}{d\log PCu}\right)$$

As $d\log E/d\log PCu > 0$ and $d\log Y_T/d\log PCu < 0$ per Proposition 1, then $d\log(e_{NT}/e_T)/d\log PCu > 0$, which means $d\log(TFP_{NT}/TFP_T)/d\log PCu > 0$ and $d\log(Y_{NT}/Y_T)/d\log PCu > 0$

C.4. Proof Proposition 3

The elasticity of real GDP, E^{real}, with respect to PCu is given by:

$$\frac{d\log E^{real}}{d\log PCu} = \phi_T\frac{d\log Y_T}{d\log P_{Cu}} + \phi_{NT}\frac{d\log Y_{NT}}{d\log PCu}$$

For this to be positive we need:

$$\frac{d\log Y_{NT}}{d\log P_{Cu}} > -\frac{\phi_T}{\phi_{NT}}\frac{d\log Y_T}{d\log PCu}$$

Substituting from the proof to Proposition 1:

$$\frac{d\log Y_{NT}}{d\log P_{Cu}} > \frac{\phi_T}{\phi_{NT}}\frac{(1-\gamma_T)}{\gamma_T}\frac{(1+\psi_e)}{\psi_e}$$

From the proof to Proposition 2 we have:

$$\frac{d\log Y_{NT}}{d\log P_{Cu}} = \frac{\psi_e\phi_{Cu}\gamma_T\gamma_{NT} - (1+\psi_e)\phi_T(1+\psi_e(1-\gamma_{NT})-\gamma_T)}{\psi_e(1+\psi_e)\gamma_T(\phi_T+\phi_{Cu})}$$

We can substitute the expression of above into the condition for $d\log E^{real}/d\log P_{Cu}$. I will solve for ϕ_{NT} after using the fact that $\phi_{Cu}+\phi_T = 1-\phi_{NT}$. I obtain:

$$\phi_{NT} > \frac{(1-\gamma_T)(1+\psi_e)^2}{\gamma_T\psi_e\left[\frac{\phi_{Cu}}{\phi_T}\gamma_{NT} - (1+\psi_e)\left(1-\frac{\gamma_{NT}}{\gamma_T}\right)\right]}$$

Appendix D: Additional Tables from Quantitative Model Section

TABLE D.1 - INDUSTRY DEFINITIONS

#	Industry	1996 IO Table	ISIC Rev. 3
1	Agriculture, hunting, forestry, and fishing	1-5	1-2,5
2	Copper	9	132
3	Other mining and quarrying	6-8,10	10-12,131,14
4	Meat products	11	1511
5	Seafood and fish products	12	1512
6	Fruit and vegetables products	13	1513
7	Oils and fats	14	1514
8	Dairy	15	152
9	Grain mill products	16	1531,1532
10	Animal feeds	17	1533
11	Bakery products	18	1541
12	Sugar	19	1542
13	Other food products	20	1543-1549
14	Liquors and spirits	21	1551
15	Wine	22	1552
16	Beer	23	1553
17	Non-alcoholic beverages	24	1554
18	Tobacco products	25	16
19	Textiles	26	17
20	Wearing apparel	27	18
21	Leather and leather products	28	1911,1912
22	Footwear	29	192
23	Wood and wood products	30	20
24	Paper and paper products	31	21
25	Printing and reproduction of recorded media	32	22
26	Coke and refined petroleum products	33	23
27	Basic chemicals	34	241

28	Other chemical products	35	242
29	Rubber products	36	251
30	Plastic products	37	252
31	Glass and glass products	38	261
32	Other non-metallic mineral products	39	269
33	Basic iron and steel	40	271
34	Non-ferrous metals	41	272
35	Metal products	42	28
36	Non-electrical machinery and equipment	43	29
37	Electrical machinery and equipment	44	30
38	Transport equipment	45	34,35
39	Furniture	46	36
40	Other manufacturing industries	47	37
41	Electricity, gas, and water	48-50	40,41
42	Construction	51	45
43	Services	52-73	50-93

TABLE D.2 - OUTPUT ELASTICITIES

Industry	Capital	Labor	Materials
Agriculture	0.251	0.247	0.502
Copper	0.397	0.141	0.462
Other mining	0.272	0.194	0.533
Meat products	0.245	0.058	0.697
Seafood products	0.198	0.104	0.698
Produce-based products	0.201	0.116	0.684
Oils and fats	0.219	0.050	0.731
Dairy	0.176	0.086	0.738
Grain mill products	0.124	0.066	0.810
Animal feeds	0.057	0.032	0.911
Bakery products	0.269	0.135	0.596
Sugar	0.349	0.051	0.600
Other food products	0.317	0.093	0.590
Liquors	0.217	0.108	0.675
Wine	0.278	0.103	0.619
Beer	0.239	0.105	0.656
Non-alcoholic beverages	0.319	0.094	0.587
Tobacco products	0.773	0.023	0.203
Textiles	0.263	0.157	0.580
Wearing apparel	0.206	0.199	0.596
Leather products	0.305	0.145	0.550
Footwear	0.291	0.175	0.535
Wood products	0.316	0.117	0.567
Paper products	0.294	0.091	0.614
Printing	0.283	0.234	0.483
Refined petroleum products	0.383	0.020	0.597
Basic chemicals	0.250	0.108	0.642
Other chemical products	0.313	0.160	0.527
Rubber products	0.181	0.190	0.629
Plastic products	0.123	0.156	0.721
Glass products	0.308	0.154	0.538
Other non-metallic mineral products	0.314	0.133	0.553

Basic iron and steel	0.321	0.163	0.515
Non-ferrous metals	0.302	0.112	0.586
Metal products	0.293	0.161	0.546
Non-electrical machinery	0.104	0.175	0.721
Electrical machinery	0.125	0.191	0.685
Transport equipment	0.273	0.153	0.574
Furniture	0.321	0.153	0.526
Other manufacturing industries	0.214	0.211	0.575
Electricity, gas, and water	0.410	0.092	0.498
Construction	0.259	0.278	0.463
Services	0.341	0.275	0.385

TABLE D.3 - 1996 CONSUMPTION AND GDP SHARES

Industry	Consumption	GDP Share
Agriculture	0.044	0.058
Copper	0.000	0.060
Other mining	0.000	0.012
Meat products	0.045	0.008
Seafood products	0.005	0.006
Produce-based products	0.010	0.005
Oils and fats	0.006	0.001
Dairy	0.021	0.004
Grain mill products	0.007	0.002
Animal feeds	0.001	0.001
Bakery products	0.027	0.007
Sugar	0.005	0.002
Other food products	0.014	0.006
Liquors	0.004	0.001
Wine	0.004	0.003
Beer	0.004	0.001
Non-alcoholic beverages	0.015	0.004
Tobacco products	0.013	0.007
Textiles	0.007	0.007
Wearing apparel	0.050	0.005
Leather products	0.003	0.001
Footwear	0.020	0.004
Wood products	0.000	0.010
Paper products	0.009	0.012
Printing	0.007	0.010
Refined petroleum products	0.021	0.015
Basic chemicals	0.001	0.003
Other chemical products	0.036	0.013
Rubber products	0.001	0.002
Plastic products	0.006	0.004
Glass products	0.001	0.001
Other non-metallic mineral products	0.002	0.009

Basic iron and steel	0.000	0.004
Non-ferrous metals	0.000	0.003
Metal products	0.005	0.011
Non-electrical machinery	0.010	0.003
Electrical machinery	0.018	0.001
Transport equipment	0.028	0.005
Furniture	0.008	0.004
Other manufacturing industries	0.009	0.001
Electricity, gas, and water	0.020	0.030
Construction	0.001	0.100
Services	0.512	0.553

Appendix E: Quantitative Model Solution

E.1. Households

The household's problem is:

$$
\begin{aligned}
V_t\left(B_t, \{N_{jt}\}_{j=1}^J\right) &= \max_{\mathbf{v}_t} \frac{u\left(C_t, \{N_{jt}\}_{j=1}^J, \{e_{jt}\}_{j=1}^J\right)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t\left[V_{t+1}\left(B_{t+1}, \{N_{jt+1}\}_{j=1}^J\right)\right] \\
\text{s.t.} \quad & B_{t+1} + \sum_{j=1}^J P_{jt} C_{jt} = (1+r_t) B_t + \sum_{j=1}^J w_{jt} e_{jt} N_{jt} + \sum_{j=1}^J \pi_{jt} \\
& C_t = \prod_{j=1}^J C_{jt}^{\alpha_j}
\end{aligned}
\tag{24}
$$

where $\mathbf{v}_t = \left\{B_{t+1}, \{C_{jt}\}_{j=1}^J, \{e_{jt}\}_{j=1}^J, \{N_{jt+1}\}_{j=1}^J\right\}$. Let $\Omega_t = C_t - \sum_{j=1}^J \frac{N_{jt}^{1+\psi_n}}{1+\psi_n} - \chi \sum_{j=1}^J N_{jt} \frac{e_{jt}^{1+\psi_e}}{1+\psi_e}$.

FOC w.r.t. C_t:

$$\Omega_t^{-\sigma} - \lambda_t = 0 \tag{25}$$

FOC w.r.t. B_{t+1}:

$$-\lambda_t + \beta \mathbb{E}_t\left[\frac{\partial V_{t+1}\left(B_{t+1}, \{N_{jt+1}\}_{j=1}^J\right)}{\partial B_{t+1}}\right] = 0 \tag{26}$$

FOC w.r.t. e_{jt}:

$$-\chi N_{jt} e_{jt}^{\psi_e} \Omega_t^{-\sigma} + \lambda_t w_{jt} N_{jt} = 0 \tag{27}$$

FOC w.r.t. N_{jt+1}:

$$\beta \mathbb{E}_t\left[\frac{\partial V_{t+1}\left(B_{t+1}, \{N_{j,t+1}\}_{j=1}^J\right)}{\partial N_{jt+1}}\right] = 0 \tag{28}$$

Envelope conditions:

$$\frac{\partial V_t\left(B_{t+1}, \{N_{jt}\}_{j=1}^J\right)}{\partial N_{jt}} = \left(w_{jt} e_{jt} - N_{jt}^{\psi_n} - \chi \frac{e_{jt}^{1+\psi_e}}{1+\psi_e}\right) \Omega_t^{-\sigma} \tag{29}$$

$$\frac{\partial V_t\left(B_t, \{N_{jt}\}_{j=1}^J\right)}{\partial B_t} = (1+r_t)\Omega_t^{-\sigma} \tag{30}$$

Combining (25), (26), and (30):

$$\Omega_t^{-\sigma} = \beta(1 + r_{t+1})\mathbb{E}_t\left[\Omega_{t+1}^{-\sigma}\right] \tag{31}$$

Combining (25) and (27):

$$e_{jt} = \left(\frac{w_{jt}}{\chi}\right)^{\frac{1}{\psi_e}} \tag{32}$$

Combining (28), and (29):

$$\mathbb{E}_t\left[\left(w_{jt+1}e_{jt+1} - N_{jt+1}^{\psi_n} - \chi\frac{e_{jt+1}^{1+\psi_e}}{1+\psi_e}\right)\Omega_{t+1}^{-\sigma}\right] = 0 \tag{33}$$

FOC w.r.t. C_{jt}:

$$C_{jt} = \alpha_j \frac{C_t}{P_{jt}} \tag{34}$$

E.2 Firms

The problem of a sector j firm can be stated as follows:

$$\begin{aligned}
\mathcal{V}_{jt}(K_{jt}) = &\max_{\mathbf{x}_{jt}} \pi_{jt} + \frac{1}{1+r_{t+1}}\mathbb{E}_t\left[\mathcal{V}_{jt+1}(K_{jt+1})\right] \\
\text{s.t.} \quad & \pi_{jt} = P_{jt}Y_{jt} - w_{jt}L_{jt} - \sum_{i=1}^{J} P_{it}M_{ij,t} - P_{jt}\left(I_{jt} + \frac{\tau}{2}\left(K_{jt+1} - K_{jt}\right)^2\right) \\
& Y_{jt} = Z_j(u_{jt}K_{jt})^{\gamma_{jk}} L_{jt}^{\gamma_{jn}} \prod_{i=1}^{J} M_{i,j,t}^{\varphi_{ij}} \\
& K_{jt+1} = I_{jt} + (1-\delta_{jt})K_{jt} \\
& \delta_{jt} = \bar{\delta}\frac{u_{jt}^{1+\psi_u}}{1+\psi_u}
\end{aligned} \tag{35}$$

where $\mathbf{x}_{jt} = \left\{L_{jt}, u_{jt}, \{M_{ij,t}\}_{i=1}^{J}, I_{jt}\right\}$

FOC w.r.t. u_{jt}:

$$u_{jt} = \left(\gamma_{jk}\frac{Y_{jt}}{\bar{\delta}K_{jt}}\right)^{\frac{1}{1+\psi_u}} \tag{36}$$

FOC w.r.t. L_{jt}:

$$L_{jt} = \gamma_{jn}\frac{P_{jt}Y_{jt}}{w_{jt}} \tag{37}$$

FOC w.r.t. M_{ijt}:

$$M_{ij,t} = \varphi_{ij} \frac{P_{jt} Y_{jt}}{P_{it}} \tag{38}$$

FOC w.r.t. I_{jt}:

$$-P_{jt}\left(1 + \tau\left(K_{jt+1} - K_{jt}\right)\right) + \frac{1}{1 + r_{t+1}} \mathbb{E}_t \left[\frac{\partial \mathcal{V}_{jt+1}(K_{jt+1})}{\partial K_{jt+1}}\right] = 0 \tag{39}$$

Envelope condition:

$$\frac{\partial \mathcal{V}_{jt}(K_{jt})}{\partial K_{jt}} = P_{jt}\left(\gamma_{jk} \frac{Y_{jt}}{K_{jt}} + 1 - \delta_{jt} + \tau\left(K_{jt+1} - K_{jt}\right)\right) \tag{40}$$

Combining (39) and (40):

$$P_{jt}\left(1 + \tau\left(K_{jt+1} - K_{jt}\right)\right) = \frac{1}{1 + r_{t+1}} \mathbb{E}_t \left[P_{jt+1}\left(\frac{\gamma_{jk} Y_{jt+1}}{K_{jt+1}} + \tau\left(K_{jt+2} - K_{jt+1}\right) + (1 - \delta_{jt+1})\right)\right] \tag{41}$$

E.3 Market clearing conditions

Labor market:

$$L_{jt} = e_{jt} N_{jt} \tag{42}$$

Non-tradable goods:

$$Y_{jt} = C_{jt} + I_{jt} + \frac{\tau}{2}\left(K_{jt+1} - K_{jt}\right)^2 + \sum_{i=1}^{J} M_{ji,t} \tag{43}$$

Balance of payments:

$$B_{t+1} + \sum_{j=1}^{J} P_{jt}\left(C_{jt} + I_{jt} + \frac{\tau}{2}\left(K_{jt+1} - K_{jt}\right)^2 + \sum_{i=1}^{J} M_{ji,t}\right) = (1 + r_t) B_t + \sum_{j=1}^{J} P_{jt} Y_{jt} \tag{44}$$

Endogenous interest-rate premium

$$1 + r_t = 1 + r^* + \phi(\exp\{\bar{B} - B_t\} - 1) \tag{45}$$

Exogenous copper price process:

$$\log P_{Cu,t} = \rho_{Cu} P_{Cu,t-1} + \varepsilon_{Cu,t} \tag{46}$$

with $\varepsilon_{Cu,t} \sim N(0, \sigma^2_{Cu})$

E.4 Equations of the Model

$$\Omega_t = C_t - \sum_{j=1}^{J} \frac{N_{jt}^{1+\psi_n}}{1+\psi_n} - \chi \sum_{j=1}^{J} N_{jt} \frac{e_{jt}^{1+\psi_e}}{1+\psi_e} \tag{47}$$

$$\Omega_t^{-\sigma} = \beta(1+r_{t+1})\mathbb{E}_t\left[\Omega_{t+1}^{-\sigma}\right] \tag{48}$$

$$e_{jt} = \left(\frac{w_{jt}}{\chi}\right)^{\frac{1}{\psi_e}} \tag{49}$$

$$\mathbb{E}_t\left[\left(w_{jt+1}e_{jt+1} - N_{jt+1}^{\psi_n} - \chi\frac{e_{jt+1}^{1+\psi_e}}{1+\psi_e}\right)\Omega_{t+1}^{-\sigma}\right] = 0 \tag{50}$$

$$u_{jt} = \left(\gamma_{jk}\frac{Y_{jt}}{\bar{\delta}K_{jt}}\right)^{\frac{1}{1+\psi_u}} \tag{51}$$

$$e_{jt}N_{jt} = \gamma_{jn}\frac{P_{jt}Y_{jt}}{w_{jt}} \tag{52}$$

$$M_{ij,t} = \varphi_{ij}\frac{P_{jt}Y_{jt}}{P_{it}} \tag{53}$$

$$Y_{jt} = Z_j(u_{jt}K_{jt})^{\gamma_{jk}}L_{jt}^{\gamma_{jn}}\prod_{i=1}^{J}M_{i,j,t}^{\varphi_{ij}} \tag{54}$$

$$P_{jt}\left(1+\tau\left(K_{jt+1}-K_{jt}\right)\right) = \frac{1}{1+r_{t+1}}\mathbb{E}_t\left[P_{jt+1}\left(\frac{\gamma_{jk}Y_{jt+1}}{K_{jt+1}} + \tau\left(K_{jt+2}-K_{jt+1}\right) + (1-\delta_{jt+1})\right)\right] \tag{55}$$

$$\delta_{jt} = \bar{\delta}\frac{u_{jt}^{1+\psi_u}}{1+\psi_u} \tag{56}$$

$$C_{jt} = \alpha_j\frac{C_t}{P_{jt}} \tag{57}$$

$$K_{jt+1} = I_{jt} + (1-\delta_{jt})K_{jt} \tag{58}$$

$$Y_{jt} = C_{jt} + I_{jt} + \frac{\tau}{2}\left(K_{jt+1}-K_{jt}\right)^2 + \sum_{i=1}^{J}M_{ji,t} \quad j \in \mathcal{J}^{NT} \tag{59}$$

$$B_{t+1} + \sum_{j=1}^{J}P_{jt}\left(C_{jt} + I_{jt} + \frac{\tau}{2}\left(K_{jt+1}-K_{jt}\right)^2 + \sum_{i=1}^{J}M_{ji,t}\right) = (1+r_t)B_t + \sum_{j=1}^{J}P_{jt}Y_{jt} \tag{60}$$

$$1 + r_t = 1 + r_t^* + \phi(\exp\{\bar{B} - B_t\} - 1) \tag{61}$$

$$\mathbb{E}_t[\log P_{Cu,t+1}] = \rho_{Cu}\log P_{Cu,t} \tag{62}$$

There are 29 tradable and 14 non-tradable industries. Hence, we have $43 \times 9 + 43^2 + 14 + 4 = 2254$ equations. It will be helpful to reduce the dimensionality of the system if possible.

E.5 Solution of the Model

E.5.1 Steady State

$$\Omega = C - \sum_{j=1}^{J} \frac{N_j^{1+\psi_n}}{1+\psi_n} - \chi \sum_{j=1}^{J} N_j \frac{e_j^{1+\psi_e}}{1+\psi_e} \tag{63}$$

$$1 = \beta(1+r) \tag{64}$$

$$e_j = \left(\frac{w_j}{\chi}\right)^{\frac{1}{\psi_e}} \tag{65}$$

$$w_j e_j - N_j^{\psi_n} - \chi \frac{e_j^{1+\psi_e}}{1+\psi_e} = 0 \tag{66}$$

$$u_j = \left(\gamma_{jk} \frac{Y_j}{\bar{\delta} K_j}\right)^{\frac{1}{1+\psi_u}} \tag{67}$$

$$e_j N_j = \gamma_{jn} \frac{P_j Y_j}{w_j} \tag{68}$$

$$M_{ij} = \varphi_{ij} \frac{P_j Y_j}{P_i} \tag{69}$$

$$Y_j = Z_j (u_j K_j)^{\gamma_{jk}} L_j^{\gamma_{jn}} \prod_{i=1}^{J} M_{ij}^{\varphi_{ij}} \tag{70}$$

$$K_j = \frac{\gamma_{jk} Y_j}{r + \delta_j} \tag{71}$$

$$\delta_j = \bar{\delta} \frac{u_j^{1+\psi_u}}{1+\psi_u} \tag{72}$$

$$C_j = \alpha_j \frac{C}{P_j} \tag{73}$$

$$K_j = I_j + (1-\delta_j) K_j \tag{74}$$

$$Y_j = C_j + I_j + \sum_{i=1}^{J} M_{ji} \quad \text{if sector is non-tradable} \tag{75}$$

$$\sum_{j=1}^{J} P_j \left(C_j + I_j + \sum_{i=1}^{J} M_{ji} \right) = rB + \sum_{j=1}^{J} P_j Y_j \tag{76}$$

$$1 + r = 1 + r^* + \phi(\exp\{\bar{B} - B\} - 1) \tag{77}$$

Solution

From (77):

$$1 + r^* + \phi(\exp\{\bar{B} - B\} - 1) = \frac{1}{\beta}$$

Solving for $\bar{B}$:

$$B = \bar{B} - \log\left(1 + \frac{1}{\phi}\left(\frac{1}{\beta} - 1 - r^*\right)\right) \tag{78}$$

Combining (65) and (68):

$$e_j = \left[\frac{\gamma_{jn} P_j Y_j}{\chi N_j}\right]^{\frac{1}{1+\psi_e}}$$

Combining (65) and (66):

$$N_j = \left[\frac{\psi_e}{1+\psi_e} \chi e_j^{1+\psi_e}\right]^{\frac{1}{\psi_n}}$$

Substituting for e_j from the expression from above and solving for N_j:

$$N_j = \left[\frac{\psi_e}{1+\psi_e} \gamma_{jn} P_j Y_j\right]^{\frac{1}{1+\psi_n}} \tag{79}$$

Substituting for N_j in the expression for e_j from above:

$$e_j = \left[\frac{(\gamma_{jn} P_j Y_j)^{\psi_n}}{\chi^{1+\psi_n}} \frac{1+\psi_e}{\psi_e}\right]^{\frac{1}{(1+\psi_n)(1+\psi_e)}} \tag{80}$$

Therefore, L_j is given by:

$$L_j = \left[\left(\frac{\psi_e}{1+\psi_e}\right)^{\psi_e} \frac{(\gamma_{jn} P_j Y_j)^{1+\psi_e+\psi_n}}{\chi^{1+\psi_n}}\right]^{\frac{1}{(1+\psi_n)(1+\psi_e)}} \tag{81}$$

Combining (67) and (71):

$$u_j = \left(\frac{1+\psi_u}{\psi_u} \frac{r}{\bar{\delta}}\right)^{\frac{1}{1+\psi_u}} \tag{82}$$

Hence:

$$\delta_j = \frac{r}{\psi_u} \tag{83}$$

And:

$$K_j = \frac{\psi_u}{1+\psi_u}\frac{\gamma_{jk}Y_j}{r} \tag{84}$$

Combining (67), (68), (69), (70), and (71):

$$\begin{aligned} Y_j = \left(P_j^{\tilde{\gamma}_j} Z_j\right)^{\frac{1}{1-\tilde{\gamma}_j}} \times \left(\left(\frac{\psi_u}{1+\psi_u}\frac{1}{r}\right)^{\psi_u} \frac{\gamma_{jk}^{1+\psi_u}}{\bar{\delta}}\right)^{\frac{\gamma_{jk}}{(1+\psi_u)(1-\tilde{\gamma}_j)}} \\ \times \left(\left(\frac{\psi_e}{1+\psi_e}\right)^{\psi_e} \frac{\gamma_{jn}^{1+\psi_e+\psi_n}}{\chi^{1+\psi_n}}\right)^{\frac{\gamma_{jn}}{(1+\psi_e)(1+\psi_n)(1-\tilde{\gamma}_j)}} \times \prod_{i=1}^{J}\left(\frac{\varphi_{ij}}{P_i}\right)^{\frac{\varphi_{ij}}{1-\tilde{\gamma}_j}} \end{aligned} \tag{85}$$

where:

$$\tilde{\gamma}_j = \gamma_{jk} + \frac{\gamma_{jn}(1+\psi_e+\psi_n)}{(1+\psi_e)(1+\psi_n)} + \sum_{i=1}^{J}\varphi_{ij}$$

For tradable firms we have P_j given, so we can solve directly for Y_{jt}. For non-tradable firms we need to clear the market. This means:

$$Y_j = C_j + I_j + \sum_{i=1}^{J} M_{ji}$$

From (73) and (76):

$$C = rB + \sum_{j=1}^{J} P_j\left(Y_j - I_j - \sum_{i=1}^{J} M_{ji}\right)$$

From (74):

$$I_j = \delta_j K_j = \frac{\gamma_{jk}Y_j}{1+\psi_u}$$

Thus:

$$C = rB + \sum_{j=1}^{J} P_j\left(\left(1-\frac{\gamma_{jk}}{1+\psi_u}\right)Y_j - \sum_{i=1}^{J}\varphi_{ji}\frac{P_iY_i}{P_j}\right)$$

Simplifying a bit further:

$$C = rB + \sum_{j=1}^{J}\left(1-\frac{\gamma_{jk}}{1+\psi_u}\right)P_jY_j - \sum_{j=1}^{J}\sum_{i=1}^{J}\varphi_{ji}P_iY_i \tag{86}$$

So for non-tradable sectors:

$$Y_j = \alpha_j\frac{C}{P_j} + \frac{\gamma_{jk}Y_j}{1+\psi_u} + \sum_{i=1}^{J}\varphi_{ji}\frac{P_iY_i}{P_j}$$

Simplified:

$$\left(1-\frac{\gamma_{jk}}{1+\psi_u}\right)Y_j=\alpha_j\frac{C}{P_j}+\sum_{i=1}^{J}\varphi_{ji}\frac{P_iY_i}{P_j}$$

Then we have a system of $J^{NT}+1$ equations to solve for the J^{NT} prices of non-tradable goods and the steady-state level of the consumption composite, C. Once we have the prices of all sectoral goods and the consumption composite level, we can solve for all variables in this economy.

E.5.2 Dynamics

I will log-linearize equations (47) to (62). Let $\hat{X}_t$ denote the log deviation of variable X from its steady-state value.

$$\hat{\Omega}_t=\omega_c\hat{C}_t-\sum_{j=1}^{J}\omega_{nj}\hat{N}_{jt}-\sum_{j=1}^{J}\omega_{ej}\left(\hat{N}_{jt}+(1+\psi_e)\hat{e}_{jt}\right) \tag{87}$$

where $\omega_c=C/\Omega$, $\omega_{nj}=N_j^{1+\psi_n}/(\Omega(1+\psi_n))$, and $\omega_{ej}=\chi N_j e_j^{1+\psi_e}/(\Omega(1+\psi_e))$.

$$-\sigma\hat{\Omega}_t=\hat{R}_{t+1}-\sigma\mathbb{E}_t\left[\hat{\Omega}_{t+1}\right] \tag{88}$$

where $R_t=1+r_t$.

$$\hat{e}_{jt}=\frac{1}{\psi_e}\hat{w}_{jt} \tag{89}$$

Equation (50) can be log-linearized in the following way. First, I will rewrite it as:

$$\mathbb{E}_t\left[w_{jt+1}e_{jt+1}\Omega_{t+1}^{-\sigma}\right]=\mathbb{E}_t\left[\left(N_{jt+1}^{\psi_n}+\frac{\chi}{1+\psi_e}e_{jt+1}^{1+\psi_u}\right)\Omega_{t+1}^{-\sigma}\right]$$

After applying log-linearization rules to the above equation I get:

$$\mathbb{E}_t\left[\hat{w}_{jt+1}\right]=\frac{\psi_e\psi_n}{1+\psi_e}\hat{N}_{jt+1} \tag{90}$$

$$\hat{u}_{jt}=\frac{1}{1+\psi_u}\left(\hat{Y}_{jt}-\hat{K}_{jt}\right) \tag{91}$$

$$\hat{e}_{jt}+\hat{N}_{jt}=\hat{P}_{jt}+\hat{Y}_{jt}-\hat{w}_{jt} \tag{92}$$

$$\hat{M}_{ij,t}=\hat{P}_{jt}+\hat{Y}_{jt}-\hat{P}_{it} \tag{93}$$

$$\hat{Y}_{jt} = \gamma_{jk}\left(\hat{u}_{jt} + \hat{K}_{jt}\right) + \gamma_{jn}\left(\hat{e}_{jt} + \hat{N}_{jt}\right) + \sum_{i=1}^{J} \varphi_{ij}\hat{M}_{ij} \tag{94}$$

$$\begin{aligned}\hat{P}_{jt} + \tau K_j\left(\hat{K}_{jt+1} - \hat{K}_{jt}\right) =& \mathbb{E}_t\left[\hat{P}_{jt+1}\right] - \hat{R}_{t+1} + \frac{r(1+\psi_u)}{(1+r)\psi_u}\left(\mathbb{E}_t\left[\hat{Y}_{jt+1}\right] - \hat{K}_{jt+1}\right) \\ &+ \frac{\tau K_j}{1+r}\left(\mathbb{E}_t[\hat{K}_{jt+2}] - \hat{K}_{jt+1}\right) - \frac{r}{(1+r)\psi_u}\mathbb{E}_t[\hat{\delta}_{jt+1}]\end{aligned} \tag{95}$$

$$\hat{\delta}_{jt} = (1+\psi_u)\hat{u}_{jt} \tag{96}$$

$$\hat{C}_{jt} = \hat{C}_t - \hat{P}_{jt} \tag{97}$$

$$\hat{I}_{jt} = \frac{1}{\delta_j}\hat{K}_{jt+1} - \frac{1-\delta_j}{\delta_j}\hat{K}_{jt} + \hat{\delta}_{jt} \tag{98}$$

$$\hat{Y}_{jt} = s_{cj}\hat{C}_{jt} + s_{kj}\hat{I}_{jt} + \sum_{i=1}^{J} s_{mji}\hat{M}_{ji,t} \tag{99}$$

where $s_{cj} = C_j/Y_j$, $s_{kj} = I_j/Y_j$, and $s_{mji} = M_{ji}/Y_j$. This equation holds for every $j \in \mathcal{J}^{NT}$.

$$\begin{aligned}\varsigma_b\hat{B}_{t+1} =& R\varsigma_b\left(\hat{R}_t + \hat{B}_t\right) - \hat{C}_t \\ &+ \sum_{j=1}^{J}\left(\varsigma_{yj}(\hat{P}_{jt} + \hat{Y}_{jt}) - \varsigma_{kj}(\hat{P}_{jt} + \hat{I}_{jt}) - \sum_{i=1}^{J}\varsigma_{mji}(\hat{P}_{jt} + \hat{M}_{ji,t})\right)\end{aligned} \tag{100}$$

where $\varsigma_b = B/C$, $\varsigma_{yj} = P_jY_j/C$, $\varsigma_{kj} = P_jI_j/C$, and $\varsigma_{mji} = P_jM_{ji}/C$.

$$\hat{R}_t = \frac{R^*}{R}\hat{R}_t^* - \frac{\phi\exp\{\bar{B} - B\}B}{R}\hat{B}_t$$

I will assume that $\beta R^* = 1$ at the steady state, which implies $B = \bar{B}$, $R = R^*$, and that $R_t^* = R^*\ \forall t$. Thus:

$$\hat{R}_t = -\frac{\phi B}{R}\hat{B}_t \tag{101}$$

Finally, the copper price process is:

$$\mathbb{E}_t[\hat{P}_{Cu,t+1}] = \rho_{Cu}\hat{P}_{Cu,t} \tag{102}$$

System reduction

I will reduce the system to a system of equations in terms of the exogenous state variable, $\hat{P}_{Cu,t}$, the endogenous state variables $\{\hat{B}_t\}$, $\{\hat{K}_{jt}\}$, $\{\hat{N}_{jt}\}$, and the control variables, $\{\hat{C}_t\}$ and $\{\hat{P}_{jt}\}_{j\in\mathcal{J}^{NT}}$.

Substituting (87) into (88), and (101) into (88):

$$\begin{aligned}\gamma\left(\omega_c\hat{C}_t-\sum_{j=1}^{J}(\omega_{nj}+\omega_{ej})\hat{N}_{jt}-\sum_{j=1}^{J}\omega_{ej}(1+\psi_e)\hat{e}_{jt}\right)&=\frac{\phi\bar{B}}{R^*}\hat{B}_{t+1}\\+\gamma\left(\omega_c\mathbb{E}_t[\hat{C}_{t+1}]-\sum_{j=1}^{J}(\omega_{nj}+\omega_{ej})\mathbb{E}_t[\hat{N}_{jt+1}]-\sum_{j=1}^{J}\omega_{ej}(1+\psi_e)\mathbb{E}_t[\hat{e}_{jt+1}]\right)&\end{aligned} \tag{103}$$

Combining (89) and (92):

$$\hat{w}_{jt}=\frac{\psi_e}{1+\psi_e}\left(\hat{P}_{jt}+\hat{Y}_{jt}-\hat{N}_{jt}\right) \tag{104}$$

Hence:

$$\hat{e}_{jt}=\frac{1}{1+\psi_e}\left(\hat{P}_{jt}+\hat{Y}_{jt}-\hat{N}_{jt}\right) \tag{105}$$

From (91):

$$\hat{u}_{jt}+\hat{K}_{jt}=\frac{1}{1+\psi_u}\hat{Y}_{jt}+\frac{\psi_u}{1+\psi_u}\hat{K}_{jt} \tag{106}$$

From (105):

$$\hat{e}_{jt}+\hat{N}_{jt}=\frac{1}{1+\psi_e}\left(\hat{P}_{jt}+\hat{Y}_{jt}\right)+\frac{\psi_e}{1+\psi_e}\hat{N}_{jt} \tag{107}$$

Substituting (106), (107), and (93) into (94), and solving for $\hat{Y}_{jt}$ we get:

$$\hat{Y}_{jt}=\Gamma_{pj}\hat{P}_{jt}+\Gamma_{kj}\hat{K}_{jt}+\Gamma_{nj}\hat{N}_{jt}-\sum_{i=1}^{J}\Gamma_{ij}\hat{P}_{it} \tag{108}$$

where:

$$\begin{aligned}\Gamma_{pj}=&\frac{\gamma_{jn}(1+\psi_u)+(1+\psi_e)(1+\psi_u)\sum_{i=1}^{J}\varphi_{ij}}{(1+\psi_e)(1+\psi_u)(1-\sum_{i=1}^{J}\varphi_{ij})-\gamma_{jk}(1+\psi_e)-\gamma_{jn}(1+\psi_u)}\\\Gamma_{kj}=&\frac{\gamma_{jk}(1+\psi_e)\psi_u}{(1+\psi_e)(1+\psi_u)(1-\sum_{i=1}^{J}\varphi_{ij})-\gamma_{jk}(1+\psi_e)-\gamma_{jn}(1+\psi_u)}\\\Gamma_{nj}=&\frac{\gamma_{jn}(1+\psi_u)\psi_e}{(1+\psi_e)(1+\psi_u)(1-\sum_{i=1}^{J}\varphi_{ij})-\gamma_{jk}(1+\psi_e)-\gamma_{jn}(1+\psi_u)}\\\Gamma_{ij}=&\frac{(1+\psi_e)(1+\psi_u)\varphi_{ij}}{(1+\psi_e)(1+\psi_u)(1-\sum_{i=1}^{J}\varphi_{ij})-\gamma_{jk}(1+\psi_e)-\gamma_{jn}(1+\psi_u)}\end{aligned}$$

We can then re-write (103) after applying (108) to (104) and (105):

$$\begin{aligned}
&\gamma\omega_c\hat{C}_t-\gamma\sum_{j=1}^{J}\omega_{ej}(\Gamma_{pj}+1)\hat{P}_{jt}-\gamma\sum_{j=1}^{J}\omega_{ej}\Gamma_{kj}\hat{K}_{jt}-\gamma\sum_{j=1}^{J}(\omega_{nj}+\omega_{ej}\Gamma_{nj})\hat{N}_{jt}\\
&+\gamma\sum_{j=1}^{J}\omega_{ej}\sum_{i=1}^{J}\Gamma_{ij}\hat{P}_{it}\\
&=\frac{\phi\bar{B}}{R^*}\hat{B}_{t+1}+\gamma\omega_c\mathbb{E}_t[\hat{C}_{t+1}]-\gamma\sum_{j=1}^{J}\omega_{ej}(\Gamma_{pj}+1)\mathbb{E}_t[\hat{P}_{jt+1}]-\gamma\sum_{j=1}^{J}\omega_{ej}\Gamma_{kj}\hat{K}_{jt+1}\\
&-\gamma\sum_{j=1}^{J}(\omega_{nj}+\omega_{ej}\Gamma_{nj})\hat{N}_{jt+1}+\gamma\sum_{j=1}^{J}\sum_{i=1}^{J}\omega_{ej}\Gamma_{ij}\mathbb{E}_t[\hat{P}_{i,t+1}]
\end{aligned}\tag{109}$$

Re-writing (90):

$$(\Gamma_{pj}+1)\mathbb{E}_t\left[\hat{P}_{jt+1}\right]+\Gamma_{kj}\hat{K}_{jt+1}+\left(\Gamma_{nj}-1-\psi_n\right)\hat{N}_{jt+1}-\sum_{i=1}^{J}\Gamma_{ij}\mathbb{E}_t[\hat{P}_{it+1}]=0\tag{110}$$

Re-writing (97):

$$\begin{aligned}
0=&\mathbb{E}_t\left[\hat{P}_{jt+1}\right]-\hat{P}_{jt}-\hat{R}_{t+1}-\tau K_j\left(\hat{K}_{jt+1}-\hat{K}_{jt}\right)\\
&+\frac{r}{(1+r)}\left(\Gamma_{pj}\mathbb{E}_t\left[\hat{P}_{jt+1}\right]+(\Gamma_{kj}-1)\hat{K}_{jt+1}+\Gamma_{nj}\hat{N}_{jt+1}-\sum_{i=1}^{J}\Gamma_{ij}\mathbb{E}_t[\hat{P}_{i,t+1}]\right)\\
&+\frac{\tau K_j}{1+r}\left(\mathbb{E}_t[\hat{K}_{jt+2}]-\hat{K}_{jt+1}\right)
\end{aligned}\tag{111}$$

Re-writing (99):

$$\begin{aligned}
&(\Gamma_{pj}+s_{cj}+\sum_{i=1}^{J}s_{mji})\hat{P}_{jt}\\
&=s_{ci}\hat{C}_t+\frac{s_{kj}}{\delta_j}\hat{K}_{j,t+1}-\left(\Gamma_{kj}+s_{kj}\frac{(1-\delta_j)}{\delta_j}\right)\hat{K}_{j,t}\\
&+\sum_{i=1}^{J}s_{mji}\left((\Gamma_{pi}+1)\hat{P}_{i,t}+\Gamma_{ki}\hat{K}_{i,t}+\Gamma_{ni}\hat{N}_{i,t}-\sum_{h=1}^{J}\Gamma_{hi}\hat{P}_{h,t}\right)
\end{aligned}\tag{112}$$

Re-writing (100):

$$
\begin{aligned}
&\zeta_B \hat{B}_{t+1} = \zeta_B(R - \phi \bar{B}) - \hat{C}_t \\
&+ \sum_{j=1}^{J} (\zeta_{yj}(\Gamma_{pj} + 1) - \zeta_{kj}) \hat{P}_{jt} + \sum_{j=1}^{J} \left(\zeta_{yj}\Gamma_{kj} + \zeta_{kj} \frac{(1-\delta_j)}{\delta_j} \right) \hat{K}_{jt} + \sum_{j=1}^{J} \zeta_{yj}\Gamma_{nj}\hat{N}_{jt} \\
&- \sum_{j=1}^{J} \frac{\zeta_{kj}}{\delta_j} \hat{K}_{j,t+1} - \sum_{j=1}^{J}\sum_{i=1}^{J} (\zeta_{yj}\Gamma_{ij} + \zeta_{mji}(\Gamma_{pi}+1))\hat{P}_{i,t} - \sum_{j=1}^{J}\sum_{i=1}^{J} \zeta_{mji}\Gamma_{ki}\hat{K}_{i,t} - \sum_{j=1}^{J}\sum_{i=1}^{J} \zeta_{mji}\Gamma_{ni}\hat{N}_{i,t} \\
&+ \sum_{j=1}^{J}\sum_{i=1}^{J}\sum_{h=1}^{J} \zeta_{mji}\Gamma_{hi}\hat{P}_{h,t}
\end{aligned}
\tag{113}
$$

Therefore, equations (109)-(113) comprise the system of $2+2J+J^{NT}$ equations that with the J^T exogenous price processes solve the dynamics of the system.

At this stage, the dynamics of the system can be solved using standard linear rational expectations tool kits as described in Blanchard and Khan (1980) and Klein (2000). My calculations are based on the algorithms described in Klein (2000)