

FRAÇÕES e EQUAÇÕES

01. OS AXIOMAS

“
Premissa considerada necessariamente evidente e verdadeira, fundamento de uma demonstração, porém ela mesma indemonstrável, originada, segundo a tradição racionalista, de princípios inatos da consciência”



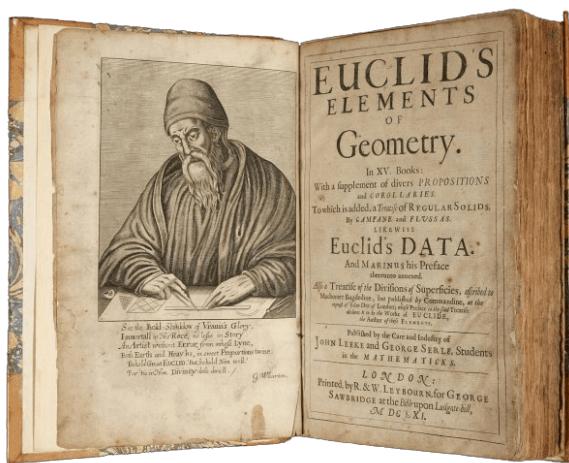
AXIOMA
ou **POSTULADO**

- . Os axiomas são as regras do jogo matemático.
- . Tudo deve respeitar os axiomas
- . Cada área da matemática terá seus próprios axiomas.



As noções comuns de Euclides

→ Matemático grego
(Alexandria)
→ 300 a.C.



"OS ELEMENTOS"

um dos
tratados
gregos mais
antigos

tratado
sobre geometria
plana

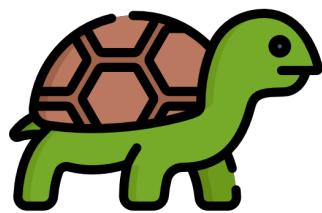
ao enunciar os
axiomas ele
separa alguns, as
"noções comuns"



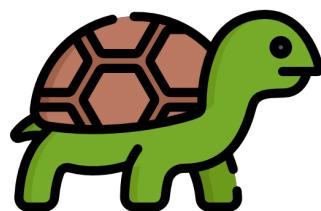
#01

Coisas iguais às mesmas coisas são iguais entre si.

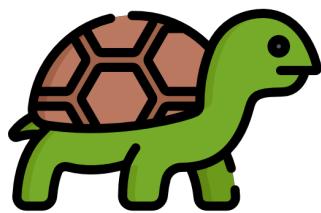
ARMANDO



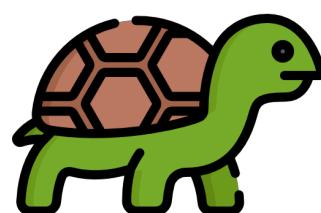
CADU



BILU



CADU



Conclusão:

se $A = C$ e $B = C$

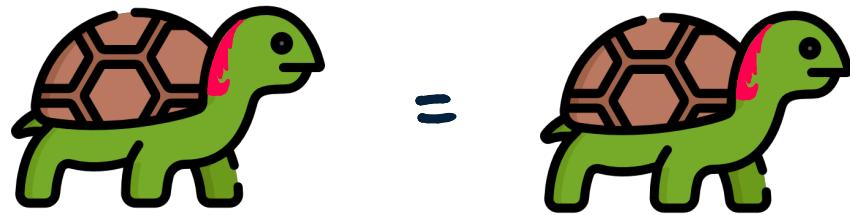
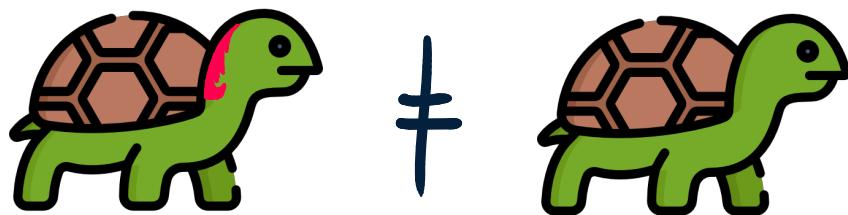
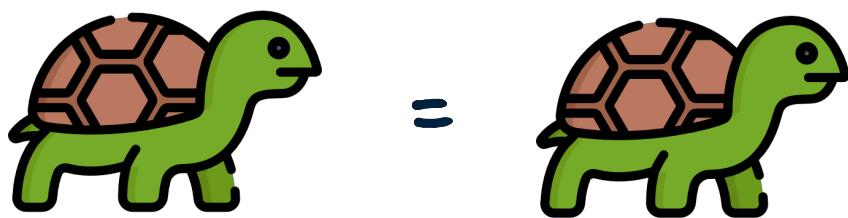
então $A = C$



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#02

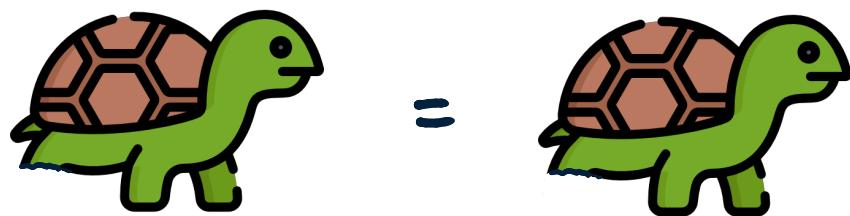
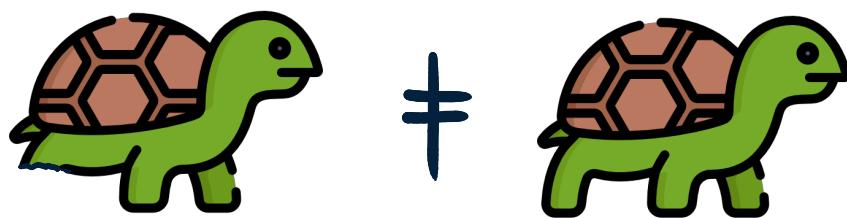
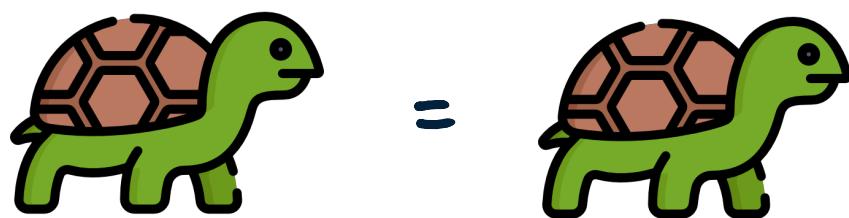
Coisas iguais adicionadas a coisas iguais
formam coisas iguais.



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#03

Coisas iguais subtraídas de coisas iguais
formam coisas iguais.



EXEMPLO 01

Encontre o valor de x em :

$$5x - 8 = 12$$

R :

$$\cancel{5x - 8 + 8} = 12 + 8$$

$$5x = 20$$

$$\frac{5x}{5} = \frac{20}{5}$$

$$x = 4$$



EXEMPLO 02

Encontre o valor de x em :

$$15x - 3 = 5x + 32$$

R :

$$15x - 3 + 3 = 5x + 32 + 3$$

$$15x = 5x + 35$$

$$15x - 5x = \cancel{5x} + 35 - \cancel{5x}$$

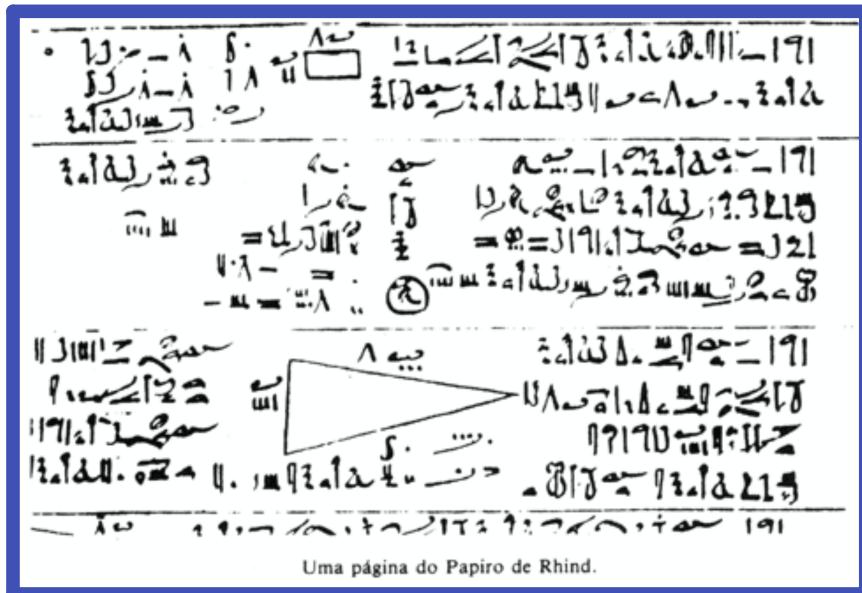
$$10x = 35$$

$$\frac{10x}{10} = \frac{35}{10}$$

$$x = 3,5$$



A matemática do egito antigo



Papiro de Ahmes (Rhind)

- 1650 a.C.
- Egito



EXEMPLOS DO PAPIRO

Qual é o número que somado à sua terça parte dá 8?

R : REGRA DA FALSA POSIÇÃO :

PASSO 01 : fazer um chute ($x = 3$)

$$3 + \frac{1}{3} \cdot 3 = 8$$

4 ≠ 8 X

PASSO 02 : ver se funcionou

PASSO 03 : fazer outro chute

$$6 + \frac{1}{3} \cdot 6 = 8$$

X = 6 ✓

Atualmente

$$x + \frac{x}{3} = 8$$

$$3 \cdot x + \cancel{3 \cdot \frac{x}{3}} = 3 \cdot 8$$

$$3 \cdot x + x = 24$$

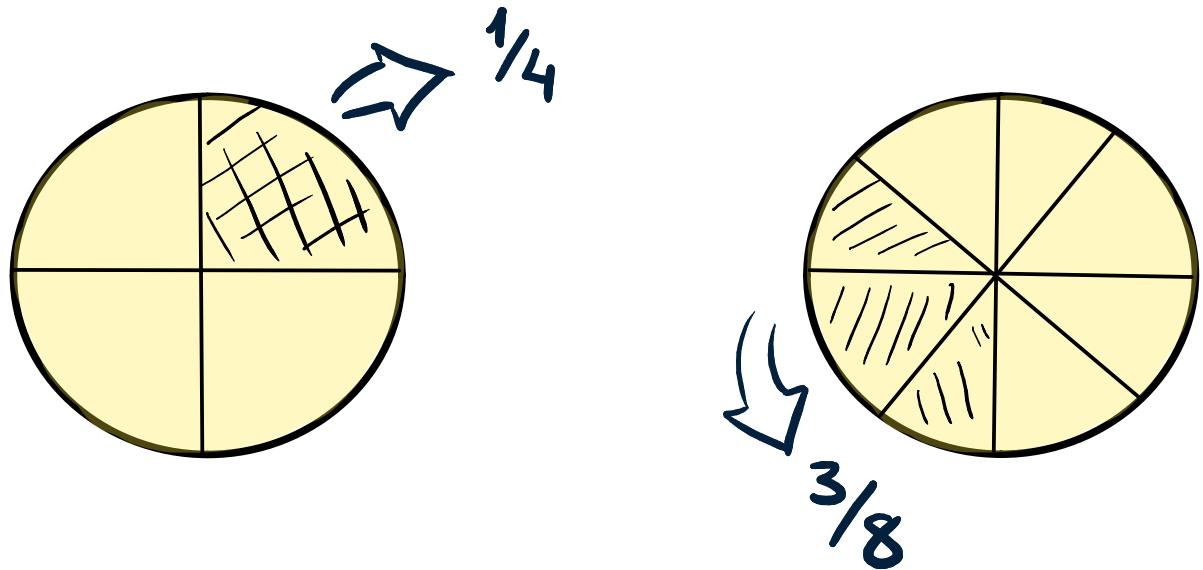
$$4x = 24$$

$$\cancel{\frac{4x}{4}} = \frac{24}{4}$$

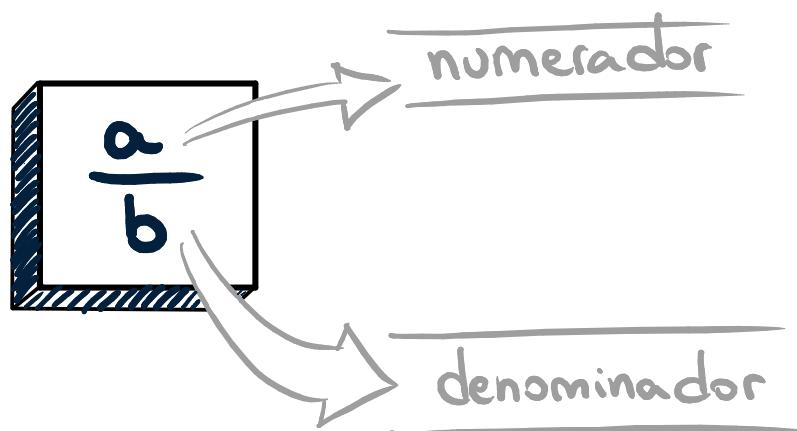
$$x = 6$$



02. FRACÔES

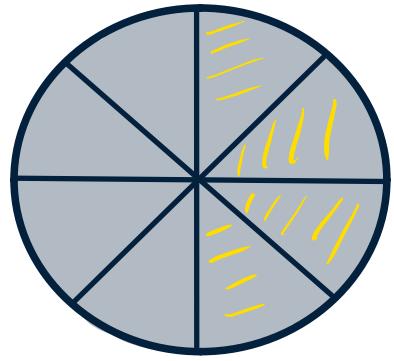
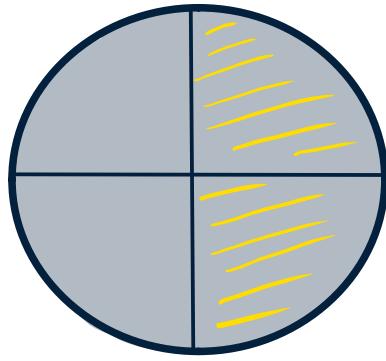
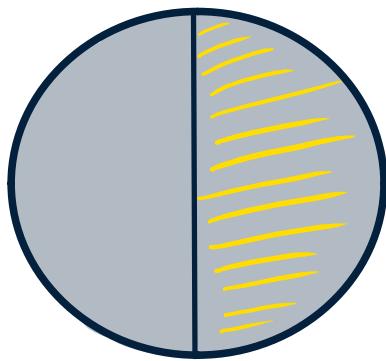


• Fracão é todo número da forma:



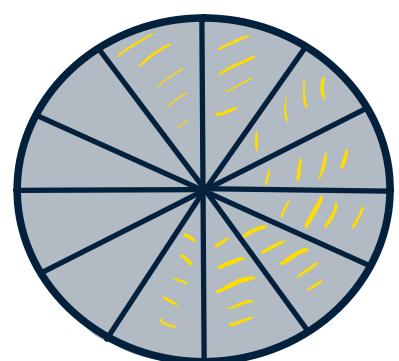
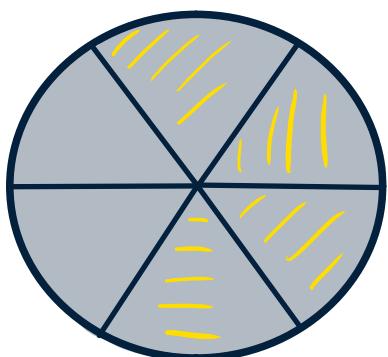
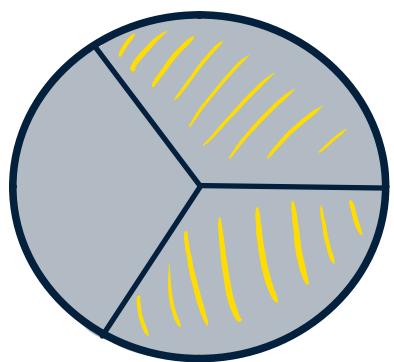
Transformações com frações

Uma fração não se altera quando os seus dois membros são multiplicados ou divididos pelo mesmo número



$$\frac{1}{2} \xrightarrow[\substack{\times 2 \\ \times 2}]{} = \frac{2}{4} \xrightarrow[\substack{\times 2 \\ \times 2}]{} = \frac{4}{8}$$





$$\frac{2}{3}$$

=

$$\frac{4}{6}$$

=

$$\frac{8}{12}$$



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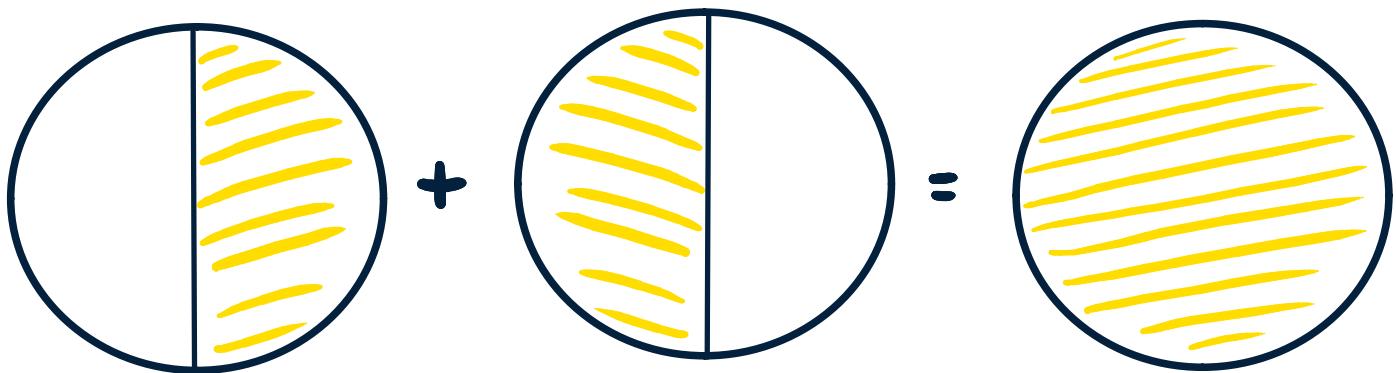
EXEMPLOS

$$(i) \quad \frac{3}{2} = \frac{15}{10} = \frac{9}{6} = \frac{30}{20}$$

$$(ii) \quad \frac{7}{3} = \frac{14}{6} = \frac{21}{9} = \frac{35}{15}$$



Adição e Subtração de frações



$$\frac{1}{2} + \frac{1}{2} = 1$$

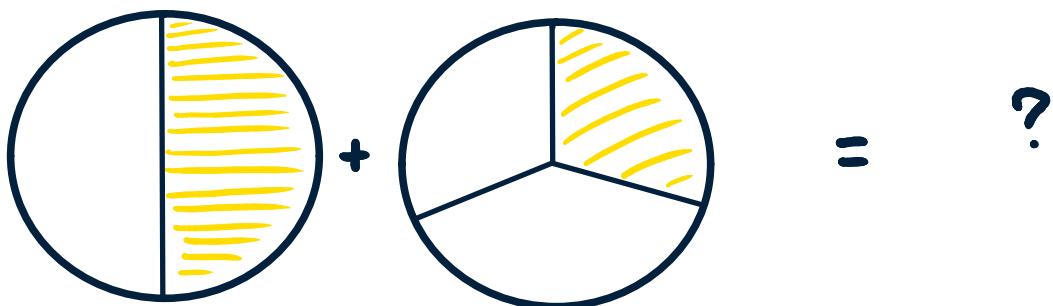
$$\rightsquigarrow \frac{1}{2} + \frac{1}{2} = \frac{1+1}{2+2} = \frac{2}{4} = \frac{1}{2} \text{ (X)}$$



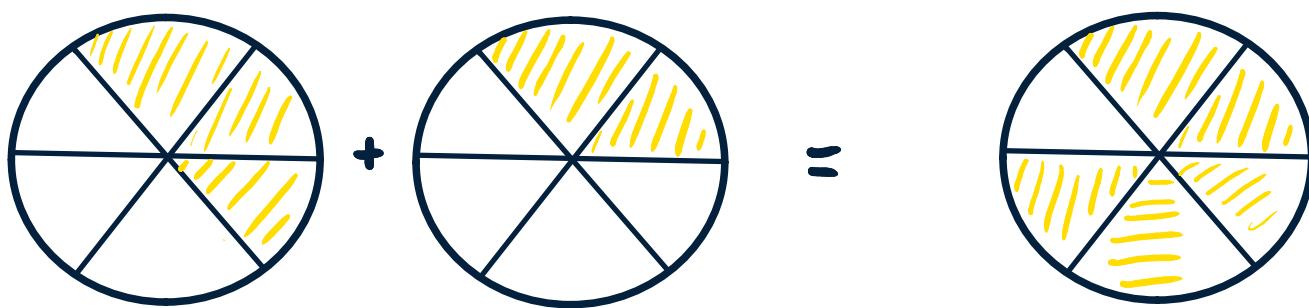
ATENÇÃO : $\frac{a}{b} + \frac{c}{d} \neq \frac{a+c}{b+d}$



→ Para somar/subtrair frações é preciso que elas estejam na mesma base!
denominador



$$\frac{1}{2} + \frac{1}{3} = ?$$



$$\frac{1 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2} =$$

$$\frac{3}{6} + \frac{2}{6} = \frac{3 + 2}{6} = \frac{5}{6}$$



De modo abstrato :

$$\frac{1}{2} + \frac{1}{3} =$$

$$= \frac{1 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2} =$$

$$= \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$$



EXEMPLO 01

$$\frac{1}{5} - \frac{1}{7} =$$

$$= \frac{1 \times 7}{5 \times 7} - \frac{1 \times 5}{7 \times 5} =$$

$$= \frac{7}{35} - \frac{5}{35} =$$

$$= \frac{7 - 5}{35} = \underbrace{\frac{2}{35}}$$



EXEMPLO 02

$$\frac{1}{3} - 1 + 0,5 =$$

$$= \frac{1}{3} - 1 + \frac{1}{2} =$$

$$= \frac{1 \times 2}{3 \times 2} - 1 + \frac{1 \times 3}{2 \times 3} =$$

$$= \frac{2}{6} - \frac{6}{6} + \frac{3}{6}$$

$$= \frac{2 - 6 + 3}{6} = \frac{-1}{6}$$



EXEMPLO 03

$$\frac{4}{7} - 5x + \frac{x}{2} =$$

$$= \frac{4 \times 2}{7 \times 2} - 5x + \frac{x \times 7}{2 \times 7} =$$

$$= \frac{8}{14} - \frac{5x \cdot 14}{1 \cdot 14} + \frac{7x}{14}$$

$$= \frac{8}{14} - \frac{70x}{14} + \frac{7x}{14}$$

$$= \frac{8 - 70x + 7x}{14} =$$

$$= \frac{8 - 63x}{14}$$

||



ALERTA

$$x + \frac{x}{2} = 3$$

CAMINHO 01

$$x + \frac{x}{2} = 3$$

$$\cancel{2 \cdot x} + \frac{x}{2} = 3$$

$$\frac{2x + x}{2} = 3$$

$$\cancel{2} \cdot \frac{3x}{2} = 3 \cdot \cancel{2}$$

$$\cancel{\frac{3x}{3}} = \frac{6}{3}$$

$$\boxed{x = 2}$$

CAMINHO 02

$$x + \frac{x}{2} = 3$$

$$\cancel{2 \cdot x} + \cancel{2 \cdot \frac{x}{2}} = 3 \cdot \cancel{2}$$

$$2x + x = 6$$

$$\cancel{\frac{3x}{3}} = \frac{6}{3}$$

$$\boxed{x = 2}$$

CAMINHO 03

$$x + \frac{x}{2} = 3$$

$$\cancel{4 \cdot x} + \cancel{4 \cdot \frac{x}{2}} = 3 \cdot \cancel{4}$$

$$4x + 2x = 12$$

$$\cancel{\frac{6x}{6}} = \frac{12}{6}$$

$$\boxed{x = 2}$$



Multiplicação e Divisão de frações

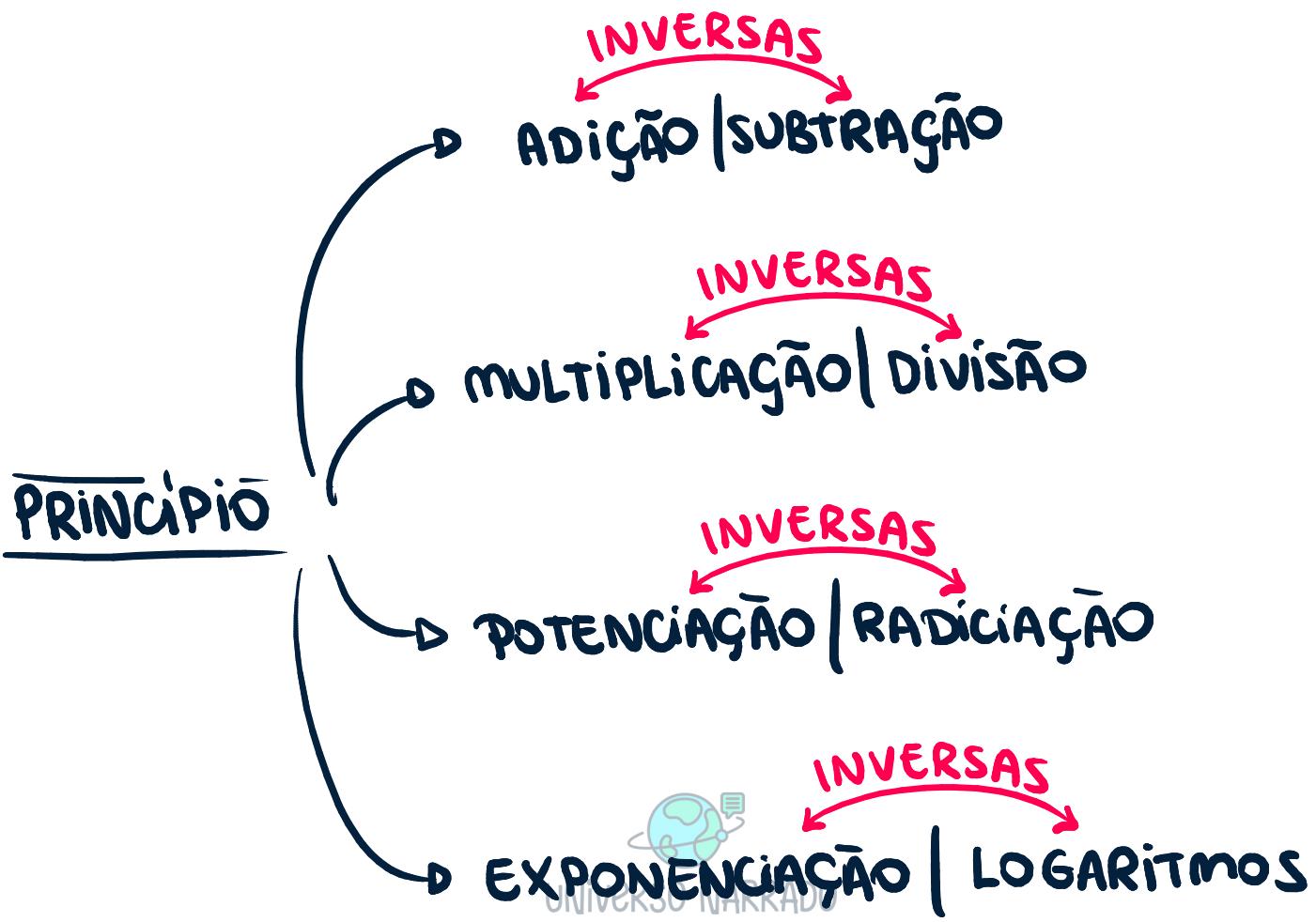
Exemplo:

CAMINHO 01 :

$$\frac{8}{2} \cdot \frac{15}{3} = 4 \cdot 5 = 20$$

CAMINHO 02 :

$$\frac{8}{2} \cdot \frac{15}{3} = \frac{120}{6} = 20$$



MULTIPLICAÇÃO

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

$$(i) \quad \frac{4}{8} \cdot \frac{5}{15} \quad \begin{matrix} \xrightarrow{\hspace{1cm}} \\ \xrightarrow{\hspace{1cm}} \end{matrix} \quad \frac{4 \cdot 5}{8 \cdot 15} = \frac{20}{120} = \frac{1}{6}$$
$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$(ii) \quad \frac{5}{25} \cdot \frac{3}{6} = \frac{1}{5} \cdot \frac{1}{2} = \underline{\underline{\frac{1}{10}}}$$



DIVISÃO

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

MOTIVO

$$\frac{\frac{3}{8}}{\frac{5}{7}} = ?$$

$$\frac{\frac{3}{8}}{\frac{5}{7}} = \frac{\frac{3}{8} \cdot \frac{7}{7}}{\frac{5}{7} \cdot \frac{7}{7}} = \frac{\frac{3 \cdot 7}{8} \cdot \frac{1}{5}}{5 \cdot \frac{1}{5}} =$$

$$= \frac{\frac{3}{8} \cdot \frac{7}{5}}{1} = \frac{3}{8} \cdot \frac{7}{5}$$



$$(i) \frac{\frac{25}{3}}{\frac{5}{9}} =$$

$$= \frac{25}{3} \cdot \frac{9}{5} = \frac{\cancel{25}^5 \cdot \cancel{9}^3}{\cancel{3} \cdot \cancel{5}} = \underline{\underline{15}}$$

$$(ii) \frac{\frac{21}{3}}{\frac{7}{4}} =$$

$$= \frac{\cancel{21}^3}{\cancel{3}} \cdot \frac{4}{\cancel{7}} = \frac{12}{3} = \underline{\underline{4}}$$

$$(iii) \frac{3}{5} \div \frac{1}{5} = \frac{3}{\cancel{5}} \cdot \frac{\cancel{5}}{1} = \underline{\underline{3}}$$



MULTIPLICAR CRUZADO

$$\frac{x}{3} = \frac{10}{6}$$

$$6 \cdot \frac{x}{3} = \frac{10}{6} \cdot 6 \quad \therefore \cancel{3 \cdot 6 \cdot \frac{x}{3}} = 10 \cdot 3$$

$$6x = 10 \cdot 3$$

$$\frac{x}{3} \times \frac{10}{6} \rightarrow 6x = 10 \cdot 3$$

$$\frac{6x}{6} = \frac{30}{6}$$

$$x = 5$$



RESUMO

PRINCIPIO 01: MANIPULAÇÃO DE EQUAÇÕES

→ Se iguais são feitos a iguais eles permanecem iguais

$$\square + 1 = 1 + \square$$

PRINCIPIO 02: MANIPULAÇÃO DE FRAÇÕES

→ Todo número multiplicado por 1 é igual a ele mesmo : $a \cdot 1 = a$

$$\frac{2}{5} \cdot \frac{2}{2} = \frac{4}{10} \cdot \frac{3}{3} = \frac{12}{30} (\dots)$$

