rk4_solve_ivp_benchmark

September 2, 2020

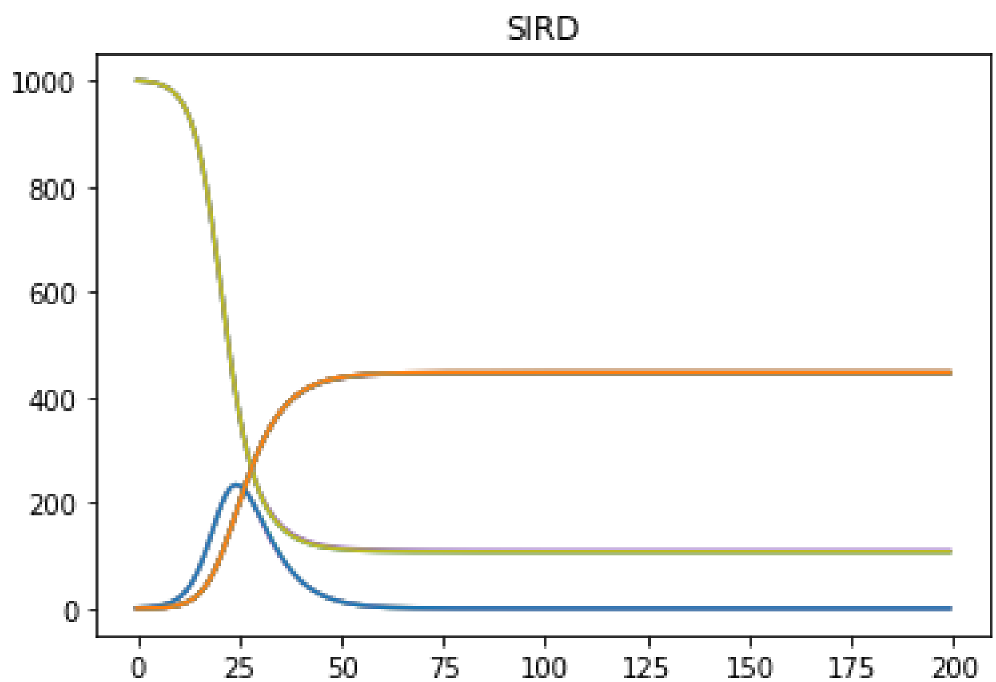
[39]:
```python
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Created on Tue Sep  1 18:43:52 2020

@author: joaop
"""

# Import libraries
import numpy as np
import matplotlib.pyplot as plt
import time
import numba
from numba import njit
import warnings
from epidemic_model_classes import *
from scipy.integrate import solve_ivp

# Deactivate numba deprecation warnings
warnings.simplefilter('ignore', category=numba.NumbaDeprecationWarning)
warnings.simplefilter('ignore', category=numba.NumbaPendingDeprecationWarning)
```

[40]:
```python
t = np.linspace(1, 200, 200) # Days
n = 1000 # Repetitions

functions = [SIRD, SEIRD, SEIHRD, SEIARD] # Models

t_rk = np.zeros(len(functions)) # Time array for rk4
t_sivp_rk = np.zeros(len(functions)) # Time array for solve_ivp(RK45)
t_sivp_ls = np.zeros(len(functions)) # Time array for solve_ivp(LSODA)
```

[41]:
```python
c = 0 # Counting functions

for func in functions:

    # Generate initial conditions
    y0 = np.zeros(func.ncomp)
```

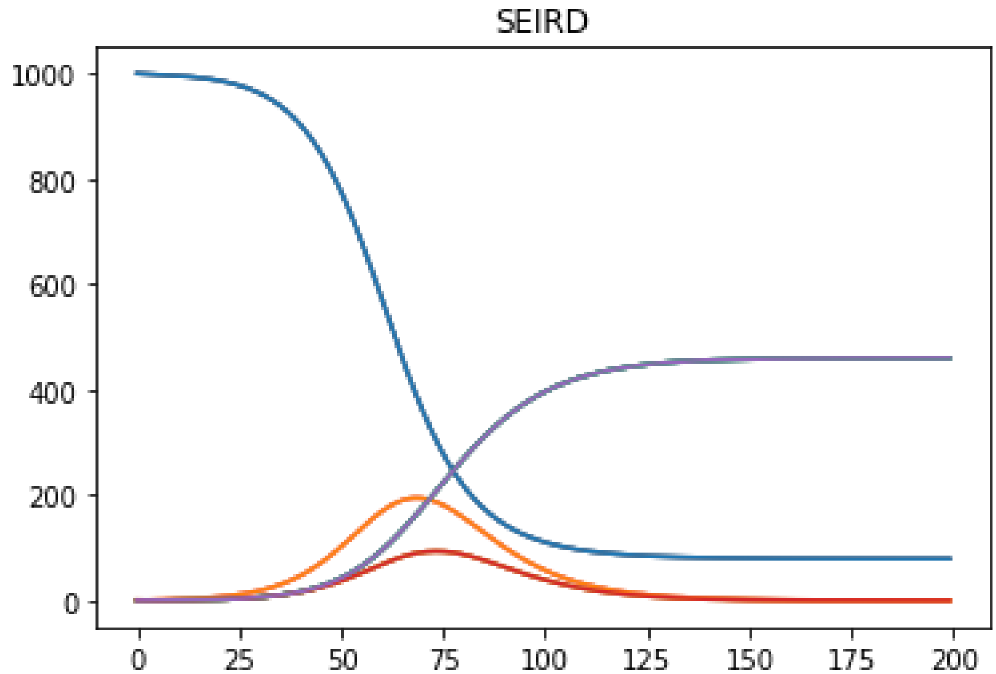
```
    y0[0] = 1000
    y0[-2] = 1

    # Generate parameters
    p = np.ones(func.nparams)/10
    p[0] = 0.5
    for i in range(len(func.params)):
        if func.params[i] == r"$N$":
            p[i] = 1000
        if func.params[i] == r"\tau_{h}":
            p[i] = 10
        if func.params[i] == r"\tau_{t}":
            p[i] = 20

    # Calculate and plot solutions for diffrent methods
    rk = rk4(func.model, y0, t, p, h=1.)
    sivp_rk = solve_ivp(func.model, [t[0], t[-1]], y0, t_eval=t, method="RK45",␣
↪args=[p])
    sivp_ls = solve_ivp(func.model, [t[0], t[-1]], y0, t_eval=t,␣
↪method="LSODA", args=[p])

    plt.plot(rk)
    plt.plot(sivp_rk.y.T)
    plt.plot(sivp_ls.y.T)
    plt.title(func.name)
    plt.show()

    # Print distance between results of different methods
   ␣
↪print("\n######################################################################\n")
    print(func.name, "\n")
    print("rk4 - solve_ivp(RK45) distance: %.4f" % np.sum(np.abs(rk-sivp_rk.y.
↪T)))
    print("rk4 - solve_ivp(LSODA) distance: %.4f" % np.sum(np.abs(rk-sivp_ls.y.
↪T)))
    print("solve_ivp(LSODA) - solve_ivp(RK45) distance: %.4f\n" % np.sum(np.
↪abs(sivp_ls.y.T-sivp_rk.y.T)))

    # Calculate execution time
    t_rk[c] = time.time()
    for i in range(n):
        rk = rk4(func.model, y0, t, p, h=1.)
    t_rk[c] = time.time()-t_rk[c]

    t_sivp_rk[c] = time.time()
    for i in range(n):
```

```
        sivp_rk = solve_ivp(func.model, [t[0], t[-1]], y0, t_eval=t,␣
↪method="RK45", args=[p])
    t_sivp_rk[c] = time.time()-t_sivp_rk[c]

    t_sivp_ls[c] = time.time()
    for i in range(n):
        sivp_ls = solve_ivp(func.model, [t[0], t[-1]], y0, t_eval=t,␣
↪method="LSODA", args=[p])
    t_sivp_ls[c] = time.time()-t_sivp_ls[c]

    # Print execution time results
    print("rk4 execution time: %.6f s" % (t_rk[c]/n))
    print("solve_ivp(RK45) execution time: %.6f s" % (t_sivp_rk[c]/n))
    print("solve_ivp(LSODA) execution time: %.6f s" % (t_sivp_ls[c]/n))

    c += 1
```

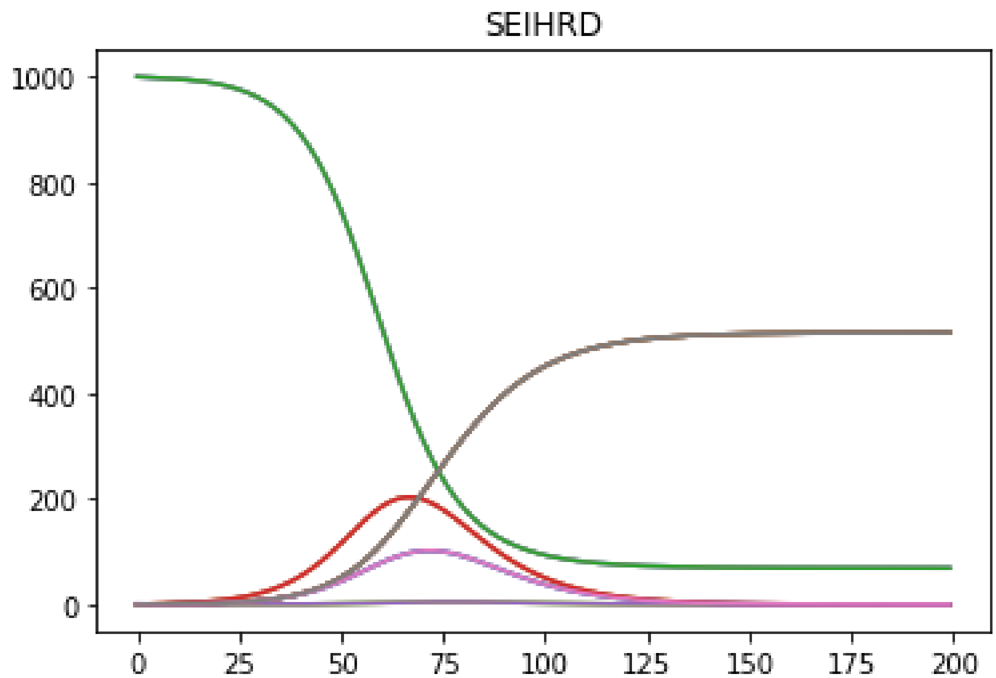



```
####################################################################

SIRD

rk4 - solve_ivp(RK45) distance: 936.0098
rk4 - solve_ivp(LSODA) distance: 8.2085
solve_ivp(LSODA) - solve_ivp(RK45) distance: 935.3987
```

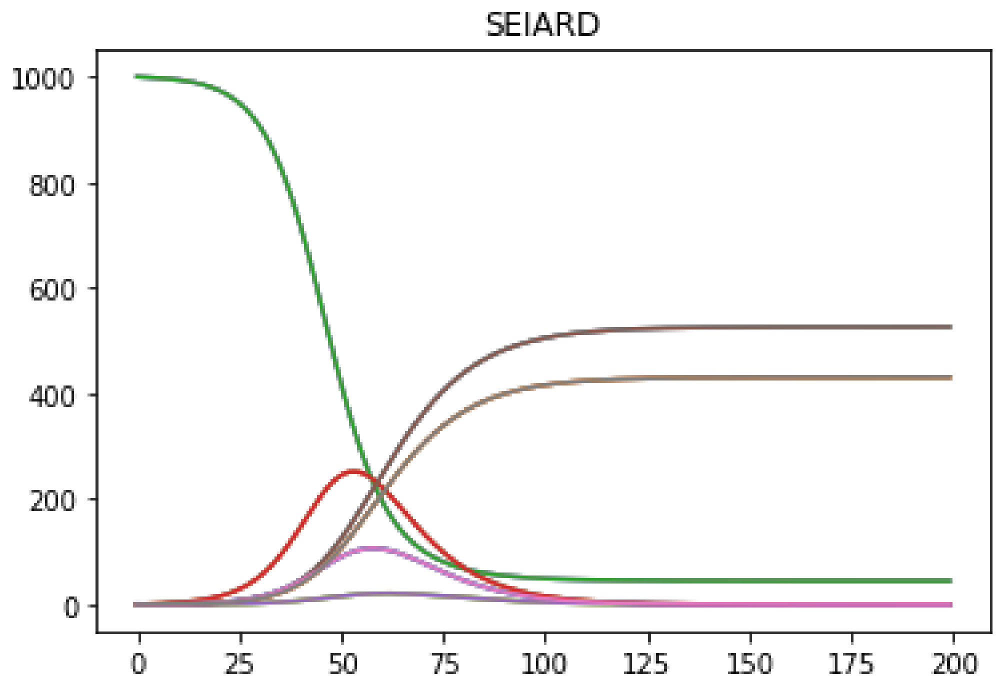
```
rk4 execution time: 0.000296 s
solve_ivp(RK45) execution time: 0.007813 s
solve_ivp(LSODA) execution time: 0.007765 s
```




```
####################################################################

SEIRD

rk4 - solve_ivp(RK45) distance: 13.4316
rk4 - solve_ivp(LSODA) distance: 26.4327
solve_ivp(LSODA) - solve_ivp(RK45) distance: 26.0336

rk4 execution time: 0.000302 s
solve_ivp(RK45) execution time: 0.006406 s
solve_ivp(LSODA) execution time: 0.007810 s
```



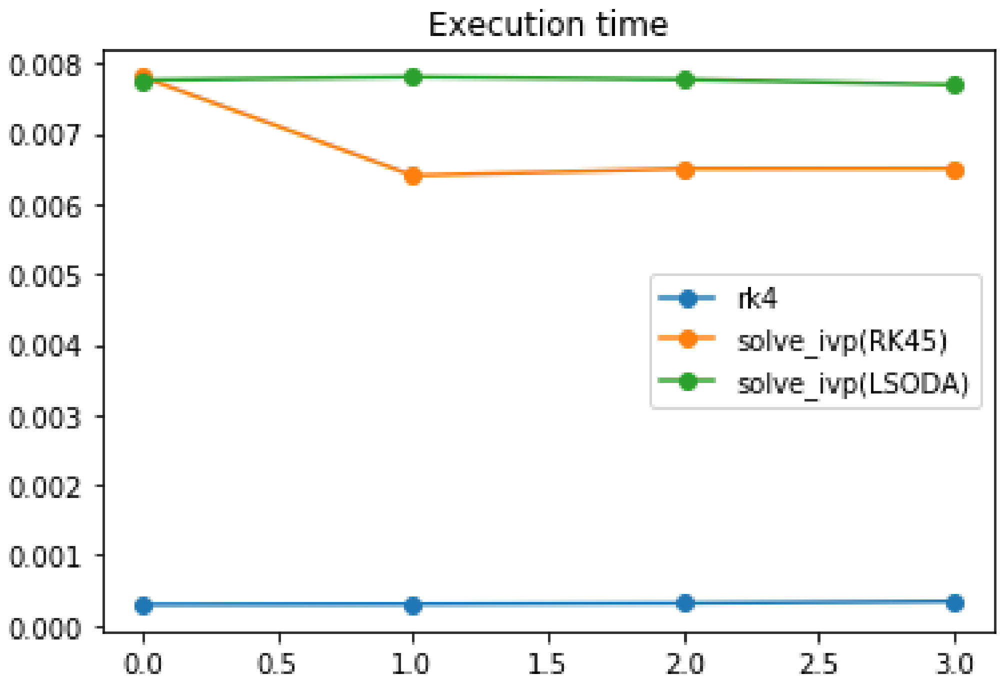

```
######################################################################

SEIHRD

rk4 - solve_ivp(RK45) distance: 11.4011
rk4 - solve_ivp(LSODA) distance: 48.3669
solve_ivp(LSODA) - solve_ivp(RK45) distance: 46.3574

rk4 execution time: 0.000314 s
solve_ivp(RK45) execution time: 0.006501 s
solve_ivp(LSODA) execution time: 0.007774 s
```




```
######################################################################

SEIARD

rk4 - solve_ivp(RK45) distance: 18.5001
rk4 - solve_ivp(LSODA) distance: 17.4302
solve_ivp(LSODA) - solve_ivp(RK45) distance: 25.1771

rk4 execution time: 0.000341 s
solve_ivp(RK45) execution time: 0.006502 s
solve_ivp(LSODA) execution time: 0.007698 s
```

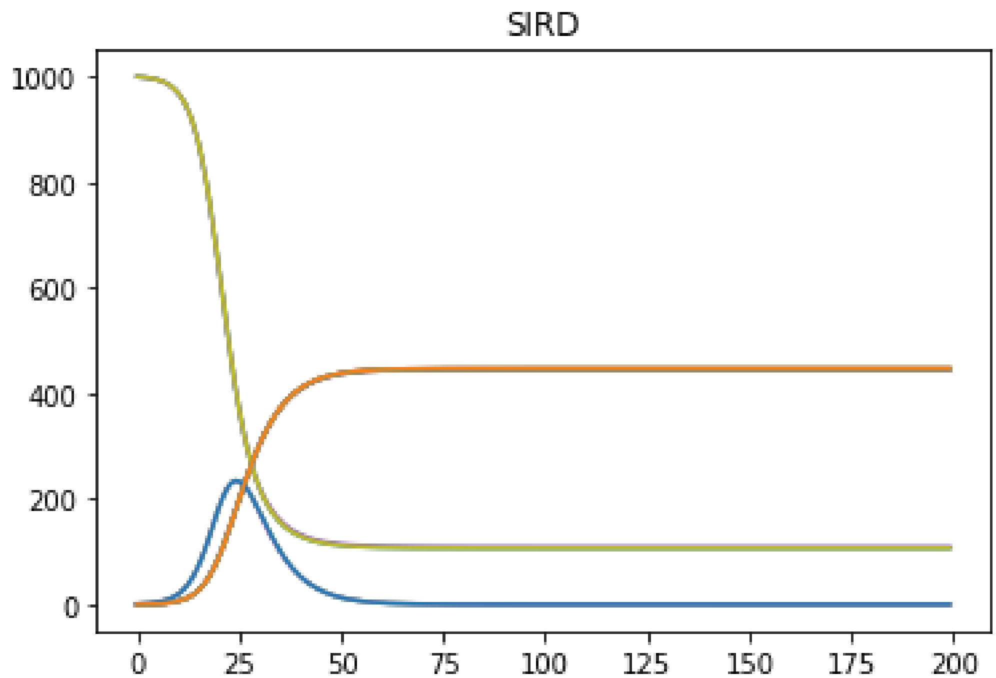
[42]:

```
plt.plot(t_rk/n, linestyle="-", marker="o", label="rk4")
plt.plot(t_sivp_rk/n, linestyle="-", marker="o", label="solve_ivp(RK45)")
plt.plot(t_sivp_ls/n, linestyle="-", marker="o", label="solve_ivp(LSODA)")
plt.legend()
plt.title("Execution time")
plt.show()
```

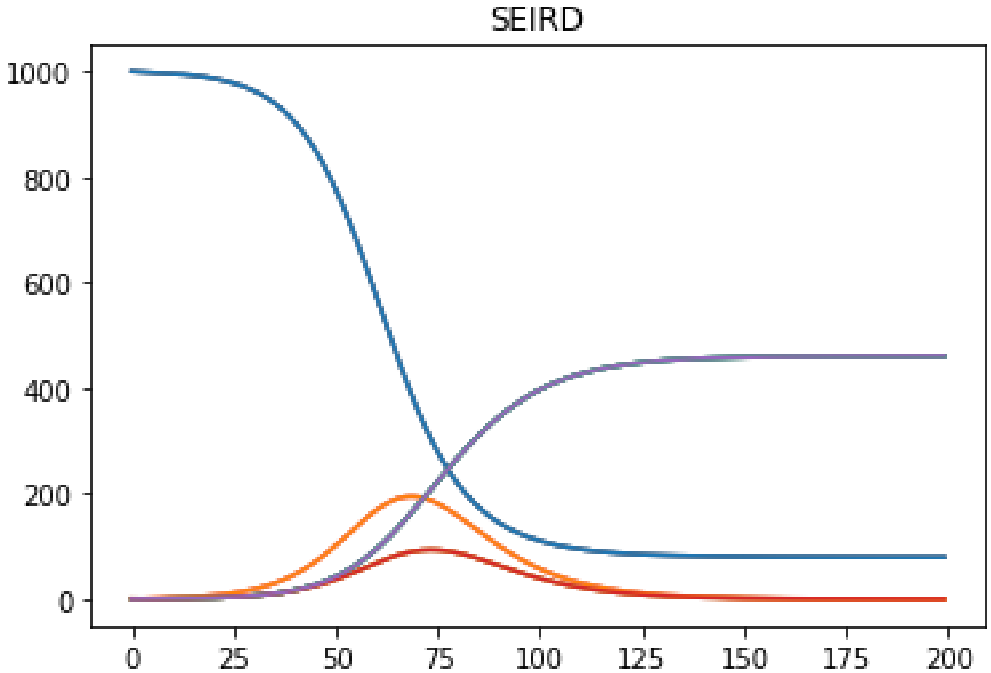


[43]:
```
# Repeating for a smaller time step (0.01) in rk4

c = 0 # Counting functions

for func in functions:

    # Generate initial conditions
    y0 = np.zeros(func.ncomp)
    y0[0] = 1000
    y0[-2] = 1

    # Generate parameters
    p = np.ones(func.nparams)/10
    p[0] = 0.5
    for i in range(len(func.params)):
        if func.params[i] == r"$N$":
            p[i] = 1000
        if func.params[i] == r"\tau_{h}":
            p[i] = 10
        if func.params[i] == r"\tau_{t}":
            p[i] = 20

    # Calculate and plot solutions for diffrent methods
    rk = rk4(func.model, y0, t, p, h=1.)
```

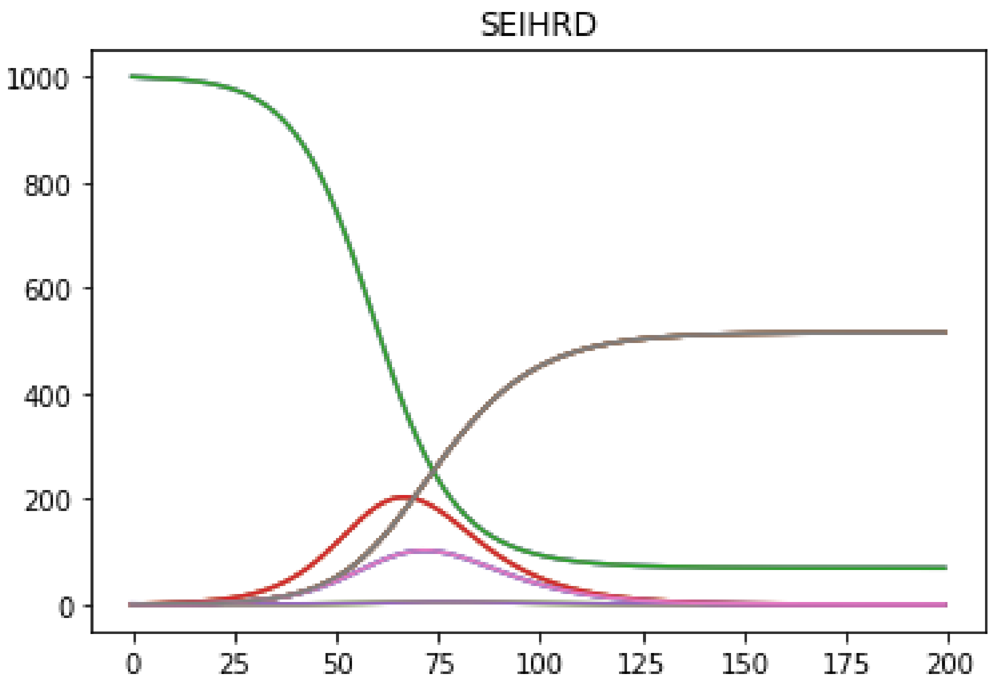
```python
    sivp_rk = solve_ivp(func.model, [t[0], t[-1]], y0, t_eval=t, method="RK45",␣
↪args=[p])
    sivp_ls = solve_ivp(func.model, [t[0], t[-1]], y0, t_eval=t,␣
↪method="LSODA", args=[p])

    plt.plot(rk)
    plt.plot(sivp_rk.y.T)
    plt.plot(sivp_ls.y.T)
    plt.title(func.name)
    plt.show()

    # Print distance between results of different methods
   ␣
↪print("\n######################################################################\n")
    print(func.name, "\n")
    print("rk4 - solve_ivp(RK45) distance: %.6f" % np.sum(np.abs(rk-sivp_rk.y.
↪T)))
    print("rk4 - solve_ivp(LSODA) distance: %.6f" % np.sum(np.abs(rk-sivp_ls.y.
↪T)))
    print("solve_ivp(LSODA) - solve_ivp(RK45) distance: %.6f\n" % np.sum(np.
↪abs(sivp_ls.y.T-sivp_rk.y.T)))

    # Calculate execution time
    t_rk[c] = time.time()
    for i in range(n):
        rk = rk4(func.model, y0, t, p, h=0.01)
    t_rk[c] = time.time()-t_rk[c]

    t_sivp_rk[c] = time.time()
    for i in range(n):
        sivp_rk = solve_ivp(func.model, [t[0], t[-1]], y0, t_eval=t,␣
↪method="RK45", args=[p])
    t_sivp_rk[c] = time.time()-t_sivp_rk[c]

    t_sivp_ls[c] = time.time()
    for i in range(n):
        sivp_ls = solve_ivp(func.model, [t[0], t[-1]], y0, t_eval=t,␣
↪method="LSODA", args=[p])
    t_sivp_ls[c] = time.time()-t_sivp_ls[c]

    # Print execution time results
    print("rk4 execution time: %.6f s" % (t_rk[c]/n))
    print("solve_ivp(RK45) execution time: %.6f s" % (t_sivp_rk[c]/n))
    print("solve_ivp(LSODA) execution time: %.6f s" % (t_sivp_ls[c]/n))

    c += 1
```



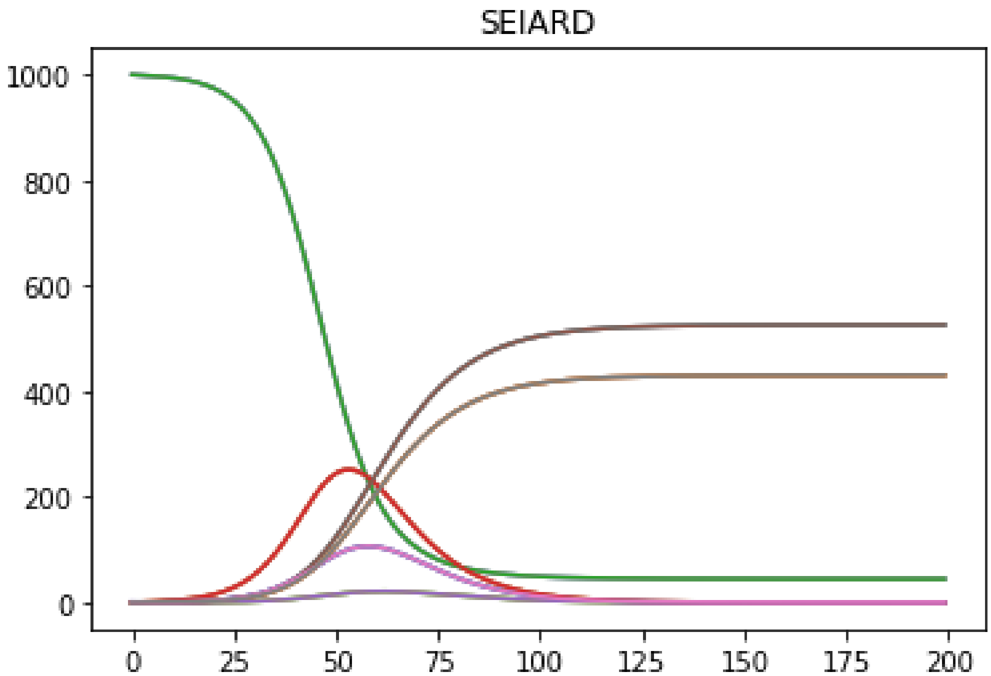

```
####################################################################

SIRD

rk4 - solve_ivp(RK45) distance: 936.009789
rk4 - solve_ivp(LSODA) distance: 8.208510
solve_ivp(LSODA) - solve_ivp(RK45) distance: 935.398726

rk4 execution time: 0.024033 s
solve_ivp(RK45) execution time: 0.006707 s
solve_ivp(LSODA) execution time: 0.007508 s
```




```
#################################################################

SEIRD

rk4 - solve_ivp(RK45) distance: 13.431635
rk4 - solve_ivp(LSODA) distance: 26.432703
solve_ivp(LSODA) - solve_ivp(RK45) distance: 26.033583

rk4 execution time: 0.025456 s
solve_ivp(RK45) execution time: 0.006879 s
solve_ivp(LSODA) execution time: 0.007955 s
```



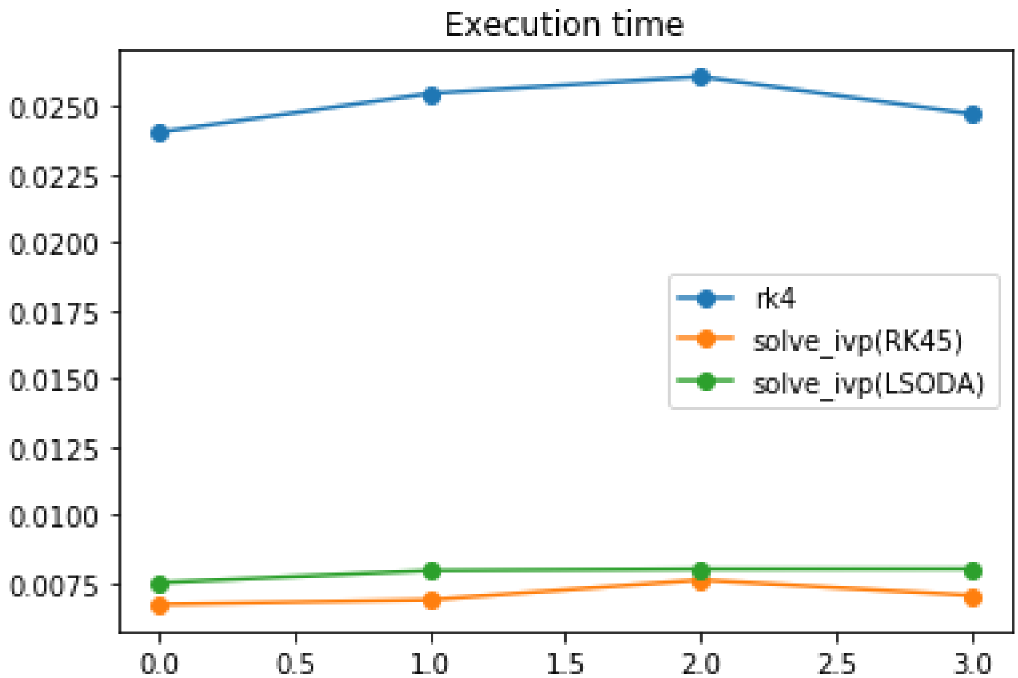

```
######################################################################

SEIHRD

rk4 - solve_ivp(RK45) distance: 11.401142
rk4 - solve_ivp(LSODA) distance: 48.366888
solve_ivp(LSODA) - solve_ivp(RK45) distance: 46.357365

rk4 execution time: 0.026070 s
solve_ivp(RK45) execution time: 0.007607 s
solve_ivp(LSODA) execution time: 0.008009 s
```




```
######################################################################

SEIARD

rk4 - solve_ivp(RK45) distance: 18.500141
rk4 - solve_ivp(LSODA) distance: 17.430164
solve_ivp(LSODA) - solve_ivp(RK45) distance: 25.177137

rk4 execution time: 0.024704 s
solve_ivp(RK45) execution time: 0.007039 s
solve_ivp(LSODA) execution time: 0.008018 s
```

[44]:

```
plt.plot(t_rk/n, linestyle="-", marker="o", label="rk4")
plt.plot(t_sivp_rk/n, linestyle="-", marker="o", label="solve_ivp(RK45)")
plt.plot(t_sivp_ls/n, linestyle="-", marker="o", label="solve_ivp(LSODA)")
plt.legend()
plt.title("Execution time")
plt.show()
```



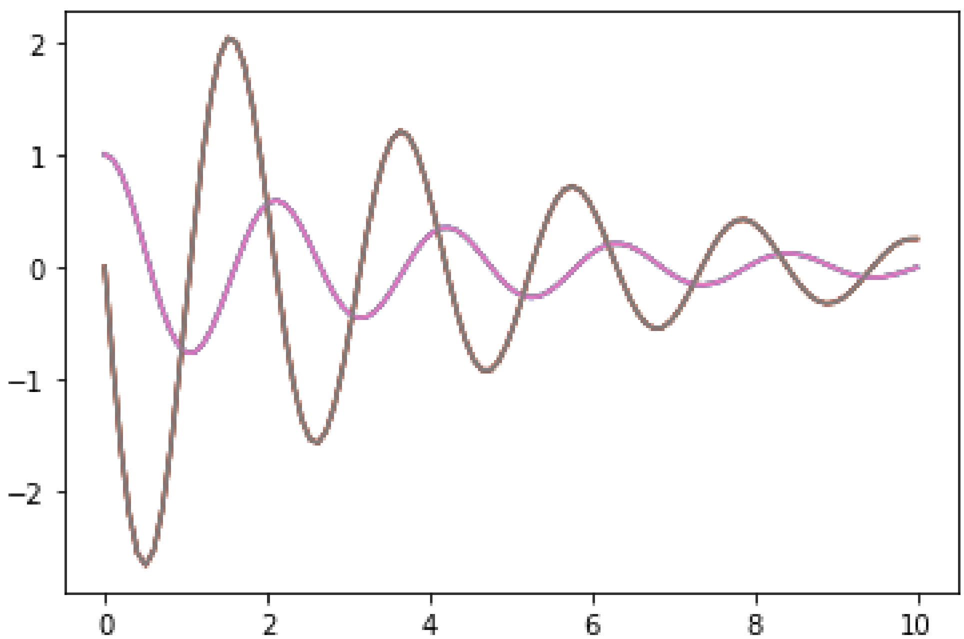
[45]:
```
# Benchmark comparing to a damped harmonic oscillator's analytic solution

# ODE system definition
@njit
def f(t, x, p):

    return np.array([x[1], -p[0]**2*x[0]-p[1]*x[1]])

t = np.linspace(0, 10, 100) # Time array

y0 = np.array([1.,0.]) # Initial conditions
p = np.array([3., 0.5]) # parameters (frequency and dissipation const.)
```

[46]:
```
# Numerical solutions
rk_sol = rk4(f, y0, t, p, 1e-2)
sivp_sol_rk = solve_ivp(f, [t[0], t[-1]], y0, t_eval=t, args=[p])
sivp_sol_ls = solve_ivp(f, [t[0], t[-1]], y0, t_eval=t, args=[p],␣
 ↪method="LSODA")

# Analytic solution
w1 = np.sqrt(p[0]**2-p[1]**2/4)

x1 = np.exp(-p[1]*t/2)*(np.cos(w1*t) + p[1]/2/w1*np.sin(w1*t))
x2 = -p[1]/2*x1 + np.exp(-p[1]*t/2)*(p[1]/2*np.cos(w1*t)-w1*np.sin(w1*t))
```

```
anal_sol = np.concatenate((x1, x2)).reshape((2,len(t))).T
```

[47]:
```
# Plotting, we see all results overlap
plt.plot(t, rk_sol)
plt.plot(t, sivp_sol_rk.y.T)
plt.plot(t, sivp_sol_ls.y.T)
plt.plot(t, anal_sol)
```

[47]:
```
[<matplotlib.lines.Line2D at 0x7fb9a887b990>,
 <matplotlib.lines.Line2D at 0x7fb9a7a39e90>]
```



[53]:
```
# Calculate execution time
t_rk = time.time()
for i in range(n):
    rk_sol = rk4(f, y0, t, p, 1e-2)
t_rk = time.time()-t_rk

t_sivp_rk = time.time()
for i in range(n):
    sivp_sol_rk = solve_ivp(f, [t[0], t[-1]], y0, t_eval=t, args=[p])
t_sivp_rk = time.time()-t_sivp_rk

t_sivp_ls = time.time()
for i in range(n):
```

```
    sivp_sol_ls = solve_ivp(f, [t[0], t[-1]], y0, t_eval=t, args=[p],␣
 ↪method="LSODA")
t_sivp_ls = time.time()-t_sivp_ls

# Print execution time and distances between analytic and numerical results
print("rk4 execution time: %.6f" % (t_rk/n))
print("solve_ivp(RK45) execution time: %.6f" % (t_sivp_rk/n))
print("solve_ivp(LSODA) execution time: %.6f" % (t_sivp_ls/n))
print("\nrk4 - analytic_sol distance:", np.sum((rk_sol-anal_sol)**2))
print("solve_ivp(RK45) - analytic_sol distance:", np.sum((sivp_sol_rk.y.
 ↪T-anal_sol)**2))
print("solve_ivp(LSODA) - analytic_sol distance:", np.sum((sivp_sol_ls.y.
 ↪T-anal_sol)**2))
print("\nrk4 - solve_ivp(RK45) distance:", np.sum((rk_sol-sivp_sol_rk.y.T)**2))
print("rk4 - solve_ivp(LSODA) distance:", np.sum((rk_sol-sivp_sol_ls.y.T)**2))
print("solve_ivp(RK45) - solve_ivp(LSODA) distance:", np.sum((sivp_sol_rk.y.
 ↪T-sivp_sol_ls.y.T)**2))
```

```
rk4 execution time: 0.001135
solve_ivp(RK45) execution time: 0.012888
solve_ivp(LSODA) execution time: 0.019942

rk4 - analytic_sol distance: 7.776660732980828e-06
solve_ivp(RK45) - analytic_sol distance: 0.00032558129854945797
solve_ivp(LSODA) - analytic_sol distance: 0.002955447446804876

rk4 - solve_ivp(RK45) distance: 0.0003329727965266321
rk4 - solve_ivp(LSODA) distance: 0.0030306543430492416
solve_ivp(RK45) - solve_ivp(LSODA) distance: 0.003822990073739528
```