FEM 1D | integrar em 0 a L | $\left.\begin{matrix} V_L \\ V_R \end{matrix}\right\}$ cond de contorno ou incógnitas ou dados | $\varepsilon = (x - x_L)/L_K$

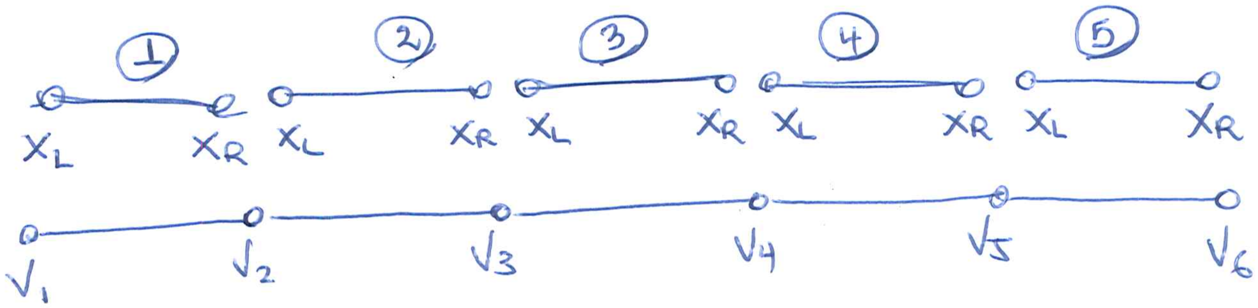
$$W = -\int_0^L \left\{ g v^2 + \frac{1}{R}\left(\frac{dv}{dx}\right)^2 \right\} dx$$

$$v(x) = a_L(x) V_L + a_R(x) V_R$$

$$a_L = \frac{x_2 - x}{x_2 - x_1}$$

$$a_R = \frac{x - x_1}{x_2 - x_1}$$

$$W = \sum_{K=1}^{5} W_K$$



$$W_K = -\int_{x_L}^{x_R} \left\{ g(a_L V_L + a_R V_R)^2 + \frac{1}{R}(a_L' V_L + a_R' V_R)^2 \right\} dx$$

$$W_K = -\int_{x_L}^{x_R} \left\{ g [V_L \; V_R] \begin{bmatrix} a_L a_L & a_L a_R \\ a_R a_L & a_R a_R \end{bmatrix} \begin{bmatrix} V_L \\ V_R \end{bmatrix} + \frac{1}{R} [V_L \; V_R] \begin{bmatrix} a_L' a_L' & a_L' a_R' \\ a_R' a_L' & a_R' a_R' \end{bmatrix} \begin{bmatrix} V_L \\ V_R \end{bmatrix} \right\} dx$$

$$W_K = -[V_L \; V_R] \left\{ \int_{x_L}^{x_R} \left(g \overbrace{\begin{bmatrix} a_L a_L & a_L a_R \\ a_R a_L & a_R a_R \end{bmatrix}}^{T} + \frac{1}{R} \overbrace{\begin{bmatrix} a_L' a_L' & a_L' a_R' \\ a_R' a_L' & a_R' a_R' \end{bmatrix}}^{S} \right) dx \right\} \begin{bmatrix} V_L \\ V_R \end{bmatrix}$$

$$W_K = -[V_L \; V_R] \left\{ g \underbrace{\begin{bmatrix} \int_{x_L}^{x_R} a_L a_L dx & \int_{x_L}^{x_R} a_L a_R dx \\ \int_{x_L}^{x_R} a_R a_L dx & \int_{x_L}^{x_R} a_R a_R dx \end{bmatrix}}_{T} + \frac{1}{R} \underbrace{\begin{bmatrix} \int_{x_L}^{x_R} a_L' a_L' dx & \int_{x_L}^{x_R} a_L' a_R' dx \\ \int_{x_L}^{x_R} a_R' a_L' dx & \int_{x_L}^{x_R} a_R' a_R' dx \end{bmatrix}}_{S} \right\} \begin{bmatrix} V_L \\ V_R \end{bmatrix}$$

$$W_K = -[V_L \; V_R]\left\{\frac{1}{R}S + gT\right\}\begin{bmatrix} V_L \\ V_R \end{bmatrix}$$

i	j		
0	0	L	L
0	1	L	R
1	0	R	L
1	1	R	R

$$S_{ij} = \int_{x_L}^{x_R} a_i' a_j' \, dx \qquad T = \int_{x_L}^{x_R} a_i a_j \, dx$$

$$W_K = -[V_L \; V_R]\, M \begin{bmatrix} V_L \\ V_R \end{bmatrix} \qquad \begin{bmatrix} V_L \\ V_R \end{bmatrix} = \mathbf{V}_K \qquad M = \left[\frac{1}{R}S + gT\right]$$

$$W_K = -V_K^T M_{K?} V_K$$

Lembrar S é das derivadas dos a_s

Normalizar a variável x do elemento, pelo compr. do mesmo

Fazemos $\varepsilon = (x - x_L)/L_K$ onde $L_K = x_R - x_L$

$$a_1(x) = \frac{x_R - x}{x_R - x_L} = \frac{1}{L_K}(x_R - \varepsilon L_K - x_L) = 1 - \varepsilon$$

$$\boxed{a_1(\varepsilon) = 1 - \varepsilon}$$

$$a_2(x) = \frac{x - x_L}{x_R - x_L} = \frac{1}{\cancel{L_K}}(\varepsilon \cancel{L_K} + \cancel{x_L} - \cancel{x_L}) = \varepsilon$$

$$\boxed{a_2(\varepsilon) = \varepsilon}$$

$$x = x_L + L_K \varepsilon$$

$$\frac{dx}{d\varepsilon} = L_K \qquad dx = L_K \, d\varepsilon \qquad \boxed{\frac{d\varepsilon}{dx} = \frac{1}{L_K}}$$

$$\boxed{\frac{dV}{dx} = \frac{V_R - V_L}{L_K}}$$

$$\boxed{V = a_1(\varepsilon)V_1 + a_2(\varepsilon)V_2}$$

$$\boxed{\frac{dV}{d\varepsilon} = V_R - V_L}$$

Vamos mudar as variáveis de integração em S e T para a nova variável normalizada

Comecemos com T

$$L_K = x_R - x_L$$

$$x = x_L + (x_R - x_L)\varepsilon$$

$$dx = L_K\, d\varepsilon$$

$$\frac{d\varepsilon}{dx} = \frac{1}{L_K}$$

x	ε
x_L	0
x_R	1

$$T_{ij}^{(K)} = \int_{x_L}^{x_R} a_i(x)\, a_j(x)\, dx$$

$$T_{ij}^{(K)} = \left[\int_0^1 a_i(\varepsilon)\, a_j(\varepsilon)\, d\varepsilon\right] L_K \quad \text{ou} \quad \boxed{T_{ij}^{(K)} = L_K\, T_{ij}^{(\varepsilon)}}$$

$$S_{ij}^{(K)} = \int_{x_L}^{x_R} \frac{da_i}{dx} \cdot \frac{da_j}{dx} \cdot dx$$

$$S_{ij}^{(K)} = \int_0^1 \frac{da_i}{d\varepsilon} \cdot \frac{d\varepsilon}{dx} \cdot \frac{da_j}{d\varepsilon} \cdot \frac{d\varepsilon}{dx} \cdot L_K\, d\varepsilon$$

$$S_{ij}^{(K)} = \int_0^1 \frac{da_i}{d\varepsilon} \cdot \frac{da_j}{d\varepsilon} \cdot \frac{1}{L_K} \cdot \frac{1}{\cancel{L_K}} \cdot \cancel{L_K} \cdot d\varepsilon$$

$$S_{ij}^{(K)} = \frac{1}{L_K} \int_0^1 \frac{da_i}{d\varepsilon} \cdot \frac{da_j}{d\varepsilon} \cdot d\varepsilon \quad \text{ou} \quad \boxed{S_{ij}^{(K)} = \frac{1}{L_K}\, S_{ij}^{(\varepsilon)}}$$

Vamos agora calcular as matrizes normalizadas

Comecemos com T (lembrando $a_L = 1-\varepsilon$ e $a_R = \varepsilon$)

$$T = \begin{bmatrix} \int_0^1 a_L a_L\, d\varepsilon & \int_0^1 a_L a_R\, d\varepsilon \\ \int_0^1 a_R a_L\, d\varepsilon & \int_0^1 a_R a_R\, d\varepsilon \end{bmatrix} = \begin{bmatrix} \int_0^1 (1-\varepsilon)^2 d\varepsilon & \int_0^1 \varepsilon(1-\varepsilon)d\varepsilon \\ \int_0^1 \varepsilon(1-\varepsilon)d\varepsilon & \int_0^1 \varepsilon^2 d\varepsilon \end{bmatrix}$$

Realizando as integrações elemento a elemento:

$$\boxed{T_\varepsilon = \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix}}$$ o que nos leva à: $T_K = L_K \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix}$

ou: $T_K = \frac{L_K}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Façamos o mesmo para S

$$S_\varepsilon = \begin{bmatrix} \int_0^1 a_1' a_1' d\varepsilon & \int_0^1 a_1' a_2' d\varepsilon \\ \int_0^1 a_2' a_1' d\varepsilon & \int_0^1 a_2' a_2' d\varepsilon \end{bmatrix}$$

Lembrando

$a_1(\varepsilon) = 1 - \varepsilon$

$a_1'(\varepsilon) = -1$

$a_2(\varepsilon) = \varepsilon$

$a_2'(\varepsilon) = 1$

Logo

$$\boxed{S_\varepsilon = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$ ou $S_K = \frac{1}{L_K} S_\varepsilon$

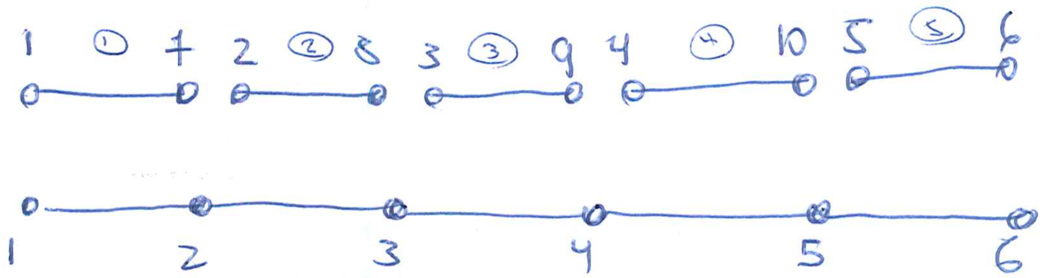
Lembrando, a solução original era:

$$W_K = -[V_L \; V_R]\left\{\frac{1}{R} S_K + g T_K\right\}\begin{bmatrix} V_L \\ V_R \end{bmatrix}$$

Substituindo $S_K = \frac{S_\varepsilon}{L_K}$ e $T_K = L_K T_\varepsilon$

$$\boxed{W_K = -[V_L \; V_R]\left(\frac{1}{R L_K} S_\varepsilon + g L_K T_\varepsilon\right)\begin{bmatrix} V_L \\ V_R \end{bmatrix}}$$

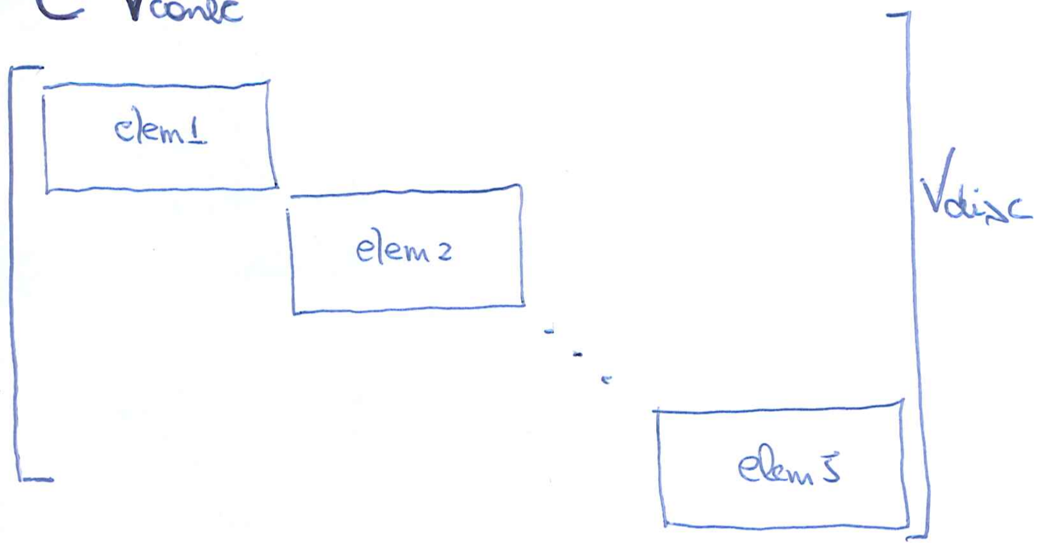
Calculamos a matriz de conexão para o grid abaixo



$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix} = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

$$V_{disc} = C \, V_{conec}$$

$W = -V_{disc}^T$



$M \leftarrow (10 \times 10)$ disconectado

$$elem\,1 = \frac{S}{R_1 L_1} + g_1 L_1 T \leftarrow (2 \times 2)$$

$$elem\,2 = \frac{S}{R_2 L_2} + g_2 L_2 T \leftarrow (2 \times 2)$$

etc...

logo $\underset{(escalar)}{W} = -\underset{1\times 10}{V_{disc}^T} \, \underset{\substack{10\times 10 \\ disc.}}{M} \, \underset{10\times 1}{V_{disc}} = -V_{conec}^t C^t \underset{disc}{M} C V_{conec}$

$$W = -V_{con}^{t}\, C^{t} M_{disc.} C V_{con}$$

Onde cada elemento tem $M_e = \dfrac{S}{R L_e} + g_e L T$ Supondo todos os elementos com R e g const. e mesmo tamanho L_e

Formando uma matriz $M_{disc.}$ bloco diagonal

Calculando para todos os elementos

$$\frac{1}{LR}\left[C^{t} S_e C\right] \quad e \quad g\left[C^{t} T_e C\right]$$

$$\frac{1}{L_e R}\begin{bmatrix} 1 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & -1 & 2 & -1 & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 1 \end{bmatrix} \qquad \frac{g L_e}{6}\begin{bmatrix} 2 & 1 & & & & \\ 1 & 4 & 1 & & & \\ & 1 & 4 & 1 & & \\ & & 1 & 4 & 1 & \\ & & & 1 & 4 & 1 \\ & & & & 1 & 2 \end{bmatrix}$$

$$W = -V_{con}^{t} \cdot \left[\frac{C^{t} S_e C}{R} + g\, C^{t} T C\right] V_{con}$$

para $L_e = 1$ $R = 1$ $g = 0,1$

Teremos:

Isto é apenas a matriz entre colchetes

NÃO É W AINDA

$$-\begin{bmatrix} -1,2 & 0,9 & 0 & 0 & 0 & 0 \\ 0,9 & -2,4 & 0,9 & 0 & 0 & 0 \\ 0 & 0,9 & -2,4 & 0,9 & 0 & 0 \\ 0 & 0 & 0,9 & -2,4 & 0,9 & 0 \\ 0 & 0 & 0 & 0,9 & -2,4 & 0,9 \\ 0 & 0 & 0 & 0 & 0,9 & -1,2 \end{bmatrix}$$

Revisar este final

vide próxima página

~~As derivadas serão os próprios coef. e teremos como sistema a própria matriz acima,~~ e $\boxed{V_6 = 1}$

Ver explicação na próxima página

condição inicial ou de contorno arbitrária

Supondo que o potencial possa variar em 1 a 5 e em 6 seja fixo (cond. inicial) calculemos $\dfrac{\partial W}{\partial V_k}$ para os $N-1$ nós (neste caso 5) em que ele varia e igualamos a 0 as derivadas (da física, sabemos que a distrib. de potencial é tal que minimiza a perda de potência na linha.)

$$Q(x) = x^T A x$$ h é escalar

$$Q(x+h) = (x+h)^T A (x+h)$$

$$Q(x+h) = (x^T + h^T) A (x+h)$$

$$Q(x+h) = (x^T A + h^T A)(x+h)$$

drop

$$Q(x+h) = x^T A x + h^T A x + x^T A h + h^T A h$$

$$Q(x+h) - Q(x) = \underset{1\times n}{h^T} \underset{n\times n}{A} \underset{n\times 1}{x} + \underset{1\times n}{x^T} \underset{n\times n}{A} \underset{n\times 1}{h}$$

$$Q(x+h) - Q(x) = x^T A^T h + x^T A h$$

$$Q(x+h) - Q(x) = x^T (A^T + A) h$$

$$\frac{\partial Q}{\partial x} = \underset{1\times n}{x^T} (\underset{n\times n}{A^T} + \underset{n\times n}{A}) \leftarrow \text{ROW!}$$

$$\left(\frac{\partial Q}{\partial x}\right)^t = \left[x^T (A^T + A)\right]^T$$

$$\left(\frac{\partial Q}{\partial x}\right)^t = (A^T + A) x \leftarrow \text{COLUMN}$$

Derivada de uma quadrática

Implementei o sistema de equações na planilha

$$\frac{\partial W}{\partial V_K} = 0 \quad \text{para } K = 1 \ldots 5$$

Para $V_6 = Z \leftarrow$ condição de contorno

7

FEM 1D - Adaptação Silvester p/ chapa e controle

$$\begin{bmatrix} S_{ff} & S_{fp} \\ S_{pf} & S_{pp} \end{bmatrix} \begin{bmatrix} U_f \\ U_p \end{bmatrix} = \begin{bmatrix} K_{ff} & K_{fp} \\ K_{pf} & K_{pp} \end{bmatrix} \begin{bmatrix} \mathbb{1} \\ \mathbb{1} \end{bmatrix} + \begin{bmatrix} R_{ff} & R_{fp} \\ R_{pf} & R_{pp} \end{bmatrix} \begin{bmatrix} T_f \\ T_p \end{bmatrix}$$

$$\overset{3\times3}{S_{ff}}\,\overset{3\times1}{U_f} + \overset{3\times2}{S_{fp}}\,\overset{2\times1}{U_p} = \overset{3\times3}{K_{ff}}\,\overset{3\times1}{\mathbb{1}} + \overset{3\times2}{K_{fp}}\,\overset{2\times1}{\mathbb{1}} + \overset{3\times3}{R_{ff}}\,\overset{3\times2}{T_f} + \overset{3\times2}{R_{fp}}\,\overset{3\times1}{T_p}$$

$$\overset{2\times3}{S_{pf}}\,\overset{3\times1}{U_f} + \overset{2\times2}{S_{pp}}\,\overset{2\times1}{U_p} = \overset{3\times3}{K_{pf}}\,\overset{3\times1}{\mathbb{1}} + \overset{2\times2}{K_{pp}}\,\overset{3\times1}{\mathbb{1}} + \overset{2\times3}{R_{pf}}\,\overset{3\times1}{T_f} + \overset{2\times2}{R_{pp}}\,\overset{2\times1}{T_p}$$

$\cancel{S_{ff}U_f}$

$$S_{ff}U_f - R_{fp}T_p = K_{ff}\mathbb{1} + \underset{}{\overset{0}{K_{fp}}}\mathbb{1} + \overset{0}{R_{ff}}T_f - S_{fp}U_p$$

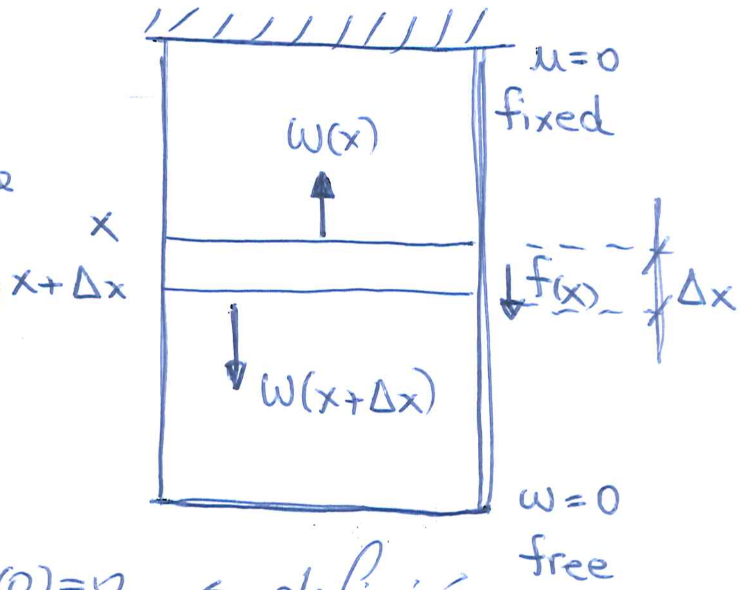
$$S_{pf}U_f - R_{pp}T_p = \underset{0}{K_{pf}}\mathbb{1} + K_{pp}\mathbb{1} + \underset{0}{R_{pf}}T_f - S_{pp}U_p$$

$$\begin{bmatrix} S_{ff} & -R_{fp} \\ S_{pf} & -R_{pp} \end{bmatrix} \begin{bmatrix} U_f \\ T_p \end{bmatrix} = \begin{bmatrix} K_{ff} & K_{fp} \\ K_{pf} & K_{pp} \end{bmatrix} \begin{bmatrix} \mathbb{1} \\ \mathbb{1} \end{bmatrix} + \begin{bmatrix} R_{ff} & -S_{fp} \\ R_{pf} & -S_{pp} \end{bmatrix} \begin{bmatrix} T_f \\ U_p \end{bmatrix}$$

$$\begin{bmatrix} U_f \\ T_p \end{bmatrix} = \begin{bmatrix} S_{ff} & -R_{fp} \\ S_{pf} & -R_{pp} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} K_{ff} & K_{fp} \\ K_{pf} & K_{pp} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} R_{ff} & -S_{fp} \\ R_{pf} & -S_{pp} \end{bmatrix} \begin{bmatrix} T_f \\ U_p \end{bmatrix} \right\}$$

Strang cap 3

$u(0)=0$



$$e(x) = Au(x) = \frac{du}{dx}$$

(o alongamento da barra é igual a variação do comprimento do mesmo)

ou

$$e(x) = \frac{u(x+\Delta x) - u(x)}{\Delta x}$$

$$e(x) = \frac{du}{dx} = Ax \qquad u(0)=0 \quad \leftarrow \text{definição}$$

$$w(x) = c(x)\, e(x) \quad \leftarrow \text{Lei de Hooke}$$

a força a que está submetido um ponto x da barra é $\propto$ ao seu alongamento (Δ ref dis)

$$w(x) = c(x)\frac{du}{dx}$$

$$w(x) = c(x)\, u'(x)$$

$$w(x) = w(x+\Delta x) + f(x)\Delta x$$

$$-\frac{w(x+\Delta x) - w(x)}{\Delta x} = f(x)$$

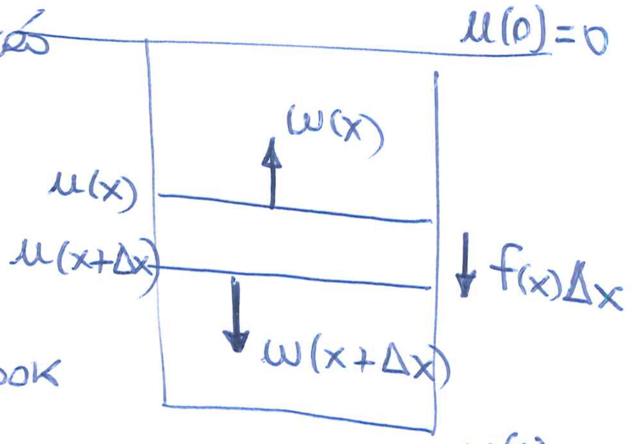
$$-\frac{dw}{dx} = f(x)$$

$$A^T w = f(x)$$

observação: Strang define d/dx (derivação) como A.

$-d/dx$ (−derivada) com A^T.

$u(x) \rightarrow$ deslocamento em relação à posição original



$e(x) = \dfrac{du}{dx}$ ← definição de along.

$w(1)=0$

$c(1)\,u'(1)=0$

$w(x) = c(x)\,e(x)$

← Lei de Hook

$$w(x) = w(x+\Delta x) + f(x)\Delta x$$

$$\frac{-\left[w(x+\Delta x) - w(x)\right]}{\Delta x} = f(x)$$

$$f(x) = -\frac{dw}{dx} = -\frac{d\,c(x)e(x)}{dx}$$

Definindo

$A.u = du/dx$

$A^T u = -du/dx$

$$f(x) = -\frac{d}{dx}\,c(x)\left[\frac{du(x)}{dx}\right]$$

$$f(x) = A^T C A u$$

$$Ku = A^T C A u = f$$

→ deslocamento em relação à posição original

$u(0)=0$

$c(1)\,u'(1)=0$

→ força por unidade de comprimento

$$f = -\frac{d c}{dx}\left[\frac{du}{dx}\right]$$

$$f v = -\frac{d}{dx}\left[c\frac{du}{dx}\right] v$$

$$\left[c u' v\right]_{x=0}^{x=1} - \int c \frac{du}{dx}\frac{dv}{dx}\,dx = -\int f v\,dx$$

$$\underbrace{c(1)\,u'(1)}_{0}\,v(1) - c(0)\,u'(0)\,\underbrace{v(0)}_{0}$$

BOUNDARY COND　　　　POR ESCOLHA NOSSA

$$\downarrow$$

$$\int c\frac{du}{dx}\cdot\frac{dv}{dx}\,dx = \int f v\,dx$$

$$U(x) = U_1 V_1(x) + U_2 V_2(x) + U_3 V_3(x) + \dots + U_n V_n(x)$$

$$\int c(x)\cdot\left(\sum U_j V_j'(x)\right)\frac{dV_i(x)}{dx}\,dx = \int f V_i(x)\,dx$$

$$K_{ij} = \int c(x)\,V_i'(x)\,V_j'(x)\,dx \qquad KU = F$$

MATRICIAL strong Exemplo 3.4

$cu' = w \qquad CAu = w$

$-w' = f \qquad A^T w = f \qquad u(0) = 0$

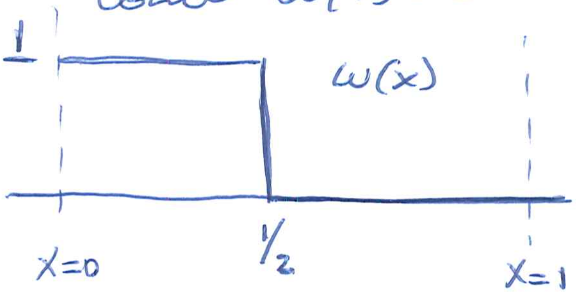
$c(1)u'(1) = 0$ ou $w(1) = 0$

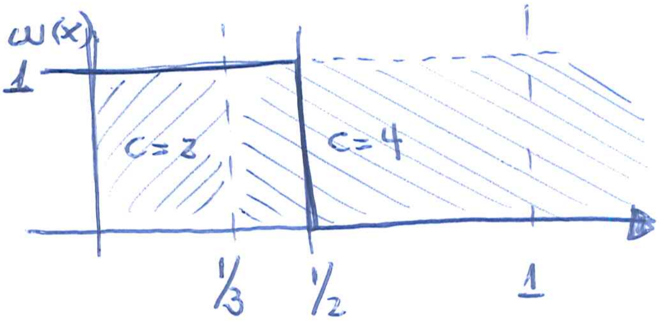
Point load $f(x) = \delta(x - \frac{1}{2})$ e $c = 2$ até $\frac{1}{3}$ e $c = 4$ depois

$$w = -\int_0^1 f(x)\,dx = -\int_0^1 \delta(x - \tfrac{1}{2})\,dx = -1$$

salto de -1 em $x = \frac{1}{2}$

como $w(1) = 0$

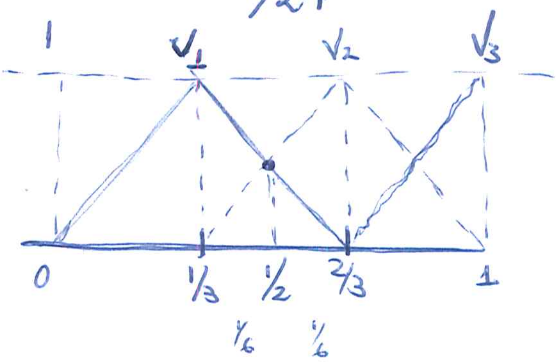




$cu' = w$

$$u = \int \frac{w\,dx}{c} = \int_0^{1/3} \frac{1\,dx}{2} + \int_{1/3}^{1/2} \frac{1\,dx}{4} + \int_{1/2}^{1} \frac{0\,dx}{4} = \frac{5}{24}$$

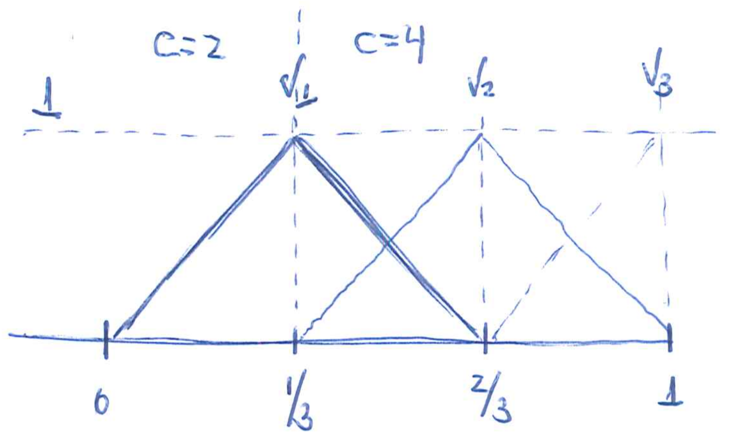
$u = \frac{5}{24}$



$$\int_0^{2/3} \delta(x - \tfrac{1}{2}) V_1(x)\,dx = V_1(\tfrac{1}{2}) = \tfrac{1}{2}$$

$$\int_{1/3}^{2/3} \delta(x - \tfrac{1}{2}) V_2(x)\,dx = V_2(\tfrac{1}{2}) = \tfrac{1}{2}$$

$$\int_{2/3}^{1} \delta(x - \tfrac{1}{2}) V_3(x)\,dx = 0$$



$$\int c\, V_i' V_j' \, dx$$

$$V_i' = 3 \text{ ou } -3 \ \forall i$$

$$\int_0^{1/3} 2 \cdot 3^2 dx + \int_{1/3}^{2/3} 4 \cdot 3^2 dx = \frac{2 \cdot 9}{3} + \frac{4 \cdot 9}{3} = \boxed{18 = K_{11}}$$

$$\int_{1/3}^{1} 4 \cdot 3^2 dx = 4 \cdot 9 \cdot 2/3 = \boxed{24 = K_{22}}$$

$$\int_{2/3}^{1} 4 \cdot 3^2 dx = 4 \cdot 9 \cdot 1/3 = \boxed{12 = K_{33}}$$

$$\int c V_1' V_2' dx = \int_{1/3}^{2/3} (-3) \cdot (3) \cdot 4 \, dx = -\frac{9 \cdot 4}{3} = \boxed{-12 = K_{12}}$$

$$\int c V_1' V_3' dx = \boxed{0 = K_{13}} \qquad \int_{2/3}^{1} c V_2' V_3' dx = \int_{2/3}^{1} 4 \cdot (-3) \cdot (3) dx = \boxed{-12 = K_{23}}$$

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \longrightarrow \begin{bmatrix} 18 & -12 & 0 \\ -12 & 24 & -12 \\ 0 & -12 & 12 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 4/24 \\ 5/24 \\ 5/24 \end{bmatrix}$$

$\boxed{31.1}$ $\boxed{31.2}$ chapter $\boxed{31.1}$ $\boxed{31.2}$

(1)

$\Delta = 1,25$

$T(0,t) = 75$

$T(10,t) = 150$

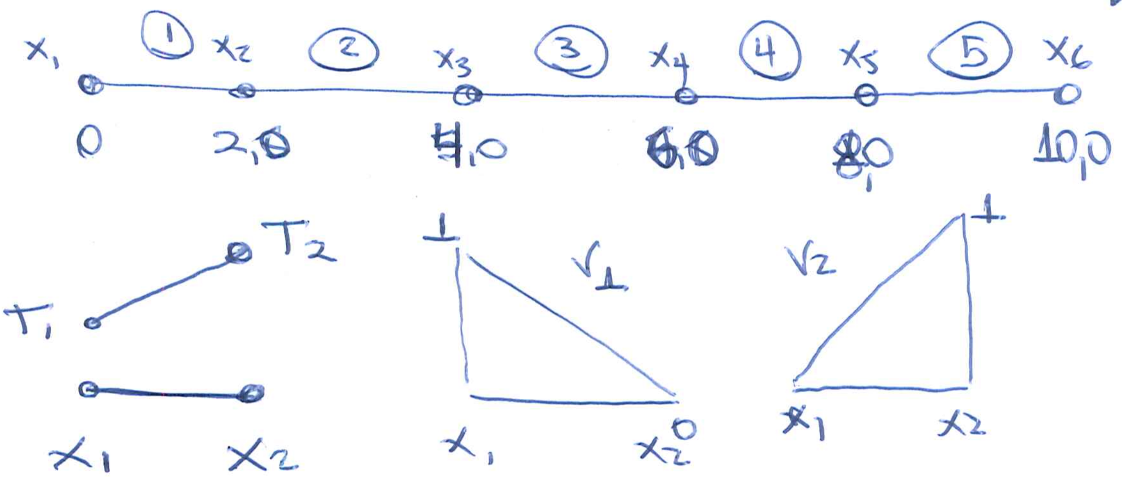
heat source of 15

$$\frac{d^2T}{dx^2} = -15$$

Exata $T(x) = -7{,}5x^2 + 82{,}5x + 75$

$\boxed{31.1}$

$\boxed{31.2}$



$$\int \frac{d^2T}{dx^2} v_i(x)\,dx = -15\int_{x_1}^{x_2} dx$$

$$\Big[v_i(x)T'(x)\Big]_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{dT}{dx}\frac{dv_i}{dx} = -15\left[x_2 - x_1\right]/2$$

(1) $$\Big[\underbrace{V_1(x_2)}_{0}T'(x_2) - \underbrace{V_1(x_1)}_{1}T'(x_1)\Big] - \frac{dT}{dx}\Big[V_1\Big]_{x_1}^{x_2} = -\frac{15}{2}\left[x_2 - x_1\right]$$

$$\Big[\underbrace{V_2(x_2)}_{1}T'(x_2) - \underbrace{V_2(x_1)}_{0}T'(x_1)\Big] - \frac{dT}{dx}\Big[V_2\Big]_{x_1}^{x_2} = -\frac{15}{2}\left[x_2 - x_1\right]$$

②

$$-T'_{x_1} - \frac{T_2-T_1}{x_2-x_1}\left[\underbrace{V_1(x_2)}_{0} - \underbrace{V_1(x_1)}_{1}\right] = \frac{-15}{2}\left[x_2-x_1\right]$$

$$+T'_{x_2} - \frac{T_2-T_1}{x_2-x_1}\left[\underbrace{V_2(x_2)}_{1} - \underbrace{V_2(x_1)}_{0}\right] = \frac{-15}{2}\left[x_2-x_1\right]$$

$$-T'_{x_1} + \frac{T_2-T_1}{x_2-x_1} = -\frac{15}{2}\left[x_2-x_1\right]$$

$$+T'_{x_2} - \frac{T_2-T_1}{x_2-x_1} = -\frac{15}{2}\left[x_2-x_1\right]$$

considerando que $x_2 - x_1 = 2{,}0$ em todos os elementos

$$-T'_{x_1} + \frac{T_2-T_1}{2} = -\frac{15}{\cancel{2}}\cdot\cancel{2}$$

$$+T'_{x_2} - \frac{T_2-T_1}{2} = \frac{-15}{\cancel{2}}\cdot\cancel{2}$$

$$0{,}5\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}T_1\\ T_2\end{bmatrix} = \begin{bmatrix}-15+T'_{x_1}\\ -15-T'_{x_2}\end{bmatrix}$$

$$\begin{bmatrix}-0{,}5 & 0{,}5\\ 0{,}5 & -0{,}5\end{bmatrix}\begin{bmatrix}T_1\\ T_2\end{bmatrix} = \begin{bmatrix}-15+T'_{x_1}\\ -15-T'_{x_2}\end{bmatrix}$$

Matriz C

$$
\begin{array}{c c}
 & \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{bmatrix} T_{1\_1} \\ T_{1\_2} \\ T_{2\_1} \\ T_{2\_2} \\ T_{3\_1} \\ T_{3\_2} \\ T_{4\_1} \\ T_{4\_2} \\ T_{5\_1} \\ T_{5\_2} \end{bmatrix} =
& \begin{bmatrix}
1 & & & & & \\
 & 1 & & & & \\
 & 1 & & & & \\
 & & 1 & & & \\
 & & 1 & & & \\
 & & & 1 & & \\
 & & & 1 & & \\
 & & & & 1 & \\
 & & & & 1 & \\
 & & & & & 1
\end{bmatrix}
\end{array}
$$

Matriz de Rigidez

$$
\begin{bmatrix} -0{,}5 & 0{,}5 \\ 0{,}5 & -0{,}5 \end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} =
\begin{bmatrix} -15 + T_1' \\ -15 - T_2' \end{bmatrix}
$$

$$
\begin{bmatrix} 1 & x \\ 1 & 0 \\ 1 & 2 \\ 1 & 4 \\ 1 & 6 \\ 1 & 8 \\ 1 & 10 \end{bmatrix}
\longrightarrow
\begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 2 \\ 1 & 4 \\ 1 & 4 \\ 1 & 6 \\ 1 & 6 \\ 1 & 8 \\ 1 & 8 \\ 1 & 10 \end{bmatrix}
$$

Tabela de Elementos

Matriz S (Bloco diagonal)

$$
\begin{bmatrix}
-0{,}5 & 0{,}5 & & & & & & & & \\
0{,}5 & -0{,}5 & & & & & & & & \\
 & & -0{,}5 & 0{,}5 & & & & & & \\
 & & 0{,}5 & -0{,}5 & & & & & & \\
 & & & & -0{,}5 & 0{,}5 & & & & \\
 & & & & 0{,}5 & -0{,}5 & & & & \\
 & & & & & & -0{,}5 & 0{,}5 & & \\
 & & & & & & 0{,}5 & -0{,}5 & & \\
 & & & & & & & & -0{,}5 & 0{,}5 \\
 & & & & & & & & 0{,}5 & -0{,}5
\end{bmatrix}
$$

Fazendo $(C^t S C)T = R + F$

$$\begin{bmatrix} -0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 \\ 0{,}5 & -1 & 0{,}5 & 0 & 0 & 0 \\ 0 & 0{,}5 & -1 & 0{,}5 & 0 & 0 \\ 0 & 0 & 0{,}5 & -1 & 0{,}5 & 0 \\ 0 & 0 & 0 & 0{,}5 & -1 & 0{,}5 \\ 0 & 0 & 0 & 0 & 0{,}5 & -0{,}5 \end{bmatrix} \begin{bmatrix} 75 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ 150 \end{bmatrix} = \begin{bmatrix} -15+T_1' \\ -30 \\ -30 \\ -30 \\ -30 \\ -15-T_2' \end{bmatrix}$$

Resolvemos o sistema obtendo

$T_1' = 82{,}5$

$T_2 = 210$

$T_3 = 235$ duzentos trinta e cinco

$T_4 = 300$ trezentos

$T_5 = 255$

$T_6' = -67{,}5$

CHAPRA CANALE
EX 32.1
Reação química

$$V\frac{\Delta c}{\Delta t} = Qc - Q[c + c_x \Delta x] - DAc_x + DA[c_x + c_{xx}\Delta x] - KVc$$

$\cancel{Vc_t = Q}$ moles/h que entram

moles/h que saem

moles/m^3 c → soluto (sólido)

m^3/h q → solvente (líquido)

$$\underbrace{Vc_t}_{\text{taxa de variação da concentração ao longo do tempo}} = qc - q\underbrace{[c + c_x\Delta x]}_{c(x+\Delta x)} - DAc_x + DA\underbrace{[c_x + c_{xx}\Delta x]}_{c_x(x+\Delta x)} - KVc$$

$-KVc$ → perde por causa da reação química

$$c_t = -\frac{q}{V}c_x\Delta x + \frac{DA}{V}c_{xx}\Delta x - \frac{KV}{V}c$$

$$c_t = \frac{DA\Delta x}{V}c_{xx} - \frac{q\Delta x}{V}c_x - Kc$$

quando estes $\Delta x \to 0$, os coeficientes tornam-se constantes

$$c_t = Dc_{xx} - \frac{q\Delta x}{V}c_x - Kc$$

$$U = \frac{q}{A_c} \quad \frac{m^3/h}{m^2} = \frac{m}{h}$$

$$c_t = Dc_{xx} - Uc_x - Kc$$

$$\underbrace{\frac{m^2}{h}\cdot\frac{moles}{m^5}}_{\text{Dispersão}} \quad \underbrace{\frac{m}{h}\cdot\frac{mols}{m^4}}_{\text{Velocidade}} \quad \underbrace{\frac{1}{h}\,\frac{mols}{m^3}}_{\text{Decaimento}}$$

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} - U\frac{\partial c}{\partial x} - Kc$$

moles/hora

$$V\frac{\Delta c}{\Delta t} = Q\,c(x) - Q\left[c(x) + \frac{\partial c(x)}{\partial x}\Delta x\right]$$

$\frac{moles}{hora}$; $\frac{m^3}{h}$; $\frac{moles}{m^3}$; $\frac{moles/m^3}{m}\cdot m$

$$- D A_c \frac{\partial c(x)}{\partial x}$$

$\frac{m^2}{h}\cdot m^2$; $moles/m^3/m$

$$+ D A_c \left[\frac{\partial c(x)}{\partial x} + \frac{\partial}{\partial x}\frac{\partial c(x)}{\partial x}\Delta x\right]$$

$\frac{m^2}{h}$; m^2 ; $\frac{moles/m^3}{m}$; $\frac{1}{m}$; $\frac{moles/m^3}{m}\cdot m$

$$- K V c$$

$h^{-1}\cdot m^3 \frac{moles}{m^3}$

$\Delta x = 2{,}5$

$D = 2$

$U = 1$

$K = 0{,}2$

$C_{in} = 100$

$c(x,0) = 0$

$c'(L,t) = 0$

V = Volume (m^3)

Q = Flow Rate (m^3/h)

c = concentr (moles/m^3)

D = dispers. coef. (m^2/h)

A_c = Área transversal do tanque (m^2)

K = first order decay coefficient (h^{-1})

$$V\frac{\Delta c}{\Delta t} = Q\,c(x) - Q\left[c(x) + \frac{\partial c(x)}{\partial x}\Delta x\right] - DA_c\frac{\partial c(x)}{\partial x} + DA_c\left[\frac{\partial c(x)}{\partial x} + \frac{\partial}{\partial x}\frac{\partial c(x)}{\partial x}\Delta x\right] - KVc$$

$$\frac{\Delta c}{\Delta t} = -\frac{Q}{V}\frac{\partial c(x)}{\partial x}\Delta x + \frac{DA_c}{V}\frac{\partial}{\partial x}\frac{\partial c(x)}{\partial x}\Delta x - \frac{KVc}{V}$$

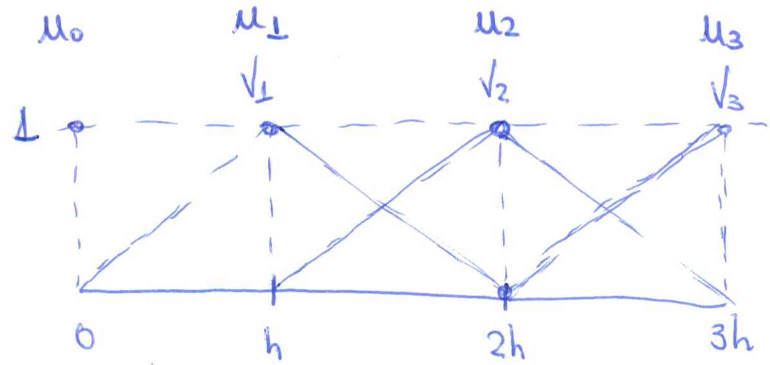
$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} - U\frac{\partial c}{\partial x} - Kc$$

Em steady state:

$$0 = Dc_{xx} - Uc_x - Kc$$

A variação na concentração ao longo do tempo é proporcional à dispersão e inversamente proporcional ao fluxo e à reação química

~~ONE~~ 9.1.1
AFEM



$f = 1 - x$

$$\int f v_i \, dx = \int c u' v' \, dx$$

$$\int_0^h (1-x)\cdot\frac{x}{h}\,dx + \int_h^{2h} (1-x)\cdot -\frac{(x-2h)}{h}\,dx = c\int_0^h u'\cdot\frac{1}{h}\,dx + c\int_h^{2h} u'\cdot -\frac{1}{h}\,dx$$

$$h - h^2 = \frac{c}{h}\left[u(h) - u(0)\right]$$

$$- \frac{c}{h}\left[u(2h) - u(h)\right]$$

x	y
h	1
2h	0

$$y = -\frac{1}{h}(x - 2h)$$

$$(h-h^2)\frac{h}{c} = 2u(h) - u(2h) - \underset{\downarrow 0}{u(0)}$$

$$\boxed{\frac{h^2 - h^3}{c} = 2U_1 - U_2}$$

$$\int_h^{2h} (1-x)\cdot\frac{(x-h)}{h}\,dx + \int_{2h}^{3h} (1-x)(-1)\frac{(x-3h)}{h}\,dx = c\int_h^{2h} u'\cdot\frac{1}{h}\,dx + c\int_{2h}^{3h} u'\cdot -\frac{1}{h}\,dx$$

$$h - 2h^2 = \frac{c}{h}\left[u(2h) - u(h) \quad u(2h) - u(3h)\right]$$

$$\boxed{\frac{h^2 - 2h^3}{c} = 2U_2 - U_1 - U_3}$$

$$\int_{2h}^{3h} (1-x)\frac{(x-2h)}{h}\,dx = c\int_{2h}^{3h} u'\frac{1}{h}\,dx$$

$$-\frac{(8h^2 - 3h)}{6} = \frac{c}{h}\left[u(3h) - u(2h)\right]$$

$$\boxed{-\left[8h^3 - 3h^2\right]/(6c) = U_3 - U_2}$$

AFEM 9.1.1

$$c\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} h^2 - h^3 \\ h^2 - 2h^3 \\ (3h - 8h^2)/6 \end{bmatrix}$$

$$\frac{c}{h}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} h - h^2 \\ h - 2h^2 \\ (3 - 8h)/6 \end{bmatrix}$$

$h = 1/3$

$$\frac{c}{h}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1/3 - 1/9 \\ 1/3 - 2/9 \\ (3 - 8/3)/6 \end{bmatrix}$$

$$c\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1/9 - 1/27 \\ 1/9 - 2/27 \\ (1 - 8/9)1/6 \end{bmatrix}$$

$$c\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 2/27 \\ 1/27 \\ 1/54 \end{bmatrix} = \begin{bmatrix} 4/54 \\ 2/54 \\ 1/54 \end{bmatrix}$$

$$54c\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

$u_1 = 7 . 54c$

$u_2 = 10 . 54c$

$u_3 = 11 . 54c$