The Calculus of a Stockholder Under a Simple Proxy Voting Regime

Gustavo Pellegrini Dias

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Abstract

At corporate elections, stockholders can choose between representing themselves or delegating their votes to a trustee. If a stockholder is interested in maximizing the probability that their company will make the correct business decision, when should they delegate? An intuitive strategy is for stockholders to vote only when they believe to be the most informed about the business decision at stake. And, when that is not the case, they should delegate their vote to the stockholder they believe to be the most competent. It is hard to think of a better plan. But, contrary to intuition, such a plan exists: stockholders should condition their competence on the probability that their vote will make any difference in the results of the election. By following the same intuitive rule while accounting for the probability of being a pivotal stockholder, stockholders correct for the bias towards delegation inherent to the more intuitive strategy. I discuss how this finding can explain observations of "over-delegation" in recent proxy voting experiments, and why companies may prefer to ascribe fixed actions to stockholders rather than allow them to choose their own actions with some probability.

1 Introduction

Proxy voting has become an indispensable tool for public companies. It helps them meet the minimum quorum required for corporate elections, and foster a more democratic impression of the company's governance. From the stockholders' perspective, the possibility of delegating votes is also valuable as it allows them not to spend too much time pondering over the business decisions while not having their interests misrepresented. The implementation of proxy voting may vary slightly from company to company. The Delaware Code outlines in its statutes the standard for the delegation of votes in corporate elections. According to Chapter 1, Subchapter VII, Section 212 of the Delaware General Corporation Law:

> *"One stockholder or 2 or more stockholders may by agreement in writing deposit capital stock of an original issue with or transfer capital stock to any person or persons, or entity or entities authorized to act as trustee, for the purpose of vesting in such person or persons, entity or entities, who may be designated voting trustee, or voting trustees, the right to vote thereon for any period of time determined by such agreement, upon the terms and conditions stated in such agreement"* (Delaware Code, Chapter 1, Subchapter VII, § 212).

The Code describes a simple mechanism of transference of voting weights on top of the usual weighted majority rule that makes up a corporate election. To illustrate this mechanism, suppose stockholders A and B own 2 and 1 votes respectively, and stockholder A chooses to delegate all their votes to stockholder B. Stockholder B now owns 3 votes, and stockholder A owns 0 votes. That means that stockholder A's opinion will have a weight of 0 in the weighted majority rule that informs the company's final decision. Stockholder B's opinion will have a weight of 3.

But when is it optimal for a stockholder to give up their voting power? To answer this question, a stockholder must know the value of the information they hold. And for stockholders who are aligned with the incentives of the corporation, their information is valuable in so far as it contributes to the majority of stockholders making the best decision for the company. Therefore, this paper investigates the decision of the stockholder in a common utility model, where the utility function of all stockholders is represented by the probability that the majority of votes opt for the correct alternative in a set of binary options.

In this context, the research presented here shows that stockholders systematically undervalue the information they possess. I provide a new solution that corrects for this bias and that stockholder's can use to make a more accurate decision between voting and delegating. I also show how voting systems where stockholders have an ex-ante determined action outperform systems where they are allowed to choose their own actions, especially when it comes to small populations of stockholders.

2 Literature Overview

The Condorcet Jury Theorem (from here on referred to as CJT), one of the earliest results in the political economy literature, provides a compelling statistical principle of how, under certain conditions, the chance that a majority selects the best alternative (when such alternative exists) converges to 1 as the size of the voter population grows. This theorem underscores the foundational perspective on the aggregation of individual judgments into a "wisdom of the crowd". Here's the original exercise that led to the CJT: Consider a committee who is faced with the decision to convict or acquit a defendant. The only information available derives from the independent competencies of an unspecified amount of committee members. If we make the assumption that the least competent committee member can perform better than random choice, then it follows from the Law of Large Numbers (LLN) that the number of committee members making the correct decision will converge to some proportion of the committee population above 50%. Consequently, the probability that more than 50% of the committee will make the correct decision converges to 100%. In other words, as the population of committee members grows, the probability that their majority is correct converges to 1.

Austen-Smith and Banks 1996 develop a corollary of the CJT that is relevant to the context of proxy voting: One can always find a number of voters for whom the probability that a majority of them selects the correct alternative is greater than the probability that any single voter in that population selects the correct alternative. So, continuing the exercise in the previous paragraph, as long as the most competent committee member is not perfectly competent, we can always find a number of less competent committee members for which the result of a majority rule among them will always outperform the decision of the single expert committee member. These results suggest that there exists a trade-off between quality and quantity of information that committee members hold.

In T. Feddersen and Pesendorfer 1998, the authors investigate whether requiring committee members to reach a unanimous decision reduces the probability of convicting an innocent defendant and increases the probability of acquitting a guilty defendant. Specifically, they are interested in testing whether this claim holds true even if committee members are allowed to engage in strategic behavior. They reach the counter intuitive conclusion that imposing the unanimity restriction actually increases both the likelihood of convicting an innocent defendant and the likelihood of acquitting a guilty defendant. In fact, they find that the likelihood of committing both errors only increases as the number of committee members increases. They show how the majority rule performs much better when it comes to diminishing the likelihood of both of these errors. This study underscores the complexity of voting dynamics and the relevance of population size in informing individual decision. My analysis also draws inspiration from T. J. Feddersen and Pesendorfer 1996, an earlier study by the same authors which introduces the calculus of a pivotal stockholder and develops the concept of Swing Voter's Curse, providing information aggregation arguments for a voter's decision to abstain from an election.

In the 21^{st} century, proxy voting entered the spotlight thanks to advancements on blockchain technology that allow for greater accountability of decisions made by decentralized authorities. This push towards less centralized decision-making is best represented by the development of Liquid Democracy. Liquid Democracy is a system that combines instant referendums with proxy voting. In Liquid Democracy, a voter is always able to choose between the policy options available or choose a proxy representative to make the decision using their voting power. Therefore, instead of electing a subset of voters to represent the population on a set of topics every 4 years, Liquid Democracy allows for "modular congresses" to be composed for every new debate. Early applications of Liquid Democracy include the Google Votes experiment (Hardt and Lopes 2015), where Google employees were asked to review food trucks using Liquid Democracy. In that occasion, researchers observed a specialization effect where votes on vegetarian food trucks were delegated to vegetarian employees. The German Pirate Party used Liquid Democracy as their official internal decision-making tool from 2009 to 2015, abandoning it after concerns that Liquid Democracy had led the party to a situation where a few voters concentrated most of the voting power. In 2016, Democracy Earth, a political advocacy group, ran a mock Liquid Democracy referendum on the peace agreements between the Colombian government and the FARC guerrillas.

The notable surge in Liquid Democracy's popularity has been closely followed by scholars in the computer science and economics circles. Revel *et al.* 2022 find that for voter competencies drawn from an arbitrary distribution whose support is bounded away from 1, a constant fraction of the total population is needed to maximize the probability that the representatives make the correct decision on behalf of the entire population. This result strengthens the notion that congresses should be much larger than they currently are in modern democracies, and that decentralized voting systems could improve over the current version of representative democracy many countries have. Other papers offer a more cautionary perspective. Campbell *et al.* 2022 focuses on comparing the performance of majority rule with abstention *versus* Liquid Democracy. They study a model where voters share a common utility and are trying to maximize the chance that the majority will pick the ex-post correct alternative out of 2 options. Through two experiments, the authors show that even though Liquid Democracy could theoretically improve over majority rule with abstention, voters in Liquid Democracy exhibit a tendency to over-delegate, choosing to vote independently more than the optimal quantity. In applied settings, therefore, majority rule with abstention has shown better results than decision mechanisms that allow for proxy-voting. Explanations as to why the "over-delegation" phenomenon occurs mostly center around the costs of voting. However, the results of my analysis suggest a new explanation based on the failure of a pivotal stockholder to account for how their pivotality condition affects the probability their peers are making the correct decision.

3 The Model

This section presents the formal model of proxy voting. The model is constructed to explore the dynamics of the company's decision-making under ex-ante uncertainty and the trade-off between the quality and quantity of information suggested by the CJT and its corollary.

- **Set of Stockholders:** Consider a set of stockholders of size $n \in \mathbb{N}$, with $n > 2$ and n odd, that deliberates to reach a decision on whether to approve or not a business decision. Index stockholders by $I = \{1, 2, \ldots, i, \ldots, n\}$.

- **Set of States and Alternatives:** There exists a set of possible states of the economy denoted by $Z = \{0, 1\}$. Similarly, there exists a set of alternatives, representing the company's possible business decisions, denoted by $X = \{0, 1\}$.

- **Set of Actions:** Stockholders can take one of two actions, represented by the set $A = \{v, \delta\}$, reflecting their decision to vote independently (v) or vote by proxy (δ).

- **Nature's Role:** The true state of the economy, $z \in Z$, is determined by a uniformly random draw by nature, reflecting the inherent ex-ante uncertainty in the election's outcome.

- **Stockholder Types:** I distinguish stockholders between two types within the set $T = \{e, \overline{e}\}$, where $\overline{e}$ denotes non-experts with a common accuracy $p \in (0.5, 1)$ and e denotes experts with a strictly higher competence level $q \in (p, 1)$. Note that even though $\overline{e}$ stockholders share the accuracy p, each p is independent from each other so that the probability 2 $\overline{e}$ are simultaneously correct is equal to p^2.

- **Utility Functions:** All stockholders, regardless of type, share a common utility function $u(x, z)$, which is 1 if their decision matches the true state ($x = z$) and 0 otherwise.

- **Decision Rule:** The company's decision is determined by a weighted majority rule, where each stockholder i has a weight ω_i.

Throughout all the analysis contained in this paper it is assumed for simplicity that there exists only 1 expert stockholder and $n-1$ non-expert stockholders. By focusing on the trade-off between adding one more weight to the most expert stockholder *versus* adding a new non-expert stockholder, this assumption makes us ready to study the trade-off between the quality of a source and quantity of independent sources.

4 Analysis

I start my analysis by acknowledging that any stockholder can only have a preference over their actions if the action they take can affect the company's decision. A stockholder that finds themselves in such position is referred to as the pivotal stockholder. In this section, I turn to the analysis of the problem faced by this pivotal stockholder and their equilibrium solution. An important idea is that, in equilibrium, no stockholders will choose a strictly dominated strategy. Observe that if the pivotal stockholder is the expert stockholder then, in equilibrium, they will always choose action υ, since voting independently strictly dominates delegating to any equal or less competent stockholder. If the pivotal stockholder is a non-expert stockholder then, in equilibrium, they will never choose to delegate to another non-expert stockholder as this strategy is strictly dominated by both delegating to the expert stockholder and voting independently. Therefore, for the remainder of my analysis I will focus on studying the decision of a non-expert pivotal stockholder between voting independently or using the expert stockholder as their trustee. As detailed in the description of my model, non-expert stockholders have an inherit competence p and cannot change the probability they will vote correctly. Nonetheless, they still have a relevant decision to make. They must consider how voting independently or voting by proxy affects the probability that the majority of the stockholders will be correct. Recall that all stockholders have the same utility function. Therefore, a pivotal stockholder i is always interested in maximizing the probability that the majority makes the correct decision. So I will use the probability that the majority is correct as their objective function. The first part of this section is dedicated the construction of this function.

4.1 Constructing the Pivotal Stockholder's Objective Function

When the expert stockholder has the majority of votes it is easy to calculate the probability that the majority is correct. Either the expert stockholder is correct and consequently the majority is correct, or the expert is wrong and consequently the majority is incorrect. Therefore, this case can be summarized by

$$\mathbb{P}\left(\text{Majority is Correct}|w_e \geq \frac{n+1}{2}\right) = q. \tag{1}$$

When the expert stockholder does not have the majority of votes, then the scenarios in which the majority is correct become a bit more involved. If the expert is correct, then I need another $\frac{n+1}{2} - \omega_e$ non-expert stockholders to be correct in order to form a correct majority. This case can be modeled as

$$\mathbb{P}\left(\text{Majority is Correct}|\omega_e < \frac{n+1}{2}, e \text{ is correct}\right) =$$

$$q\left[\sum_{j=\frac{n+1}{2}-\omega_e}^{n-\omega_e}\binom{n-\omega_e}{j}p^j(1-p)^{n-\omega_e-j}\right]. \tag{2}$$

Now, if the expert stockholder does not have the majority of votes and is incorrect, I need $\frac{n+1}{2}$ non-expert stockholders to be correct in order to form a correct majority. This case is defined by the expression below.

$$\mathbb{P}\left(\text{Majority is Correct}|\omega_e < \frac{n+1}{2}, e \text{ is incorrect}\right) =$$

$$(1-q)\left[\sum_{j=\frac{n+1}{2}}^{n-\omega_e}\binom{n-\omega_e}{j}p^j(1-p)^{n-\omega_e-j}\right]. \tag{3}$$

Given the probabilities described by Equations 1, 2, and 3, the probability that the majority is correct is given by their sum

$$\mathbb{P}(\text{Majority is correct}) =$$

$$q\left[\sum_{j=\max\left\{\frac{n+1}{2}-\omega_e,0\right\}}^{n-\omega_e}\binom{n-\omega_e}{j}p^j(1-p)^{n-\omega_e-j}\right]$$
$$+\,(1-q)\left[\sum_{j=\frac{n+1}{2}}^{n-\omega_e}\binom{n-\omega_e}{j}p^j(1-p)^{n-\omega_e-j}\right]. \tag{4}$$

Equation 4 completely describes the objective function of the pivotal stockholder in my model. Note that I added the term $\max\left\{\frac{n+1}{2}-\omega_e,0\right\}$ to account for the case where $\omega_e > \frac{n+1}{2}$ and e is correct. In the next section I use equation 4 to setup the problem faced by the pivotal stockholder.

4.2 The Pivotal Stockholder's Optimization Problem

I begin the setup of the pivotal stockholder optimization problem by noting that once one is aware of the CJT, it is intuitive to propose that, for distinct sizes of stockholder populations, there exist different optimal amounts of weight that should be allocated to the expert stockholder. In fact, it seems reasonable to suppose that this optimal weight lies somewhere between absolute power to the expert and a majority rule where all stockholders have the same weight. The setup of my model allows the company to make a decision informed only by the stockholder with the best information by setting the weight of the expert $w_e \geq \frac{n+1}{2}$. It also allows the company to make a decision informed by a majority rule between all available sources by distributing 1 vote to each stockholder. But more importantly, the company is able to reach strategies in between these two extremes, where the most informed stockholder may have a higher weight than others yet not hold absolute power.

Given equation 4 and the constraints of my model, I can characterize the pivotal stockholder's problem as the following maximization problem:

$$\max_{\{\omega_e^*\}} \{\mathbb{P}(\text{Majority is Correct})\}$$

Subject to:

$$\sum_{i=1}^{n} \omega_i = n,$$

$$\omega_e \geq 1,$$

$$\omega_i = \{0, 1\} \ \forall \ i \neq e.$$

Assuming a uniform distribution of parameters p and q, I can use expected values to generate the utility of stockholders as a function of the weight of the expert stockholder. These curves are plotted in Figure 1 for different values of n. These curves suggest that for any n the expected relationship between the probability that the majority is correct and the optimal weight of the expert stockholder follows a single-peaked pattern. Observe that, for $n > 3$, the peak of the curve seems to always fall in between the two extreme solutions. For the values of n selected in Figure 1, it looks like the optimal value of ω_e changes very slowly with respect to n.

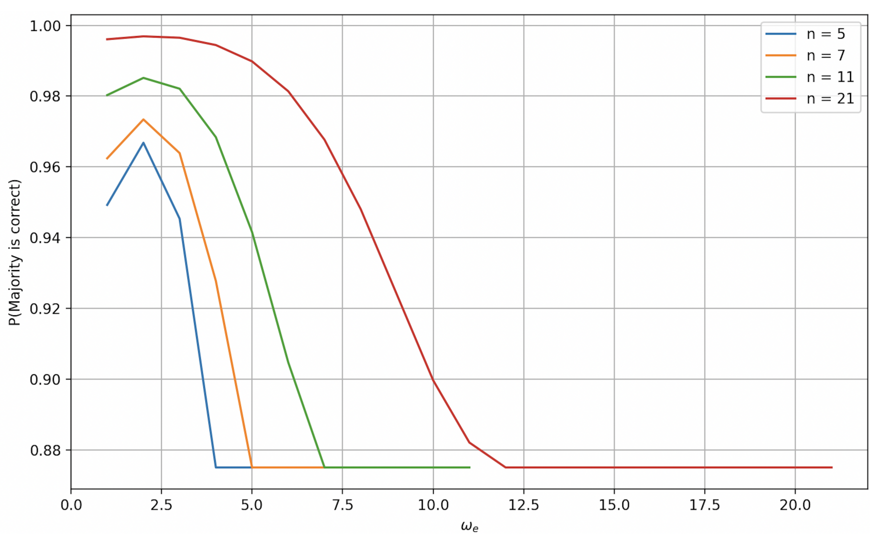

Figure 1: $\mathbb{E}[\mathbb{P}(\text{Majority is Correct})]$ as a Function of ω_e for Select Values of n.

4.3 The Decision of the Pivotal Stockholder

In this section I am interested in determining when the non-expert pivotal stockholder i prefers action υ over action δ. This decision is informed by the difference of the utilities derived from each action. Therefore, i's optimal choice of action is represented by

$$A^*(i) = \begin{cases} \upsilon \text{ if } u_i(\upsilon) > u_i(\delta), \\ \delta \text{ if } u_i(\upsilon) < u_i(\delta), \\ \text{Indifferent otherwise.} \end{cases} \tag{5}$$

I can start analyzing the decision of pivotal stockholder i by determining the probability that i finds themselves to be the pivotal stockholder. In my model, i can only be pivotal when there are exactly $\frac{n-1}{2}$ stockholders voting for convicting the defendant and $\frac{n-1}{2}$ voting for acquitting the defendant. Evidently, this can only happen when $\omega_e < \frac{n+1}{2}$, so we do not need to consider any scenarios involving $\omega_e \geq \frac{n+1}{2}$. Then, there are two distinct occasions in which the non-expert stockholder is pivotal: one in which the expert stockholder is correct, and another in which the expert stockholder is incorrect. In the case when the expert stockholder is correct we require that exactly $\frac{n-1}{2} - \omega_e$ non-expert stockholders be correct and, consequently, $\frac{n-1}{2}$ non-expert stockholders be incorrect. I can represent this scenario as

$$\mathbb{P}(i \text{ is pivotal}|e \text{ is correct}) = \binom{n - \omega_e - 1}{\frac{n-1}{2}} p^{\frac{n-1}{2} - \omega_e} + (1-p)^{\frac{n-1}{2}}. \tag{6}$$

Similarly, in the case when the expert stockholder is incorrect, I require that exactly $\frac{n-1}{2} - \omega_e$ non-expert stockholders are also incorrect and, consequently, $\frac{n-1}{2}$ non-expert stockholders are correct.

$$\mathbb{P}(i \text{ is pivotal}|e \text{ is incorrect}) = \binom{n - \omega_e - 1}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2} - \omega_e}. \tag{7}$$

By summing equations 6 and 7 using the probabilities that the expert stockholder is correct and incorrect as weights for each of the corresponding terms, I get the total probability that a non-expert stockholder i is pivotal. This is represented by equation 8 below

$$\begin{aligned} \mathbb{P}(i \text{ is pivotal}) = q &\left[\binom{n - \omega_e - 1}{\frac{n-1}{2}} p^{\frac{n-1}{2} - \omega_e} (1-p)^{\frac{n-1}{2}} \right] \\ +(1-q) &\left[\binom{n - \omega_e - 1}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2} - \omega_e} \right]. \end{aligned} \tag{8}$$

Equation 8 fully captures the probability that i is a pivotal stockholder. Note that $\mathbb{P}(i \text{ is pivotal})$ is not influenced by the $\mathbb{P}(i \text{ is correct}) = p$. The next step is to find the utility of i associated with choosing action υ *versus* choosing δ given that i is pivotal. Let's start by calculating the utility of voting.

$$u_i(\upsilon) = \mathbb{P}(\text{Majority is Correct}|A(i) = \upsilon). \tag{9}$$

And

$$\mathbb{P}(\text{Majority is Correct}|A(i) = \upsilon) = \mathbb{P}(i \text{ is correct}|i \text{ is pivotal}). \tag{10}$$

Since the probability that i is pivotal is independent of the probability i is correct, then I have that the utility of i from choosing action υ is

$$u_i(\upsilon) = p. \tag{11}$$

Now I calculate $u_i(\delta)$. Naturally, the utility derived from delegating varies depending on whether the expert is correct or incorrect. I account for that in the equations below

$$u_i(\delta) = \mathbb{P}(\text{Majority is Correct}|A(i) = \delta). \tag{12}$$

$$\mathbb{P}(\text{Majority is Correct}|A(i) = \delta) = \mathbb{P}(e \text{ is correct}|i \text{ is pivotal}). \tag{13}$$

From Bayes' Rule I have that

$$\mathbb{P}(e \text{ is correct}|i \text{ is pivotal}) = \frac{\mathbb{P}(i \text{ is pivotal}|e \text{ is correct})\mathbb{P}(e \text{ is correct})}{\mathbb{P}(i \text{ is pivotal})}. \tag{14}$$

From equation 8 and from the definitions of my model I have

$$\mathbb{P}(e \text{ is correct}|i \text{ is pivotal}) = \frac{q\left[\binom{n-\omega_e-1}{\frac{n-1}{2}}p^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}}\right]}{q\left[\binom{n-\omega_e-1}{\frac{n-1}{2}}p^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}}\right] + (1-q)\left[\binom{n-\omega_e-1}{\frac{n-1}{2}}p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}-\omega_e}\right]}$$

$$= \frac{\binom{n-\omega_e-1}{\frac{n-1}{2}}\left[qp^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}}\right]}{\binom{n-\omega_e-1}{\frac{n-1}{2}}\left(\left[qp^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}}\right] + (1-q)\left[p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}-\omega_e}\right]\right)}$$

$$= \frac{qp^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}}}{qp^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}} + (1-q)\left[p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}-\omega_e}\right]}.$$

So i's utility from delegating is equal to

$$u_i(\delta) = \frac{qp^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}}}{qp^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}} + (1-q)\left[p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}-\omega_e}\right]}. \tag{15}$$

The pivotal stockholder would be indifferent between voting independently or voting by proxy as long as $u_i(\upsilon) = u_i(\delta)$. This means they will be indifferent whenever $p = \mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$ or, from equation 15, whenever

$$p = \frac{qp^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}}}{qp^{\frac{n-1}{2}-\omega_e}(1-p)^{\frac{n-1}{2}} + (1-q)\left[p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}-\omega_e}\right]}. \tag{16}$$

If $p > \mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$, the pivotal stockholder will prefer to vote independently. If $p < \mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$, the pivotal stockholder will prefer to vote by proxy. Having established the relationship that orients the decision of the non-expert pivotal stockholder, I conclude the analysis section of my paper and move on to an exploration of the results of my analysis.

5 Results

Figure 2 plots both p and $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$ as a function of the share of votes to the expert, $\frac{\omega_e}{n}$. From figure 2 I can dissect the many moving parts that make up the decision of the pivotal stockholder. Let's start by identifying the point where the horizontal line p crosses the blue curve $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$. It is here that I find the share of votes $\frac{\omega_e^*}{n}$ that makes i indifferent between voting independently or voting by proxy, that is, the optimal weight for the expert.

As $q \to 1$, the curve that represents $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$ approaches a constant $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal}) = 1$. As $q \to 0.5$, the optimal share of votes to the expert goes to 0.

As I move p, two effects are observed. The first is a change in $\frac{\omega_e^*}{n}$ due to the vertical shift of p. That is, all else constant, if p shifts down, $\frac{\omega_e^*}{n}$ increases. If p shifts up, $\frac{\omega_e^*}{n}$ decreases. The second effect is on the shape of $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$. As p increases, I observe a linearization of $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$. Therefore, as p increases, there are two combined effects that make $\frac{\omega_e}{n}$ attain its optimal at lower values. As p increases, I observe a curvilinearization of $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$. So as p decreases, there also exists two simultaneous effects that make $\frac{\omega_e}{n}$ attain its optimal at higher values.

At $\frac{\omega_e}{n} = 0$, the $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$ approaches q. As I change n, I observe no effects on p or $\mathbb{P}(e \text{ is correct}|i \text{ is pivotal})$. This means that the value of ω_e^* does not change with respect to n. Therefore, holding all other parameters constant,

$$\lim_{n\to\infty} \frac{\omega_e^*}{n} = 0. \tag{17}$$

Figure 3 provides a visualization of how the optimal allocation of share of votes to the expert varies as n grows for $p = 0.75$ and $q = 0.875$. It illustrates

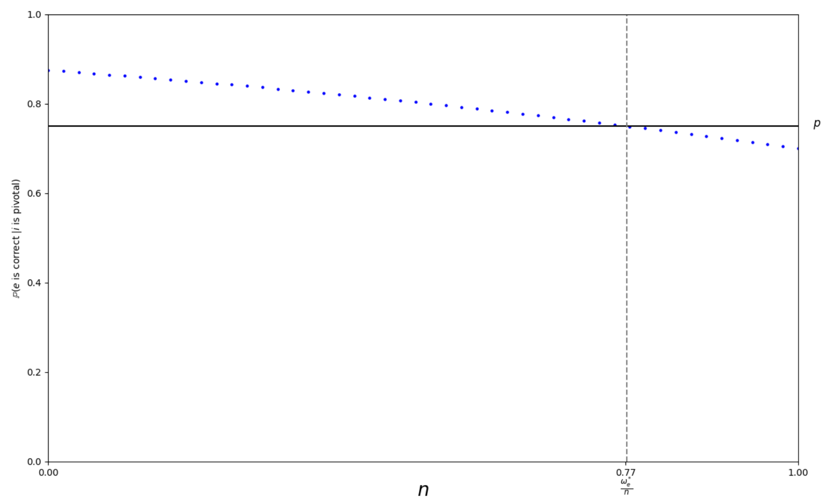

Figure 2: Trade-Off Faced by a Pivotal Stockholder ($p = 0.75$, $q = 0.875$)

perhaps the most important result of this paper: the probability of the expert being correct given that i is pivotal drops very quickly as the population size increases. Here's an intuition for why this phenomenon happens: First, consider what happens to the share of stockholders that receive the correct signal as the population of stockholders increases. As n goes to infinity, the share of stockholders that receive the correct signal converges to p as demonstrated by

$$\lim_{n\to\infty} \frac{p(n-1)+q}{n} \implies \lim_{n\to\infty} \frac{np-p+q}{n} = p. \tag{18}$$

From the construction of my model, I know that $p > 0.5$. So as the population increases, the population of correct stockholders should be strictly greater than 50% of the stockholder population. Now, in a situation where i is pivotal, it must be that at least $\frac{n-1}{2}$ stockholders received a signal opposite to the one the expert received. As the population grows and i remains pivotal, the proportion of signals that are opposite to the expert's signal converges to 50% of all realized signals. I can see that in the following equation

$$\lim_{n\to\infty} \frac{\left(\frac{n-1}{2}\right)}{n} = \frac{1}{2}. \tag{19}$$

So from the fact that $n \to \infty$ I have at the same time that the proportion of the population that is correct is greater than 50%, and that the proportion of the population that received signals opposite to the expert stockholder is at least 50%. Therefore, as $n \to \infty$, the expert stockholder must belong to the incorrect minority. This result suggests a severe limitation to the ability of

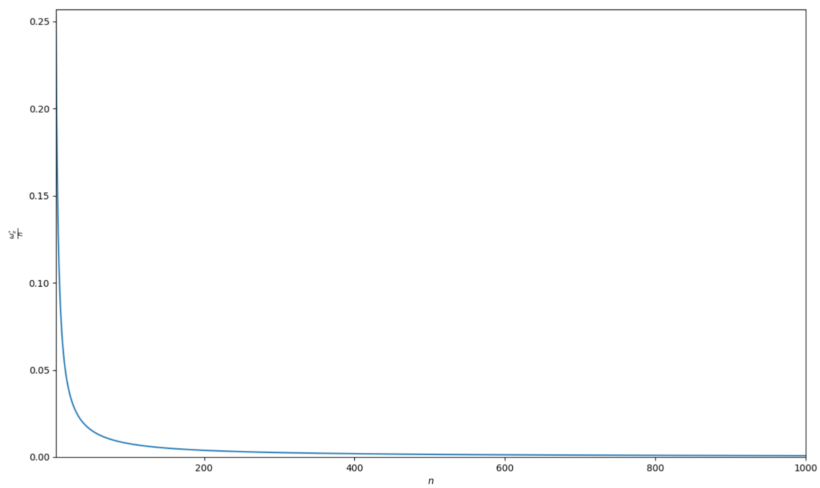

Figure 3: $\frac{\omega_e^*}{n}$ as a Function of n

proxy voting to improve on the aggregation of information, especially when it comes to large populations of stockholders.

Another way to visualize the decision of the pivotal stockholder is by plotting i's indifference curves between delegating and voting for every value of ω_e over the pq-plane. In Figure 4 I plot the indifference curves for $\omega_e = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ to illustrate the movement of i's indifference curve as ω_e grows.

These curves hold for any value of n. Each of them plots the combinations of (p, q) for which the pivotal stockholder i is indifferent between ω_e and $\omega_e + 1$. If the specific curve represents a value of $\omega_e > \frac{n-1}{2}$, then it has no meaning since this condition violates the concept that i is a pivotal stockholder. In that sense, these curves represent the combinations of (p, q) for which ω_e is a sufficient and necessary for the non-expert pivotal stockholder to be indifferent between voting or delegating. Any (p, q) above this specific curve would lead i to delegate their vote, and any (p, q) below it would lead the i for the pivotal stockholder to prefer to delegate. I observe that, as ω_e grows, i's indifference curve moves closer to the lines $p = 0.5$ and $q = 1$, the limits of the region of feasible (p, q) to the left and to the top. This shows again how the higher the number of stockholders delegating gets, the lower the probability that the pivotal stockholder will prefer to delegate.

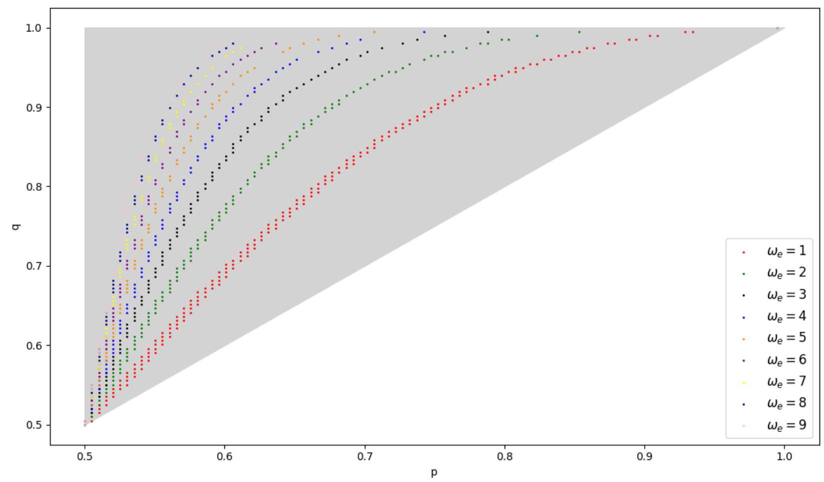

Figure 4: Pivotal Stockholder's Indifference Curves for $0 < \omega_e \leq 9$

6 Discussion

6.1 An Explanation to Over-Delegation

A main finding from the two experiments conducted by Campbell *et al.* 2022 is that participants in Liquid Democracy tend to delegate their votes more frequently than the optimal amount of delegation. The authors use this fact to explain why outcomes from majority voting rule and majority voting with abstention outperform decisions made through Liquid Democracy. However, they do not provide a compelling explanation for why this phenomenon occurs. My work may offer such an explanation.

The insight that the pivotal stockholder's calculus between voting and delegating depends on the probability that the expert stockholder is correct conditional on the fact that they are pivotal suggests a reason why this bias towards delegating may be happening. Humans are limited in their abilities to recognize and use probability appropriately in their every day life. It is very plausible that when deciding whether to vote or to delegate, participants in the Liquid Democracy experiment made their decisions based only on their beliefs about the difference between the face value competence of other stockholders and the face value of their own competence.

6.2 Coordination Problems

From Figure 4, notice that a coordination problem arises given a proxy-voting setting. Suppose that all stockholders are aware of the distribution of competence levels among the population. Each of them can use Figure 4 to identify the optimal ω_e that the expert should have. All stockholders are commonly interested in giving the expert that exact amount of votes, so what's the problem? There still exists uncertainty about which stockholders should delegate and which should vote. If they are not allowed to communicate, then some coordination strategy must be adopted to reach the optimal at equilibrium.

A possible symmetric equilibrium would be for all stockholders to choose to delegate with probability $\frac{\omega_e}{n}$. No stockholders would be interested in deviating from this strategy and, for a large enough population, the amount of stockholders delegating would approach the optimal value very closely.

Another solution that would reach the optimal at equilibrium would for ω_e stockholders to always delegate and $n - \omega_e$ to always vote. Again, no stockholders would have the incentive to move away from their own strategy. This asymmetric equilibrium describes a solution to the coordination problems through the specialization of stockholders. Naturally, the asymmetric equilibrium would always outperform the symmetric equilibrium since the former attains the optimal ω_e at every n, whereas the latter approaches the optimal value of ω_e, but never attains it exactly.

7 Conclusion

In this paper I explore the dynamics of proxy voting through a corollary of the Condorcet Jury Theorem, offering a fresh perspective on the strategic calculus of pivotal stockholders. I go into the intricacies of the choice stockholders face between voting independently or delegating their vote to a more informed peer. This decision-making process is analyzed within a common interest model that incorporates the varying competencies of stockholders, the influence of an expert stockholder, and the collective goal of reaching an accurate majority decision.

My analysis reveals that the decision of a pivotal stockholder to delegate or vote directly hinges on the interplay between their own competence, the competence of the expert stockholder, and the distribution of delegated votes. The findings suggest that as the size of the voting body increases, the optimal share of votes for the expert stockholder diminishes, highlighting a potential limitation in the scalability of proxy voting systems for large electorates. Furthermore, I address the phenomenon of over-delegation observed in empirical studies of proxy voting, proposing that a failure to account for conditional probabilities may lead stockholders to delegate more often than is theoretically optimal.

My discussion extends to the coordination challenges inherent to proxy voting, contrasting symmetric and asymmetric equilibria. While a symmetric strategy, where stockholders delegate with a certain probability, may approach optimality in large populations, an asymmetric strategy, predicated on a fixed division of stockholders into delegates and direct representation, could more reliably achieve optimal outcomes. By elucidating the strategic considerations of pivotal stockholders and the coordination dilemmas posed by proxy voting, I provide valuable insights into the potential and limitations of this voting system.

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