

Lending Rate Caps in Emerging Markets: Good for Growth? *

Gustavo Joaquim [†]
gustavo.joaquim@bos.frb.org
FRB of Boston

Damiano Sandri
dsandri@imf.org
IMF

October 2, 2020

Abstract

In many emerging markets, governments try to increase credit access and stimulate economic growth by imposing caps on lending rates. We analyze these policies by extending workhorse models with financial frictions to include a banking sector with market power. Caps are beneficial as they reduce credit costs but are also harmful as they crowd out risky borrowers which can access credit only at high interest rates, and thus have an ambiguous effect in current output and capital accumulation. To prevent crowding out of risky borrowers, in some emerging markets banks are permitted to charge uncapped rates on a share of their loans. This allows banks to service risky borrowers but generates capital misallocation since banks provide capped loans to less productive borrowers, while charging higher rates to more productive ones. In a calibrated version of the model, we show that the optimal policy to maximize steady state welfare involves relatively high caps on a large share of bank loans. The optimal policy decreases output today, but increases capital accumulation through a lower cost of credit and thus output in the future. The model also reveals that caps may have a perverse effect of reinforcing market power in the banking sector since they may force less profitable banks to exit the market. Thanks to tractable aggregation properties, the framework can be used to analyze a broad set of alternative credit policies.

JEL Codes: E20, G21, O16

Keywords: Financial frictions, banks, credit policies, market power

*We thank Giovanni Dell’Ariccia, Alp Simsek, Antonio Spilimbergo, Robert Townsend, Bernardus van Doornik and seminar participants at the IMF and MIT for very helpful comments. **The views expressed in the paper are those of the authors and do not necessarily represent the views of the IMF, its Executive Board, or IMF management. The views also do not indicate concurrence by the Federal Reserve Bank of Boston, the principals of the Board of Governors, or the Federal Reserve System..**

[†]I thank the Alfred P. Sloan Foundation and the Macro Financial Modeling (MFM) project for the support of my research.

1 Introduction

In many emerging markets, governments try to increase credit access and stimulate economic growth by imposing caps on lending rates. For example, [Ferrari, Masetti and Ren \(2018\)](#) find that out of sample of 69 non-advanced economies, 51 countries impose restrictions on lending rates.¹ These restrictions are generally adopted with the intent to stimulate credit often driven by the idea that the banks charge unfairly high lending rates. Are these restrictions warranted or can they generate severe unintended consequences? More specifically, what are the effects on the level and allocation of credit? We address these questions by extending workhorse models with financial frictions in the corporate sector to include market power in banking.

We build on a growing literature that analyzes the effects of limited credit access on investment and productivity.² These models feature financial frictions that limit the extent to which entrepreneurs can raise external finance, leading to under-investment and capital misallocation. Several studies have already used these models to analyze various policy measures to ease financial frictions and speed up economic development. For example, [Buera and Shin \(2013\)](#) show that credit subsidies to productive firms can be beneficial in the short-run, but can become counter-productive if they stay in place for too long. [Song, Storesletten and Zilibotti \(2011\)](#) consider a broad range of policies used in China, among which interest rate policies and controls on deposit and lending rates. Finally, [Itskhoki and Moll \(2018\)](#) show that pro-business policies, including credit subsidies, are optimal at an early stage of development. By boosting profits, these policies allow entrepreneurs to increase investment through higher retained earnings in the corporate sector.

The literature provides, however, limited insights on the effects of lending rate caps. First, it generally considers credit subsidies that are financed through taxation, while lending rate caps impose costs on the banking sector in terms of foregone profitability. Second, most papers, such as [Itskhoki and Moll \(2019\)](#), assume that credit subsidies apply equally to all firms or are granted to specific firms by public authorities, contrary to existent empirical evidence. For example, using individual firm-loan data from the Brazilian credit registry, [Bonomo, Brito and Martins \(2015\)](#) find that subsidized loans are provided mostly to larger, older, and less risky firms. Similarly, [Safavian and Zia \(2018\)](#)

¹The study documents an extensive use of lending rate caps even in advanced economies, but these are generally set at non-binding levels in the context of anti-usury laws.

²This literature includes [Banerjee and Newman \(1993\)](#), [Galor and Zeira \(1993\)](#), [Aghion and Bolton \(1997\)](#), [Cagetti and De Nardi \(2006\)](#), [Jeong and Townsend \(2007\)](#), [Buera, Kaboski and Shin \(2011\)](#), [Song, Storesletten and Zilibotti \(2011\)](#), [Buera and Shin \(2013\)](#), [Sandri \(2014\)](#), [Midrigan and Xu \(2014\)](#), and [Moll \(2014\)](#).

show that the introduction of interest rate caps in Kenya in 2016 led to a shift in lending away from small and medium firms towards safer corporate borrowers using both bank-level panel data and surveys conducted with bank managers. We bring together the micro evidence of the effect of lending caps of [Bonomo, Brito and Martins \(2015\)](#) and others ³ to a macro, dynamic, general equilibrium model to evaluate the effect of lending caps from an aggregate perspective.

To properly capture the effects of lending rate caps and, more generally, any credit policy on credit allocation, we extend workhorse models of financial frictions in the corporate sector. Our model features three main economic agents: managers, banks and households. As our focus is on the credit allocation across firms and the role of the banking sector, we allow for a rich heterogeneity in the corporate sector and consider a simple household block for our economy. Managers control firms that combine capital and labor to produce output, and use external finance to scale up their production subject to a collateral constraint. Managers are heterogeneous in three dimensions: collateralizable capital, productivity and risk, and obtain external finance from the banking sector, which can exercise some market power and can strategically choose which firms to allocate subsidized or cheaper credit to. The financial sector chooses how much credit to provide and at what lending rate for each firm type subject to the credit policy defined by the government.

Given a bank lending policy, managers obtain external finance, produce and then save a share of their profits to be used as collateralizable capital in the future and redistribute the rest in dividends to a representative household. In our main results, we assume that bank market power does not affect the volume of credit statically, but only the cost of finance, as there is some theoretical and empirical ambiguity in the effect of bank competition on credit provision (e.g. see [Petersen and Rajan \(1995\)](#), [Carlson, Correia and Luck \(2019\)](#)) but we relax this assumption in our analytical and numerical results. Finally, we close the model with a representative household that inelastically supplies labor to the managers and solves a standard unconstrained savings-consumption problem.

Our analysis of the model is given in three steps. First, we assume that there is a generic credit policy in place that heterogeneously affects the leverage and profit level of each type of manager in our economy. We show that under broad conditions, we can compute the effect of these policies on aggregate output (or representative household's welfare) through a set of sufficient statistics. The average productivity in the economy where

³For the effect of lending caps in consumer loans in Chile, see [Cuesta and Sepulveda \(2018\)](#). For the effect of usury laws: [Temin and Voth \(2008\)](#), [Benmelech and Moskowitz \(2010\)](#), [Zinman \(2010\)](#), [Rigbi \(2013\)](#), [Melzer and Schroeder \(2017\)](#) or, more generally, price regulations in financial products: [Agarwal et al. \(2014\)](#), [Benetton \(2018\)](#), [Nelson \(2017\)](#).

each manager's productivity level is weighted by their leverage is a sufficient statistic for the effect of a generic credit policy on output. The intuition behind this result is that the economy is better off in the aggregate when capital is allocated at productive managers. Moreover, to maximize output in the steady state, an optimal credit policy maximizes the difference between the productivity weighted leverage with the average cost of finance in the economy. A dynamically optimal credit policy must balance the static effects (discussed on the previous paragraph) with an increase the ability of the financially constrained corporate sector to increase its savings and thus produce more in the future through a reduction in the cost of finance, as in [Itskhoki and Moll \(2018\)](#). Together, these two results provide a framework for academics and policy makers alike to understand the effect of any credit policy that affects firms heterogeneously.

Second, we use our analytical results on a generic credit policy to analyze the effect of lending caps. We start by considering the particular case in which lending caps apply to all loans, in line for example with the regulations adopted by Kenya in 2016. Consistent with the micro evidence in [Cuesta and Sepulveda \(2018\)](#) and others, a cap on all loans in our model has two effects. First, it excludes risky managers from the credit market. As banks cannot be compensated to lend for risky managers, they do not offer loans above a given level of riskiness. Second, it reduces the interest rate for safer managers. Statically, only the former channel affects output, which implies that no cap (i.e., laissez-faire) is the output maximizing policy in the short run. Dynamically, however, the reduction in cost of finance from the cap is significant and the optimal policy is to implement a cap that is binding (that is, lower than the rate that would be charged for a manager absent the cap). A tighter policy (lower cap) is optimal for larger levels of market power in banks in the baseline.

Given that a lending cap on all loans excludes risky managers from the credit market, some governments regulate the banking sector by imposing a lending cap on a proportion of the banks' lending portfolio. This is the approach followed for example by the Brazilian authorities that require banks to lend at least a share of their balance sheets at low rates ([Santos \(2016\)](#)). By imposing lending caps only on a fraction of the loan portfolio, the idea is that the policy is more efficient since it allows banks more flexibility to charge high rates on some borrowers, while still providing lower rates for others. This policy reduces the problem of excluding risky managers from credit markets that a cap on all loan implies. Under a cap on a share of loans, only high risk-low productivity managers are excluded. However, the downside is that a cap on a share of loans induces the banking sector to create a capital misallocation. Banks charge less for low productivity managers below the cap, and more to those above the cap, which tend to be more produc-

tive, that is: a cap on share of loans introduces a cross subsidy in credit markets where high productivity managers subsidize low productivity managers.

Third, we calibrate our model to the Brazilian economy to understand the quantitative effect of lending caps on productivity and growth. We choose Brazil as around 50% of all credit in Brazil is subsidized or provided through government programs, and 86% of it is subject to some form of restriction on interest rates ([Santos, 2016](#)). Our calibration exercise provides two valuable lessons. First, cross-subsidy and subsequent misallocation from imposing a cap on a share of loans implies that the optimal policy is to set a looser cap on all loans, even if some risk managers are excluded from the credit market. This indicates that the misallocation generated by the banking sector in the previous paragraph is quantitatively important. Second, we show that the optimal policy can recover around 18% of the output in steady state difference between laissez-Faire and perfect competition.

We also evaluate a potential feedback channel to the policy to the level of bank market power in the economy. A policy that introduces a constraint in the pricing decisions of the financial sector limits its profitability, which could in turn encourage exit and less competition dynamically. We introduce this feedback channel in our model and show that if competition in the bank sector is sensitive enough to the credit policy, there is no role for lending caps to increase growth. Finally, we consider alternative policies, such as cap on interest rate income (after default is taken into account) and show that a calibrated interest rate income is Pareto improving over a lending cap. Under a cap on interest income, banks can still lend to risky managers as the cap is only the repayments received by the bank. This mitigates the negative side of a lending cap and can increase welfare for all agents in the economy compared to the optimal cap.

This paper is structured as follows. In [Section 2](#) we present the setting of our model and show the aggregate effect of a generic credit policy. In [Section 3](#), we explore specifically the effect of lending caps on all and only on a share of loans. [Section 4](#) provides a quantitative evaluation on a calibrated version of our model.

2 Aggregate Effects of Credit Policies

In this section we present the setting of our model and what are the aggregate effects of credit policies. We set our model in continuous time. Our economy is characterized by three types of agents. First, there are managers, which are heterogeneous in their productivity, access to collateralizable capital and safety. Managers choose labor and capital subject to a financial constraint that capital in production is limited by collateralizable

capital. Managers in our model produce like the entrepreneurs in [Itskhoki and Moll \(2018\)](#), but are risk neutral as in [Bernanke, Gertler and Gilchrist \(1999\)](#). The addition of the heterogeneity in safety is motivated by the data, as there is evidence that safer, more productive firms are the ones contemplated with subsidized loans ([Bonomo, Brito and Martins \(2015\)](#)). Managers exogenously save a share of the profits of production. Second, there is a financial sector. The financial sector in our model is composed by one representative bank that faces a downward sloping, constant elasticity demand for bank credit. We show that the elasticity of demand captures well the idea of market share and is equivalent to a Cournot model without policy interventions. The representative bank offers leverage to managers, but it charges them to do so. Banks are not subject to any macro-prudential or balance sheet regulations, and make their lending decisions statically. We close our economy using a representative worker that supplies labor inelastically, owns all firms in the economy and partially owns banks. The representative worker can save/borrow between periods.

In this framework, we characterize how a general credit policy affects total capital, output and aggregate wealth of the economy over time. Our result is close in spirit to [Itskhoki and Moll \(2018\)](#), but we focus not only on different policies, but also on the an empirically consistent *heterogeneous* exposition each firm has to credit policies, while [Itskhoki and Moll \(2018\)](#) explores only the inter-temporal distortions in the economy. We demonstrate that statically, the planner wants to maximize the total leverage plus the covariance of leverage and productivity. Our previous analysis on total credit will speak to the first term, while now we also have to take into account the second term, which is the misallocation in this economy. In the steady state, however, the planner also want to minimize how much productive agents pay for financing. In particular, to guarantee a faster accumulation process for capital, it is important that the most productive agents not only leverage up to produce more given capital, but that they are also the ones that pay relatively less for capital. The optimal policy must be a combination of the static policy with a term that captures the covariance of productivity and relative cost of finance between different productivity levels.

2.1 Setting

Managers. Our economy is populated by managers, which take input decisions for firms. Managers are heterogeneous in their productivity, z , and their access to collateralizable capital, a . Differently from [Buera and Shin \(2013\)](#), [Moll \(2014\)](#), [Itskhoki and Moll \(2018\)](#), and other models in the macro development literature, we assume managers are also het-

erogeneous in σ , their riskiness. In particular, we assume that each manager production function assumes a CRS Cobb-Douglas form, as in Eq.(1) ⁴

$$A\vartheta\{(zk)^\alpha n^{1-\alpha}\}, \quad \alpha \in (0, 1) \quad (1)$$

where ϑ is distributed with c.d.f. F_ϑ , and A is the aggregate deterministic productivity. In particular, we make the following simplifying assumption for the distribution F_ϑ

$$\vartheta = \begin{cases} 1 + \sigma, & \text{with probability } \frac{1}{1+\sigma} \\ 0, & \text{with probability } \frac{\sigma}{1+\sigma} \end{cases} \quad (2)$$

that is, either the project is successful and the outcome is multiplied by $1 + \sigma$ or the project fails and generates zero output. The stock of capital is not affected if the project fails. Managers are risk neutral with respect to the project risk ϑ . With this particular distribution of F_ϑ , we have that $E(\vartheta) = 1$ and $Var(\vartheta) = \sigma$.

Managers face a contracting friction of $k \leq \lambda a$, with $\lambda > 1$. In this project, we do not attempt to model this constraint. ⁵ We assume that the posted collateral is to avoid default on the capital stock - which does not happen in equilibrium. Managers can only default on income generated by capital (when the project fails), which is completely non-recoverable. Let F_h denote the distribution of $h \in \{z, \sigma, a\}$, i.i.d. across characteristics. ⁶ Given an interest rate r^l on a loan, wages w , collateralizable capital a and a productivity risky profile $\theta \equiv \{z, \sigma\}$, the *expected* production profit of a manager is given by Eq. (3)

$$\pi^e(r^l, w|a, \theta) \equiv \max_{k \leq \lambda a, n} \int [A\vartheta\{(zk)^\alpha n^{1-\alpha}\}^\nu - r^l k - wn] dF_\vartheta(\vartheta) \quad (3)$$

Define also the self financing profits of a manager, i.e., when $k \leq a$ as in Eq. (4)

$$\hat{\pi}^e(r, w|a, \theta) \equiv \max_{k \leq a, n} \int [A\vartheta\{(zk)^\alpha n^{1-\alpha}\}^\nu - rk - wn] dF_\vartheta(\vartheta) \quad (4)$$

The solution to the problem of each manager in terms of input choices is given by (all

⁴ As will be seen, the CRS case together with the linearity in the financial constraint allows for aggregation in this model. The marginal value of capital in this case is the same for all types within a given level of productivity. This means that there is no micro distortion from different firms facing different levels of interest rate. In a more general mode, as in our quantitative section, we allow for decreasing returns to scale, where micro distortions from different MPK 's interact with the macro distortions of this section.

⁵ This could be motivated by a loan-to-value restriction, a moral hazard problem or other contracting constraints. As discussed in Moll (2014), what is crucial for our results is that the financial constraint is linear in a .

⁶ We consider a model with persistence in our quantitative exercise.

derivations are in Appendix B):

$$k(a, \theta) = \begin{cases} \lambda a, & z \geq \frac{r^l}{\kappa(1+\sigma)} \\ 0, & z < \frac{r^l}{\kappa(1+\sigma)} \end{cases} \quad \text{and } l(a, \theta) = \left[\frac{1-\alpha}{w} \right]^{1/\alpha} z k(a, \theta) \quad (5)$$

Expected profits are given by Eq.(6).

$$\pi^e(r^l, w|a, \theta) = a \int \max\{\vartheta z \kappa - r^l, 0\} dF_\vartheta(\vartheta) = \lambda a \max\left\{z \kappa - \frac{r^l}{1+\sigma}, 0\right\} \quad (6)$$

where $\kappa(w) \equiv \alpha \left[\frac{1-\alpha}{w} \right]^{\frac{1-\alpha}{\alpha}}$. The marginal value of capital is always $z\kappa$ and, thus, there is no loss of redistributing capital within the same productivity type, z . In the case of self-finance:

$$\hat{\pi}^e(r, w|a, \theta) = a \max\{z\kappa - r, 0\} \quad (7)$$

Note that Eq. (6)-(7) are linear in collateralizable capital a . This comes from the combination from CRS and a linear constraint $k \leq \lambda a$ in collateralizable capital.

To facilitate the exposition, we propose the following change of variables: instead of focusing on interest rates charged by banks, we focus on end-of-period profits for the firms. This is consistent with the industrial organization literature that focuses on utility in models of discrete choice (Berry, Levinsohn and Pakes (1995)), since the end of period profits is what matters for managers in the production process. Note that there is a one-to-one relationship between them in our model, as shown in Eq.(6). Define the following interest rate function, $r^{pc}(\sigma)$ as the one that makes the bank indifferent (i.e., have zero expected profits) to lending to a type σ as in Eq. (8). This is the rate that would prevail under perfect competition in our economy.

$$\frac{1}{1+\sigma} r^{pc}(\sigma) - r = 0 \Rightarrow r^{pc}(\sigma) = (1+\sigma)r \quad (8)$$

The interest rate required is the deposit rate in the economy, r , times a risk premium based on σ . Given a level of interest rates r^l , the representative bank's expected profits in a given loan are given by

$$\begin{aligned} \lambda a \left[\frac{1}{1+\sigma} r^l - r \right] &= \pi^e(r^{pc}(\sigma), w|a, \theta) - \pi^e(r^l, w|a, \theta) \\ &= a \left\{ \pi^e(r^{pc}(\sigma), w|1, \theta) - \pi^e(r^l, w|1, \theta) \right\} \end{aligned} \quad (9)$$

Note that the linearity in a and production profits translates in our model to a linearity

in bank profits on collateralizable capital a . Therefore, the solution that follows is the same for all levels of wealth and we simply omit it from the notation. The profit of the financial sector is given by Eq. (10). It is the outcome of Eq. (9) multiplied by the downward sloping demand, \mathcal{D} . The demand is parametrized by μ , the market power, as becomes clear later. For each level θ , the bank must choose a level of end of period profits $\pi^w(\theta)$ that maximizes Eq.(10). We assume the bank observes θ , the problem for each θ can be written separately.

$$\pi^{w,*}(\theta, \mu) \equiv \arg \max_{\pi} [\pi^e(r(\sigma)|\theta) - \pi] \mathcal{D}(\pi | \theta, \mu) \quad (10)$$

where we assume the demand function, \mathcal{D} , has the following constant elasticity functional form as in Eq. (11)

$$\mathcal{D}(\pi | \theta, \mu) = \Gamma(\mu) \left[\frac{\pi - \hat{\pi}^e(r, w|\theta)}{\pi^e(r(\sigma), w|\theta) - \hat{\pi}^e(r, w|\theta)} \right]^{\frac{1}{\mu}-1} \quad (11)$$

A few comments on this functional form are in order. First, the second term in \mathcal{D} is always between zero and one, i.e., represents a share. The share of entrepreneurs of type θ that uses the financial sector services in increasing in the end of period wealth offered by banks with respect to their outside option (financial autarky), over the difference between perfect competition and financial autarky. Moreover, we place a function of market power μ , $\Gamma(\mu)$ controlling the scale of demand for the representative bank. We do so to keep the effect market power has on access to credit from a single bank as a general function, since there is no causal robust empirical result on the sign of this relation ⁷. Note that in this formulation of the model, managers borrow as much as they can in the intensive margin (up until the leverage constraint), but the demand is downward sloping in the extensive margin. This can be microfounded with heterogeneous preferences or costs over using bank services, see for instance [Joaquim, Townsend and Zhorin \(2019\)](#) that develop a spatial model of competition among financial service providers. It is also consistent with previous work on the field, as [Rice and Strahan \(2010\)](#), which concludes that small firms are more likely to borrow following the US economy banking deregulation episode, but that lending volume does not change per loan. Specifically for Brazil, [Joaquim and Doornik \(2019\)](#) show that the number of loans increases by approximately 2.5 % with a reduction of .1 in HHI in a given market in Brazil, while the average loan does not significantly change.

Market power, μ , pins down the curvature of this demand. If μ is higher, the elasticity

⁷See, for instance, [Petersen and Rajan \(1995\)](#), [Zarutskie \(2006\)](#), [Rice and Strahan \(2010\)](#) and others.

of demand is lower (more inelastic), which means the representative bank can extract more surplus from the intermediation gains.

$$\frac{dD}{d\pi} \frac{[\pi - \hat{\pi}^e(r, w|\theta)]}{D} = \frac{1}{\mu} - 1$$

This interpretation is clear when we consider the solution to the unconstrained bank problem, i.e., without any credit policy, in Lemma 1 (proofs of all Lemmas in Appendix C). Note that market power is the weight in a convex combination between perfect competition and autarky outcomes (Eq.(13)). If market power is higher, end of period profit of managers are closer to autarky outcomes. A different way to see this is to consider the total gains from intermediation by unit of collateralizable capital (which is not a function of a) as in Eq.(12)

$$g(\theta) \equiv a^{-1} [\pi^e(r^{pc}(\sigma), w|\theta) - \hat{\pi}^e(r, w|\theta)] \quad (12)$$

Market power, μ pins down how much of the intermediation gains stay with the representative bank and how much goes for agents, as Eqs.(13)-(14). This result is more general than our model. With the demand function proposed in Eq.(11), does not matter when intermediation gains come from, they are divided between banks and managers as in Lemma 1.

Lemma 1. Unconstrained Bank Problem. *The optimal end of period profits offered by the bank to managers are*

$$\begin{aligned} \pi^{w,*}(\theta, \mu) &= (1 - \mu)\pi^e(r^{pc}(\sigma), w|\theta) + \mu\hat{\pi}^e(r, w|\theta) \\ &= (1 - \mu)g(\theta) + \hat{\pi}^e(r, w|\theta) \end{aligned} \quad (13)$$

and bank profits by each loan offered are

$$\pi^b(\pi^{w,*}(\theta, \mu)) = \mu g(\theta) \quad (14)$$

In Appendix B.2, we show that the outcome in Lemma 1 is equivalent to a Cournot model with $\mu = \frac{1}{B+1}$, where B is the number of banks, that is: for more banks in a market, the market power of each is smaller. We opt for the representative bank for simplicity when we do introduce credit policies.

Workers, Dynamics and Equilibrium. We now present the dynamic choices and behavior of agents in our economy and close the model with the representative worker.

We assume that z is Pareto distributed with parameter η_z , that is $F_z(z) = 1 - z^{-\eta_z}$, i.i.d. over time and independent of collateralizable capital and risk. We assume that collat-

eralizable capital pays a rental rate of r , but depreciates at a rate δ . At the end of each period, the manager and banks distributes a share ∇ of their profits for workers in the form of dividends. For notation purposes, let \mathcal{X} denote aggregate collateralizable capital of all managers in the economy

$$\mathcal{X} \equiv \int a dF_a(a) \quad (15)$$

where we simply integrated over a as θ, a are independent (as θ is i.i.d. over time).

For simplicity, we assume that there is a representative worker, who supplies labor inelastically (normalized to one), and solves a consumption-savings problem as in Eq. (17). As we are not concerned with the redistribute effect of lending caps (but simply their aggregate effects), and we assume the worker receives the same share of profits (as dividends) from managers and the financial sector, its income \mathcal{W} is proportional to output \mathcal{Y} and given by

$$\mathcal{W} = \alpha_W \mathcal{Y}, \quad \alpha_W = (1 - \alpha) + \alpha \nabla \quad (16)$$

The planner problem to maximize welfare of the representative can be then written as simply maximizing output. The representative's worker problem is

$$\max_{c(t)} \int e^{-\rho t} u(c(t)) dt \quad (17)$$

s.t.

$$c(t) + \dot{b} = rb + \mathcal{W} = rb + \alpha_W \mathcal{Y} \quad (18)$$

We focus here on a fixed interest rate r , given exogenously⁸. To guarantee the existence of a finite steady-state and a simple solution to the workers problem, we assume $\delta > r = \rho$. In this case, workers consume a fixed level c^w , since their FOC is given by

$$\frac{\dot{u}_c}{u_c} = \rho - r = 0$$

Therefore, an equilibrium is an interest rate r , and a path for wages, $w(t)$, such that given wages and interest rate, the representative bank offers contracts as in Eq.(13), the representative worker consumes c^w and the labor market clears, that is

$$\int_{a,z,\sigma} l(a, \theta) dF_a(a) dF_z(z) dF_\sigma(\sigma) = 1 \quad (19)$$

where $l(a, \theta)$ is given by Eq.(5).

⁸As in [Itskhoki and Moll \(2018\)](#), it is possible to extend the analysis to a closed economy, where capital demand and supply must be equalized.

2.2 Aggregation Results

We explore now the aggregate effects of a credit policy. We first define the new notation and show a few basic results in this setting. We then prove our the aggregation results.

Let τ denote a generic credit policy. Define $\lambda(z | w, \tau)$ as in Eq.(20)⁹

$$\lambda(z | w, \tau) \equiv \int_{\sigma} (\lambda - 1) \iota(\sigma, z | w, \tau) dF_{\sigma}(\sigma) + 1 \quad (20)$$

where $\iota(\sigma, z | w, \tau)$ is the share of entrepreneurs of type σ, z that participate in intermediation. Eq. (20) is the average leverage of agents of a given level of productivity. Note that our model provides a micro based structure for $\lambda(z | w, \tau)$ through a non-competitive banking sector and credit policy. However, the results presented in this section would apply in a very general settings. We could have simply started solving the model saying that each type z can leverage up to $\lambda(z | w, \tau)$ without defining the role of the financial sector and still derive all of our results. The extra level of heterogeneity, which here we consider idiosyncratic risk σ , could be very general. One particular case of interest is that it could political connections that impact the probability of receiving subsidized credit and, thus, would affect sector leverage, as in [Lazzarini et al. \(2015\)](#). Equivalently, define the analogous $r(z | w, \tau)$ as the average cost per unit of capital of entrepreneurs in a given level of productivity.

We assume that no loans are made with interest rates below r .¹⁰ Define the following cut-off level \underline{z}

$$\underline{z} \equiv \kappa(w)^{-1} r \quad (21)$$

The productivity level \underline{z} is the manager that is indifferent between producing or not if it faces the rate d and is completely safe. Once we define \underline{z} as in Eq. (21), we can re-define the $\lambda(\cdot), r(\cdot)$ functions as functions of τ and \underline{z} , that is

$$\lambda(z | w, \tau) = \lambda(z | w(\underline{z}), \tau)$$

where $w(\underline{z})$ is the one that solve Eq. (21) for a given \underline{z} . In what follows, we simply write

⁹We use here our assumption that σ, z are independent. We could extend our analytical analysis relaxing this assumption as long as we put some structure in the dependence. For instance, one case where everything goes through is when z changes the distribution of σ as

$$F_{\sigma|z}(\sigma | z) = 1 - (1 + \sigma)^{\eta_{\sigma} + \eta_{z\sigma} \log(z)}$$

¹⁰We show a model that relaxes this assumption in Appendix E.

for simplicity¹¹

$$\lambda(z | \underline{z}, \tau)$$

Let $\tilde{z} \equiv \frac{z}{\underline{z}}$. The key assumption of this section is that $\lambda(z | \underline{z}, \tau)$ satisfies

$$\lambda(z | \underline{z}, \tau) = \tilde{\lambda}(\tilde{z} | \tau) \quad (22)$$

and the same for $r(z | w, \tau)$. In words, this means we can write it all the relevant information in terms of ratios with respect to the level \underline{z} . This assumption, coupled with the Pareto distribution for z , is what is able to generate our results. One case where this assumption is satisfied is if $\iota(\cdot)$ depends on z and w via gains from intermediation. We can re-write the gains from intermediation as a function of \tilde{z}

$$g(z, w) = (\lambda - 1)[z\kappa(w) - r] = (\lambda - 1)d \left[\frac{z}{\underline{z}} - \frac{r}{d} \right] \equiv g(\tilde{z}) \quad (23)$$

Define the quantities in Eqs.(24)-(26). Economically, θ_λ captures the average leverage of a firm in the economy *conditional* on producing, while ζ_λ is the productivity weighted leverage. As will be clear later, they are very related to output and TFP in our economy.

$$\theta_\lambda(\tau) \equiv \mathbb{E} \left[\tilde{\lambda}(\tilde{z} | \tau) | \tilde{z} > 1 \right] \quad (24)$$

$$\zeta_\lambda(\tau) \equiv \mathbb{E} \left[\tilde{\lambda}(\tilde{z} | \tau) \tilde{z} | \tilde{z} > 1 \right] \quad (25)$$

$$\zeta_r(\tau) \equiv \mathbb{E} \left[\tilde{r}(\tilde{z} | \tau) \tilde{\lambda}(\tilde{z} | \tau) | \tilde{z} > 1 \right] \quad (26)$$

Analytically, Eq.(22) guarantees that $\{\theta_\lambda(\tau), \zeta_\lambda(\tau), \zeta_r(\tau)\}$ are finite and not a function of \tilde{z} , that is, we can separate our analysis in \tilde{z} (and wages) and in the aggregates θ_λ , ζ_λ , and ζ_r (for a mathematical proof of this result, see Appendix B.3)

We start by aggregating over the input choices for managers from Eq.(5). Let \mathcal{Y} denote aggregate output. As in Moll (2014), we can write the an aggregate production function for the economy that has the same form of each firm individual function, with a TFP that captures the misallocation term of capital due to financial frictions and access to credit in our economy, that is, Managers' input choices imply (Appendix B.4)

$$\mathcal{Y} = \mathcal{Z} \mathcal{K}^\alpha \mathcal{L}^{1-\alpha} \quad (27)$$

where Z is the endogenous TFP, given by $\mathcal{Z} \equiv A \left(\frac{\zeta_\lambda(\tau)}{\theta_\lambda(\tau)} \underline{z} \right)^\alpha$ and \mathcal{K} and \mathcal{L} is the aggregate

¹¹Although writing both functions as λ is abuse of notation, we prefer to do it to avoid introducing new notation.

capital and demand for labor, respectively. Moreover, aggregate capital is given by

$$\mathcal{K} = \theta_\lambda(\tau) \mathcal{X} \underline{z}^{-\eta} \quad (28)$$

More useful for our characterizations, however, is Lemma 2. This is our extension of Lemma 2 in [Itskhoki and Moll \(2018\)](#). The idea is that in a given period \mathcal{X} is given. Therefore, we can re-compute the values of aggregate capital \mathcal{K} , the cut-off participation type \underline{z} , and equilibrium wages w as a function of \mathcal{X} .

Lemma 2. *Manager's input choices and the labor market clearing condition imply that*

$$\mathcal{Y} = \mathcal{A}[\zeta_\lambda(\tau) \mathcal{X}]^\gamma \quad (29)$$

where $\mathcal{A} = \left(\frac{\alpha A}{r}\right)^{\gamma\eta/\alpha}$ and $\gamma \equiv \frac{\alpha/\eta}{\alpha/\eta + (1-\alpha)}$. Wages w , production profits of managers Π^M , and of the representative bank Π^B are respectively given by

$$w = (1-\alpha)\mathcal{Y}, \quad \Pi^M = \alpha \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)}\right] \mathcal{Y} \quad \text{and} \quad \Pi^B = \alpha \left[\frac{\zeta_r(\tau) - \theta_\lambda(\tau)}{\zeta_\lambda(\tau)}\right] \mathcal{Y} \quad (30)$$

From the results of Lemma 2, we can infer the effects of the credit policy statically in the representative worker end of period wealth. We can also solve for the dynamics of \mathcal{X} , which given Lemma 2, characterize the equilibrium in the economy. In particular, we can write Eq.(31) as the evolution of collateralizable capital in the corporate sector

$$\dot{\mathcal{X}} = \hat{\alpha} \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)}\right] \mathcal{Y} + (r - \delta) \mathcal{X} \quad (31)$$

where $\hat{\alpha} = \alpha_W - (1-\alpha)$ represents the non-wage share of output workers receive, and the steady state level of collateralizable capital as Eq.(32)

$$\mathcal{X}^{ss} = \left\{ \frac{\hat{\alpha} \mathcal{A} \zeta_\lambda(\tau)^\gamma \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)}\right]}{\delta - r} \right\}^{\frac{1}{1-\gamma}} \quad (32)$$

which characterizes the steady level of the other aggregate outcomes using the results from Lemma 2.

Optimal Policy. We consider a Planner that is interested in maximizing the welfare of the representative worker. Our result on maximizing current and steady state worker income is in Lemma 3. From Lemma 2, we know that maximizing the income of the representative worker is equivalent to maximizing output. For that, managers must be able to

leverage up their production and those that do leverage it up by more should be the most productive ones. This is what the term in Eq. (33) is capturing: the leverage weighted productivity is what determines output.

Lemma 3. *Optimal Policy.* *Maximizing output today requires that the planner maximizes leverage weighted productivity, that is, $\zeta_\lambda(\tau)$, that is*

$$\arg \max_{\tau} \mathcal{Y} = \arg \max_{\tau} \mathbb{E}[\tilde{\lambda}(z | \tau)z] \quad (33)$$

Moreover, maximizing output in the steady state, \mathcal{Y}^{ss} , requires that the planner maximizes leverage weighted productivity and minimizes the cost of finance, that is

$$\underbrace{\mathbb{E}[\tilde{\lambda}(z | \tau)z]}_{\text{Allocation/Leverage}} - \underbrace{\mathbb{E}[\tilde{\lambda}(z | \tau)\tilde{r}(z | \tau)]}_{\text{Cost of Finance}} \quad (34)$$

To make this point more clearly, we can re-write Eq.(33) as

$$\mathbb{E}[\tilde{\lambda}(z | \tau)] + \mathbb{C}(\tilde{\lambda}(z | \tau), z) \quad (35)$$

That is, the sum of the average leverage and a correlation term - which captures the degree of misallocation in the economy (if the covariance is low, misallocation is large).

We can go beyond the result in Lemma 3 to characterize how output depends on the credit policy quantitatively. The productivity weighted leverage is a sufficient statistic in our model to determine the elasticity of output to credit policies, as indicated by Eq.(36).

$$\frac{d \ln \mathcal{Y}}{d \ln \tau} = \gamma \cdot \frac{d \ln \zeta_\lambda(\tau)}{d \ln \tau} \quad (36)$$

The first part of Lemma 3 and Eq.(36) is silent about the cost of finance: as the representative worker receives the same share of dividends from the corporate and financial sector, it does not matter how the production surplus is divided among them. In the second part of Lemma 3, we show that this is *not* the case when the planner maximizes steady state income of the representative worker. As the corporate sector is constrained, lower interest rates (i.e., a lower cost of finance) now allow for a faster capital accumulation process, which leads to a faster transition to the steady state. Note that the optimal policy is not to maximize collateralizable capital, but a combination of static allocation and cost of finance.

As in the static case, we can also show how output in the steady state quantitatively

changes with credit policies, as in Eq.(37).

$$\frac{d \ln \mathcal{Y}^{ss}}{d \ln \tau} = \frac{\gamma}{1 - \gamma} \cdot \frac{d \ln [\zeta_\lambda(\tau) - \zeta_r(\tau)]}{d \ln \tau} \quad (37)$$

In Appendix D, we explore how the optimal policy evolves over time. Intuitively, the optimal policy over time is a combination of maximizing capital accumulation and the static outcome. The relative weight given for each is the co-state, which captures the marginal value of collateralizable capital in the economy. If the co-state is large, i.e., the economy has a small level of collateralizable capital and its marginal value is large, we have that the optimal policy would focus on reducing cost of finance and increasing capital accumulation in the beginning of a development process, reverting to focus relatively more in the static policy of Lemma 3 over time.

3 Lending Rate Caps

We use our analytical results for a general credit policy from Section 2 to characterize the aggregate effects of lending rate caps. First, we consider the case with a single lending cap on all loans. Our model implies that riskier managers will be priced out of the market due to the cap, but it will also decrease the average cost of finance for those below. This is consistent with the micro evidence of Cuesta and Sepulveda (2018) to consumers in Chile. In our economy, lending caps decrease output statically by reducing the credit in the economy,¹² but help capital accumulation and thus can increase steady state output. We show that under broad assumptions, the optimal cap on all loans to maximize steady state output is binding (lower than the unconstrained rate for a strictly positive share of managers).

Second, we analyze the case of lending caps on a share of loans. Contrary to a cap on all loans, a cap on a share of loans does not price out every manager above a risk level. We show that those priced out are only those that are not productive enough to for their riskiness level to receive loans above the cap - which potentially reduces the downside of a cap on all loans. However, we show that interest rates *increase* for those above the cap - which are the high risk - high productive managers - and *decreases* for all of those below the cap (not only the managers to which the cap is binding). This implies that there is a cross-subsidy in our economy from productive to unproductive managers, which generates a misallocation in the economy.

¹²Under the assumption that bank market power does not distort credit allocation in the market. We come back to this assumption in Section 3.3.

3.1 Cap on All Loans

We analyze now the effect of an interest rate cap \bar{r} for loans made for all types θ . All loans made by the representative bank must have $r^l \leq \bar{r}$. We focus first on credit allocation and interest rates. We then apply our aggregation results from Section 2 to compute the optimal cap.

Before analyzing the aggregate effects of interest rate caps, we must compute the interest rates for economy with and without the caps. From Lemma 1, the implied interest rate for type θ is given by Eq. (38), which is increasing in risk σ and in productivity, z . Denote $r^{w,*}(\theta, \mu)$ as the interest rate consistent with end of period wealth $\pi^{w,*}(\theta, \mu)$ in Lemma 1, that is

$$r^{w,*}(\theta, \mu) \equiv (1 + \sigma) \left[r + \mu \frac{g(z)}{\lambda} \right] \quad (38)$$

which is increasing in market power, μ , productivity (through intermediation gains), z , and riskiness, σ . Note that the gains from intermediation $g(z)$ is a function only of z - and not of θ in our model, such that interest rates only depend on riskiness through the probability of repayment.

There are three cases to consider to solve now the constrained problem of the bank. If $r^{w,*}(\theta) \leq \bar{r}$, the bank charges the same interest rate. If $r^{w,*}(\theta) > \bar{r}$, then the bank offers the loan at rate \bar{r} if it makes a profit from it. For the bank to make a profit lending at \bar{r} , it must be the case that

$$\frac{1}{1 + \sigma} \bar{r} - r \geq 0 \Leftrightarrow \sigma \leq \frac{\bar{r}}{r} - 1 \equiv \bar{\sigma}$$

Thus, the bank simply adopts a cut-off strategy. If the manager has $\sigma > \bar{\sigma}$, the bank does not extend a loan. Otherwise, it lends at the minimum of \bar{r} and $r^{w,*}(\theta, \mu)$, the unconstrained rate. Lemma 4 presents this result.

Lemma 4. Interest Rate Cap on All Loans. *With an interest rate cap on all loans, \bar{r} , banks charge*

$$r^{all}(\theta, \mu) = \min \{ \bar{r}, r^{w,*}(\theta, \mu) \} \quad (39)$$

if $\sigma \leq \bar{\sigma} \equiv \frac{\bar{r}}{r} - 1$, and do not provide loans for $\sigma > \bar{\sigma}$.

Some managers will face the same interest rates than before, but some now will receive the interest rate at the cap. Let $\sigma^{all}(z, \mu, \bar{r})$ solve Eq. (40)

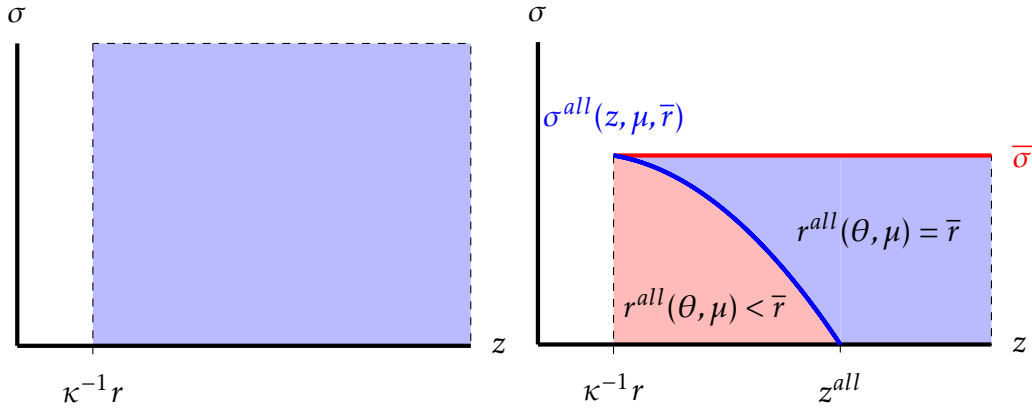
$$(1 + \sigma^{all}(z, \mu, \bar{r})) \left[r + \mu \frac{g(z)}{\lambda} \right] = \bar{r} \Rightarrow \sigma^{all}(z, \mu, \bar{r}) = \frac{\bar{r}}{r + \mu \frac{g(z)}{\lambda}} - 1 \quad (40)$$

If $\sigma \in [\sigma^{all}(z, \mu, \bar{r}), \bar{\sigma}]$ for a given z , the loan made to (z, σ) is constrained, i.e., the interest

rate on this loan is \bar{r} .

We present two figures to illustrate the differences in allocation and interest rates between the equilibrium without any policy intervention (Lemma 1) and with an interest rate caps on all loans (Lemma 4). In the unconstrained case, the loans are made for all managers above $z \geq \kappa^{-1}r$, which is the left panel of Figure 1. In the unconstrained case, the bank can be fully compensated for the idiosyncratic risk, such that it does not matter in the allocation. With a cap on all loans, however, we have that only managers with $\sigma \leq \bar{\sigma}$ receive a loan, which is the total shaded area in the right panel of Figure 1. The curve $\sigma^{all}(z, \mu, \bar{r})$ shows the border of those for which the cap is actually binding $r^{all}(\theta) = \bar{r}$.

Figure 1: Credit Allocation: Unconstrained (left) vs Cap of \bar{r} on all loans (right)

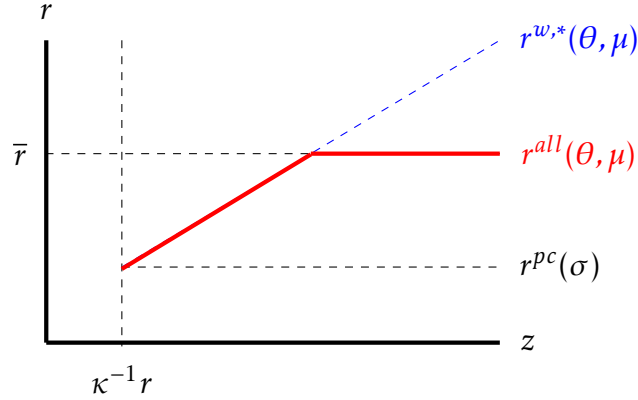


Note: Credit allocation in Laissez-faire (left) vs under the cap \bar{r} for all loans (Lemma 4). The curve $\sigma^{all}(\cdot)$ defines the set of managers for who the cap is in fact binding, as in Eq.(40).

Figure 1 shows the allocation of capital with and without an interest cap on all loans, but that is only part of the effect of the policy. In Figure 2, we show the policy effect on interest rates charged by banks. We plot interest rates as a function of z for $\sigma < \bar{\sigma}$. The blue dashed line is the unconstrained interest schedule (increasing in productivity, since intermediation gains are increasing in productivity), while the black line is the perfect competition interest rates, $r^{pc}(\sigma) = (1 + \sigma)r$. The constrained interest rate schedule, as can be seen, is given by $r^{all}(\theta, \mu) = \min \{\bar{r}, r^{w,*}(\theta, \mu)\}$. The region where the $r^{all}(\theta, \mu) < r^{w,*}(\theta, \mu)$, that is, where the cap is in fact binding, is precisely the region where the corporate sector has now access to cheaper credit and thus can increase their profits and accumulate capital faster.

Welfare. As interest rates offered do not have an effect on lending in the intensive margin (since managers scale production up to the constraint), we can write (note that we replace

Figure 2: Interest Rates: Unconstrained vs Cap of \bar{r} on all loans



Note: Interest rate in three different settings. $r^{pc}(\sigma) = (1 + \sigma)r$ is the perfect competition interest rate, $r^{w,*}(\cdot)$ is the unconstrained (laissez-faire case), as in Lemma 1, and $r^{all}(\cdot)$ is the one under the cap for all loans - from Lemma 4.

τ as \bar{r} here)

$$\iota(\sigma, z \mid w, \bar{r}) = \begin{cases} 1, & \text{if } \sigma \leq \bar{\sigma} = \frac{\bar{r}}{r} - 1 \\ 0, & \text{if } \sigma > \bar{\sigma} \end{cases} \quad (41)$$

for $z \geq \kappa^{-1}(w)r$ and zero otherwise. In this case, we have that (Appendix C.5)

$$\zeta_\lambda(\bar{r}) = \frac{\eta_z}{\eta_z - 1} [(\lambda - 1)F_\sigma(\bar{\sigma}) + 1] \quad (42)$$

From Eq.(42), it is easy to see that the policy that maximizes output today is to set $\bar{r} = +\infty$, that is, no-intervention. This is the power of Lemma 3: once we can compute ζ_λ and ζ_r for a given policy, we know the general equilibrium effects of the policy on output.

No intervention is optimal to maximize current output due to market power in our economy distorting only the distribution of aggregate output, not the level, and an intervention of $\bar{r} < \infty$ leads the representative bank to stop lending for managers with $\sigma > \bar{\sigma}$. Thus, there is no gain for the policy - only an allocative loss. This result is summarized in the first part Lemma 5.

Lemma 5. Optimal Cap on All Loans. *To maximize output \mathcal{Y} , we have that:*

1. **Statically.** *The optimal policy is to set $\bar{r} = \infty$ (no intervention).*
2. **Steady State.** *The optimal policy can be written as a multiplier of the deposit rate $\bar{r}_{SS}^*(\mu) \equiv r \cdot \tilde{r}_{SS}^*(\mu)$, where the optimal cap is (i) lower than laissez-faire rate for a strictly positive share of managers, (ii) decreasing in market power μ and (iii) $\lim_{\mu \rightarrow 0} \bar{r}_{SS}^*(\mu) = \infty$.*

Dynamically, however, reducing the total cost of finance is important to allow managers

to accumulate collateralizable capital over time. The trade-off is in Figure 1. Reducing the cap \bar{r} increases the static allocative loss by making managers with $\sigma > \bar{\sigma}$ lose access finance - which itself also reduces capital accumulation. However, it also guarantees that those in the blue region, with $r^{all}(\theta, \mu) = \bar{r}$ face a smaller interest rate than in the non-intervention case.

The second part of Lemma 5 shows that the optimal cap can be written as a multiple of the deposit rate, r . To maximize steady state output, the optimal policy is to set a cap that is lower than the Laissez-Faire rate for some managers, i.e., the policy is binding. Moreover, the optimal cap on all loans is decreasing in market power in the financial sector μ . Intuitively, the gains from the cap are the difference between market interest rates and the cap - and thus increasing in μ , while the loss are $(1 - \mu)$ times the gains from intermediation, which is decreasing in μ . In particular, for $\mu = 0$ - a perfectly competitive financial sector, no intervention also maximizes output in the steady state, that is: the cap in our economy is a solution to the market power distortion in the financial sector.

Our result is complementary to the optimal policy of [Itskhoki and Moll \(2018\)](#). [Itskhoki and Moll \(2018\)](#) show that due to the financial frictions, the laissez fare equilibrium is not the first best. There is an incentive to tax workers in the beginning of the development period to allow for capital accumulation, while firms should be taxed later on to compensate workers. Their focus is on this intertemporal tradeoff. We could include this in our model by allowing the cap to be set below r and tax workers to pay for this subsidy. We focus however in the case where the planner is only setting up the cap - and considering their cross-section heterogeneous effect (as seen in Figure 1).

We can also show that there is always a role for the policy if the Planner can choose a path $\bar{r}(t)$ over time, that is, the optimal policy always involves a binding cap for some managers (Appendix D). Through a development process where collateralizable capital \mathcal{X} is being accumulated, we have that the optimal path for the cap is increasing. A lower cap has a higher marginal value for the economy in early stages of development due to its effect on capital accumulation (through lower costs of finance). However, the optimal dynamic cap is larger than the steady state cap for $t \rightarrow \infty$. The cap that maximizes the steady state output does not take into account the fact that a lot of managers lose access to finance in the short run, which can increase the transition period to the steady state. As $t \rightarrow \infty$, the optimal policy is decreasing in market power in the financial sector, μ . The intuition is the same as in Lemma 5. Overall, as the optimal path features mostly the same qualitative channels as our steady state analysis, we focus in the main text in the effect of lending caps on steady state levels, and leave the transition dynamics analysis to the appendix.

3.2 Cap on a Share of Loans

We consider now the case the bank must make a share ω of lending volume below the cap. The problem of the bank now is to solve the problem in Eq. (43) subject to (44)-(45) (we hide the dependence on of the problem on parameters, such as μ , for clarity). This problem introduces an interaction between loans made below and above the cap, since the balance sheet constraint in Eq. (44) must be satisfied. The financial sector can choose two contracts by type: one below the cap, $r^e(\theta)$ and one free, $r^n(\theta)$. The financial sector offers the contract below the cap for a share $e(\theta)$ for managers of type θ , and the contract above the cap for a share $n(\theta)$.

$$\begin{aligned} \max_{r^e(\theta), r^n(\theta), e(\theta), n(\theta)} \quad & \int_z \int_{\sigma} e(\theta) \left[\frac{r^e(\theta)}{1 + \sigma} - r \right] D(r^e(\theta)) dF_{\sigma}(\theta) dF_z(z) + \\ & \int_z \int_{\sigma} n(\theta) \left[\frac{r^n(\theta)}{1 + \sigma} - r \right] D(r^n(\theta)) dF_{\sigma}(\theta) dF_z(z) \end{aligned} \quad (43)$$

subject to

$$(1 - \omega) \int_z \int_{\sigma} e(\theta) D(r^e(\theta)) dF_{\sigma, z}(\theta) = \omega \int_{z, \sigma} n(\theta) D(r^n(\theta)) dF_{\sigma}(\theta) dF_z(z) \quad (44)$$

and

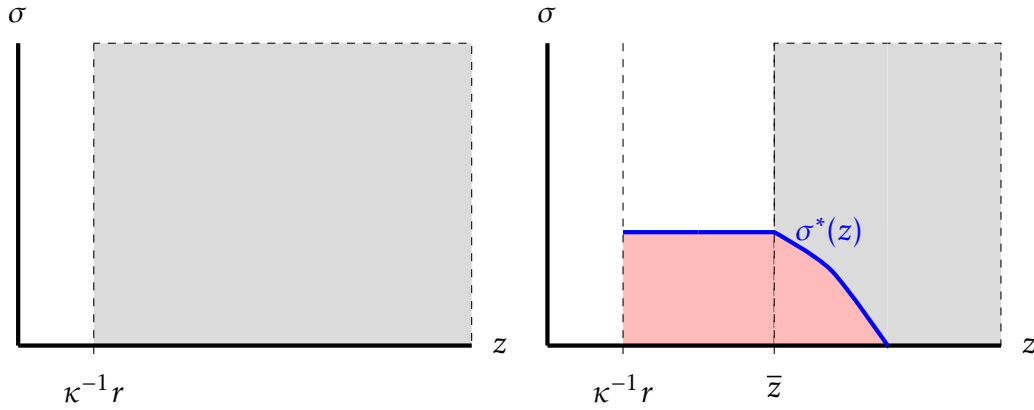
$$r^e(\theta) \leq \bar{r}, \quad e(\theta) + n(\theta) \leq 1 \quad \text{and} \quad e(\theta), n(\theta) \geq 0 \quad (45)$$

Due to constraint in Eq.(44), we cannot fully characterize the analytical solution to the problem of the bank. We proceed in two ways. In Appendix E, we present a simplified version of the bank problem where we impose more constraints in terms of the bank ability to choose interest rates, which illuminates some of the mechanisms in the general version of the problem. Second, we can show key characteristics of the allocation and interest rates under a cap on a share of loans. We present the formal allocation and pricing results in Appendix C.6, and focus here on the pictorial representation.

Figure 3 shows the credit allocation under a cap on a share of loans. The financial sector lends to a manager below the cap if this manager is relatively safe and unproductive, that is, the region below a curve $\sigma^*(z)$. The intuition is that given the financial sector inability to be compensated for risk and the possibility of default under the cap, it prefers to lend to safer types. On the other hand, the financial sector can make more profits by lending to productive types above the cap. The portion above \bar{z} where $\sigma^*(z)$ is decreasing is exactly

where this safety-productivity tradeoff is binding - and the financial sector is willing to lend below the cap for productive types of $z > \bar{z}$ given that they are relatively safe ($\sigma < \sigma^*(z)$). Risk managers with a low productivity (below \bar{z}) are now not productive enough for the financial sector to lend to given that it would to provide additional loans below the cap (and potentially make a loss at them). These are the managers that are now excluded from the credit market. The cap on a share of loans can thus potentially reduce the negative effect of a cap on all loans by excluding only a share of risk managers (those with lower productivity) and allowing productive and risky managers to receive loans above the cap.

Figure 3: Credit Allocation: Unconstrained (left) vs Cap \bar{r} of a share ω of loans (right)



Note: Credit allocation in Laissez-faire (left) vs under the cap \bar{r} for a share ω of total credit. The curve $\sigma^*(z)$ defines the set of managers that receive loans below the cap (for managers of type z, σ such that $\sigma < \sigma^*(z)$). See Appendix C.6 for more details.

The other difference in a cap on a share of loans with respect to the cap on all loans is the interest rate effect. We characterize the interest under the policy of a cap \bar{r} for a share ω of total lending in Lemma 6. For each loan above the cap, the bank must also make loans below the cap to compensate and satisfy the balance sheet constraint. Loans made above the cap thus have an extra marginal cost, which is that they make it harder to satisfy the balance sheet requirement, while the opposite is true for loans below the cap (since it relaxes the constraint). As can be seen in Eq.(46), the difference in interest rates from those below and above the cap is a direct function of the shadow price of the constraints on Eq.(44).

The equilibrium interest rate function $r^g(\theta, \mu)$ has a discontinuity, starting from the first type that is strictly above the cap. This implies that there is a *cross-subsidization* of interest rates: relatively risky and productive managers pay higher interest rates, while safe and unproductive managers face lower interest rates. This introduces a misallocation in our economy. This is shown pictorially in Figure 4, where we plot interest rates

as a function of risk, σ , for a level of productivity z that has both constrained and unconstrained managers.¹³

Lemma 6. *The interest rate policy under a cap \bar{r} for a share ω of all loans can be written as*

$$r^g(\theta, \mu) = \begin{cases} r^{w,*}(\theta, \mu) + \omega \Upsilon(\theta, \omega), & \text{if } r^g(\theta, \mu) > \bar{r} \\ \min \{r^g(\theta, \mu) - (1 - \omega) \Upsilon(\theta, \omega), \bar{r}\}, & \text{o.w.} \end{cases} \quad (46)$$

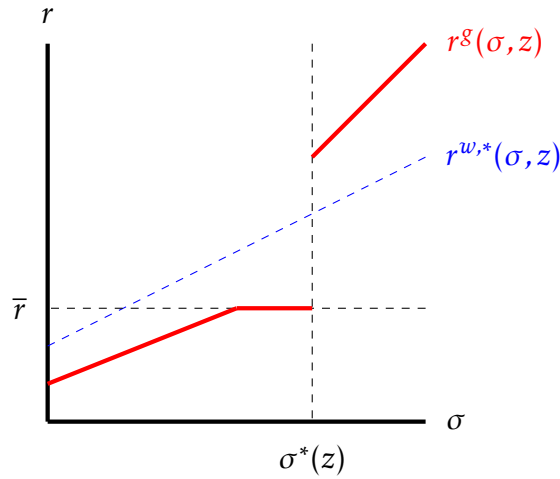
where $\Upsilon(\theta, \omega)$ are the cross-subsidies, given by

$$\Upsilon(\theta, \omega) \equiv \lambda^{-1}(1 + \sigma)(1 - \mu)\xi(\omega, \mu)$$

and $\xi(\omega, \mu)$ is the Lagrange multiplier on Eq.(44).

Proof. See Appendix C.6. ■

Figure 4: Interest Rates: Unconstrained ($r^{w,*}$) vs Under a cap \bar{r} for a share ω of loans (r^g)



Lemma 6 and Figure 4 imply that free market spreads, that is, of loans made above the cap \bar{r} , will be larger than in the unconstrained case (given the jump in interest rates). In a simplified case we study in Appendix E, we show that the relation between the share of loans below the cap and spreads is non-monotone, but increases for small values of ω .

Welfare. As in the case with a cap for all loans (or, equivalently, $\omega = 1$), the optimal policy that maximizes output today is combination of \bar{r} and ω that are not binding, that is, would be satisfied by the unconstrained solution of the bank (Appendix B.5). We can

¹³As will become clear later, we have that for large enough z , managers below a function $\sigma^*(z)$ receive loans below the cap, while those above $\sigma^*(z)$ receive loans above the cap.

illustrate this result in Figure 3. For intermediate values of productivity, managers (z, σ) above $\sigma^*(z)$ lose access to finance, which decreases the ability of managers to leverage their capital. As a result, productivity weighted leverage in this economy is reduced.

The steady state and dynamics are however much more complex under a cap on a share of loans than under the cap for all loans, since our solution for interest rates in Lemma 6 is left implicit with respect to Υ . Nonetheless, our theoretical analysis is still enlightening in the effects of this policy. To maximize steady state output, we should maximize profits in the corporate sector and facilitate capital accumulation (Lemma 3). Introducing a cap on a share of loans, as seen in Figure 4, creates winners and losers of this policy: those receiving loans below the cap are better off and can save more, while those receiving loans above the cap are worse off and can save less. At the same time, however, the cap on a share of loans reduces the negative static distortion of pricing out risky managers from the market.¹⁴ We evaluate the net effect of these two channels in a quantitative version of our model in Section 4.

We have assumed so far that productivity z and risk σ are not persistent, that is, that they are independent over time. Note that the cap on all loans benefits disproportionately those with higher levels of productivity, since they are those that the cap is binding for. At the same time, a cap for a share of loans benefits disproportionately more those with lower levels of productivity (through the cross-subsidization mechanism). Thus, the policy today can change productivity weighted leverage $\zeta_\lambda(\tau)$ in the future positively or negatively, since it will change the covariance of leverage and productivity in our economy. For this reason, we also consider persistence in productivity in our quantitative analysis on Section 4.

3.3 Intensive Margin Channel

We have assumed through Section 3 that the only effect market power has in our economy is to change interest rates and extensive margin access to credit, that is, we considered only which types of managers will have access to credit, but assumed that market power and interest rates had no effect on the share of managers of a given type with access to credit. We focus now on changes in the share of managers of a type with access to credit through changes in interest rates.¹⁵ In the unconstrained case, the level of demand in

¹⁴We show in Appendix B.5 that these effects cancel out in the aggregate, apart from those managers where the cap is in fact binding. This means that in our model, although the policy affects interest rates of all managers, the effect on those at the cap is a sufficient statistic to evaluate welfare and to compute the optimal steady state and dynamic policies.

¹⁵Note that we use the intensive vs extensive margin in our setting in terms of types of managers, that is: extensive margin refers to different types of managers and intensive to the share of managers of a given

equilibrium is given by

$$\mathcal{D}(\pi^{w,*} \mid \theta, \mu) = \Gamma(\mu)(1 - \mu)^{\frac{1}{\mu}-1} \equiv \varphi(\mu) \quad (47)$$

The function $\varphi(\mu) \in (0, 1)$ captures the intensive margin effect of market power on quantities. For the Cournot model with \mathcal{B} banks, for instance, we would have that

$$\varphi(\mu) = \frac{\mathcal{B}}{\mathcal{B} + 1}$$

That is, a share $1/\mathcal{B}$ of managers receive credit from each bank, and an equal share $1/\mathcal{B}$ does not receive credit *at the equilibrium level of interest rates*. However, for higher or lower interest rates, such as those below the cap, we have that intensive margin demand (i.e., between managers of a given type) will not be given by $\varphi(\mu)$, and that it will be affected by the policy.

For concreteness, consider the case of the cap for all loans. In Section 3.1, we assumed that $\varphi(\mu) = 1$ for all μ , that is: that there was no effect of the cap on the share of managers of a given type θ that use bank credit (intensive margin). The only difference in credit access of the cap is in the extensive margin, given by $\sigma > \bar{\sigma}$ in Figure 1. Thus, to maximize productivity weighted leverage, which depends on ability to access credit, output is maximized under laissez-faire (Lemma 5). With $\varphi(\mu) < 1$, however, we have that the cap can *increase* total credit in the economy (or at least mitigate the total reduction) through increases in *intensive margin* access where managers face lower interest rates compared to the unconstrained case (where $r^{all}(\theta, \mu) = \bar{r}$).

It is clear from Figure 1 that the net effect on total credit given the loss in the extensive margin and gain in the intensive margin is indeterminate and depends on the parameters of the model. If the mass of managers above $\bar{\sigma}$ is large, the extensive margin channel dominates, for instance. See Appendix B.6 for an analytical version of this result. We will consider both cases in our quantitative assessment. Overall, considering that interest rates change credit demand within a given type θ of managers will make a lending cap (on all or a share of loans) more useful: not only it will affect the cost of finance and capital accumulation dynamically, but it will also mitigate (or eventually invert) the potential negative effect on total credit and output statically.

type. For each individual manager, given the constant returns to scale production function and linear financial friction, there is no intensive margin effect for a change in interest rates.

4 Quantitative Evaluation

In this section we calibrate our model to understand the quantitative effect of lending caps on aggregate variables. The difference between our analytical model is that we assume that productivity z and risk σ follow a Ornstein-Uhlenbeck process (instead of being Pareto distributed). First, we show that the optimal cap on all loans is around 1.7 times the deposit rate in the economy, and that it is able to recover approximately 18% of the difference between perfect competition and laissez-faire in our benchmark economy. Although a tighter cap effectively would increase capital in the steady state, it decreases TFP by pricing out risky but productive entrepreneurs of the market. The optimal cap can be determined by our key aggregators from the theoretical model - even if they are derived under different assumptions on the distribution of risk and productivity in the economy. Moreover, the additional policy tool of using the cap on a share of loans is not useful to increase welfare. The upside of a cap on a share of loans of reducing the number of managers excluded from the credit market does not compensate from the increased cost of finance for those that receive loans above the cap. Finally, we show that our results are robust to the financial sector not observing productivity level of managers.

Second, we explore a feedback channel between the policy and market power. Although a lending cap can be welfare enhancing, it also reduces the profits of the financial sector. A reduced profitability can induce exit and, as a result, increase market power in the future. We show that if market power increases significantly after the introduction of a lending cap, all the gains can be reversed and a lending cap is always welfare reducing.

Third, we explore two alternative policies. A cap on interest rate *income* - after defaults are computed, can restore the perfect competition outcomes in our economy. We show that a cap on income is Pareto improving: fixing the profit levels of banks, it increases output by not excluding risky managers from the credit market. Further, we show that a tax on bank profits redistributed back to managers can also be effective to increase capital accumulation and output, but this result must be carefully interpreted since we do not model aggregate risk or macroprudential policy restricting lending by the financial sector.

4.1 Setting

Risk and Productivity Process. Differently from our analytical model, we parametrize the distribution of risk and productivity with distributions given by Eq.(48) and Eq.(49)

$$d \ln z = (\mu_z - \nu_z) \ln z \, dt + s_z \sqrt{\nu_z} \, dW \quad (48)$$

$$d \ln \sigma = (\mu_\sigma - \nu_\sigma) \ln \sigma \, dt + s_\sigma \sqrt{\nu_\sigma} \, dW \quad (49)$$

where ν_z and ν_σ are the inverse of the persistence of, respectively, productivity z and risk σ . The parameters μ_z and s_σ are the mean and standard deviation of \ln of z (and the same for σ). Eq.(48) and Eq.(49) gives the following stationary distribution

$$\varphi(z, \sigma) \propto \frac{1}{z\sigma} \exp \left[-\frac{(\ln z - \mu_z)^2}{s_z^2} - \frac{(\ln \sigma - \mu_\sigma)^2}{s_\sigma^2} \right] \quad (50)$$

Calibration. We calibrate our economy using mainly the parameter values for a general emerging market of Moll (2014). The key parameter in our paper different from the literature is the market power in the financial sector μ . To calibrate this parameter, we solve the model for different levels of μ *without* any credit policy and compute the implied spread on loan operations in the steady state with the invariant distribution $\psi(\theta)$ for z and σ , as in Eq.(51) ¹⁶

$$\mathcal{S}(\mu) = \int \frac{r^{w,*}(\theta, \mu)}{r} d\psi(\theta) \quad (51)$$

where $r^{w,*}(\theta, \mu)$ is the one defined in Eq.(38). We choose μ such that $\mathcal{S}(\mu) = \frac{18.3}{11.23} = 1.6$, where 18.3 is the spread on loans to firms in free market operations in Brazil for 2011-2016 according to the Brazilian Central Bank and 11.23 p.p. is the average policy rate for the period. ¹⁷This results in $\mu \approx .33$. This is equivalent to two banks competing in Cournot (as seen in Lemma B.1). This is consistent with the evidence in Joaquim and Doornik (2019) that show that the median number of banks in banking markets in Brazil is approximately two. For details on the Calibration and the numerical solution of the model, see Appendix F.

Key Variables and Intensive Margin Channel. Throughout this section we use $\tilde{r} = \bar{r}/r$, i.e., a multiplier of the deposit rate in our economy, as the choice variable for the lending cap. We opt for \tilde{r} instead of \bar{r} to make statements relative to deposit rates, which are

¹⁶We opt to compute the spread in our models as a ratio for the deposit rate (and not a subtraction) due to the fact that (i) our model is not parametrized any levels and (ii) the policy rate in Brazil from 2011-2016 was on average 11.23 p.p. - which is not a number we take into account in the model.

¹⁷Source: Brazilian Central Bank FAQ's on spreads from 2011-2016.

different in different economies. Moreover, our results in Lemma 5 show that \tilde{r} is the relevant statistic for welfare.

For output \mathcal{Y} and other aggregate variables, we measure the effect of a credit policy on levels and on the relative effect of the policy in terms of recovering the difference between perfect competition and laissez-faire, that is

$$\hat{\mathcal{Y}} \equiv \frac{\mathcal{Y}(\tilde{r}) - \mathcal{Y}^{LF}}{\mathcal{Y}^{PC} - \mathcal{Y}^{LF}} \quad (52)$$

where \mathcal{Y}^{LF} and \mathcal{Y}^{PC} are, respectively, output without any intervention and under perfect competition. We denote $\hat{\mathcal{Y}}$ as the *relative output*.

Finally, we conduct our analysis with and without the intensive margin channel of credit we describe in Section 3.3. Without the intensive margin channel, market power in the financial sector distorts only interest rates, but not the allocation under laissez-faire. With the intensive margin channel, market power in the financial sector distorts both the allocation of credit and interest rates.

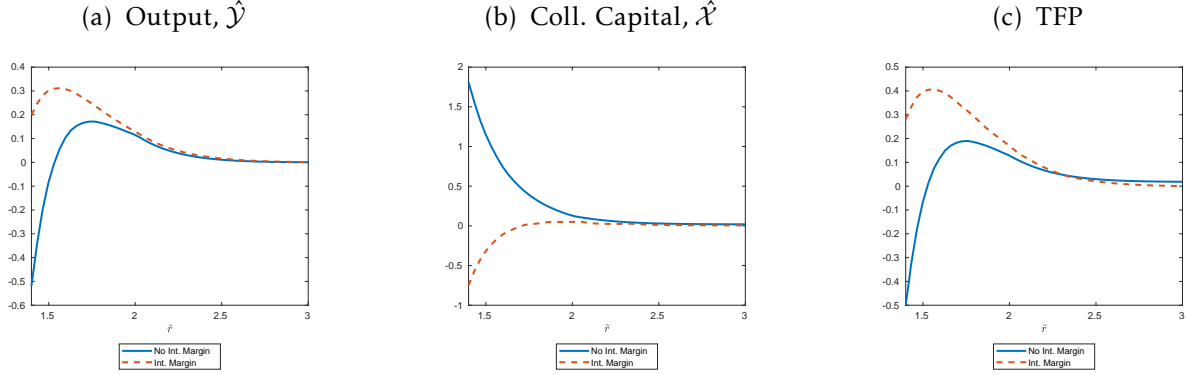
4.2 Steady State

We start our analysis with the steady state effects of credit policies in an economy without persistence in z and σ , that is, where the distribution of risk and productivity in the economy is given by Eq.(50). We return to the role of the persistent at the end of the section.

Cap on All Loans. Figure 5 plots the effect of a cap on all loans on the relative output (Eq. 52). For completeness, we also present the results in Table 1. The optimal cap on all loans is approximately $\tilde{r} \approx 1.75$ without the intensive margin channel and around 1.55 with it. With the intensive margin channel, the market power distortion is more detrimental to growth (since it affects both interest rates and lending volume), and the loss of a tighter cap is smaller, and thus a more stringent policy is optimal. Without the intensive margin channel, the cap works in our economy by both increasing the level of collateralizable \mathcal{X} and TFP at the optimum. The optimal cap can recover $\approx 18\%$ between the difference of laissez-faire and perfect competition, which is an expressive number given that it is one unidimensional instrument (not an optimal schedule). We summarize this results in Table 1.

An important caveat for policy makers, is that the effect of a cap on all loans is extremely asymmetric around the optimum and that a cap policy should be designed conservatively. For instance, a cap of 1.5 times the deposit rate versus the optimal 1.7 can

Figure 5: Aggregate Relative Effects of a cap on all loans, $\tilde{r} = \frac{\bar{r}}{r}$



Note: Steady State Output with no persistence in z and σ . Output shown as percentage between laissez-faire and perfect competition, as Eq.(52). The parameters used are in Table 3. See Appendix F for details on computation. The no intensive margin case is where market power in intermediation distorts only rates, but not the quantity of credit. The intensive margin case is where market power in intermediation distorts both interests rate up and quantity of credit down statically. See Section 3.3 for details.

reduce output by 30%. This indicates that policy makers should err in the side of a looser cap - since the effects of a slightly tighter than optimal cap can be significantly negative. The same insight can be derived from analyzing the sufficient statistics of the aggregate effect of credit policies from Lemma 3. We plot ζ_λ, ζ_π in Figure 6.

Without the intensive margin channel, the cap only reduces ζ_λ - the leverage weighted productivity in the economy. Dynamically, maximizing profits in the corporate sector are what matter to maximize output, as in the left panel of Figure 6. With the intensive margin channel, the cap is beneficial statically and dynamically (in the values plotted), and they both max out at roughly the same \tilde{r} . Notice that ζ_λ and ζ_π are aggregates leverage and profitability of those that are producing (taking into account those producing under autarky), which can be estimated with access to firm level data by policymakers.

Cap on a Share of Loans. In our parametrization, it is optimal to have $\omega \in (0, 1)$ if the cap \tilde{r} is too tight, but the optimal cap on all loans still increases output by more, as can be seen in Figure 7. This is not surprising, given the misallocation that this policy generates by having less productive managers subsidizing more productive managers. Although we focus in here in loans below and above the cap in a single sector economy, our results would also apply for an economy with multiple sectors where one sector is subsidized. The key dynamics in the model is the Lagrange Multiplier ξ in Lemma 6. Loans made below the cap help the constraint be satisfied, and thus are made at cheaper rates. The opposite is true for loans above the cap. This misallocation dynamically reduces the effect

Table 1: Optimal Cap and Aggregate Outcomes for a cap on all loans: Steady State

	No Intensive Margin	Intensive Margin
\tilde{r}^*	1.75	1.55
\mathcal{Y}^{LF}	1.000	0.9203
\mathcal{Y}^{PC}	1.0607	1.0607
\mathcal{Y}^*	1.0104	1.0028
\mathcal{X}^{LF}	0.2087	0.2455
\mathcal{X}^{PC}	0.2284	0.2289
\mathcal{X}^*	0.2164	0.2264
TFP^{LF}	0.9879	0.9214
TFP^{PC}	1.0396	1.0406
TFP^*	0.9977	0.9913

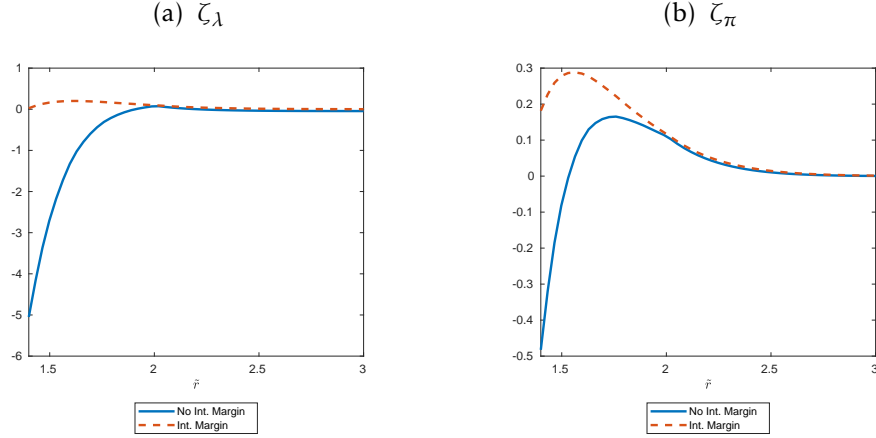
Note: Optimal cap \tilde{r}^* and steady state Output \mathcal{Y} , Collaterizable Capital \mathcal{X} and TFP for laissez-faire (LF), perfect competition (PC) and at the optimal cap (*) with no persistence in z and σ . Output under laissez-faire \mathcal{Y}^{LF} normalized to one. The parameters used are in Table 3. See Appendix F for details on computation. The no intensive margin case is where market power in intermediation distorts only rates, but not the quantity of credit. The intensive margin case is where market power in intermediation distorts both interests rate up and quantity of credit down statically. See Section 3.3 for details.

of a cap on a share of loans - and it is better for output to set a less tight cap (larger) on all loans. Finally, we plot in Figure A.1 the values for ζ_λ and ζ_π , and show that the effects of the alternative policy of a lending cap on a share of loans can also be understood by Lemma 3.

In Figure 8 we can see the implied spreads on loans made above the cap for different values of ω . As the share ω increases, each loan above the cap becomes marginally more costly to be done by the bank, in which case interest rates on these loans increase by more. In Appendix G we show the numerical allocation of loans below and above the cap and that the shadow price on the balance sheet constraint of the problem of the bank is always increasing in ω , such that the discontinuity in interest rates is always larger. This causes spreads to increase when ω increases (even as the riskiness of managers below and above the cap is non-monotone in the share ω , as shown in Appendix G). In Brazil, for instance, approximately 35% of loans are made under a lending cap. Our model qualitatively explains why balance sheet requirements on banks increases spreads in the free market for credit (i.e., loans bove the cap).

Non-Observed Productivity. The channels highlighted on this paper depend on the fact that interest rates are heterogeneous across agents, but it does *not* depend on interest rates being increasing in productivity (conditional on repayment probability). To show this, we conduct the following experiment within the model. We assume the bank has no signal or information about any z , and is simply randomizing interest rates at the

Figure 6: Productivity Weighted Leverage (ζ_λ) and profits of corporate sector (ζ_π) with a cap on all loans, $\tilde{r} = \frac{\tilde{r}}{r}$



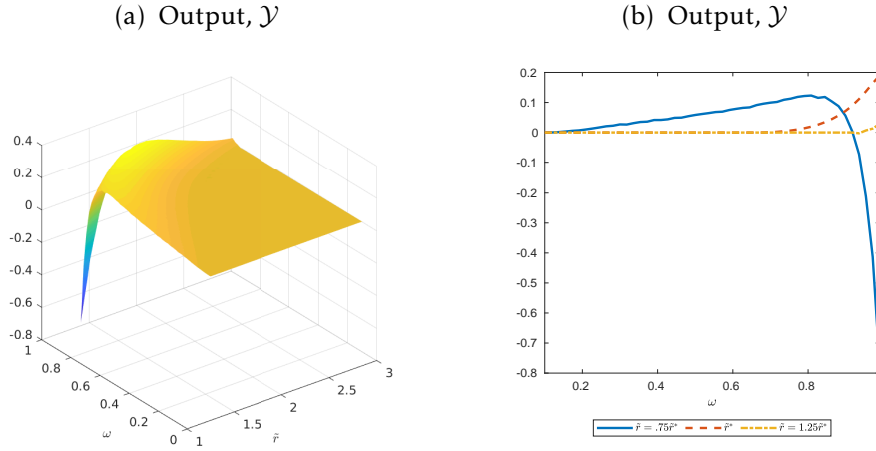
Note: Steady State ζ_λ - productivity weighted leverage - and ζ_π - profits in the corporate sector - in our economy no persistence in z and σ . See Eqs.(182)-(183) in Appendix H for details. Variables shown as percentage between laissez-faire and perfect competition, as Eq.(52). The parameters used are in Table 3. See Appendix F for details on computation. The no intensive margin case is where market power in intermediation distorts only rates, but not the quantity of credit. The intensive margin case is where market power in intermediation distorts both interests rate up and quantity of credit down statically. See Section 3.3 for details.

productivity dimension. Furthermore, we assume that managers produce using the bank loan even if they have a bad draw from the bank randomization - i.e., get a higher interest rate than the one their reservation one. These two assumptions are extreme, but they illustrate that the mechanisms we highlight in this paper are applicable in more general settings. The cap on all loans is still effective in this setting, as can be seen in Figure A.2. The magnitude of the policy effect in this case is much larger due to fact that the laissez-faire output is much lower, while the perfect competition one does not change (since risk-adjusted interest rates were all r). For the policy to be effective it is necessary to have variation in interest rates, which in turn leads to a number of managers exactly at the cap - and thus able to save and produce more in the future. The specifics of how productivity and interest rates are related are not fundamental for our qualitative insights.

4.3 The Role of Persistence

We extend our analysis to allow manager's productivity and risk to be persistent over time (but still i.i.d. in the cross-section). If a policy is successful in redistributing resources for more productive managers, for instance, the model with persistence indicates that the policy should be used more (a tighter cap, for instance). In Appendix 2 we derive the

Figure 7: Effect on Output of a cap \tilde{r} on a share ω of loans



Note: Steady State Output with no persistence in z and σ as a function of cap $\tilde{r} = \bar{r}/r$ and the share of loans that must be below the cap ω . Output shown as percentage between laissez-faire and perfect competition, as Eq.(52). The parameters used are in Table 3. See Appendix F for details on computation. We assume here the 'no intensive margin' case: market power in intermediation distorts only rates, but not the quantity of credit.

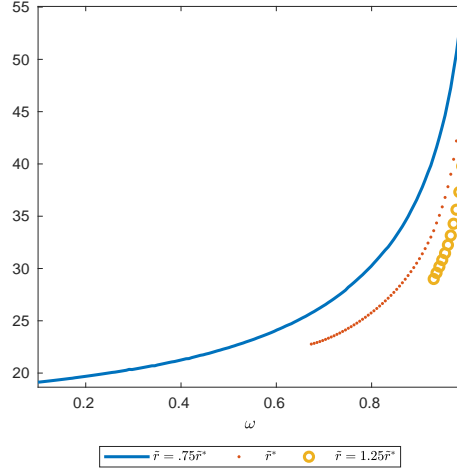
analogous of the aggregation Lemma 2 with persistence.

We focus on this section on the numerical results. The key difference in this section is that z and σ are persistent, as in Eqs.(48)-(49). We choose 75% of the collateralizable capital in the laissez-faire steady state as our initial point in the transition. To facilitate exposition, we compute the Net Present Value (NPV) of a variables. For output, for instance

$$\mathcal{Y}^{NPV} = \int_0^\infty e^{-rt} \mathcal{Y}(t) dt \quad (53)$$

We still use our metrics of Eq.(52), but now on NPVs. The results are in Figure A.4. The results are quantitatively similar to those in the steady state, iid case of the previous section. The reason is that although the policy can be more useful to increase output now, the gap between perfect competition and laissez-faire also is (Table 2). The share of the gap that can be recovered then remains close in the two cases. To separate the static and dynamic effects, we plot on the left panel of Figure A.4 the initial and final output without the intensive margin channel. The cap on all loans reduces credit and output today, but increases it over time eventually enough to recover from the static misallocation. Finally, we plot the path of Y, \mathcal{K} and TFP in Figure A.5. As predicted in our analytical model, the total effect on output of a cap on all loans increases both capital accumulation and TFP, and the differences are increasing over time.

Figure 8: Effect on Spreads of loans above the cap \tilde{r} on a share ω of loans



Note: Spreads in the Steady State with no persistence in z and σ as a function of cap $\tilde{r} = \bar{r}/r$ and the share of loans that must be below the cap ω . The parameters used are in Table 3. See Appendix F for details on computation. We assume here the 'no intensive margin' case: market power in intermediation distorts only rates, but not the quantity of credit.

4.4 Endogenous Market Power

All policies discussed in this paper reduce the profitability of the financial sector, since they impose a constraint in the bank problem. Therefore, by introducing a lending cap to mitigate the effects of market power in the economy, the Planner reduces the profitability of the financial sector. This reduction in profitability itself can lead to exit of banks, which increases market power and the need to tighten the cap. We introduce now this feedback channel in our model to understand the effects of the credit policies with an market power that itself depends on the policies.

To capture this channel, we assume that market power in the steady state is given by Eq.(54). We provide in Appendix I a model that can justify the signs in Eq.(54), but Eq.(54) is strictly a reduced form approach to capture this feedback channel. We opt for a simple reduced form version of this effect over a full blown model structural model of entry/exit due to the fact that we do not want to take a stand on entry/operating costs of banks, time frame their decisions is made, political economy channels, the role public banks etc., but rather how the policy affects it. The sensitivity of the market power to the policy is simply captured by the parameter ψ_r .

$$\mu = \mu_0 + (1 - \mu_0)e^{-\psi_r^{-1}(\bar{r}-r)} \quad (54)$$

The results for $\psi_r \in \{0, .75, 1.5\}$ with $\mu_0 = .25$ are in Figure A.3. We focus on the steady

state effects with no persistence in z or σ for simplicity. The effect of the cap on all loans now is reduced for $\psi_r = .75$. For $\psi_r = 1.5$, a cap on all loans is now always output reducing, since market power significantly increases as a response to the cap. This is true for both the model with and without the intensive margin channel. This shows that the feedback channel has the potential to revert the output gains from the optimal cap on all loans - and thus a policy maker should not only rely on interest rate caps, but also to promote policies that increase competition in the financial sector.

4.5 Alternative Policies

Given our mixed results in terms of the effectiveness of the lending cap, are there any other non-tax based policies that can be adopted to increase growth? We focus on two alternatives: an interest income cap and a tax on the profits of the representative bank. As this policies can potentially work differently with and without persistence in productivity and risk, we focus on the net present value of output in the version of our model with persistence.

Interest Income Cap. Consider the case where the Planner imposes a cap on interest *income*, after default is taken into account. Under Laissez-Faire, we have that the interest rates are given by

$$r^{w,*}(\theta, \mu) \equiv (1 + \sigma) \left[r + \mu \frac{g(z)}{\lambda} \right] \quad (55)$$

Denote the aggregate interest income (as a percentage of total capital) of the bank by $\mathcal{I}(\mu)$. Integrating is given by

$$\mathcal{I}(\mu) = r + r\mu \frac{\lambda - 1}{\lambda} [\zeta_\lambda(\tau) - 1] \quad (56)$$

Note that in this case the Planner can completely undo the market power distortion by imposing a cap on interest income of exactly the deposit rate r . Within our framework, interest rates are high due to risk and high intermediation gains. The Planner is not concerned with high interest rates due to risk, as they would appear even in the competitive case. With a single cap on all or a share loans to undo market power, however, the Planner forces the bank to reduce lending for risky managers at the same time it allows more productive managers to obtain cheaper loans. With a cap on interest rate income (after default), the Planner can target exactly the market power distortion *without* the adverse effect of reduced lending for risky managers.

The results are in Figure A.6. The income cap can be used at much tighter levels than the absolute cap, and can re-store perfect competition levels of output for values close to r . As our model does not take into account other channels through which bank profitabil-

ity affects aggregates (macroprudential regulations, stability etc.), we focus our analysis on the cap on income that would deliver the same profit level for banks as the cap on all loans (the optimal one). In our benchmark parametrization, the Planner could implement an income cap and gain .5% of welfare without changing the profits of banks.

Bank Profit Taxation. We consider now the case where the Planner directly taxes profits of the financial sector and redistributes it to the corporate sector. Bank profits are given by

$$\Pi^B = \alpha \left[\frac{\zeta_r(\tau) - \theta_\lambda(\tau)}{\zeta_\lambda(\tau)} \right] \mathcal{Y} \quad (57)$$

A share $(1 - \nabla)$ of profits is distributed back to workers. Of the remaining, we suppose the government taxes a share τ_b of the profits of the representative bank to redistribute to workers at the end of period (after production is undertaken). The capital accumulation equation becomes (Appendix H)

$$\dot{\mathcal{X}} = \hat{\alpha} \left[\frac{\zeta_\pi^p(\tau, w)}{\zeta_\lambda^p(\tau, w)} + \tau_b \frac{\zeta_r(\tau) - \theta_\lambda(\tau)}{\zeta_\lambda(\tau)} \right] \mathcal{Y} + (r - \delta) \mathcal{X} \quad (58)$$

and savings for each manager

$$\zeta_s(\theta, \tau, w) \equiv \hat{\alpha} \pi(\theta, \tau) + (r - \delta) + \tau_b \left[\frac{\zeta_r(\tau) - \theta_\lambda(\tau)}{\zeta_\lambda(\tau)} \right] \mathcal{Y} \quad (59)$$

Without persistence and no intensive margin quantity of credit distortion, taxing banks and redistributing all of the profit lump sum to the corporate sector re-stores the perfect competition outcome. This comes from the fact that there is no static distortion from market power in our economy (see Lemma 5). With persistence, however, a lump sum redistribution scheme cannot restore the perfect competition path, since it does not redistribute profits proportionally to productivity (which is the case in the perfect competition economy), and thus allows more productive managers to save relatively more under perfect competition. Still, as can be seen in Figure A.7, taxing banks and redistribute the profits to the corporate sector is an efficient way of increasing output in our model.

The results in Figure A.7 are most likely an upper bound for the effectiveness of bank taxation. First, a tax τ_B can significantly reduce the profitability of banks - and thus the market power of the financial sector in the long-run. Second, our model does not feature macroprudential regulations or balance sheet constraints from the bank perspective - which tilts the optimal policy to increase capital in the corporate sector. A model that

includes both market power in the lending side and constraints from banks is such that keeping profits in the financial sector are useful to increase its equity.

5 Conclusion

We investigate the effect of lending caps on economic growth when the banking sector has market power. For that, we expand the macro-development model of [Itskhoki and Moll \(2019\)](#) to include managers that are heterogeneous in collateralizable capital and productivity (as in the literature), but also riskiness, and a financial sector that is non-competitive. Under a few functional form assumptions, we can understand the effect of any credit policy with heterogeneous effects in the corporate sector. We show that productivity weighted leverage is a sufficient statistic for the effect of a credit policy on output today. On the other hand, a mix of productivity weighted leverage and the cost of finance is optimal for output in the steady state, given that a lower cost of finance allows the financially constrained corporate sector to accumulate capital faster over time.

We apply our analytical results to lending rate caps. Caps on interest rates can be beneficial for economic growth since they reduce the cost of finance. However, caps can also be harmful since they crowd out risky firms which can access credit only at high interest rates to compensate lenders for the risk of default. To prevent the latter effect, in some emerging markets banks are permitted to charge uncapped rates on a share of their loans. This allows banks to service risky borrowers, but the bank optimal choice over who to lend below the cap generates capital misallocation since banks provide capped loans to less productive borrowers, while charging higher rates to more productive ones.

We calibrate our model to the Brazilian economy to quantitatively assess the positive and negative effect of lending caps. First, a cap on all loans optimally calibrated can cause growth and recover approximately 18% of the difference between laissez-faire and a perfectly competitive banking system. The effect of a cap on all loans is asymmetric, with potential large negative effects for tighter than optimal caps, which indicates policy makers should choose a cap conservatively in case of uncertainty. Second, the misallocation generated by a cap on a share of loans is severe, and the optimal policy is to set a specific cap on all loans.

Our analysis also provides important caveats and an alternative. If the intervention in the financial sector causes it to become significantly more concentrated as a result (through the loss of profits), the gains from introducing a cap can be reduced and, eventually, any cap can hinder growth. Finally, we show that an alternative policy of introducing interest income (after default is accounted) mitigates the negative effects of a hard

cap and is Pareto increasing.

References

- Agarwal, Sumit, Souphala Chomsisengphet, Neale Mahoney, and Johannes Stroebe.** 2014. "Regulating consumer financial products: Evidence from credit cards." *The Quarterly Journal of Economics*, 130(1): 111–164.
- Aghion, Philippe, and Patrick Bolton.** 1997. "A theory of trickle-down growth and development." *The Review of Economic Studies*, 64(2): 151–172.
- Angeletos, George-Marios, and Vasia Panousi.** 2011. "Financial integration, entrepreneurial risk and global dynamics." *Journal of Economic Theory*, 146(3): 863–896.
- Banerjee, Abhijit V, and Andrew F Newman.** 1993. "Occupational choice and the process of development." *Journal of political economy*, 101(2): 274–298.
- Benetton, Matteo.** 2018. "Leverage regulation and market structure: An empirical model of the UK mortgage market." *Available at SSRN 3247956*.
- Benmelech, Efraim, and Tobias J Moskowitz.** 2010. "The political economy of financial regulation: Evidence from US state usury laws in the 19th century." *The journal of finance*, 65(3): 1029–1073.
- Bernanke, Ben S, Mark Gertler, and Simon Gilchrist.** 1999. "The financial accelerator in a quantitative business cycle framework." *Handbook of macroeconomics*, 1: 1341–1393.
- Berry, Steven, James Levinsohn, and Ariel Pakes.** 1995. "Automobile prices in market equilibrium." *Econometrica*, 7(1): 841–890.
- Bonomo, Marco, Ricardo D Brito, and Bruno Martins.** 2015. "The after crisis government-driven credit expansion in Brazil: A firm level analysis." *Journal of International Money and Finance*, 55: 111–134.
- Buera, Francisco J, and Yongseok Shin.** 2013. "Financial frictions and the persistence of history: A quantitative exploration." *Journal of Political Economy*, 121(2): 221–272.
- Buera, Francisco J, Joseph P Kaboski, and Yongseok Shin.** 2011. "Finance and development: A tale of two sectors." *American Economic Review*, 101(5): 1964–2002.

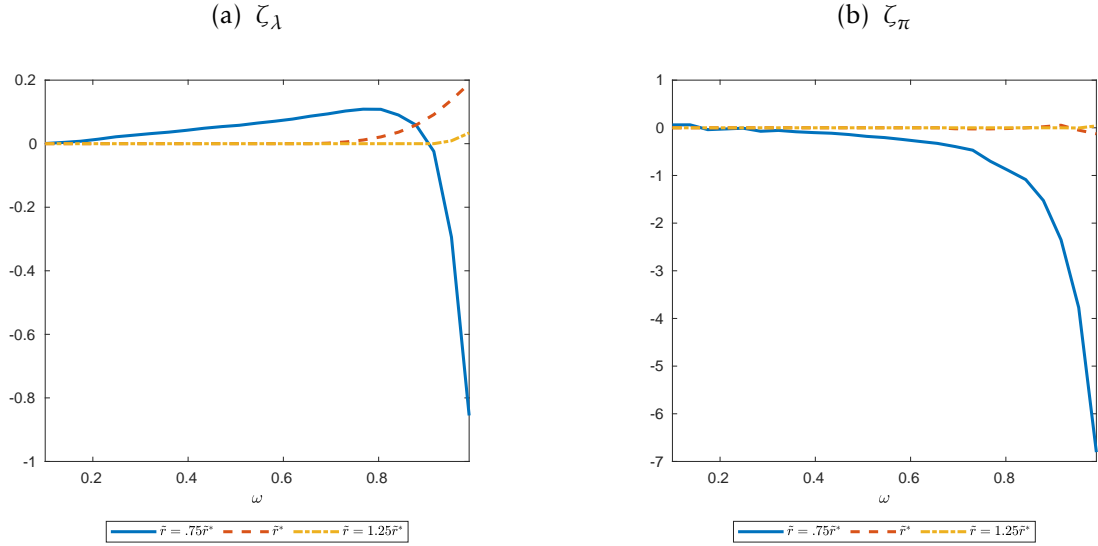
- Cagetti, Marco, and Mariacristina De Nardi.** 2006. "Entrepreneurship, frictions, and wealth." *Journal of political Economy*, 114(5): 835–870.
- Carlson, Mark A, Sergio Correia, and Stephan Luck.** 2019. "The effects of banking competition on growth and financial stability: Evidence from the national banking era." *Available at SSRN 3202489*.
- Cuesta, Jose Ignacio, and Alberto Sepulveda.** 2018. "Price Regulation in Credit Markets: A Trade-off between Consumer Protection and Credit Access." *Available at SSRN 3282910*.
- Ferrari, Aurora, Oliver Masetti, and Jiemin Ren.** 2018. "Interest Rate Caps: The Theory and The Practice." *World Bank Policy Research Working Paper 8398*.
- Galor, Oded, and Joseph Zeira.** 1993. "Income distribution and macroeconomics." *The review of economic studies*, 60(1): 35–52.
- Itskhoki, Oleg, and Benjamin Moll.** 2018. "Optimal Development Policies with Financial Frictions." *Econometrica*, x(x): x.
- Itskhoki, Oleg, and Benjamin Moll.** 2019. "Optimal Development Policies with Financial Frictions." *Econometrica*, Forthcoming.
- Jeong, Hyeok, and Robert M Townsend.** 2007. "Sources of TFP growth: occupational choice and financial deepening." *Economic Theory*, 32(1): 179–221.
- Joaquim, Gustavo, and Bernardus Van Doornik.** 2019. "Bank Competition and Economic Activity: Evidence from Brazil." *Working Paper*.
- Joaquim, Gustavo, Robert M Townsend, and Victor Zhorin.** 2019. "Optimal Contracting and Spatial Competition Among Financial Service Providers." *Working Paper*.
- Lazzarini, Sergio G, Aldo Musacchio, Rodrigo Bandeira-de Mello, and Rosilene Marcon.** 2015. "What do state-owned development banks do? Evidence from BNDES, 2002–09." *World Development*, 66: 237–253.
- Melzer, Brian, and Aaron Schroeder.** 2017. "Loan contracting in the presence of usury limits: Evidence from automobile lending." *Consumer Financial Protection Bureau Office of Research Working Paper*, , (2017-02).
- Midrigan, Virgiliu, and Daniel Yi Xu.** 2014. "Finance and misallocation: Evidence from plant-level data." *American economic review*, 104(2): 422–58.

- Moll, Benjamin.** 2014. "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" *American Economic Review*, 104(10): 3186–3221.
- Nelson, Scott Thomas.** 2017. "Private information and price regulation In the US credit card market." PhD diss. Massachusetts Institute of Technology.
- Petersen, Mitchell A, and Raghuram G Rajan.** 1995. "The effect of credit market competition on lending relationships." *The Quarterly Journal of Economics*, 110(2): 407–443.
- Rice, Tara, and Philip E Strahan.** 2010. "Does credit competition affect small-firm finance?" *The Journal of Finance*, 65(3): 861–889.
- Rigbi, Oren.** 2013. "The effects of usury laws: Evidence from the online loan market." *Review of Economics and Statistics*, 95(4): 1238–1248.
- Safavian, Mehnaz, and Bilal Zia.** 2018. "The Impact of Interest Rate Caps on the Financial Sector." *World Bank Policy Research Working Paper 8393*.
- Sandri, Damiano.** 2014. "Growth and capital flows with risky entrepreneurship." *American Economic Journal: Macroeconomics*, 6(3): 102–23.
- Santos, Fernando Kuwer dos.** 2016. "Earmarked credit and misallocation: evidence from Brazil."
- Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti.** 2011. "Growing like china." *American economic review*, 101(1): 196–233.
- Temin, Peter, and Hans-Joachim Voth.** 2008. "Interest rate restrictions in a natural experiment: Loan allocation and the change in the usury laws in 1714." *The economic journal*, 118(528): 743–758.
- Zarutskie, Rebecca.** 2006. "Evidence on the effects of bank competition on firm borrowing and investment." *Journal of Financial Economics*, 81(3): 503–537.
- Zinman, Jonathan.** 2010. "Restricting consumer credit access: Household survey evidence on effects around the Oregon rate cap." *Journal of banking & finance*, 34(3): 546–556.

Appendix

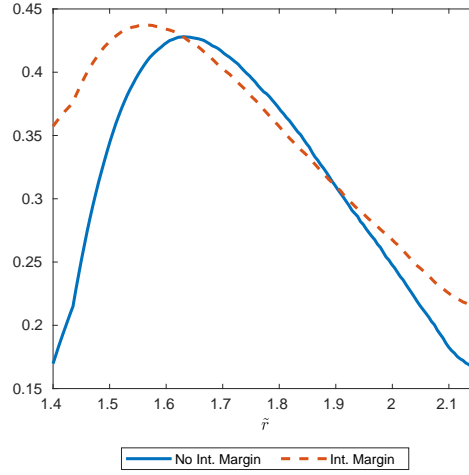
A Additional Figures and Tables

Figure A.1: Productivity Weighted Leverage (ζ_λ) and profits of corporate sector (ζ_π) with a cap on share ω of loans



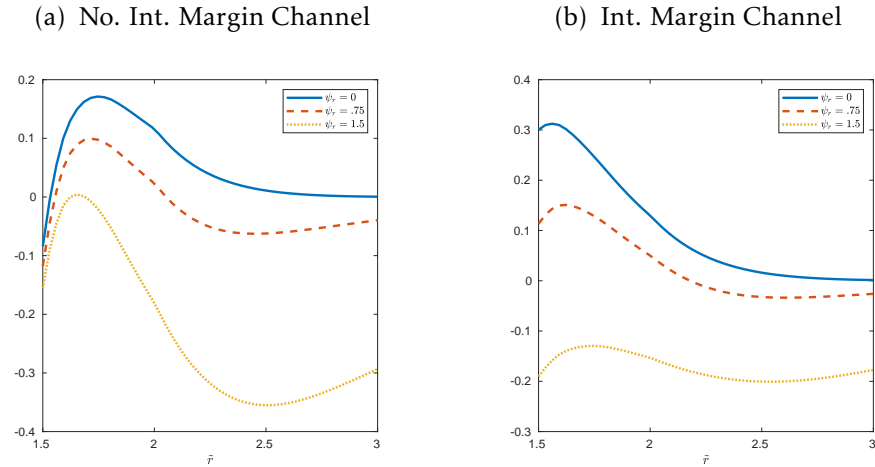
Note:

Figure A.2: Effect on Output of a cap on all loans, $\tilde{r} = \frac{\bar{r}}{r^*}$



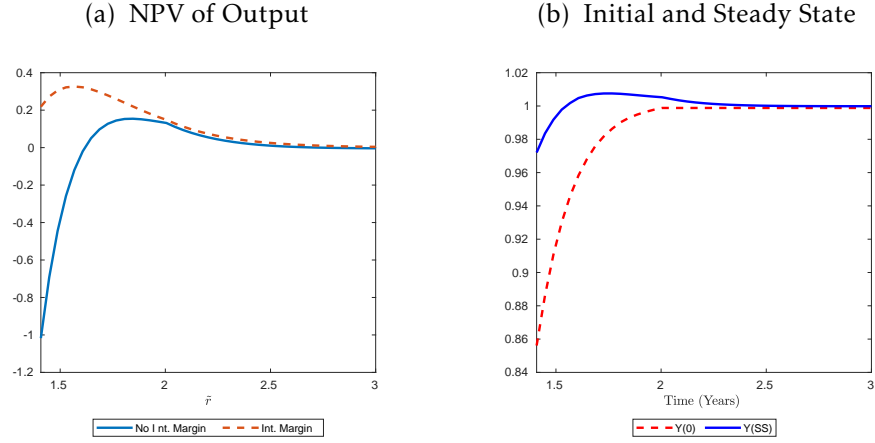
Note: Steady State Output with no persistence in z and σ , with randomized inter rates on productivity level. Output shown as percentage between laissez-faire and perfect competition, as Eq.(52). The parameters used are in Table 3. See Appendix F for details on computation.

Figure A.3: Effect on Steady State Output of a cap on all loans \tilde{r} with the Feedback Channel



Note: Steady state output (normalized as in Eq.(52)) when μ is given by Eq.(54), for $\psi_r \in \{0, .75, 1.5\}$ with $\mu_0 = .25$. The parameters used - apart from μ - are in Table 3. See Appendix F for details on computation. Productivity z and σ are i.i.d. between themselves and over time, and are distributed as Eq.(50). We use here the no intensive margin case: market power in intermediation distorts only rates, but not the quantity of credit. The intensive margin case is where market power in intermediation distorts both interests rate up and quantity of credit down statically.

Figure A.4: Effect on NPV, Initial and Steady State levels of Output of a cap on all loans,
 $\tilde{r} = \frac{\bar{r}}{r}$



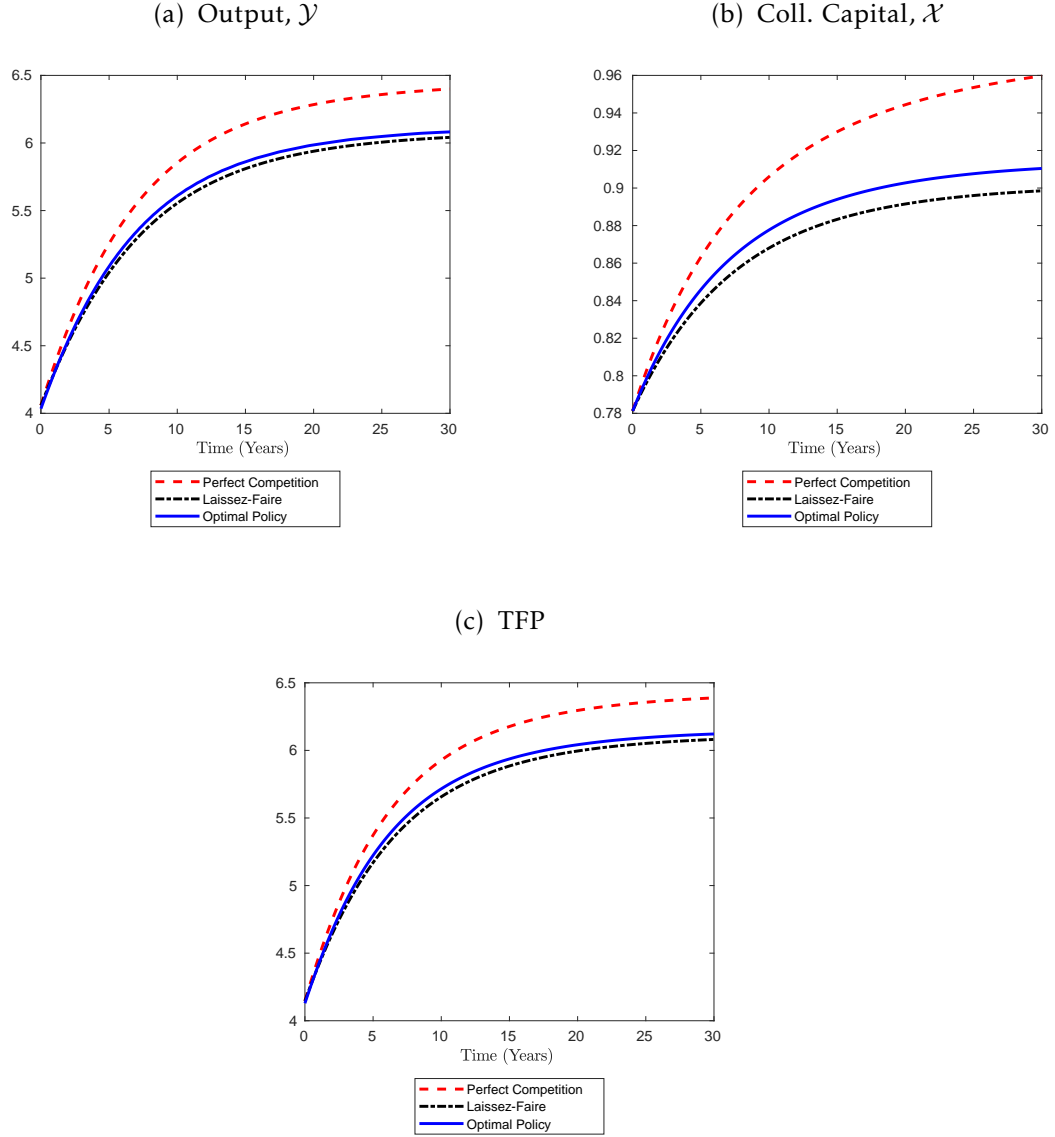
Note: Net Present Value (Eq. (53)), initial and steady state output as a function of a cap $\tilde{r} = \bar{r}/r$ on all loans. The NPV is normalized as in Eq.(52), while initial and steady state outputs are normalized by the laissez-faire level y^{LF} . The parameters used are in Table 3. See Appendix F for details on computation. The no intensive margin case is where market power in intermediation distorts only rates, but not the quantity of credit. The intensive margin case is where market power in intermediation distorts both interests rate up and quantity of credit down statically. See Section 3.3 for details.

Table 2: Optimal Cap and Aggregate Outcomes for a cap on all loans: Dynamics

	No Intensive Margin	Intensive Margin
\tilde{r}^*	1.8459	1.5673
$\mathcal{Y}^{LF}(NPV)$	1.0000	0.8802
$\mathcal{Y}^{PC}(NPV)$	1.0496	1.0496
$\mathcal{Y}^*(NPV)$	1.0077	0.9355
$\mathcal{Y}^{LF}(0)$	1.0000	0.7942
$\mathcal{Y}^{PC}(0)$	1.0000	1.0000
$\mathcal{Y}^*(0)$	0.9928	0.9459
$\mathcal{Y}^{LF}(SS)$	1.4885	1.3522
$\mathcal{Y}^{PC}(SS)$	1.5767	1.5767
$\mathcal{Y}^*(SS)$	1.4988	1.4911

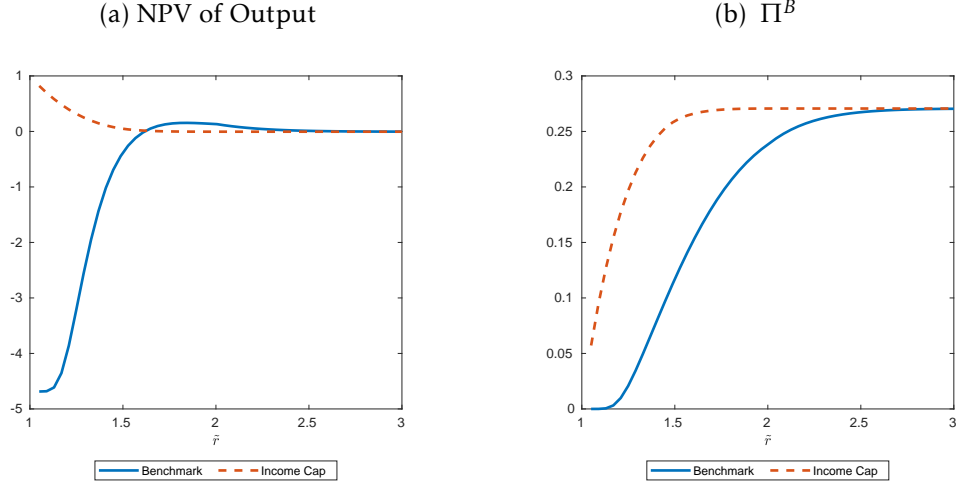
Note: Optimal cap \tilde{r} , Net Present Value (Eq. (53)), initial and steady state output under laissez-faire (LF), perfect competition (PC) and at the optimum ($*$). The NPV is normalized as s.t. $\mathcal{Y}^{LF}(NPV) = 1$, while initial and steady state outputs are normalized by the laissez-faire level $\mathcal{Y}^{LF}(0)$. The parameters used are in Table 3. See Appendix F for details on computation. The no intensive margin case is where market power in intermediation distorts only rates, but not the quantity of credit. The intensive margin case is where market power in intermediation distorts both interests rate up and quantity of credit down statically. See Section 3.3 for details.

Figure A.5: Path of Aggregate Outcomes of a cap on all loans \tilde{r}



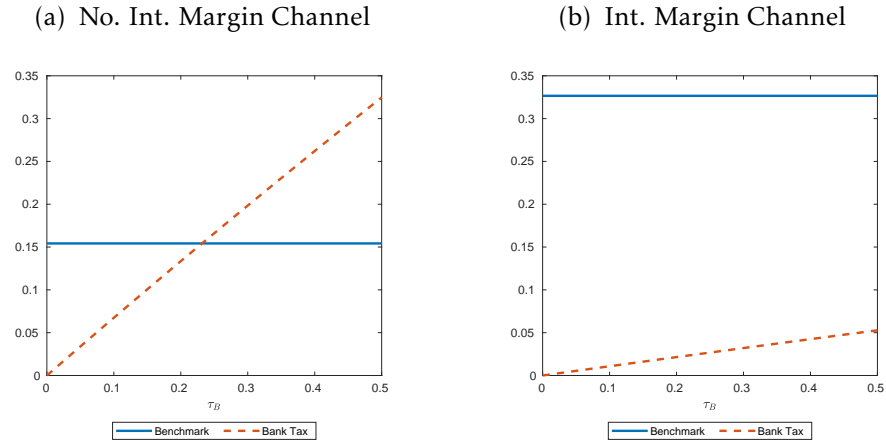
Note: Path of output \mathcal{Y} , collateralizable capital \mathcal{K} and TFP over 30 years. The parameters used are in Table 3. See Appendix F for details on computation. We assume here the no intensive margin case: market power in intermediation distorts only rates, but not the quantity of credit.

Figure A.6: Effect on NPV of Output and Bank Profits Π^B of Income Cap \tilde{r}



Note: Net Present Value (Eq. (53)) of output as a function of the cap \tilde{r} . The NPV is normalized as in Eq.(52). The parameters used are in Table 3. See Appendix F for details on computation. We use here the no intensive margin case where market power in intermediation distorts only rates, but not the quantity of credit. Productivity z and risk σ are persistent according to Eqs.(48)-(49). Income cap is defined as a cap on $\mathcal{I}(\mu)$ of Eq. (56).

Figure A.7: Effect on NPV of Output of a Bank Profit Tax τ_B Redistributed back to Managers



Note: Net Present Value (Eq. (53)) of output as a function a tax on bank's profits τ_B , and collateralizable capital \mathcal{K} evolves as in Eq. (58). The NPV is normalized as in Eq.(52). The parameters used are in Table 3. See Appendix F for details on computation. Productivity z and risk σ are persistent according to Eqs.(48)-(49). The no intensive margin case is where market power in intermediation distorts only rates, but not the quantity of credit. The intensive margin case is where market power in intermediation distorts both interests rate up and quantity of credit down statically.

B Derivations and Additional Results

B.1 Eq.(5)

Taking the FOC in the profit maximization problem of the manager, Eq. (3), with respect to l :

$$(1 - \alpha)A(zk)^\alpha l^{-\alpha} = w \Rightarrow l(a, \theta) = \left(\frac{(1 - \alpha)A}{w} \right)^{1/\alpha} zk(a, \theta) \quad (60)$$

In the expected profit function :

$$\pi^e(r^l, w|a, \theta) = A(zk)^\alpha l^{1-\alpha} - wl - \frac{r^l}{1 + \sigma} k = \left[zA \left(\frac{(1 - \alpha)A}{w} \right)^{(1-\alpha)/\alpha} - zw \left(\frac{(1 - \alpha)A}{w} \right)^{1/\alpha} - \frac{r^l}{1 + \sigma} \right] k \quad (61)$$

$$= \left[z\alpha A^{1/\alpha} \left(\frac{(1 - \alpha)}{w} \right)^{(1-\alpha)/\alpha} - \frac{r^l}{1 + \sigma} \right] k = \left[\kappa(w)z - \frac{r^l}{1 + \sigma} \right] k \quad (62)$$

$$(63)$$

Therefore, as long as $\kappa(w)z > \frac{r^l}{1 + \sigma}$, the manager wants to scale the production up to the collateral constraint. That is the consequence of constant returns to scale: the profit is linear in k , and thus managers either bind at their financial constraints or do not produce at all. Finally, the expected output if a manager is in fact producing

$$y(a, \theta) = A(zk)^\alpha l^{1-\alpha} = \lambda zaA [(1 - \alpha)A/w]^{\frac{1-\alpha}{\alpha}} = \frac{\kappa(w)}{\alpha} \lambda za \quad (64)$$

As expected, risky does not affect output in Eq.(64), but it does affect participation for a given interest rate.

B.2 Cournot Equivalence

Lemma B.1. Cournot Equivalence. *The outcome in Lemma 1 is the same as in the unique Nash Equilibrium in symmetric Cournot game with B banks and demand in Eq. (65) for all banks b when $\mu = \frac{1}{1+B}$.*¹⁸

$$\mathcal{D}^C \left(\pi^{w,b}, \{ \pi^{w,-b} \} \mid \theta, \mu \right) = \pi^{w,b} - \frac{1}{B} \hat{\pi}^e(r, w|\theta) - \frac{1}{B} \sum_{j \neq b} \pi^{w,j} \quad (65)$$

where $-b$ represents the set of all banks that are not b .

Proof. With the demand system in Eq.(65), a demand for a given bank is higher(i) the lower the interest rate it offers, (ii) the lower the autarky profits, (iii) the higher the interest rates of its competitors. In the

¹⁸For simplicity, we assume $D^C \in [0, 1]$, which does happen in equilibrium, so we don't write this constraint explicitly.

demand system in Eq. (65), market power comes fully from the number of banks, \mathcal{B} . For simplicity, we use π^e as profits under perfect competition and $\hat{\pi}^e$ for autarky profits for a manager of a given type θ and collateralizable capital a . As with our result in Lemma 1, we can separate the problem in different productivity-risk profile θ 's and collateralizable capital a .

The problem of the bank is given by (66).

$$\max_{\pi^{w,b}} \left[\pi^e - \pi^{w,b} \right] \left[\pi^{w,b} - \frac{\hat{\pi}^e}{\mathcal{B}} - \frac{1}{\mathcal{B}} \sum_{j \neq B} \pi^{w,j} \right] \quad (66)$$

Taking the FOC (sufficient, due to strict concavity):

$$-2\pi^{w,b} + \frac{\hat{\pi}^e}{\mathcal{B}} + \frac{1}{\mathcal{B}} \sum_{j \neq B} \pi^{w,j} + \pi^e = 0 \quad (67)$$

In a symmetric equilibrium $\pi^{w,b} = \pi^{w,*}, \forall b$:

$$-2\pi^{w,*} + \frac{\mathcal{B}-1}{\mathcal{B}} \pi^{w,*} + \frac{\hat{\pi}^e}{\mathcal{B}} + \pi^e = 0 \Rightarrow \pi^{w,*} = \frac{1}{1+\mathcal{B}} \hat{\pi}^e + \frac{\mathcal{B}}{1+\mathcal{B}} \pi^e \quad (68)$$

which is equivalent to Eq.(13) with $\mu = \frac{1}{1+\mathcal{B}}$. Moreover, note that:

$$\partial_{\pi^{w,b}} \partial_{\pi^{w,-b}} \left\{ \left[\pi^e - \pi^{w,b} \right] \left[\pi^{w,b} - \frac{\hat{\pi}^e}{\mathcal{B}} - \frac{1}{\mathcal{B}} \sum_{j \neq B} \pi^{w,j} \right] \right\} = \frac{1}{\mathcal{B}} > 0 \quad (69)$$

The game is strictly supermodular. As the choice variable is unidimensional and the game is symmetric, any equilibrium must be symmetric. The only symmetric equilibrium that satisfies the optimality condition of each FSP is the one derived. ■

B.3 Aggregates and \tilde{z}

Proof. If z is pareto distributed, the conditional distribution of $z > \underline{z}$ is also Pareto (with the same shape parameter, η_z). Therefore:

$$\theta_\lambda(\tau) \equiv \mathbb{E} \left[\tilde{\lambda}(\tilde{z} | \tau) | \tilde{z} > 1 \right] = \mathbb{E} \left[\tilde{\lambda}(x | \tau) \right]$$

where x is a r.v. that has a Pareto distribution with shape parameter η_z . Therefore, $\theta_\lambda(\tau)$ is not a function of z , only a function of η_z (and other parameters and policies). Moreover, as $\tilde{\lambda}(x | \tau) < \lambda$, we know that $\theta_\lambda(\tau) < \infty$. The proof is equivalent for the other terms. ■

B.4 Eqs. (27)-(28)

Proof. From Eq.(5), the optimal input choice made by managers implies that aggregate capital is

$$\mathcal{K} \equiv \int k(a, z, \sigma) dF_a(a) F_z(z) F_\sigma(\sigma) = \mathcal{X} \int_{z \geq \underline{z}} \lambda(\bar{z} | \tau) dF_z(z) = \theta_\lambda(\tau) \mathcal{X} \underline{z}^{-\eta} \quad (70)$$

Moreover, aggregate labor demand is given by

$$\mathcal{L} = [(1 - \alpha)A/w]^{1/\alpha} \zeta_\lambda(\tau) \underline{z}^{1-\eta} \mathcal{X} = [(1 - \alpha)A/w]^{1/\alpha} \underline{z} \chi_\lambda(\tau) \mathcal{K} \quad (71)$$

$$= \left[\frac{\kappa(w)}{\alpha A} \right]^{1/(1-\alpha)} \underline{z} \chi_\lambda(\tau) \mathcal{K} \quad (72)$$

where $\chi_\lambda \equiv \zeta_\lambda / \theta_\lambda$. Therefore:

$$\kappa(w) = \alpha A \mathcal{L}^{1-\alpha} \mathcal{K}^{\alpha-1} (\underline{z} \chi_\lambda(\tau))^{\alpha-1}$$

Moreover, we know that aggregate output can be written as

$$\mathcal{Y} \equiv \int y(a, z, \sigma) dF_a(a) dF_z(z) dF_\sigma(\sigma) = \frac{\kappa(w)}{\alpha} \zeta_\lambda(\tau) \underline{z}^{1-\eta} \mathcal{X} = \frac{\kappa(w)}{\alpha} \chi_\lambda(\tau) \underline{z} \mathcal{K} \quad (73)$$

Replacing $\kappa(w)$ in the above equation yields

$$\mathcal{Y} = A \mathcal{L}^{1-\alpha} \mathcal{K}^{\alpha-1} (\underline{z} \chi_\lambda(\tau))^{\alpha-1} \underline{z} \chi_\lambda(\tau) \mathcal{K} = \mathcal{Z} \mathcal{K}^\alpha \mathcal{L}^{1-\alpha}, \text{ where } \mathcal{Z} = A (\underline{z} \chi_\lambda(\tau))^\alpha \quad (74)$$

■

B.5 Welfare Effect of Lending Cap on a Share of Loans

Lemma B.2. 1. To maximize output \mathcal{Y} today, a optimal policy pair (ω, \bar{r}) is such that Eq.(44) and Eq.(45) would be satisfied by the unconstrained solution of the bank (*Laissez-faire*).

2. Under a cap \bar{r} in share ω , even though interest rates change for all managers, it is a sufficient statistic for changes in effective cost of finance (include defaults) the effect on those that the constraints bind, and is proportional to the Lagrange multipliers $\nu(\theta)$ of Eq.(45), that is

$$\int_\sigma \int_z \left[\frac{r^g(\theta, \mu) - r^{w,*}}{1 + \sigma} \right] dF_z(z) dF_\sigma(\sigma) = \lambda^{-1} \omega \int_\sigma \int_z \mathbb{1}_{r^g(\theta, \mu) = \bar{r}} \nu(\theta) dF_z(z) dF_\sigma(\sigma) \quad (75)$$

Proof. Maximizing output is equivalent to maximizing $\zeta_\lambda(\bar{r}, \omega)$. We know that:

$$\zeta_\lambda(\bar{r}, \omega) \leq \lim_{\bar{r} \rightarrow \infty} \zeta_\lambda(\bar{r}, \omega) = \lambda \int_{z > \bar{z}} \frac{z}{\bar{z}} dF_z(z) \quad (76)$$

since for $\zeta_\lambda(\bar{r}, \omega)$ we integrate only under $\sigma^*(z)$. From the proof of Lemma 6, we have that

$$\begin{aligned}
& \int_{\sigma} \int_z \left[\frac{r^g(\theta, \mu) - r^{w,*}}{1 + \sigma} \right] dF_z(z) dF_{\sigma}(\sigma) = \\
& = \int_{\sigma} \int_z \mathbb{1}_{r^g(\theta, \mu) \leq \bar{r}} \left[\frac{r^g(\theta, \mu) - r^{w,*}}{1 + \sigma} \right] dF_z(z) dF_{\sigma}(\sigma) + \int_{\sigma} \int_z \mathbb{1}_{r^g(\theta, \mu) > \bar{r}} \left[\frac{r^g(\theta, \mu) - r^{w,*}}{1 + \sigma} \right] dF_z(z) dF_{\sigma}(\sigma) \\
& = \lambda^{-1} \omega (1 - \mu) (1 - \mu) \xi - \lambda^{-1} \omega (1 - \mu) (1 - \mu) \xi + \int_{\sigma} \int_z \mathbb{1}_{r^g(\theta, \mu) = \bar{r}} \nu(\theta) dF_{\sigma}(\sigma) \\
& = \lambda^{-1} \omega \int_{\sigma} \int_z \mathbb{1}_{r^g(\theta, \mu) = \bar{r}} \nu(\theta) dF_z(z) dF_{\sigma}(\sigma)
\end{aligned} \tag{77}$$

■

B.6 Intensive Margin Channel and Output

Lemma B.3. *If $1 + \sigma$ is Pareto distributed with shape parameter η_{σ} and we take $\eta_{\sigma} \rightarrow \infty$, then $\exists \bar{r}$ such that productivity weighted leverage $\mathbb{E}[\tilde{\lambda}(z | \bar{r})z]$ is larger with $\bar{r} < \infty$ than at $\bar{r} = \infty$. Thus optimal policy to maximize \mathcal{Y} is to set a binding cap. Alternatively, if $\eta_z \rightarrow \infty$, productivity weighted leverage is larger with $\bar{r} = \infty$.*

Proof. We can write the extensive margin loss as

$$L^E(\bar{r}) \equiv (1 - F_{\sigma}(\bar{\sigma}))(\lambda - 1) = (\lambda - 1)\bar{r}^{-\eta_{\sigma}} \tag{78}$$

While we can write the intensive margin gain as

$$\begin{aligned}
G^I(\bar{r}) & \equiv \int_{\sigma \leq \bar{\sigma}} \varphi(\mu) \left[\left((1 - \mu)^{-1} \left[1 - \frac{\bar{r}}{g(z)/\lambda} \right] \right)^{\mu^{-1}-1} - 1 \right] \mathbb{P}[z \geq z^{\sigma}] dF_{\sigma}(\sigma) \\
& = \varphi(\mu) \int_{\sigma \leq \bar{\sigma}} \lambda_{\mu}^{\eta_z} \left(\lambda_{\mu} - 1 + \frac{\bar{r}}{1 + \sigma} \right)^{-\eta_z} \left[\left((1 - \mu)^{-1} \left[1 - \frac{\bar{r}}{g(z)/\lambda} \right] \right)^{\mu^{-1}-1} - 1 \right] dF_{\sigma}(\sigma)
\end{aligned} \tag{79}$$

where z^{σ} and λ_{μ} is as in the proof of Lemma 5. If $\eta_z \rightarrow \infty$, the loss is finite, while the gain converges to zero. Note also that

$$\begin{aligned}
G^I(\bar{r}) & > \varphi(\mu) \int_{\sigma \leq \bar{\sigma}} \lambda_{\mu}^{\eta_z} \left(\lambda_{\mu} - 1 + \bar{r} \right)^{-\eta_z} \left[\left((1 - \mu)^{-1} \left[1 - \frac{\bar{r} - 1}{g(z)/\lambda} \right] \right)^{\mu^{-1}-1} - 1 \right] dF_{\sigma}(\sigma) \\
& = \varphi(\mu) \lambda_{\mu}^{\eta_z} \left(\lambda_{\mu} - 1 + \bar{r} \right)^{-\eta_z} \left[\left((1 - \mu)^{-1} \left[1 - \frac{\bar{r} - 1}{g(z)/\lambda} \right] \right)^{\mu^{-1}-1} - 1 \right] [1 - \bar{r}^{-\eta_{\sigma}}]
\end{aligned} \tag{80}$$

If $\eta_{\sigma} \rightarrow \infty$, we have that $G^I(\bar{r}) - L^E(\bar{r}) > 0$.

■

C Proofs

C.1 Lemma 1

Proof. The problem of the bank is (in a simplified notation)

$$\max_{\pi^w} [\pi^e - \pi^w] \left(\frac{\pi^w - \hat{\pi}^e}{\pi^e - \hat{\pi}^e} \right)^{\frac{1}{\mu} - 1} \quad (81)$$

The FOC (sufficient, due to strict concavity in W) is given by

$$\frac{[\pi^{w,*} - \hat{\pi}^e]}{[\pi^e - \hat{\pi}^e]} = \left(\frac{1}{\mu} - 1 \right) [\pi^e - \pi^{w,*}] \Rightarrow \pi^{w,*} = \mu \hat{\pi}^e + (1 - \mu) \pi^e \quad (82)$$

Replacing back at the profit function, the profit per loan (ignoring the demand):

$$\pi^b(\pi^{w,*}(\theta, \mu)) = \pi^e - \pi^{w,*} = \mu g(\theta) \quad (83)$$

■

C.2 Lemma 2.

Proof. We keep implicit the dependence of the key terms on τ to simplify the notation. From the managers' input choices, we have that individual labor demand is given by

$$\frac{wl(\theta)}{1 - \alpha} = y \Rightarrow \mathcal{L}w = (1 - \alpha)\mathcal{Y}$$

Note that the entry tradeoff for $\sigma = 0$ in z is given by:

$$\left(\frac{1 - \alpha}{w} A \right)^{\frac{1 - \alpha}{\alpha}} = \frac{r}{\alpha A \underline{z}}$$

Moreover, we can solve for the entry trade-off in terms of \mathcal{X}, \mathcal{Y} :

$$\frac{w\mathcal{L}}{1 - \alpha} = \mathcal{Y} = \left(\frac{1 - \alpha}{w} A \right)^{\frac{1 - \alpha}{\alpha}} \zeta_\lambda \underline{z}^{1 - \eta} \mathcal{X} = \frac{r}{\alpha A \underline{z}} A \zeta_\lambda \underline{z}^{1 - \eta} \mathcal{X} \quad (84)$$

$$\Rightarrow \underline{z}^\eta = \zeta_\lambda \frac{r}{\alpha} \frac{\mathcal{X}}{\mathcal{Y}} \quad (85)$$

given the entry trade-off in terms of \mathcal{X}, \mathcal{Y} , we can solve for output as a function of labor, \mathcal{L} , and collateralizable capital \mathcal{X} , and not capital \mathcal{K} as in Appendix B.4. In this case:

$$\mathcal{Y} = \mathcal{Z}\mathcal{K}^\alpha \mathcal{L}^{1-\alpha} = A(\mathcal{Z}\mathcal{X}_\lambda)^\alpha (\theta_\lambda \mathcal{X} \mathcal{Z}^{-\eta})^\alpha \mathcal{L}^{1-\alpha} \quad (86)$$

$$= A(\zeta_\lambda)^\alpha \mathcal{Z}^{\alpha-\eta\alpha} \mathcal{X}^\alpha \mathcal{L}^{1-\alpha} = A(\zeta_\lambda)^\alpha \left[\zeta_\lambda \frac{r}{\alpha} \frac{\mathcal{X}}{\mathcal{Y}} \right]^{\alpha/\eta-\alpha} \mathcal{X}^\alpha \mathcal{L}^{1-\alpha} \quad (87)$$

$$= A(\zeta_\lambda)^\alpha (\zeta_\lambda)^{\alpha/\eta-\alpha} \mathcal{X}^{\alpha/\eta} \left[\frac{r}{\alpha} \frac{1}{\mathcal{Y}} \right]^{\alpha/\eta-\alpha} = A(\zeta_\lambda)^{\alpha/\eta} \left[\frac{r}{\alpha} \frac{1}{\mathcal{Y}} \right]^{\alpha/\eta-\alpha} \quad (88)$$

$$\mathcal{Y} = \mathcal{A}[\zeta_\lambda \mathcal{X}]^\gamma \quad (89)$$

Aggregate profits of managers are given by

$$\Pi^M = \int \pi^{w,*}(a, z, \sigma) dF_a(a) dF_z(z) dF_\sigma(\sigma) = \mathcal{X} \mathcal{Z}^{-\eta} r \left[\frac{\zeta_\lambda - \zeta_r}{\theta_\lambda} \right] \theta_\lambda = \mathcal{K} \left[\frac{\zeta_\lambda - \zeta_r}{\theta_\lambda} \right] \quad (90)$$

Aggregate profits of the representative bank

$$\Pi^B = \int \pi^b(\pi^{w,*}(a, z, \sigma)) dF_a(a) dF_z(z) dF_\sigma(\sigma) = r\mathcal{K} \left[\frac{\zeta_r}{\theta_\lambda} - 1 \right] \quad (91)$$

We know that $\alpha\mathcal{Y} = \Pi^B + \Pi^M + r\mathcal{K}$. Thus:

$$\alpha\mathcal{Y} = r\mathcal{X}_\lambda \mathcal{K} \Rightarrow r\mathcal{K} = \theta_\lambda \zeta_\lambda^{-1} \alpha\mathcal{Y} \quad (92)$$

which implies that

$$\Pi^M = \alpha \left[1 - \frac{\zeta_r}{\zeta_\lambda} \right] \mathcal{Y} \quad \text{and} \quad \Pi^B = \alpha \left[\frac{\zeta_r(\tau) - \theta_\lambda(\tau)}{\zeta_\lambda(\tau)} \right] \mathcal{Y} \quad (93)$$

Representative Worker's Income. By definition, $\alpha\mathcal{Y} = \Pi^B + \Pi^M + r\mathcal{K}$. As we assume the representative worker retains ∇ of all profits from the corporate and financial sector (including capital renters), we have that

$$\alpha_W = (1 - \alpha) + \nabla\alpha$$

■

C.3 Lemma 3

Proof. As \mathcal{X} is given in the static problem, we have from Lemma 2 that

$$\arg \max_{\tau} \mathcal{Y} = \arg \max_{\tau} \zeta_\lambda(\tau) \quad (94)$$

Which is equivalent to maximize productivity weighted leverage, that is

$$\mathbb{E}[\tilde{\lambda}(z | \tau)z] \quad (95)$$

and, in particular, we have from Lemma 2

$$\frac{d \ln \mathcal{Y}}{d \ln \tau} = \gamma \cdot \frac{d \ln \zeta_\lambda(\tau)}{d \ln \tau} \quad (96)$$

We can compute the steady state level of income for a worker from the steady state level in collateralizable assets of Eq.(32) in Eq.(29)

$$\mathcal{Y}^{ss} = \mathcal{A} \left[\zeta_\lambda(\tau) \left\{ \frac{\hat{\alpha} \mathcal{A} \zeta_\lambda(\tau)^\gamma \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)} \right]}{\delta - r} \right\}^{\frac{1}{1-\gamma}} \right]^\gamma \quad (97)$$

Thus

$$\arg \max_{\tau} \mathcal{Y}^{ss} = \arg \max_{\tau} \zeta_\lambda(\tau) \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)} \right] = \arg \max_{\tau} [\zeta_\lambda(\tau) - \zeta_r(\tau)] \quad (98)$$

Which is equivalent to maximize profits in the corporate sector, that is

$$\mathbb{E}[\tilde{\lambda}(z | \tau)(z - \tilde{r}(z | \tau))] \quad (99)$$

and, in particular, we have from Lemma 2

$$\frac{d \ln \mathcal{Y}^{ss}}{d \ln \tau} = \frac{\gamma}{1-\gamma} \cdot \frac{d \ln [\zeta_\lambda(\tau) - \zeta_r(\tau)]}{d \ln \tau} \quad (100)$$

■

C.4 Lemma 4

Proof. The problem of the bank can still be separated by (a, θ) . If $r^{w,*}(\theta) \leq \bar{r}$, the bank charges the same interest rate, that is, the lending rate cap is not binding in the problem of

$$\max_{\pi^w} [\pi^e - \pi^w] \left(\frac{\pi^w - \hat{\pi}^e}{\pi^e - \hat{\pi}^e} \right)^{\frac{1}{\mu}-1} \quad (101)$$

s.t.

$$(1 + \sigma) [z\kappa - \lambda^{-1} \pi^w(\theta, \mu)] \leq \bar{r} \quad (102)$$

where the constraint comes from the transformation of end of period profits to interest rate equivalents as

in Eq.(103).

$$r^{w,*}(\theta, \mu) \equiv (1 + \sigma) \left[z\kappa - \lambda^{-1} \pi^{w,*}(\theta, \mu) \right] \quad (103)$$

We know that if the constraint is not binding, the bank makes positive profits - so it still provides the loan. If the constraint is binding, the bank must offer the loan at \bar{r} . The bank will make positive profits if the expected return of the loan is positive, that is

$$\frac{1}{1 + \sigma} \bar{r} - r \geq 0 \Leftrightarrow \sigma \leq \frac{\bar{r}}{r} - 1 \equiv \bar{\sigma}$$

Thus, the bank simply adopts the cut-off strategy described in the text. If the manager has $\sigma > \bar{\sigma}$, the bank does not extend a loan. Otherwise, it lends at the minimum of \bar{r} and $r^{w,*}(\theta, \mu)$, the unconstrained rate. ■

C.5 Lemma 5

Proof. Static. We start by computing $\zeta_\lambda(\bar{r})$, which according to Lemma 3 is all we need to know do determine current output. We have that:

$$\begin{aligned} \zeta_\lambda(\bar{r}) &= \lambda \int_z \int_{\sigma \leq \bar{\sigma}} z dF_\sigma(\sigma) F_z(z) + \lambda \int_z \int_{\sigma > \bar{\sigma}} z dF_\sigma(\sigma) F_z(z) \\ &= \frac{\eta_z}{\eta_z - 1} \{ \lambda F_\sigma(\bar{\sigma}) + (1 - F_\sigma(\bar{\sigma})) \} \\ &= \frac{\eta_z}{\eta_z - 1} [(\lambda - 1) F_\sigma(\bar{\sigma}) + 1] \end{aligned} \quad (104)$$

Given that maximizing current output is equivalent to maximizing $\zeta_\lambda(\bar{r})$, we have that

$$\bar{r}_T^* = \arg \max_{\bar{r}} F_\sigma \left(\frac{\bar{r}}{r} - 1 \right) \quad (105)$$

As F_σ is increasing, we have that the optimum level for the cap is the highest possible.

Let $\tilde{r} = \bar{r}/r$. If $(1 + \sigma)$ is Pareto distributed with parameter η_σ , we have that

$$\zeta_\lambda(\bar{r}) = \frac{\eta_z}{\eta_z - 1} [(\lambda - 1)(1 - \tilde{r}^{-\eta_\sigma}) + 1] \quad (106)$$

Thus, the elasticity of productivity weighted leverage to the credit policy is given by

$$\partial_{\bar{r}} \zeta_\lambda(\bar{r}) \cdot \frac{\bar{r}}{\zeta_\lambda(\bar{r})} = \frac{(\lambda - 1) \eta_\sigma \tilde{r}^{-\eta_\sigma}}{(\lambda - 1)(1 - \tilde{r}^{-\eta_\sigma}) + 1} \quad (107)$$

Steady State. From our result of Lemma 3, we know that maximizing steady state output is equivalent to maximizing $\zeta_\pi(\bar{r}) \equiv \zeta_\lambda(\bar{r}) - \zeta_r(\bar{r})$. To facilitate the exposition, we compute $\zeta_\pi(\bar{r}) - \zeta_\pi(\infty)$, that is, the

difference to the no-intervention value. We can decompose this difference in two terms:

$$\zeta_{\pi}(\bar{r}) - \zeta_{\pi}(\infty) = G(\bar{r}, \mu) - L(\bar{r}, \mu) \quad (108)$$

where $G(\cdot)$ corresponds to the gains of the policy, namely the blue part of panel (b) if Figure 1, while $L(\cdot)$ corresponds to the loss, namely the white part of the same figure. We start by $L(\cdot)$, the losses. The losses are the surplus after banks are paid of managers when they have access to finance (no intervention) to the case that they don't (with intervention), that is

$$\begin{aligned} L(\bar{r}, \mu) &\equiv r(1 - F_{\sigma}(\bar{\sigma}))(1 - \mu) \frac{\lambda - 1}{\lambda} \int_z (z - 1) dF_z(z) \\ &= r(1 - F_{\sigma}(\bar{\sigma}))(1 - \mu) \frac{\lambda - 1}{\lambda} \frac{1}{\eta_z - 1} \end{aligned} \quad (109)$$

The gains are harder to compute, since we only want to consider the region above $\sigma^{all}(\cdot)$ for each z . For each level of sigma, we can define the gains $G_{\sigma}(\bar{r}, \mu)$ as

$$G_{\sigma}(\bar{r}, \mu) \equiv r \frac{1}{1 + \sigma} \left((1 + \sigma) \left[1 + \mu \frac{\lambda - 1}{\lambda} \mathbb{E}[z - 1 | z > z^{\sigma}] \right] - \bar{r} \right) \mathbb{P}[z \geq z^{\sigma}] \quad (110)$$

$$= r \left(1 - \mu \frac{\lambda - 1}{\lambda} + \frac{\eta_z}{\eta_z - 1} \left[\frac{\tilde{r}}{1 + \sigma} - 1 \right] + \frac{\eta_z}{\eta_z - 1} \mu \frac{\lambda - 1}{\lambda} - \frac{\tilde{r}}{1 + \sigma} \right) \cdot \mathbb{P}[z \geq z^{\sigma}] \quad (111)$$

$$= r \frac{1}{\eta_z - 1} \left(\left[\mu \frac{\lambda - 1}{\lambda} - 1 \right] + \frac{\tilde{r}}{1 + \sigma} \right)^{1 - \eta_z} \cdot \left(\mu \frac{\lambda - 1}{\lambda} \right)^{\eta_z} \quad (112)$$

where $\tilde{r} \equiv \frac{\bar{r}}{r}$, z^{σ} is the inverse of $\sigma^{all}(\cdot)$, that is

$$z^{\sigma} = (\lambda - 1)^{-1} \mu^{-1} \lambda \left[\frac{\tilde{r}}{1 + \sigma} - 1 \right] - 1 = \left(\mu \frac{\lambda - 1}{\lambda} \right)^{-1} \left[\frac{\tilde{r}}{1 + \sigma} + \left(\mu \frac{\lambda - 1}{\lambda} \right) - 1 \right]$$

and

$$\mathbb{E}[z - 1 | z > z^{\sigma}] = \frac{\eta_z}{\eta_z - 1} z^{\sigma} - 1$$

Note that in this case we can write both the gains and losses in terms of \tilde{r} , which we use now as our choice variable. We can write the FOC of the gain as

$$\partial_{\tilde{r}} G(\tilde{r}, \mu, \sigma) = - \left(\left[\mu \frac{\lambda - 1}{\lambda} - 1 \right] + \frac{\tilde{r}}{1 + \sigma} \right)^{-\eta_z} \frac{1}{1 + \sigma} \cdot \left(\mu \frac{\lambda - 1}{\lambda} \right)^{\eta_z} \quad (113)$$

which implies

$$\partial_{\tilde{r}, \mu} G(\tilde{r}, \mu, \sigma) > 0 \quad (114)$$

Thus, the FOC of $\zeta_\pi(\bar{r})$ can be written as

$$\int_{\sigma \leq \bar{\sigma}} \partial_{\bar{r}} G(\bar{r}, \mu, \sigma) dF_\sigma(\sigma) + G(\bar{r}, \mu, \bar{\sigma}) f_\sigma(\bar{\sigma}) + f_\sigma(\bar{\sigma})(1 - \mu) \frac{\lambda - 1}{\lambda} \frac{1}{\eta_z - 1} \quad (115)$$

$$= \int_{\sigma \leq \bar{\sigma}} \partial_{\bar{r}} G(\bar{r}, \mu, \sigma) dF_\sigma(\sigma) + \frac{f_\sigma(\bar{\sigma})}{\eta_z - 1} \left[\frac{\lambda - 1}{\lambda} \right] \quad (116)$$

Which yields

$$-\left(\mu \frac{\lambda - 1}{\lambda}\right)^{\eta_z} \int_0^{\bar{\sigma}} \left(\left[\mu \frac{\lambda - 1}{\lambda} - 1 \right] + \frac{\bar{r}}{1 + \sigma} \right)^{-\eta_z} \frac{1}{1 + \sigma} dF_\sigma(\bar{r}) + \frac{f_\sigma(\bar{r})}{\eta_z - 1} \left[\frac{\lambda - 1}{\lambda} \right] = 0 \quad (117)$$

Note that the from Eq.(114), Eq.(117) implies that the optimal \bar{r} is decreasing in μ . Moreover, note that if $\mu = 0$, there are no gains from the policy, and thus no intervention is optimal.

For notation purposes, let $\lambda_\mu \equiv \mu \frac{\lambda - 1}{\lambda}$. We can write Eq. (117) as

$$\lambda_\mu^{\eta_z} \int_0^{\bar{\sigma}} \left(\lambda_\mu - 1 + \frac{\bar{r}}{1 + \sigma} \right)^{-\eta_z} (1 + \sigma)^{-\eta_\sigma - 2} d\sigma = \frac{\bar{r}^{-\eta_\sigma - 1}}{\eta_z - 1} \left[\frac{\lambda - 1}{\lambda} \right] \quad (118)$$

We know that

$$\lambda_\mu^{\eta_z} \left(\lambda_\mu - 1 + \bar{r} \right)^{-\eta_z} \leq \lambda_\mu^{-\eta_z} \left(\lambda_\mu - 1 + \frac{\bar{r}}{1 + \sigma} \right)^{-\eta_z} \leq 1 \text{ for } \sigma \leq \bar{\sigma} \quad (119)$$

At the upper bound of the inequalities in Eq.(119), we have a lower bound for the optimal policy

$$-\frac{(\bar{r}^{lb})^{-\eta_\sigma - 1}}{\eta_\sigma + 1} + \frac{1}{\eta_\sigma + 1} = \frac{(\bar{r}^{lb})^{-\eta_\sigma - 1}}{\eta_z - 1} \left[\frac{\lambda - 1}{\lambda} \right] \Rightarrow \bar{r}^{lb} = \left\{ 1 + \frac{\eta_\sigma + 1}{\eta_z - 1} \left[\frac{\lambda - 1}{\lambda} \right] \right\}^{1/(\eta_\sigma + 1)} \quad (120)$$

At the lower bound, we have an upper bound for the optimal policy

$$\bar{r} = \left\{ 1 + \frac{\eta_\sigma + 1}{\eta_z - 1} \lambda_\mu^{-\eta_z} \left(\lambda_\mu - 1 + \bar{r} \right)^{\eta_z} \left[\frac{\lambda - 1}{\lambda} \right] \right\}^{1/(\eta_\sigma + 1)} \quad (121)$$

$$= \left\{ 1 + \frac{\eta_\sigma + 1}{\eta_z - 1} \lambda_\mu^{-\eta_z} \bar{r}^{\eta_z} \left(\frac{\lambda_\mu - 1}{\bar{r}^{ub}} + 1 \right)^{\eta_z} \left[\frac{\lambda - 1}{\lambda} \right] \right\}^{1/(\eta_\sigma + 1)} \quad (122)$$

$$= \left\{ \bar{r}^{-\eta_z} + \frac{\eta_\sigma + 1}{\eta_z - 1} \lambda_\mu^{-\eta_z} \left(\frac{\lambda_\mu - 1}{\bar{r}} + 1 \right)^{\eta_z} \left[\frac{\lambda - 1}{\lambda} \right] \right\}^{1/(\eta_\sigma + 1 - \eta_z)} \quad (123)$$

$$\equiv \bar{r}^{ub} \quad (124)$$

Therefore, the solution to Eq. (117), \bar{r}_{SS}^* is such that

$$1 < \bar{r}^{lb} < \bar{r}_{SS}^*(\mu) < \bar{r}^{ub}(\mu) < \infty \quad (125)$$

and both the optimal policy and the upper bound are decreasing in μ . ■

C.6 Lemma 6

Interest Rates. We focus on the FOC w.r.t to end of period profits (instead of interest rates). Let ξ be the Lagrange Multiplier of Eq.(44) and $\nu(\theta)$ the cap constraints. In a shortened notation

$$-D + \partial_{\pi} D [\pi^e - \pi] + \xi ((1 - \omega)e(\sigma) - \omega n(\sigma)) \partial_{\pi} D + \nu(\theta) \lambda^{-1} \quad (126)$$

If $e(\theta) = 0$ and $n(\sigma) = 1$

$$-D + \partial_{\pi} D [\pi^e - \pi] - \xi \omega(z) \varepsilon = 0 \quad (127)$$

Therefore:

$$-\pi + \hat{\pi}^e + \varepsilon [\pi^e - \pi] - \zeta \omega(z) \partial_{\pi} D = 0 \Rightarrow \pi = \mu \hat{\pi}^e + (1 - \mu) \pi^e - (1 - \mu) \omega \zeta \quad (128)$$

Now, if $e(\theta) = 1$ and $n(\sigma) = 0$, we can redo the computations above to recover

$$\pi = \mu \hat{\pi}^e + (1 - \mu) \pi^e + (1 - \mu)(1 - \omega) \zeta + \nu(z) (1 + \sigma) \lambda^{-1} \quad (129)$$

If $\nu(\theta) = 0$ (the cap constraint is not binding for this type)

$$\pi = \mu \hat{\pi}^e + (1 - \mu) \pi^e + (1 - \mu)(1 - \omega) \xi \quad (130)$$

if $\nu(\theta) > 0$

$$\pi = \pi^e - \lambda \left[\frac{\bar{r}}{1 + \sigma} - r \right] \quad (131)$$

Note that

$$\lambda^{-1} \nu(z) = \mu \frac{g(z)}{\lambda} - \frac{\bar{r}}{1 + \sigma} + r - \lambda^{-1} (1 - \mu)(1 - \omega) \xi \quad (132)$$

We have that $\nu(z) > 0$ whenever

$$\mu \frac{g(z)}{\lambda} > \frac{\bar{r}}{1 + \sigma} - r - \frac{(1 - \mu)(1 - \omega) \xi}{\lambda} = \frac{\bar{r}}{1 + \sigma} - r - (1 - \omega) \Upsilon(\theta, \omega) \quad (133)$$

Let $z^{l, \sigma}$ be such that the inequality above holds as an equality. The points (z, σ) where $z \geq z^{l, \sigma}$ and loans are made below the cap, are those where the cap is actually binding - i.e., those that are in fact receiving loans at the cap.

Allocation. Taking the FOC w.r.t. to $e(\theta), n(\theta)$, we can pin down the allocation. Consider first the case

where $e(\theta) + n(\theta) = 1$, $e(\theta) > 0$. Deriving with respect to $e(\theta)$ delivers

$$\left[\frac{r^e(\theta)}{1+\sigma} - r \right] \mathcal{D}(r^e(\theta)) - \left[\frac{r^n(\theta)}{1+\sigma} - r \right] \mathcal{D}(r^n(\theta)) + \lambda^{-1} \xi (1-\omega) \mathcal{D}(r^e(\theta)) + \lambda^{-1} \xi \omega \mathcal{D}(r^n(\theta)) = 0$$

thus

$$\left[\frac{r^e(\theta)}{1+\sigma} - r + \lambda^{-1} \xi (1-\omega) \right] \mathcal{D}(r^e(\theta)) = [\mu g(z) + \lambda^{-1} \mu \xi \omega] \mathcal{D}(r^n(\theta)) \quad (134)$$

If $r^e(\theta) < \bar{r}$, we can re-write the equation above as

$$[\mu g(z) + \mu \xi \omega] \mathcal{D}(r^e(\theta)) = [\mu g(z) + \mu \xi \omega] \mathcal{D}(r^n(\theta))$$

This equality is impossible to satisfy, since $\mathcal{D}(r^e(\theta)) > \mathcal{D}(r^n(\theta))$. In particular, we have that

$$\mathcal{D}(r^e(\theta)) = \begin{cases} \varphi(\mu) \left(1 + \frac{(1-\omega)\xi}{g(z)} \right)^{\mu^{-1}-1}, & \text{if } r^e(\theta) < \bar{r} \\ \frac{\varphi(\mu)}{(1-\mu)^{\mu^{-1}-1}} \left(1 + \frac{r - \bar{r}}{g(z)/\lambda} \right)^{\mu^{-1}-1}, & \text{if } r^e(\theta) = \bar{r} \end{cases} \quad (135)$$

$$\mathcal{D}(r^n(\theta)) = \varphi(\mu) \left(1 - \frac{\omega \xi}{g(z)} \right)^{\mu^{-1}-1} \quad (136)$$

Therefore, it must be that $r^e(\theta) = \bar{r}$ at $\sigma^*(z)$. Note that for it to be possible to satisfy Eq. (134), it must be that

$$\frac{\bar{r}}{1+\sigma} - r + \lambda^{-1} \xi (1-\omega) > 0 \quad (137)$$

and $g(z) > \omega \xi$ (substituting in the demand). If $e(\theta) + n(\theta) < 1$, the indifference between offering a loan below the cap and not offering at all is given by deriving with respect to $e(\theta)$, which delivers

$$\frac{r^e(\theta)}{1+\sigma} - r + \lambda^{-1} \xi (1-\omega) = 0$$

Again, it must be that $r^e(\theta) = \bar{r}$ in this case, or this equation is not possible to be satisfied. Remember that we assume exogenously that these loans are not made below r .

Equivalently, the bank is indifferent between offering a loan or not above the cap when

$$\mathcal{D}(r^n(\theta)) = 0 \Rightarrow g(z) = \omega \xi$$

For notation purposes, denote \bar{z} to be the productivity level such that $g(z) = \omega \xi$ and $\sigma^*(z)$ as the border of the indifference region of the bank. Note that the problem of the bank is linear in $e(\theta), n(\theta)$, thus if Eq. (137) and $g(z) < \omega \xi$ it must be the case that $e(\theta) = 1, n(\theta) = 0$, since the FOC w.r.t. to $e(\theta)$ would always be positive. Note also that Eq.(137) is decreasing in θ , so at this section, any point below the one that makes the Eq.(137) hold with an equality must also have $e(\theta) = 1, n(\theta) = 0$.

Equivalently, if Eq. (137) and $g(z) \geq \omega \xi$ it must be the case that $e(\theta) + n(\theta) = 1$, and the curve is character-

ized by Eq. (134). Note that the LHS is decreasing in σ (through the demand channel also), while the RHS is increasing in z . Thus, the border $\sigma^*(z)$ is decreasing in this region.

For points that violate Eq. (137) and have $g(z) < \omega\xi$, we have that $n(\theta)$ can be anything, since the demand is zero anyway.

D Dynamic Policy

We focus now on to the optimal policy to maximize the utility of the representative worker over time, that is, Eq.(17). As we assume $\rho = r$, this is the same as maximizing the current value of income. Before presenting our result, we define $\mathcal{S}(\tau)$ as in Eq.(138). This is the term that captures managers savings from production to the next period (it is the term that multiplies \mathcal{X}^γ in Eq.(32)). To maximize savings and capital accumulation, the planner would maximize $\mathcal{S}(\tau)$.

$$\mathcal{S}(\tau) \equiv \hat{\alpha} \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)} \right] \mathcal{A}[\zeta_\lambda(\tau)]^\gamma \quad (138)$$

Our result for the dynamics is in Lemma D.1. The optimal policy over time is a combination of maximizing capital accumulation and the static outcome. The relative weight given for each is the co-state, which captures the marginal value of collateralizable capital in the economy. If the co-state is large, i.e., the economy has little capital, we have that the optimal policy would focus on reducing cost of finance and increasing capital accumulation in the beginning of a development process, reverting to focus relatively more in the static policy of Lemma 3 over time.

Lemma D.1. Dynamics. *The solution to the dynamic problem of maximizing Eq.(17) choosing a path $\{\tau(t)\}_t$ corresponds to the globally stable saddle path ODE system of Eqs.(139)-(141), where h is the co-state. In particular, in a capital accumulation path ($\dot{\mathcal{X}} > 0$), the optimal policy involves focusing on capital accumulation in the beginning of the development process (Eq. (138)), and in increasing productivity weighted leverage (as in Eq. (33)) later on.*

$$0 = \partial_\tau [\zeta_\lambda(\tau)^\gamma] + h \cdot \partial_\tau \mathcal{S}(\tau) \quad (139)$$

$$\dot{h} = -\gamma \mathcal{X}^{\gamma-1} \zeta_\lambda(\tau)^\gamma + [\delta - \gamma \mathcal{X}^{\gamma-1} \mathcal{S}(\tau)] h \quad (140)$$

$$\dot{\mathcal{X}} = \mathcal{S}(\tau) \mathcal{X}^\gamma + (r - \delta) \mathcal{X} \quad (141)$$

Proof. If we allow $\tau(t)$ to vary, we can write the dynamic problem as

$$\max_{\tau(t)} \int e^{-\rho t} \ln c(t) \quad (142)$$

s.t.

$$\dot{\mathcal{X}} = \hat{\alpha} \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)} \right] \mathcal{Y} + (r - \delta) \mathcal{X} \quad (143)$$

$$c + \dot{b} = rb + \alpha_W \mathcal{Y} \quad (144)$$

Which is equivalent to maximizing discounted value (as $\rho = r$) of output Y , that is

$$\max_{\tau(t)} \int e^{-\rho t} [\zeta_\lambda(\tau) \mathcal{X}]^\gamma dt \quad \text{s.t.} \quad \dot{\mathcal{X}} = \hat{\alpha} \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)} \right] \mathcal{A}[\zeta_\lambda(\tau) \mathcal{X}]^\gamma + (r - \delta) \mathcal{X} \quad (145)$$

We write the current value Hamiltonian as

$$\mathcal{H} = [\zeta_\lambda(\tau)\mathcal{X}]^\gamma + h[\mathcal{S}(\tau)\mathcal{X}^\gamma + (r - \delta)\mathcal{X}] \quad (146)$$

where \mathcal{H} is the current value Hamiltonian, and $\mathcal{S}(\tau) \equiv \hat{a} \left[1 - \frac{\zeta_r(\tau)}{\zeta_\lambda(\tau)} \right] \mathcal{A}[\zeta_\lambda(\tau)]^\gamma$. The necessary conditions for an optimum are given by

$$\mathcal{H}_\tau = 0 \Rightarrow \gamma [\zeta_\lambda(\tau)]^{\gamma-1} \partial_\tau \zeta_\lambda(\tau) + h \cdot \partial_\tau \mathcal{S}(\tau) = 0 \quad (147)$$

$$\mathcal{H}_\mathcal{X} = \rho h - \dot{h} \Rightarrow \gamma \mathcal{X}^{\gamma-1} \zeta_\lambda(\tau)^\gamma + [\gamma \mathcal{X}^{\gamma-1} \mathcal{S}(\tau) - \delta] h = -\dot{h} \quad (148)$$

$$\dot{\mathcal{X}} = \mathcal{S}(\tau)\mathcal{X}^\gamma + (r - \delta)\mathcal{X} \quad (149)$$

We want to analyze the system above in terms of (\mathcal{X}, h) using a phase diagram. The locus of $\dot{\mathcal{X}} = 0$ is given by \mathcal{X}^{ss} . For $\mathcal{X} > \mathcal{X}^{ss}$, we have that $\dot{\mathcal{X}} < 0$ (and the opposite for $>$). For $\dot{h} = 0$

$$\dot{h} = 0 \Rightarrow h^{ss} = \frac{\gamma [\mathcal{X}^{ss}]^{\gamma-1} \zeta_\lambda(\tau)^\gamma}{\delta - \gamma [\mathcal{X}^{ss}]^{\gamma-1} \mathcal{S}(\tau)} = \left[\hat{a} \mathcal{A} \frac{\zeta_\pi(\tau)}{\zeta_\lambda(\tau)} \right]^{-1} \frac{\gamma}{\delta(1 - \gamma) + \gamma r} \quad (150)$$

which is decreasing in \mathcal{X} . We plot the phase diagram in Figure D.1. The state space can be divided in four quadrants. It is easy to see that $\dot{h} > 0$ for all points northeast of the $\dot{h} = 0$ locus, and $\dot{\mathcal{X}} > 0$ for all points west of $\dot{\mathcal{X}} = 0$. It then follows that the system is saddle-path stable. With Inada conditions on the utility function, the saddle path is the unique solution to the problem. Over the path of $\dot{\mathcal{X}} > 0$ (a development path), we have that $\dot{h} < 0$ and, thus, the optimal policy shifts from maximizing savings to minimizing static misallocation (Eq. 147). ■

D.1 Optimal Path of a Lending Cap on All Loans

Lemma D.2. Optimal Path. *The optimal path for the cap $\{\bar{r}_D^*(t; \mu)\}$ is (i) binding for some managers $\forall t$, but larger than $\bar{r}_{SS}^*(\mu)$ for $t \rightarrow \infty$, (ii) increasing through the development process (when $\dot{\mathcal{X}} > 0$), and (iii) decreasing in μ for $t \rightarrow \infty$.*

Proof. We can rewrite $\mathcal{S}(\bar{r})$ as

$$\mathcal{S}(\bar{r}) = \hat{a} \mathcal{A}[\zeta_\lambda(\bar{r}) - \zeta_r(\bar{r})] \zeta_\lambda(\bar{r})^{\gamma-1} \quad (151)$$

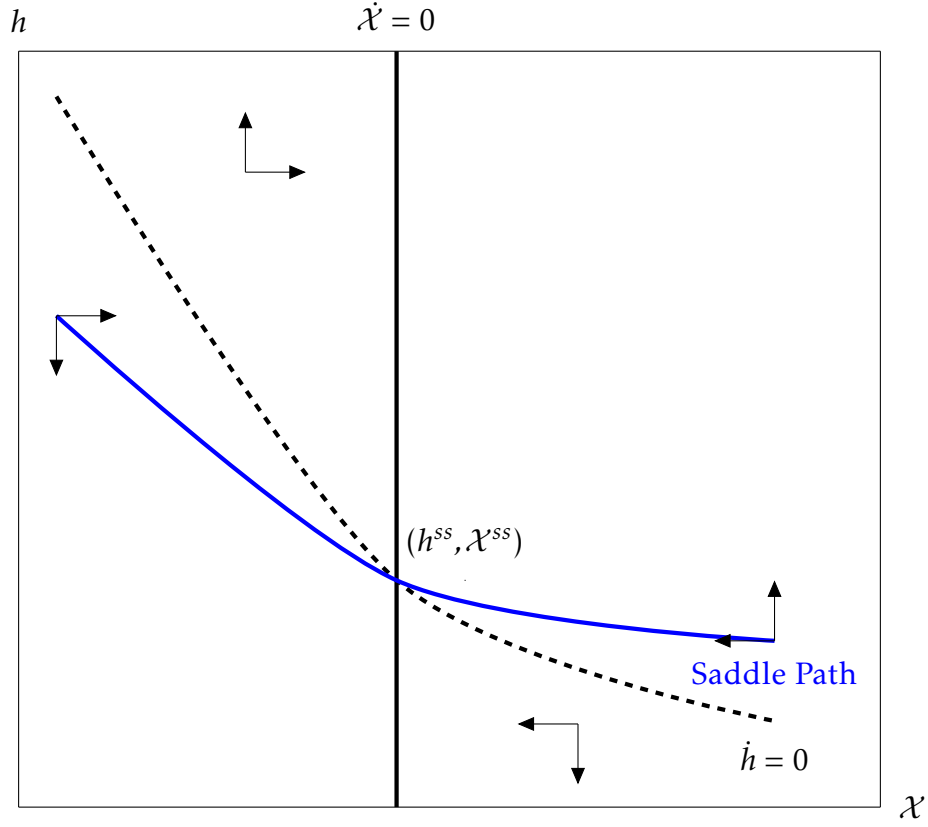
Therefore

$$\partial_{\bar{r}} \mathcal{S}(\bar{r}) = \hat{a} \mathcal{A} \left[\partial_{\bar{r}} \zeta_\pi(\bar{r}) \cdot \zeta_\lambda(\bar{r})^{\gamma-1} - (1 - \gamma) \zeta_\pi(\bar{r}) \zeta_\lambda(\bar{r})^{\gamma-2} \cdot \partial_{\bar{r}} \zeta_\lambda(\bar{r}) \right] \quad (152)$$

From Eq.(117),

$$\bar{r} > \bar{r}_{SS}^* \Rightarrow \partial_{\bar{r}} \mathcal{S}(\bar{r}) < -(\eta_\sigma + 1)^{-1}$$

Figure D.1: Optimal Credit Policy: Dynamics



Taking limits on Eq. (107)

$$\lim_{\bar{r} \rightarrow \infty} \partial_{\bar{r}} \zeta_{\lambda}(\bar{r}) = 0$$

Therefore, any point in the trajectory that satisfies Eqs.(139)-(141) must have

$$\tilde{r}_D^*(t) < \infty$$

that is, some intervention is always optimal. In the steady state the policy must be such that $\partial_{\bar{r}} \mathcal{S}(\bar{r}) < 0$, i.e., not only focused on savings.

Note also that we can rewrite Eq.(139) as

$$\gamma \partial_{\tau} \zeta_{\lambda}(\tau) + h \hat{\alpha} \mathcal{A} \partial_{\tau} \zeta_{\pi}(\tau) + h(\gamma - 1) \hat{\alpha} \mathcal{A} \zeta_{\pi}(\tau) [\zeta_{\lambda}(\tau)]^{-1} \partial_{\tau} \zeta_{\lambda} = 0 \quad (153)$$

Which implies

$$\left[1 - \gamma - \tilde{h}^{-1} \frac{\tilde{\zeta}_{\lambda}(\bar{r})}{\tilde{\zeta}_{\pi}(\bar{r})} \left(\frac{\delta}{\delta - r} - \gamma \right) \right] \partial_{\bar{r}} \ln \zeta_{\lambda}(\bar{r}) = \partial_{\bar{r}} \ln \zeta_{\pi}(\bar{r}) \quad (154)$$

where $\tilde{x} = x(t)/x^{ss}$ represents percentage deviations from the steady state. We know that $\tilde{h} \rightarrow 1$, $\tilde{\zeta}_{\lambda}(\bar{r}) \rightarrow 1$

and $\tilde{\zeta}_\pi(\bar{r}) \rightarrow 1$ when $\dot{\lambda} > 0$, which guarantees that

$$1 - \gamma - h^{-1} \frac{\tilde{\zeta}_\lambda(\bar{r})}{\tilde{\zeta}_\pi(\bar{r})} \left(\frac{\delta}{\delta - r} - \gamma \right) = 1 - \frac{\delta}{\delta - r} < 0$$

Thus, at $t \rightarrow \infty$

$$\partial_{\bar{r}} \ln \zeta_\pi(\bar{r}) < 0$$

which implies the optimal policy is larger than the steady state one as $t \rightarrow \infty$. Finally, as $\partial_{\mu, \bar{r}} \ln \zeta_\pi(\bar{r}) < 0$ for $t \rightarrow \mu$, the optimal policy is decreasing in μ . ■

E Simplified Problem with a Cap on a Share of Loans

We consider now a simpler case where the cap only applies for a share of loans that allows for a tighter analytical solution. There are two simplifications: the ability of the financial sector to change rates after the policy is implemented and no heterogeneity in productivity. The government imposes that a share ω of loans must be made at exactly $\bar{r} \in (0, r]$ and that the rates for those above the cap cannot change from the unconstrained case. The constraint of $\bar{r} < r$ implies that the financial sector is making a loss at this loans, while $\bar{r} > 0$ implies the bank still prefers to receive the interest rate on the loan (the principal is always repaid). This simplification is useful to discuss only the allocation of loans (without changes of rates) for different levels of risk, which is one key innovation of our model.

The problem of the bank now is to solve the problem in Eq. (155) (we hide the dependence on of the problem on parameters, such as μ , for clarity).

$$\max_{n(\sigma), e(\sigma) \geq 0} \int_{\sigma} \left\{ \varphi(\mu) \mu g(z) n(\sigma) + \left[\frac{\bar{r}}{1 + \sigma} - r \right] e(\sigma) \right\} F_{\sigma}(\sigma) \quad (155)$$

subject to

$$\frac{\omega(z)}{1 - \omega(z)} \varphi(\mu) \int_{\sigma} n(\sigma) dF_{\sigma}(\sigma) = \int_{\sigma} e(\sigma) dF_{\sigma}(\sigma) \quad (156)$$

And

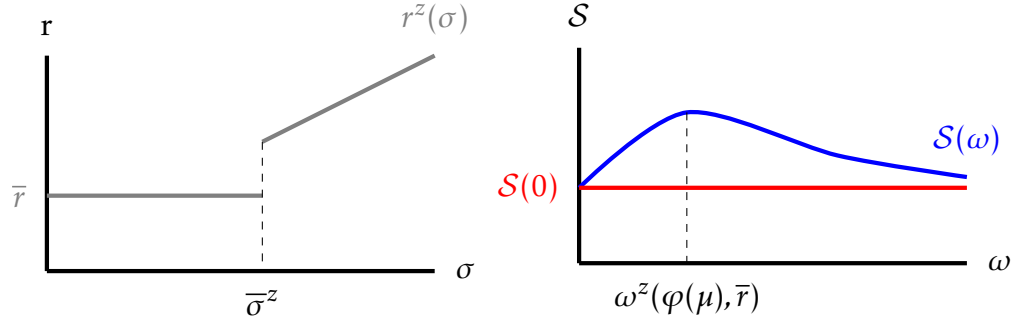
$$e(\sigma) + n(\sigma) \leq 1, \quad \forall \sigma \quad (157)$$

A few comments are in order. The choices made to one specific risk level are not independent of others, since the balance sheet constraint in Eq. (156) must be satisfied. The financial sector must choose now the share $e(\sigma)$ of agents at level σ to lend at \bar{r} , and the share $n(\sigma)$ to lend the unconstrained rate from Lemma 4. The objective function in Eq. (155) already substitutes and profits for managers that receive loans at unconstrained rates. We also assume that the relative share of managers of a given type that takes the contract in equilibrium between the constrained and unconstrained case is given by $\varphi(\mu)$ (see Section 3.3).

We characterize the solution of the problem of the bank in Lemma E.1. The problem of the bank can be transformed in a problem where the financial sector chooses $\bar{\sigma}^z$, below which $e(\sigma) = 1$ and $n(\sigma) = 0$, i.e., all managers receive offers of $r = \bar{r}$ for $\sigma < \sigma^*$. For $\sigma > \bar{\sigma}^z$, however, only a share $s \leq 1$ of managers receive the possibility of taking loans, that is, $e(\sigma) = 0$, $n(\sigma) = s$.¹⁹ This threshold $\bar{\sigma}^z$ is non-monotone in ω . For small values of ω (in the first case of Lemma E.1), $\bar{\sigma}^z$ and the total lending in the economy is increasing. On the other hand, if the value of ω is above a threshold (second case of Lemma E.1), the financial sector reduces its balance sheets to preserve profitability, i.e.: offering less loans (a small share) above the cap,

¹⁹We assume that this share is homogeneously distributed among types, but the bank is different over this distribution in the maximization problem.

Figure E.1: Interest rates and Spread as a function of risk, σ , and share of subsidized loans, ω



but to focus loans below the cap on the safer agents.

Moreover, we show that the threshold level for ω , the point which the financial sector starts to reduce its balance sheet, is increasing in \bar{r} . If the cap imposed is not too stringent (i.e., the losses for banks in this case, since we assume $\bar{r} < r$), there is more space to ask banks to lend at the rate. The threshold ω is, however, decreasing in the quantity distortion through competition, $\varphi(\mu)$, since the bank must be lending enough to cover for the balance sheet constraint. Therefore, this indicates that a mix of the two policy instruments (\bar{r}, ω) could be useful to increase total credit.

Figure E.1 pictorially illustrates the results in Lemma E.1 on the interest rates and spreads. Spreads in the constrained case are always larger than in the unconstrained case for loans made above \bar{r} , as in Figure E.1, where we plot interest rates as a function of ω . As the free market now is only composed of managers with $\sigma > \bar{\sigma}$, then spreads of free loans are simply a function of the pool of borrowing agents, which are riskier now. Note that spreads are non-monotone in the policy. When the government increases the share ω sufficiently, $\bar{\sigma}^z$ is decreasing and so is the spread (the second case in Lemma E.1). As in the case of spreads, total credit in this economy is also non-monotone in ω . For small values of ω , the financial sector satisfies the constraint without having to change much of the allocation, that is, the profits on free loans are enough to cover the losses. Differently from the spread, however, values of ω sufficiently high decrease lending compared to the unconstrained allocation.

Lemma E.1. Allocation. *There is a cut-off $\omega^z(\varphi(\mu), \bar{r})$ is the balance sheet requirement such that*

1. *For $\omega \leq \omega^z(\varphi(\mu), \bar{r})$, the financial sector offers loans at \bar{r} for $\sigma \leq \bar{\sigma}^z$ and non-subsidized loans otherwise, where $\bar{\sigma}^z$ solves*

$$\frac{F_{\sigma}(\bar{\sigma}^z)}{1 - F_{\sigma}(\bar{\sigma}^z)} = \frac{\omega}{1 - \omega} \varphi(\mu)$$

2. *For $\omega > \omega^z(\varphi(\mu), \bar{r})$, the financial sector offers loans at \bar{r} for $\sigma \leq \bar{\sigma}^z$ and non-subsidized loans for a share s of other managers, where $\bar{\sigma}^z$ is given by*

$$\bar{\sigma}^z = \frac{\bar{r}}{r - \mu \frac{g(z)}{\lambda} \frac{1-\omega}{\omega}} - 1$$

and the share s is given by

$$s = \frac{1-\omega}{\omega} \varphi(\mu)^{-1} \frac{F(\bar{\sigma}^z)}{1-F(\bar{\sigma}^z)}$$

where $\omega^z(\varphi(\mu), \bar{r})$ is increasing in both arguments.

Spreads. Observed spreads for loans made at free market rates, $\mathcal{S} \equiv \mathbb{E}[(r^{w,*}(\sigma) - r) \mid \sigma > \bar{\sigma}^z]$, are always larger than in the case of $\omega = 0$, increasing for $\omega \leq \omega^z(\mu, \varphi(\mu), \bar{r})$ and decreasing for $\omega > \omega^z(\mu, \varphi(\mu), \bar{r})$.

Total Credit. Define the difference in total credit as $\Delta T^z(\omega)$ for the credit under a share ω of capped loans at \bar{r} minus the credit in the unconstrained problem ($\omega = 0$). Then, $\Delta T^z(\omega)$ is increasing for $\omega \leq \omega^z(\mu, \varphi(\mu), \bar{r})$, decreasing for $\omega > \omega^z(\mu, \varphi(\mu), \bar{r})$, eventually reaching $\Delta T^z(\omega) < 0$ for ω close enough to one.

Proof. Problem of the financial sector is given by

$$\max_{n(\sigma), e(\sigma) \geq 0} \int_{\sigma} \left\{ \varphi(\mu) \mu g(z) n(\sigma) + \left[\frac{\bar{r}}{1+\sigma} - r \right] e(\sigma) \right\} dF_{\sigma}(\sigma) \quad (158)$$

subject to

$$\frac{\omega(z)}{1-\omega(z)} \varphi(\mu) \int_{\sigma} n(\sigma) dF_{\sigma}(\sigma) = \int_{\sigma} e(\sigma) dF_{\sigma}(\sigma) \quad (159)$$

And

$$e(\sigma) + n(\sigma) \leq 1, \quad \forall \sigma \quad (160)$$

The idea is to show that if there is σ_1, σ_0 with $e(\sigma_1) = 1$ and $e(\sigma_0) = 0$ there is an alternative plan, which still satisfies the constraint on the balance sheet, that delivers a higher profit. Assume simply that there are two types (σ_1, σ_0) that receive positive weights. Consider a weight reshift, that is, the changes in the profit function from providing subsidies for a type σ_0 instead of σ_1 . This change, C is given by:

$$\begin{aligned} C &= \Pi^b(W^*(\sigma_1, \mu) \mid \theta, \mu) + \Pi^b(\bar{W}(\sigma_0, \mu) \mid \theta, \mu) \\ &\quad - \Pi^b(W^*(\sigma_0, \mu) \mid \theta, \mu) - \Pi^b(\bar{W}(\sigma_1, \mu) \mid \theta, \mu) \\ &= \int_{\sigma_0}^{\sigma_1} \partial_{\sigma} \Delta \Pi^b(\sigma) d\sigma \end{aligned}$$

where: $\Delta \Pi^b$ is the difference between the free market and subsidized profits for a given type. Therefore, $\partial_{\sigma} \Delta \Pi^b(\sigma) > 0 \Rightarrow C > 0$. Therefore:

$$\partial_{\sigma} \Delta \Pi^b(\sigma) = \partial_{\sigma} \left[\varphi(\mu) \mu g(z) - \frac{\bar{r}}{1+\sigma} + r \right] = \frac{\bar{r}}{(1+\sigma)^2} > 0 \quad (161)$$

This implies that there is threshold rule and that for any point below the threshold, we have that $e(\sigma) = 1$. Note that for points above the threshold, we will have that $e(\sigma) = 0$, but $n(\sigma)$ is still undetermined. Take

the FOC w.r.t. to $n(\sigma)$ ($l_\omega, l_1(\omega)$ the multipliers)

$$\varphi(\mu)\mu g(z) + \frac{\omega}{1-\omega}\varphi(\mu)l_\omega + l_1(\omega)$$

which is not a function of σ . Therefore, the financial sector is indifferent between who gets the free market loans above the cut-off. Thus, we can re-write the problem as choosing the cut-off, $\bar{\sigma}^z$ as in Eq. (162). The equation for the share, s , of agents above the cut-off that receive loans is obtained by solving the balance sheet constraint. (162).

$$\max_{\bar{\sigma}^z} \varphi(\mu)\mu g(z)s(1 - F_\sigma(\bar{\sigma}^z)) + \int_{\sigma < \bar{\sigma}^z} \left[\frac{\bar{r}}{1+\sigma} - r \right] dF_\sigma(\sigma) \quad (162)$$

subject to the share s of agents above $\bar{\sigma}^z$ that guarantees the balance sheet condition is satisfied is given by

$$s = \min \left\{ \frac{1-\omega}{\omega} \varphi(\mu)^{-1} \frac{F(\bar{\sigma}^z)}{1 - F(\bar{\sigma}^z)}, 1 \right\} \quad (163)$$

Lets solve a relaxed version of the problem, where we relax the constraint that $s \leq 1$. Denote the argmax of this relaxed problem as $\bar{\sigma}^r$. The problem of the financial sector be simplified to

$$\max_{\bar{\sigma}^r} \frac{1-\omega}{\omega} \mu g(z) F_\sigma(\bar{\sigma}^r) + \int_{\sigma < \bar{\sigma}^r} \left[\frac{\bar{r}}{1+\sigma} - r \right] dF_\sigma(\sigma) \quad (164)$$

Taking the FOC

$$\frac{1-\omega}{\omega} \mu g(z) f(\bar{\sigma}^r) + \left[\frac{\bar{r}}{1+\bar{\sigma}^r} - r \right] f(\bar{\sigma}^r) = 0$$

Therefore

$$\frac{\bar{r}}{1+\bar{\sigma}^r} = r - \mu g(z) \frac{1-\omega}{\omega}$$

Furthermore, define $\bar{\sigma}^s$ as the one that would guarantee $s = 1$, that is

$$\frac{1-\omega}{\omega} \varphi(\mu)^{-1} \frac{F_\sigma(\bar{\sigma}^s)}{1 - F_\sigma(\bar{\sigma}^s)} = 1$$

The solution is thus given by

$$\bar{\sigma}^z = \min \{ \bar{\sigma}^s, \bar{\sigma}^r \} \quad (165)$$

since for $\sigma < \bar{\sigma}^r$, the FOC is positive. The only reason to not go over to $\bar{\sigma}^r$ is if the constraint of $s = 1$ is binding, in which case the solution is given by the smaller $\bar{\sigma}^s$ value. Moreover, note that for $\omega \rightarrow 0$, only $\bar{\sigma}^s$ is well defined. For $\omega \rightarrow 1$, $\bar{\sigma}^s > \bar{\sigma}^r$. As $\bar{\sigma}^r$ is decreasing in ω and $\bar{\sigma}^s$ is increasing in ω , it must be the case that there is a unique $\omega^z(\varphi(\mu), \bar{r})$ s.t.

$$\bar{\sigma}^z = \begin{cases} \bar{\sigma}^s, & \omega \leq \omega^z(\mu, \varphi(\mu)) \\ \bar{\sigma}^r, & \omega > \omega^z(\mu, \varphi(\mu)) \end{cases} \quad (166)$$

where $\omega^z(\varphi(\mu), \bar{r})$ solves

$$\frac{\bar{r}}{r - \mu \frac{g(z)}{\lambda} \frac{1-\omega}{\omega}} = 1 + F_{\sigma}^{-1} \left(\frac{1}{\frac{1-\omega}{\omega} \varphi(\mu) + 1} \right)$$

which is increasing in $\varphi(\mu), \bar{r}$. The results on spreads and capital are trivial given the equation for σ_z and its non-monotonicity, as explained in the main text. ■

F Calibration and Numerical Solution

In this section we describe how we compute the equilibrium. The code is inspired in [Moll \(2014\)](#). We first discuss how to compute the steady state for a given distribution $\psi(\theta)$ and then the transition dynamics.

F.1 Steady State.

The objective is to find $\{\mathcal{X}^{SS}, w^{SS}\}$, which jointly with Lemma [H](#), the condition $\dot{\mathcal{X}} = 0$ and the distribution $\psi(\theta)$ determine the equilibrium. We take $\psi(\theta)$ as given to compute the steady-state. For the i.i.d. case, $\psi(\theta)$ is the multiplication of the invariant distributions of the O-U processes for z and σ . For a given wage, w , we can compute the aggregators ζ_λ and ζ_π . We use $\dot{\mathcal{X}} = 0$ to recover the level of collateralizable capital consistent with this wage as in Eq.([167](#)). Given this level of collateralizable capital, we use the difference $(1 - \alpha)\mathcal{Y} - w$ to compute the excess demand/supply of labor. We find the equilibrium level of wages - and thus all else in the economy - through a bisection method.

$$\mathcal{X}^{SS} = \left\{ \frac{\hat{\alpha}}{\delta - r} \frac{\zeta_\pi^p(\tau, w)}{\zeta_\lambda^p(\tau, w)} A(\underline{z}(w)[1 - F(\underline{z}(w))]) \zeta_\lambda^p(\tau)^\alpha \right\}^{1/(1-\alpha)} \quad (167)$$

F.2 Transition Dynamics.

The objective is to find the sequences $\{\psi(\theta, t), X(t), w(t)\}$, which jointly with Lemma [H](#) determine fully the equilibrium. We approximate this functions at $n = 1, \dots, N$ points in the time dimension. More specifically, we use $T = 100$ and $N = 500$. We approximate productivity z and riskiness σ using equispaced grids. For z , we denote a point in the grid by z_j , $j = 1, \dots, J$, with $J = 400$, while for σ we denote σ_r , $r = 1, \dots, R$, with $R = 400$. Let Δt be the distance between two time grid points, and Δz and $\Delta \sigma$ the equivalent for z and σ . For simplicity, we adopt the notation $\psi_{j,r}^n \equiv \psi(z_j, \sigma_r, t^n)$.

Before diving into the details of the algorithm, we want to focus on the idea. From Lemma [H](#), we know that given $\{X^n, \psi_{j,r}^n, w^n\}$, we can compute the change in $\{X^n, \psi_{j,r}^n\}$ and thus how the economy evolves over time. For any point in time, we use $\{X^n, \psi_{j,r}^n\}$ and compute the equilibrium wages (and the other variables) by using the bisection method, as in the case for the steady states. Given the equilibrium wages, we use the differential equations in Eqs.([182](#))-([183](#)) to compute $\{X^{n+1}, \psi_{j,r}^{n+1}\}$ and, therefore, wages in the following period. Therefore, the key question is how we take Eqs.([182](#))-([183](#)) to the computer.

Step 1. Equilibrium wages w^n . Given $\{X^n, \psi_{j,r}^n\}$, compute outcome of the bank problem and total output. We compute the demand for labor from:

$$l^{d,n} = \frac{1 - \alpha}{w} \mathcal{Y}^n$$

We then use the bisection method to find the equilibrium wages in time n , w^n .

Step 2. Collaterizable Capital. We use a forward-difference approximation for \mathcal{X} , that is

$$\mathcal{X}(t^n) \approx \frac{\mathcal{X}^{n+1} - \mathcal{X}^n}{\Delta t}$$

Therefore, from Eq.(180), we can write:

$$\mathcal{X}^{n+1} = \Delta t \left[\hat{\alpha} \frac{\zeta_{\pi}^{p,n}}{\zeta_{\lambda}^{p,n}} \frac{\mathcal{Y}^n}{\mathcal{X}^n} - (r - \delta) \right] \mathcal{X}^n + \mathcal{X}^n$$

Step 3. Collaterizable Capital Shares. From the PDE 181, we can write:

$$\partial_t \psi(\theta, t) = A(\theta, t) \psi(\theta, t) + B_Z(z) \partial_z \psi(\theta, t) + B_{\sigma}(\sigma) \partial_{\sigma} \psi(\theta, t) + C_Z(z) \partial_z^2 \psi(\theta, t) + C_{\sigma}(\sigma) \partial_{\sigma}^2 \psi(\theta, t) \quad (168)$$

Where

$$A(\theta, t) = \hat{\alpha} \zeta_s(\theta, \tau, w) - \frac{\dot{\mathcal{X}}}{\mathcal{X}} - \mu'_z(z) - \mu'_{\sigma}(\sigma) + .5 \left(s_z^2 \right)'(z) + .5 \left(s_{\sigma}^2 \right)'(\sigma) \quad (169)$$

$$B_v(v) = -\mu_v(v) + \left(s_v^2 \right)'(v), \quad v \in \{z, \sigma\} \quad (170)$$

$$C_v(v) = .5 \left(s_v^2 \right)(v), \quad v \in \{z, \sigma\} \quad (171)$$

We use a forward-difference approximation in the time dimension and central-difference approximations in the θ -dimension, that is

$$\partial_t \psi_{j,r}^n \approx \frac{\psi_{j,r}^{n+1} - \psi_{j,r}^n}{\Delta t}, \quad \partial_z \psi_{j,r}^n \approx \frac{\psi_{j+1,r}^{n+1} - \psi_{j-1,r}^n}{2\Delta z}, \quad \partial_{\sigma} \psi_{j,r}^n \approx \frac{\psi_{j,r-1}^{n+1} - \psi_{j,r+1}^n}{2\Delta \sigma} \quad (172)$$

$$\partial_z^2 \psi_{j,r}^n \approx \frac{\psi_{j+1,r}^{n+1} - \psi_{j,r}^n + \psi_{j-1,r}^n}{(\Delta z)^2}, \quad \partial_{\sigma}^2 \psi_{j,r}^n \approx \frac{\psi_{j,r+1}^{n+1} - \psi_{j,r}^n + \psi_{j,r-1}^n}{(\Delta \sigma)^2} \quad (173)$$

We can thus write 168 as

$$\begin{aligned} \frac{\psi_{j,r}^{n+1} - \psi_{j,r}^n}{\Delta t} &= A_{j,r}^{n+1} \psi_{j,r}^n + B_{z,j}^n \frac{\psi_{j+1,r}^{n+1} - \psi_{j-1,r}^n}{2\Delta z} + B_{\sigma,r}^n \frac{\psi_{j,r-1}^{n+1} - \psi_{j,r+1}^n}{2\Delta \sigma} \\ &\quad + C_{z,j}^n \frac{\psi_{j+1,r}^{n+1} - \psi_{j,r}^n + \psi_{j-1,r}^n}{(\Delta z)^2} + C_{\sigma,r}^n \frac{\psi_{j,r+1}^{n+1} - \psi_{j,r}^n + \psi_{j,r-1}^n}{(\Delta \sigma)^2} \end{aligned} \quad (174)$$

Which can be written as

$$\psi_{j-1,r}^{n+1} x_{z,j} + \psi_{j,r-1}^{n+1} x_{\sigma,j} + \psi_{j,r}^{n+1} y_{j,r} + \psi_{j+1,r}^{n+1} z_{z,j} + \psi_{j,r+1}^{n+1} z_{\sigma,j} = \psi_{j,r}^n \quad (175)$$

for

$$y_{j,r} = 1 - A_{j,r} \Delta t + 2C_{z,j} \frac{\Delta t}{(\Delta z)^2}, \quad 2C_{\sigma,r} \frac{\Delta t}{(\Delta \sigma)^2}$$

and

$$x_{z,j} = B_{z,j} \frac{\Delta t}{2\Delta z} - C_{z,j} \frac{\Delta t}{(\Delta z)^2}, \quad z_{z,j} = -B_{z,j} \frac{\Delta t}{2\Delta z} - C_{z,j} \frac{\Delta t}{(\Delta z)^2}$$

and analogously for σ . We impose the boundary conditions that $\psi_{1,1} = 0$, and that $\Delta\sigma\Delta z \sum_{j,r} \psi_{j,r}^{n+1} = 1$. Eq.(175) can be written as a linear system of $J \times R - 1$ equations and $J \times R - 1$ unknowns. Let $AA_{r,j}$ be the rj -est row of the matrix with the coefficients. The system can be written as

$$AA \times \begin{pmatrix} \psi_{2,1}^{n+1} & \psi_{3,1}^{n+1} & \cdots & \psi_{J,1}^{n+1} & \psi_{2,2}^{n+1} & \cdots & \psi_{J,R}^{n+1} \end{pmatrix}' = \begin{pmatrix} \psi_{2,1}^n & \psi_{3,1}^n & \cdots & \psi_{J,1}^n & \psi_{2,2}^n & \cdots & \psi_{J,R}^n \end{pmatrix}'$$

for a sparse transition matrix AA . To solve for $\{\psi_{j,r}^{n+1}\}$ given $\{\psi_{j,r}^n\}$ we then invert matrix AA numerically. For the Ornstein-Uhlenbeck process we use in this paper, we have that

$$\mu(z) = \nu_z \left(-\ln z + s_z^2/2 \right) z \quad \text{and} \quad s(z) = \nu_z z^2 s_z^2/2$$

and analogously for σ .

F.3 Parametrization

We parametrize our economy according to Table 3. We calibrate the market power parameter μ to match spreads in free market credit operations for firms in Brazil (18.33 p.p.), as described in the main text. The marginal productivity of capital, interest rates and depreciation and dividend distribution are chosen as standard values in the literature. We follow Moll (2014) to calibrate the financial friction and the process for z . The parameter ν_z , which appears in Eq.(48) is

$$\nu_z = -\ln \rho_z$$

where $\rho_z = e^{-s} \text{corr}(\ln z(t), \ln z(t+s))$ is the autocorrelation of $\ln z(t)$. We calibrate $\rho_z = .95$.

We assume that z is distributed between $[z_{min}, z_{max}]$ to solve the model. The distribution of σ is analogous. We use the same persistence as in z , and a proportional variance given the size of the grids. We calibrate the mean to match the parametrization in Angeletos and Panousi (2011) of undiversifiable risk of around .35. Finally, we assume that σ is distributed between $[0, 1]$ to solve the model. That is, there is a riskless manager and a manager that defaults 50% of the time. Our results are not sensitive for changes in the other parameters in the risk distribution.

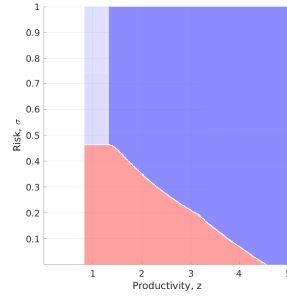
Table 3: Parameters in Quantitative Exercise

Parameter	Model	Value
Brazil Specific (Match 18.3 p.p. Spreads)		
μ	Market Power	.33
Standard		
α	Mg. Prod of k	.4
r	Deposit/Discount Rate	.03
δ^*	Depreciation	.06
∇	Share of Profits Distributed as Dividends	.1
Moll (2014)		
λ	Fin Friction	1.8
ρ_z	z auto-correlation	.95
s_z^2	Variance of stationary dist. of $\ln z$	1.94
μ_z	Mean of $\ln z$	0
z_{min}	Min of z	0
z_{max}	Max of z	5.3
Entrepreneur Risk (Angeletos and Panousi, 2011)		
μ_σ	Mean $\ln \sigma$	-1.07
Risk Distribution		
ρ_σ	σ autocorrelation	.95
s_σ^2	Variance of stationary dist. of $\ln \sigma$.4
σ_{min}	Min of σ	0
σ_{max}	Max of σ	1

G Quantitative Allocation: Cap on a Share of Loans

Figure G.1 displays the allocation of loans below and above the cap for a given level of wages. As in our analytical session, safer and less productive borrowers are the ones that receive subsidized loans.

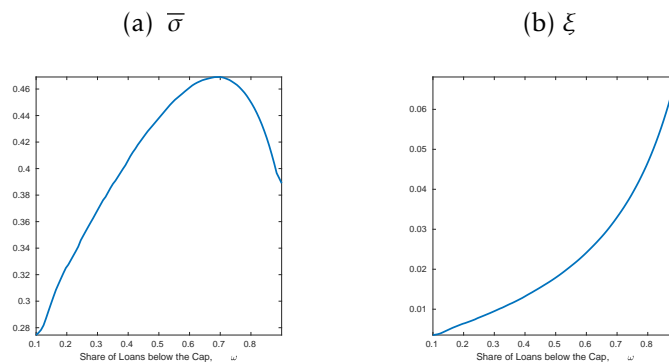
Figure G.1: Allocation under a cap for $w = 3$, $\omega = .7$, $\tilde{r} = 1.2$



Note: Allocation under a cap $\tilde{r} = 1.2$ for a share $\omega = .7$ of all loans. This is the quantitative version of Figure 3. Output shown as percentage between laissez-faire and perfect competition, as Eq.(52). The parameters used are in Table 3. We use wages $w = 3$ to generate this figure.

If the share of loans below the cap ω changes, the maximum risk level $\bar{\sigma}_B$ that receives a loan below the cap changes as in Figure G.2 (as in Appendix E). The Lagrange Multiplier ξ on the constraint - which determines the cross-subsidization between loans below and above the cap, as in Lemma 6 and Figure 4 is always increasing on ω . To guarantee that the constraint is satisfied for larger values of ω , the financial sector must increase this level of transfers to guarantee more loans are made below the cap.

Figure G.2: Maximum risk σ below the cap (left) and Lagrange Multiplier (right)



Note: maximum level of risk below the cap $\bar{\sigma}$ and Lagrange Multiplier ξ when varying the share ω of loans below a cap $\tilde{r} = 1.2$, with wages $w = 3$. See Lemma 6 for the relation of ξ and the cross-subsidization between managers.

H Aggregation with Persistence

Denote the joint distribution of productivity, risk and collateralizable at time t $g_t(\theta, a)$ is the joint distribution of types θ and wealth at time t . Let the wealth share with type θ be given by, where

$$\psi(\theta, t) \equiv \mathcal{X}(t)^{-1} \int_0^\infty a g_t(\theta, a) da \quad (176)$$

where \mathcal{X} is the total value of collateralizable value in the economy, that is

$$\mathcal{X}(t) = \int a dG_t(\theta, a) \quad (177)$$

Moreover, we assume $\theta = \{z, \sigma\}$ follows a diffusion process as in Eq. (178):

$$d\theta = \mu_\theta(\theta)dt + s_\theta(\theta)W \quad (178)$$

We assume that z, σ are uncorrelated, such that the covariance matrix be the two-by-two covariance matrix of the process, denoted by s_θ^2 , is diagonal.

Lemma H.1 shows how macro aggregates and $\psi(\theta, t)$ evolve over time. There are two significant differences from our analytical model that are worth highlighting. First, the policy itself will affect wealth shares, and all of the aggregation is conditional on wealth shares in the beginning of a period. Therefore, we would not be able to derive the optimal policy analytically taking into account this change in the wealth shares. Second, our analytical model is easy to solve due to the fact that we can write the terms relevant for macro aggregates in terms of $\tilde{z} = z/\underline{z}$. This is no longer the case here, due to the fact that the shares $\psi(\theta, t)$ may or not be written in terms of ratios. The terms $\zeta_\lambda^p(\tau, w)$ are a function of wages w and therefore in equilibrium a function of output \mathcal{Y} and collateralizable capital \mathcal{X} . This is the source of the difference in the results of how \mathcal{Y} depends on \mathcal{X} .

Lemma H.1 gives us a way of solving for our model numerically. For given values of $\mathcal{X}(0)$ and $\psi(\theta, 0)$, we find wages that clear the labor market and compute the evolution of $\mathcal{X}(t)$ and $\psi(\theta, t)$ over time. We detail the numerical method in more details in Appendix F.

Lemma H.1. *Given a path for wages, $w(t)$, wealth shares $\psi(\theta, t)$, and collateralizable capital $\mathcal{X}(t)$, an equilibrium is characterized by*

$$\mathcal{Y} = A \left(\underline{z}(w) [1 - F(\underline{z}(w))] \zeta_\lambda^p(\tau) \mathcal{X} \right)^\alpha \quad (179)$$

where $\underline{z}(w) = \kappa(w)^{-1}r$, and $\mathcal{X}(t)$ evolves over time as in Eq. (180)

$$\dot{\mathcal{X}} = \hat{\alpha} \frac{\zeta_\pi^p(\tau, w)}{\zeta_\lambda^p(\tau, w)} \mathcal{Y} + (r - \delta) \mathcal{X} \quad (180)$$

The shares $\psi(\theta, t)$ evolve over time as in Eq.(181)

$$\begin{aligned} \partial_t \psi(\theta, t) = & \left[\hat{\alpha} \zeta_s(\theta, \tau, w) - \frac{\dot{\mathcal{X}}}{\mathcal{X}} \right] \psi(\theta, t) - \partial_z [\psi(\theta, t) \mu_z(z)] - \partial_\sigma [\psi(\theta, t) \mu_\sigma(\sigma)] \\ & + \frac{1}{2} \partial_{z^2} [s_z^2(z) \psi(\theta, t)] + \frac{1}{2} \partial_{\sigma^2} [s_\sigma^2(\sigma) \psi(\theta, t)] \end{aligned} \quad (181)$$

where $\zeta_s(\theta, \tau, w) \equiv \hat{\alpha} \pi(\theta, \tau) + (r - \delta)$ is the savings of managers of type θ and the terms $\{\zeta_\lambda^p, \zeta_\pi^p\}$ are defined in Eqs.(182)-(183)

$$\zeta_\lambda^p(\tau, w) \equiv \int_z \int_\sigma [(\lambda - 1) \iota(z, \sigma | w, \tau) + 1] \frac{z}{\underline{z}} \psi(\theta) dz d\sigma \quad (182)$$

$$\zeta_\pi^p(\tau, w) \equiv \int_z \int_\sigma (r)^{-1} \pi(\theta, \tau, w) \psi(\theta) dz d\sigma \quad (183)$$

Proof. Statics. From Eq.(5), the optimal input choice made by managers implies that aggregate capital is

$$\mathcal{K} \equiv \int k(a, z, \sigma) dG(\theta, a) = \mathcal{X} \int_\theta \lambda(\theta | w, \tau) \psi(\theta) d\theta = \theta_\lambda^p(\tau) \mathcal{X} [1 - F(\underline{z})] \quad (184)$$

Moreover, aggregate labor demand is given by

$$\mathcal{L} = [(1 - \alpha)A/w]^{1/\alpha} \zeta_\lambda^p(\tau) \mathcal{X} [1 - F(\underline{z}(w))] \underline{z} = [(1 - \alpha)A/w]^{1/\alpha} \chi_\lambda^p(\tau) \underline{z} \mathcal{K} = \left[\frac{\kappa(w)}{\alpha A} \right]^{1/(1-\alpha)} \chi_\lambda^p(\tau) \underline{z} \mathcal{K} \quad (185)$$

Therefore:

$$\kappa(w) = \alpha A \mathcal{L}^{1-\alpha} \mathcal{K}^{\alpha-1} \left(\underline{z} \chi_\lambda^p(\tau) \right)^{\alpha-1}$$

Moreover, we know that aggregate output can be written as

$$\mathcal{Y} \equiv \int y(a, z, \sigma) dG(\theta, a) = \frac{\kappa(w)}{\alpha} \zeta_\lambda^p(\tau) \underline{z} \mathcal{X} [1 - F(\underline{z})] = \frac{\kappa(w)}{\alpha} \chi_\lambda^p(\tau) \underline{z} \mathcal{K} \quad (186)$$

Replacing $\kappa(w)$ in the above equation yields

$$\mathcal{Y} = A \mathcal{L}^{1-\alpha} \mathcal{K}^{\alpha-1} \left(\underline{z} \chi_\lambda^p(\tau) \right)^{\alpha-1} \underline{z} \chi_\lambda(\tau) \mathcal{K} = A \left(\underline{z} [1 - F(\underline{z}(w))] \zeta_\lambda^p(\tau) \mathcal{X} \right)^\alpha \quad (187)$$

with $\mathcal{L} = 1$ in equilibrium. Note that the entry tradeoff for $\sigma = 0$ in z is given by:

$$\left(\frac{1-\alpha}{w} A \right)^{\frac{1-\alpha}{\alpha}} = \frac{r}{\alpha A \underline{z}}$$

Moreover, we can solve for the entry trade-off in terms of \mathcal{X}, \mathcal{Y} :

$$\frac{w\mathcal{L}}{1-\alpha} = \mathcal{Y} = \left(\frac{1-\alpha}{w}A\right)^{\frac{1-\alpha}{\alpha}} \zeta_{\lambda} \underline{z}(1-F(\underline{z}))\mathcal{X} = \frac{r}{\alpha A \underline{z}} A \zeta_{\lambda} \underline{z}(1-F(\underline{z}))\mathcal{X} \quad (188)$$

$$\Rightarrow \zeta_{\lambda}(1-F(\underline{z}))\mathcal{X} = \frac{\alpha}{r}\mathcal{Y} \quad (189)$$

Dynamics. Aggregate profits of managers are given by

$$\Pi^M = \int \pi^{w,*}(a, z, \sigma) dG(\theta, a) = \mathcal{X} [1 - F(\underline{z}(w))] \left[\frac{\zeta_{\lambda} - \zeta_r}{\theta_{\lambda}} \right] \theta_{\lambda} = \mathcal{K} \left[\frac{\zeta_{\lambda} - \zeta_r}{\theta_{\lambda}} \right] \quad (190)$$

Repeating the same steps as in the proof of Lemma 2, we have that aggregate collateralizable capital \mathcal{X} in our economy evolves as

$$\dot{\mathcal{X}} = \hat{\alpha} \frac{\zeta_{\pi}^p(\tau)}{\zeta_{\lambda}^p(\tau)} \mathcal{Y} + (r - \delta)\mathcal{X} \quad (191)$$

Moreover, note that we can write the Kolmogorov-Forward equation for g_t (using Eq.(178)) as

$$\begin{aligned} \partial_t g_t(a, \theta) = & -\partial_a [a \zeta_s(\theta, \tau, w) g_t(a, \theta)] - \partial_z [g_t(a, \theta) \mu_z(z)] - \partial_{\sigma} [g_t(a, \theta) \mu_{\sigma}(\sigma)] \\ & + \frac{1}{2} \partial_{z^2} [s_z^2(z) g_t(a, \theta)] + \frac{1}{2} \partial_{\sigma^2} [s_{\sigma}^2(\sigma) g_t(a, \theta)] \end{aligned} \quad (192)$$

where $\zeta_s(\theta, \tau)$ are the savings relative to collateralizable capital managers at of type θ sustain. From our definition of $\psi(\theta, t)$

$$\partial_t \psi(\theta, t) = \mathcal{X}(t)^{-1} \int_0^{\infty} a \partial_t g_t(a, \theta) da - \frac{\dot{\mathcal{X}}}{\mathcal{X}} \psi(\theta, t) \quad (193)$$

Integrating by parts

$$-\int_0^{\infty} a \partial_a [a \zeta_s(\theta, \tau, w) g_t(a, \theta)] da = \zeta_s(\theta, \tau, w) \psi(\theta, t) \quad (194)$$

Thus

$$\begin{aligned} \partial_t \psi(\theta, t) = & \left[\zeta_s(\theta, \tau, w) - \frac{\dot{\mathcal{X}}}{\mathcal{X}} \right] \psi(\theta, t) - \partial_z [\psi(\theta, t) \mu_z(z)] - \partial_{\sigma} [\psi(\theta, t) \mu_{\sigma}(\sigma)] \\ & + \frac{1}{2} \partial_{z^2} [s_z^2(z) \psi(\theta, t)] + \frac{1}{2} \partial_{\sigma^2} [s_{\sigma}^2(\sigma) \psi(\theta, t)] \end{aligned} \quad (195)$$

■

I Endogenous Market Power

In this section we focus on profit levels in the intermediation market. Each bank in the economy has to solve a profit maximization problem as in Eq.(65) with \mathcal{B} of banks in the market

$$\Pi^B(\mathcal{B}, \tau) \equiv \max_{\{\pi^w(\theta)\}} \int_a \int_z \int_\sigma [\pi^e - \pi^w] \left\{ \pi^w - \frac{1}{\mathcal{B}} \hat{\pi}^e - \frac{1}{\mathcal{B}} \sum_{j \neq b} \pi^{w,j} \right\} dF_\sigma(\sigma) dF_z(z) dF_a(a) \quad (196)$$

s.t. $\Psi(\{\pi^w(\theta)\}, \tau) \leq 0$, the constraints imposed by the policy (such as the cap).

Taking the FOC, we have that

$$-2\pi^{w,b} + \frac{\hat{\pi}^e}{\mathcal{B}} + \frac{1}{\mathcal{B}} \sum_{j \neq B} \pi^{w,j} + \pi^e + \psi(\theta, a) = 0 \quad (197)$$

where $\psi(\theta, a) = \gamma(\theta, a) \partial_{\pi^w(\theta)} \Psi(\{\pi^w(\theta)\}, \tau)$ with $\gamma(\theta)$ begin the Lagrange Multiplier. Assuming a symmetric equilibrium $\pi^{w,b} = \pi^{w,*}, \forall b$ ²⁰

$$-2\pi^{w,*} + \frac{\mathcal{B}-1}{\mathcal{B}} \pi^{w,*} + \frac{\hat{\pi}^e}{\mathcal{B}} + \pi^e + \psi(\theta, a) = 0 \Rightarrow \pi^{w,*} = \frac{1}{1+\mathcal{B}} \hat{\pi}^e + \frac{\mathcal{B}}{1+\mathcal{B}} \pi^e + \frac{\mathcal{B}}{1+\mathcal{B}} \psi(\theta, a) \quad (198)$$

thus

$$\Pi^{B,*}(\mathcal{B}, \tau) = \int_z \int_\theta [\pi^e - \pi^{w,*}(\mathcal{B})] \frac{1}{1+\mathcal{B}} [g(z) + \psi(\theta, a)] dF_z(z) F_\sigma(\sigma) \quad (199)$$

which is decreasing in \mathcal{B} (even without taking into account the reduction in demand).

If banks have a fixed costs c_E of operating, the entry/exit equilibrium is satisfied when

$$\Pi^{B,*}(\mathcal{B}+1, \tau) < c_E \text{ and } \Pi^{B,*}(\mathcal{B}, \tau) > c_E \quad (200)$$

Therefore, if τ reduces profits $\forall \mathcal{B}$, it eventually decreases the number of banks in equilibrium in the market.

²⁰This assumption is true if the constraints define a complete lattice. This is easily verifiable for the cap on all loans. If the set Ψ defines a complete lattice, we have that the game between banks is strictly supermodular. From Vives (1999), as our game is symmetric, only symmetric equilibria exists.