Targets and inflation dynamics[*][†]

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Abstract

Brazil has experienced crucial changes in its inflation process since the adoption of inflation targeting in mid 1999. This article addresses changes in the analytical framework employed to track the inflation dynamics, specifically the relevance of an explicit target for inflation. A New-Keynesian Phillips curve (NKPC) is derived incorporating indexation not only to past inflation but also to inflation targets, generalizing the Woodford (2003) hybrid curve.

In our modeling, firms that do not optimally set their prices in a given period adjust them only by indexing their previous prices to a weighted average of the inflation target and lagged inflation. In such a framework, the impact of inflation targets on agents' decisions regarding the supply side can be analytically measured by the parameter associated to the inflation target. It is shown that inflation target affects the welfare-based monetary policy objective function by penalizing deviations of actual inflation from target instead of from zero. This result establishes the micro foundation basis for ad-hoc loss functions as indicated in traditional literature. Therefore, the inflation target also affects the optimal target criterion.

We also present a microfounded specification to model inflation expectations, and conclude that when firms attribute a high weight to the government's inflation target when setting their own prices, exchange rate and demand shocks are unable to alter significantly inflation expectations. Such a result gives some light to empirical ad hoc assessment conducted in traditional literature.

Our empirical evidence shows that even after major shocks, the target ability of anchoring inflation has been restored. Although not formally tested, such a fact followed the monetary authorities' firm commitment to meet the inflation targets, reinforced by the government's necessary support through a consistent fiscal policy.

Keywords: Inflation targets; inflation dynamics; inflation inertia; inflation expectations; unit labor cost; microfounded macroeconomic models; optimal monetary policy.

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1 Introduction

What are the consequences of adopting an explicit target for inflation? Since the early 1990s, a growing number of central banks in industrial and emerging countries have considered this question, including the US[1].

The inflation targeting literature points out that much of its benefits can be attributed to its impact on inflation expectations[2]. Woodford (2004) argues that the most important achievement of inflation-targeting central banks has not been the reorientation of the goals of monetary policy toward a stronger emphasis on controlling inflation, but rather the development of an approach to the conduct of policy that focuses on a clearly defined target. According to him, one important advantage of commitment to an appropriately chosen policy rule is that it facilitates public understanding of policy, which is crucial in order for monetary policy to be most effective[3].

This seems to be the case in Brazil. As pointed out by Cerisola and Gelos (2005), the adoption of an explicit and public target for inflation influenced the expectations of private agents. The authors examine the macroeconomic determinants of survey inflation expectations in Brazil since the adoption of inflation targeting in 1999. The results suggest that the inflation targeting framework has helped anchor expectations, with the dispersion of inflation expectations declining considerably. They also find that the inflation target has been instrumental in shaping expectations while the importance of past inflation in determining expectations appears to be relatively low.

The success of inflation targeting is solidly based on credibility. Private agents must believe that the central bank will act consistently within the inflation targeting framework. During the process of credibility construction in the context of large shocks, however, even a strong response by the monetary authority, will not be able to prevent expectations to deviate from the targets. In this case communication with the public so as to explain the reasons of the breach becomes crucial. Furthermore, it is important that expectations converge to the target over a certain time horizon. Minella et al (2003) presented some evidence on the behavior of the Central Bank of Brazil, private agents' expectations, and changes in inflation dynamics. The authors conclude that the Central Bank of Brazil has been reacting strongly to expected inflation. It conducts monetary policy in a forward-looking manner, and responds to inflationary pressures. Although the actual inflation has been above the upper limit of the tolerance interval in 2001 and 2002, the inflation-targeting regime has been successful in anchoring expectations, with a substantial reduction in the degree of inflation persistence after inflation targeting was adopted. This is a direct consequence of credibility gains that the Central Bank of Brazil has obtained since the implementation of the new regime. Credibility, however, is still under construction as it takes time to achieve and great effort to preserve.

Next section summarizes Brazilian inflation targeting framework. In section 3 we present the analytical framework that captures the changes in inflation dynamics highlighted in the empirical works mentioned before. The adoption of an explicit target for inflation together with a perceived commitment of the monetary authority to pursue this target were, to a large extend, responsible for the remarkable change in the inflation process during the period, mainly in regards to inflation persistence. Section 4 present an extension to a small-open economy and derives a theoretical equation for expectations on future inflation, giving some light to empirical ad hoc assesment, conducted in traditional literature. Then, the section assesses the credibility process using the framework presented and test the robustness of such a theoretical formation

[1] According to Mishkin (2004a), the Federal Reserve should adopt inflation targeting.

[2] See Mishkin and Schmidt-Hebbel (2001) for a survey of early experiences with inflation targeting. Ball and Sheridan (2003) present a more pessimistic view from experience to date.

[3] In Woodford's (2004) own words "For not only do expectations about policy matter, but, at least under current conditions, very little else matters."

role of expectations. Section 5 concludes and points out the challenges ahead.

2 Inflation targeting in Brazil

Soon after changing to a floating exchange rate regime in 1999, Brazil has adopted an explicit inflation targeting framework as part of an extensive program of economic reforms. This development ended a period during which the exchange rate had been the main anchor for monetary policy[4]. In June 1999, a presidential decree established the new monetary framework, whose key points are listed bellow:

- Adoption of an explicit target for inflation;
- Inflation targets as well as the respective tolerance intervals were to be set by the National Monetary Council on the basis of a proposal by the Minister of Finance[5];
- Targets to be set no later than June 30, two years in advance;
- The Central Bank was given the responsibility to implement the policies necessary to achieve the targets;
- The Central Bank were to issue a quarterly inflation report that would provide information on the performance of the framework, the results of monetary policy actions, and the outlook for inflation; and
- In case the targets were breached, the Central Bank's Governor would be required to issue an open letter to the Minister of Finance explaining the causes of the breach, the measures to be adopted to ensure that inflation returns to the tolerated levels, and the period of time that will be needed for these measures to take effect.

The mounting uncertainties after the floating of the real in early 1999 enticed the implementation of a more strict inflation targeting framework, one that would represent a firm commitment to prevent inflation from getting out of control. Also, the relatively loose fiscal stance at the outset of the new regime as well as the lack of formal operational autonomy of the Central Bank presented additional challenges to the conduct of monetary policy, in particular to the construction of credibility. In order to deal with these concerns, the Central Bank has adopted a flexible and accountable approach in conducting policy. For instance, even when the targets were breached and revised, the process was undertaken in a very transparent manner through open letters from the Central Bank. As noted by Mishkin (2004b), the role of the Central Bank in this accomplishment provides a good example for other emerging markets considering adopting inflation targeting: the way the Central Bank articulated the reasons why the initial inflation target was missed, how it responded to the shock, and how it planned to return to its longer-run inflation goal.

The new regime has been tested in a number of different ways during its short lifetime, with the intensity and frequency of shocks being unprecedented. Despite challenging conditions the new monetary framework has proven to be an effective guide for expectations. Even when current inflation deviated from the established targets, monetary policy under inflation target was, most of the times, capable of keeping inflation expectations in line with the official inflation targets. In the following section we will formally analyze how the adoption of an explicit target for inflation affects inflation dynamics and monetary policy.

[4]For an extensive analysis about inflation targeting in Brazil see Bogdanski et al (2000), Bogdanski et al (2001), Minella et al (2002), and Minella et al (2003).

[5]Brazilian inflation targets, tolerance bands, and outcomes are showed in Figure 1 in the Appendix.

3 Changes in the inflation process: analytical framework

The standard approach to characterize the inflation process is some kind of Phillips curve relation, specifically the so-called "New-Keynesian" Phillips curve (NKPC), which can be derived from first principles based on assumption about nominal rigidities[6] and possesses the well know specification

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{1}$$

where x_t is a measure of the output gap, π_t is the inflation rate and E_t is the expectation operator based on date t information set[7]. The parameter κ measures the combined impact of nominal and real rigidities while $\beta \in (0, 1)$ is the discount coefficient of the representative household. This specification has two sound characteristics: (i) it is derived from first principles and thus can deals with the Lucas (1976) critic, and (ii) it emphasizes the importance of the future state of the economy on current inflation. Iterating (1) forward one obtains

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t x_{t+j} \tag{2}$$

The message of equation (2) is straightforward: if the monetary authority stabilizes all future stream of output gaps it also stabilizes inflation. Not only the current conditions of the economy are relevant but also the economic outlook.

Although appealing, the NKPC has problems when faced with the facts, specifically because the absence of any inertial component. As we can see from equation (2), the NKPC is completely forward-looking, while it does not seem to be the case of inflation in any country[8]. In order to make the NKPC compatible with the data, some sort of backward looking behavior should be introduced. Assume that the fraction of firms that do not optimally readjust their prices in a certain period use a cost free rule-of-thumb based on lagged inflation. As a result, Phillips curve assume the so-called hybrid specification as follows

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \beta E_t \left[\pi_{t+1} - \gamma \pi_t \right] \tag{3}$$

where $0 \leq \gamma \leq 1$ measures the degree of indexation to the most recently available inflation measure.[9]. Iterating this equation forward one obtains

$$\pi_t = \gamma \pi_{t-1} + \kappa \sum_{j=0}^{\infty} \beta^j E_t x_{t+j} \tag{4}$$

Now, even if the central bank stabilizes the output gap from now on, the same would not occur with current inflation since it is influenced by its own recent history.

A typical exercise undertaken to measure the relative importance of the past versus future components of inflation is to estimate equation (3) and compare the weights associated with the backward versus forward looking components. Although relevant, it does not provide evidence of the impact of the adoption of an official target for inflation, after all, there are other events occurring in the economy that should induce the private sector to act in a more forward looking fashion. In the next section, we will present a straightforward extension of the hybrid model that can be used to assess the relevance of adopting an official inflation target.

[6] The model of nominal rigidities proposed by Calvo (1983) was used in this case.

[7] For a step-by-step derivation in a general-equilibrium model see Woodford (2003).

[8] Some references are in Woodford (2003), Galí and Gertler (1999) and Fuhrer and Moore (1995), among others.

[9] A detailed derivation of the hybrid curve is presented in Woodford (2003). A slightly different derivation, together with its empirical relevance in the US, is presented in Galí and Gertler (1999).

3.1 Targets and inflation dynamics

Why should agents only consider lagged inflation in their rule-of-thumb decisions? In an inflation targeting economy it is natural to assume that pricing decisions should also incorporate the inflation target[10]. In such a context, consider the following generic case[11] that generalizes the hybrid curve (3).

$$\pi_t - \pi_t^* = \kappa x_t + \beta E_t \left[\pi_{t+1} - \pi_{t+1}^*\right] \tag{5}$$

where now π_t^* equals

$$\pi_t^* = \lambda \pi_t^o + (1-\lambda)\,\pi_{t-1} \tag{6}$$

where $0 \leq \lambda \leq 1$ is the weight associated to the inflation target π_t^o for the current period, which, in the case of Brazil, is defined at least one year in advance by an institution other than the Central Bank of Brazil, the National Monetary Council, and so it is not affected by the monetary policy[12]. Although tempting, the association of λ with credibility should be made with care since there are other aspects that cannot be captured by the model. First of all, any measure of credibility should be policy-dependent[13]. Even considering the proposed time-dependent model as a first stylized representation of a richier state-dependent approach, it is clear that the timing between monetary policy changes and its effects on the economy may cause λ reductions even though both the target and the monetary authority may be full credible. Another limitation is the absence of fiscal considerations, which can be taken into account in any measure of credibility. A more subtle issue is how to distinguish policy-making credibility from target credibility since they are mutually dependent.

The consequences of this extension on the inflation dynamics can be assessed by iterating (5) forward to obtain a result analogous to (4)

$$\pi_t = \pi_t^* + \kappa \sum_{j=0}^{\infty} \beta^j E_t x_{t+j} \tag{7}$$

Although similar in form, equations (3) and (5) are different in spirit in a crucial aspect. Now, if the central bank stabilizes all the future sequence of output gaps, inflation converges to its target in the long-run provided that $\lambda > 0$. Otherwise the model resembles the hybrid curve with $\gamma = 1$[14].

3.2 Targets and optimal monetary policy

The adoption of an explicit target for inflation also affects optimal monetary policy. It can be shown that in order to maximize the welfare of the representative household the monetary authority should minimize

$$W_0 = -\Omega \sum_{t=0}^{\infty} \beta^t \left[(\pi_t - \pi_t^*)^2 + \phi x_t^2\right] \tag{8}$$

[10] As an example of a similar idea, Yun (1996) assumes that prices are automatically increased at some rate $\bar{\pi}$ between occasions on which they are reconsidered. In that case, however, $\bar{\pi}$ is not directly associated with the inflation target but instead it is the actual long-run average rate of inflation of the economy.

[11] Its formal analitycal derivation is presented in Appendix B.

[12] Note that equation (5) nest the hybrid curve as a particular case in which inflation target is zero in all periods.

[13] Although not considered as a direct credibility measure, λ evolution should also be state-dependent, so our time-invariant λ shall be analyzed with caution in the light of the Lucas critic.

[14] Empiric evidence towards $\gamma = 1$ is presented by Giannoni and Woodford (2003) in a complete model of the monetary transmission mechanism without inflation targets.

where Ω is a constant and the weight ϕ is based on the deep parameters of the economy, specifically, $\phi = \frac{\kappa}{\theta}$, where κ is the coefficient associated with output in the NKPC and θ represents the elasticity of substitution between goods in the economy[15].

The minimization of (8) subject to the constraint represented by the NKPC (5) generate the following criterion

$$\pi_t - \pi_t^* + \frac{\phi}{\kappa}(x_t - x_{t-1}) = 0 \tag{9}$$

This so-called *optimal target criterion*[16] indicates that deviations of the inflation rate from π_t^* should be accepted as long as they are negatively proportional to output gap variations, $x_t - x_{t-1}$, over the same period.

This criterion nests the standard and the hybrid problem as particular cases. For instance, under the hybrid curve paradigm with $\gamma = 1$, equation (9) becomes

$$\pi_t - \pi_{t-1} + \frac{\phi}{\kappa}(x_t - x_{t-1}) = 0 \tag{10}$$

indicating that the acceptable inflation forecast for the current period should depend not only on the expected variation in output gap, but also on recent past inflation rate: a higher existing inflation rate justifies a higher expected near-term inflation rate, in the case of any given output-gap forecast. The above equation can be presented in an intuitive form

$$\pi_t + \frac{\phi}{\kappa}x_t = \Pi_{-1} \tag{11}$$

indicating that in each period the right hand side of this last equation should equal the target $\Pi_{-1} = \pi_{-1} + \frac{\phi}{\kappa}x_{-1}$. This result, presented in Giannoni and Woodford (2003), has now a natural interpretation: without a target anchoring firm's pricesetting behavior, the best monetary policy can do is to keep Π_t unchanged.

Remember that such conclusions would also be made under our proposed generalizing approach in the particular case in which $\lambda = 0$. But the opposite scenario $\lambda = 1$ induces an essentially different criterion:

$$\pi_t + \frac{\phi}{\kappa}(x_t - x_{t-1}) = \pi_t^o \tag{12}$$

Now we have a direct relation between optimal policy and the explicit inflation target: the monetary policy should be managed in order to keep the left-hand-side of (12) equal to the official target.

In the next section we show an extension of our model for the case of a small-open economy. We also derive a microfounded formation rule for inflation expectations, which predicts that when firms attribute a high weight to the government's inflation target when setting their on prices, exchange rate and demand shocks are unable to alter significantly inflation expectations.

Next, we use the Brazilian inflation targeting period (1999-2004) to assess the relevance of the model. Close attention will be devoted to the evolution of the target parameter λ, assessed with recursive estimation. Then, we use its time varying estimates in order to test the robustness of the microfounded inflation expectation formation role.

[15] This loss function derivation and analysis are also presented in Appendix B. For an extensive explanation about deriving microfunded welfare based central bank loss functions, see Woodford (2003). Basically, it is a second order Taylor approximation of the welfare function around an efficient steady state. Its functional form is crucially influenced by the assumed source of price stickiness, in this case, the Calvo's (1983) model.

[16] Following Giannoni and Woodford (2003), we obtain an optimal target criterion under a timeless perspective, in which the central bank is supposed to have been always committed to such a rule in the past, so it is time consistent. In alternative approaches, the central bank is supposed to commit to an optimum rule from period zero on. In such approaches, optimal rules are not time consistent, for allowing incentives that induce the central bank to break its vow in initial period, keeping its commitment only from the following period on. Hence, such a rule cannot be perceived as credible by economic agents.

4 Assessing the role played by the inflation target

Dealing with the pass-through from depreciation to inflation has been one of the main challenges to the inflation-targeting regime in emerging markets economies. Brazil is not an exception, as can be seen by the evolution of the nominal and real exchange rate presented in Figures 3 and 4 respectively[17]. Emerging markets seem to be more sensitive to the effects of financial crises than industrialized countries. Exchange rate market volatility generates frequent revisions of inflation rate expectations and may result in non-fulfillment of inflation targets.

In order to analyze the impact of λ during the inflation targeting period we estimate an open economy NKPC based on the following equation[18]

$$\pi_{H,t} - \pi_t^* = \tilde{\kappa} x_t + \beta E_t \left[\pi_{H,t+1} - \pi_{t+1}^*\right] \tag{13}$$

This curve can be derived from a small-open economy model analogous to the one presented by Galí and Monacelli (2004). Note that this new specification is very similar to the one valid for closed economies. The only difference is the appearance of $\pi_{H,t}$, the inflation rate of domestically produced and consumed goods prices due the fact that domestic firms decide on domestic prices.

Furthermore, one may obtain an expression relating the market inflation rate, π_t, that aggregates domestic and imported goods price inflation, to $\pi_{H,t}$

$$\pi_{H,t} = \pi_t - \frac{\delta}{1-\delta} \Delta q_t \tag{14}$$

where q_t is the real exchange rate gap from its steady state value, so Δq_t measures its depreciation rate, and $\delta \in (0,1)$ is a parameter inversely related to the home bias on consumer decisions about his consumption basket. The lower is δ, the less is the consumption of imported goods.

Therefore, one may combine these expressions and obtain

$$\pi_t = \pi_t^* + \tilde{\kappa} x_t + \beta E_t \left[\pi_{t+1} - \pi_{t+1}^*\right] + \frac{\delta}{1-\delta} E_t \left[\Delta q_t - \beta \Delta q_{t+1}\right] \tag{15}$$

Hence, considering the definition of real exchange rate and π_t^*, these last expressions may be combined into the following result, where π_t^f represents the imported goods inflation rate, denominated in foreign currency

$$\begin{aligned} \pi_t &= \lambda \pi_t^o + (1-\lambda)\pi_{t-1} + \beta E_t \left[\pi_{t+1} - \lambda \pi_{t+1}^o - (1-\lambda)\pi_t\right] \\ &\quad + \frac{\delta}{1-\delta} E_t \left[\Delta e_t + \pi_t^f - \pi_t - \beta\left(\Delta e_{t+1} + \pi_{t+1}^f - \pi_{t+1}\right)\right] + \tilde{\kappa} x_t \end{aligned} \tag{16}$$

In line with frontier econometric analysis for microfounded nominal rigidities models, we considered a more fundamental specification for our NKPC. Indeed, the output gap term was only shown in our previous equation due to a slightly strong hypothesis regarding to the functional form of firms production function. As a matter of fact, if such an assumption were not made, we would obtain a very similar NKPC in which the output gap term would be replaced by the firms' average marginal cost, as follows:

$$\begin{aligned} \pi_t &= \lambda \pi_t^o + (1-\lambda)\pi_{t-1} + \beta E_t \left[\pi_{t+1} - \lambda \pi_{t+1}^o - (1-\lambda)\pi_t\right] \\ &\quad + \frac{\delta}{1-\delta} E_t \left[\Delta e_t + \pi_t^f - \pi_t - \beta\left(\Delta e_{t+1} + \pi_{t+1}^f - \pi_{t+1}\right)\right] + \xi mc_{H,t} \end{aligned} \tag{17}$$

[17] For a specific analysis about exchange rate pass-through in Brazil see Belaisch (2003).

[18] For ease of reference, all data series' descriptions, together with its sources, are defined in Table 2 in the Appendix.

where $mc_{H,t}$ is the real marginal cost in terms of domestic prices. The relation between $mc_{H,t}$ and the real marginal cost mc_t in terms of overall aggregated prices is the following

$$mc_{H,t} = mc_t + \frac{\delta}{1-\delta} q_t$$

Therefore

$$\begin{aligned}
\pi_t &= \lambda \pi_t^o + (1-\lambda)\pi_{t-1} + \beta E_t \left[\pi_{t+1} - \lambda \pi_{t+1}^o - (1-\lambda)\pi_t\right] \\
&+ \frac{\delta}{1-\delta} E_t \left[\Delta e_t + \pi_t^f - \pi_t - \beta\left(\Delta e_{t+1} + \pi_{t+1}^f - \pi_{t+1}\right)\right] \\
&+ \xi mc_t + \xi \frac{\delta}{1-\delta} q_t \qquad (18)
\end{aligned}$$

4.1 The role of inflation targets on inflation expectations

Monetary policy ultimately aims at maintaining inflation controlled at low levels, regardless the way it is implemented, being that by targeting money, exchange rate, or, more explicitly, inflation. In pursuing that goal the role of inflation expectations becomes crucial. Most micro-founded macroeconomic models attach considerable importance to the role of central banks in measuring and shaping inflation expectations towards their targets, whether explicit or not.

But which role do the targets play in shaping inflation expectations? Recursively solving equation[19] (16) and after some algebraic manipulation[20], we obtain the following specification for $E_t\pi_{t+1}$, which sheds some light on this question.

$$\begin{aligned}
E_t\pi_{t+1} &= (1-\lambda)^2 \pi_{t-1} + \lambda(1-\lambda)\pi_t^o + \lambda\pi_{t+1}^o + (1-\lambda)\left[\frac{\delta}{1-\delta}\Delta q_t + \tilde{\kappa} x_t\right] + \\
&+ \frac{\delta}{1-\delta} E_t \Delta q_{t+1} + \tilde{\kappa}\left(1 - \lambda + \beta^{-1}\right) E_t \sum_{j=1}^{\infty} \beta^j x_{t+j} \qquad (19)
\end{aligned}$$

Such a result is also presented in Alves et al (2005), in which the authors assess the evolving inflation process in Brazil.

Note that inflation expectations $E_t\pi_{t+1}$ positively depends: on lagged inflation rates, on the inflation target path, on current output gap, on current real exchange rate variation and on expectations about future output gaps and real exchange rate variations. Note that if households were habit-persistent, current output gap would depend on lagged output gap and on lagged real interest rates, thereby $E_t\pi_{t+1}$ would also depend on such variables.

According to equation (19), $\lambda = 1$ implies that $E_t\pi_{t+1}$ neither depends on lagged inflation rates nor on the current output gap (or lagged in case of habit-persistence) and real exchange variations. Rather, it depends on predetermined inflation targets, with π_{t+1}^o coefficient achieving its maximum when $\lambda = 1$, and on expected future real exchange rate variations and output gaps. On the other extreme, $\lambda = 0$ implies that lagged inflation rates, current real exchange rate variations and output gaps play an important role in forming inflation expectations, but the inflation targets are not relevant anymore in that regard.

So, the closer to the unit is λ, the less current output gap and real exchange rate variation affect inflation expectations. In other words, as long λ is close to 1 current exchange rate and output gap shocks have a little effect on $E_t\pi_{t+1}$. In that case, aggregated demand

[19] We chose such an specification for pragmatic reasons: it has a term with the output gap, which is important since many central banks and economic agents use such an activity variable for empirical analysis.

[20] Its derivation is shown in Appendix C.

and the foreign sector are only relevant through the future expectations channel $E_t \Delta q_{t+1}$ and $E_t \sum_{j=1}^{\infty} \beta^j x_{t+j}$, which are expected to smoothly evolve and remain closer to zero than do their current realizations.

Thus, if λ is close enough to 1, $E_t \pi_{t+1}$ shall be pegged to the inflation targets and empirical evidence will point out to the relevance of inflation targets in driving inflation expectations.

We test the robustness of such a formation role further on in subsection 4.6, where we compare our microfounded formation role with three ad hoc empirical formation roles for Brazilian inflation.

4.2 Econometric approach

In order to implement econometric analysis with quarterly Brazilian data, it was necessary to create an inflation target measure for the period ranging 1995Q1 to 1998Q4, previous to the inflation targeting adoption. During such period, the official exchange rate policy consisted of an intended nominal devaluation of 7.5 percent, on an exchange rate crawling peg regime, as explained in Bogdanski et al (2000). Additionally, such period was characterized by a near time invariant real exchange rate. Therefore, it was if there was an official inflation target resulting from maintaining the real exchange rate in a constant level even though nominal exchange rate was set to devaluate at the defined rate. Thus, we could create an inflation target series for the considered period as follows:

$$\overbrace{\Delta q_t}^{0} = \overbrace{\Delta e_t}^{\frac{1}{4}\log(1.075)} + \pi_t^f - \pi_t^o$$

then

$$\pi_t^o = \frac{1}{4} \log(1.075) + \pi_t^f \tag{20}$$

From 1999Q1 and 1999Q2, there was not an official nominal anchor, so we imposed the restrictions $\lambda_{1999Q1} = 0$ and $\lambda_{1999Q2} = 0$. From 1999Q3 on, Central Bank of Brazil has been pursuing end-of-year inflation targets, so there are no quarterly official inflation targets. Therefore, the simplest solution was to consider that the quarterly inflation targets were fixed, in each year, at a quarter of the corresponding end-of year official values.

As the inflation rate π_t, we considered the Brazilian broad consumer price index (IPCA), used to gauge Brazilian inflation targets. In order to filter the strong seasonal pattern present until 1998Q4, we considered a filtered series, obtained by the general full sample asymmetric band-pass (from 3.1 to 4 quarters) frequency filter of Christiano and Fitzgerald (2003). Since Brazilian inflation seasonal pattern seemed to change over time and was difficult to extract due to strong external shocks that affected Brazilian IPCA inflation rate in 2001 and 2002, we believed this filtering process was appropriated due to the fact that this asymmetric filter is time varying with its parameters depending on the data and changing for each observation. Figure 2 shows the obtained filtered inflation.

As the foreign inflation rate measure π_t^f, we considered the US export price index variation rate (all commodities). Therefore, real exchange rates were constructed considering those inflation rates. As in Goldfajn and Werlang (2000), detrended (Hodrick-Prescott Filter)[21] real exchange rate was considered as a proxy to q_t, as illustrated in Figure 4.

As a measure of firms aggregated marginal cost, we constructed the proxy $mc_t = w_t L_t / \tau Y_t$, where w_t is the real wage index, L_t is the occupied labor force, τ is the income labor share and Y_t is the GDP. Since the only uninterrupted Brazilian series of labor force is the one released

[21] See Hodrick and Prescott (1997). For an interesting generalization, also see Araújo, Areosa, and Rodrigues Neto (2003).

by Seade[22] for São Paulo's metropolitan region, mc_t was constructed with data proceeding from that region. For quarterly São Paulo's GDP measure, we interpolated its annual share to the Brazilian GDP and applied the resulting quarterly share to quarterly Brazilian GDP[23]. Regarding to the income labor share, we considered $\tau = 0.6$, the Brazilian adjusted estimative made by Gomes et al (2002).

Hence, we estimated the following structural equation with a Two Stage Least Squares procedure, since there were contemporaneous and future regressors that could be correlated with the error term ε_t. The instrumental list included $\{\pi_{t+j}\}_{j=-1}^{-4}$, $\left\{\pi_{t+j}^{f}\right\}_{j=0}^{-2}$, $\{\Delta e_{t+j}\}_{j=-1}^{-2}$, q_{t-1}, $\left\{\pi_{t+j}^{o}\right\}_{j=0}^{1}$, mc_{t-1}, and $\left\{SoT_{t+j}^{CBond}\right\}_{j=-1}^{-3}$, the Brazilian C-Bond spread over (US) treasury. The intercept coefficient was imposed in order to capture possible estimating level bias in the latent marginal cost measure and in the real exchange rate gap. If such term was neglected, parameter estimates could be biased due to the omitted variable specification.

$$\begin{aligned}
\pi_t &= A_0 + A_1\pi_t^o + (1-A_1)\,\pi_{t-1} + A_2 E_t\left[\pi_{t+1} - A_1\pi_{t+1}^o - (1-A_1)\,\pi_t\right] \\
&\quad + A_3 E_t\left[\Delta e_t + \pi_t^f - \pi_t - A_2\left(\Delta e_{t+1} + \pi_{t+1}^f - \pi_{t+1}\right)\right] \\
&\quad + A_4 mc_t + A_5 q_t + \varepsilon_t
\end{aligned} \tag{21}$$

The parameters estimates in the full sample are presented in Table 1. As can be seen, the parameter associated with the target is quantitatively relevant and statistically significant, with a value of almost 0.70. Note also that the short-term exchange rate pass-through coefficient is both significant and low, with a value of about 5 percent. This estimative is in line with the values estimated in the recent literature of most countries where a floating exchange regime is adopted. Indeed, it means that Brazilian economy is still essentially closed, due the fact that the home bias parameter $\delta = \frac{0.053}{1+0.053} = 0.05$. Another parameter that is also significant is A_5, regarded to the real exchange rate gap. Remember that this variable is a component of the real marginal costs in terms of domestic consumed products $mc_{H,t}$. As the results indicate, this variable is relevant to explain Brazilian consumer inflation, although neglected in some empirical works in the literature that are used to consider only the unit labor cost mc_t. On the other hand, our econometric procedure rejected the hypothesis that our measure of unit labor cost is relevant, for its coefficient A_4 is not significant. Nevertheless, a Wald test do not reject the null hypothesis that $A_4 = A_3/A_5$ (p-value = 0.25), as should be if the structural Phillips curve presented in equation (18) above performs fine to model actual Brazilian supply side. Note also that the preferences intertemporal discounting parameter β, represented by A_2, was sub estimated by our regression, since one expected it to be close to the unity. This result may suggest that Brazilian agents are impatient in their consuming decisions, but it may be only a consequence of correlated regressors. Since our empirical evidence points out that inflation target are an important variable in explaining current inflation, future inflation should be also explained by future inflation targets and so should be current expectation regarding future inflation. Hence, $E_t\pi_{t+1}$ must be correlated with π_{t+1}^o and some estimation problems may arise. On the other hand, some results points out that using different unit labor cost measures would modify our estimative of β, due to spurious in-sample correlation between $E_t\pi_{t+1}$ and mc_t. Although our estimatives of λ seemed to be robust to such sources of problem, future extensions should concern more attention on this issue.

In order to map the evolution of the weight associated with the target we perform recursive

[22] Fundação Sistema Estadual de Análise de Dados (www.seade.gov.br). A more traditional measure for labor force is released by Instituto Brasileiro de Geografia e Estatística (IBGE). But its series methodology changed in 2001, and regardless the fact that the old series is updated only until December 2002, the new series was not applied to previous years, starting in January 2001.

[23] See Appendix for details.

estimations of our Phillips curve[24]. The evolution of the parameter λ is showed in Figure 5, with a confidence interval of 2 standard errors. In almost all sub samples, this parameter was significant in explaining inflation.

The recursive estimates of λ indicate that the weight associated with the official inflation target increases in periods when inflation expectations are close enough to the official targets. The period can be separated in three sub-samples: (i) 1999 – 2000, the implementation phase, (ii) 2001 – 2002, the stress test, (iii) 2003 – 2004, the restoration of credibility.

4.3 Implementing IT: 1999 - 2000

During the first two years, λ increased with regularity from closed to 0.50 in mid 1999 to approximately 0.75 at the end of 2000. Despite the adoption of IT in Brazil had occurred in the midst of a foreign exchange crisis, the transition to the new regime in 1999 was relatively smooth. Against all pessimistic views, inflation at the end of 1999 reached the one-digit level mark (8.9 percent), while annual GDP grew by almost 1 percent (0.8 percent). The response of the Brazilian government and Central Bank to the crisis combined fiscal consolidation, a strong commitment with price stability, and external financial support.

The exchange rate was rapidly stabilized, market interest rates fell, and inflation expectations were brought under control, which allowed the Central Bank to lower interest rates quite aggressively (from 45 percent to below 20 percent in a seven-months period). Besides confidence's strength, the extensive amount of exchange rate hedging, which isolates the non-financial private sector from the exchange rate devaluation, helps to explain the remarkably small impact on output that followed the overshooting of the real. However, the transition to a floating exchange rate regime left some scars: public debt increased and became more sensitive to exchange rate changes.

After the initial transition phase, with the normalization of financial conditions and under the effects of significant interest rate cuts, inflation ended 2000 right at the 6 percent mid-point target, with robust economic growth of 4.4 percent. However, during 2000 a series of important shocks occurred, notably: oil prices had double since 1999 while the prices of technology firms sharply felt, with the meltdown of NASDAQ. At the same time, monetary policy conditions were tightened in the U.S. with fed funds rate being raised to 6.5 percent in May 2000, from 5.5 percent at the end of 1999.

Although the overall performance in 2000 was auspicious, the accelerate rate of growth of the Brazilian economy at the margin combined with US and Global slowdown pointed to difficulties ahead. Brazilian economic recovery that began at the end of 1999, was based on strong credit expansion, increasing exports of industrial goods, and agricultural prices recovery. This recovery, however, combined with increasing oil prices and US slowdown, adversely affected the balance of trade, which entered negative territory (12 months) in September 2000 after a period of recovery following the depreciation of the real in early 1999. Brazilian core IPCA inflation started to show a growth trend after November 2000.

4.4 Inflation targeting under stress: 2001 - 2002[25]

After an initial increase in mid of 2001 - almost reaching the theoretical upper limit of 1 - λ suffers two considerable reductions: (i) from 1 to 0.75, at the end of 2001, and (ii) the collapse from almost 0.90 to 0.25 during the second half of 2002.

[24] We considered performing Kalman filter analyses in order to model λ as a time varying parameter. However the results were considerably unstable.

[25] See Bevilaqua and Loyo (2005) for a detailed description about Brazil's stress test of inflation targeting.

The year of 2001 was marked by a series of adverse shocks, most notably: the Argentina default, officially announced in 2001Q4, the energy crises in Brazil, and the September 11 attack. In the beginning of the year, consumer price inflation came above expectations, while core inflation trend was incompatible with the 4 percent inflation target for the year. After reducing the Selic rate[26] to 15.25 percent in January, Central Bank started in March the first monetary policy tightening cycle of the inflation targeting regime. After an initial 50 basis points increase, the tightening cycle was interrupted only in July, with the Selic rate reaching 19 percent. The policy rate remained unchanged from August 2001 to February 2002, when the Central Bank began the easing process, although just from a brief period of time. The series of adverse events produced during 2001 significant exchange rate depreciation, hovering around 20 percent. At the end of 2001, inflation reached 7.7 percent (3.7 percentage points above the 4 percent target) and the economy grew 1.3 percent.

Even tough the target was not reached, the results obtained in face of an extremely adverse scenario were satisfactory, revealing the inflation targeting regime as an effective and flexible framework to pin down expectations. Inflation expectations for 2002, gauged at the end of 2001, were still below 5 percent. The way monetary policy was conducted with the swift reaction after the September 11 terrorist attacks kept expectations under control and made economic agents believe that the 2001 adverse inflationary shock would be dissipated during the following year.

The year 2002 began with the view that the end of the energy crisis combined with an improved international environment would allow some flexibility in the conduct of monetary policy. In fact, a considerable exchange rate appreciation occurred (from a 2.80 R$/US$ just after September 11 to 2.40 R$/US$ in the beginning of May 2002). In this context, the monetary policy was relaxed in the beginning of the year with the Selic rate being reduced from 19 percent in February to 18 percent in June. However, later in the year, the uncertainty associated with the presidential election sets off an unprecedented confidence crisis leading to sharp exchange rate depreciation and to a very unfavorable debt-dynamics. During that time, a number of arguments arose suggesting that particular circumstances distorted the transmission mechanism of monetary policy, which was then bound for defeat against inflation. Blanchard (2003) went as far as suggesting that monetary policy might actually work in reverse, given Brazil's large stock of public debt. Increases in interest rates would worsen debt dynamics, increasing country risk and making the exchange rate depreciate; with the pass-through from depreciation, tight money would make inflation higher rather than lower. Although theoretically plausible, Blanchard's empirical evidence is helped by not allowing monetary policy to have any impact on inflation through domestic aggregate demand. Indeed, as pointed out by Bevilaqua and Loyo (2005), Brazil did succeed in securing disinflation through monetary tightening, with a perceptible contribution from the aggregate demand transmission channel.

The commitment assumed by the new President to sustain sound macroeconomic policies, combining fiscal discipline, a floating exchange rate regime, and the inflation targeting framework, was crucial to dissipate the fear associated with changes in the course of the economy and related to debt sustainability. From September 2002 to December of the same year the Central Bank increased its policy rate from 18 percent to 25 percent. However, the sharp exchange rate depreciation during the year yielded a considerable increase of inflation, which ended 2002 at 12.5 percent, and a modest GDP growth of 1.9 percent. Although the inflation targeting regime was unable to anchor expectations during that year, the months that fol-

[26] The Selic interest rate, the Central Bank's primary monetary policy instrument, is the average interest rate on overnight inter-bank loans collateralized by government bonds that are registered with, and traded on, the Sistema Especial de Liquidação e Custódia (Selic). The Central Bank of Brazil Monetary Policy Committee (COPOM) establishes a target for the Selic interest rate and the Central Bank's open market desk executes regular liquidity management operations in the domestic money market with the goal of keeping the daily Selic interest rate at the target level.

lowed this episode proved that inflation targeting has been a useful framework to align market expectations with government objectives.

4.5 Reconstructing credibility: 2003 - 2004

Since the beginning of 2003 the target parameter displays a consistent recovery, reaching almost 0.75 at the end of 2004. In January 2003, the Central Bank sent an open letter to the Minister of Finance explaining why the inflation targets were breached, and made explicit estimates of the size of the shocks and their persistence. The Central Bank added to the original inflation target for 2002 (4 percent) part of the breach experienced in the previous year, to account for inertia effects (inflation carryover from the 2002 shock) and for the impact on administered prices that, by contract provisions, are adjusted according to past inflation. These two effects let the Central Bank to adjust the inflation target for 2003 to 8.5 percent. The Central Bank made explicit reference to the fact that, after the sharp increase in inflation in 2002, attempting to achieve the original inflation target of 4 percent for 2003 would require a sizeable output sacrifice. Inflation in 2003 came down by more than 3 percentage points, ending up at 9.3 percent, close to the adjusted target, and GDP declined by a modest 0.2 percent. Of course, the Central Bank has not been able to do this on its own. The new government not only supported the inflation targeting regime but also pursued tight spending policies that have resulted in a primary budget surplus in 2003 of 4.3 percent of GDP.

In 2004 GDP expanded vigorously with growth reaching almost 5 percent and with employment increasing at two-digit rate. However, the strong economic recovery in 2004 required a gradual but firm response of the Central Bank to fight emerging inflationary pressures and prevent these pressures from contaminating inflationary expectations. From September 2004 to May 2005 the Central Bank raised its policy rate by 3.75 percentage points to 19.75 percent. Moreover, the government announced in September 2004 a change in the primary surplus target for 2004, from 4.25 to 4.5 percent of GDP. Inflation, despite some acceleration during the second half of 2004, ended the year at 7.6 percent, above the 5.5 percent target, but within the tolerance interval.

In September 2004, the Central Bank communicated to the public the new operational target for 2005 of 5.1 percent. When it became clear to the Central Bank that the 5.5 percent target for 2004 would not be fulfilled and it was possible to project with greater accuracy the 2004 deviation, the Central Bank established 5.1 percent as its operational target to be pursued in 2005.

4.6 Testing the robustness of the microfounded inflation expectation formation role

As depicted in Figure 5, note that λ fell significantly in the end of 2002 and remained low for about four quarters. Thus, according to our previous result, $E_t\pi_{t+1}$ must have been more sensitive to exchange rate variations and past inflation during 2002Q4 to 2003Q4. Simultaneously, such a period was characterized by high inflation rates and real depreciations.

The presented microfounded theory would predict that inflation expectations must have departed from the vicinities of inflation targets during the mentioned period. Indeed, Figure 6 shows that the Brazilian 12-month ahead expected IPCA inflation[27] adheres quite well to the inflation target throughout the sample, except during the period from September 2002 to July 2003, in line with the theoretical model predictions.

[27] The Investor Relations Group of Central Bank of Brazil surveys inflation expectations, among other indicators. Until November 2001, 12-month ahead inflation expectation was not available, so we interpolated end-of-year inflation expectation series.

Regarding ad hoc empirical approaches, Minella et al (2003) found that expected inflation reacts significantly to the inflation targets, whose coefficients were estimated around 1. On the other hand, past inflation rates' coefficient were low. In their study, 12-month ahead inflation expectations (survey) are regressed against their own lags, lagged 12-month inflation rate, 12-month ahead inflation targets, lagged 12-month exchange rate variation, lagged interest rates and lagged Brazilian risk premium (Embi plus), in four different specifications. When ending the sample in mid 2002, just before the confidence crises, past inflation is no longer significant. Figure 7 and 8 show their recursive estimates of past inflation and exchange rate variation coefficients.

In another similar approach, Cerisola and Gelos (2005) suggest that the inflation targeting framework has helped anchor expectations in Brazil and that past inflation's importance appears to be relatively low. The authors examine the macroeconomic determinants of survey inflation expectations in Brazil since the adoption of inflation targeting. They also find that apart from the inflation target, the stance of fiscal policy, as proxied by the ratio of the consolidated primary surplus to GDP, has been instrumental in shaping expectations.

Carvalho and Bugarin (2005) assess the formation rule of inflation expectations in Mexico, Brazil and Chile in recent years. Among other results, they find that credibility was high in Chile and Mexico, with still some room for improvement. In Brazil, credibility was severely undermined during the shock period, in 2002 and early 2003, when the framework failed to coordinate expectations. They also build a theoretical model to investigate credibility in monetary policy when inflation targets are not set by the monetary authority, as is the case of Brazil and Chile, and there is uncertainty about the preferences of the central banker. The model shows that inflation targets have a role in anchoring expectations. It also shows that in countries with greater dispersion in central bankers' preferences, strong-type central bankers have to be more conservative in order to persuade society of their commitment to controlling inflation.

Therefore, it is possible to infer that our microfounded modeling is also an important contribution to explain how expectations on future inflation reacts to current shocks and to inflation targeting, just in line with some empirical evidence suggests.

5 Concluding remarks and challenges ahead

We develop and estimate a structural model of inflation to measure the relevance of an explicit target for inflation. A New-Keynesian Phillips curve (NKPC) is derived incorporating inflation targets generalizing the Woodford's (2003) hybrid curve.

On the theoretical side it is shown that the inflation target affects the welfare-based monetary policy objective function by penalizing deviations of actual inflation from its implicit target instead of from zero. This result establishes the micro foundation basis for ad-hoc loss functions as indicated in the traditional literature. Therefore, the official inflation target also affects the optimal target criterion.

Moreover, we present a microfounded formation rule for inflation expectations, which predicts that when firms attribute a high weight to the government's inflation target when setting their on prices, exchange rate and demand shocks are unable to alter significantly inflation expectations, giving some light to empirical ad hoc assesment.

On the empirical side, we test the model using the Brazilian inflation targeting period (1999-2004). We find that the parameter associated with the target is quantitatively relevant and statistically significant. Recursive estimates indicate that the parameter associated with the target increases in periods when inflation expectations are close enough to the official targets.

The empirical evidence also shows that even after major shocks, the target ability of anchoring inflation has been restored. Although not formally tested, such a fact followed the monetary authorities' firm commitment to meet the inflation targets, reinforced by the government's necessary support through a consistent fiscal policy.

There are three possible extensions where future investigation would be quite useful. First, the same analysis should be performed based on other countries data in order to verify if the result that the official target is relevant for inflation is robust. Second, if firms pay close attention to the inflation target, the timing and magnitude of an individual firm's price adjustment depends on the monetary authority actions. The effects of nominal disturbances on aggregate real activity will also be policy-dependent, since the price level depends on the weight firms associate with the targets. As a result the weight associated with the target should not be time-dependent but instead state-dependent. The challenge is how to make such transition in a tractable manner. Finally, the model ignores the role of fiscal policy in the process. Only a model that consider the interrelation between monetary and fiscal policies can draw a comprehensive picture of the credibility mechanism. In particular, the consequences for monetary policy of an adoption of a fiscal target can be directly assessed.

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A Appendix: Figures and tables

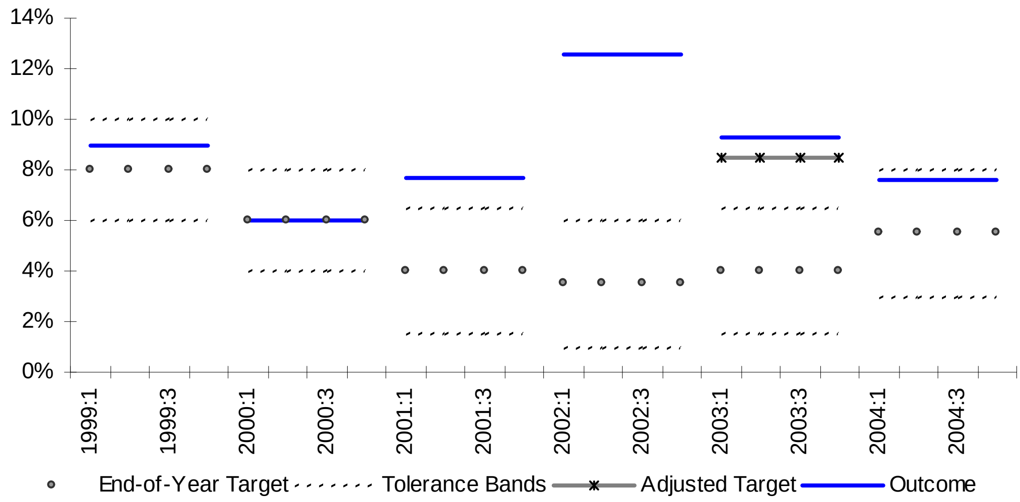

Figure 1: Brazilian inflation targets, tolerance bands, and outcomes

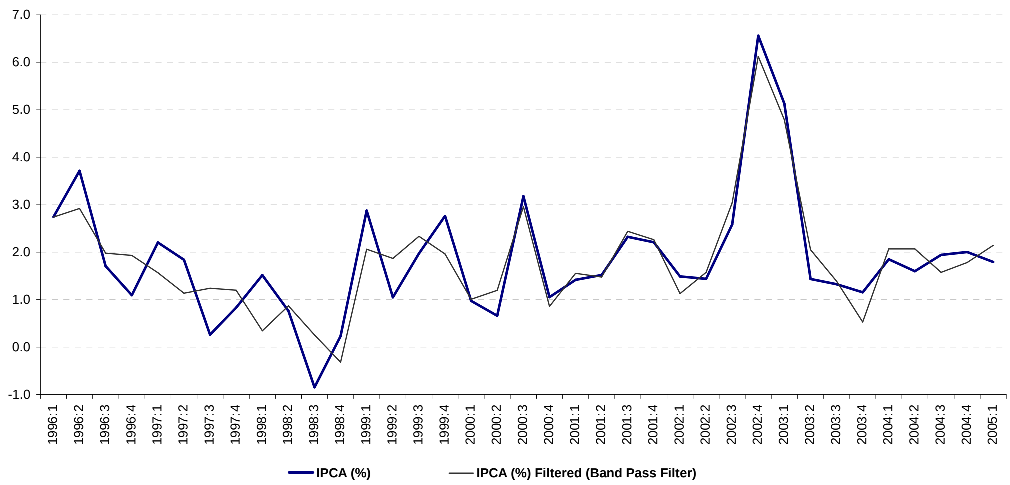

Figure 2: Quarterly Brazilian IPCA

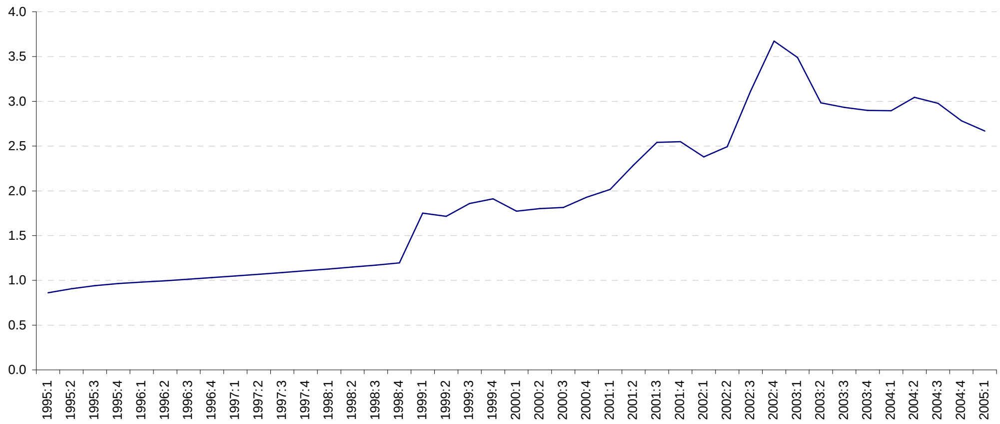

Figure 3: Evolution of the nominal exchange rate (Brazilian R$ / US$)

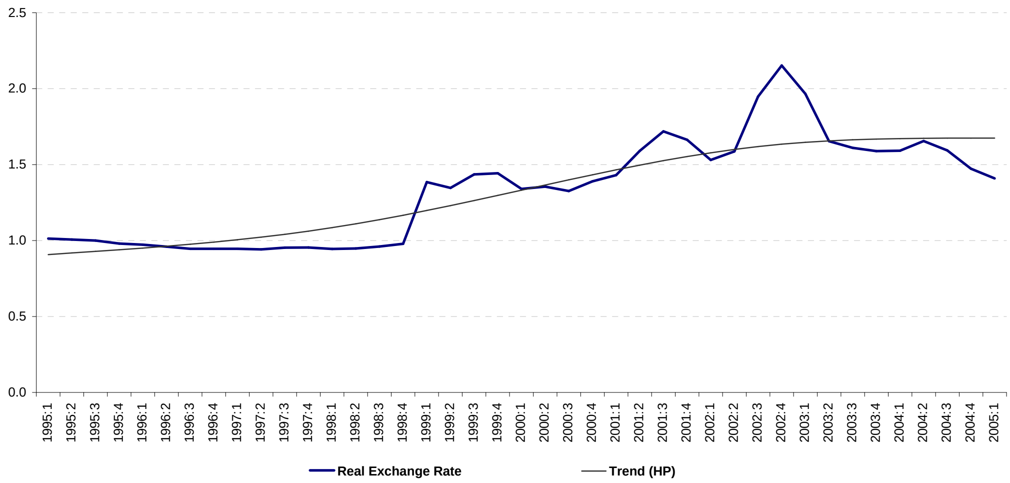

Figure 4: Evolution of the real exchange rate in Brazil

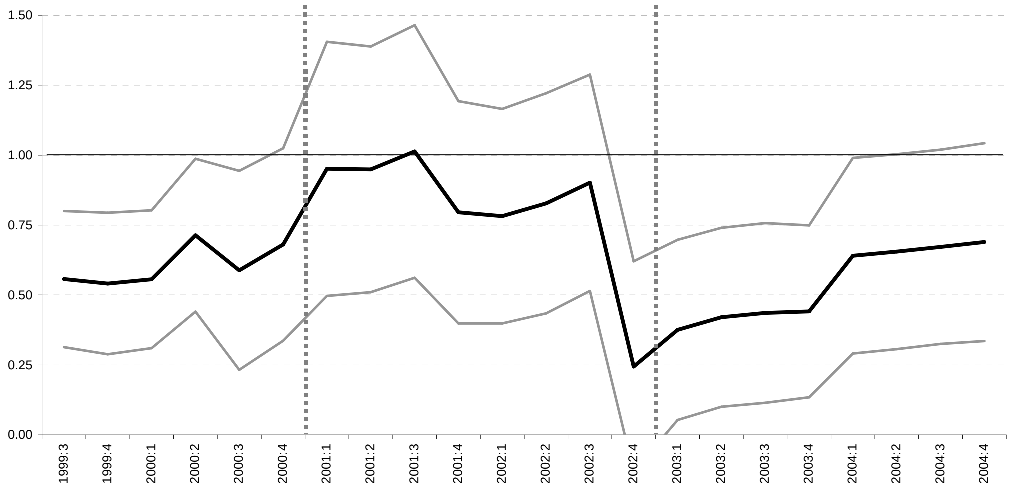

Figure 5: Evolution of the target parameter (λ): Recursive estimates

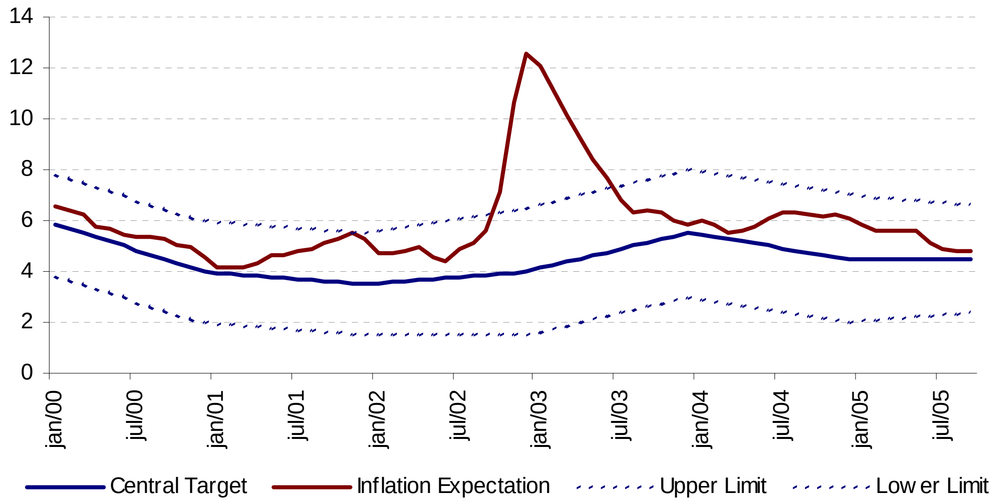

Figure 6: 12-Month Ahead Inflation Expectations and
12-Month Ahead Inflation Targets (Central and Tolerance Band)

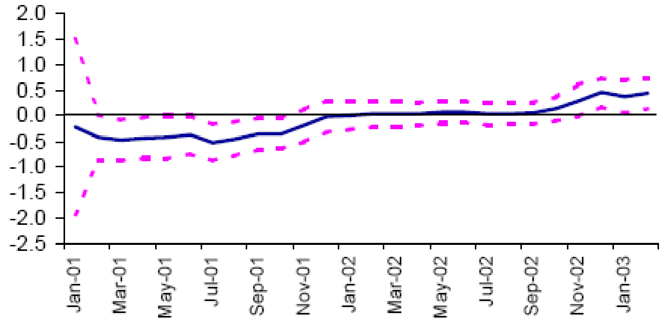

Figure 7: Recursive Estimates of the Coefficient on Past Inflation

Source: Minella et al (2003)

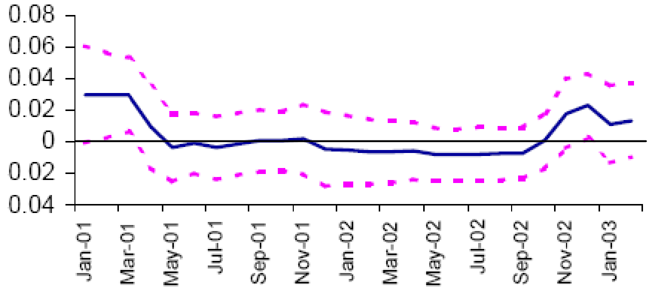

Figure 8: Recursive Estimates of the Coefficient on Exchange Rate Depreciation.
Source: Minella et al (2003)

Table 1: Structural Estimates of Eq. (21)

	Method 2SLS; 1996Q1 TO 2004Q4			
	Coef	SE	t	p
A_0	0.002	0.002	1.303	0.203
A_1	0.689	0.177	3.896	0.001
A_2	0.297	0.269	1.104	0.279
A_3	0.053	0.031	1.714	0.097
A_4	-0.113	0.107	-1.050	0.302
A_5	0.049	0.023	2.130	0.041
$R^2 = 0.557 \quad R^2\ Adj. = 0.484$				
Breusch-Godfrey LM Test[28](2 lags): $n \cdot R^2 = 3.86$; $p = 0.15$ $\{Dist.\chi^2\,(2)\}$				
White Heteroskedasticity Test: $F = 0.53$; $p = 0.91$ $\{Dist.F\,(24, 11)\}$				

[28]Our LM test was carried out regressing (OLS) the residuals against its lags (t-1) and (t-2) and against the resulting vector of regressors from Eq. (21), replacing expectation and contemporaneous variables, except for the pre-determined inflation target, with the corresponding laggeg variables: $\pi_t^o - \pi_{t-1}$, $\pi_{t+1}^o - \pi_{t-1}$, $\Delta e_{t-1} + \pi_t^f - \pi_{t-1}$, π_{t-1}, mc_{t-1} and q_{t-1}.

Table 2: Data Series and Sources

Serie	Description	Source
π_t^o	Inflation targets	BCB[a]
e_t	Nominal exchange rate (R$/US$)	BCB[a]
π_t	Inflation rate series	IBGE[b]
Y_t	São Paulo's metropolitan region GDP	IBGE[b] (Quarterlized)
L_t	Labor force series for São Paulo	Seade[c]
w_t	Average real wage series for São Paulo	Seade[c]
π_t^f	Foreing inflation rate	U.S. Department of Labor[d]
SoT_t^{CBond}	Brazilian C-Bond spread over US treasury	Ipea Data[e]
mc_t	Unit labor cost	$= w_t L_t / \tau Y_t$; $(\tau = 0.6)$
Δq_t	Real exchange rate variation	$= \Delta e_t + \pi_t^f - \pi_t$

[a]Banco Central do Brasil (www.bcb.gov.br)
[b]Instituto. Brasileiro de Geografia e Estatística (www.ibge.gov.br)
[c]Fundação Sistema Estadual de Análise de Dados (www.seade.gov.br/produtos/ped)
[d]Bureau of Labor Statistics (www.bls.gov)
[e]Ipea Data (www.ipeadata.gov.br)

Obtaining Quarterly São Paulo's GDP

IPEA and IBGE released annual GDP (1970, 1975, 1980, 1985, 1996, 1999, 2000, 2001 and 2002) from several cities in Brazil. In particular, we could aggregate GDP from São Paulo's metropolitan region[29] and determine its share to Brazilian GDP, for each year. Since this indicator are not supposed to present a volatile pattern from a particular quarter to the following one, except from seasonal idiosyncrasies, we generated quarterly shares minimizing the sum of squared second differences constrained to the restriction that their annual average should equal the previously estimated occurred shares for each one of the cited years. Following, we applied this estimated quarterly shares to actual seasonally adjusted quarterly Brazilian GDP. For years 2003, 2004 and 2005, for which such cities GDP were not released, we considered the same quarterly share estimated to 2002Q4.

B Appendix: The closed-economy model

In our simple economy, there is a representative household whose problem is to intertemporally decide about its consumption basket among differentiated goods in a continuum of unit mass

[29]São Paulo's metropolitan region encompasses 38 cities. The complete list of cities is available at www.seade.gov.br.

$c_t(i)$, $\forall i \in (0,1)$, and about its supplied labor $h_t(i)$. For each good i, there is only one firm i, specialized only in that type of production. For so, each firm hires the specialized labor $h_t(i)$, at wage $w_t(i)$. Suppose, as usual, that individual firms's decisions have no influence on wages

One aggregates consumption C_t in (22) as in Dixit and Stiglitz (1977), assuming constant substitution elasticity, where $\theta > 1$.

$$C_t \equiv \left[\int_0^1 c_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{22}$$

Define $\mathrm{u}(C;\xi)$ as the concave consumption utility function, and $v(h;\xi)$ as the convex labor disutility function. Both are strictly increasing in their first arguments. For each of them, ξ is a vector of preference shocks.

One may define the aggregated price index P_t as follows. Note that each consumer gets utility only by the quantity of aggregated consumption C_t, independently on how the distribution of the consumption basket are set among the continuum of goods. Hence, the *hicksian* demand for each good i is determined by the following spending minimization problem, given a certain consumption level $\overline{C}$.

$$\begin{array}{ll} \min\limits_{\{c_t(i)\}} & \int_0^1 p_t(i)\ c_t(i)\, di \\ s.t. & \left[\int_0^1 c_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \geq \overline{C} \end{array} \tag{23}$$

Where $p_t(i)$ is the unit price for good type i. Optimal solution implies that P_t the demand function must satisfy (24) and (25), respectively.

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{1/(1-\theta)} \tag{24}$$

$$c_t(i) = C_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \tag{25}$$

Assuming that financial assets are evenly share among all consumers in period zero, complete markets imply in identical budget restrictions for every household, so that the representative household faces the following intertemporal problem, in case of money absence.

$$\begin{array}{ll} \max\limits_{\{C_t;\ h_t(i)\}} & E_0 \sum\limits_{t=0}^{\infty} \beta^t \left[u(C_t;\xi_t) - \int_0^1 v(h_t(i);\xi_t)\, di \right] \\ s.t. & P_t C_t + E_t(Q_{t,t+1} W_{t+1}) \leq W_t + \int_0^1 w_t(i)\ h_t(i)\, di + \int_0^1 \Pi_t(i)\, di \end{array} \tag{26}$$

Where W_t is the nominal financial wealth held by the household in the beginning of period t, $Q_{t,t+1}$ is the stochastic discounting factor that must exist under absence of arbitrage and β is the preference discounting factor. Finally, $\Pi_t(i)$ represents the nominal profit from selling each good i.

A necessary condition for the existence of a solution of such an intertemporal problem is the following tranversality inequality, where $Q_{t,T} = \prod\limits_{s=t}^{T-1} Q_{s,s+1}$.

$$W_{t+1} + E_{t+1} \sum_{T=t+1}^{\infty} Q_{t+1,T} \left[\int_0^1 w_T(i)\, h_T(i)\, di + \int_0^1 \Pi_T(i)\, di \right] \geq 0$$

In equilibrium, all aggregated production must be consumed ($C_t = Y_t$). So, one obtains the standard Euler equation (27) and the expression (28) for the real wage $w_t(i)/P_t$ of type i.

$$(1+i_t)^{-1} = E_t Q_{t,t+1} = \beta E_t \left[\frac{u_C(Y_{t+1};\xi_{t+1})}{u_C(Y_t;\xi_t)} \frac{P_t}{P_{t+1}} \right] \tag{27}$$

$$\frac{w_t(i)}{P_t} = \frac{\upsilon_h(h_t(i))}{u_C(Y_t)} \tag{28}$$

One may log-linearize the Euler equation and obtain the standard IS equation (29), relating the output gap $x_t = \widehat{Y}_t - \widehat{Y}_t^n = \log\left(\frac{Y_t}{Y_t^n}\right)$ to real ex-ante interest rate $\widehat{r}_t$. Here, Y_t^n is the natural product, the one prevailing under flexible prices. The "hat" symbol refers to log-deviations from steady state levels.

$$x_t = x_{t+1} - \sigma(\widehat{r}_t - \widehat{r}_t^n) \tag{29}$$

Where the natural real interest rate equals

$$\widehat{r}_t^n = \sigma^{-1}\left[\left(g_t - \widehat{Y}_t^n\right) - E_t\left(g_{t+1} - \widehat{Y}_{t+1}^n\right)\right]$$

Additionally, $\widehat{r}_t = \left(\widehat{i}_t - E_t\pi_{t+1}\right)$, $g_t = -\frac{u_{C\xi}(\overline{Y};0)}{u_{CC}(\overline{Y};0)}\frac{\xi_t}{\overline{Y}}$ and $\sigma^{-1} = -\frac{u_{CC}(\overline{Y};0)}{u_C(\overline{Y};0)}\overline{Y}$ is the steady state relative risk aversion index.

Regarding to firm decisions, the core of our modeling analysis, assume that each firm production function assumes the functional form shown in (30), where A_t models an exogenous time varying technology parameter and $f(\cdot)$ is a concave and increasing function. Therefore, the only cost source for each firm is labor cost, as follows:

$$y_t(i) = A_t f(h_t(i)) \tag{30}$$

$$Cost_t(i) = w_t(i) \cdot h_t(i)$$

In such a context, since firms face monopolistic competition with individual demand for each specialized good defined in (25), their problem under flexible price setting is the one depicted in (31),

$$\begin{aligned} \max_{\{p_t(i)\}} \quad & \Pi_t(i) = p_t(i)\cdot y_t(i) - w_t(i)\cdot f^{-1}\left(A_t^{-1}\cdot y_t(i)\right) \\ s.t. \quad & y_t(i) = Y_t\left(\frac{p_t(i)}{P_t}\right)^{-\theta} \end{aligned} \tag{31}$$

Optimal solution implies the traditional result in which all firms apply the same markup term $\mu = \theta/(\theta-1) > 1$ to their marginal costs. So, in a flexible price setting equilibrium, all firms face the same marginal cost in all periods and they all set the same optimum price:

$$p_t^* = P_t = \mu \cdot MC_t(i)$$

Where $MC_t(i)$ is the nominal marginal cost of each firm.

An equivalent result may be written as $\mu \cdot mc_t(i) = 1$, where $mc_t(i) = MC_t(i)/P_t$ is the real marginal cost.

Note that $mc_t(i)$ can be also represented as $mc_t(i) = mc[y_t(i), Y_t; \xi_t] = \frac{\widetilde{\upsilon}_y(y_t(i);\xi_t)}{u_C(Y_t;\xi_t)}$, where $\widetilde{\upsilon}(y_t(i);\xi_t) = \upsilon\left[f^{-1}\left(A_t^{-1}\cdot y_t(i)\right);\xi_t\right]$. Thus, one implicitily defines the previously mentioned natural product Y_t^n, the aggregated product level prevailing under an equilibrium of flexible prices, as follows:

$$\mu \cdot mc\left[Y_t^n, Y_t^n; \xi_t\right] = 1 \tag{32}$$

Similarly, one implicitly defines the steady state product $\overline{Y}$ as:

$$\mu \cdot mc\left[\overline{Y}, \overline{Y}; 0\right] = 1 \tag{33}$$

On the other hand, under a price stickiness environment, we state that, in every period, each firm faces the same probability $(1-\alpha)$ of setting an optimal price and α of readjusting their prices according to the following rule of thumb:

$$p_t(i) = p_{t-1}(i) \cdot \overline{\Pi^*}_t \tag{34}$$

Where $\lambda \in (0,1)$, $\overline{\Pi^*}_t = \overline{\Pi^o}_t^{\lambda} \cdot \overline{\Pi}_{t-1}^{1-\lambda}$, $\overline{\Pi}_t = \frac{P_t}{P_{t-1}}$, $\overline{\Pi^o}_t = (1 + \overline{\pi}_t^o)$ and $\overline{\pi}_t^o$ is the official inflation target rate for period t.

Therefore, each firm supposed to optimally set its price in period t solves the following problem:

$$\begin{aligned}
\max_{\{p_t(i)\}} \quad & E_t \sum_{T \geq t}^{\infty} \alpha^{T-t} Q_{t,T} \cdot \Pi\left[p_t(i), P_T; \xi_T\right] \\
s.t. \quad & \Pi\left[p_t(i), P_T; \xi_T\right] = y_T(i) \cdot p_t(i) \prod_{\tau=t}^{T-1} \overline{\Pi^*}_{\tau+1} - Cost_T(i) \\
& Cost_T(i) = w_T(i) \cdot f^{-1}\left(A_T^{-1} \cdot y_T(i)\right) \\
& y_T(i) = Y_T \left(\frac{p_t(i)}{P_T} \prod_{\tau=t}^{T-1} \overline{\Pi^*}_{\tau+1} \right)^{-\theta}
\end{aligned} \tag{35}$$

The sufficient[30] first order condition is:

$$E_t \sum_{T \geq t}^{\infty} \alpha^{T-t} Q_{t,T} \cdot \Pi_p\left[p_t^*, P_T; \xi_T\right] = 0 \tag{36}$$

Where

$$\Pi_p\left[p_t^*, P_T; \xi_T\right] = (1-\theta) Y_T \left(\frac{P_T}{p_t^*}\right)^{\theta} \prod_{\tau=t}^{T-1} \left(\overline{\Pi^*}_{\tau+1}\right)^{1-\theta} \left[1 - \left(\frac{P_T}{p_t^*}\right) \prod_{\tau=t}^{T-1} \left(\overline{\Pi^*}_{\tau+1}\right)^{-1} \mu\, mc_T(i)\right] \tag{37}$$

Since the term inside the brackets equals zero in the steady state, we may log-linearize the whole expression around the steady state values as follows[31]:

$$\Pi_p\left[p_t(i), P_T; \xi_T\right] \approx (1-\theta) \overline{Y} \left[\log\left(\frac{p_t^*}{P_T}\right) + \sum_{\tau=t}^{T-1} \pi_{\tau+1}^* - \widehat{mc}_T(i)\right]$$

Where $\widehat{mc}_t(i) = \log(\mu\, mc_t(i))$, $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$, $\pi_t^o = \log\left(\overline{\Pi^o}_t\right) = \log(1 + \overline{\pi}_t^o)$ and $\pi_t^* = \log\left(\overline{\Pi^*}_t\right) = \lambda \pi_t^o + (1-\lambda)\pi_{t-1}$, as previously shown in (6).

Since $mc\left[y_t(i), Y_t; \xi_t\right] = \frac{\widetilde{v}_y(y_t(i);\xi_t)}{u_C(Y_t;\xi_t)}$, we log-linearize it as follows:

[30] The assumed regularity conditions of preferences and production function implies that the first order condition are sufficient to describe the maximum point.

[31] Remember that (33) directly implies $\mu \cdot \overline{mc} = 1$.

$$\begin{aligned}
\widehat{mc}_t(i) &\approx \mu\frac{\widetilde{v}_{yy}\left(\bar{Y};0\right)\overline{Y}}{u_C\left(\bar{Y};0\right)}\hat{y}_t(i) - \mu\frac{\widetilde{v}_y\left(\bar{Y};0\right)u_{CC}\left(\bar{Y};0\right)\overline{Y}}{\left[u_C\left(\bar{Y};0\right)\right]^2}\hat{Y}_t + \\
&\quad +\mu\left(\frac{\widetilde{v}_{y\xi}\left(\bar{Y};0\right)}{u_C\left(\bar{Y};0\right)} - \frac{\widetilde{v}_y\left(\bar{Y};0\right)u_{C\xi}\left(\bar{Y};0\right)}{\left[u_C\left(\bar{Y};0\right)\right]^2}\right)\xi_t \\
&= \omega\hat{y}_t(i) + \mu\,\overline{mc}\,\sigma^{-1}\hat{Y}_t + \mu\,\overline{mc}\cdot\left(\frac{\widetilde{v}_{y\xi}\left(\bar{Y};0\right)}{\widetilde{v}_y\left(\bar{Y};0\right)} - \frac{u_{C\xi}\left(\bar{Y};0\right)}{u_C\left(\bar{Y};0\right)}\right)\xi_t \\
&= \omega\hat{y}_t(i) + \sigma^{-1}\hat{Y}_t + \left(\omega\frac{\widetilde{v}_{y\xi}\left(\bar{Y};0\right)}{\widetilde{v}_{yy}\left(\bar{Y};0\right)\bar{Y}} + \sigma^{-1}\frac{u_{C\xi}\left(\bar{Y};0\right)}{u_{CC}\left(\bar{Y};0\right)\bar{Y}}\right)\xi_t
\end{aligned}$$

Thus

$$\widehat{mc}_t(i) \approx \omega\hat{y}_t(i) + \sigma^{-1}\hat{Y}_t - \left(\omega q_t + \sigma^{-1}g_t\right) \tag{38}$$

Where we defined $\omega \equiv \mu\frac{\widetilde{v}_{yy}\left(\overline{Y};0\right)\overline{Y}}{u_C\left(\overline{Y};0\right)} = \frac{\widetilde{v}_{yy}\left(\overline{Y};0\right)\overline{Y}}{\widetilde{v}_y\left(\overline{Y};0\right)}$, $q_t \equiv -\frac{\widetilde{v}_{y\xi}\left(\bar{Y};0\right)}{\widetilde{v}_{yy}\left(\bar{Y};0\right)\bar{Y}}\xi_t$ and $g_t \equiv -\frac{u_{C\xi}\left(\bar{Y};0\right)}{u_{CC}\left(\bar{Y};0\right)\bar{Y}}\xi_t$.

We may also log-linearize (22) as $\hat{Y}_t \approx \int_0^1 \hat{y}_t(i)\,di$. Thus, we aggregate $\widehat{mc}_t(i)$ as follows:

$$\widehat{mc}_t = \int_0^1 \widehat{mc}_t(i)\,di \approx \left(\omega + \sigma^{-1}\right)\hat{Y}_t - \left(\omega q_t + \sigma^{-1}g_t\right) \tag{39}$$

Log-linearizing the demand function in period T, we have $\hat{y}_T(i) = \hat{Y}_T + \theta\log\left(P_T/p_T(i)\right)$. However (34) implies $p_T(i) = p_t(i)\prod_{\tau=t}^{T-1}\overline{\Pi^*}_{\tau+1}$, for $T > t$ Hence it is easy to obtain $\widehat{mc}_T(i) \approx \widehat{mc}_T + \omega\theta\log\left(P_T/p_t^*\right) - \omega\theta\sum_{\tau=t}^{T-1}\pi^*_{\tau+1}$.

Thus we log-linearize (36) as follows:

$$E_t\sum_{T\geq t}^{\infty}(\alpha\beta)^{T-t}\left[(1+\omega\theta)\log\left(\frac{p_t^*}{P_T}\right) + (1+\omega\theta)\sum_{\tau=t}^{T-1}\pi^*_{\tau+1} - \widehat{mc}_T\right] = 0$$

Thus

$$\log\left(p_t^*\right) = \frac{1-\alpha\beta}{1+\omega\theta}E_t\sum_{T\geq t}^{\infty}(\alpha\beta)^{T-t}\left[\widehat{mc}_T + (1+\omega\theta)\left(\log\left(P_T\right) - \sum_{\tau=t}^{T-1}\pi^*_{\tau+1}\right)\right]$$

Leading the previous relation in one period, applying the expectance operator and considering the iterating expectatives property:

$$\alpha\beta E_t\log\left(p^*_{t+1}\right) = \frac{1-\alpha\beta}{1+\omega\theta}E_t\sum_{T\geq t+1}^{\infty}(\alpha\beta)^{T-t}\left[\widehat{mc}_T + (1+\omega\theta)\left(\log\left(P_T\right) - \sum_{\tau=t+1}^{T-1}\pi^*_{\tau+1}\right)\right]$$

Hence

$$\log\left(p_t^*\right) - \alpha\beta E_t\log\left(p^*_{t+1}\right) = \frac{1-\alpha\beta}{1+\omega\theta}\widehat{mc}_t + (1-\alpha\beta)\log\left(P_t\right) - (1-\alpha\beta)\sum_{T\geq t+1}^{\infty}(\alpha\beta)^{T-t}E_t\pi^*_{t+1}$$

or

$$\log\left(p_t^*\right) - \alpha\beta E_t \log\left(p_{t+1}^*\right) = \frac{1-\alpha\beta}{1+\omega\theta}\widehat{mc}_t + (1-\alpha\beta)\log\left(P_t\right) - \alpha\beta E_t \pi_{t+1}^* \tag{40}$$

Moreover, from (24) and (34), the aggregated price P_t may be represented as follows:

$$P_t = \left[(1-\alpha)\cdot\left(p_t^*\right)^{1-\theta} + \alpha\cdot\left(P_{t-1}\cdot\overline{\Pi^*}_t\right)^{1-\theta}\right]^{1/(1-\theta)} \tag{41}$$

We log-linearize it and obtain $\log\left(P_t\right) = (1-\alpha)\log\left(p_t^*\right) + \alpha\log\left(P_{t-1}\right) + \alpha\pi_t^*$. Such an expression is equivalent to:

$$\log\left(p_t^*\right) = \log\left(P_t\right) + \frac{\alpha}{(1-\alpha)}\left(\pi_t - \pi_t^*\right)$$

Thus

$$E_t \log\left(p_{t+1}^*\right) = E_t \log\left(P_{t+1}\right) + \frac{\alpha}{(1-\alpha)} E_t\left(\pi_{t+1} - \pi_{t+1}^*\right)$$

Substituting in (40), we obtain the following New-Keynesian Phillips Curve (NKPC):

$$\pi_t - \pi_t^* = \gamma\widehat{mc}_t + \beta E_t\left[\pi_{t+1} - \pi_{t+1}^*\right] \tag{42}$$

Where we defined $\gamma \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\theta)}$.

Note that the NKPC's most fundamental structural form regards the real marginal cost, in log terms, $\widehat{mc}_t$. Nevertheless, traditional empirical evidence consider the output gap x_t instead of $\widehat{mc}_t$. Actually, it is standard to rewrite (42) considering an output measure if the production function assumes the functional form shown in (30), as follows:

From (32) and (38), we obtain $\left(\omega q_t + \sigma^{-1} g_t\right) \approx \left(\omega + \sigma^{-1}\right)\hat{Y}_t^n$. Hence, from (39), the following expression may be considered:

$$\widehat{cm}_t = \left(\omega + \sigma^{-1}\right)\cdot x_t$$

Therefore, the NKPC could also represented as follows, as previously shown in (5).

$$\pi_t - \pi_t^* = \kappa\cdot x_t + \beta E_t\left[\pi_{t+1} - \pi_{t+1}^*\right] \tag{43}$$

Where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\cdot\frac{\omega+\sigma^{-1}}{1+\omega\theta}$.

From now on, one shows how the adoption of an explicit inflation target affects the optimal monetary policy that maximizes the representative household welfare W_0, represented bellow. Actually, one shows that its second order approximation assumes the opposite form of the traditional central bank loss function.

$$W_0 = E_0\sum_{t=0}^{\infty}\beta^t U_t \tag{44}$$

Where $U_t = u\left(C_t;\xi_t\right) - \int_0^1 \upsilon\left(h_t(i);\xi_t\right)di$.

As in Woodford (2003), a second order approximation of (44) can be firstly represented as follows[32]:

[32] Such an approximation independs on the actual source of price stickiness and its derivation is very standard in recent monetary policy analysis. Therefore, we skip its analytical steps.

$$U_t = -\frac{\overline{Y}\overline{u_C}}{2}\left[\left(\omega+\sigma^{-1}\right)\left(x_t-\widehat{Y^*}\right)^2+\theta\left(1+\omega\theta\right)Var_i\log\left(p_t\left(i\right)\right)\right]+tip+O\left(\left\|\phi_y;\widetilde{\xi}\right\|^3\right) \quad (45)$$

Where $\widetilde{\xi_t}$ is the aggregated exogenous shock vector[33], tip colects the policy-independent terms and Y^* is the efficient production level, that globally maximazes U_t in every period, and $\widehat{Y^*} = \log\left(\frac{Y^*}{\overline{Y}}\right)$. Such a production level implicitly solves:

$$mc\left[Y^*, Y^*; 0\right] = 1 \quad (46)$$

Note that the efficient production does not necessarily equal to the steady state production level, for it is defined as in (33). Hence, the inefficiency parameter ϕ_y, that must be small enough to permit such an approximation, is defined as:

$$\phi_y \equiv mc\left(Y^*, Y^*; 0\right) - mc\left[\overline{Y}, \overline{Y}; 0\right] = 1-\mu^{-1}$$

Despite the fact that such an approximation independs on the addopted price stickiness source, the variance of $\log\left(p_t\left(i\right)\right)$, in i, depends.

Note that (41) can be log-linearized as follows:

$$\log\left(P_t\right) \approx (1-\alpha)\cdot\log\left(p_t^*\right)+\alpha\cdot\left[\log\left(P_{t-1}\right)+\pi_t^*\right]$$

Defining $\overline{P_t} \equiv E_i\log\left(p_t\left(i\right)\right) \approx (1-\alpha)\cdot\log\left(p_t^*\right)+\alpha\cdot\left[E_i\log\left(p_{t-1}\left(i\right)\right)+\pi_t^*\right]$, one obtains:

$$\overline{P_t}-\left[\overline{P}_{t-1}+\pi_t^*\right] \approx (1-\alpha)\cdot\left[\log\left(p_t^*\right)-\left(\overline{P}_{t-1}+\pi_t^*\right)\right]$$

One now may adress the variance issue, defining $\Delta_t \equiv Var_i\log\left(p_t\left(i\right)\right)$.

Since $\left(\log\left(P_{t-1}\right)+\pi_t^*\right)$ independs on i, Δ_t may also be represented as:

$$\begin{aligned}
\Delta_t &= Var_i\left[\log\left(p_t\left(i\right)\right)-\left(\overline{P}_{t-1}+\pi_t^*\right)\right] \\
&= E_i\left[\log\left(p_t\left(i\right)\right)-\left(\overline{P}_{t-1}+\pi_t^*\right)\right]^2-\left[E_i\log\left(p_t\left(i\right)\right)-\left(\overline{P}_{t-1}+\pi_t^*\right)\right]^2 \\
&= \alpha E_i\left[\log\left(p_{t-1}\left(i\right)\right)+\pi_t^*-\left(\overline{P}_{t-1}+\pi_t^*\right)\right]^2+ \\
&\quad +(1-\alpha)\left[\log\left(p_t^*\right)-\left(\overline{P}_{t-1}+\pi_t^*\right)\right]^2-\left[\overline{P_t}-\left(\overline{P}_{t-1}+\pi_t^*\right)\right]^2 \\
&= \alpha E_i\left[\log\left(p_{t-1}\left(i\right)\right)-\overline{P}_{t-1}\right]^2+(1-\alpha)^{-1}\left[\overline{P_t}-\left(\overline{P}_{t-1}+\pi_t^*\right)\right]^2- \\
&\quad -\left[\overline{P_t}-\left(\overline{P}_{t-1}+\pi_t^*\right)\right]^2 \\
&= \alpha\left\{Var_i\left[\log\left(p_{t-1}\left(i\right)\right)-\overline{P}_{t-1}\right]+\left[E_i\log\left(p_{t-1}\left(i\right)\right)-\overline{P}_{t-1}\right]^2\right\}+ \\
&\quad +\frac{\alpha}{1-\alpha}\left[\overbrace{\overline{P_t}-\overline{P}_{t-1}}^{\approx\pi_t}-\pi_t^*\right]^2 \\
&\approx \alpha\left\{Var_i\log\left(p_{t-1}\left(i\right)\right)+0\right\}+\frac{\alpha}{1-\alpha}\left(\pi_t-\pi_t^*\right)^2 \\
&\therefore\ \Delta_t \approx \alpha\Delta_t+\frac{\alpha}{1-\alpha}\left(\pi_t-\pi_t^*\right)^2
\end{aligned}$$

Therefore:

$$\Delta_t \approx \alpha^{t-1}\Delta_{-1}+\frac{\alpha}{1-\alpha}\sum_{\tau=0}^{t}\alpha^{t-\tau}\left(\pi_\tau-\pi_\tau^*\right)^2$$

[33] It includes ξ_t and $\left(A_t-\overline{A}\right)$.

One now determines $\sum_{t=0}^{\infty}\beta^t\Delta_t$:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\Delta_t &\approx \sum_{t=0}^{\infty}\beta^t\alpha^{t-1}\Delta_{-1}+\frac{\alpha}{1-\alpha}\sum_{t=0}^{\infty}\beta^t\sum_{\tau=0}^{t}\alpha^{t-\tau}\left(\pi_\tau-\pi_\tau^*\right)^2\\ &= tip+\frac{\alpha}{1-\alpha}\sum_{\tau=0}^{\infty}\beta^\tau\left(\pi_\tau-\pi_\tau^*\right)^2\sum_{t=\tau}^{\infty}\left(\alpha\beta\right)^{t-\tau}\\ &= \frac{\alpha}{\left(1-\alpha\right)\left(1-\alpha\beta\right)}\sum_{\tau=0}^{\infty}\beta^\tau\left(\pi_\tau-\pi_\tau^*\right)^2+tip\end{aligned}$$

Changing τ by t:

$$\sum_{t=0}^{\infty}\beta^t\Delta_t\approx\frac{\alpha}{\left(1-\alpha\right)\left(1-\alpha\beta\right)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t-\pi_t^*\right)^2+tip \tag{47}$$

Since $\Delta_t\equiv Var_i\log\left(p_t\left(i\right)\right)$, (45) implies that (44) may be rewritten as follows:

$$W_0=-\Omega\sum_{t=0}^{\infty}\beta^tL_t+tip+O\left(\left\|\phi_y;\widetilde{\xi}\right\|^3\right) \tag{48}$$

Where L_t is the traditional central bank loss function, as previously shown in (8):

$$L_t=\left[\left(\pi_t-\pi_t^*\right)^2+\phi x_t^2\right] \tag{49}$$

But now, the relative weight of output gap is endogenous and depend on its coefficient in the NKPC $\phi=\kappa/\theta$. The level parameter Ω is defined as $\Omega=\frac{\overline{Y u_C}}{2}\phi\left(\omega+\sigma^{-1}\right)$.

Note that, as long as $\overline{\Pi^*}_t$ is not a function of any variable regarding a particular firm i, our results ultimately independs on the way $\overline{\Pi^*}_t$ is defined[34]. Otherwise, $\left(\log\left(P_{t-1}\right)+\pi_t^*\right)$ will depend on i, and the result on Δ_t will not be achieved.

C Appendix: Formation rule of inflation expectations

Recursively solving equation (16), we easily obtain the following result:

$$\pi_t=\left(1-\lambda\right)\pi_{t-1}+\lambda\pi_t^o+\frac{\delta}{1-\delta}\Delta q_t+\tilde{\kappa}E_t\sum_{j=0}^{\infty}\beta^jx_{t+j} \tag{50}$$

Lagging the former equation in one period, we obtain an expression for π_{t-1} to be substituted in (50), so we rewrite it as follows:

$$\begin{aligned}\pi_t &= \left(1-\lambda\right)^2\pi_{t-2}+\lambda\left(1-\lambda\right)\pi_{t-1}^o+\lambda\pi_t^o+\left(1-\lambda\right)\left[\frac{\delta}{1-\delta}\Delta q_{t-1}+\tilde{\kappa}x_{t-1}\right]+\\ &\quad+\frac{\delta}{1-\delta}\Delta q_t+\tilde{\kappa}\left(1-\lambda\right)E_{t-1}\sum_{j=1}^{\infty}\beta^jx_{t-1+j}+\tilde{\kappa}\beta^{-1}E_t\sum_{j=1}^{\infty}\beta^jx_{t-1+j}\end{aligned} \tag{51}$$

Leading (51) in one period, we asssess an expression for π_{t+1}:

[34]Therefore, another indexation rule of thumb is likely to generate the same result.

$$\pi_{t+1} = (1-\lambda)^2 \pi_{t-1} + \lambda (1-\lambda) \pi_t^o + \lambda \pi_{t+1}^o + (1-\lambda) \left[\frac{\delta}{1-\delta} \Delta q_t + \tilde{\kappa} x_t \right] + + \frac{\delta}{1-\delta} \Delta q_{t+1} + \tilde{\kappa} (1-\lambda) E_t \sum_{j=1}^{\infty} \beta^j x_{t+j} + \tilde{\kappa} \beta^{-1} E_{t+1} \sum_{j=1}^{\infty} \beta^j x_{t+j} \tag{52}$$

Considering then the iterating expectatives property, we obtain the following expression for $E_t \pi_{t+1}$:

$$E_t \pi_{t+1} = (1-\lambda)^2 \pi_{t-1} + \lambda (1-\lambda) \pi_t^o + \lambda \pi_{t+1}^o + (1-\lambda) \left[\frac{\delta}{1-\delta} \Delta q_t + \tilde{\kappa} x_t \right] + + \frac{\delta}{1-\delta} E_t \Delta q_{t+1} + \tilde{\kappa} \left(1-\lambda+\beta^{-1}\right) E_t \sum_{j=1}^{\infty} \beta^j x_{t+j} \tag{53}$$