Notes on Everything

(*The most futile and most crucial note document)*

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§§.b1.

“The Science of Mechanics: A Critical and Historical Account of Its Development” (1883), Ernst Mach

1. The principle of the Lever. The principle of the Inclined Plane. The two principles are related. The relationship is established by the use of pulleys.

2. The lever-arm law has always been magic for me until I read Archimedes’s proof, and later this book. The equivalence with the center of gravity (not center of mass) was a hidden obstacle.

3. The impossibility of a Perpetuum Mobile. <assumption>

4. Symmetry: assuming all the determining causes are accounted for, a symmetrical state cannot turn into an asymmetrical one. <assumption>
    - This (logical?) fact(?) is used extensively in Euclid’s Elements without it featuring as a *common notion*. Basic and fundamental examples are right angles, isosceles (the famous ‘pons asinorum’) and equilateral triangles.

5. Archimedes’s and Stevin’s proofs of the two principles deduce the lever-arm law starting from a symmetrical state, but in one step turn it into an asymmetrical one by executing an unjustified mass replacement. The replacement is only justified in a purely symmetric case. This was criticized by Mach.

6. The center of gravity plays a crucial rule. In fact, it is equivalent to the lever-arm law. The lever-arm law is empirical. Pulley systems are conceptually equivalent to lever systems.

7. For a simple symmetrical pulley in equilibrium, the static momenta determine the equilibrium hence require equal weights. However, the fact that the force holding the pulley has to equal the sum of the two weights is assumed throughout but not proved. This is another implicit empirical law. <unsure>
   In analytical dynamics those two equilibria are exactly the split into pure force and pure torque.

8. The force holding the pulley being equal to the sum of the weights in 10. can be at least made intuitive by the mental experiment of gradually bringing the two weight together, binding them, in essence reducing the pulley to a particle held in equilibrium.

9. Another empirical law is the orthogonality of the static momentum's arm. In a pure pulley, this issue does not occur, but it does in more general cases. The law is attributed to Leonardo Da Vinci (p.20) with the remark 'The method by which Leonardo arrived at this view is difficult to discover.' We therefore consider it empirical.

10. The lever-arm law is therefore at this stage fully empirical, both in its 'scalar' expression and its spatial character of orthogonality (much later cross product). Exactly how did Euler originally formulate the laws without a cross product? <open>

11. Only with the assumption 11. can the section of virtual displacements be understood. It starts with several arrangements of pulleys in which the weight needed for the 'last' pulley is only a fraction of the weight on the first, in contrast with the simplest case where the weights are equal.

12. The correct interpretation of 'possible approach to and recession from the centre of the earth.' (p.51) is important. 'possible' here is the expression of 'virtual' and building statics upon dynamics. The equilibrium is not only determined by the masses, and we see that clearly in the various pulley examples in 14. What is missing? It is the virtual displacement (in other words also virtual work? <unsure>), it is the equilibrium of the dynamical situations of both weights, in a vague form, but brilliantly captured. To emphasize, it is empirically and even logically (given the lever-arm law) clear that weight alone is not the determining factor.

13. One should not read more into p.50 than it is. It was a great observation of an empirical fact, to be generalized later by Lagrange and others.

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§§.b2.

"A Treatise On The Analytical Dynamics of Particles And Rigid Bodies, With an Introduction to the Problem of Three Bodies" (1917), E.T Whittaker

1. The examples used for the concept of work: rough plane, smooth plane, rigid connection are to be remembered.

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§§.b3.
"Analysis by Its History" (2008), Ernst Hairer, Gerhard Wanner

1. One part of a memorable quote from the book that has stuck with me for quite a while, is: *What is a derivative? a limit.* I think this should be refined to make the relationships between the various topics studied in real analysis even clearer. Even though a limit is a 'first class citizen', to relate derivative more intimately to the rest, one should add "What is a limit? It is the point of convergence of an infinite sequence, given that the sequence does converge, which is not a given, and in fact, the conditions of it doing so in the most general cases constitutes the main part of basic real analysis". It is also helpful to keep in mind, as a relation from the physical concepts to derivative, the thought of tangent, characteristic triangle (17th century), per example for velocity or acceleration, in one or multiple dimensions, whose sharpest expression known to man is found in real analysis.

2. While Dedekind cuts might be simpler and more elegant and to the point for real numbers, Cauchy sequences are the ones that can be generalized to general sets and metric spaces as defining completeness.

3. Always remember the historical chain: Gauss/Fundamental Theorem of Algebra → Intermediate value theorem → rigor → Cauchy → Analytic proof, which requires at least continuity, limits and real numbers.

I. Sequences, Real Numbers, Series

1. We should be grateful for the term 'sequence' and its formal definition. Historically, this was the fuzzy concept of 'quantity' or 'variable quantity' (p.172)

2. A sequence makes a process (induces an ordering) out of structures of any dimension, so every vectors in Cn or whole functions can then be 'ordered'.

3. Every Cauchy seq. converges is in general not true. Take per example the state of matters before the construction of reals. The convergent did not 'exist'. Cauchy introduced the sequences in 1821, while the construction of real numbers happened later in 1872 (Cantor, Dedekind) (is this correct?)
   In fact, this criterion makes for an excellent definition of when a space is complete. (as in Zakon §§17 Def. 2) ([0,1] in $\mathbb{R}$ is a complete space).

4. A Cauchy seq. is NOT a seq. where s_{n+2} is closer to s_{n+1} than s_{n+1} is to s_n. It is stricter and more useful than that.

5. A series is NOT an seq. and both are assumed infinite.

6. We know that concepts on finite creatures do not in general apply to infinite one. In this specific case, we consider the commutativity of addition as it occurs in a series. 'a+b=b+a' but '$a_1+a_2+a_3+...+b_1+b_2+b_3+...$!= $a_1+b_1+a_2+b_2+...$' as there is no logical necessity for this unless established. In fact, '$a_1+a_2+a_3+...$' is simply bad and confusing notation borrowed from the finite world. A better notation is the one from analysis.
    a. Let us classify. ∞+ is in general not 'commutative', when it is, we call the series 'absolutely convergent'. And this is a good name because the condition for the 'commutativity' has to do with absolute values.
    b. Does this concept have a similar one for sequences? Is this not about being able (or not) to 'isolate' multiple sequences/series compounded in one? For sequences this reminds us of ordinal numbers: Imagine the seq. '-1,1,-1+ε,1-ε,-1+2ε,1-2ε,...' compared to '-1,-1+ε,-1+2ε,...(0),1,1-ε,1-2ε,...(0)' or '-1,-1+ε,-1+2ε,...(0),...,1-ε,1'

7. Uniform convergence is related to the simultaneous convergence of ∞many points: convergence of functions. <Rel. Metric spaces history quote TODO.>

8. I found an excellent quote summarizing the history of infinite series in analysis. [q.3]

II. Continuity

1. Dedekind felt that filling the gaps made the real numbers 'continuous' (complete). In this section, we are defining and treating continuity with functions in mind (in the restricted context of real numbers). The two continuities are related conceptually, but not at all the same, and we should not forget about the former one.

2. Bolzano wished to remove the physical concept of 'motion' from analysis. Expressions such as 'approaches' are to be replaced. This motivates his definitions.

3. Because of the infinite divisibility of the real number line (or even rationals) we have to work with intervals. Finite ideas such as 'Closest neighbor', 'Smallest distance' can at best serve as a motivation.

4. Continuity as defined is not exclusively about the absence of gaps. It really is, as Cauchy said, that we wish to find an interval in the domain (input) that produces a interval in the range (output) that is as small as desired. This does disallow 'gaps' but not only that. Situations such as sin(1/x) near zero are also disallowed since no matter how small the input interval is, the output will vary 'wildly' (between -1 and 1). Indeed, using a dynamic intuitive visualization, we see that any finite movement around x=0 results in the output moving an infinite distance.

   a. Note that for sin(1/x), any point with x=0 and y in [-1,1] 'feels' like a cluster point, hence, the function does not 'converge' to 0. This can be made precise by considering sequences of the form { $f(x_n)$ with $x_n=|1/n|$ } as $n\to\infty$?

5. It is sometimes better to use a dynamic horizontal visualization of the ε/δ definition that the usual vertical one. We use two parallel vertical lines, the top one for the range, the lower for the domain. Assuming a motion along the domain (in spite of Bolzano), we require that the resulting motion along the range is continuous: can be achieved finitely (see sin(1/x) note above) and without removing the pencil (gaps). This helps bring to intuition of continuity implying boundedness in the "Hauptlehrstaz" [III.].
   a. Imagine this process for f(x)=c, f(x)=n.x, $f(x)=x^2$, f(x)=sin(x), f(x)=1/(x-1)

6. Guided by the physical intuition above, it feels that the definition should proceed from small input intervals to small output intervals, But as we are not in a finite world, we cannot require that the 'smallest interval in the domain' correspond to the 'smallest interval in the range', in fact, it does not even make sense. It has to be clear that the structure of our infinite world does not allow for such a definition.
   a. We might think that we can use a limit to express the 'smallest interval' by defining continuity in the following way: $\lim_{(h\to 0)} f(x_0+h)-y_0 = 0$ means continuity at y_0. This is not bad but needs clarification. In fact, this is the definition of the limit of a function, which is introduced by Hairer after the ε/δ definition, but that order should be reversed.
   b. In the definition above, $\lim_{(h\to 0)}$ needs a definition on its own since it does not correspond to the already established limits of sequences or series, where we let the index n go to infinity. How does it relate to the ε/δ definition? [7.]

7. Let us examine and relate the multiple equivalent (iff.) definitions of the limit of a function.
   a. p.205 contains a definition that is ε/δ free and that related to limits of sequences: For every seq. $\{x_n\}$ in the range, $\lim_{(n\to\infty)} f(x_n)=f(x_0)$ if $\lim_{(n\to\infty)} x_n=x_0$. This defines continuity, but it can also be modified to define the limit of the function and continuity from that. So here it is, an ε/δ free definition. What is wrong with it? It contains two limits, where the ε/δ definition contains none and is therefore probably more practical in many cases! This definition corresponds to an old idea of using the sequence { $x_n=x_0+1/n$ }.
   b. p.209 contains Weierstrass's definition that is basically the one above: $\lim_{(x\to x0)}$ f(x)=y0. As we explained, $\lim_{(x\to x0)}$ has to be defined, and this brings in the ε/δ component.
   c. p. 210 contains W's definition (1872), this time of continuity (based on the limit of func. def. above). Here again it reminds of an old idea of two sidedness, sandwiching the y_0 point from both sides. It is then proved that this definition, called a theorem, is equivalent to the ε/δ definition. With the ε in W's definition corresponding to a 2ε in B's (1817). In W's definition x0 must be a cluster point a subtle but important technicality to work around empty sets and vacuous truth.

d. Clearly, what is the definition and what is an equivalent theorem is a matter of choice and taste.
e. From the definitions available, we see that we have the choice between limits of sequences or intervals (that define limits in sets).
f. In the ε/δ definition, the free choice of ε>0 provides the conceptual link to 'smallest interval'.

III. "Hauptlehrsatz"

1. "Hauptlehrsatz" (Principal theorem) is the name given by W in his 1861 lectures (published by Cantor 1870) to the theorem that $f:[a,b]\to\mathbb{R}$ and continuous implies it is bounded, has a min. and max.
   a. Every such function passes through any point between a and b adds to this. B's theorem and a main motivation <rel. algebra>
   b. Every such function is also uniformly continuous (a coming topic), hence integrable in the Riemann/Darboux sense.
   c. Clearly, such functions form quite a special class.

2. Note the importance of the closed interval condition. It is essential and should never be forgotten. Also, it is quite intuitive (see p. 207)
   a. The point of it is to 'commit' f to the two real values. An open interval fails in exactly that. It is not that the theorem cannot ever be true for an open interval, but such an interval can allow either unboundedness or missing extrema or both to sneak in: This happens if the point removed is an extremum (e.g as min. 0 for x^2), or is a point where the function is undefined in R, and ±∞ in R* (e.g: 0 for 1/x).
   b. One the examples in 207 treats [a,∞) as an interval, but this reminds us of the importance (Zakon) to define intervals and maybe redefine them in each section as such an interval is not of finite diameter.
   c. Once the function is 'committed' the continuity 'handles the rest'.
   d. Note the use of closed intervals in this and B/W supermum proofs, and the use of strict inequality in other cases like clustering and ε/δ. {WEAK}

3. The proof of the "Hauptlehrsatz" deserves analysis. Before that can be done, the B/W sup. theorem appears in a new light and must be analyzed further as it is deeper than it appears. It highlights yet another intuition about bounded infinite sets in the sense that there is *no escape from clustering despite the infinity*. Additionally, the former proof seems to be based on the interplay of the latter between domain and range, with the continuous function as the bridge. The B/W Theorem is analyzed in the next section (III+), which should be read before the rest of this section.

4.

III+. Analysis of 'A bounded set has [a] cluster[s] (per example a lub) in ℝ'

1. The proof by bisection that has a specific initial discomfort associated with the choice being made at every step (This is NOT the axiom of choice at play here <check>). However, a bit of thought shows that this is no different than processes such as the approximation of √2, where are each step, a function is applied to get a new estimate. Here a function is applied, but it is an 'algorithmic' kind of function: choose new bounds based on the center of the previous bound's relation to the interval. Therefore, the inf.many choices can be accepted under the usual umbrella of 'finite descriptions of inf. processes'.
    a. The intervals do get smaller and smaller and that can be said with confidence.
    b. Another conceptual difficulty is the assumption that we can determine whether some point is or is not an upper bound of some set (in a metric space to be more general). We do assume here that it is. However, note that here, unlike the case of approximation, we can never be sure to be getting closer to some real answer because we are dealing with a yes/no problem. We cannot establish the answer to any 'degree of desired accuracy'. A satisfactory resolution [7.] is given later in this note (existence/computability/construtivity).
    c. A less satisfactory initial resolution is the following: The difficulties can be removed if we assume that the set is given in such a way that the question of an upper bound can always be answered finitely. Is this true in general? Is this an implicit assumption about any given set in a metric space? Are there sets that do not have this property and that we are ignoring for now because they are out of scope (Brouwer and lawless sequences)?

2. To analyze the matter, let us compare the Cauchy/Bolzano/Weierstrass way (A.H) to the Dedekind way (Zakon).

3. For the C/B/W way, we note the following.
    a. The proof of existence of lub by B. first extracts a C-seq (Cauchy sequence) from a set using a choice procedure. The 'approximation intuition' works well for the C-seq convergence proof (p.181) and the proof pauses no problems. Specifically because of the sequential process-like nature. There is no discomfort and it may very well be 'applied', we can keep moving towards a guaranteed better lub during the process.
    b. The problem occurs during the extraction of the sequence from the set. This does not necessarily have an 'applied' form because of the yes/no problem described above. In short until resolution: The seq. seems not to be a problem, the set does.

4. For the D way, the proof of the completeness of the field constructed with Dedekind cuts

is the related proof. This is achieved by a definition of the lub that is the union of an inf. family of sets, each with inf.many elements. Given the lub the rest follows so the main problem if any resides here. Such a union has again no process-like nature and comes with the same discomfort as above. At first I thought this is related to the axiom of choice but it is not at all (see [7.])

5. One possible resolution is to claim that any finite description is acceptable, no matter the 'applicability', the 'process-like' nature, the possibility of actually finding a result given enough time (human bias?). This would remove the problem.
    a. Let us compare the def. of lub in D. to the applied example of proving that √2 is the lub of { x | x^2 <= 2 with x rational }. Here we show that the finite description of √2 : 'The/A number that squared gives 2', provides the means to show that it is necessarily the lub of the set. This is unproblematic: √2 is larger than any x and no number less than √2 can be larger than all x. This is quite different from the def. and we did not directly use the definition and the union of a family, although an indirect relationship is visible. We were able to find a finite equiv. proof for the def. that is inf. in character.
    b. The assumption then is that given any such finite description of a set and a finite description of a number, we can always relate the two descriptions to prove the relation of lub, or simply of a bound, or more simply of inclusion or exclusion in the set. Is this a reasonable assumption? Can we simply assume it and declare that 'execution' is a 'technical problem'? That sounds reasonable enough and a subject that is possibly treated within mathematical logic and advanced set theory. In any case, the resolution of this is also the resolution of the C/B/W proofs.

6. These difficulties might be the reason that would prevent others from creating such theories and are very understandable, is this what we mean by 'genius' in this case? In other words, any person that would try to proceed logically would stop short, unless! They can use the split attitude to their advantage, or have mastery of mathematical logic. Where the creators aware of this? Or were they bold or naive? <rel. Thinking>

7. The Resolution.
   After thought, the problem lies in the confusion of a number of things. Existence, computability, decidability, constructivity.
    a. The comfort of the C-seq comes from the fact that it is computable.
    b. The union needed in the D lub is guaranteed by the 'Axiom of Union'. It is an axiom and therefore the construction is impossible without this specific axiom. Should this axiom be accepted? Here we can adopt the split attitude and say both yes and no. However, we must note here that saying no is reducing analysis to less than it can be. The axiom of union merely guarantees the existence of the union set. And this is subtle: It is all about existence, and not computability, not a process-like procedure that can be applied, and not a constructive point of view (vague). This is merely about existence. If we want to take analysis to the highest

level possible, to be able to given finite claims and theorem about as many creatures as possible, we should be acceptful to all such axioms as long as they do not lead to contradiction and paradoxes (that is the task of logic and meta-mathematics and is outside of the scope).

c. The yes/no problem is a problem of decidability, and the resolution is here one of non-constructive? nature. If we accept that the answer must either be yes or no and cannot be undecidable, then this is good enough for us. There is no need for an applicable process.
d. We can surely classify real analysis, and decide there are other kinds of analysis that obey more stringent requirements and there indeed are (construcitvistic?). After that, using the split mentality <rel. Understanding>, we can again fully embrace it, but the analysis of these problems is crucial and increases the understanding indefinitely.
e. The problem of existence is solved by analysis. The problem of finding a specific limit, etc. is another one, checking is yet another (simpler) one. Analysis is not a method for finding such answers, and that is why it can be so encompassing, and must be.
f. I know about computability/decidability/constructivism from my Crysis readings (see Library), but I have not formally studied them. The analysis above brings me finally to a concrete connection (that I established by pure thinking) with these topics. For the foreseeable future, I will not have time to delve deeper, but the motivation in form of a problem that needs solving (like my initial $\sqrt{2}$ for analysis) is now there and this is a crucial step. At some point, a serious study will begin, motivated by exactly the questions above, and will add to the notes on everything. The decidability of set membership is one specific and basic question, or better said, the classification of sets under this questions. To reiterate: We can posit existence using axioms, and this is all we need for the all englobing analysis. Further classification of these existent objects in terms of decidability falls under logic.

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§§.1.
The Historic Method

1. Just like with mathematics, and $\sqrt{2}$ being the initial seed for our quest for demystification, for complete understanding, for the development of a philosophy of thought (extensively analyzed by Christoph Sigwart in “Logic”), it turns out that the situation is not different in physics in general and mechanics in particular. <relation> between studying physics and mathematics.

2. Even the slightest attempt at truly understanding of ‘Newton’s laws’ as taught in schools, shows the usual immense amount of mystification, decontextualization, de-philosophication, and the stripping of all meaning. Reducing them to the lowest level of ‘understanding’, a level that is in fact detrimental to understanding and could be safely considered a *negative* level.

3. The educational system is fundamentally broken. It adopts for explaining, the same strategy that parents employ with a two year old. Upon complaining to its father that it does not want to go to the nursery, the child gets the following response “You have to dear, because daddy has to go to work”. Upon pointing at a person making a strange face and inquiring about it, it gets the explanation “Because the man is angry”. Such answers are always accepted. The child does not go on to ask “Why does daddy have to go to work?” and then “Why do we need money?”, ultimately demanding to understand the whole economic system. It is thus safe to say that the child did not *understand* much about the world by accepting the simple answers. In the same way, a student taught about statics in secondary education, does not *understand* much. If so, the student would have to ask a chain of critical questions whose answers would amount to reading works such as Mach’s (§§.b1). This phenomenon is by no means limited to secondary education. As a result, one never *understands* anything within the current system. *Understanding* is only accessible to those who strive for a holistic view, and obtain it independently, outside the system. This view must include the organic, slow-stepping, man-created, crooked-path, historical point of view, alongside the most modern and rigorous treatment, both are indispensable. Per example one cannot claim to *understand* anything about physics if one does not *understand* number, which is out of the question without modern real analysis, which is totally out of context (and hence not understandable) without the history of *number*, starting from at least the Greek times. This is not the only way in which the system is broken. Due to the fallacious argument of not wanting to frustrate students, books are written in such a way that ‘difficult’ and fundamental proofs are presented within the material, while the simpler applications are used as exercises (even the ‘difficult’ ones are chosen this way). Why are the fundamental proofs and some of the exercises ‘difficult’? they are not. They only appear to be because there is a long story that needs to be told before introducing them. The adjective ‘difficult’ is the wrong one. Forcefully removed from context, with all traces of the crime explained away by the ‘two year old’ method is the correct one. The result of this sad situation is that the student is left with a mere illusion of understanding; a most dangerous illusion of grave consequences, carried along dormant until *understanding* is actually needed, on which occasion the illusion manifests itself, much too late for recovery. Thought is aborted. Common sense, sensing a vacuum of fundamental facts, commits suicide. Instruction should take as long as needed, including all the historical and philosophical context. Only then is it better than the ‘two year old’ method and has a chance of leading to *understanding*.

4. When a thinker creates a theory, theorem, proof, etc. he has already reached a stage of

very intimate familiarity with the subject. He has experienced it intensely and for a prolonged period of *intellectual time*. He is familiar with it as he is with every-day objects or more. Thus, he *understands* it much better than those who have not done that. In general, the studier will not have this understanding, he has not swum in the subject. This unfamiliarity plays a major part in creating the sense of magic and wonder of the solution proposed. Examples are Newton's mechanics to somebody who has never tried to personally make scientific sense of mechanics. Or variational dynamics to someone who has never tried to personally make methodic sense of solving problems using Newtonian mechanical means. The historic method solves these problems, proportionally to the time it is given. [vq.4]

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§§.2.
Physics

1. Newton's classical mechanics for somebody of this era with a typical general education, should only be attacked with the help of Max Jammer's books, which philosophically and historically elucidate the concepts of time, space and force. One can take as a proof of the negative level of the standard approach the fact that even an incomplete reading of these books provides a very path to relativity which is otherwise completely obfuscated. The inseparability of the two from an 'understanding of everything' point of view is the reason for putting this note under 'physics' and not 'mechanics'

2. The roots of the concepts of conservation of energy, before its mathematical explanation by Noether, comes from thermodynamical considerations. This is why, for a sane historical perspective, we should keep in mind the inherent difficulties of the old argumentations and proofs and also not shy away from basic thermodynamics, for they are an integral part of the play. An excellent popular account on this is Mach's 'On the Conservation of Energy' in "Popular Scientific Lectures".

3. Similarly to 2. Hydrodynamics should not be kept at a distance (Bernoulli, Euler, …). Here again (§§.b1.13), we need to find out what was used instead of the modern cross product. <open>

4. Mach's chapter on symmetry in "Popular Scientific Lectures" brought to light a lifelong confusion about symmetry and rotation. two symmetric figures cannot be brought to match by rotation, and this is something that was somehow confused in my mind.
    a. The distinction between rotation and improper rotation gains in intuitive clarity. The fact that the determinant of a nonsingular matrix can either be positive or negative, although clear from the onset, also gains a bit more intuition. I had

been bothered by the fact that in more than three dimensions, one can have more than one permutation, hence there are many different symmetries. How come we reduce them to only two classes? This is simply a property of the determinants in classifying odd and even permutations. It is related to symmetry, but is not the only property of it. Separating between the number of possible permutations (which fits into two classes in three dimensions but has more to it in larger ones) and the determinant is important.
   b. After the obvious distinction between rotation and symmetry was established in my mind it became clear that symmetry groups and rotation groups are distinct and of course they are. In the back of my mind lie groups were rotation groups (same as symmetry groups). This was rectified despite the fact that I have not yet started studying them.

5. After arriving at the thought of minimizing potential energy [Mechanics.7] I wondered if it is empirical <open>. I also found out that there is a number of principles that have to be distinguished <open>: “Principle of minimum total potential energy” (Physics, Chemistry, etc.) , “Principle of minimum energy” (Thermodynamics), “Principle of least action” (Classical mechanics).
   a. To my surprise the principle of least action is equivalent to minimization of potential energy (q.1).
   b. It seems that I am right about it being empirical. (q.2)

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§§.3.
Mechanics

1. Ernst Mach’s famous work is also an excellent source of understanding. It highlights the development and casting of an ‘axiomatic system’ (remember Euclid) for classical mechanics while pointing to the usually ignored (at the level of general education) aspect of empirical versus logical laws.

2. Mach’s criticism of the law of the lever proofs is eye opening and of great value to our philosophy of thought. The criticism has been debated at length as is described in modern articles (Van Dyck, M. “On the epistemological foundations of the law of the lever”, Paolo Palmieri. “The empirical basis of equilibrium: Mach, Vailati, and the lever”, etc.). The conclusion that I support is that the law of the lever was never proven, and cannot be, it is not *a priori* but empirical.
   Note. In the first mentioned article, the nail is hit on the head by providing mathematical arguments showing exactly what is being generalized and what is assumed by analyzing the function: $p.F(d) = p/2.F(d + h) + p/2.F(d - h)$.

3. Newton's three laws are therefore not sufficient to work out all mechanical problems.

4. The empiricity of law of the lever demystifies immediately the concepts of center of gravity (distinct conceptually from center of mass which is more of a geometric problem with no empirical content once space and mass has been defined) and the moment of inertia. The qualitative properties of the center of gravity: it's existence, uniqueness are never proved. Assuming those properties, a very restricted law of the lever can be deduced by a symmetric lever in equilibrium if we add some additional very easy to accept assumptions. However, any breaking of the symmetry (by splitting a mass into to, or by merging two masses into one on one side of a symmetric lever with two weights on each side) does not logically follow, and in fact, contains the implicit assumption of what there is to prove (as Mach explains in detail). Therefore, the law of the lever is never proved. It is easy to see that the law of the lever is a concept that is identical to that of the center of gravity, since given a way to calculate a center of gravity, would mean being given a way to prove the law of the lever.

   This means that the known formula for the center of gravity is based on the assumption of the law of the lever, and this completely demystifies it, makes it's formula extremely intuitive (an incremental process from an atomic point of view turned into an integral) and shows its empirical and not logical basis. The moment of inertia follows.

5. Can we do better? Can we reduce the law of the lever to a more basic empirical fact? The answer is yes, and it brings us back to our original quest. The law of the lever can be arrived at from the conservation of angular momentum, which can be proven from Noether's theorem, which shows that the assumption that physical laws are rotationally invariant implies conservation of energy and momentum.

   We have already established the path to reaching Noether's theorem, and this is one more reason for pursuing it with all available energy.

   Note: "From Summetria to Symmetry: The Making of a Revolutionary Scientific Concept", by Bernard R. Goldstein, might be worthwhile reading. <relation>

6. It makes sense to reduce statics to a special case of dynamics. For this a way to related them must be established. Hence there is no escape of a concept of virtual displacements. Note that from the onset, when constraints are considered (e.g: smooth plane, rough plane, rigid bar between two particles (§§.b2.1)), they from the 'static' part of the dynamic problem. Virtual displacements conceptually is the obvious way to build statics on top of dynamics, contrary to the historical development which by itself makes sense since it is easier to analyze complex static situations than dynamic ones.

7. Thinking about virtual displacements led me to the thought that Newton's equations of motion do not explain why motion happens. Two separated static particles can stay so without violating the equations, everything that should be conserved indeed is. The

cause of movement is outside the equations of motion. Thinking about this naturally leads to an (probably empirical <open>) fact that the system tends to minimize its ‘potential energy’. Already here we see how potential energy could rise to become a crucial concept. While the equations of motion constraint the ‘how’, minimization of potential ‘explains to a first degree’ the ‘why’. This historical seed of this is pinpointed in (§§.b1.p50).

8. It become clear to me why the mechanics books I have (Lanczos, Arnold, etc.) have the prerequisites they do. Thanks to the historic method, it is obvious that, just like with real analysis, there is an understandable urge to build mechanics on as elementary and rigorous grounds as possible. Noether’s theorem among others provided such grounds, and all the mathematical background for it is a prerequisite. Such a modern treatment is indeed the most epistemological of all possible ones so far when one *understands* it. Additionally, I see how such books on classical mechanics are mere ‘applied mathematics’, if one forgets the very strong ties between the historical development of the two disciplines.

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§§.4.
Gyroscope

1. Has to be completely understood. <open>

2. The old book 'Spinning Tops and Gyroscopic Motion, John Perry', might be of value here. In addition many books with interesting titles like this one are found at the back of 'A Treatise on Algebraic Plane Curves, Coolidge'.

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§§.5.
Statics

1. For a static system, there is no motion, hence no displacement, hence no work, no exchange of energy. This is equivalent to saying that internal forces do no work. More precisely, no internal force does work. This does not imply that an external force always does work. It may or may not. Consider as a system the rigid body falling. The force (exchange of motion) of gravity does work. The same body in contact with it’s ‘gravity partner’ could be static and this time the external force of gravity does no work.

2. What is then the definition of internal force? In a system, it must be any force on one particle for which there is an equal force (Newton's third) on a partner particle pairwise. In other words, an internal force is a force from a pair of forces, which are both forces of one exchange of acceleration between two particles of the system. Remember that we extracted the force concept to simplify certain calculations and laws and make them more elegant by being able to work on individual particles of systems without their 'partners' or 'fields' which are their counterparts in the motion or energy exchange. Internally to the system, each pairwise exchanges are then split into a pair of forces, each of which is then called an internal force.

3. In the case of a system being a rigid body static in contact with its gravity partner, the internal forces are the ones that enforce the rigidity. There is an external gravity force acting on each particle and an external contact force of contact, also acting on *each* particle. The presence of both these forces results in each of them doing no work.The situation here is complicated and require analytical work that is not directly reducible to a non-mathematical (we mean rhetorical) version.

4. In the falling case of the rigid body, if the system considered is the body, the external forces do work. If it is the pair of bodies, this external force becomes internal and, assuming no other bodies around, the internal forces again do no work, including the one that was in the previous case considered external.

5. In the absence of gravity, the object being absolutely static (assume Newton's absolutes), are there internal forces? one cannot say, since

6. There should be no confusion about 1. and the feeling of exercising muscles when holding an object. That system is thermodynamical and hence heat, chemical reactions, etc. have to be included.

7. Let us translate what it means for a force to do or not do work to our 'exchange' language.
    a. Exchanges are zero-sum.
    b. A system on which work is done exchanges energy with another system, hence, gains or loses energy.
    c. When two bodies exchange acceleration, they exchange energy and conversely.
    d. A force either does work or not. All this means is that an exchange of acceleration either happens between two particles of the system, or between a particle of the system and one of another. In the first case, the system's energy remains unchanged. An example of this is particles inside a freely rotating rigid body. In the second, all energy exchange scenarios are possible, depending on the totality of interactions between particles of the system with particles outside the system. Examples of this are a body falling, a body rising and a body in contact with it's gravity partner. This is made clearer by considering the case of two lonely particles where exchange of acceleration and energy are clearly

inseparable or by the fact that Newton's second is equivalent to conservation of energy. We must here emphasize once more that the force and work language allows the considerations of unary calculations and fields and are hence indispensable.

e. An external force does no work when its work is counterbalanced by the work of another external force.
f. Force and work in everyday language are perfectly ambiguous.

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§§.6.
Convincing

1. People who regard correctness as unnecessary or inefficient.
    a. Appeal to the authority of Russell, and explain that what is *obvious* and *intuitive* may very well, after rigorous examination turn out to be true, but also false or conditionally so (vq.1).

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§§.7.
Understanding, Knowing, Thinking

1. Understanding has to be distinguished from familiarity no matter how prolonged and intimate. Per example, almost everyone thinks they understand counting numbers and arithmetic at the latest after secondary education, but in reality, they don't. They are merely very familiar with such numbers. This illusion of understanding dominates our lives (vq.2). Having realized this, we can achieve the same kind of comfortable familiarity with any concept, no matter how removed from 'every-day' experience by simply thinking about the concept, encountering it mentally, as often we do encounter everyday objects (vq.3).

2. Understanding Physics must begin with the conviction that what we actually observe and feel is merely a projection onto our infinitely narrow and biased senses, followed by the resolution to reconstruct the source of the projection.

3. When a thinker creates a theory, theorem, proof, etc. he has already reached a stage of very intimate familiarity with the subject. He has experienced it intensely and for a prolonged period of intellectual time. He is familiar with it as he is with every-day objects or more. Thus, he understands it much better than those who have not done that. In

general, the studier will not have this understanding, he has not swum in the subject. This unfamiliarity plays a major part in creating the sense of magic and wonder of the solution proposed. Examples are Newton's mechanics to somebody who has never tried to personally make scientific sense of mechanics. Or variational dynamics to someone who has never tried to personally make methodic sense of solving problems using Newtonian mechanical means. The historic method (§§.1) solves these problems, proportionally to the time it is given.

4. A major triumph of mathematics in terms of application to physics in the very general sense (e.g Greek physics is Euclidean geometry) is the additional to our language of new constructs that allow us to handle quantitative reasoning with the needed exactness. To this it also added the calculus, to handle the relation of change with the needed exactness. Until this happened, when we saw a rock falling, all we could say is that it is going down and not up, fast or slow, but not much more at all. It is easy to fall into the illusion that we could in fact say more in terms of exact quantity. Without mathematics, (and Galileo to steer the attention to the *how* and *how much* from the *why*) we really could not have said much. When write down equations (sentences in mathematical language with deeply nested and recursive meanings) about the rock falling, we are bringing in mental sceneries of measuring rods, clocks, geometric relations, quantities, variation) all together, in a mathematically rhetorical way, and we can even apply logic to them to say more about them. That is indeed an evolution. We must note here that living in a certain age is a necessary condition to having learned these great abilities, but it is not sufficient: Most people can successfully visit life and never acquire or need them.

5. Understanding leads to knowing. Knowing enough can approximate understanding by exhaustion. To recover facts from understanding requires thinking, possibly a lot of it, hence knowing is a crucial shortcut. Thinking can be based purely on knowing or purely on understanding but preferably both. It is probably that genius requires both lots of knowing and lots of understanding (Knowledge rich, bias free). When, given a topic, a questionable proposition is presented as obviously true (e.g axiom of choice in the Dedekind and Bolzano-Cauchy supremum existence proofs, etc.) and that proposition is a fundamental building block, then the right way forward, if the proposition is unresolved after enough thinking, is splitting. The mind is split into two, one that believes it, and one that does not. The believing part can easily assimilate the whole topic, and it is important that believing is made as honest as possible, and even understood intuitively so that it can be used effectively. The disbelieving part will have to do with an incomplete theory, and, given the luxury of sufficient time, analyze the consequences. Splitting is not the same as controlled schizophrenia, but cs is used to be able to work on both split parts with no problems. Trust in the fact that the proposition has been accepted by many helps justifying the believing. Critical attitude and the real understanding help justify the counterpart. This is the fastest and best way to handle such situations.

6. Given the relation we found between knowing and understanding, it is clear that a

perfect memory would give an immense thinking advantage. From the phenomena of autism and genius we know that perfect memory, or at least much better than normal memory, is possible. The distinction between detail and no-detail is the key. We are told that it is important to lose detail to not be overwhelmed by sensory information, and that is reasonable. However, a switch to switch between this mode and a 'now blindly remember everything on this page, book, ...' would be nice but is not there for normal people. I wish there was a way to switch off detail loss by some mental technique.

7. Exhaust, exhaust, exhaust.

8. Thinking: Reducing a wall to rubble and walking on it.
    a. In the sense of thinking about a problem, analyzing it.
    b. This occurred to me after the resolution of the conceptual problems <rel.>, which occurred after an intellectual upgrade and yet another approach to studying (criss-cross, relational, all-inclusive, exhaustive, comprehensive, clear) triggered by reading Mach, Russell, Sigwart (Logik), etc.

9. During the *mathematics for its own sake* (split) mode, there is a very simple and extremely motivating way to proceed in even the most (so called) abstract scenarios. The richness of a theory and the surprises revealed by its important theorems always enhance the understanding of *essence*. In this case of the interplay between a chosen axiomatic systems, especially in comparison to neighboring systems. A very basic example is note 8.1 [8.1].

10. A specific tactic related to 'Controlled Schizophrenia' is the 'Pendulum tactic' during which in one single thinking session the mind swings between two extremes: The purely abstract (for itself and by itself) and the purely applied. <rel. MCMM-I.2>

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§§.8.
Group Theory

1. The first beautiful fact about groups is that groups axioms suffice to show that when an identity element exists it must be unique, and when an inverse exists, it must be unique as well. Even this basic fact shows how deep and important it is the full chart the landscape of Mathematics. One would think the first is a property of the natural numbers under multiplication and therefore has its essence in the arithmetic of counting numbers. One would also think similarly about the second property with naturals under addition or rationals under division. But no, the essence of this is purely group theoretical, occurring at a ‘lower level’ so to speak: The natural numbers from groups, but there are much simpler and also finite sets that form groups.

I do not know why this exact fact became clear to me at this stage, while reading <TODO> and not while reading elements of algebra or other snippets about groups in the past. It may be the exposition, but surely also the fact that I can now finally think (to a limited degree).

Quotes

1. *Euler continued to write on the topic; in his Reflexions sur quelques loix generales de la nature (1748), he called the quantity "effort". His expression corresponds to what we would now call potential energy, so that his statement of least action in statics is equivalent to the principle that a system of bodies at rest will adopt a configuration that minimizes total potential energy.*

2. *Inasmuch as Newton's laws are inductive generalizations, and these principles are equivalent to Newton's laws, it's obvious that these principles, too are inductive generalizations. Thermodynamics further generalizes, in a way, these originally mechanical principles when it comes to the context of studies of heat, mechanical work, their relationships, and effects. Qua inductive generalization, these principles have no prior deductive proof. Their truth is established using the method of induction. However, qua their use as starting points in performing analysis, their status may be taken as that of postulates.*
   Note. 'Inductive' as in inductive reasoning not inductive proof, hence empirical.

3. *Infinite series, and their analogues-integral representations, became fundamental tools in mathematical analysis, starting in the second half of the seventeenth century. They have provided the means for introducing into analysis all of the so-called transcendental functions, including those which are now called elementary (the logarithm, exponential and trigonometric functions). With their help the solutions of many differential equations, both ordinary and partial, have been found. In fact the whole development of mathematical analysis from Newton up to the end of the nineteenth century was in the closest way connected with the development of the apparatus of series and integral representations. Moreover, many abstract divisions of mathematics (for example, functional analysis) arose and were developed in order to study series.*
   In the development of the theory of series two basic directions can be singled out. One is the justification of operations with infinite series, the other is the creation of techniques for using series in the solution of mathematical and applied problems. Both directions have developed in parallel. Initially progress in the first direction was significantly smaller, but, in the end, progress in the second direction has always turned out to be of greater difficulty.
   It would be a mistake to think that the justification of operations with series interested our

predecessors less than us, or that they valued techniques more highly than rigour. Newton's proofs were completely rigorous, and he was reluctant to publish an insufficiently justified theory of fluxions. In my opinion, the small advances in the justification of operations with infinite series is explained by the absence of a suitable language in which to conveniently speak of these operations, and the creation of a language requires incomparably greater efforts than the proof of individual results. As a rule, the creation of a language is the work of several generations. In this respect we can refer to the example of Euler, whose research affected his contemporaries by its depth and non-triviality, but shocked them with its lack of rigour. To a modern reader the arguments of Euler do not seem to be so very non-rigorous. Simply, Euler already understood the principle of analytic continuation (for single-valued analytic functions), but the absence of a suitable language prevented him from transmitting this understanding to his contemporaries.
In the mid nineteenth century there was already a completely modern under- standing of a convergent series which allowed one to prove the required results with complete rigour and to distinguish valid arguments from invalid ones. However, left over from the seventeenth and eighteenth centuries were many puzzling unjustified arguments which, for all their lack of justification, led to true results by significantly briefer routes. The expansion of the main points of these arguments and the creation of new means of justifying operations with divergent series and integrals was one of the basic achievements of the last century. A short account of the stages in the development of the modern approach to these questions forms the content of the first chapter of this article.
The second chapter is devoted to the second direction; techniques for using series and integral representations in mathematical analysis. The selection of the material for this chapter presented a most difficult problem, and the chosen solution is purely subjective. I have desisted from an attempt to list results, since this route would have required a much larger volume and would have ended only with the production of a reference book; completely useless for reading. A unique opportunity for me, it would appear, to give an exposition of fundamental methods. However, even this path has its own obstacles. The fact is that almost every method which has been used in analysis has generated, in its applications to different objects, extensive theories. Some of these theories have been successfully concluded, some are being rapidly developed and some have come to a dead end. In any of these cases a detailed story of these theories is inadvisable. I have decided to recount in this article only those analytic methods which have not yet been developed into a general theory. Almost all of it is around 100 years old (or more), but is familiar only to sophisticated analysts. To establish the authorship of these methods is most often impossible; they represent the birth of "mathematical folklore". (Encyclopedia of Mathematical Sciences, Analysis I, Integral Representations and Asymptotic Methods, R.V. Gamkrelidze (Ed.))

Validating Quotes

These quotes are such that they validate ideas I arrived at by thinking, and later found (always better) explained or expressed by an authority.

1. *Obviousness is always the enemy to correctness.* (r.1)

2. *Let us suppose that one of them has been released, and compelled suddenly to stand up, and turn his neck round and walk with open eyes towards the light; and let us suppose that he goes through all these actions with pain, and that the dazzling splendour renders him incapable of discerning those objects of which he used formerly to see the shadows. What answer should you expect him to make, if someone were to tell him that in those days he was watching foolish phantoms, but that now he is somewhat nearer to reality, and is turned towards things more real, and sees more correctly; above all, if lie were to point out to him the several objects that are passing by, and question him, and compel him to answer what they are? Should you not expect him to be puzzled, and to regard his old visions as truer than the objects now forced upon his notice? Yes, much truer.* (r.1.p5)

3. *Hence, I suppose, habit will be necessary to enable him to perceive objects in that upper world.* (r.1.p5)

4. *It has seemed to me for a long time that commutative algebra is best practiced with knowledge of the geometry that played a great role in its formation: in short, with a view toward algebraic geometry. Most texts on commutative algebra adhere to the tradition that says a subject should be purified until it references nothing outside itself. There are good reasons for cultivating this style; it leads to generality, elegance, and brevity, three cardinal virtues. But it seems to me unnecessary and undesirable to banish, on these grounds, the motivating and fructifying ideas on which the discipline is based.* (Commutative Algebra: with a View Toward Algebraic Geometry, David Eisenbud)

5. *As soon as one is convinced of the correctness and the power of this calculus and has mastered it,he is apt to be reminded of what Jacobi had to say about the significance of algorithms (see A.KNESER, "Euler an die Variationsrechnung", Festschrift zur Feier des 20. Geburtstages Leonhard Euler, Teubner Verlag, 1907,p.24): "da es … (because in mathematics we pile inferences upon inferences, it is a good thing whenever we can subsume as many of the mas posible under one symbol. For once we have understood the true significance of an operation, just the sensible apprehension of its symbol will suffice to obviate the whole reasoning process that earlier we had to engage anew each time the operation was encountered)."* (The Theory of Complex Functions, Remmert)

   Of course, we disagree that there is any 'true significance' and an 'obviation of a reasoning process', but we have had this idea a very long time ago (not documented).

References

1. Russell, Bertrand. Mysticism and Logic: And Other Essays. 1918.