G says it is not provable that G."

The possibility of an (indirectly) self referencing formal language

Is it simply by making both propositions and their numbers present in the formal system, using a conveniently defined oredering similar to the ordering that makes rationals or integers countable?

All things being equal, there is no necessity for a one-to-one correspondence between questions and answers.

Any theory with sufficient recursive resources can refer to its own sentences in a cleverly indirect way, without blurring metalanguage and object language. By recursively enumerating the well-formed formulas of the theory we can speak unambiguously, in first-order terms, about '*the formula with (Gödel) number n*'. This is the trick behind Gödel's Incompleteness Theorems (see Section 11.3 and Appendix C); via a diagonalization argument we prove the existence of a sentence which (indirectly) asserts its own unprovability, the sentence '*the Gödel number of this sentence is the Gödel number of an unprovable sentence*' substituting the '*I am lying*' of the liar paradox. Gödel explicitly mentions the analogy in his 1931 paper[2] – '*...the analogy of this result with the antinomy of Richard is immediately evident; there is also a close relation to the liar paradox*'.

far-ranging examples. Given the success and wide scope of PM and ZF one might be led to believe that *all* questions which are expressible in the underlying code of these formal systems can be decided in them. This is not so, and more surprisingly it can be shown that in both systems there are propositions about the *natural numbers* which cannot be proved true or false: these systems are 'incomplete'. Furthermore, this is not a peculiarity of the given systems but a phenomenon which holds for a very broad class of formal systems.

- The Gödel sentence does not make reference to itself in the simple ways suggested by the its informal versions. The numeral for G—and the sentence G itself—correspond precisely in that there is a one-to-one coding between them, but they do not have the same "meaning." They are "extensionally equivalent" but not "intensionally equivalent."

I have argued (Good, 1967) that the assertion that *human logic can do some things that a Turing machine cannot do* cannot be proved by means of Gödel's theorem. Lucas (1967) misrepresents me when he says that I 'deny that there

Gödel developed a numbering system that allowed him to include formal mathematical proofs into numeric expressions. The numeric expressions—or Gödel numerals—can be recognised and manipulated within the Principia system in the same way that any numerals could be. Gödel numerals allowed the Principia—and formal systems generally—to look at themselves and to say things about themselves. Through this method, Gödel wanted to find out whether the formal system of Principia Mathematica could prove itself consistent.

He discovered that no such proof exists. There is no way from the Principia to prove itself consistent. Gödel went further and used his method to prove that no complete formalisation of arithmetic exists at all (Gödel's first theorem). As a corollary, he returned to the consistency question and showed that no consistent formal system of arithmetic could be proved consistent using only its own methods of proof (Gödel's second theorem).