Note. pg 314, par. 3. 'what is far more...'. It is as if the greeks were not really doing geometry at all, but alg. desguised as geom.
one might say that just as with proj. geom, circles appear as special cases of conic proj., or a pentag. a special case of a hex. (pg.300), or involution and harm sets as projecting to bisections, greek geom is but a proj. of alg. into a plane or a 3d space.

todo. add pg 319 to 'amazing greeks', it's not only me!

erratum. pg 321, in three unknowns ??? for ...

note. synth geom to coord. geom is lin. alg to matrix alg. in the respect: choice of basis/coord sys

note. pg 323, very enlightening about terminology 'mess'

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is area 'defined' ? yes, why does it work then? from discrete to cintinuous, semantics, rhetroric
an area of (8 meters) x (2 area units per meter) is 16 area units.
analogically to area under vekocity, that is dist/sec --> started by galileo
this comes from averages, rhe sjmplest form area: both dimensions 'constant'
this is easily and finiteky computable, we try to express all in terms of it (a choice, a reasonable choice)
so again it is all about finite xomputability. computers took over num. comp. the known closed forms of calc. are finite symbolic comoutable
one day comput. will tajw over that too, the last step is deciding what is more or less relevant to attack. human math as we know it will be dead.
we can always (sometimes) 'find averages' that is 'find closed form answer' that is true answer, the limit of any num. comp. process
about computab.: in mathematics calculus, alg, we deal with 'simple' finite computable objects, that is one reason for why we need to model the world close to these simple (=finite, computable numericakky or symbolically) objects:curve regression, approx. , etc...

we wokred somhard to get the right symbolism, computer gets it for free

quote pg. 349 lin 4-6
q.350.16

calxukus is included in R becuase of limits. in many cases it is the simpler lub that we use: convergence of an infinite limit process

p.351. this 'pre-calculus' method is very 'euclidean-geom' like, if we add cavalieri's theorem as an 'axiom'. very synthetic indeed. (add to pre-calc. collection?)

p.352. why did they study series sums? integrals!
also note 'analytic geometrization' of exhaustion to get a 'method' of course only poss. with alg. and theory if funcs to the rescue

Q. what did wallis do exactly? p.353.-1
Q. check p.355.10 or p.354.-2
Q. rectification of ellipse. Yet another conic special! learn more p.355\3 (paragraph 3)
Q. p.356\1.-5

note how integ. using csrt. coords is simpler than echaust since one axis is a line. in some cases it might be more difficult though.
also note that length of cyrv seems at fiest sught more compl. than area under curve

note how after in eucl. geom we built many 'invis.' constructions, with exhaustion we start to 'analyze' the shapes more closely,
cutting them to pieces, or approx. their contours with lines at points.

diff. from neighb. features to pt. feature. integ. from 'pt.' or 'neighborhood' festures to global? features

indeed after even a bit of thinking, diff between anti-diff and integ is obvious, diff between area and anti-diff.
it is very wrong to relate them mentally a priori.

note. tang. indic. 'dir' of curvature (curve 'below' tang. locally)
note. 2d phys actually 3d prob. x,y,t . imagine 2d car, stopped moving on x axis for a whyle-> x,y graph a 'problem'
but sometimes we can proj 3d to 2d prob.

note on tang evolu (p.344) from 1. greek (static) to 2. roberval (dynamic/phys.) to 3. fermat infintes (limit)
big jump 1->2: 'change generates the curve'. in 3. measure the change using infin. on the curve.
in general this is all about a curve being gen'd with 'sthg wrp. sthg' (multi-dim). a curve that is not such, is not differentiable?
of course we mean 'continuous' 'gapless' change, change that is 'smooth iver the reals/'inputs'

integ. is then the accum of the changes and one chane can be obtained back by the diff between two accumulations: ftc.

given real analysis (limits) basic calculus is trivial, given a proper phil/histor. background crucial to justify/support/demyst. the definitions themselves.
in fact, the 'tricky?' part becomes (as usual with axiom. system) justifying the rel. (is this math. concept what really models best...? why? what do we require from the condcpt?) to the applications/phys world.

p.347. kepler's idea worded diff. and in hindsight is : if we (the change at a point) approach zero (for swirch of signs=> extrem. that is point at which the change is zero) BUT never overshoot it (the point) the changes MUST be getting smaller since the infinitely many-pts monotonousls approach the max pt.
if not, we would 'overshoot'
note that this does not woek for a peak formed by two straihr linws,,e.g area of equi. triangle.it works when the curve must flatten at the extremum, hence the approach of dy=0 for any dx

pg. 348/2 did kepler then calculate pi? find out... was it an approx? (maybe in stillwells book?)

note. diff is rate of chnage dy/dx (vanish. ratio) integ is not dy.dx but S(dy.dx) which brings us to area. what abiut simoly dy.dx this is always zero.
but S is not because of 'non-zero measure?' any finite sum of dy.dx would be zero when dx->0
definite area came firsf, jump to indef is quite big, why does it work, how does it work around the origin and with neg areas?

notice that perimeter involved the pyth triangkes infinites. from the tangent. in a way S(dy^2+dx^2) is that so?

p.350. use cavalieri's princ to prove area under stright line

tangents. if we want to link the definition of rate of change (limit) to geom tangent, additionally to the hist. persp. from the book we can think of two addit.ideas.
1. close to dyn. view of tangent. for any fct. that is buildable by a finiteky comp. rate of change. the tan. is the const. rate of change curve (line, hyperplane)
that we would get at pt. if we 'freeze' the rate of chg. at the point, make it ct. for the rest of the curve.
2. of course we think of tan. not touching twice, but of course, within some small neighbhood (note that probabaly we canget some pathological c fct where the rate of change osxialltes so much that the neighb is never found?). now this idea is best captured imagining the circle with the obvious synthetic tangent. this is nice becuase it also fits the conceot (that i finakky undertsnad) of making the circle with a certai radius a "measure' of curvature (a very interesting topic from now on)

erratum. p.368.-5 missing end of sentence point.

logatirhms beatifully relate products to sums (through a table and reverse lookup) [the history of mathemarics, burton p.354]. Does this make them significant in number theoery?

'oneandallsophisms.Aftertwothousandyearsofcontinual
refutation,these*sophismswerereinstated,andmadethe
foundationofamathematicalrenaissance,byaGerman professorwhoprobablyneverdreamedofanyconnexion
betweenhimselfandZeno.Weierstrass,bystrictlybanishing

allinfinitesimals,hasatlastshownthatweliveinan unchangingworld,andthatthearrow,ateverymomentofits flight,istrulyatrest.TheonlypointwhereZenoprobably erredwasininferring(ifhedidinfer)that,becausethere isnochange,theworldmustbeinthesamestateatonetime asatanother.Thisconsequencebynomeansfollows,andin thispointtheGermanprofessorismoreconstructivethanthe ingeniousGreek.Weierstrass,beingabletoembodyhis opinionsinmathematics,wherefamiliaritywithtruheliminatesthevulgarprejudicesofcommonsense,hasb eenableto givetohispropositionstherespectableairofplatitudes;and iftheresultislessdelightfultotheloverofreasonthanZeno's bolddefiance,itisatanyratemorecalculatedtoappeasethe massofacademicmankind...'
1 BertiandRussell,ThePrinciplesofMathematics,vol.i,1903,pp. 347,348 ---> read russel's elaboration on this, a good memory peg.

'Fomenko' quoting hilbert: the tendency to abstraction has led to grand systematic constructions of algebraic geometry,Riemann geometry and topology where the --methods-- of abstract reasoning, symbolism and analysis are widely used.

p.1158. An obvious but excellent rephrasing of one-to-one corresp. in topolog. ' in a way that does not create new points or fuse existing ones'. About continuity he says 'carries nearby points to nearby points'

chap 50. is the missing chapter in our historic understandig of the generalizations in Zakon I (e.g metric spaces, continuity in general, etc...) and its relation to our new fascination topology that seems so related and touching a deep but intuitive concept that needed a sophisitcated formalism to be captured.
on the one hand abstracting manifolds from coordinates, and classufying them, also relating them : cube=sphere<>torus. Rn is always there (and therefore R1, our basic and essential formal understanding of continuity, first absolutely: gapless line as in dedeking, then half-absolute in eps-delta, then general/relative: homeomorphism with eps-delta a hom. between a curve and R1)

note. what i just said about coordinates is said much better in this quote i found on p.1163. 'as far back as 1679 Leibniz, in his Characteristica Geometrica, tried to formulate basic geometric properties of geometrical figures, to use special symbols to represent them, and to combine these properties under operations to so as to produce others. He called his study analysis situs or geometria situs. He explained in a letter to Hugens of 1679 that he was not satisfied with the coordinate geometry treatment of geometric figues because, beyond the fact that the method was not direct or pretty; it was concerned with magnitude,whereas "i believe we lack another analysis properly geometric or linear which expresses location [situs] directly as algebra expresses magnitude." Leibniz's few examples of what he proposed to build still involved metric properties even though he aimed at geometric algorithms that would furnish solutions of purely geometric problems. Perhaps because Leibniz was vague about he kind of geometry he sought, Huygens was not enthusiastic about his idea and his symbolism. To the extent that he was at all clear, Leibniz envisioned what we now call combinatorial topology.'

p.162.-5 'the subject of point set topology has continued to be enormously active. It is relatively easy to introduce variations, specializations, and generalizations of the axiomatic bases of the various types of spaces. Hundreds of concepts have been introduced and theorems established, though the ultimate value of these concepts is dubious in most cases. As in other fields, mathematicians have not hesitated to plunge freely and broadly into point set topology'

p.1170 'before he undertook the combinatorial theory we are about to describe, poincare contributed to another area of topology, the qualitative theory of differential equations (chap 29, sec, 8). That work is basically topological because it is concerned with the form of the integral curves and the nature of the singular points. the contribution to combinatorial topology was stimulated by the problem of determining the structure of the four-dimensional 'surfaces' used to represent algebraic functions f(x,y,z)=0 wherein x,y and z are complex.He decided that a systematic study of the analysis situs of general n-dimensional figures was necessary. After some notes in the Comptes Rendus of 1892 and 1893, he published a basic paper in 1895, followed by five lengthy supplements running until 1904 in various journals. He regarded his work on combinatorial topology as a systematic way of studying n-dimensional geometry, rather than as a study of topological invariants' note. it is true that as soon as we go to 4+d we cannot represent our geometry figuratively, and analytically was no success (and poor?). How then??? topology is the most natural answer!

p.1170 'in his 1985 paper poincare tried to approach the theory of n-dimensional figures by using their analytical representations. he did not make much progress this way, and he turned to a purely geometric theory of manifolds, which are generalizations of Riemann surfaces'

p.1173 'it is clear from even these very simple examples that the betti numbers and torsion coefficients of a geometric figure do somehow distinguish one figure from another, as the circular ring is distinguished from the circle'

p.1777 quote the beginning and end of 'fixed point theorems' another full circle for us??

p.1776 notez 35 and 36 are by themselves motivations since it is so strange that they not be true in general and this contributes highly to understanding n-dim spaces!