'Preliminary Notes On Everything' contains candidates for notes. Some notes in this document are either superseded by better ones, or found to be wrong after more analysis, or are too vague or badly formulated. However, it is important to note all these 'negative results' and keep them because they help converge to the correct ones in situations where the correct ones are not easily reachable.

Symbols: ∞ ɸ ^{2} § → Δ † § α

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§ On Thinking.

1. Given the relation we found between knowing and understanding, it is clear that a perfect memory would give an immense thinking advantage. From the phenomena of autism and genius we know that perfect memory, or at least much better than normal memory, is possible. The distinction between detail and non-detail is the key. We are told that it is important to lose detail to not be overwhelmed by sensory information, and that is reasonable. However, a switch to switch between this mode and a 'now remember everything on this page, book, ...' would be nice but is not there for normal people. Is there a way to switch off detail loss by some technique?

* Why do we need the visual real number line along with a static of dynamic point scene to think about limiting processes and convergence in analysis. It seems that what we need is a picture, but that is not the real need. To pass from idea A to idea B through a link of transformation L, we must have all of A,L,B in mind simultaneously (maybe B only partially since it is still forming). This seems impossible when thinking verbally (is it really?). It is very possible with a mental image because of the parallel treatment of it, or at least the working of very short term memory giving the illusion of parallelism. Is this the only real possibility of thinking from A to B we have? Can formal symbols be treated like these images? Maybe sometimes and with training. Are the symbols stripped of 'meaning' and reduced to the allowed transformations on them at the time of processing? Is this related to the speed of visual processing compared to the storage time-frame of short term memory? Pronouncing a word mentally might already too slow for it to fit in this memory while another linking work follows? How do we get around this limitation? At the same time, we know we DO use verbal thinking. This is unclear. {WEAK}

* All modes of thinking are valuable: Visual, formal/symbolic/manipulatory, relational, causal, dynamic/process/sequential, exhaustive/random/exploratory.
    - Since the possibilities of symbolic transformations are usually large, it is a good idea to guide this mode of thought with strong relational thinking, or wishful (proof) thinking (Euler mode).
    - The exhaustive mode should not be excluded or forgotten (Euler mode). {WEAK}

* Exhaust, exhaust, exhaust.

* Why do we consider differentiation, integration, etc. ? We do because of applied mathematics, and not by exhaustion of mathematics. On a lower level, limit, number, etc. are also motivated by applied mathematics. They exist purely by and for themselves, but of all existing creatures, the choice is necessarily motivated by the applied, directly or indirectly. {VAGUE}

    - Are the 'details' of the proofs (e.g in analysis for 'convergence', 'completeness', etc.) details? Or are they the 'essence' of number, function, etc.? They would be if they were the 'only essence' but are they<open>? I think not, or let us simply say, some may, some may be not. Those that are not we could think of as 'accidentally true', but motivated by applied mathematics. Feeling accidental makes them appear as details and therefore difficult to remember without effort [1.] We have to 'be the number', 'be the limit', 'be the point of continuity', 'be the sequence' in order to internalize the essence of the details/proofs. Is this 'retainable' for the long term? Why does it feel desirable? To allow deep and flowing thinking. It is strange that they feel like details, whereas the theorems that they prove do not. {VAGUE}

* Thinking: Reducing a wall to rubble and walking on it.
    - In the sense of thinking about a problem, analyzing it.
    - This occurred to me after the resolution of the conceptual problems <rel.>, which occurred after an intellectual upgrade and yet another approach to studying (criss-cross, relational, all-inclusive, exhaustive, comprehensive, clear) triggered by reading Mach, Russell, Sigwart (Logik), etc.

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§ On Writing.

* Before 'writing' an idea, 'meta-think' about it. What are the 'objects'? What relations do we want to express? What are the words that best capture the relations? What is the sentence structure that captures the relations? {REFORMULATE}

* Remember Russell. Shortest clearest meaning. {WEAK}

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§ Notes on A.H

* Always remember the historical chain: Gauss/Fundamental Theorem of Algebra → Intermediate value theorem → rigor → Cauchy → Analytic proof, which requires at least continuity, limits and real numbers.

* Hairer says 'What is a limit? A number'. We must add: 'What is a number? A limit (in the Cauchy sense)'. However, some numbers are exclusively accessible as limits, others are also accessible by other means: induction for integers, field axioms for rationals. {WEAK}

* We should be grateful for the term 'sequence' and its formal definition. Historically, this was the fuzzy concept of 'quantity' or 'variable quantity' (p.172)

* A sequence can be visualized either two dimensionally and statically (index, value) or one dimensionally and dynamically (moving values). Another possibility is a cute Sheep herd, either looking at their next neighbors or at some cluster point. {SHALLOW}

* A sequence makes a process (induces an ordering) out of structures or any dimension, so every vectors in Cn or whole functions can then be 'ordered'.

* Every Cauchy seq. converges is in general not true. Take per example the state of matters before the construction of reals. The convergent did not 'exist'. Cauchy introduced the sequences in 1821, while the construction of real numbers happened later in 1872 (Cantor, Dedekind) (is this correct?)
In fact, this criterion makes for an excellent definition of when a space is complete. (as in Zakon §§17 Def. 2) ([0,1] in R is a complete space).

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§ Notes on A.H (III.1)

* A Cauchy seq. is NOT a seq. where sn+2 is closer to sn+1 than sn+1 to sn. It is stricter and more useful than that.

* An series is NOT an seq. and both are assumed ∞

* We know that concepts on finite creatures do not in general apply to infinite one. In this specific case, we consider the commutativity of addition as it occurs in a series. 'a+b=b+a' but 'a1+a2+a3+...+b1+b2+b3+... != a1+b1+a2+b2+...' as there is no logical necessity for this unless established. In fact, 'a1+a2+a3+...' is simply bad and confusing notation borrowed from the finite world. A better notation is the one from analysis.
    - Let us classify. ∞+ is in general not 'commutative', when it is, we call the series 'absolutely convergent'. And this is a good name because the condition for the 'commutativity' has to do with absolute values.
    - Does this concept have a similar one for sequences? Is this not about being able (or not) to 'isolate' multiple sequences/series compounded in one? For sequences this reminds us of ordinal numbers: Imagine the seq. '-1,1,-1+e,1-e,-1+2e,1-2e,...' compared to '-1,-1+e,-1+2e,...(0),1,1-e,1-2e,...(0)' or '-1,-1+e,-1+2e,...(0),...,1-e,1'
    - Uniform convergence is related to the simultaneous convergence of ∞many points: convergence of functions. <Rel. Metric spaces history quote TODO.>

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§ Notes on A.H (III.3)

* Dedekind felt that filling the gaps made the real numbers 'continuous' (complete). In this section, we are defining and treating continuity with functions in mind (in the restricted context of real numbers). The two continuities are related conceptually, but not at all the same, and we should not forget about the former one.

* Bolzano wished to remove the physical concept of 'motion' from analysis. Expressions such as 'approaches' are to be replaced. This motivates his definitions.

* Because of the infinite divisibility of the real number line (or even rationals) we have to work with intervals. Finite ideas such as 'Closest neighbor', 'Smallest distance' can at best serve as a motivation.

* Continuity as defined is not exclusively about the absence of gaps. It really is, as Cauchy said, that we wish to find an interval in the domain (input) that produces a interval in the range (output) that is as small as desired. This does disallow 'gaps' but not only that. Situations such as sin(1/x) near zero are also disallowed since no matter how small the input interval is, the output will vary 'wildly' (between -1 and 1). Indeed, using a dynamic intuitive visualization, we see that any finite movement around x=0 results in the output moving an infinite distance.
    - Note that for sin(1/x), any point with x=0 and y in [-1,1] 'feels' like a cluster point, hence, the function does not 'converge' to 0. This can be made precise by considering sequences of the form { f(xn) with xn=|1/n| } as n→∞ ?

* It is sometimes better to use a dynamic horizontal visualization of the e-d definition that the usual vertical one. We use two parallel vertical lines, the top one for the range, the lower for the domain. Assuming a motion along the domain (in spite of Bolzano), we require that the resulting motion along the range is continuous: can be achieved finitely (see sin(1/x) note above) and without removing the pencil (gaps). This helps bring to intuition the necessary boundedness in the "Hauptlehrstaz" (see below).
    - Imagine this process for f(x)=c, f(x)=n.x, f(x)=x*x, f(x)=sin(x), f(x)=1/(x-1)

* Guided by the physical intuition above, it feels that the definition should proceed from small input intervals to small output intervals, But as we are not in a finite world, we cannot require that the 'smallest interval in the domain' correspond to the 'smallest interval in the range', in fact, it does not even make sense. It has to be clear that the structure of our infinite world does not allow for such a definition.
    - We might think that we can use a limit to express the 'smallest interval' by defining continuity in the following way: lim(h→0) f(x0+h)-y0 = 0 means continuity at y0. This is not bad but needs clarification. In fact, this is the definition of the limit of a function, which is introduced by Hairer after the e-d definition, but that order should be reversed.
    - In the definition above, lim(h→0) needs a definition on its own since it does not correspond to the already established limits of sequences or series, where we let the index n go to ∞. How does it relate to the e-d definition? Let us examine the multiple equivalent (iff.) definitions of the limit of a function.
        . p.205 contains a definition that is e-d free and that related to limits of sequences: For every seq. {xn} in the range, lim(n→∞) f(xn)=f(x0) if lim(n→∞) xn=x0. This defines continuity, but it can also be modified to define the limit of the function and continuity from that. So here it is, an e-d free definition. What is wrong with it? It contains two limits, where the e-d definition contains none and is therefore probably more practical in many cases! This definition corresponds to an old idea of using the sequence { xn=x0+1/n }.
        . p.209 contains Weierstrass's definition that is basically the one above: lim(x→x0) f(x)=y0. As we explained, lim(x→x0) has to be defined, and this brings in the e-d component.
        . p. 210 contains W's definition (1872), this time of continuity (based on the limit of func def. above). Here again it reminds of an old idea of two sidedness, sandwiching the y0 point from both sides. It is then proved that this definition, called a theorem, is equivalent to the e-d definition. With the eps. in W's definition corresponding to a 2eps in B's (1817). In W's definition x0 must be a cluster point a subtle but important technicality to work around empty sets and vacuous truth.
        . Clearly, what is the definition and what is an equivalent theorem is a matter of choice and taste.
        . From the definitions available, we see that we have the choice between limits of sequences or intervals (that define limits in sets).
        . In the e-d definition, the free choice of eps>0 provides the conceptual link to 'smallest interval'.

- add note about equiv def with $n \to \infty$

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§ Notes on A.H (III.3."Hauptlehrsatz")

* "Hauptlehrsatz" (Principal theorem) is the name given by W in his 1861 lectures (published by Cantor 1870) to the theorem that $f:[a,b] \to R$ and continuous implies it is bounded, has a min. and max.
  - Every such function passes through any point between a and b adds to this. B's theorem and a main motivation <rel. algebra>
  - Every such function is also uniformly continuous (a coming topic), hence integrable in the Riemann/Darboux sense.
  - Clearly, such functions form quite a special class.

* First of all we note the importance of the closed interval condition. It is essential and should never be forgotten. Also, it is quite intuitive (see p. 207)
  - The point of it is to 'commit' f to the two real values. An open interval fails in exactly that. It is not that the theorem cannot ever be true for an open interval, but such an interval can allow either unboundedness or missing extrema or both to sneak in: This happens if the point removed is an extremum (e.g as min. 0 for x^2), or is a point where the function is undefined in R, and $\pm\infty$ in R* (e.g: 0 for $1/x$).
    . As a side note, one the examples in 207 treats $[a,\infty)$ as an interval, but this reminds us of the importance (Zakon) to define intervals and maybe redefine them in each section as such an interval is not of finite diameter.
    . Once the function is 'committed' the continuity 'handles the rest'.
    . Note the use of closed intervals in this and B/W supermum proofs, and the use of strict inequality in other cases like clustering and e-d. {WEAK}
* The proof of the "Hauptlehrsatz" deserves analysis. Before that can be done, the B/W sup. theorem appears in a new light and must be analyzed further as it is deeper than it appears. It highlights yet another intuition about bounded infinite sets in the sense that there is no escape from clustering despite the infinity. Additionally, the former proof seems to be based on the interplay of the latter between domain and range, with the continuous function as the bridge.

* Let us analyze the B/W theorem (bounded set has cluster[s])
  - The proof by bisection that has a specific initial discomfort associated with the choice being made at every step (This is probably NOT the axiom of choice at play here <todo>). However, a bit of thought shows that this is no different than processes such as the approximation of sqrt(2), where are each step, a function is applied to get a new estimate. Here a function is applied, but it is an 'algorithmic' kind of function: choose new bounds based on the center of the previous bound's relation to the interval. Therefore, the inf.many choices can be accepted under the usual umbrella of 'finite descriptions of inf. processes'.
    . The intervals do get smaller and smaller and that can be said with confidence.
    . Another conceptual difficulty is the assumption that we can determine whether some point is or is not an upper bound of some set (in a metric space to be more general). We do assume here that it is. However, note that here, unlike the case of approximation, we can never be sure to be getting closer to some real answer because we are dealing with a yes/no problem. We cannot establish the answer to any 'degree of desired accuracy'. A satisfactory <resolution> is given later in this note (existence/computability/construtivity).
      .. A less satisfactory initial resolution is the following: The difficulties can be removed if we assume that the set is given in such a way that the question of an upper bound can always be answered finitely. Is this true in general? Is this an implicit assumption about any given set in a metric space? Are there sets that do not have this property and that we are ignoring for now because they are out of scope (Brouwer and lawless sequences)?
  - To analyze the matter, let us compare the Cauchy/Bolzano/Weierstrass way (A.H) to the Dedekind way (Zakon).
    . For the C/B/W way, we note the following.
      .. The proof of existence of lub by B. first extracts a C-seq (Cauchy sequence) from a set using a choice procedure. The 'approximation intuition' works well for the C-seq convergence proof (p.181) and the proof pauses no problems. Specifically because of the sequential process-like nature. There is no discomfort and it may very well be 'applied', we can keep moving towards a guaranteed better lub during the process.
      .. The problem occurs during the extraction of the sequence from the set. This does not necessarily have an 'applied' form because of the yes/no problem described above. In short until resolution: The seq. seems not to be a problem, the set does.

. For the D way, the proof of the completeness of the field constructed with Dedekind cuts is the related proof.

.. This is achieved by a definition of the lub that is the union of an inf. family of sets, each with inf.many elements. Given the lub the rest follows so the main problem if any resides here. Such a union has again no process-like nature and comes with the same discomfort as above. At first I thought this is related to the axiom of choice but it is not at all (see <resolution>)

. One possible resolution is to claim that any finite description is acceptable, no matter the 'applicability', the 'process-like' nature, the possibility of actually finding a result given enough time (human bias?). This would remove the problem.

.. Let us compare the def. of lub in D. to the applied example of proving that sqrt(2 is the lub of { x | x^2 <= 2 with x rational }. Here we show that the finite description of sqrt(2 : 'The/A number that squared gives 2', provides the means to show that it is necessarily the lub of the set. This is unproblematic: sqrt(2 is larger than any x and no number less than sqrt(2 can be larger than all x. This is quite different from the def. and we did not directly use the definition and the union of a family, although an indirect relationship is visible. We were able to find a finite equiv. proof for the def. that is inf. in character.

.. The assumption then is that given any such finite description of a set and a finite description of a number, we can always relate the two descriptions to prove the relation of lub, or simply of a bound, or more simply of inclusion or exclusion in the set. Is this a reasonable assumption? Can we simply assume it and declare that 'execution' is a 'technical problem'? That sounds reasonable enough and a subject that is possibly treated within mathematical logic and advanced set theory. In any case, the resolution of this is also the resolution of the C/B/W proofs.

- These difficulties might be the reason that would prevent others from creating such theories and are very understandable, is this what we mean by 'genius' in this case? In other words, any person that would try to proceed logically would stop short, unless! They can use the split attitude to their advantage, or have mastery of mathematical logic. Where the creators aware of this? Or were they bold or naive?

- **<Resolution>** We have obviously been confusing a number of things. Existence, computability, decidability, constructivism.

. The comfort of the C-seq comes from the fact that it is computable.

. The union needed in the D. lub is guaranteed by the 'Axiom of Union'. It is an axiom and therefore the construction is impossible without this specific axiom. Should this axiom be accepted? Here we can adopt the split attitude and say both yes and no. However, we must note here that saying no is reducing analysis to less than it can be. The axiom of union merely guarantees the existence of the union set. And this is subtle: It is all about existence, and not computability, not a process-like procedure that can be applied, and not a constructive point of view (vague). This is merely about existence. If we want to take analysis to the highest level possible, to be able to given finite claims and theorem about as many creatures as possible, we should be as accepting to any such axioms as long as they do not lead to contradiction and paradoxes (that is the task of logic and meta-mathematics and is outside of the scope).

. The yes/no problem is a problem of decidability, and the resolution is here one of non-constructive? nature. If we accept that the answer must either be yes or no and cannot be undecidable, then this is good enough for us. There is no need for an applicable process.

. We can surely classify real analysis, and decide there are other kinds of analysis that obey more stringent requirements and there indeed are (construcitvist?). After that, using the split mentality, we can again fully embrace it, but the analysis of these problems is crucial and increases the understanding indefinitely.

. The problem of existence is solved by analysis. The problem of finding a specific limit, etc. is another one, checking is yet another (simpler) one. Analysis is not a method for finding such answers, and that is why it can be so encompassing, and must be.

* TODO nested intervals and Cauchy seq.
* TODO "Hauptlehrsatz" analysis the interplay between domain and range.

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§ Notes on Metric Spaces

* Quote from TODO.

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§ Notes on Z.A.I (Zakon, Analysis vol. I)

-- 17.Nov.2013

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§ On Thinking.

* A specific tactic related to 'Controlled Schyzophrenia' is the 'Pendulum tactic' during which in one single thinking session the mind swings between two extremes: The purely abstract (for itself and by itself) and the purely applied. <rel. MCMM-I.2>

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§ Notes on MCMM-I

§.II Analysis

 * TODO add notes from email? about the existence of a derivative in the formal sence is the right classification of function where avg velocity makes at all sense in the first place! This is a possible resolution of our problem.
 * TODO paradox between conservation of energy and discrete/computing universe.

 * (p66) "Mathematical analysis is the branch of mathematics that provides methods for quantitative investigation of various processes of change, motion and dependence of one magnitude on another". Although purposfully crude, the by now understood extensive generality of analysis puts the expression 'dependence of one magnitude on another' under a new light.

 * (p67) Let us compare the example given about gravity (determining the 'instantaneous' velocity during vertical free fall) to our doubts about the 'meaning' of derivative and tangent. The example is down to earth and makes a lot of sense, and this hints leads us to think that our intuitive problem with the formal definition of derivative is essentially a intuitive problem about infinity, i.e., the inifinte divisibility of intervals.

 * (p68) If we think about what we are doing here to determine the force acting on a wall of a water reservoir using calculus, it turns out to be a mental travesti at very first sight, and this is surely an intuitive obstacle that prevents many from taking this path of thinking to resolve this problem. The reason for the unituitiveness comes once more from our finitist biases, in the sense that the thinking process we are about to describe is never used and indeed a sure dead end (and therefore to be avoided) when dealing with finite (discrete) problems. It feels wrong to start on purpose with a 'wrong' statement (that has an error term in it) and to expect that by manipulating it we might arrive at the right result (despite the fact that this has been used by humans since a long time ago e.g., the Greeks). This strange method evokes paradox-like descriptions like: The perfect approximation, the non-approximate approximation. Passing to friendlier ones like: The limit of an approximation, the removal of error in an initial approximation, the refinement of error in an approximation, the transformation of an initial approximation to strictly better ones, the mindful addition of a finite (but precisely expressible) error followed by its progressive elimination from an expression. Even friendlier ways to say it are: The decomposition of a complex problem into simpler ones, but this time an infinitely of finitely describable ones. An even better way to see it, and which brings in the limit very clearly is: Relating a clearly definite finite creature (the weight of the column of water at a 'point') to a necessarily infinite one (the force on the surface formed by all the points in a wall') by a process called limit which, when it converges, logically identifies a unique answer (in this case a number). This last sentence might as well be the essence of a large part of basic analysis.

 * Another latent discomfort that this reading elucidates is one of absolute and relative differences. Clearly $\lim_{h\to 0} f(x+h)-f(x)$ is zero. Nevertheless $\lim_{h\to 0} (f(x+h)-f(x)) / h$ must not necessarily be so. Again this is a finitist bias, and it is very obvious that the above is true in infinitesimal analysis by considering even the simplest function such as $f(x)=x$. It is amazing how long this latent discomfort has survived.
    - The usage of the absolute difference limit is silent when comparing two functions. For all functions it is zero, at least for all functions where it makes sense, which brings us to its usage for defining continuity. It is there where it has meaning. In this case, it is not the value of the difference that we are interested in, but in the possibility of obtaining any desired (as small as desired) difference by perturbing x.
    - As for the relative difference, we may say that the hint to form this ratio comes readily from intuition, in terms of direction when moving, that is the direction of the velocity vector, and also from tangents. We may imagine the point $f(x)$ as not only a point,

but one endowed with a direction, just like a human (not a fully symmetrical creature, i.e., not a sphere and not a point) moving 'forward'.

- TODO add note from email? about singularities! this is related to dividing by almost 'zero', to neighborhoods, to local behvior, to topology.

* (p67) "uav = Δs/Δt = gt + (1/2).g.Δt . Letting Δt approach zero we obtain an average velocity which approaches as close as we like to the true velocity at the point A".

- I find 'true velocity' to be nowhere properly defined. Such sentences make it sound like we know what 'true/instantaneous velocity" is, either intuitively, or that we have defined it, or maybe even that we understand it philosophically. But we know that philosophically the topic of change is not settled, that intuitively it is vague even qualitatively due its inseparability with infinity. The definition given is one of a limit, which renders sentences like the above circular in their attempt to explain true velocity.

- Average velocity seems less probelmatic, maybe precisely because it need not involve limits (at first sight, before needing to measure some square root of two distance) is purely arithmetical. The average velocity mulitplied by time gives the position difference.

- As we know from the definitions of a function limit, they ultimately involve a sequence. TODO fix this: the essence of the limit of a function is that of a decreasing h, this can be turned into a sequence and is then equivalent (iff.) but this is only a variation in desription amenable to easier analysis. We also feel nested intervals at play here, or simply any strictly decreasing function of h (e.g 1/x sampled, giving the sequene definition). TODO <rel. to function limit discussion and resolve>. Note that for integration, the presence of a sequence occurs more naturally (TODO with resolution)

- The average velocity is easily accepted as 'uav = gt + (1/2).g.Δt'. This is the velocity at a specific point (time t) and is a function of Δt. In other words, for each Δt there is generally a different velocity.

- Conceptually we could say that the 'true velocity' is the velocity with which the 'particle' (TODO!!! slope/direction vs. speed!!! slope in x/t graph is 'speed' !! latent confusion!!) will continue with if abruptly any 'constraints' disappeared (TODO vague!! in which graph???). In that case, we can use the 'left' limit and get 'uav = gt - (1/2).g.Δt' which must give the same result (by the definition of a limit with is 'double sided' by default). Very informally, is this the velocity at which the particle would continue? Any specific Δti, with a velocity of gt - (1/2).g.Δti' is contradicted by any different Δtj! 'Geometrically' the tangent would intersec the trajectory for any specific Δti. The only uav that cannot be contradicted is one for which no 'next smaller' Δti exists which brings us to the problem of infinite divisibility and its resolution by limits. Visualizing the problem for a curve going downwards, we see a sumpremum-like situation concerning the 'ultimate' Δti, and we assume that at an ultimate stage, the slope function is 'nice' and non-oscillating(†). At that stage, for every two distinct Δt, if one is smaller than the other, the slope at it is higher. If we now exlude all the specific Δti velocities, and we exclude all slopes that are clearly larger than all of them, we arrive at the true slope in a manner exactly corresponding to the definition of a supremum, and that reminds of the methods of Archimedes. A natural classifiction of functions 'nice' with regards to slope ensues, and clearly, they all have to be 'continuous' to begin with (the absolute changes must always exist) (TODO check).

- When this limit exists, it is clearly special amongst all others in many senses.

  . Closest to all with no 'tangent intersection'.

  . Upped bound (least).

  . Independence of Δti and in fact of any punctual change of the function outside the point itself.

  . (††) Any Δti-specific velocity 'misses' the behavior in the interval [0,Δti], in other words, an infinity of functions have the same Δti-velocity at that point but are arbitrarily different. Therefore Δti-specific velocities cannot in general give 'complete' information about a function, they cannot completely characterize it. This means that we cannot 'recover' the full function(†††) from these velocities, unlike the 'true' velocities as proven in analysis by integration(††) (up to a constant).

  . We cannot 'integrate' the Δti-specific 'derivatives', can we even express them?

  . Maybe most importantly (and this occured to me one day after the initial note) that it is only for the functions that admit a 'true' velocity (differentiable) can we talk about the meaningfulness of the average velocities, because for all other functions, the average velocities are so 'wild' and 'incoherent' that they loose (intuitive, physical) meaning. This is a very strong and strict relationship and even very intuitive.

We conclude that the justifiction 'true velocity' is a mathematical one and not in principle physical <TODO rel. energy conserv paradox>. It is nevertheless true that the more precisely we can physically measure Δt, the closer the measured velocity gets to the 'true' one.

  . The physical measurement is only meaningful when the function is well behaved and quite monotonous at the Δt resolution.

. In the cases of functions that are so 'oscillatory' that they are beyong the physical measurement resolution, the correspondence to 'true' velocity breaks down, and this is surely not uncommon during physical investigation on the 'border' between classical mechanics and newer developments. As a barbarically crude example, imagine an oscillation period of 10^-100 sec.

. Talking about 'material particles', we ultimately get out of classical mechanics but our conclusions still apply.

- We have here done more than justify 'true velocity' for a specific example and freed it from the original meaning of 'velocity'.

- We must at once and forever stop being irritated by the zero in lim h→0 and this is fully justified by this note. <rel. singularity TODO>

- At least for the time being, we have precisely separated the mathematics from the 'intuition' (or better said described the actual relationship), from 'justification', and from 'origins'. We have also established their relationship 'meta-proved'. The fundamental theorem of calculus appears in a new light, especially because the note ††(††) and we see the >>necessity<< of limits without which the theorem would not work at all. This is obvious, but in fact subtle.

- (†) We can see how the existence of a derivative means 'eventually monotonous', 'eventually non-oscillatory' (warning: subtle with the example of sin(1.nx)/n). We can take this a bit further and say 'eventually limiting to a straight line'.

- (p119) (†††) Our analysis of the unique ability of the derivative among the discrete forms to recover the full information about a function has several consequences.

. The FTC establishes more precisely the 'recovery' and also the opposite direction: differentiation. When possible, the primitive is then pinpointed up to a constant. This relates very tightly the differentiation and integration. In other words, up to to constant, describing the derivative or describing the primitive amount to describing the 'same thing'. They are almost two eexpressions of the same function (again, when possible). The immediate practical use is that when the derivative of some function is easier to describe than the function itself, we are very close to describing the function. This is exactly the idea applied to the determination of volumes in p119-120 or force in p67. Note that in these examples, what we have easy access to is the differntiated form (surely a common situation) and not the derivative itself. We can however easily extract the derivative. The important difference between the two is analyzed in the next note.

* (p119) In the previous note, we precisely described the difference between the average velocities (Δti-specific) and the derivative, the sampled and the limit. It gets much better: A precise relation beween the two forms is obtainable, very interesting and useful. It is in principle related to our idea of 'recovery'. Since we can recover the whole function, we can also recover information about the Δti-specific form. In other words, we can precisly relat the derivative form to any of the approximate forms of which there is an infinity. The discrete Δti-forms are from now on called differentials, usually denoted by Δy = f(x+Δx)-f(x) for an increment of Δx with y=f(x). The discrete rate of change is the Δy/Δx, let us denote it by F'.

- There 'error' between F' and f' is α = F' - f', and is a function of both Δx and x. It follows from our definitio of f' that lim Δx→0 Δy/Δx = f'. Therefore, lim Δx→0 α = lim Δx→0 Δy/Δx - f' = 0. Since α tends to 0 as Δx does, at good enough approximation, both of them are less than one, and so their product α.Δx is less than Δx, it tends to zero faster than Δx, and α.Δx tends to become negligible compared to Δx. So if Δx is called first order infinitesimal, α.Δx is of lower order, all in 17th century parlance which can be rigorously proved using limits. We have actually established the sought for precise relationship already. Δy = f'.Δx + α.Δx with α=g(x,Δx). What this means is that if we have an expression of Δy (a differential) as a function of Δx, and group all factors of power Δx^1 together, the sum of these factors must equal f'. We say that f' is the linear, or first order, component of Δy. By 'the linear' we means the 'only' since α.Δx must be of lower order. This 'detail' is crucial.

- Note. 17th century parlance 'tends to' is very intuitive and powerful. It hints to a simple arithmetic with limits that has to be used whenever the rules for it (established by modern analysis) hold.

- Relating this to our previous note, it is because of this precise relationship that we can determine volumes, starting with differentials.

- Note. Differentials are thoroughly (more thoroughly than one could imagine) treated in Euler's <TODO> which I studied for a short while. It makes sense to try to make them the finitist base of real analysis.

* Relating the two preceding notes, and coming back to the advantage of the rate of change limit compared to the change with respect to 'recovery' and differentiation of functions, is it not so that, except from being relating to the intuitive concept of velocity, there is

maybe no reason to give the exact form of derivative the exclusivity of essence in calculus. Is it not maybe true that any function g of the form $g(f(x+\Delta x) - f(x))$ that has a limit not equal to zero has an equal claim to a related FTC? We note that all higher order derivatives are a special case of this form. Is this in any way related to the concept of convolution?
<TODO>