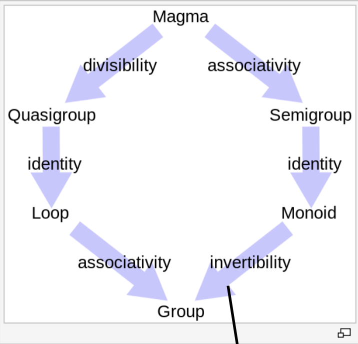



## Algebraic structures

**Group-like** [hide]

Semigroup / Monoid
Racks and quandles
Quasigroup and loop
Abelian group · Magma · Lie group
*Group theory*

**Ring-like** [hide]

Ring · Rng · Semiring · Near-ring ·
Commutative ring · Domain · Integral domain ·
Field
*Ring theory*

**Lattice-like** [hide]

Lattice · Semilattice
Map of lattices
*Lattice theory*

**Module-like** [hide]

Module
Group with operators
Vector space

**Algebra-like** [hide]

Algebra
Associative · Non-associative · Lie algebra ·
Graded · Bialgebra

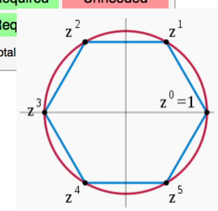
V · T · E

**Group-like structures**

| | Totality[α] | Associativity | Identity | Divisibility | Commutativity |
|---|---|---|---|---|---|
| Semicategory | Unneeded | Required | Unneeded | Unneeded | Unneeded |
| Category | Unneeded | Required | Required | Unneeded | Unneeded |
| Groupoid | Unneeded | Required | Required | Required | Unneeded |
| Magma | Required | Unneeded | Unneeded | Unneeded | Unneeded |
| Quasigroup | Required | Unneeded | Unneeded | Required | Unneeded |
| Loop | Required | Unneeded | Required | Required | Unneeded |
| Semigroup | Required | Required | Unneeded | Unneeded | Unneeded |
| Monoid | Required | Required | Required | Unneeded | Unneeded |
| Group | Required | Required | Required | Required | Unneeded |
| Abelian Group | Required | Required | Required | Req | |

^α Closure, which is used in many sources, is an equivalent axiom to total

Semicategory




An algebraic structure that can be defined by identities is called a **variety,** and these are sufficiently important that some authors consider varieties the only object of study in universal algebra, while others consider them an object.[citation needed]

Restricting one's study to varieties rules out:

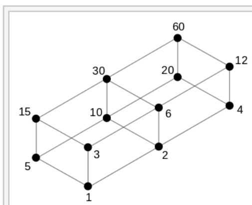
- Predicate logic, notably quantification, including universal quantification (∀), except before an equation, and existential quantification ( ∃ )
- All relations except equality, in particular inequalities, both $a \neq b$ and order relations

In this narrower definition, universal algebra can be seen as a special branch of model theory, typically dealing with structures having operations only (i.e. the type can have symbols for functions but not for relations other than equality), and in which the language used to talk about these structures uses equations only.

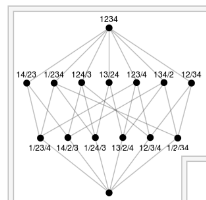
Not all algebraic structures in a wider sense fall into this scope. For example ordered groups are not studied in mainstream universal algebra because they involve an ordering relation.

A more fundamental restriction is that universal algebra cannot study the class of fields, because there is no type (a.k.a. signature) in which all field laws can be written as equations (inverses of elements are defined for all *non-zero* elements in a field, so inversion cannot simply be added to the type).

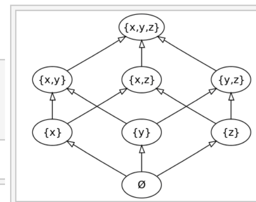
his restriction is that the structures studied in universal algebra can be defined in any category that has example, a topological group is just a group in the category of topological spaces.



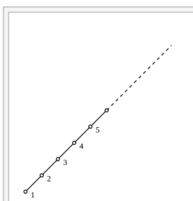

**Pic.2:** Lattice of integer divisors of 60, ordered by "*divides*".



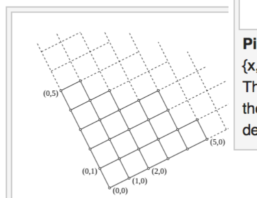

**Pic.3:** Lattice of partitions of {1,2,3,4}, ordered by "*refines*".



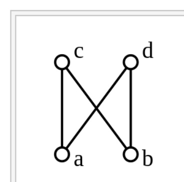

**Pic.1:** Lattice of subsets of {x,y,z}, ordered by "*is subset of*". The name "lattice" is suggested by the form of the Hasse diagram depicting it.



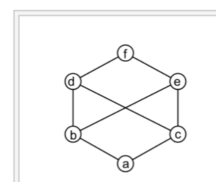
**Pic.4:** Lattice of positive integers, ordered by ≤.



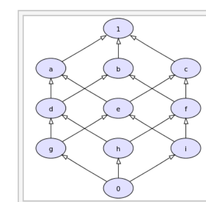
**Pic.5:** Lattice of nonnegative integer pairs, ordered componentwise.




**Pic.6:** Non-lattice poset: *c* and *d* have no common upper bound.




**Pic.7:** Non-lattice poset: *b* and *c* have common upper bounds *d*, *e*, and *f*, but none of them are the least upper bound.



**Pic.8:** Non-lattice poset: *a* and *b* have common lower bounds 0, *d*, *g*, *h*, and *i*, but none of them are the greatest lower bound.

and 3, none of which is the

http://math.chapman.edu/~jipsen/structures/doku.php
http://math.chapman.edu/~jipsen/structures/doku.php/varieties