Smooth Manifolds in Short

$$[\text{Cover}] \quad \doteqdot \quad \frac{\bigcup\left[\,\circ_{\left(C_{\mathcal{P}_X}\right)}\,\right] = X}{C \text{ is a Cover } \parallel \ {}_{\widehat{X}}C}$$

$$[\text{Open Cover}] \quad \doteqdot \quad {}_{\left(\widehat{X},\mathcal{O}\right)}C$$

$$[\text{(Open) Subcover}] \quad \doteqdot \quad {}_{\left(\widehat{X},\mathcal{O}\right)}S\left[{}_{(\widehat{X},\mathcal{O})}C\right]$$

$$[\text{Basis}] \quad \doteqdot \quad \frac{\left[{}_{\widehat{X}}B\right] + \left[\dfrac{\left[{}^{\forall}_{\bullet}\right]_{(X\,,\,\Delta_B \cap \nabla_B)}}{(\bullet)\ {}^{\exists}\ [\circ]_{(\Delta \cap \nabla\,,\,B)}}\right]}{B \text{ is a Basis } \parallel \ {}_{\underline{X}}B}$$

[Locally Countable]

$$[\text{(Second) Countable}] \quad \doteqdot \quad \frac{{}_{\underline{X},\mathbb{N}}\overset{\exists}{[B]}}{X \text{ is a second countable } \parallel \ {}_{\underline{\mathbb{N}}}X}$$

$$[\text{Lemma A.4 (Prop A.16)}] \quad \Rightarrow \quad \frac{{}_{\underline{\mathbb{N}}}X}{{}_{(\widehat{\mathcal{O}},\mathbb{N})}\left[s^{\exists}\right]\left[{}_{\widehat{\mathcal{O}}}\mathcal{C}^{\forall}\right]}$$

$$[\text{Compact}] \quad \doteqdot \quad \frac{{}_{\mathbb{n}}\left[\widehat{\mathcal{O}}^{\,\exists}\right]_{\left[\widehat{\mathcal{O}}^{\,\forall}\right]}}{X \text{ is compact } \parallel \ {}_{\mathbb{n}}X}$$

[Lemma 1.6]

$$[\text{Point-grained (Hausdorff)}] \quad \doteqdot \quad \frac{\left[\left(\bullet_{X}^{\forall}\right)\mathcal{O}_{X}^{\exists}\right] \not\cap \left[\left(\bullet_{X}^{\forall}\right)\mathcal{O}_{X}^{\exists}\right]}{X \text{ is a point-grained } \Big\| \ {}_{(\bullet)}X}$$

$$[\text{Locally n-Euclidean}] \quad \doteqdot \quad \frac{\left[\left(\bullet_{X}^{\forall}\right)_{\mathcal{O}}U_{X}^{\exists}\right] \xleftrightarrow{\dot{\varphi}^{\exists}} [\mathcal{O}_{\mathbb{R}^n}]}{X \text{ is locally Euclidean } \Big\| \ {}_{\dot{\mathcal{E}}_n}X}$$

$$[\text{Manifold}] \quad \doteqdot \quad \frac{\left[{}_{\dot{\mathcal{E}}_n}X\right] + \left[{}_{(\bullet)}X\right] + \left[{}_{\underline{\mathbb{N}}}X\right]}{X \text{ is a manifold with chart } {}_{\circledcirc}(U,\varphi) \ \Big\| \ {}_{[\mathcal{M}^{n}_{(U,\varphi)}]}X}$$

$$[\text{Coordinate function}] \quad \doteqdot \quad \frac{\left[\mathcal{M}_{(-,\varphi)}\right]: \ [\varphi = (x^{i/n})]}{x^{i/n} \text{ are coordinate functions}}$$

$$[\text{Smoothly compatible}] \quad \doteqdot \quad \frac{\left[\mathcal{M}_{(\triangle,\curlywedge),(\triangledown,\Upsilon)}\right]: \ \left[\ {}^{[\triangle\cap\triangledown]}_{\underset{\sim}{\leftrightarrow}}[\curlywedge/\Upsilon]_{[\triangle\cap\triangledown]} \ \vee \ [\triangle \not\cap \triangledown] \ \right]}{\text{The charts are smoothly compatible } \ \Big\| \ (\triangle,\curlywedge)\underset{\sim}{\leftrightarrow}(\triangledown,\Upsilon)}$$

$$[\text{Atlas}] \quad \doteqdot \quad \frac{\bigcup\left[\triangle^{\forall}_{[{}_{\circledcirc}(\triangle,-)_{A}]}\right] = X}{A \text{ is an atlas } \ \| \ {}_{\mathcal{A}}A}$$

$$[\text{Smooth atlas}] \quad \doteqdot \quad \frac{{}_{\mathcal{A}}A\text{:} \ \left[\ \circledcirc_{A}^{\forall} \underset{\sim}{\leftrightarrow} \circledcirc_{A}^{\forall} \ \right]}{A \text{ is a smooth atlas } \ \Big\| \ {}_{\underset{\sim}{\mathcal{A}}}A}$$

Bibliography

Lee, John M. 2003. "Smooth Manifolds." In ***Introduction to Smooth Manifolds***, 1–29. Springer.