Rigid Body Dynamics Treatments
Volume I

I Introductory Misconceptions

1. How do the proofs that implicit integration is unconditionally stable offer any guarantees to RBD methods, given that we approximate the derivatives? Is there ***a treatment to be done, concerning perturbations to the derivative and their effects on the stability of implicit integration***? Another direct and simple question is: can we apply the kind of stability analysis in http://www.it.uu.se/edu/course/homepage/bridging/ht13/Stability_Analysis.pdf to the default RBD method?

Interrupt. We should go through mhs, compare with http://qucs.sourceforge.net/tech/node24.html and see whether there indeed is a missing component. It also seems that the usual proofs of stability of implicit integration only apply to the 'test equation' as seen in http://www.it.uu.se/edu/course/homepage/bridging/ht13/Stability_Analysis.pdf and http://engineering.utsa.edu/~foster/ME4603/files/integration.pdf. This is generalized in google books: 'Computer Methods for Ordinary Differential Equations and Differential' which we should check as a reference, but note that even there we do not leave the realm of linear ODEs!. This also look interesting: http://www2.mpia-hd.mpg.de/~dullemon/lectures/fluiddynamics08/chap_9_impl_incompr.pdf. In relation to the possible treatment we find "The comparison of the long-time behaviour of dynamical systems and their numerical approximations is not straightforward since in general such methods only converge on bounded time intervals. However, one can still compare their asymptotic behaviour using the global attractor, and this is now standard in the deterministic autonomous case. For random dynamical systems there is an additional problem, since the convergence of numerical methods for such systems is usually given only on average. In this paper the deterministic approach is extended to cover stochastic differential equations, giving necessary and sufficient conditions for the random attractor arising from a random dynamical system to be upper semi-continuous with respect to a given family of perturbations or approximations. In the theory of deterministic dynamical systems globally attracting sets play a central role. Usually they occupy a restricted portion of the original phase space, and the hope is that the dynamics restricted to the attractor—a natural way to understand the idea of "the asymptotic dynamics of the system"—is easier to understand than the full dynamics of the system. They have also been used as a way to compare the asymptotic dynamics of numerical approximations with those of the original system, since generally error estimates are only valid on bounded time intervals (see [31] and references therein)." http://www.sciencedirect.com/science/article/pii/S0022039602000384

2. How does PBD actually work, specifically, with a pendulum, if we do not keep velocities, the pendulum slows down exponentially and never reaches the bottom, if we keep the velocities, they are also all vertical and don't help. There must be a misconception on our part at work here.

3. Where exactly do constraints come into Newton's laws. Before talking about Euler's rigid bodies, where is the formalization of a 'pendulum' with its constraint having to be satisfied along with the 'equations of motion'?

The answer is that 'by studying the model of the system', we have to explicitly find the values and directions of the forces at each point, the forces on each body are added together giving the total force of the body, from which its motion is derived using Newton's law linking motion (acceleration) to force. Per example, in the case of a simple pendulum, we 'know' that the string cancels every force that would case the body from leaving the circular trajectory, hence cancels any forces perpendicular to the circle, hence cancels that part of the gravity-induced force perpendicular to the circle. By trigonometry, this gives the total force on a body at an arbitrary point as function of the angle. This is explained here http://physics.stackexchange.com/questions/165012, here http://physics.stackexchange.com/questions/133091 from which at the very least we have to internalize this: At the very least, we need to memorize this:

In the ***Newtonian approach*** to mechanics, these systems are treated by introducing variables $F_1, F_2, \ldots$ representing the unknown forces, and solving the system of equations for the unknown forces and accelerations. This procedure might be complicated; moreover, we are not always interested in the magnitudes of these unknown forces.In the Lagrangian approach, there are two straightforward ways to treat constrained systems:

1. The ***method of solving the constraints***. In this method, we introduce the ***generalized coordinates*** in such a way that the constraints are automatically satisfied. For example, suppose a point mass is constrained to move along a circle of radius RR. We might describe this situation by saying that the Cartesian coordinates x, y are constrained to satisfy the equation $x^2 + y^2 = R^2$. Now we can introduce the angle ϕ as the "generalized coordinate" and express the Cartesian coordinates of the point mass as $x = R\cos\phi, y = R\sin\phi$. These coordinates solve the constraint for all ϕ . The power of the Lagrangian approach is that any generalized coordinates are good enough; so we can now directly write the Lagrangian in terms of the function $\phi(t)$ and forget about the fact that the system is constrained. We shall automatically obtain the correct equations of motion.
2. The ***method of Lagrange multipliers*** . In this method, we do not try to introduce new generalized coordinates that solve the constraints. (This may be difficult; not all algebraic equations can be solved!) Instead, we formulate the variational problem in the presence of constraints: the correct trajectory $q_i(t)$ is such that the action functional has an extremum while the constraint equations are satisfied. For example, if a point mass is constrained to move along a circle of radius RR, but is otherwise unforced, then the Lagrangian is $L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$ and the constraint is $x^2 + y^2 = R^2$. The correct trajectory $x(t), y(t)$ will be such that the integral $\int L dt$ has the minimum value while the constraint holds at every t. Thus we need to solve the problem of conditional minimization.

Note that for the single, and even double pendulum, there is a nicely worked out example using both (Lagrange) methods here: http://www.phys.lsu.edu/faculty/gonzalez/Teaching/Phys7221/DoublePendulum.pdf. So one can indeed find the explicit expressions for the string tensions even in the double pendulum case. For both cases, there is a differential equation (Eqn. 1) (not related to the tensions) that has no analytic solution except at small angles. For the single pendulum we arrive at

$$l\theta'' = g\sin\theta \tag{1}$$

$$ml\theta'^2 = T \tag{2}$$

Probably, in general we cannot guarantee that we will be able to obtain explicit differential equations for where the internal forces do not figure, but this is just a guess. Could it be that this is only true for holonomic ("only algebraic relationships between the coordinates, not involving inequalities or derivatives") constraints? After a while, we answered this completely in speed_proof.pdf

Clearly, this does not allow a methodic treatment. All that we want is plug what the constraint implies, what it constrains the motion to and not to have to fiddle with forces. But this is exactly the Lagrangian/Hamiltonian formulation. This is pointed clearly here http://physics.stackexchange.com/questions/161791 and here https://en.wikibooks.org/wiki/Classical_Mechanics/Constrained.

4. Given the above treatment, is not Lagrange's method 1 merely the idea of re-parametrising coordinates and basically reducing to working out Newton's method but in a more convenient setting? We obviously still need to find the internal forces, if only to plug them into equations and cancel them? In the case of the pendulum, two cartesian coordinates are replaced by a distance and angle, but the distance being constant, we win a simplification, is that all there is to it for method 1?

5. By now we need to know all this: http://www.lecture-notes.co.uk/susskind/classical-mechanics/ and all this https://www.amazon.com/gp/product/046502811X.

6. Given the above question and answer, what kind of dynamics are we doing according to mhs?

7. Reviving the session least_action at index 3, we find that we missed the continuation of the history of Lagrange's derivation of the LE equations (leading to the Hamiltonian / Lagrangian). This is treated in quite some detail in [@c1] as pointed out by [@c2, p.36]. [@c2, p.38] Also points out that, summarizing [@c1], that it was Hamilton who "[1834] realized that Lagrange's equations of motion were equivalent to a variational principle", so it is important to note the presence of Hamilton as a torch-bearer , where Lagrange himself did not realize this (although he 'should have' according to [@c1]). This is an added note much later: Hairer, has a very nice in-short two-page treatment of this in [Numerical Geometric Integration]! Additionally, [SODE-I, p.202] is enlightening.

We also remember Landau's justification in [@c3] which does not at all pass through Newton's, but from the assumption that 'classical mechanics minimizes' and invariances, he arrives to deriving the specific Lagrangian for a system of particles.

The language used in [@c1, p.228-230] describes what Lagrange did, in modern language (differential forms, etc.). This shows we need to continue our project on differential forms, noting that [@c3] dispenses of this language until its last part and [@c2] does so as well until part 3, where we finally he see ***geodesics*** come in comparing: "We have already seen the simple example of the curve that minimizes the distance between two points in Euclidean space—unsurprisingly a straight line" and "After such a parameterization, minimizing the energy is equivalent to minimizing the length, and this is how we proceed henceforth. We are now in a position to derive the geodesic equations that characterize the curves that minimize the length/energy between two arbitrary points on a smooth manifold $\mathcal{M}$".

8. Basic questions about the LE equations and the related variational version (7):

1. Staring at http://www.phys.lsu.edu/faculty/gonzalez/Teaching/Phys7221/DoublePendulum.pdf p.6, how is LE derived from NE, LE from NE? We need to remember this.
2. In which way should have Lagrange known the relation to the variational version? Some if that might already be captured in our written notes on functional analysis.
3. Gather proofs of LE vs. NE vs. Variational Lagr vs. Variational Hamilt.
4. To do this, we need to complete vec-calc, integration, functional analysis, etc., but as long as we can read, we are happy in the context of this treatment.
5. Since we are not quizzed, we do not train to commit through exercise, so let us at least do that using notation and comprehensive diagrams.

II Treat PBD I

9. Let us ***repeat analytically the derivation*** in [@c4] of constraint projection, ***relate it to the Lagrangian multipliers for once*** (are the Lagrangian multipliers here related to those in 2?), and come back to [@c5]. Finally, ***add this to hinges_py***.

10. We start by treating the multiplier question. The fact that physics can be described by constrained optimization of the Lagrangian is 'physics'. The fact that constrained optimization can be turned into unconstrained (giving rise to multipliers) is mathematics. We will first treat the mathematical part, and then potentially the equivalence with the physics part to other physics formulations (which would be done mathematically), alternatively, one could take this as the defined model for physics. ***Pfad***:

1. proof_lagrmul

11. Note this: https://en.wikipedia.org/wiki/Verlet_integration

III Web Sessions

III.1 Misc

Symplectic integrator - Wikipedia Energy drift - Wikipedia constrained newtonian mechanics - Google Search Constraint (classical mechanics) - Wikipedia Classical Mechanics/Constrained - Wikibooks, open books for an open world newtonian mechanics solution double pendulum - Google Search Double Pendulum – from Eric Weisstein's World of Physics www.itp.uni-bremen.de/prichter/download/ForcesDoublePendulum.pdf www.phys.lsu.edu/faculty/gonzalez/Teaching/Phys7221/DoublePendulum.pdf Double pendulum | Lecture 5 - Classical Mechanics - Susskind Lectures - Lecture Notes susskind classical mechanics - Google Search Principle of least action | Lecture 3 - Classical Mechanics - Susskind Lectures - Lecture Notes susskind lectures classical mechanics pdf - Google Search References | The Theoretical Minimum The Theoretical Minimum: What You Need to Know to Start Doing Physics: Leonard Susskind, George Hrabovsky: 9780465028115: Amazon.com: Books

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