Notes on 'The Liar'

[p.47] This paragraph is a nice goal of ('ability to read' class) understanding, especially Dana Scott's 'trick'.

'The proof of the Embedding Theorem, while a bit tricky in detail, is simple enough to describe. First Aczel isolates an equiv- alence relation =A on graphs which holds between two graphs just in case they represent the same set. For example, all the graphs in Figure 2 are =.4, as are the four graphs from Figures 4 and 6. This allows each set in Aczel's universe to be represented by an equivalence class of graphs from the wellfounded universe. There is a slight hitch, though, since each set is actually depicted by a proper class of graphs, and to carry out the proof in ZFC~ one has to work with sets. To do this Aczel borrows a trick of Dana Scott's, and represents each set b by the set Gj, of those graphs of minimal rank in the cumulative hierarchy that depict it. Since every graph is, by the axiom of choice, isomorphic to a graph on some set of ordinals, Gj, will always be nonempty.6 Then, using the class of sets of the form G&/=A, Aczel is able to show (1) that all the axioms of ZFC/AFA are true (using the natural interpretation of membership), and (2) that every wellfounded set is uniquely represented in the resulting model.'