Speed Proofs

I Proof List

1. MVT

2. IVT

3. FTC

4. Lebesgue Integral

5. Mixed partials

6. Lagrange multipliers

7. Stochastic gradient descent

8. Implicit integrator

9. Generalized Navier Stokes

10. FTC using NSA

11. Nonsolvability quintic

12. Euler equations of motion (including inertia tensor) from Newton equations of motion

13. The cartesian product of two topological spaces is a topological space?

14. Rotation has no global parametrization

15. The equivalence of these three to the Leibniz Integral rule (wikipedia)

1. The interchange of a derivative and an integral (Leibniz rule)
2. The change of order of partial derivatives
3. The change of order of integration (Fubini's theorem)

II QL (FOL) Prerequisites

16. Trying our first speed-proof (the limit of a sum of sequences is the sum of the limits), we find that we seem to need to use 'semantic' of quantifiers to proceed. We do not want that. And indeed, we have a hole in our knowledge in the sense that we never reached FOL/QL exercises in our logic studies. We try to patch the gap here.

17. Good resources seem to be: [@c1, p.75], [@c2], [@c4], [@c6] as well as our classics [@c5] and TODO(book at home). Remember that 'flag-style' notation we learned from [@c3].

III A look at (H. Enderton and Enderton 2001)

18. Now this is a nice quote that we had been looking for in ages:

> As the preceding example indicates, the generalization and deduction theorems (and to a smaller extent the corollaries) will be very useful in showing that certain formulas are deducible. But there is still the matter of strategy: For a given and φ, where should one begin in order to show that ! φ? One could, in principle, start enumerating all finite sequences of wffs until one encountered a deduction of φ from . Although this would be an effective procedure (for reasonable languages) for locating a deduction if one exists, it is far too inefficient to have more than theoretical interest.

and

> One technique is to abandon formality and to give in English a proof that the truth of implies the truth of φ. Then the proof in English can be formalized into a legal deduction. (In the coming pages we will see techniques for carrying out such a formalization in a reasonably natural way.)

but also, validating our syntactic goals:

> There are also useful methods based solely on the syntactical form of φ.

With our goal in mind, let us work through section 2.4 (A Deductive Calculus) with exercises, it seems highly relevant. It is very important to note at this point the difference between Goedel's completeness theorem of FOL and his incompleteness theorem! [1] [2] [3]

1. FOL, a deductive system, is complete

   > A deductive system is called complete if every logically valid formula is the conclusion of some formal deduction, and the completeness theorem for a particular deductive system is the theorem that it is complete in this sense.

2. Every formal system containing basic arithmetic (a formal theory) is incomplete

   > A complete theory is a theory that contains every sentence or its negation. Importantly, one can find a complete consistent theory extending any consistent theory.

Moreover

> A set of axioms is (syntactically, or negation-) complete if, for any statement in the axioms' language, that statement or its negation is provable from the axioms (Smith 2007, p. 24). This is the notion relevant for Gödel's first Incompleteness theorem. It is not to be confused with semantic completeness, which means that the set of axioms proves all the semantic tautologies of the given language. In his completeness theorem, Gödel proved that first order logic is semantically complete. But it is not syntactically complete, since there are sentences expressible in the language of first order logic that can be neither proved nor disproved from the axioms of logic alone: for example, "the flower is pretty".

[1] https://en.wikipedia.org/wiki/Gödel%27s_completeness_theorem#Relationship_to_the_incompleteness_theorem

[2] https://en.wikipedia.org/wiki/Model_theory#Using_the_compactness_and_completeness_theorems

[3] https://en.wikipedia.org/wiki/Gödel%27s_incompleteness_theorems

> In a mere system of logic it would be absurd to expect syntactic completeness. But in a system of mathematics, thinkers such as Hilbert had believed that it is just a matter of time to find such an axiomatization that would allow one to either prove or disprove (by proving its negation) each and every mathematical formula.
>
> A formal system might be syntactically incomplete by design, such as logics generally are. Or it may be incomplete simply because not all the necessary axioms have been discovered or included. For example, Euclidean geometry without the parallel postulate is incomplete, because it is not possible to prove or disprove the parallel postulate from the remaining axioms. Similarly, the theory of dense linear orders is not complete, but becomes complete with an extra axiom stating that there are no endpoints in the order. The continuum hypothesis is a statement in the language of ZFC that is not provable within ZFC, so ZFC is not complete. In this case, there is no obvious candidate for a new axiom that resolves the issue.
>
> The theory of first-order Peano arithmetic is consistent, has an infinite but recursively enumerable set of axioms, and can encode enough arithmetic for the hypotheses of the incompleteness theorem. Thus, by the first incompleteness theorem, Peano Arithmetic is not complete. The theorem gives an explicit example of a statement of arithmetic that is neither provable nor disprovable in Peano's arithmetics. Moreover, this statement is true in the usual model. Moreover, no effectively axiomatized, consistent extension of Peano arithmetic can be complete.

This is very important to get back to as a general high level idea to memorize, but we do not waste time on it right now. Reading along, while very nice, this is too difficult for a first touch, we try to gentler [@c4] instead.

IV A look at (Goldrei 2005)

19. We are lead to Peter Suber's course [@c7] which is a gem, and also leads us to the book [@c8] (which seems to be an old classic), whose chapter 4 starts off in a great way.

V A look at (Suber) and (Copi 1954)

20. In [@c8, p65] we understand that

1. Quantifier logic adds a universe
2. Magic things (propositional functions) that pick out binary information about the universe (predicates)
3. Predicates are simply (propositional) functions. They are applied to 'variables', but we can see that as simply a meta-language (infinitary) that can look individuals up in infinite tables. So 'instantiation' is nothing but 'application' and variable is nothing but 'member of the universe'
4. An 'individual' is simple a tag attached to an individual (a thing) in the universe. But how can we point an individual? We can't, we never really do, we usually only need a symbol that might point to an individual such that constraints (predicates and combinations of them) do hold for that individual.

21. What one needs to keep in mind, which we have never done for FOL, is to keep track of what is being said in this chapter with the goal on mind of: how are we going to formalize it, and given this, what does it tell us about our goal: the inverse process of proving, in terms of 'strategies'.

22. Note that [@c7] has a dedicated section on 'strategies'. So does [@c4] as far as we remember.

23. The book's notation is quite nice and compact. Maybe we should augment with notation indicating free variables! By the way, in the definition of limit of a sequence, is the sequence a free variable? That is both strange and interesting. If yes, that makes definitions not fully 'formal objects' (maybe that is why they needed additional treatment in one of our favorite 'typed lambda calculus and logic' books?)

24 (@c8, p66)**.** is a very good page to stop being confused by the concept of an empty universe and its relation to quantifiers. An empty universe is a possible universe and it is special in the way the relation between the two quantifiers is in it. So it is a big deal to first show that a universe is not empty, in particular, that a set-theoretic set is not empty.

- In a non-empty universe
$$(x)(Mx) \implies (\exists x)(Mx)$$
- In an empty universe
$$(x)(Mx) \not\Longrightarrow (\exists x)(Mx)$$

25. In this notation, Socrates is a human becomes

$$Hs$$

26. To internalize the fact that 'All men are mortal' translates to

$$(x)[Hx \supset Mx],$$

all we need to do is to remember the universe, and that without further assumptions, it does hold types more general (e.g 'Thing') than the largest type at question (e.g 'Creature').

27 (@c8, p60)**.** shows an example of 'flag notation' and of 'assumption discharge', the use of a slanted line demarcates the end of assumptions as explained in [@c8, p31].

28. We note that already with the easy to understand 'universal generalization', we are talking about an arbitrary individual. Therefore, it is now less surprising the see how 'universal generalization' starts with an arbitrary symbol. The idea is that no assumptions are made about that symbol: it does not take part of any other (implicit) 'constraints'. This is how, for now, we model 'assume any arbitrary v'.

29. ??? We note that UI and UG together tell us that there is a true equivalence between 'for all' and 'for arbitrary':

$$\frac{(x)\Phi x}{\Phi v} \ \& \ \frac{\Phi v}{(x)\Phi x}.$$

Arguably, what is 'confusing' here is the word 'instantiation'. We would think that we are 'picking one individual', and we both are and are not. We are 'focusing' on one individual, while at the same time, we have not determined 'exactly' which one it is and 'pinpointed it', and that's precisely because we do not want to... We do not want to commit to any additional properties of some single individual. This leaves us at another confusion as to what exactly the difference is between the two versions. The quantifier seems to still be there, only 'implicitly'. We have to keep an eye on this in proofs to understand it better.

The answer is in (31). The 'operational' value of UI is that the rules of inference do not, purely syntactically, work with quantifiers. Per example, the transitivity of implication (Hypothetical syllogism). To use it formally, we cannot do that within a quantifier, so we get rid of the quantifier first using UI, and we can recover it using UG. The proof given in the next note is the perfect minimal example.

30. We note that the 'definitions' used at [@c8, p72] are 'preliminary' because they treat free and bound variables naively, dealing with only one variable. That is the only 'flaw' in them that is fixed in the later better definitions at [@c8, p93].

31. We note the proof at [@c8, p73]. The nice and easy usage of UI and UG is almost too nice and too easy. But it does work and must be internalized! Operationally, it is a matter of removing and adding parentheses to work with the formulas 'inside', without the quantifiers, temporarily. Indeed, here are nice quotes about 'arbitrary': [4] [5] [6]

> What does that mean? It means we really could have proven P(x) for any value of x, not just x=c; the same proof would apply! So we have actually shown that (∀ x)P(x).

> It's not "I pick a random c and if it's true for c, then it's true for all x". It's "If I know it's true for c even if I don't know which c I have, then it's true for all x we pick up."

> The reason this rule is valid is because we assume absolutely nothing about the element x.

32. To prove the invalidity of an argument already in PL [@c8, p45], we have to proceed by trial and error if we do not want to exhaust a truth table. Note that to prove the invalidity of an argument is not the same as proving its negative is valid! The argument could be invalid without its negation being valid. Note that we are talking about arguments here, it is not directly clear what a negation is, is it the negation of every premise and the conclusion? Note that all of the premises and conclusion are compound statements, and the argument is invalid if the statements, not the propositional variables, are true, with the conclusion statement being false. In any case, we need to remember the truth-functional definition of invalid argument for PL. As we said this is usually done by trial and error and already foreshadows that giving counterexamples in QL will also be reduced to trial and error.

33. There is an explicit section that talks about substitution: [@c8, p18]. It also talks informally about using substitution instances to decide which argument forms are valid and invalid. In short, we can do substitutions because that's what we 'mean' with a logic, that's our intended use, that's the way we are modeling things. There can only be a justification for this, no 'proof'.

34. The 19 rules so far (up to section 3.3) are incomplete. To that come two new rules (expressed in terms of natural deduction: to be able to prove implications by discharge) which make the set complete. This claim is made at [@c8, p.56]. The proof is postponed by the author until chapter 7. But for our speed proving purposes, we will not care at the moment. WARNING! The first 9 rules are different from the rules 10-19. The former are 'inference rules', while the latter, while being inference rules, are special in that they are 'replacement rules'. Operationally, the latter are 'bidirectional'.

[4] https://math.stackexchange.com/questions/1889107/why-does-universal-generalization-work-the-rule-of-inference
[5] https://math.stackexchange.com/questions/2468781/how-does-universal-generalization-agree-with-facts
[6] https://math.stackexchange.com/questions/997037/if-true-for-the-general-element-then-true-for-all-whats-this/997889#997889

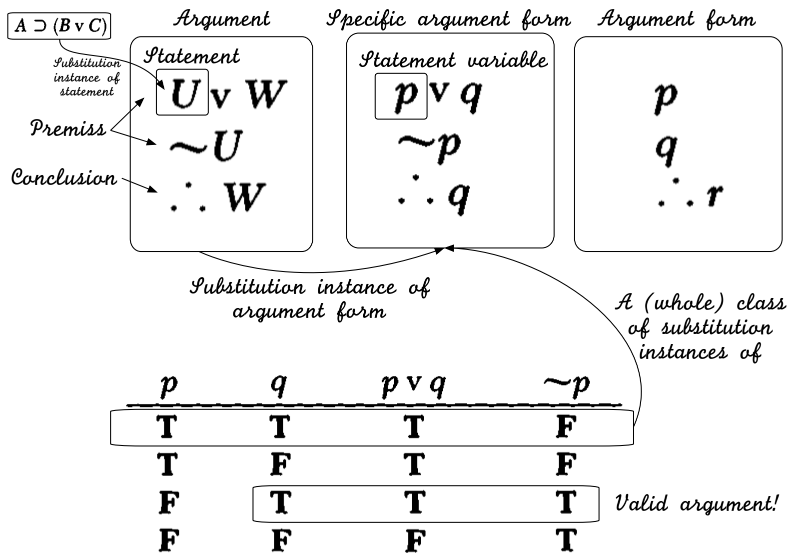

Figure 1

35. What does (39) tell us in terms of proof technique? It is important! It tells us that one cannot formally prove

$$A \supset B$$
$$\therefore A \supset (A\dot{B})$$

without assumption and discharge (conditional proof)! One could try forever, get stuck, give up! Conditional proof is NOT equivalent to any of the other 19 rules. Informally, one can see why the argument is valid, and the conditional proof is exactly the formalization of the kind of thinking used to 'semantically' see why it is.

VI I Can Not

36. Formally explain the difference between holonomic and nonholonomic systems and why Lagrange multipliers work for the former and not the latter. In fact, most treatments are either non-rigorous, or based on specific examples (see tag 'lagr'). The only book that seems to have enough rigor is [@c9], which only treats holonomic constraints and defines then in a very rigorous way. It also points to a rigorous treatment of nonholonomic constraints in [@c10], [@c10b]. The author of [@c11] seems to be an authority on the problem, and here is a nice quote from him:

> The d'Alembert–Lagrange principle (DLP) is designed primarily for dynamical systems under ideal geometric constraints. Although it can also cover linear-velocity constraints, its application to non-linear kinematic constraints has so far remained elusive, mainly because there is no clear method

> whereby the set of linear conditions that restrict the virtual displacements can be easily extracted from the equations of constraint. On recognition that the commutation rule traditionally accepted for velocity displacements in Lagrangian dynamics implies displaced states that do not satisfy the kinematic constraints, we show how the property of possible displaced states can be utilized ab initio so as to provide an appropriate set of linear auxiliary conditions on the displacements, which can be adjoined via Lagrange's multipliers to the d'Alembert–Lagrange equation to yield the equations of state, and also new transpositional relations for nonholonomic systems.

There is an article on the history of the subject is [@c12] which says:

> The basic difference between nonholonomic dynamics and common Lagrangian one lies in the fact that the equations of constraints, written in terms of generalized coordinates qj and generalized velocities $\dot{q}j$ in the following form:
>
> $$f_i(q, \dot{q}, t) = 0, i = 1, ..., k, q = (q1, ..., qn),$$
>
> cannot be presented in the final (integral) form
>
> $$F_i(q, t) = 0$$
>
> that sets limits only to the generalized coordinates. In this sense, one says that the constraints are nonintegrable (differential). According to Herz, they also can be called nonholonomic.

An alternative and maybe different (and more canonical?) view of the problem is at [@c13]. From [@c14] we quote

> D'Alembert's principle, which gives a complete conceptual solution to problems of classical mechanics, hinges upon the first-order virtual work done by the impressed (given) forces and that done by the forces of inertia (Lanczos, 1970). The former can often be expressed in terms of the variation of a potential energy function (Lanczos, 1970). By integrating with respect to time, the virtual work done by the forces of inertia can be transformed into a true variation (Rosenberg, 1972). Thus for holonomic systems, D'Alembert's principle can be reformulated as Hamilton's variational principle, which re- quires that a definite integral be stationary (Lanczos, 1970). The set of Lagrangian equations of motion that follow remain invariant under arbitrary, one-to-one point transformations.
>
> It was in 1829 that Gauss (1829) gave an aesthetic and ingenious reinterpretation of D'Alembert's principle, changing it into a true minimum principle. This principle is applicable to systems with general constraints, including configuration constraints (Rosenberg, 1972). Gauss argued that the deter- mination of the motion of an n-degree-of-freedom system in which positions and velocities were known, hinged o n o u r ability to determine the accelerations under the given applied forces. He formulated the principle of "least constraint" for describing the motion of mechanical systems. This principle is closely analogous to his celebrated "method of least squares," a method he developed and applied to the adjustment of errors in measurements. Unlike Hamilton's principle, the principle of least constraint has the additional advantage of not requiring any integration in time. Hertz gave a geometrical interpretation of Gauss's principle for the special case when the

> impressed forces vanish (Hertz, 1917). He showed that in this case Gauss' s "constraint" can be interpreted as the geodesic curvature of the configuration point in 3«-dimensional space. Appell and Gibbs (see Pars, 1979) further extended the principle to apply to nonholonomic conditions and in cases where it maybe advantageous to use kinematical variables (Lanczos, 1970).
>
> They used the idea of pseudo-coordinates (see, Pars 1979) 2 which has, more recently, been again explored by Shan (1975) . Synge (1926) has also provided an alternative set of equations of motion of nonholonomic systems in terms of the geometry of the resultant trajectories. As such, his formulation is dif- ficult to directly apply to engineering problems. From a practical standpoint, however, the computational difficulties of directly solving a minimization problems at each instant of time to describe the motion of a mechanical system made Gauss's principle unattractive at the time. This caused mechanicians of the late 18th, 19th, and 20th centuries to expound on, and mainly utilize the methods of Jacobi and Hamilton in the solution of problems in mechanics. Modern day texts in classical mechanics usually concentrate on these two latter approaches (e.g., Arnold, 1980), often relegating Gauss's principle to the position of a theoretically insightful approach, yet practically speaking, an unusable novelty.
>
> In this paper we show that with our improved understanding of generalized inverses of rank-deficient matrices, Gauss's principle may offer a new, direct and oftentimes simpler ap- proach to handling complex problems in mechanical systems

After some study, given our improved understanding of multivariable calculus, we found these useful resources, and a short study gives the following conclusions, that are nowhere mentioned in exactly this form, but usually a bit differently:

1. Working within Cartesian coordinates, we can show that the L-E (Lagrange Euler equations) are equivalent to Newton's.
2. We can show that given any set coordinates, transforming it (by a change of coordinates that is nice: invertible) to another set of coordinates, If the L-E equations are satisfied for one set, then they are also satisfied for the other. This means that we can change coordinates at will and find a trajectory that satisfies the L-E equations, and that trajectory, re-parametrized to any other coordinate set, will be such that its re-parametrization also satisfies the L-E equations.
3. Combining (1) and (2) above, this means that the L-E equations allow us to solve for the trajectory independently of choice of coordinates.
4. In particular, (3) tells us that if we have a set of constraints that are of such kind that they can be expressed as merely reducing the number of degrees of freedom (and not another more complicated expression), we can change to the reduced coordinates and solve the L-E equations there. This is how the Lagrangian formalism automatically handle such kinds of constraints. We call such constraints holonomic (or ideal) constraints (these are the constraints that 'do no work' (not all constraints are so, see TODO), these are the constraints that can be expressed with certain kinds of equalities, these are the constraints that are rigorously defined a bit differently and mathematically in [@c9]). We find that the most rigorous and in that sense 'best' book to study this is Bloch and Marsden's [@c9]. Other kinds of coordinates are called non-holonomic. Some of them can be twisted to use to the holonomic treatment, but in general, there is no general canonical way to handle them as there is to handle the holonomic ones.
5. In (4), we showed how to pass to reduced coordinates to solve a holonomically constrained system. In this way, we bypass needing to look at constraints and their forces. But if we are interested in calculating these

forces, we can express the problem differently, not reducing the coordinates by getting rid of degrees of freedom, but instead using Lagrange multipliers: TODO. This leads us directly into the rigid-body Baraff approach.

6. Optionally, we can link the L-E equations to the 'principle of least action'. This means that we can show that the L-E equations are merely yet another manifestation of a 'least action'. This is not directly clear from the L-E equations, but one can prove that if one does take a certain integral called action over a certain function called the Lagrangian, the 'least action' of this is indeed equivalent to the condition of the L-E equations holding, and by that, we show that the L-E equations are indeed an instance of a 'least action' principle.
7. Note that a valid change of coordinates requires a non-zero determinant of Jacobian. For a change which removes degrees of freedom, the Jacobian is not square. This is glossed over in TODO. One then must use what is equivalent to a pseudo-inverse. A nice reference on this is [The Change of Variables Formula Using Matrix Volume].

VII Can I?

37. Operationally describe (and hence internalize) each basic valid rule of inference [@c8, p32] for PL? I solved [@c8, 3.1.I-III] for that purpose. I have internalized and recognize the rules, not sure if I can fully list them. Naming them is something I do not care about, but I have given them useful shortcut names that describe their operational aspects. (Big black notebook).

38. Form a sentence that uses the expression 'fixed probability distribution', where I have confidence that the sentence is correct. (see caption of first figure in wiki article on arithmetic coding).

39. Remember the example of which simple valid argument cannot be proven with the 19 rules alone, but also needs the rule of 'conditional proof'? ('assume and discharge'):

$$A \supset B$$
$$\therefore A \supset (A \cdot B)$$

VIII I Have

40. Summarized the first 9 inference rules of [@c8] in an operational format:

$$\cancel{\mathbf{p}} \ni \mathbf{q} \quad \left\| \quad \begin{array}{l} \mathbf{p} \supset \mathbf{q} \\ \mathbf{p} \\ \hline \qquad \mathbf{q} \end{array} \right. \tag{1}$$

$$\cancel{\mathbf{p}}^{\mathbf{q}} \quad \left\| \quad \begin{array}{l} \mathbf{p} \\ \mathbf{p} \supset \mathbf{q} \\ \hline \qquad \mathbf{q} \end{array} \right. \tag{1'}$$

$$\overset{\sim}{\downarrow} \mathbf{p} \ni \cancel{\mathbf{q}} \quad \left\| \quad \begin{array}{r} \mathbf{p} \supset \mathbf{q} \\ \sim \mathbf{q} \\ \hline \sim \mathbf{p} \qquad \end{array} \right. \tag{2}$$

$$\sim \cancel{\mathbf{p}}^{\mathbf{q}} \quad \left\| \quad \begin{array}{r} \sim \mathbf{p} \\ \mathbf{q} \supset \mathbf{p} \\ \hline \sim \mathbf{q} \qquad \end{array} \right. \tag{2'}$$

$$\mathbf{p} \ \supset \ \cancel{\mathbf{q}}^{\mathbf{r}} \quad \left\| \quad \begin{array}{c} \mathbf{p} \supset \mathbf{q} \\ \mathbf{q} \supset \mathbf{r} \\ \hline \mathbf{p} \supset \mathbf{r} \end{array} \right. \tag{3}$$

$$\cancel{\mathbf{q}}^{\mathbf{p}} \ \supset \ \mathbf{r} \quad \left\| \quad \begin{array}{c} \mathbf{q} \supset \mathbf{r} \\ \mathbf{p} \supset \mathbf{q} \\ \hline \mathbf{p} \supset \mathbf{r} \end{array} \right. \tag{3'}$$

$$\cancel{\mathbf{p}}\ \cancel{\vee}\ \mathbf{q} \qquad \Bigg\| \qquad \begin{array}{l} \mathbf{p} \vee \mathbf{q} \\ \sim \mathbf{p} \\ \hline \mathbf{q} \end{array} \tag{4}$$

$$\cancel{\sim}\ \cancel{\mathbf{p}}^{\mathbf{q}} \qquad \Bigg\| \qquad \begin{array}{l} \sim \mathbf{p} \\ \mathbf{p} \vee \mathbf{q} \\ \hline \mathbf{q} \end{array} \tag{4'}$$

$$(\cancel{\mathbf{p}} \cancel{\supset} \mathbf{q})\ \cancel{\wedge}^{\vee}\ (\cancel{\mathbf{r}} \cancel{\supset} \mathbf{s}) \qquad \Bigg\| \qquad \begin{array}{l} (\mathbf{p} \supset \mathbf{q}) \wedge (\mathbf{r} \supset \mathbf{s}) \\ \mathbf{p} \qquad\quad \vee \quad \mathbf{r} \\ \hline \qquad \mathbf{q} \quad \vee \qquad\quad \mathbf{s} \end{array} \tag{5}$$

$$\cancel{\mathbf{p}}^{\mathbf{q}} \vee \cancel{\mathbf{r}}^{\mathbf{s}} \qquad \Bigg\| \qquad \begin{array}{l} \mathbf{p} \qquad\quad \vee \quad \mathbf{r} \\ (\mathbf{p} \supset \mathbf{q}) \wedge (\mathbf{r} \supset \mathbf{s}) \\ \hline \qquad \mathbf{q} \quad \vee \qquad\quad \mathbf{s} \end{array} \tag{5'}$$

$$(\overset{\sim}{\downarrow}\mathbf{p} \cancel{\supset} \cancel{\mathbf{q}})\ \cancel{\wedge}^{\vee}\ (\overset{\sim}{\downarrow}\mathbf{r} \cancel{\supset} \cancel{\mathbf{s}}) \qquad \Bigg\| \qquad \begin{array}{l} (\mathbf{p} \supset \mathbf{q}) \wedge (\mathbf{r} \supset \mathbf{s}) \\ \qquad \sim \mathbf{q} \quad \vee \qquad \sim \mathbf{s} \\ \hline \sim \mathbf{p} \qquad \vee \sim \mathbf{r} \end{array} \tag{6}$$

$$\sim \cancel{\mathbf{p}}^{\mathbf{q}} \vee \sim \cancel{\mathbf{r}}^{\mathbf{s}} \qquad \Bigg\| \qquad \begin{array}{l} \qquad \sim \mathbf{p} \quad \vee \qquad \sim \mathbf{r} \\ (\mathbf{q} \supset \mathbf{p}) \wedge (\mathbf{s} \supset \mathbf{r}) \\ \hline \sim \mathbf{q} \qquad \vee \sim \mathbf{s} \end{array} \tag{6'}$$

$$\mathbf{p} \wedge \cancel{\mathbf{q}} \qquad \Bigg\| \qquad \begin{array}{l} \mathbf{p} \wedge \cancel{\mathbf{q}} \\ \hline \mathbf{p} \end{array} \tag{7}$$

$$\mathbf{p}\ \cancel{,}^{\wedge}\ \mathbf{q} \qquad \Bigg\| \qquad \begin{array}{l} \mathbf{p} \\ \quad \mathbf{q} \\ \hline \mathbf{p} \wedge \mathbf{q} \end{array} \tag{8}$$

$$\mathbf{p}\ \overset{\vee \mathbf{q}}{\downarrow} \qquad \Bigg\| \qquad \begin{array}{l} \mathbf{p} \\ \hline \mathbf{p} \vee \mathbf{q} \end{array} \tag{9}$$

IX I Can

41. Translate the combinatorial of $()$, $(\exists)$, $\sim$, Φ into english. The combinations are:

1. $(x)\Phi x$
2. $(x) \sim \Phi x$
3. $(\exists x)\Phi x$
4. $(\exists x) \sim \Phi x$

and the relations are that 1) and 4) are 'contradictory' and 2) and 3) are 'contradictory'. Note that we are used to this but in fact, why is not 2) the negation of 1)? It 'is' it is called the 'contrary' (it does also 'contradict it' but much more strongly). When we do logical negation, we mean the 'contradictory'.

42. Define 'Universal Instantiation'

$$\frac{(x)\Phi x}{\Phi v} \textit{ (for any individual } v\textit{)}$$

43. Define 'Universal Generalization'

$$\frac{\Phi v}{(x)\Phi x} \textit{ (for an arbitrary individual } v\textit{)}$$

44. Use the 9 basic PL inference rules to prove simple propositions (prove the validity of simple propositional arguments), purely syntactically. I solved [@c8, 3.1.I-III] for that purpose.

45. TODO: Use the 19 basic (and redundant) rules to prove simple propositions by doing [@c8, 3.2.I-III].

46. TODO: Prove invalidity by 'truth-table' counter-example by doing [@c8, 3.3.I].

47. TODO: The same feat as above but with the complete set of 21 rules as poses in [@c8, p.56].

48. TODO: The same feat as above but with QL.

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