1) TODO: copy our trigger paper

2) TODO: add note about geometry's rise, topology, necessity of some projection of any system, etc.

3) Note that, like in 1) we finally realize what Cartesian Geometry. 'Greek' Geometry is just like our closed 'vector space'. To peek into it, we use coordinates, and that is what Analytic Geometry is.

* We also realize, that it seems nobody really understands what exactly happens in terms of 'intuition', and everybody just repeats the usual explanation. Per example, the excellent Chvatal [LP] says: [p250] 'The resulting one-to-one correspondence between points of the plane and ordered pairs [xx, x2] of real numbers creates a correspondence between plane geometry and certain parts of algebra. This correspondence can be exploited in both directions. On the one hand, it elucidates algebraic arguments by providing them with intuitive geometric interpretations; on the other hand, it makes it possible to investigate problems of geometry by applying algebraic techniques. In this chapter, we shall use the correspondence in both ways. In the beginning, we shall illustrate in intuitive geometric terms the algebraic techniques developed earlier in this book. Later, we shall reverse the direction and see that certain classical results on the geometry of convex sets follow routinely from the algebraic theorems of Chapters 9 and 16.'

What is missing is the realizations that follow.

* Even though Gaussian elimination, that is, the method of substitution, has been know since antiquity, it was for a long time not seen as Geometric, but that is not the whole story. The mind approaches this line per line. Each line is indeed seen 'geometrically', as numbers on a real line, that have to be moved left or right until some constraint is satisfied. Algebraically, a symbol game can be played, resulting in substitution. What is introduced by 'Geometry' is in fact the viewing of this 'numbers on the real line' game as something totally different. We embed the problem into the largest multiply determined problem with no unneeded determinations. So for n variables, the space Rn. This is a totally different mind view, and the key in it is that it enables us to see the problem in terms of 'left of, right of, on, inside, between, border of'. Of course, only in two dimensions (at most three, but that is already more difficult, but we know by 2) that this can already be enough, as the concepts we just mentioned are mostly luckily preserved). What we have done is taken the exact set that satisfies the constraint, and showed it in another (not necessarily it's 'real') space, that can be surveyed. What happens then is that, even when looking at the original symbol line, we start using the same concepts in it, even though we were so weak as not to see them before, and we needed the visual example slapping us in the face first. The concepts also include intersections, unions (corner points [p253]). It is rather sad that we do not see this immediatly, and that, our basic thinking is the one line manipulation one. But we should not even see as sad, but learn from it. All views are the right views, and one should learn to see all the applicable thought concepts to a new problem as soon as possible (e.g in Calculus, multi-var calculus, etc.). Lastly, imagine the usefulness of the Geometric view if your mind could survey the plane only 'point by point', we would not be much better off, and in fact, we would be better of sticking to symbolic manipulation. Hence, as we already know, this is nothing but lifting the human mind out of its native misery.

4) What the misery of the mind related to 3) also reveals is that the solution to the lin. ineq. problem happens by moving values on the real line, line by line, this is so strongly embedded in the native mind. Even after linear algebra, it clings to it, and even when executing it, does not see that the actual solution relies on modifying the 'problem' (the costant factors), not concerned about the 'variables' at all. Of course, we know that this is the methodical search for a correspondence (that we can compute), to a simpler problem whose solution is easy to find, but that is another note.

5) Finally we find out that singular, degenerate, and non-regular (and therefore regularization as we heard of it in the n-body problem) are exactly the same. The context in which they appear is of course very general (http://en.wikipedia.org/wiki/Degenerate_form). Additionally, it is quite understandable how this affects algorithms. As soon as there is no clear way to make a decision (very roughly, related to any map that is not one-to-one), there is a potential of cycling, and therefore, of never terminating. In a more general case this could also happen with vanishing derivatives, etc.

6) A clear proof that even the almost complete blindness, that is, the access of memory one bit of information at a time, is capable of all thinking and calculation, is the success of 'Turing Machines'. Hence one should not be discrouraged by the size of a (mathematical) problem, since, at the extreme, one can always work on very small sub-problems, if keeping a map to crawl along when needed, bit by bit.

7) On Pivoting. Although pivoting seems completly understood since it is simply a linear map,

it should be 'taken seriously', and not thought of as a mere tool. The essence of it is the essence of the 'grand strategy' of the whole class of pivoting methods, that of walking the space of extremal solutions, using some local strategy, by -- and this is the important part -- the fixing of some variables, and the satisfaction of the dependent ones, while approaching an easily decidable problem, moving from problem to problem, the difference between which is the difference in choice about what is fixed and what not, the translation of one problem to another being the pivot operation, and recorded in some way, so that the solution can be readily transformed. The pivotal method, using the full constraints (or their modifications) is therefore extremal. The subtelty that has to be taken seriously is exactly this choice switch.

8) As soon as we think about cones and orthogonal spaces, we have to remember the native mind, and its geometric projections, and that the 'spaces and structures' we 'see' are the fuzzy expression of exactly the stressed concepts that seem 'so obvious', with regrads to cones and convexity. Indeed, before the geometric image, is the stage of total randomness, of, per example an orthogonal space, which, without any knowledge could just be any random set of vectors. The geometric image gives this space some regularity in the mind, but the algebraic theory of convex polyhedra is the most general image, and it should be the one strived for, so that it is directly usable in the mind, without 'eliminating' the geometric one.

8) A mathematician is a child reconstructing in his mind a beautiful projection of the infinite mathematical space's structure using mental Lego blocks.

9) Nering maintains the importance of not giving the vector space Rn with the standard basis any special treatment, we call this vector space with this choice of basis the 'intuitive vsisual Rn', in the sense that it is what (fortunately or unfortunately) springs to mind whenever we think of finite dimensional vector. Nering's opinion, as noble and as it is, is also marmoric. He almost contradicts himself giving more reasons for the natural point of view. Also, we see the danger of claiming that most of the rework is about change of basis: This makes problems solving seem like a game of basis choices, play with magic, and based in linear algebra. However, this is not true; even though we fill in matrices to create a change of basis, the change of basis matrices themselves are almost never a linear function of the state of the problem at the stage. Instead, anything goes there. We do need linear algebra for what it does, especially dimension considerations. But we should not confuse it with matrix theory, and we should not confuse matrix theory with the problem solutions themselves, which usually use quite 'random' processes of building the matrices. The quotations from Nering are the following. [p18]

'The set of n-tuples together with the rules for addition and scalar multiplication forms a vector spaceinitsown right. However, when a basis is chosen in an abstract vector space V the correspondence described above establishes an isomorphism between V and F n . In this context we consider F n to be a representation of V. Because of the existence of this isomorphism a study of vector spaces could be confined to a study of coordinate spaces. However, the exact nature of the correspondence between V and F n depends upon the choice of a basis in V. If another basis were chosen in V a correspondence between the a g V and the «-tuples would exist as before, but the correspondence would be quite different. We choose to regard the vector space V and the vectors in V as the basic concepts and their representation by n-tuples as a tool for computation and convenience. There are two important benefits from this viewpoint. Since we are free to choose the basis we can try to choose a coordinatization for which the computations are particularly simple or for which some fact that we wish to demonstrate is particularly evident. In fact, the choice of a basis and the consequences of a change in basis is the central theme of matrix theory. In addition, this distinction between a vector and its representation removes the confusion that always occurs when we define a vector as an n-tuple and then use another n-tuple to represent it.'

'Only the most elementary types of calculations can be carried out in the abstract. Elaborate or complicated calculations usually require the introduction of a representing coordinate space. In particular, this will be required extensively in the exercises in this text. But the introduction of coordinates can result in confusions that are difficult to clarify without extensive verbal description or awkward notation. Since we wish to avoid cumbersome notation and keep descriptive material at a minimum in the exercises, it is helpful to spend some time clarifying conventional notations and circumlocutions that will appear in the exercises. The introduction of a coordinate representation for V involves the selection of a basis {<x x , . . . , a n } for V. With this choice a x is represented by (1, 0, ... , 0), a 2 is represented by (0, 1 , 0, . . . , 0), etc. While it may be necessary to find a basis with certain desired properties the basis that is introduced at first is arbitrary and serves only to express whatever problem we face in a form suitable for computation. Accordingly, it is customary to suppress specific reference to the basis given initially. In this context it is customary to speak of "the vector {a x , a 2 , . . . , a„)" rather than "the vector a whose representation with respect to the given basis {a 19 . . . , a n }is(a 1 , a 2 , . . . , a n)." Such short-cuts may be disgracefully inexact, but they are so common that we must learn

how to interpret them.'

10) Related to 9) reconsider 'Gaussian elimination'. It can be carried away without mention of linear algebra nor bases, let alone choices of bases. Why is the final form any more usable than the first? This cannot be separated from our Turing-machineness. We can only do the computation instruction by instruction. There is nothing else that is practically the essence of this method. Using determinants is similar. Of course linear algebra is there to teach us about the whole world in which we are operating, and that, per example, the number of solutions is always some dimension of the field us: R0,R1,... and never anything else. If we really want to explain this method as closely to linear algebra as possible, we must first notice this fact, and then explain that we procede by injective change of basis, such as, the vector x we seek, after the last change, is such that, in the new basis, is such that the numerical matrix allows sequential instruction reconstruction. So instead of the intuitive view of moving between problems, we get the view of changing bases, keeping the creatures we seek the same. All while noting that as soon as the numerical matrix and right-hand-side are there, we are already bound into (quite arbitrary) choices if we want to relate the problem to linear algebra. Considering the problem in R1 sheds a nice light on it, not forgetting that in algebra, the specific complete ordered field has a natural unity element (from the natural numbers), which then fixes the whole field. Denying the naturality of the 'intuitive visual Rn' is denying the naturality of the counting unity as manifested in R, we can do it, but the we should spend some time explaining what is done. In the case of LP, we do not end up with a single instruction construction of the solution, but we get an finite enumerative indication of termination, and with it the solution (both in the tableau).

11) Finally, we understand what this means and why it was said by Chvatal: [p253] 'Although the problem asks for an optimal solution selected from an infinite set of feasible solutions, the simplex method examines only a finite number of them: the basic ones. In every basic solution, three of the five variables xb x2,. . ., x5 are nonbasic. These three variables are set at zero; in fact, setting them at zero determines uniquely the values of the remaining two variables. In geometric terms, this means that every point representing a basic feasible solution is uniquely determined as the intersection of three facets. Such points fit our intuitive notion of corner-points or, by another name, vertices, of the polyhedron. Hence, at least in this example, the basic feasible solutions (an algebraic concept) correspond to the vertices (a geometric concept) and vice versa.'
We notice explicitely how indeed, in the dictionary, we always have three zeros corresponding to a choice of two basis vectors for the solution. It is important to embody this because one would intuitively think that in the problem posed, one would need three bases for the optimal solution.

12) Generate linear algebra and matrix theory notes.
a) Add trivializers: lin map is map bet. Vectors, matrix is map between coords. Basis is an biject (internal) map between vectors and coords (between space and Rn) . see the basis map explic. That is: a vec is mapped to a coord tuple, another basis (map) may map it to another coord (tuple), and the tuple from the former basis map may be mapped to another vector.
b) So then given two spaces and two basis maps and a matrix (tuple coord map), there is a number of situations we can achieve by using different basis maps, adjusting the matrix, or not, according to what is needed. Eg changing the input basis map, and wishing that the vector map is kept the same, the matrix has to be adjusted, because it maps input coords to the same vectors as before, but the coords are mapped by the new basis map (invertible) to potentially different vectors (at least some vecs must be differently mapped). It might be useful to enumerate the other situations, and maybe even their combinations.
c) Nering p53. This statement now poses no conceptual difficulty at all: 'we may also ask how much simplification of the matrix representing a linear transformation a of U into V can be affected by a change of basis in V alone.

13) Completely understanding Nering's view of the Simplex algorithm [p239-252] is very fruitful and important. Especially [p245-246]. There it is important to keep in mind that the solution is one of coordinates, since it is the solution of a matrix problem. In that sense, one of the functions of linear algebra here is illustrative.