

In Whittaker's 'From Euclid to Eddington', page 20, it is mentioned that Euclid's definition of a point is usually criticized as lacking, it defines what a point is not 'that which has no part'. I always felt the same, but now I realize that it is actually very good. It means that a point is such that thing which, in a specific context is considered as 'atomic', per example in the context of ancient Greeks, who did not have microscopes, a grain of sand was a good enough point. In other contexts, it might be something else, it is simply about it's relationship to other geometrical entities, in that you cannot see it as a circle per example.

This lead me to think that a point is something like a geometric infinitesimal, the enemy of standard analysis, but as we know, put on solid basis in this century. 'as small as needed' is the idea.

But are points then non archimedean as non standard analysis describes infinitesimals (since no number multiplied by an infinitesimal gives a non infinitesimal (see Zakon per example on the archimedean property) I guess the answer is yes, since no number of finite points surely for Euclid as well, would have formed a continuous line.

Am I the only one who thought of this? probably not. I found a similar idea here: <http://sci.tech-archive.net/Archive/sci.math/2005-10/msg02928.html> (see <1. Re: Infinity)

"Then, Euclid's points, Leibniz' infinitesimals, and Schmieden and
> > Laugwitz, are points on a line as beads on a string. "

I also found a slightly related topic $\langle x+dx=x \rangle$ I did not know Leibniz's infinitesimals could be considered Archimedean..

After this, I went back to Hilbert's 'The Foundations of Geometry' to see his definition of point, but of course, with his axiomatic approach, there is no definition of a point at all.

The discussion about abstracting systems away from reality and put the relationship in the hands of philosophy is an important and recurrent one. But that is outside the scope of this thought.