title: Notes on Library 3 author: Jad Nohra geometry: margin=3cm mainfont: 'Charter'

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Quest 3: 'Instantaneous Velocity (and nsa)'

This quest is started in the 'mhs' notebook at page [6'].

- See the notebook at page [8] for a derivation of commutativity and additivity of infinitesimal rotations.

Miscellaneous

- In 'mhs' we explained we will use spectral analysis to show that rotation has an axis instead of Euler's geometric (with his time's meaning) proof. We forgot to mention we would have significant difficulties doing it Euler's way. Of course we know he or Huygens were masters of this kind of reasoning, being the tool of their time, but not of ours (if we simplify matters). This gives us an additional characterization of the 'Algebra-Geometry' topic. We know that the strength of visualization, diagrams, geometry is sometimes that it allows to see 'everything at once'. We also remember Euler's remark about wanting to make an encumbered diagram clearer by omitting some parts of it (or was it Lagrange), which makes seeing 'everything at once' a weakness, and a virtue of analytic geometry being exactly seeing things a detail at a time (in sequence). This brings us to the point of this note: Geometry sees the forest, algebra sees the tree. Depending on what is needed, one better use the right translation. But what if there was a way to also algebrize the way to see the forest, seeing it all at once, algebraically. A sort of algebraic level of detail. Is this something that geometric algebra gives us?!

- We seem to have lost from files during the library restructuring, specificially the 'NumercialInteg' directiory from Library1, containing the invaluable Henrici books, and also the 'ghost' references from Hairer. Here are the notes from that directory:

  - another topic: search: stability of numerical simulation of dynamical systems
    - Dynamical Systems and Numerical Analysis, Volume 8, By Andrew Stuart, A. R. Humphries
    - http://www.inf.ethz.ch/personal/cellier/Lect/NSDS/Lect*nsds*index_engl.html
  - ghost node finite difference
  - differential equations ghost node
  - http://www.math.pku.edu.cn/teachers/lizp/courses/Numerical*PDE/Numerical%20PDE%20Lecture%20Slides/numpde*lecture*2*c1.pdf
  - Hairer, ASYMPTOTIC EXPANSIONS FOR REGULARIZED STATE-DEPENDENT NEUTRAL DELAY EQUATIONS
  - Dissipation: Hairer:
  - http://www.researchgate.net/publication/200032546*Geometric*Numerical*Integration-Structure-Preserving*Algorithms*for*Ordinary*Differential*Equations
  - Semi-implicit schemes: http://www.atmos.umd.edu/~ekalnay/syllabi/AOSC614/

  These files will be tagged: numerical,integ

Quest 4: Searching for a good path into category theory (again), including motivations.

- Trinitarianism seems beautiful and proper. Especially interesting is the table at http://ncatlab.org/nlab/print/computational+trinitarianism
- How does smooth analysis related to Hilbert spaces? The only related book we found was the (missing) book: Smooth analysis in Banach spaces By Petr Hájek, Michal Johanis
- Our proper path seems to be the following: Geometric Algbera contains answers directly related to our initial quest:

  - The wikipedia article on algebraic topology mentions: "One can use the differential structure of smooth manifolds via de Rham cohomology, or ... , sheaf cohomology to investigate the solvability of differential equations defined on the manifold in question.". It also talks of proofs of the fixed point thm and the bairy ball thm.
  - This directly relates also to our 'translations', what is the language to talk about these things.
  - But algebraic topology led to cat, sheaves and topi. A very good historical introduction liking those, also to logic can be found in no other than McLane's 'Sheaves in Geometry and Logic: A First Introduction to Topos Theory'. We should use this as a map.

- The map above links our quest also to intuitionistic logic and smooth infinitesimal analysis. Everything comes together, really everything.
  - Both 'Syhthetic Differential Geometry' books seem good actual pathways.
  - 'An informal introduction to topos theory' might also help, at least in terms of intro-gold
  - In the end, we have an alg langg for diff geom, without the set centered hassle, what more can one want?
- Nonstandard Analysis Applied to Advanced Undergraduate Mathematics, p20, seems to be gold.
- Why are there modelS of nsa, modelS for sia (Models for Smooth Infinitesimal Analysis), but not modelS of real analysis. Only ZFC? This brings us to study model theory a bit more directly.
  - see tag 'model'
  - Some books could provide more direct access.
  - Along the way, we stumble upon Algebraic books like 'Algebraic Systems', pointed at from 'INEVITABILITY OF INFINITESIMALS'(missing) (rel 'Inevitability of Things Non-Standard') (vquote kind of title).
  - Playful seems to be: 'Fun With Nonstandard Models'
  - Also, we might have found a good analysis book that is a standard as can be but not as 'modern' as Zakon's and one that puts some topology at the start, fixing one of our complaints about Zakon's: 'Real Analysis and Foundations' Krantz
  - 'Alternative Axiomatic Set Theories' (plato.stanford) also seems useful at this stage.
  - 'Set Theoretical Aspects of Real Analysis' helps us shed light on the 'models' by first looking at real analysis from a related? perspective. It seems this is intro-gold.
  - After a while we posed the valid question 'algebraic model of differential geometry', and found multiple gems, some with intro-gold:'Algebraic Models in Geometry', 'An algebraic model of transitive differential geometry', 'Basic Concepts of Synthetic Differential Geometry', 'Model Theory and Differential Algebraic Geometry'. We tag them with alg-geom.
- From reading of the many sources under 'cat' and 'model' we realize once again that our blocker is basics of logic from a notational point of view. Let us try to fix this using 'Handbook of Logic'. But in this sense, 'Potter's 'Reason's Nearest Kin'' that we already started is a good dialtectical historical introduction, faithful to our method.
- After some thought, it seems we have had an implicit problem, which we were aware of but now we can link it to the formal modelling/translation level. That of simultaneity. We have considered this while planning to read Jammer's 'Concept of Simultaneity'. Also, we by now do not care if the world is or is not simultaneous, but consider the difference between the simultaneuous solution of two linear equations, and the simultaneous solution of the laws of physics between two pairs of particles having one particle in common. We did not 'mind' the first one but we did mind the second. The problem is of course time and simultaneity. Of course, we remember how relieved we were at one point when we learned that 'gravity' needs time to travel. But all this is by now irrelevant. The question now is, how is is that we formally are allowed to express the simultaneity of the satisfaction of this 'conflicting' law on the two pairs. Are there systems that disallow this? What is the formalism of simultaneity, can we disallow automatic disjunction of propostions? or is the matter totally different? On a related note, we feel there is a necessity that any law is broken on some level. It seems that this is true in Quantum Physics, where conservation of energy happens within (immesuable) time. But consider the following: Is there an approximate model in the real world of a differential equation that has no solution at all? Can we force such an equation by building a system? What would happen? Maybe we are also still struggling with additivity of forces on a single particle. But the formalization of this is simply the use of vectors for forces. Maybe our question is 'is the application of force in newtonian physics an event? a causal event? a simulatenous causal event? a causal even due to an infinitesimal violation? Maybe what nsa tells us is that we can formalize this causality and still keep the classical part intact? These seem to address this: (quest 5)
  - 'Newtonian Mechanics: Particles and Forces'
    - "Contra Forces: Unobservable, Redundancy Argument, Vicious Regress, Causal Overdetermination, Force-Free Theories Pro Forces: Experience of Forces, Forces as Dispositions, Forces as Causal Relations, Forces as Aspects, Forces as Intermediaries Ontologies without Forces".
    - Note that this document makes us aware of yet another formulation of classical mechanics: Hamilton–Jacobi (HJ). This brings us to the idea that all these formulations are 'coordinate selections on a meta-level' of the same 'asbtract' structure.
    - Is not the source of belief in quantum time the kind of 'vicious regress' problem mentioned?
  - 'Causal Fundamentalism in Physics' "Causality in physics has had bad press in philosophy at least since Russell's famous 1913 remark: "The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm" (Russell 1913, p. 1). Recently Norton (2003 and 2006) has launched what would seem to be the definite burial of causality in physics. Norton argues that causation is merely a useful folk concept, and that it fails to hold for some simple systems even in the supposed paradigm case of a causal physical theory – namely Newtonian mechanics. ... The purpose of this article is to argue against this devaluation of causality in physics. I shall try to defend that Norton's charges against causality in Newtonian mechanics are flawed, and I will also suggest how the central causal message of Newtonian mechanics may proliferate into its supposed successor theories, namely special (and to some extent general) relativity and quantum mechanics."
- A related obstacle is semantic versus syntactic, here we read the following:
  - "So far, so obvious. And of course by simply inspecting the truth table for A→A we can see that it's a tautology; this is the semantic approach, while the derivation shown on the Wikipedia page is a syntactic approach. The problem with the truth table approach is that it doesn't scale to first-order logic: what's the truth table for ∀xP(x)? Assuming we assigned

constants to every element of the domain, we could posit some kind of analogue for a truth table, but if the domain were infinite then the truth table would be infinitely wide! This is why semantics for first-order logic are given by model theory, not truth tables. However, the deduction theorem still holds in first-order logic (for closed formulae)." (http://math.stackexchange.com/questions/90787/whats-the-difference-between-to-implication-and-vdash-therefore/90801#90801)

- Now that we know what the question is, what to look for, we might go back to Tourlakis's book 'Lectures in Logic and Set Theory', or the chapter on Propositional Logic in '' among others.
- We now have a new tag 'semantics' with some old texts from Carnap. one seems to be intro-gold.
- Not directly related, since we mean compactness in logic, is 'http://www.mathematik.uni-muenchen.de/~pschust/publications/scrutiny.ps'

- The whole topic is very related to AC and LEM, since the compactness theorem in logic hinges on them, at least syntactically not semantically: "One can prove the compactness theorem using Gödel's completeness theorem, which establishes that a set of sentences is satisfiable if and only if no contradiction can be proven from it. Since proofs are always finite and therefore involve only finitely many of the given sentences, the compactness theorem follows. In fact, the compactness theorem is equivalent to Gödel's completeness theorem, and both are equivalent to the Boolean prime ideal theorem, a weak form of the axiom of choice.[5]"
  - This then leads to questions about this theorem in other logics, like the intuitionistic one. We have circled around this before, but a bit more blindly.
- Given that we are starting to understand semantics, the question of whether inconsistency is semantic or syntactic comes to mind. It is a valid questio and the wiki answer is:

> In classical deductive logic, a consistent theory is one that does not contain a contradiction.[1][2] The lack of contradiction can be defined in either semantic or syntactic terms. The semantic definition states that a theory is consistent if and only if it has a model, i.e. there exists an interpretation under which all formulas in the theory are true. This is the sense used in traditional Aristotelian logic, although in contemporary mathematical logic the term satisfiable is used instead. The syntactic definition states that a theory is consistent if and only if there is no formula P such that both P and its negation are provable from the axioms of the theory under its associated deductive system.
>
> If these semantic and syntactic definitions are equivalent for any theory formulated using a particular deductive logic, the logic is called complete.[citation needed] The completeness of the sentential calculus was proved by Paul Bernays in 1918[citation needed][3] and Emil Post in 1921,[4] while the completeness of predicate calculus was proved by Kurt Gödel in 1930,[5] and consistency proofs for arithmetics restricted with respect to the induction axiom schema were proved by Ackermann (1924), von Neumann (1927) and Herbrand (1931).[6] Stronger logics, such as second-order logic, are not complete.
>
> A consistency proof is a mathematical proof that a particular theory is consistent. The early development of mathematical proof theory was driven by the desire to provide finitary consistency proofs for all of mathematics as part of Hilbert's program. Hilbert's program was strongly impacted by incompleteness theorems, which showed that sufficiently strong proof theories cannot prove their own consistency (provided that they are in fact consistent).
>
> Although consistency can be proved by means of model theory, it is often done in a purely syntactical way, without any need to reference some model of the logic. The cut-elimination (or equivalently the normalization of the underlying calculus if there is one) implies the consistency of the calculus: since there is obviously no cut-free proof of falsity, there is no contradiction in general.

- We found 'Jaakko Hintikka' who seems to be a real treasure trove!!!! A philosopher logician. Almost all of his works are very relevant. We mention only two. 'The Principles of Mathematics Revisited', 'Language, Truth and Logic in Mathematics'
- Some books that hopefully directly attack our question of 'what are the semantics of arithemtic' or as it is called 'number theory', that Peano arithmetic is supposed to (incompletely) model (We finally found the exact answer for this, Notebook, p18-19). Those are: A)'The Language of Mathematics, A Linguistic and Philosophical Investigation- (2013)', B) 'Symbolic Logic, Syntax, Semantics, and Proof (2012)', C)'Computability Theory, Semantics, and Logic Programming (1987)', D)'Introduction to Semantics' [Carnap]), E) 'Language, Truth and Logic in Mathematics'
  - A.p47 Refers to Greach's donkey sentences, they are indeed beautiful, we should remember them and the wiki example.

    > In this section, we will take a sentence from a real mathematics text, and show that we cannot construct a compositional semantic representation for that sentence using first order logic. In a very loose sense, the example may be thought of as analogous to Geach's 'donkey sentences' (Geach, 1980). The observation that such sentences exist in mathematics is not novel (cf.Ranta (1994)), but to our knowledge this is the first example drawn from a mainstream mathematical text. The sentence in question, taken from Sutherland's Introduction to Metric and Topological Spaces,

- E.p2. It is easy to see why the prevalent feeling is that FOL is THE logic. Of course, there is no such thing. FOL is felt to be so because of the laws of the universe we live in, which impregnate and shape our 'common sense' based on what reasoning seems to allow us to predict and control the world, and the most essential part (that is smallest, most general, most basic, most ubiquitous as a basis) of this seems to be FOL.
- E.p4. (gold). The 'arbitrary nature of these restrictions is due to nothing but history and ... imitation! (clever not wise)'

  > The first main point I shall argue for is that the usual formulation of first-order logic incorporates completely arbitrary restrictions. As soon as you understand the usual form of first-order logic., you ipso facto understand logical ideas that take you beyond it. They should therefore be incorporated in our basic general logic on a par with the ideas of traditional first-order logic. The only reason why they have not been codified in our ground-floor general logic is a number of arbitrary nota- tional conventions which have no foundation in the true order of things or perhaps rather in the true order of logic. Hence, even though the conventional first-order logic is part of the true elementary logic, it is not all of it.

- D.p4. Finally a slow explanation of object and meta language.
- Add note about clever but not wise would never close their own door, even when it is the right, non-self-serving thing to do. Instead of stopping at inf processes, they carry on in the hope of creating their own theory.