Jad >

ThePlan

2019:
Technique is everything

2018:
-- φ & ψ is satisfied if and only if both φ and ψ are satisfied --

June 2017:
More formal than formal.
Lambda calculus, type theory, intuitionistic logic, things of beauty.

Mar 2017:
'And he that strives to touch the stars, Oft stumbles at a straw'. [Benjamin West, 'The Cave of Despair', 1772]:-

Jan 2017:
Revision of the notes from the last three years (triggered by the convergence in probability and in distribution) yield a surprising progress in mathematical logic. Von Neumann universe: demystified. Axiom of Choice (or the lack of it) finally properly trivialised. A few weeks later, Goedel statements that have long tortured us: check.

Xmas 2016:
Information theory through sigma algebra.

Summary of the end of 2016:
Swimming in eigen-heaven.

Summary of rabbit-holed middle 2016:
Juggles tensor indices with Jester.

Le juge: Accusé, vous tâcherez d'être bref.
L'accusé: Je tacherai d'être clair.
(G. Couretline).
Le juge: Bon, tâchez d'être clair.
L'accusé: je tâcherai d'être bref.
(My extension, 2016)

Summary at May Friday the 13th 2016 (a real coincidence):
Variational calculus successfully reduced to dust.

Summary of an interesting last quarter of 2015:
'The proof is complicated: it uses 8 auxiliary points and more than 70 inferences, and uses all the axioms A1-A7.' (Michael Beeson).

Summary of a strange 2014:
'It's not always like it is in the books.' (John Constantine).

The plan is knowing the prerequisites of **Mathematical Methods of Classical Dynamics (Arnold)**, and **The variational principles of mechanics (Lanczos)**
Note: If at any time new goals arise, it is easy to realign! just reread pages 193,194,229,230,291,292293 in Lanczos's book!

Goal refinements:

1. From here on named 'the goals'! A more ambitious 'form' of the goals is "What would Poincare pick up if he was alive?" is this a possible path? :
""As pointed out by Yao and Hughes [20], it would be an exciting contribution if con-vergent computational chaotic solutions of nonlinear differential equations could be obtained.
Unfortunately, such convergent solutions of chaos have never been reported.
It is even unknown whether such kind of convergent solutions (in traditional meaning)of chaos exist or not.

Besides, it is also not guaranteed whether or not a computed chaotic result obtained by the smallest time-step is closest to the exact chaotic solution of the continuous-time differential equations [18,19,21]. How can we detect are liable one from different computed chaotic results? How can we avoid the so-called computational chaos (CC) and computational periodicity (CP)?"" (Liao, On the reliability of computed chaotic solutions of nonlinear differential equations)

2. In 'chaos, an introduction to dynamical systems'
   After a readable chapter 1, chapter 2 goes 2d combining eigenvalues and Jacobian matrices, also adding topology (manifolds), explaining in p.79/-1 that the goal if chapter 10 is to show that stable and unstable mfolds are topological mfolds! Chap. 10 then proves this partially in 10.4 in a long proof. This proof and the left out part (outside scope ref. to Devaney 1986) is a perfect sub-goal.

   Note. The top. Book 'visual geom and top. has been a good companion to stillwell's book in matters of top.
   Note. The rest of the chapter 10 is extremely interesting, including the challenge 'lakes of wada' and the lab 'leaky fausset' !!!!

3. In view of the (rightful, identifying, classifying and philosophically necessary) explosion of ‘groups’ in abstract group theory, it is not wrong to guide oneself by ‘application’. Although knowing the ‘artificial’ simple templates is still useful. A perfect ‘application’ can be motivated by the following paper, which constitutes a logical continuation to our three-body-problem quest, and a link to the ‘computing age’. Luckily (but not surprisingly), it is also related to variational calculus. The paper is: “An introduction to Lie group integrators { basics, new developments and applications” (Elena Celledoni, Håkon Marthinsen, Brynjulf Owren) (http://arxiv.org/abs/1207.0069).

4. Dynamical systems: This seems to be a good paper about the 'state of affairs' around the year 2000, including talk about shattered dreams of the 60s.
   "A global view of dynamics and a conjecture on the denseness of finitude of attractors, by Jacob Palis".
   This links to Arnold, as it talks about the KAM (Kolmogorov-Arnold-Moser 1960s) theory of conservative dynamical systems.

The requisites:

1. Analysis: Differential Calculus
2. Analysis: Integral Calculus
3. Analysis: Differential Equations
4. Geometry: Vector Spaces, Vectors
5. Linear Algebra: Linear Operators
6. Linear Algebra; Quadratic Forms. Note: at last, I have found the right successor of of Hefferon's Linear Algebra. What made this take so long is that I was search for a pure linear algebra book, while, in this case the right book (thanks to internalizing and cherishing the Lakatosian argument of proof-generated concept) has to have an applied flavor. Finally I also found an author with the style, and the book is 'The Symmetric Eigenvalue Problem' by Parlett. Already the introduction fixes many wrongs and clarifies many long standing doubts (Rn versus En, angles, transposes, etc.)
7. intro to topology, for 'foundations of mechanics' book
   1. we need an intro, metric topology and then differential topology
   2. for intro and metric : http://www.topologywithouttears.net/topbook.pdf , micro-intro: http://www.math.harvard.edu/~nasko/documents/topology_and_continuity.pdf
      1. this one is too advanced and has more prereq, not an introduction: http://www.math.cornell.edu/~hatcher/
   3. for differential: geometry or topology? http://en.wikipedia.org/wiki/Differential_topology#Differential_topology_versus_differential_geometry
   4. Also useful for rotations, to have done this: "This goal is, of course, unrealizable, as it is a standard exercise in topology to show that R3 cannot be mapped into SO(3) without singularities, i.e. gimbal lock. However, as we will now show, the inevitable singularities in the exponential map are often avoidable.", from 'Practical Parameterization of Rotations Using the Exponential Map' by Grassia.
8.

If stuck possibly search for help here: http://www.saylor.org/majors/mathematics/

The Plan:

- (step 0p) is studying linear algebra which is a prereq. but we already are studying that, but we will have to target some parts of another book for some more advanced topics that are needed, like linear operators and quadratic forms. However there are some other very important motivations that link the plan to linear algebra see 'Motivation LinAlg' below.
- (step 1p) In parallel to the other steps: V. Arnold's 'Abel's theorem in problems and solutions', exists as pdf uncluding solutions, see "Motivation (Abel / Arnold)" section below!

- (step 1) The zakon line
  - http://www.trillia.com/zakon1.html this is the prereq of the zakon-analysisI with no prereq of it's own.
  - http://www.trillia.com/d7/zakon-analysisI-a4-two.pdf, covers: 4,1 (related course at http://www.saylor.org/courses/ma241/?ismissing=0&resourcetype=2), http://mathdl.maa.org/mathDL/?pa=content&sa=viewDocument&nodeId=3228
  - from http://www.trillia.com/zakon-analysisI.html
  - from http://www.trillia.com/online-math/general-mathematics.html
  - from http://www.trillia.com/online-math/index.html (EXCELLENT RESOURCE!!!)
- (step 2) http://www.trillia.com/zakon-analysisII.html, covers 2
- (step 2.5) Multivar calculus (Spivak?), then Differential Forms: Integration on Manifolds and Stokes's Theorem, by Weintraub (which has prereqs: lin alg and multivar calculus)
- (step 2.5 alt) "Differential Equations, Dynamical Systems and Linear Algebra by Smale" including prereqs?
- (step 3) http://www.mat.univie.ac.at/~gerald/ftp/book-ode/index.html (http://www.mat.univie.ac.at/~gerald/ftp/book-ode/ode.pdf), covers 3
- (step 4) http://www.trillia.com/online-math/algebra.html (TBD after choosing our heffron book), covers 5,6
- step 4.1
  - Abel's Theorem in Problems and Solutions (Arnold)
  - Ordinary Differential Equations (Arnold)
  - Geometrical Methods in the Theory of Ordinary Differential Equations (Arnold)
  - Lectures on Partial Differential Equations (Arnold)
- step 4.2 (The spivak line)
  - Calculus
  - Calculus on manifolds (This also needs lin alg as prereq)
  - A comprehensive introduction to differential geometry
  - Differential Forms: Integration on Manifolds and Stokes's Theorem, by Weintraub (which has prereqs: lin alg and multivar calculus)
  - http://www.amazon.com/Physics-Mathematicians-Mechanics-Michael-Spivak/dp/0914098322
- Complex analysis? #1 A Course of Modern Analysis. An Introduction to the General Theory of Infinite Series and of Analytic Functions; with an account of the Principal Transcendental Functions (Whittaker!)
- Geometry ??
  - Geometry [Englisch] [Taschenbuch] David A. Brannan ?
  - Felix Klein books? (Erlangen program)
  - Hilbert, The foundations of geometry
- For leisure reading: Huygens and Barrow, Newton and Hooke (Arnold again)
  - Among these are *Numbers and figures* by Rademacher and Töplitz, *Geometry and the imagination* by Hilbert and Cohn-Vossen, *What is mathematics?* by Courant and Robbins, *How to solve it* and *Mathematics and plausible reasoning* by Polya, *Development of mathematics in the 19th century* by F. Klein.
- Jump from this into numerical solutions: Solving Ordinary Differential Equations I: Nonstiff Problems and Solving Ordinary Differential Equations II: Stiff and Differential-Algebraic Problems from Hairer seem to be great.
- Next in line is time-scale calculus and related discrete mathematics topics (http://en.wikipedia.org/wiki/Time_scale_calculus http://en.wikipedia.org/wiki/Discrete_calculus http://en.wikipedia.org/wiki/Abstract_differential_geometry http://en.wikipedia.org/wiki/Discrete_mathematics)
- A very big missing topic is 'Algebraic geometry', and this seems to be quite a graduate topic. Still we MUST learn it. The prerequisites of this are at the very least "a one-year course in algebra and a little complex analysis", according to 'Elementary Algebraic Geometry (Kendig)'. The MUST was revealed when finding Coolidge's 'A Treatise on Algebraic Plane Curves' absolutely unreadable. The book is a goal book, as suggested by 'Unknown Quantity's' extracted book list. (The subpage 'Algebraic Geometry books' might be helpful).

Then on to the (step 5) Methods book and after than the (step 6) variational book.
See me in ~~5~~ 15 years.

Relevant book authors:

- Elias Zakon
- Michael Spivak
- VI Arnold
- Steven H. Weintraub
- Whittaker

Papers i would like to understand thanks to finishing the plan:

- the existence of non-collision singularities in newtonian systems (zhihong xia)
- When Chaos Meets Computers (Shujun Li)
- Computer arithmetic, chaos and fractals
- LINEAR CHAOS? Nathan S. Feldman
- http://www.scholarpedia.org/article/Hamiltonian_Systems
- basically the plan in 'short' form in one book/pdf: http://www.das.uchile.cl/~rmendez/Documents/Roger_Penrose_Road_to_reality.pdf
- Geometrical Methods In The Theory Of Ordinary Differential Equations (Arnold, again)
- http://www.scholarpedia.org/article/Hamiltonian_Systems (all articles in physics and dynamical systems sections)
    - http://gams.nist.gov/
- The 'Rotation tensors' section in 'intermediate dynamics for engineers', this requires 1.(multivar calculus+lin alg) then 2.(tensors, tensor calculus, which are outside the 'plan')
- http://projecteuclid.org/DPubS?service=UI&version=1.0&verb=Display&handle=euclid.cmp/1104203302 (Discrete versions of some classical integrable systems and factorization of matrix polynomials), a reference in Lacrousiere's thesis.
- Completely understand the banach tarski paradox, the motivation is at least a full understanding of the related axiom of choice, and the whole foundational crisis wrt cantor's diagonal argument, which I seem to (in my enfantile view) disagree with and hence bleong to the opposition that lost the battle ... see (Mathematical Infinity, Its Inventors, Discoverers, Detractors, Defenders, Masters, Victims, Users, and Spectators by Belaga and all other related documents in the library).
  (http://plato.stanford.edu/entries/set-theory/)
  This is very important not only to simply understand, but a deep understanding of this is really at the root of a real understanding of analysis (all types of it) and therefore allow finally freedom in dealing with the non discrete number systems.
- http://www.cs.cmu.edu/~scandal/alg/nbody.html, http://www.codee.org/software/
- http://arxiv.org/abs/0705.3902 (A Derivation of Einstein Gravity without the Axiom of Choice: Topology Hidden in GR)
- http://www.ihes.fr/~gromov/topics/SpacesandQuestions.pdf and many others mentioned at: http://mathoverflow.net/questions/2144/a-single-paper-everyone-should-read-closed
- A list of additional papers is also maintained in my library.
- “An introduction to Lie group integrators { basics, new developments and applications” (Elena Celledoni, Håkon Marthinsen, Brynjulf Owren) (http://arxiv.org/abs/1207.0069).

also required:

- a jump into algebra in a clearly motivated way: 'abels theorem in problems and solutions', exists as pdf uncluding solutions, and seemingly great path into nit being clueless about abstract algebra'
- Understand the mathematics behind the axial precession of earth (http://en.wikipedia.org/wiki/Axial_precession_%28astronomy%29)
-

nice to have:

- http://www.geometricalgebra.net, Geometric Algebra for Computer Science (Revised Edition): An Object-Oriented Approach to Geometry (The Morgan Kaufmann Series in Computer Graphics) [Hardcover]
- nice to measure understanding: http://www.scribd.com/doc/26818237/GOLDSTEIN-Classical-Mechanics (famous physics textbook)
- *philsci-archive.pitt.edu/4916/1/curiel-cm-lag-**not**-ham.pdf   (Classical Mechanics Is Lagrangian; It Is Not Hamiltonian†) An excellent almost non-technical 'mother structure' read.*
    - another version is: *strangebeautiful.com/papers/curiel-cm-lag-**not**-ham.pdf*
- http://www.eulerarchive.com/ , http://www.math.dartmouth.edu/~euler/docs/originals/E177.pdf (E177 -- Decouverte d'un nouveau principe de Mecanique) (among a thousand others)
- http://www.mrao.cam.ac.uk/~clifford/publications/abstracts/imag_numbs.html
- Note that Whittaker's 'Calculus of Observations' is probably an excellent entry point to numerical analysis!!! and it is from Whittaker again ...
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Post plan:

- Deep Down Things: The Breathtaking Beauty Of Particle Physics
- Fully understand the components of this thought:
  "Physics: The Wave Structure of Matter (WSM) explains Huygens' Principle

  Can we visualize a 3D universe which is finite yet unbounded? (Einstein, 1954)

  In fact it is possible for a finite spherical universe to form within an infinite Space. Unfortunately for Einstein, he incorrectly imagined a 'curved space' such that if you traveled far enough you would return to your starting point! (A rather abstract and confusing idea!) The solution is far more simple and sensible, and is found instead from Huygens' Principle.

  Christiaan Huygens found that a surface containing many separate wave sources appeared, from a distance, as a single wave front with the shape of the surface. This wave front is termed a 'Huygens combination' of the separate waves. This explains how matter's spherical In-waves are formed. The Out-waves of other SSWs (spherically distributed in the Space around us) combine to form a Huygens 'combination wave front' which forms the spherical In-wave of our wave-centers. (See diagrams below)

  Thus Smolin is correct;

  It can no longer be maintained that the properties of any one thing in the universe are independent of the existence or non-existence of everything else. It is, at last, no longer sensible to speak of a universe with only one thing in it. (Smolin, 1997)

  Most importantly, this Huygens' sharing of waves means that once you go out past a certain distance in Space (the size of our finite spherical universe) you can no longer count the Out-waves of farther distant matter as direct contributors to our In-waves, as these waves have already become part of closer wave-center's In and Out-waves and thus have already been counted as contributing to our In-waves. This sharing of waves is a profound discovery (Wolff, 1994) as it leads to the solution of the connection between the infinite and the finite, explains Mach's Principle, and explains the redshift with distance without assuming an expanding universe." (http://www.spaceandmotion.com/Physics-Christiaan-Huygens-Wave-Theory.htm)
- Physics, Topology, Logic and Computation: A Rosetta Stone (http://arxiv.org/abs/0903.0340)
  "In physics, Feynman diagrams are used to reason about quantum processes. In the 1980s, it became clear that underlying these diagrams is a powerful analogy between quantum physics and topology: namely, a linear operator behaves very much like a "cobordism". Similar diagrams can be used to reason about logic, where they represent proofs, and computation, where they represent programs. With the rise of interest in quantum cryptography and quantum computation, it became clear that there is extensive network of analogies between physics, topology, logic and computation. In this expository paper, we make some of these analogies precise using the concept of "closed symmetric monoidal category". We assume no prior knowledge of category theory, proof theory or computer science. "
- "Three-dimensional geometry and topology. 1 (1997)"
  3
  Thurston's book is also the best exception I know to my claim that the most amazing mathematicians don't write books any more. It's a very strange book, but I find it quite inspiring. – Ilya Grigoriev Jan 27 2010 at 21:15
  2
  I agree with all of this, but especially the mention of Thurston's book (it was only a bound set of notes distributed by the Princeton math department when I used it). What beautiful and inspiring stuff. – Deane Yang Jan 28 2010 at 0:57
  1
  Thurston's book is a deep,awesome,infuriating read and it HAS to be on the must read list of any mathematics or physics graduate student beyond the first year courses. – Andrew L May 28 2010 at 22:40

Here is a more detailed study plan as of Dec. 2012, copied from an email I just sent to Tom and Vlad.

> I spent the last two days deciding in more detail what to study next, and I have reached some almost definitive plan that I quite like after some failed attempts.
> The plan divides the studying into 3 groups, the basics group A, and two more advanced groups B,C that both depend on A but not on each other (not much).
> group C is related to physics and not mathematics.

I will first explain what each group contains, then the dependencies and rationale.

Group A:
1.* Book of Proof (Hammack)
2.* Linear Algebra (Hefferon)
3.^ Analysis Basics, Analysis I, Analysis II (Zakon), in parallel, support b) which starts 'in reverse' and reaches 'foundations of classical analysis' at page 172, which is where Zakon's book starts.
4. Calculus on Manifolds (Spivak)
5. Elements of Number Theory (Stillwell)
6. Elements of Algebra (Stillwell)
7. Classical Topology and Combinatorial Group Theory (Stillwell)
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a.* Journey Through Genius (Dunham)
b. Analysis by it's History (Hairer)
c. Naive Set Theory (Halmos)
d. Real Analysis (Royden)
e. Mathematics and It's History (Stillwell)
f. The evolution of Group Theory, A Brief Survey (Kleiner)
g. The Calculus, A Genetic Approach (Toeplitz)
h. The Theory of Determinants, In the Historical Order of Development (Muir)
i. Number Theory, an Approach Through History (Weil)
j. An Introduction to the Theory of Numbers, (Hardy)
k. A Course of Pure Mathematics (Hardy)
l. Number Theory and it's History (Ore)
m. Institutiones calculi differentialis (Euler, translated by Blanton)
x. (Wittgenstein)

Group B:
1. Abstract Algebra (Dummit), or Maybe Algebra (Saunders Mac Lane, Garret Birkhoff) as recommended by [UQ] instead of Van der Waerden's more 'graduate' book?
2. A comprehensive guide to Differential Geometry, All Volumes 1-5 (Spivak)
3. Differential Equations, Dynamical Systems, and Linear Algebra (Smale), What about the recent discovery: linear differential operators, Lanczos, just read the preface!!
4. Elements of Functional Analysis (Hirsch) (not sure yet)
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a. A History of Geometrical Methods (Coolidge)

Group C:
1.^ From Euclid to Newton (Whittaker)
2.^ A History of the Theories of Aether and Electricity (Whittaker)
3.^ A Treatise on the Analytical Dynamics of Particles and Rigid Bodies, with an Introduction to the Problem of Three Bodies (Whittaker)
4. Mathematical Methods of Classical Mechanics (VI Arnold)
5. Foundations of Mechanics (Abraham)
6. The Variational Principles of Mechanics (Lanczos)
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a. Concepts of Force, (Jammer)
b. A History of Mechanics (Dugas)
c. The Road to Reality (Penrose)
d. Light and Matter (Crowell)

Group D:
http://mathoverflow.net/questions/7957/books-well-motivated-with-explicit-examples
Visual Complex Analysis, Needham
Three-dimensional geometry and topology, Thurston
*Cox book Primes of the Form x^2+n*y^2*
Applied Analysis, Lanczos

.* means already finished.
.^ means already touched.
books with numbers are the actual books that will be studied.
books with letters are support books, usually historical.

Group A contains what we already are studying, Group C (except for 1 and 2) are the actual unreachable goals.

The reasons for A.1,2,3,4 are obvious.

A.5,6,7 are all transition books that are crucial as a bridge to studying the more advanced books in the other groups. They are not advanced, relatively short (<300 pages), and historically motivated, which is always crucial for me. Those books were the hardest to choose, since they are not goal books, nor obvious introductory topics, but somewhere in between, but I still wanted them to provide the 'why did we abstract this way? what were the problems that we wanted to solve?'. These questions I already have for my goal books, and my starting doubts that triggered the journey, but they do cover only a very small part of the 'why's' that are behind the bridge section. I found Stillwell to have the exact attitude I need, this shows in the content of the books, the prefaces, and that he has written 'Mathematics and it's History'. The idea is to be able to choose later more advanced books that do not need to be historically motivated anymore, like Abstract Algebra (Dummit).

The number theory book if wanted could be followed by: 'Fundamentals of Number Theory, Leveque' or 'Not always burried deep: A second course in elementary number theory, Pollack'

If I ever finish this, I will be a very pleased person for at least one day.

notes about C.4

1. "and though much less can be said about these in general, their geometric and symmetry properties can be well understood within the variational framework. This is especially true of the Lagrangian formulation and the Hamiltonian form of d'Alembert's principle of vanishing virtual work, the variational principle which
is used throughout this thesis. True, the mathematical apparatus required to use the variational formulation is more complicated than the simple vector analysis needed in analyzing Newton's three laws. The prevalence of di®erential geometry, di®erential forms, Lie groups, Lie algebras, Lie derivatives, and other tools of global analysis can be daunting, especially when several results can be derived with more pedestrian mathematical techniques, though with devilish cleverness (see the ¯rst few chapters of [22] for example). But there is a unity to the variational method which is not apparent in the Newtonian formulation, and several results which would presumably be out of reach without the bene¯t of global analysis. Of course, working backward from the answers provided by variational methods and comparing with previously known methods can provide considerable enlightenment and understanding as to what went wrong, and perhaps ideas of how to ¯x things without necessarily using all the results." (Lacoursiere, Ghosts and Machines, referring to [22] V. I. Arnold, Mathematical Methods of Classical Mechanics,)
2. "It might seem arti¯cial to look at the stability of the simple harmonic oscillator. However, as is well known in mechanics (see [105], Chapter 6, or [22] Chapter 5, and [174] Chapter V), simple harmonic motion is the most important dynamics near any equilibrium point of any system. For a complicated system, the spec- trum of frequency corresponds to the eigenvalue spectrum of the Hessian of the potential function ....,.which is expected to have vanishing ¯rst derivative at an equilibrium point ."- , say. When integrating at ¯xed time step, the danger is always present that one of the natural frequencies has a period commensu- rate with the time step and thus, it is always possible that the system become suddenly unstable. An a priori analysis of all possible natural frequencies of a system is prohibitively expensive in most cases." (Lacoursiere, Ghosts and Machines, referring to [22] V. I. Arnold, Mathematical Methods of Classical Mechanics,)

Analysis goals:
The goal is to completely demystify analysis, and this is impossible by studying classical analysis alone, nor without studying philosophy.
Studying intuitionism plays a crucial comparative role. A center figure in the comparative study is Solomon Feferman, who writes along about this, The idea is to follow the document I wrote '', but to be stated simply, one goal is totally understanding this:
'The main point of this section is to explain, without requiring a full exposition of constructive measure theory, how it is possible that Bishop's measure theory could be consistent with Church's thesis, in spite of the singular covers described above.'
For some context: 'Logicians labored in the subsequent decade to analyze what Bishop had done, by constructing suitable formal theories and studying their formal inter- pretations. This work is summarized in [1]. These studies verified (for various formal theories) that Bishop's work is indeed consistent with Church's thesis as well as with classical mathematics, and is constructive in the sense that "when a person proves an integer to exist, he or she can produce that integer". This is reflected in the "numerical existence property" of a formal theory T: if T proves ∃xA(x) then for some numeral n¯, T proves A(n¯).' ((Constructivity, Computability, and the Continuum, Michael Beeson))