On Trick and Method

Note. Once a method is finally given by someone. It sometimes goes on to be considered as ‘revolutionary’. But whether this honorary title is granted or not, does not depend on the difficulty of finding it, nor on any other characteristic. It depends solely (and sadly) on the its trivialization power. A method is a *trivializer* when it allows the solution needing mathematical layman (engineer) to solve certain problems, without understanding their mathematical depth, with no thinking effort, and more importantly, no searching effort, by directly using the method like a tool, using a procedure that can be successfully trained by repetition. For the reasons above, these methods are ‘revolutionary’: Analytic Geometry, Calculus, Fourrier Series, Fast Fourrier Transform, Linear Algebra. While the following are not: Algebraic Geometry, Lagrange’s method for solving algebraic equations, etc.

Note. It is always the case that context specific tricks are more efficient and simpler then the methods themselves. methods always requiring more advanced concepts. Maybe it is true then, that tricks are the best way to ‘get started’.

Note. Surprising is the amount of research and investigation Cardano gave the topic in his ‘Ars Magna’. The kind of investigations he embarks on do cross the mind when thinking about roots and are indeed methodical: Starting from simple polynomials like x3=a to more complex ones gradually. ‘Ars Magna’ contains innumerable pages of toil and painstaking (even if prolix) explanation, in which Cardano moves from the simplest to the most general in as tiny steps as possible. It becomes almost shameful to be able to derive the solution so easily nowadays. However, like with all historic research, it is important to remember that what he was working on was very significant at the time. We could compare it to working on mathematical models of the Big-Bang today. For him, this was no drudgery involved, every step leading to newly discovered (or invented for non-Platonists) mathematical landscape.

Note. On the meta level, if there be any method, then it is one of philosophical stance, of wanting to know why, and of extremely good observation. This is demonstrated by the Greeks, and is explained (by Heath) to be the main reason why the Greeks were able to do what they did (using this ‘method’) and other civilizations did not. In my opinion, the same can be projected to the level of an individual.
It seems unreasonable that the Greeks, for some reason, were more exact observers than other civilizations. Therefore, the right explanation is that the Greeks, by virtue of their philosophy (or by other means) learned the extremely high value of exact observation. The advantage of this point of view is that such an attitude can be controlled (and learned) on the individual level.
For references see the section ‘Greekomania’ in my ‘Notes on Euclid’s Elements’.

Examples

Diophantus and Number Theory. Resisting methodical analysis by Euler, much later brought into light with Algebraic Geometry.

> *"Also during the fifties, Euler studied Diophantus carefully, hoping to bring some order into what had all the appearances of a haphazard collection. He did succeed in isolating a few of the Greek geometer's favorite tricks, but otherwise the result of his effort was disappointing; only the later developments of algebraic geometry were to throw light into a subject which was not yet ready for even partial clarification"* [1]

Lagrange and Roots of Polynomials.

Studying the books ‘Elements of Algebra’ and ‘Analysis by its History’, solutions to quadratics and cubics appears at the beginning of both. Both books derive the solution to the cubic in slightly different ways both either using a ‘trick’ or ‘a bit of visual help’. But I was able to scribble a derivation that does not need either, after being stuck on a bit of creativity: completing the cube versus completing the square involves not directly finding the residual u in $(x+u)3-u3=d$, but seeking another simple relation between $(x+u)$ and u, which is easily obtained by comparing terms. This slight difference between the the idea for the quadratic and generalizing it to the cubic got me stuck for days. What I learned is an additional mode of thinking to add to the ‘creative’ repertoire.

I cannot once again help but wonder if there is methodical way that unites all solvable degrees. If not, then creativity and context specific observations are unavoidable in this subject.

> *“In his paper Réflexions sur la résolution algébrique des équations ("Thoughts on the algebraic solving of equations"), Joseph Louis Lagrange introduced a new method to solve equations of low degree.*
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> *This method works well for cubic and quartic equations, but Lagrange did not succeed in applying it to a quintic equation, because it requires solving a resolvent polynomial of degree at least six.[19][20][21] This is explained by the Abel–Ruffini theorem, which proves that such polynomials cannot be solved by radicals. Nevertheless the modern methods for solving solvable quintic equations are mainly based on Lagrange's method.[21]*
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> *In the case of cubic equations, Lagrange's method gives the same solution as Cardano's, where the latter may seem almost magical to the modern reader. But Cardano explains in his book Ars Magna how he arrived at the idea of considering the unknown of the cubic equation as a sum of two other quantities, by drawing attention to a geometrical problem that involves two cubes of different size. Lagrange's method may also be applied directly to the general cubic equation (1) without using the reduction to the trinomial equation (2). Nevertheless the computation is much easier with this reduced equation.*
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> *Suppose that $x0$, $x1$ and $x2$ are the roots of equation (1) or (2), and define , so that ζ is a primitive third root of unity which satisfies the relation . We now set ...*
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> *This is the discrete Fourier transform of the roots: observe that while the coefficients of the*

polynomial are symmetric in the roots, in this formula an order has been chosen on the roots, so these are not symmetric in the roots. The roots may then be recovered from the three si by inverting the above linear transformation via the inverse discrete Fourier transform, giving ..." [2]

Johan Mueller's Suspended Rod Problem. A minimization problem is solved without using Calculus. (TODO: see ‘Box of Shame’)

References

1. Andre Weil. Number theory an approach through history.
2. http://en.wikipedia.org/wiki/Cubic_function
3. Gerolamo Cardano. Ars Magna.