The Dilettante's Road to Differential Forms

I Summation Convention and Vector Algebra

1. Go through [@c1] and work all examples as exercises.

2. You can skip example 2.15.

3. We should copy and process hand-written notes from print-out and exercises [1].

4. In the beginning of section 2.4, we meet the position vector and the unit vector. Would it be useful here to add a section on 'what is a position vector on a torus'?.

II Forms on $\mathbb{R}^n$

5. Use the summation convention and the (generalized) Levi-Civita symbol, which gets us at least up to the last paragraph of 'What is a form'. Does it also work for the last paragraph?

III The Infamous 'd'

6. This is titled 'Derivatives of forms' in [@c2]

7. In the proof of Theorem 2.1. we use

$$\partial_j \partial_i f = \partial_i \partial_j f.$$

This makes the vector calculus theorem on mixed-partials a prerequisite [2].

IV Pullbacks

IV.1 Notes

8. Does it make sense to ask and derive the 'set theoretical type' of α , in 'α is a k-form on Y'?

[1] TODO

[2] dangling, use our 'Mixed Partial Derivative Intuition (join)' and [@c3]

9. The author mentions 'tangent vectors' without introducing them. Maybe that's no problem since they are not 'used'.

10. At the end of the section, the author tells us that this is nothing but changes of coordinates. would it be good to take a Torus as an exercise and find the pullback? Even if we do this, we are still having ice cream first. We can introduce the Torus in the minimality necessary and not more.

11. Probably, the properties of the given 'g^*' are the 'essence' of a change of coordinates, but this is not obvious, can it be made so?

12. Finally, why do we dangle the pullback? what will be its use? Probably, invariance under change of coordinates using the properties of the pullback? Yes, that's what the following section talks about with

$$\int_U g^*\alpha = \int_V \alpha$$

. Maybe we should subsume the two sections and explain how integration is a lot about invariance under change of coordinates, as explained in the [@c4]?

IV.2 Integration

13. At this point, solving chapter I of [@c4] is due.

14. We find [@c5], it is golden!!

15. In [@c5] the author does not have a justification for $\mathbb{R}$. In fact, maybe a very god way to simplify all the mess is to explain that the 'problem' is that we never create a specific notation on top of set theory for ordered sets, in particular for an 'ordered cartesian product'. In set theory $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$ but that is all one can express. We cannot express order. The problem is already visible in the set-theoretic treatment of tuples which necessitates an encoding step. But we never use this on the level of set products. We need an 'ordered cartesian product' and in fact, we will not even use any product sign for it, but keep it the way we intuitive see it, as a list. Like in any list, each element has an index and this allows us to express the order succinctly. So then the ordered cartesian product of two $\mathbb{R}$ sets is $(\mathbb{R}_1, \mathbb{R}_2)$. We can use this for ordered 'differential vector tuples' obtaining expressions such as (dx_1, dx_2) . When we then bring in a linear functional to act on it, the way it acts is natural. The tuple is nothing but the 'argument' to the functional, in that order.

16. In footnote 8 of [@c5], we see the author giving more rigorous definitions 2-forms. This is because one cannot prove the proofs one needs within his informal treatment using infinitesimals. But that is what could hopefully be remedied, putting the infinitesimal 'model of axiomatic differential integration' on equal footing with the other more standard models?

17. About [@c5] equation 9, we already knew that the requirement of anti-symmetry can be obtained through the need for 'degenerate' or 'linearly dependent' forms to equal the 'zero form' and hence linearly map to the 'zero scalar'. In our 'comma based' treatment, the 'basis' (dx_1, dx_1) is not a basis for dimension 2 and hence, any 'vector produced from that basis' has less than dimension 2, and we 'require'? that all such vectors map to zero?

Bibliography

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