

<http://course1.winona.edu/KSuman/Dictionary/Fill%20Ins/Monster%20Barring.htm>

jac is so simple (not nec. square matrix)

closest to line cx , best c . $0.c=0$ true, $c=0/0$ true, but both undetermining for c , hence $0/0$ 'undefined'

all of alg, anal, number the, topol appear in a new light, when we compare on the high level how we talk in them and what their essence is and why they are mathematical and how they relate

Note: our 'problem' is not infy with $1/x$ but simply a point where the fuction div is not one to one. at least this (a specific sing point) drags in open intervals into analysis.

The 'fuction' is too restr to express all tge sol of duff eq, the diff traj are too diff, we need other ways, eg topol

Need for diff fuzzy/incompl analogies for limit in terms of fin arith ---> a diff concept not logically reduc. (axiom of inf???)

Do not forget the last notes! Add on paper. In this sense the deriv is a great classifier of where avg vel is meaningful. Integ for ???

'physic' <--> converg (has meaningf avgs). Sin nx cannot be: inf length start from zero. Exist of deriv (cpnvergnce) gives mean to avgs : aftwe some point we start getting moniton better / cauchy / consiatent. Need to determ starting at what point. The lack of cobverg is an avg vel disaster. Agaib avg vel for sin nx makes no sense so phys <--> conv (vaguely) Only wheb tgere is a deriv does diff 'have meaning' anyway since otgerwize no regul, no consistency (except for special use)

P.159 noteL finally hist of oriented angle

Note. the diff between convergence (series) and approximation gt2 example. elaborate

http://en.wikipedia.org/wiki/Finite_topological_space

Note. After all the reading, the limit appears as an elementary infiniterial concept, and analysis I as quite clear and 'basic arithmetic of the infinite'-like

Is it not a good idea to study MMCM as a book?

It seems that it is as the same time specific and example oriented and abstract and advanced. Instead of swallowing whole theories/books in their encompassing character why not this approach? seriously consider this!

p.259 'Espaces vectoriels topologique' is immensely important!!!!!!

p.250 attribution.

p.176: 'la possibilité d'une représentation géométrique de tout objet susceptible de variation continue ;'

p.170.13 the 'reality of eucl geom'. saved in p174/-1.

p.172, more invariants. many...

p.172/2 that is short and good.

note how true the 'simple' is. The 'third eye' seeing what is not there, was merely a problem of language? even if yes, that language had to be developed.
note** clarify the relationship to trigger the right motivation.

p.171 WOWW, this is an example of 'solved' despite infinity.

p.166 C) wow

p.165/-1 at least that is a motivation for proj. algebra!

Fwd: Important todo quote fomenko pg.75/76 !!!

Fwd: Put in zakon notes.

chap 50. is the missing chapter in our historic understandig of the generalizations in Zakon I

(e.g metric spaces, continuity in general, etc...) and its relation to our new fascination topology that seems so related and touching a deep but intuitive concept that needed a sophisticated formalism to be captured.

on the one hand abstracting manifolds from coordinates, and classifying them, also relating them : cube=sphere<>torus. \mathbb{R}^n is always there (and therefore \mathbb{R}^1 , our basic and essential formal understanding of continuity, first absolutely: gapless line as in Dedekind, then half-absolute in ϵ - δ , then general/relative: homeomorphism with ϵ - δ a hom. between a curve and \mathbb{R}^1)

Also add note about intersecting intervals being a technicality for the IVT

And the idea of distinguishing concepts and technicalities, will still allowing concepts to hinge on technicalities without discomfort

Fwd: At any disorientation in *zakon* due to overgeneralization read Kline chap. 50 (same for topology)

Fwd: God and Newton's absolutes

'Newton's conception of absolute time, however, was primarily influenced by Isaac Barrow's *Lectiones Geometricae* (1669), which he reportedly revised and edited in the same year in which Barrow resigned his chair as Lucasian professor at the University of Cambridge to Newton. Because Newton's conception of simultaneity is intricately connected with the notion of absolute time, and because his notion of absolute time is based on Barrow's philosophy of time, it is appropriate to discuss, in brief, Barrow's theory of absolute time. Barrow's philosophy of time appears to have been strongly influenced by the philosophy of space of his colleague, the Cambridge Platonist Henry More, who was fifteen years his senior. In particular, More's conception of space as the omnipresence of God must have greatly appealed to Barrow, who had been a theologian before his appointment as professor of geometry in 1662. In addition, More's anti-Cartesian separation of space from matter and his argument for the reality of space from its measurability, as presented in his *Antidote against Atheism* (1653), must have greatly appealed to Barrow, who as a mathematician declared repeatedly that the object of science is quantity. It is not surprising, therefore, that just as More liberated space from its Cartesian bondage with matter, so Barrow disjoined time from its Aristotelian conjunction with motion.'

Fwd: 'Analysis' and alg/numbers p.143

Bien que cela sorte quelque peu de notre cadre, il importe de mentionner ici que, grâce à une topologie appropriée sur le groupe des idéles, on peut ainsi appliquer à la Théorie des nombres toute la technique des groupes localement compacts (y compris la mesure de Haar) de façon très efficace. Dans un ordre d'idées plus général

Fwd: P.146.-5, p.147

'A topology'. 'Struct' like a ;group' that is axiomatic. Many 'ways to think about' diff than 'real-world' species? Remove that bias

Fwd: Abstract usefulness, it is recurrent

Toutefois, ces topologies restaient pour la plupart des algébristes de simples curiosités, en raison du fait qu'elles sont d'ordinaire non séparées, et qu'on éprouvait une répugnance assez compréhensible à travailler sur des objets aussi insolites. Cette méfiance ne fut dissipée que lorsque A. Weil montra, en 1952, que toute variété algébrique peut être munie de façon naturelle d'une topologie du type précédent et que cette topologie permet de définir, en parfaite analogie avec le cas des variétés différentielles ou analytiques, la notion d'espace fibré [330 el; peu après, Serre eut l'idée d'étendre à ces variétés ainsi topologisées la théorie des faisceaux cohérents, grâce laquelle la topologie rend dans le cas des variétés ((abstraites)) les mêmes services que la topologie usuelle lorsque le corps de base est \mathbb{C} , notamment en ce qui concerne l'application des méthodes de la Topologie algébrique [283 a et

Platonism by extrapolation, still 'work', still 'human' in presence, is truth? Still physical in application? Totally different point of view, the body of math. Retrospect?

Mind hates futile and magic when not unavoidable (time, prior, etc.)

Real analysis for autodidacts (self-learners) with little time

<http://www.doc.ic.ac.uk/~ae/papers/longo-f.pdf>

A computable approach to measure and integration theory

Abbas Edalat

http://link.springer.com/chapter/10.1007%2F978-88-470-0784-0_12

<http://people.clas.ufl.edu/cenzer/files/n57.pdf>

<http://www.bruce-shapiro.com/math582B/notes-052307.pdf>

<http://www.sciencedirect.com/science/article/pii/S0885064X07000246>

<http://www.jstor.org/discover/10.2307/2038240?uid=2129&uid=2&uid=70&uid=4&sid=21102877387707>

P119 conclusion. Note 'topology' vs order. Does it mean completeness?

Division as a classifier? P116,17

P115 cauchy eigenval + connections

P115.12 not euler??

P113. Group rel? Cyclic \leftrightarrow generator count

P113.canon form takes a more structural meaning, link back to invariants

P113. Alg integers \leftrightarrow dioph deg > 1 makes 'vague intuitive' sense now

Expl form for all pairs p.110. Form for all prims again a kind of analysis: finite descr for all such numbs. Exist? No? Structure. Is there fin str. How does inf str look like, inter between ops and rels

what's so special about eucl metris \leftrightarrow what' ss about radicals.

A la lumière des découvertes de Galois, on s'aperçoit que le problème de la résolution « par radicaux » n'est qu'un cas particulier, assez artificiel, du problème général de la classification des irrationnelles.

En ce qui concerne tout d'abord les irrationnelles algébriques, un principe fondamental de classification était fourni par la théorie de Galois, ramenant l'étude d'une équation algébrique à celle de son groupe.

Enfin, il aborde le problème des extensions algébriques de degré infini, et constate que la théorie de Galois ne peut s'y appliquer telle quelle (un sous-groupe quelconque du groupe de Galois n'étant pas toujours identique au groupe de l'extension par rapport à une sous-extension) ; et, par une intuition hardie, il songe déjà à considérer le groupe de Galois comme groupe topologique * - idée qui ne viendra à maturité qu'avec la théorie des extensions galoisiennes de degré infini, développée par Krull en 1928 El87 dl.

that is... groups \leftrightarrow topology.

algebra \leftrightarrow analysis

Parallèlement à cette évolution se précise la notion d'élément transcendant sur un co'rps. L'existence des nombres transcendants est démontrée pour la première fois par Liouville en 1844, par un procédé de construction explicite, basé sur la théorie des approximations diophantiennes [204 cl; Cantor, en 1874, donne une autre démonstration non constructive)) utilisant des simples considérations sur la puissance des ensembles [47]; enfin, Hermite démontre en

1873 la transcendance de e, et Lindemann en 1882 celle de π par une méthode analogue à celle d'Hermite, mettant ainsi un point final à l'antique problème de la quadrature du cercle **.

Nous arrivons ainsi à l'époque moderne, où la méthode axiomatique et la notion de structure (sentie d'abord, définie à date récente seulement), permettent de séparer des concepts qui jusque-là avaient été inextricablement mêlés, de formuler ce qui était vague ou inconscient, de démontrer avec la généralité qui leur est propre les théorèmes qui n'étaient connus que dans des cas particuliers. Peano, l'un des créateurs de la méthode axiomatique. et l'un des premiers mathématiciens aussi à apprécier à sa valeur l'œuvre de

Grassmann, donne dès-1888 ([246 b], chap. IX) la définition axiomatique des espaces vectoriels (de dimension finie ou non) sur le corps des réels, et, avec une notation toute moderne, des applications linéaires d'un tel espace dans un autre ;

A tous ces exemples de « corps abstraits » viennent encore s'ajouter, au tournant du siècle, des corps d'un type nouveau très différent, les corps de séries formelles introduits par Veronese [318], et surtout les corps p-adiques de Hensel [157 f]. C'est la découverte de ces derniers qui conduisit Steinitz (comme il le dit explicitement) à dégager les notions abstraites communes à toutes ces théories, dans un travail fondamental [294 a] qui peut être considéré comme ayant donné naissance à la conception actuelle de l'Algèbre. Développant systématiquement les conséquences des axiomes des corps commutatifs, il introduit ainsi les notions de corps premier, d'éléments (algébriques) séparables, de corps parfait, définit le degré de transcendance d'une extension, et démontre enfin l'existence des extensions

algébriquement closes d'un corps quelconque.

Note the 'attitude' and the pov. i would have never discovered the gcd, just like all the babs, eggs, etc. it is something i indeed wonder about myself and here it is. 'genius' can be so subtle. what 'is' it what 'is' the problem (polys in prev note)

Les opérations arithmétiques élémentaires, et surtout le calcul des fractions, ne peuvent manquer de conduire à de nombreuses constatations empiriques sur la divisibilité des nombres entiers. Mais ni les Babyloniens (pourtant si experts en Algèbre), ni les Égyptiens (malgré leur acrobatique calcul des fractions) ne semblent avoir connu de règles générales gouvernant ces propriétés, et c'est aux Grecs que revient ici l'initiative. Leur œuvre arithmétique, dont on trouve un exposé magistral dans les Livres VII et IX d'Euclide [107], ne le cède en rien à leurs plus belles découvertes dans les autres branches des mathématiques. L'existence du p.g.c.d. de deux entiers est démontrée dès le début du Livre VII par le procédé connu sous le nom d'algo-rithme d'Euclide $D \cdot$; elle sert de base à tous les développements ultérieurs (propriétés des nombres premiers, existence et calcul du p.p.c.m., etc.) ; et le couronnement de l'édifice est formé par les deux remarquables théorèmes démontrant l'existence d'une infinité de nombres premiers (Livre IX, prop. 20) et donnant un procédé de construction de nombres parfaits pairs à partir de certains nombres premiers (ce procédé donne en fait tous les nombres parfaits pairs, comme devait le démontrer Euler). Seule l'existence et l'unicité de la décomposition en facteurs premiers ne sont pas démontrées de façon générale ; toutefois Euclide démontre explicitement que

lin eq \rightarrow vec sp. desirable but untenable and mystifying strike of genius. no path.

Bias towards radical wextract algo? Why? Bias towards no loss of info? Extract dist. Roots of $x+y=3$. why the first bias? Once removed much clearer, but needs irrationals.

Add goal more than complex on reals \rightarrow non comm \rightarrow def of algebra motivation

Transform 'details' into 'non-details' stories, local essences.

The attribution is that of a spec meta idea. Eg poly radicals: dont get blinded by the 'could be anything' real numbers, 'most general' forn. That is what the ppl get \rightarrow the 'poly' is not the problem we are solving, meta-meta, class polys, sols of polys, Refine.

Fix fct $\langle \rangle$ inf dim vect!!!!

Refine! not all norms but all 1-norms. Are there others?

Fix fix, all norms are

More q's about practical \rightarrow direction: dirac, banach spaces insuff. ...

P.21

Specificity

Whenever there is some strange spec way for sthg: circle for angles, eucl dist for analysis. One should try to show if : only one (some iff) and why or which more general categ includes this and why

More excellent quotes

P.19/-2

Interest. [5], [27]

Attribution

P.19 note 22. New topic: idea vs logical/bold execution

Attrib

P12 note 10

Quote efgarov p13.1 applied \rightarrow pure trigger / (Hypocrisy)

P12/4 (euler) 'one trick even for euler? Hiding? Knowing? Third eye: what is not there. Genius?

Vq p.7

P.8.3

Prooquote p.6.1

P.9.-4 (feeble)

Wow: meta meta: p.11.1

All this expl motiv for ineq. Integ ineq

Intuition / lang replacement after first level

Mem

Rapid succession pair memorisation

Music pairwise chain mem

Topic : pairwise chain but also topic->element pairs, not simply 'many elements' use el pairs for disambig?

Add note!

Examples of parallel process: music that is good / intricate -> simple : composer view / multiple instrument simul, long term patterns

Of course inf processes analysis

Important: combine visual parallel AND verbal description -> yet another more powerful hybrid multi-parallel processor

Not only what can be finitely said about ..., but also some contradiction free 'claims' about exist that are logically neither true nor false (axiom of union)

Existence!!! Enough for analysis (axiom of union for D , of an upper bound for CB)

Existence, determination ($\sqrt{2}$), verification, numerical computation.

http://en.wikipedia.org/wiki/Axiom_of_union

<http://ncatlab.org/nlab/show/axiom+of+union>

understand/know/trust/split (req. schyz) --> fast path .. genius

Erratum

Hairer.p207.13 [0.1] should be [0,1]

add visual thinking book to library

The writers op also applies to polished math

metric quotes

Mention should also be made of Bianchi's (1898) impressive classification of three-dimensional geometries, in which he used Lie's classification of the groups that can act on three-dimensional spaces to describe all the metrically different spaces there can be. These turned out to include several examples of spaces other than the familiar three (Euclidean, non-Euclidean, and spherical). The paper was a significant influence on Woods. Much later it was rediscovered by Abraham Taub (1951) and gave rise to the Bianchi cosmologies of modern general relativity theory. Even later, in the 1970s, the Bianchi geometries were again independently rediscovered by Thurston among his eight model geometries in three dimensions.⁴²

metric quotes

Plato's Ghost: The Modernist Transformation of Mathematics, pdf (p.222)

Thus in an unpublished manuscript on 'Formalism' of 1904 Hausdorff called upon mathematics to free itself from any kind of intuition, and in a lecture course of 1903-1904 he said:¹¹¹ 'Mathematics totally disregards the actual significance conveyed to its concepts, the actual validity that one can accord to its theorems. Its indefinable concepts are arbitrarily chosen objects of thought and its axioms are arbitrarily, albeit consistently, chosen relations among these objects. Mathematics is a science of pure thought, exactly like logic.'¹¹² Hausdorff's view quite generally was that no aspect of science can have a unique description, and mathematics should explore all possible descriptions. This is much more radical than Poincaré's conventionalism, which Hausdorff seems to have first heard about sometime after coming to his own position.

After 1911 Hausdorff's interests in mathematics deepened. He worked extensively on set theory, the first to take it up so energetically since Cantor, and his major work, important for both set theory and topology, is his *Grundzüge der Mengenlehre* (1914), very much revised when reprinted in 1927. In 1915 Hausdorff published what became known as his paradox.¹¹³ It concerns any definition of measure that satisfies Lebesgue's four axioms. He assumed, in order to derive a contradiction, that every bounded set is measurable. He took the

sphere in three-dimensional space and constructed a division of it into four disjoint sets A, B, C, D, with the property that A, B, C, and $B \cup C$ are all congruent, while D has measure zero. Because the sets A, B, and C are congruent, they have equal measure, and because they cover the sphere except for the set D of measure zero, each must have measure $1/3$. But the sets A and $B \cup C$ are also congruent and cover the sphere except for the set of measure zero, so they must have equal measure, which must therefore be $1/2$. It follows that the set A has measure $1/2$ and measure $1/3$, which is impossible. Accordingly, the assumption that there is some definition of measure according to which every bounded set is measurable is false.

More precisely, one must either agree that that's how it is—there will always be nonmeasurable sets; or one must reject the concept of measure and look for another concept altogether—but Lebesgue's four axioms are very natural ones; or one must scrutinize the proof and hope to find a flaw. This was how Borel reacted. Hausdorff had used the axiom of choice in his proof, and in the second edition of his *Lçons sur la thorie des fonctions* (1914) Borel concluded that the paradox came about not because measure was an inherently flawed concept, but because the set A was not properly defined. To construct it using the axiom of choice was for Borel no construction at all. "If one scorns precision and logic," he wrote, "one arrives at contradictions."¹¹⁴ Hausdorff, of course, begged to differ. He was not at all bothered

that any definition of the area of a set is inherently imperfect, even though this was a conclusion that could never have been dreamed of by researchers a generation earlier.

At almost the same time, and as part of the same movement among French mathematicians, Maurice Fréchet produced a similarly abstract reformulation of a more technical problem area in mathematics that was to have a decisive influence on the creation of modern topology.¹¹⁵ Fréchet was interested in the calculus of variations, a topic in which the unknown is usually a function that is required to minimize a certain integral. Examples include finding geodesics in a space, surfaces of least area with given boundaries, and harmonic functions, but many problems in physics can be expressed in the language of the calculus of variations. However, reliable techniques for solving problems of this sort had proved hard to find. It was difficult to tell a minimal value from an extremal value, and therefore a stable solution from an unstable one and the complicated theory that existed was generally agreed to be in need of improvement. Hilbert had raised it as the last of his famous Paris problems, remarking that it was "a branch of mathematics repeatedly mentioned in this lecture which, in spite of the considerable advance Weierstrass has recently given it, does not receive the general appreciation which, in my opinion, is its due."¹¹⁶

One technique much employed in contemporary calculus of variations was to find a sequence of successive approximations to the sought-for function. Fréchet proposed a simple analogy. Just as one might approximate p , say, by finding among all numbers a sequence of successive approximations to it, so one might situate the successive approximations to a sought-for function in a space of all plausibly relevant functions. There might indeed be many such spaces, depending on the properties of the functions involved, but a much graver difficulty facing Fréchet was that any such space was surely infinite, indeed infinite-dimensional, whatever that might mean. Undaunted, Fréchet proposed to show that in many contexts one could introduce a sense of distance in a space of functions and speak of a sequence of functions in this space as tending to a limit, exactly as a sequence of approximations to p tends to p . What were functions in one setting became mere points in a space in Fréchet's new view of things.

Fréchet succeeded in showing that there were many problems in the calculus of variations that could profitably be formulated his way. He found spaces of functions of various kinds, and spaces of all curves of certain kinds could be made into metric spaces: spaces in which it made sense to speak of the distance between two points, and where a limiting process could be defined. Most importantly, he showed that these spaces could be complete¹¹⁷ which means that if a sequence tends to a limit then the limiting value is also in the space.¹¹⁸ Thus in Fréchet's vision, the concept of distance is greatly generalized away from any sense of distance in any Euclidean space. Moreover, the drive was completely axiomatic in spirit. Fréchet was clear that

what was wanted was the ability to talk of distance and to take limits, in the abstract senses he had in mind, and one had this ability whenever a space satisfied certain axioms. Initially he talked of spaces of class D (or distance spaces) where it made sense to talk of distance. These spaces were later called metric spaces, admitting a distance, by Hausdorff. Then Fréchet introduced spaces of class V (for *voisinage* or neighborhood in French) where it made sense to talk of points being close, which he soon realized were equivalent to spaces of class D. Finally, Fréchet spoke of normal spaces of class V when it is possible to talk of limiting processes converging to an element of the space.

111 I am indebted to Leo Corry (2006) for calling these remarks to my attention; he notes that they come from Purkert 2002.

112 This remark recalls Poincaré's statement that mathematicians study not objects but the relations between objects; see §3.2.

113 Hausdorff 1915.

114 Borel 1914, 256, quoted in Moore 1982, 188, who also notes that a number of Italian mathematicians had already explicitly rejected the idea that one can make infinitely many arbitrary choices: Peano, Bettazzi, and Beppo Levi among them; see Moore 1982, 76–82.

115 Bearing in mind later discussions of Esperanto, I note that Frechet was a lifelong enthusiast for the language.

116 Hilbert 1901 a, in 1976, 29.

117 Strictly, sequentially compact, but the details need not concern us.

118 This need not be true: any sequence of rational numbers tending to p has a limit (p , of course), but

p is not a rational number, so it is not an element in the set of rational numbers.

Detail

Logically there is no reason to stop at details, not capacity rel otherwise no new non-details either, probably histor/habit educ

Detail is about repr. and this has to be handled, after that limits are greatly expanded

Think about deriv integ diff eq func, poly, integ, real ... Think about rel between creatures, structure, the symbol in between is the least thing to tinker about, has to be mastered nevertheless as a language

Express the rel between anal of seq of funcs and vector spaces (not finite dim), Also the rel between funcs anal and multivec anal (finite dim)

Metric

If at the essence of 'approach indefinitely' is the lim of a seq or ϵ delta for contin, etc... at the essence of this, is the tr. Ineq.

We can immediately raise this rhetoric. Idea to an abstr. lev, by defining a metric and especially the tr. Ineq.

There is the interf. between the specific need from approx process, to its formal expr, once we are in the formal domain it is a diff game. There should be the phil and math quest of the corresp between the two (take a spec ex) but once this is done we are good to go

The equiv of the certain metrics should be emphasized a lot, abs value comes back with all its intuitiv my idea of 'removing the sign by squaring comes back from the past' free analysis from pyth and euclid metric, not a logical necessity

Once the abstract view of our rhetoric of coming closer is realized, the way is also free for multivar calc, this was an important reservation, now solved

Elementary particles the simplest creatures: determining quantities? Logically Ranges of random. Also fields logically no need for spatial locality.....

The special case of orth force again. Two particles rot. No work but force
Because force orth. Momentum as a vec. Changes but its mag does not change that is one possibility for force doing no work, the other is force causes no change in mom. In other cases the mag of mom must change: only can happen in non orth (proj not zero) case

Adopt several specific high level rel views of real anal

Hairer, zakon, mine

On a lower level, to retain also details of proofs must embody fabric of functions and first retain hinge points just like simple lever proofs can be retained

Analyze the Jacobi quote at the start of AbH

Is this fair? Introduce at same time to all? What if ready next year: exp. Or iq? Fixed? 10 years later? Mortality,

[illegible]