Measure in Short

I Topology

I.1 Set limits and Toplogy, Nebenform I

$$\text{[Limit of open-set intersection]}^{1}: \quad \mathcal{T}^{\exists}: \; {}_{\mathcal{C}}\Big[\bigcap \mathcal{O}_{\mathcal{T}}^{i/\mathbb{N}}\Big]^{\exists} \;\Big\|\; \mathcal{O} \xrightarrow{\cap\mathbb{N}} (\mathcal{O} \vee \mathcal{C}) \tag{1}$$

$$\text{[`Limit of open-set intersection' complement]}: \quad \mathcal{T}^{\exists}: \; {}_{\mathcal{O}}\Big[\bigcup \mathcal{C}_{\mathcal{T}}^{i/\mathbb{N}}\Big]^{\exists} \;\Big\|\; \mathcal{C} \xrightarrow{\cup\mathbb{N}} (\mathcal{C} \vee \mathcal{O}) \tag{2}$$

$$\text{[Set limits and Topology]}: \quad (\mathcal{O} \vee \mathcal{C}) \underset{\cup\mathbb{N}}{\overset{\cap\mathbb{N}}{\rightleftarrows}} (\mathcal{O} \vee \mathcal{C}) \tag{3}$$

I.2 Set limits and Toplogy, Nebenform II

Based on [cont_short] and using its notation, with the idea from [tWoA] that limits turn strict inequalities into non-strict inequalities.

$$\text{[Open is almost (limit) closed]}: \quad {}_{\mathcal{O}}\triangle \approx {}_{\mathcal{C}}[\overline{\triangle}] \tag{4}$$

$$\text{[Closed is almost (limit) open]}: \quad {}_{\mathcal{C}}\triangle \approx {}_{\mathcal{O}}[\widehat{\triangle}] \tag{5}$$

$$\text{[Sets and Topology]}: \quad (\mathcal{O}|\mathcal{C}) \overset{\cap m}{\rightarrow} (\mathcal{O}|\mathcal{C}) \tag{6}$$

$$\text{[Set limits and Topology]}: \quad (\mathcal{O} \vee \mathcal{C}) \underset{\cup\mathbb{N}}{\overset{\cap\mathbb{N}}{\rightleftarrows}} (\mathcal{O} \vee \mathcal{C}) \tag{7}$$

$$\text{[Zero-measure boundary witness]}^{2} :\nmid \; \mu(\widetilde{\triangle}^{\exists}) > 0 \tag{8}$$

$$\text{[Zero-measure boundary condition]} \quad \Rightarrow \frac{\text{X is regular}}{\mu(\widetilde{\triangle}^{\forall}_{X}) = 0} \tag{9}$$

[1] The existence of such limits is exhibited in specific contexts with more structure than the general topology (e.g the standard topology on the reals or the standard topology on a metric space, noting that in a non-Hausdorff space similar intersections do not have to be singletons), while the case of finite intersections can be proven in general. One instance is enough.

[2] http://math.stackexchange.com/questions/200573/, http://math.stackexchange.com/questions/157255

II Measure

$$\left[\frac{\text{ℕ-arithmetic}}{\text{Sigma-algebra}}\right] \doteqdot \frac{\left[S_{\mathcal{P}_X}\right] + \left[\overline{\bigcup^{\forall}\left({}_{\mathbb{N}}\circ^{\star}_{S}\right)}^{[S]}\right] + \left[\overline{\bigcap^{\forall}\left({}_{\mathbb{N}}\circ^{\star}_{S}\right)}^{[S]}\right] + \left[\overline{-^{\forall}\left(\circ^{\star}_{S}\right)}^{[S]}\right]}{{}_{+\mathbb{N}X}S \parallel {}_{\Sigma}S} \tag{10}$$

$$[\text{Measure}] \doteqdot \frac{\left[{}^{\mathbb{R}^+}m_{+X}\right] + \left[m\bigcup = \sum m\right]}{{}_{\mu}m} \tag{11}$$

II.1 NZ Pathology

$$[\text{Non-ℕ-arithmetic}] \doteqdot \frac{X - \left[{}_{+X}S\right]}{{}_{-X}S} \tag{12}$$

$$\left[[\text{Doch} \mid \text{Non}]\text{-}\frac{\text{arithmetical}}{\text{measurable}}\right] : \quad \circ_{[+\,\mid\,-]\mathbb{N}} \tag{13}$$

$$[\text{Non-arithmetical (N) pathology}] \doteqdot \frac{\triangle_{-\mathbb{N}}}{\wr^{-}\triangle} \tag{14}$$

$$[\text{Zero-measure set}] \doteqdot \frac{\mu(\triangle) = 0}{0_{\mu}\triangle} \tag{15}$$

$$\left[\begin{array}{l}\text{Non-arithmetical zero-subset}\\ \text{(NZ) pathology}\end{array}\right] :\nmid \doteqdot \frac{\left[\triangle^{\exists}_{-X}\right] \subset \left[{}_{0_{\mu}}\circ_{[+X]}\right]}{\wr_0^{-}\triangle_{\mathcal{P}_X}} \tag{16}$$

$$\left[\frac{\text{NZ-free}}{\text{Complete}}\right] \doteqdot \frac{\wr_0^{-}\triangle^{\nexists}_{\mathcal{P}_X}}{\overline{+X}^{[/\wr_0^{-}]}} \tag{17}$$

$$[\text{Completions exist}] :{}^{3} \quad \overline{[+X]^{\forall}}^{[/\wr_0^{-}]^{\exists}} \tag{18}$$

[3] In the trivial manner, by adding the non-measurable zero-subsets as zero-measure sets.

II.2 Arithmetical Functions

$$\left[\frac{\text{Arithmetical}}{\text{Push-Pull}}\ \text{function}\right]^{4} \quad \doteqdot \frac{\begin{bmatrix} \triangle \\ {}_{+}\triangle \end{bmatrix} \overset{f}{\rightleftarrows} \begin{bmatrix} \triangledown \\ {}_{+}\triangledown \end{bmatrix}}{{}_{+\triangle\triangledown}f \;\Big\|\; {}_{+}\triangle = f^{-1}{}_{+}\triangledown \;\Big\|\; {}_{+}\triangle = {}_{+}\triangledown \backslash f} \tag{19}$$

$$\left[\frac{\text{Rebased}}{\text{Pushed}}\ \text{measure}\right] \quad \doteqdot \frac{\mu'[{}_{+}f(\triangle)] = \mu(\triangle) \;\Big\|\; \mu'(\triangledown) = \mu[{}_{+}f^{-1}(\triangledown)]}{\mu' = \mu/f \;\Big\|\; \mu' = \mu f^{-1} \;\Big\|\; \mu' = \frac{\mu}{f} \;\Big\|\; \frac{\mu'}{\mu} = f^{-1}} \tag{20}$$

II.3 Topology, OZ Pathology

$$\left[\frac{\text{Open}}{\text{Borel}}\ \mathbb{N}\text{-arithmetic}\right]^{5} \quad \doteqdot \frac{S = \overline{\{\mathcal{O}\}}^{[+\mathbb{N}]}}{{}_{+\mathbb{N}_{\mathcal{O}}}S} \tag{21}$$

$$[\text{Positive open set}] \quad \doteqdot \frac{\mu({}_{\mathcal{O}}\triangle) > 0}{{}_{\mathcal{O}^{\mu+}}\triangle} \tag{22}$$

$$[\text{Zero-measure witness}]^{6} \;:\nmid\; \mu\left({}_{[\mathcal{O}|\mathcal{C}],\ |\mathbb{R}|}\triangle^{\exists}\right) = 0 \tag{23}$$

$$[\text{Open-zeros set}]^{7} \quad \doteqdot \frac{{}_{\mathcal{O}}S_{\mathcal{P}_X} = \widehat{\bigcup [{}_{\mathcal{O}^{\mu+}}\circ^{\forall}]}}{{}_{\mu\mathcal{Z}_X}S!} \tag{24}$$

$$[\text{Support}] \quad \doteqdot \frac{{}_{\mathcal{T}}X - \mu\mathcal{Z}_X}{{}_{\mathcal{C}}X^{\mu+} + \left[\mu\left({}_{\mathcal{O}}\circ^{\forall}_{X+}\right) > 0\right]} \tag{25}$$

$$[\text{Empty support witness}] \;:\nmid\; \mu\mathcal{Z}_X^{\exists} = \emptyset \tag{26}$$

$$[\text{Full support}] \quad \doteqdot \frac{X = X^{\mu+}}{X^{\mu\pm}} \tag{27}$$

[4] Obviously this implies that f^{-1} takes arithmetical sets to arithmetical sets. The transformation on the sigma-algebras is called pull-back. The transformation on the measures is called push-forward.

III Probability

bla

Bibliography

[5] We wish to measure open sets, the rest follows by the properties of 'limits and topology' and the requirements of $\mathbb{N}$-arithmetic: http://math.stackexchange.com/questions/1748768

[6] An example of an open set is any open interval for the measure on $\mathbb{R}$ with $\begin{cases} \mu(x : x \in \mathbb{Z}) = 1 \\ 0 \text{ otherwise} \end{cases}$. The Cantor set is a closed set example. Strangely enough, one needs measure theory to 'truly' appreciate the Cantor set and vice versa. Another example is submanifolds of dimension less than n within $\mathbb{R}^n$, with Lebesgue measure.

[7] We are note sure that the union of these open sets is open, hence we wrote $\widehat{\cup}$ instead of $\cup$. This needs to be checked.