A Survey of Results Concerning Steady Solutions and the Stability of a Class of Rotating Flows

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Abstract

We review the key points concerning the linear stability of the classical von Kármán's solution of rotating disk flow, modified by the coupling, through the fluid viscosity, with concentration field of a chemical species. The results were published by Mangiavacchi *et al.* (*Phys. Fluids*, **19**: 114109, 2007) and refer to electrochemical cells employing iron rotating disk electrodes, which dissolve in the 1 **M H_2SO_4** solution of the electrolyte. Polarization curves obtained in such cells present a current instability at the beginning of the region where the current is controlled by the the hydrodynamics. The onset of the instability occurs in a range of potentials applied to the cell and disappear above and below this range. Dissolution of the iron electrode gives rise to a thin concentration boundary layer. The concentration boundary layer increases the interfacial fluid viscoity, diminishes the diffusion coefficient and couples both fields, with a net result of affecting the hydrodynamic of the problem. Since the current is proportional to the interfacial concentration gradient of the chemical species responsible by the ions transport, the instability of the coupled fields can lead to the current instability observed in the experimental setups. This work reviews the results of the linear stability analysis of the coupled fields and the first results concerning the Direct Numerical Simulation, currently undertaken in our group. We also address the steady flow developed between two coaxials counterrotating disks and close to a semi-spherical rotating electrode in elecrochemical cells.

Keywords: Rotating disk flow, rotating semi-sphere flow, hydrodynamic stability, finite element method

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1. STABILITY OF ROTATING DISK FLOW IN ELECTROCHEMICAL CELLS

We review the problem of the stability othe linear stability of the classical von Kármán's solution of rotating disk flow, modified by the coupling, through the fluid viscosity, with concentration field of a chemical species, as found in electrochemical cells using rotating disk electrodes. A constitutive equation relating the electrode viscosity to the concentration of the iron ions generated by the dissolution of the electrode, is assumed. We show that the coupling hydrodynamcic field, to the iron concentration, which carries the current changes the stability properties of the purely hydrodynamic field, reducing the critical Reynolds number to the range of values attained in electrochemical setups. Our results support also the hypothesis that oscillations of the interfacial concentration gradient are strong enough to drive the current oscillations experimentaly observed [6]. A brief review of a Finite Element Code developed at our group and numerical results concerning the base flow and some patterns emerging from the instability of the base flow are also presented.

2. THE FLOW BETWEEN COAXIAL ROTATING DISKS

The flow between coaxial rotating admits steady solutions which are governed by a generalization of von Kármaán's nonlinear time independent ODE equations for a single rotating disk, that take into account pressure variations along the radial direction. Families of solutions qualitatively similar exist for variable distances between the disks. We present some families of solutions obtained by integration of the generalized von Kármán's ODEs equations and some results of the numerical integration of the three dimensional time dependent hydrodynamic equations, also including some solutions with pattern formation beyond the stability limit of the azimuthal angle independent [1].

3. EFFECTS OF FINITE DOMAIN

Investigations performed by integration of the hydrodynamic equations, applicable to constant viscosity fluids, showed that electrochemical cells with diameter and depths a few times the electrode diamenter are already large enough prevent relevant deviations from von Kármán's flow in the neighbourhood of the electrode[3].

4. THE CONCENTRATION AND HYDRODYNAMIC FIELDS CLOSE TO A ROTATING SEMI-SPHERE ELECTRODE

Rotating disk electrodes, regularly used in elecrochemical cells present the weakness of rapidly loosing the flat shape due to the dissolution of the iron electrode in the acid solution of the electolyte. To overcome the problem rotating semi-spherical electrodes have been used. In this setup the electrode mantains the original form, though loosing mass. The flow close to a rotating semi-spherical electrode present some characteristics also found in rotating disk flow and, at the same time, those found in boundary layers. A solution in power series of the polar multiplying the functions which describe the velocity profiles in the radial direction found by Howart (1951) is well known [5], [4]. In the present work we generalize Howart's solution by coupling the hydrodynamic and the concentration fields through the same constitutive equation assumed in the case of rotating disk flow. We present The coupled solution of the boundary layer equations and the first numerical simulations of the three dimensional time dependent equations using the Finite Element Method [2].

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