



BLOOD FLOW SIMULATION USING STREAM FUNCTION-VORTICITY FEM FORMULATION

L. MARQUES¹, G. R. ANJOS² AND J. PONTES³

Universidade do Estado do Rio de Janeiro, Faculdade de Engenharia Mecânica

¹ marquesleandro67@gmail.com, ² gustavo.anjos@uerj.br and ³ jose.pontes@uerj.br

1. ABSTRACT

The present work aims at developing a computational framework to simulate coronary artery flows in cartesian coordinates. An accurate method capable of capturing the flow dynamics is strictly required. In this paper a Finite Element Method (FEM) is used to solve the governing equations of the motion of the blood flow found in coronary artery as incompressible fluid using the stream-vorticity formulation with coupled species transport equation.

Keywords: Finite Element Method, Taylor-Galerkin Method, Coronary Artery, Blood.

2. INTRODUCTION

According to the *Health World Organization*, more people die annually from the cardiovascular diseases (CVDs) than from any other cause in the world. The leading cause of these deaths is coronary artery with atherosclerosis and the main purpose of stent implementation is to attack this problem. This work aims to know how the dynamics of blood flow in coronary artery with atherosclerosis and with stents struts placed.

We consider the hydrodynamic of the blood flow in the artery in the vicinity of a stent strut as suggested by Bozsak et al. (2014) and McGinty e Pontrelli (2016) on a two-dimensional domain. The difference of artery shapes and existence of the struts have been investigated in 4 test cases in the influence on the flow dynamics and the transport of chemical species as suggested by Wang et al. (2017).

We employ the Finite Element method on an unstructured mesh created by open source software called *GMSH* for the discretization of the incompressible single-phase Navier-Stokes through the stream-vorticity function coupled with species transport equation. We apply the Taylor-Galerkin method to decrease the spurious oscillations as seen for moderate to high Reynolds numbers as proposed by Donea (1984).

3. MATHEMATICAL MODEL

A two-dimensional Finite Element Method approach is employed to analyse the dynamics of blood flow in coronary artery and possible influence of stents struts. The modelling of the problem can be described by a formulation using the vorticity and stream function applied in the conservation of momentum equation for incompressible flow and the species transport equation.

$$\frac{\partial w}{\partial t} + \mathbf{v} \cdot \nabla w = \frac{1}{Re} \nabla^2 w \quad (1)$$

$$\nabla^2 \psi = -\omega \quad (2)$$

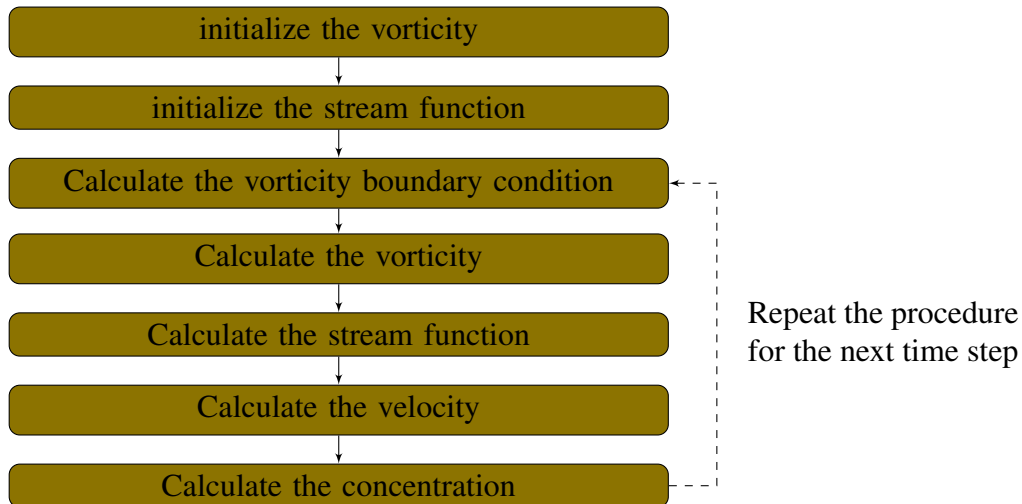
$$\mathbf{v} = \mathbf{D}\psi \quad (3)$$

$$\frac{\partial c}{\partial t} + \mathbf{v} \cdot \nabla c = \frac{1}{ReSc} \nabla^2 c \quad (4)$$

Where ω is the vorticity field, ψ is the stream function field, c is the concentration scalar field of the chemical species, $\mathbf{v} = (v_x, v_y)$ is the velocity field, $\mathbf{D} = [\partial/\partial y, -\partial/\partial x]$ is a mathematical operator, $Re = \rho u D / \mu$ is the Reynolds number, $Sc = \nu / D$ is the Schmidt number, x and y are the independent spatial variables and t is the time variable.

The spatial domain was discretized using linear triangular. For the discretization of the temporal domain was used a finite differences method with a forward difference approximation and the Taylor-Galerkin Method with omitted terms of higher order than two was used to the decrease the spurious oscillations for moderate to high Reynolds numbers as proposed by Donea (1984).

The linear system of equations that come from implementing the FEM is solved through iterative method *Conjugate Gradient Solver* available in the public library for scientific tools *SciPy* in the *Python language*. The solution algorithm used is shown below:



4. RESULTS

Numerical results are shown for several cases of blood flows in artery. The post-processing was performed by open source software *PARAVIEW*. The lumen diameter of a typical artery is about $D = 0.003\text{m}$, viscosity in the lumen are set to $\mu = 0.0035\text{Pa.s.}$ and density $\rho = 1060\text{kg/m}^3$ as suggested by Bozsak et al. (2014). According to Kessler et al. (1998), the velocity of the flow at coronary artery is $v = 12\text{cm/s.}$ Therefore, the Reynolds number is $Re = 109$. Four non-dimensional geometries where the channel length $L = 10R$ is based on the channel width $R = 1$

were used in the simulations and are shown in the Figure 1. Only half domain are shown since the results are symmetric in y-direction.

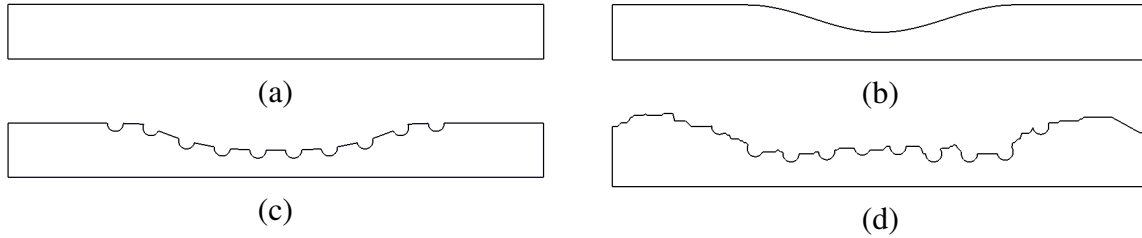


Figura 1 – Non-dimensional geometry for blood flow dynamics in coronary arteries. (a) Straight Channel (b) Curved Channel (c) Curved Channel with Stent and (d) Real Channel with Stent.

In fig 2, is shown the velocity profiles evolution in time for the geometries where Wang et al. (2017) results are compared and the velocity and concentration fields for the steady state is shown in fig 3.

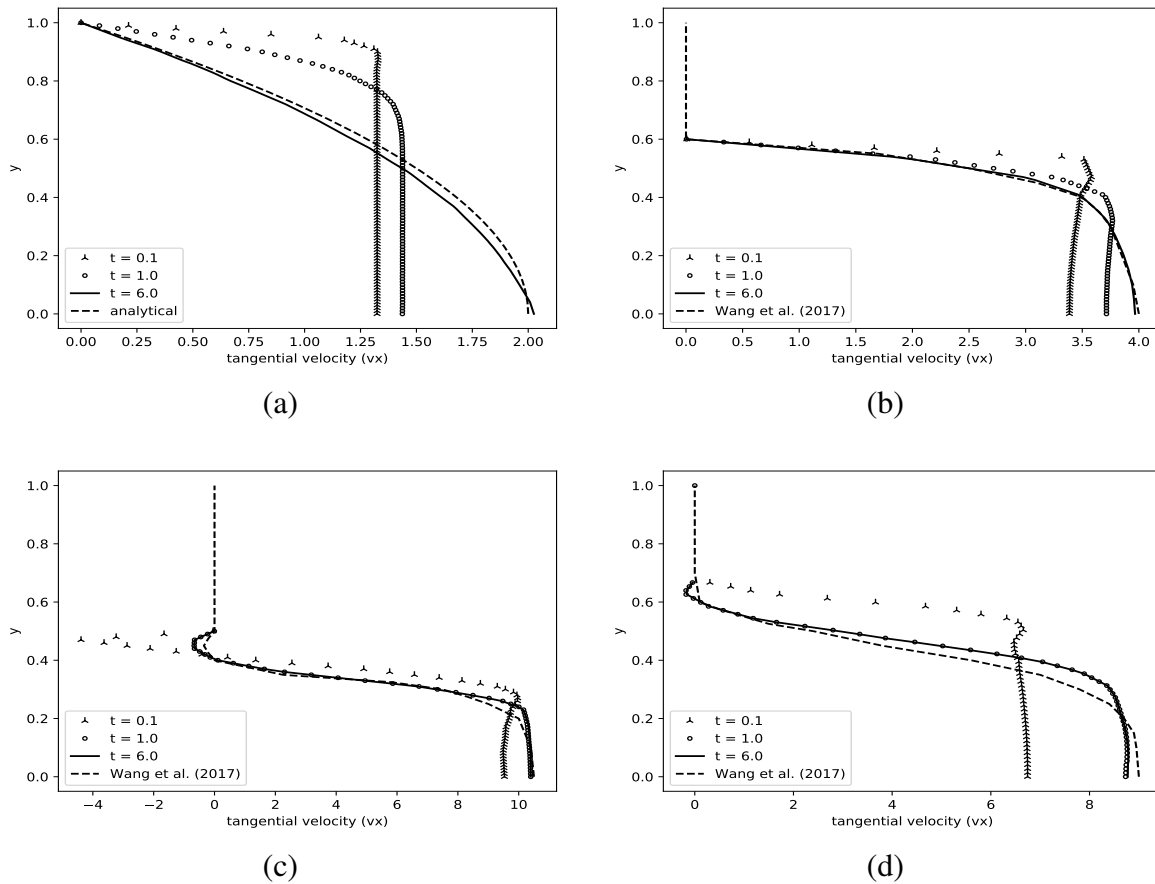


Figura 2 – Velocity field profile evolution in time for various geometries (a) Straight Channel (b) Curved Channel (c) Curved Channel with Stent and (d) Real Channel with Stent.

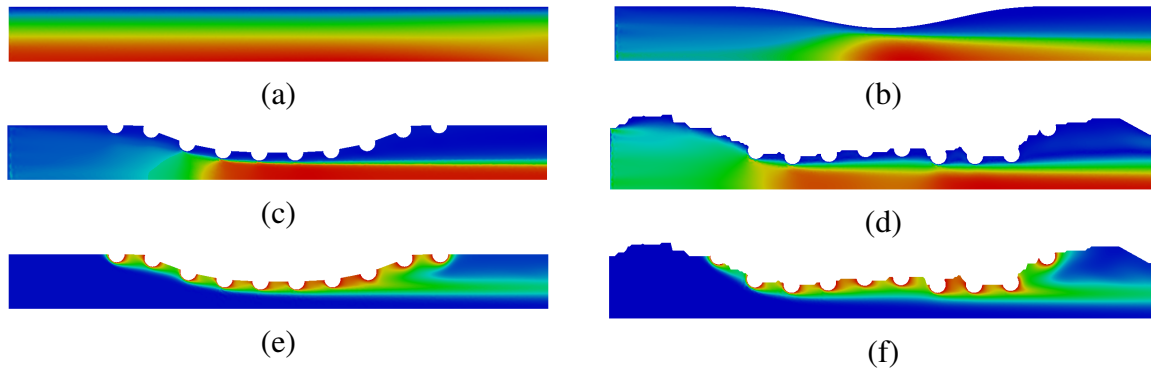


Figura 3 – Velocity field steady state: (a) Straight Channel (b) Curved Channel (c) Curved Channel with Stent and (d) Real Channel with Stent. Concentration field steady state: (e) Curved Channel with Stent and (f) Real Channel with Stent.

5. CONCLUSION

The results obtained in this paper for the velocity and concentration fields of complex geometries of a modeled coronary artery channel were similar to those presented by Wang et al. (2017). However, the numerical simulation was performed using the streamfunction and vorticity formulation by finite element method approach. The streamfunction and vorticity formulation showed an useful approximation for to calculate the velocity and concentration fields since the variables are scalars allowing then a smooth implementation.

6. REFERENCES

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