



# COUNTER-CURRENT THERMOCAPILLARY MIGRATION OF BUBBLES IN MICROCHANNELS USING SELF-REWETTING LIQUIDS

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August 19, 2015

PPG-EM Seminars: season 2015

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**Keywords:** Self-rewetting liquids; two-phase flow; thermocapillary migration.

## 1 Introduction

Thermocapillary migration is a phenomenon of bubble and drop motion driven by temperature-induced surface tension gradient. The variation of surface tension creates a tangential stress along the interface leading to interfacial flow from low to high surface tension regions, which drives fluid around on both sides of the interface. The motion of the neighboring fluid finally propels bubbles and drops in the opposite direction. This phenomenon is particularly important to understand instabilities in evaporative cooling of microelectronics due to the two-phase nature of the flow and the microscale characteristic of the system, where surface tension force become more relevant.

Since the pioneering work of Young et al. [3], that theoretically derived the terminal velocity of a bubble in a vertical temperature gradient, this subject has been extensively explored theoretically and experimentally for liquids where surface tension is a linearly decreasing function of temperature. For these liquids, bubbles migrate in direction to higher temperatures. However, little attention was given for self-rewetting liquids in this topic, i.e. binary mixtures, such as water/butanol, where surface tension is a parabolic function of temperature with a well defined minimum. These liquids have higher heat transfer coefficient than pure liquids, what make it attractive for phase-change based cooling applications. Shanahan and Sefiane [2] demonstrated that in self-rewetting liquids thermocapillary forces may drive bubbles away from high temperatures and against flow towards the surface tension minimum until reach an equilibrium position. At confinement conditions they observed that larger bubbles present sustained oscillations around the equilibrium position. The present study aim to complement this work by means of direct numerical simulations (DNS) in order to increase the comprehension of thermocapillary migration in self-rewetting fluids.

In this work, we consider an axisymmetric gas bubble of initial radius  $R$  traveling against flow in an axisymmetric horizontal channel of length  $L$  and diameter  $H$ , until reach the equilibrium position. Self-rewetting

liquid flow inside the channel with a fully developed Poiseuille velocity profile, and the channel wall is subjected to a temperature gradient,  $\Gamma = (T_{hot} - T_{cold})/L$ . The gas bubble is introduced at the downstream side of the channel (high temperatures) with zero initial velocity. We assume a thermodynamically saturated environment, with no phase change, and a negligibly small effect of gravity due to the microscale in consideration.

## 2 Model

The two-phase system is modeled with Gerris [1] using the Volume-of-Fluid method. The two fluids considered are incompressible, Newtonian and immiscible. The dimensionless governing equations for mass, momentum and energy conservation can be respectively written as:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = -\nabla p + \frac{1}{Re} \nabla \cdot \left( \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \right) + \frac{1}{We} \mathbf{f}_{st} \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{RePr} \nabla \cdot (\alpha \nabla T) \quad (3)$$

where  $\mathbf{u}$ ,  $p$  and  $T$  are the velocity, pressure and temperature fields, and  $\rho$ ,  $\mu$  and  $\alpha$  are the density, viscosity and thermal diffusivity, respectively. Time is denoted by  $t$ , and  $\mathbf{f}_{st}$  is the volumetric surface tension force. The dimensionless parameters are the Reynolds, Weber and Prandtl numbers given by  $Re \equiv \rho_L V R / \mu_L$ ,  $We \equiv \rho_L V^2 R / \sigma_0$  and  $Pr \equiv \mu_L / (\rho_L \alpha_L)$ , respectively, where  $V$  is the characteristic velocity and  $\sigma_0$  is the reference surface tension coefficient.

The two phases are modeled using a “homogeneous” approach where both fluids obey the same set of governing equations and are represented by a single fluid with different properties locally identified by a volume fraction field  $c$  that can take 1 in the liquid phase, 0 in the gas phase and between 0 and 1 at the interface. In that sense, the single fluid volume averaged density, viscosity and thermal diffusivity read as  $\rho = c + (1 - c)\rho_r$ ,  $\mu = c + (1 - c)\mu_r$  and  $\alpha = c + (1 - c)\alpha_r$ , respectively, where  $\rho_r = \rho_G / \rho_L$ ,  $\mu_r = \mu_G / \mu_L$  and  $\alpha_r = \alpha_G / \alpha_L$  are the density, viscosity and thermal diffusivity ratios and the subscript  $L$  stands for liquid and  $G$  for gas. The volume fraction field evolves in time across the domain

by the conservation equation,

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = 0 \quad (4)$$

The volumetric surface tension force in eq. (1) is denoted by  $\mathbf{f}_{st} = (\sigma \kappa \mathbf{n} + \nabla_s \sigma) \delta_s$ , where the first term is the normal component given by Laplace's formula and the second term is the tangent component given by surface tension gradient. The interface normal vector  $\mathbf{n}$  and the curvature  $\kappa$  are represented by,  $\mathbf{n} = \nabla c / |\nabla c|$  and  $\kappa = -(\nabla_s \cdot \mathbf{n})$ . The gradient operator tangent to the interface is given by,  $\nabla_s = \nabla - \mathbf{n}(\mathbf{n} \cdot \nabla)$  and the surface tension coefficient is represented by,  $\sigma = 1 - \beta_1 T + \beta_2 T^2$ , in order to model the self-rewetting fluid behaviour.

### 3 Results and Discussion

We analyse the effect of the thermal diffusion controlled by the Prandtl number, on the bubble thermocapillary motion. The base set of parameters derived from [2] is:  $Re = 4.37$ ,  $We = 4.22 \times 10^{-4}$ ,  $\rho_r = 0.001$ ,  $\mu_r = 0.01$ ,  $\alpha_r = 0.04$ ,  $\beta_1 = 0.3$ ,  $\beta_2 = 0.15$ ,  $\Gamma = 0.1$ ,  $L = 80$ ,  $H = 4$ ,  $R = 1$ ,  $U = 1$ . In Fig. 1 we can see the initial conditions, where the initial bubble position is at  $z_i = 70$ , and the reference position for the temperature corresponding to the surface tension minimum,  $T_m$ , was chose as the location where its isothermal line (black line in Fig. 1) cross the center of the channel, at  $z_m = 40$ .

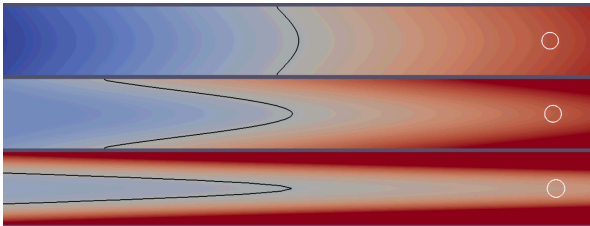


Figure 1: Initial conditions for  $Pr = 0.1$  (top),  $Pr = 1.0$  (middle) and  $Pr = 6.1$  (bottom). The black line is the isothermal line for  $T_m$ , the white line is the bubble interface and the temperature contours is shown in colour.

In Fig. 2 we show the temporal evolution of the bubble center of mass during the counter-current thermocapillary motion in a channel with flow in the positive direction of z-axis. The bubble reach the equilibrium position through a damped oscillation motion and doesn't present sustained oscillations.

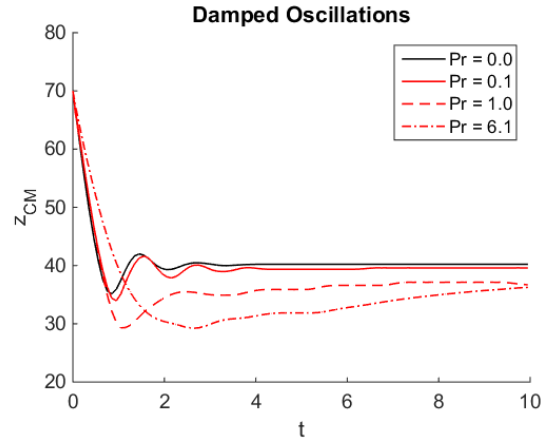


Figure 2: Temporal evolution of the bubble center of mass,  $z_{CM}$ , for different Prandtl numbers.

As we increased the Prandtl number towards realistic values ( $Pr \approx 6.1$ ), we observed two effects of the thermal diffusion on bubble's motion: first, it delays the thermocapillary effect allowing higher amplitude oscillations; second, it weakness the thermocapillary effect, resulting in lower velocities for higher Prandtl numbers, as can be seen in Fig. 2. The first effect can be interpreted as a result of the decrease in thermal diffusion retarding the change of temperature at the bubble interface. The second effect can be explained by means of the temperature gradient vector field (perpendicular to the isothermal lines) that diverges from the z-direction as we increase the Prandtl number, and consequently decrease the net thermocapillary force in the direction parallel to bubble motion.

### 4 Future Work

The model will be extended to 3D to study confinement effects and avoid strong deviation of the temperature gradient vector field from the direction parallel to bubble motion, in order to capture the sustained oscillations.

### References

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