PARTICLE TRANSPORT MODELING IN CHANNEL STEADY LAMINAR AND TURBULENT

Author: Apoena Calil and Gabriel Meletti[1] *apoenacalil@gmail.com, gabrielmeletti@gmail.com*

[1] State University of Rio de Janeiro

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1 Introduction

In this work, the physical processes responsible for the transport of particles in regular channels for turbulent flow regimes are modeled. Particle transport is a relevant area of knowledge with many applications, such as the transport of sand or paraffin in pipelines and blood flow. To account for the turbulent effects, the "zero-equation" RANS model was employed, which makes use of the concept of turbulent viscosity proposed by Boussinesq. Effects along the vertical direction (channel's depth) was neglected and the flow was considered completely developed, allowing for a 1D approach.

Particle transport was described by the BBO equation [1], through a Lagrangian approach, considering drag, lift, virtual mass and gravity as the forces which act on the particles.

The flow followed a Eulerian approach, using centered finite differences for space domain discretization and both explicit and implicit formulations were implemented. One-way coupling between flow and particles was considered, meaning that only the flow affected the particles, with no feedback from particles to the flow.

Results for the flow profile showed good results and, although preliminary, results for the particle transport showed physical coherence, in accordance to the applied forces.

2 Flow modeling

Taking into account that the flow is developed and incompressible, the RANS equations that describe the flow, already incorporating the concept of Boussinesq viscosity, is given by

momentum:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[(\nu_m + \nu_t)\frac{\partial u}{\partial y}\right] \tag{1}$$

continuity:

$$\frac{\partial u}{\partial x} = 0 \tag{2}$$

where u is de longitudinal velocity component, t is time, ρ is the density of the fluid, p is pressure, x and y are the longitudinal and orthogonal coordinates of the channel domain, ν_m is the kinematic viscosity of the fluid, designated as "molecular viscosity" in opposition to the turbulent, or apparent, viscosity, which results from the turbulence. Note that u and p actually represent the mean values of velocity and pressure fields.

The Prantdl mixing length model (algebraic model) was used for characterization of the turbulent viscosity, which means that

$$\nu_t = l_c^2 \left|\frac{\partial u}{\partial y}\right| \tag{3}$$

where l_c is the mixing length, defined as

$$l_c = \kappa y, \text{ para } y \le \delta \tag{4a}$$
$$l_c = \kappa \delta, \text{ para } y > \delta \tag{4b}$$

with κ as Von Kármán's constant and δ as the boundary layer thickness. Motion equations are solved using Centered Finite Differences for space discretization, making use of a staggered grid.

3 Particles

From the point of view of the model, particles are transported through a Lagrangian approach, so that an Eulerian-Lagrangian formulation characterizes the fluid-particles modeling. One-way coupling between flow and particles was considered, which means that particles do not make influence on the flow. The transport is modelled by a form of the BBO equation, taking into account four forces: drag, lift, virtual mass and gravity. For more details concerning the BBO equation, please refer to [1], section 4.3. The influence of the turbulent regime on the particles is simulated by means of the so-called discrete eddy model, which applies a random perturbation into the velocity components of the particles based on the magnitude of the turbulent viscosity.

The four forces taken into account in the model are expressed by the following equations:

drag force $\mathbf{F}_D$:

$$\mathbf{F}_D = 3\pi\mu D(\mathbf{u} - \mathbf{v}_p) \tag{5}$$

lift force $\mathbf{F}_L$:

$$\mathbf{F}_L = 1.61\sqrt{\mu\rho_p}D^2 \parallel \mathbf{u} - \mathbf{v}_p \parallel \frac{d\mathbf{u}}{dy}\left|\frac{du}{dy}\right|^{-0.5} \tag{6}$$

added mass force $\mathbf{F}_{AM}$:

$$\mathbf{F}_{AM} = \frac{1}{2} M \frac{d\mathbf{v}_p}{dt} \tag{7}$$

gravity force $\mathbf{F}_G$:

$$\mathbf{F}_G = M\mathbf{g} \tag{8}$$

where ρ_p, D and $\mathbf{v}_p$ are respectively the particle's density, diameter and velocity vector; $\mathbf{u}$ is velocity vector of the flow. Both $\mathbf{u}$ and $\mathbf{v}_p$ have two components in the present model.

The trajectory of the particles are computed by solving

$$\Sigma\mathbf{F} = \mathbf{F}_D + \mathbf{F}_L + \mathbf{F}_{AM} + \mathbf{F}_G = M\frac{d\mathbf{v}_p}{dt} \tag{9}$$

which is a form of the BBO equation. An additional perturbation $\mathbf{v}'_p$ is applied on the particles' velocity due to turbulence contributions. The perturbation is defined by the discrete eddy model, using Gaussian distribution to account for the random characteristic, i.e.

$$\mathbf{v}'_p = 4\zeta l_c \sqrt{2/3} \left| \frac{\partial u}{\partial y} \right| \tag{10}$$

The factor ζ is computed by a Monte Carlo like iterative procedure, in which two random values ζ and f_R are generated, with $\zeta \in [-0.5, 0.5]$ and $f_R \in [0, 1/\sqrt{2\pi}]$. If $f_R \le f$, where $f = 1/\sqrt{2\pi}\exp(-0.5\zeta^2)$ (Gaussian function), then ζ is the factor used to compute the velocity fluctuation. Otherwise, a new pair of random values ζ and f_R is generated, and the procedure is repeated until the condition $f_R \le f$ is satisfied. Equation 9 is a linear ODE that can be analytically solved for $\mathbf{v}_p$. Thus, the new position $\mathbf{x}^{n+1}$ of each particle in the time step $n+1$ is computed in a Lagrangian way using the turbulent particle velocity $\mathbf{v}_p + \mathbf{v}'_p$, i.e.,

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + (\mathbf{v}_p + \mathbf{v}'_p)\Delta t \tag{11}$$

where Δt is the time increment.

4 Results

Three results will be presented: two velocity profiles, one for steady laminar and another for steady turbulent regime, and a frame from one simulation of particles transport. Laminar profile is obtained from solving eq.1 with $\partial u/\partial t = 0$ and $\nu_t = 0$. The numerical results were compared to analytical solution. Figure 1 shows the comparison for $\mathrm{Re} = 10^3$ with a mesh of 80 nodes.

In fig. 2, the non-dimensional turbulent boundary layer is shown, for $\mathrm{Re} = 10^5$ and a mesh of 800 nodes. Comparison is qualitative good.

Now, 20 particles were randomly distributed in the channel, with null initial velocity, in a turbulent flow regime. Figure 3 shows a frame of the simulation.

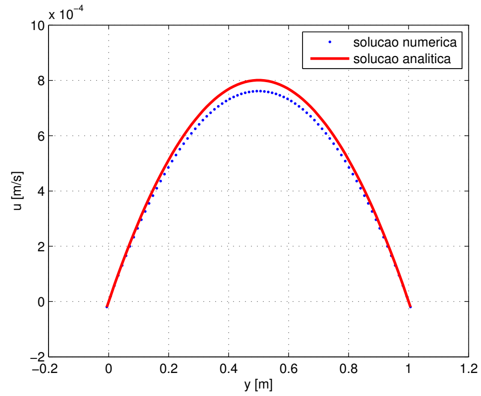

Figure 1: Numerical and analytical laminar profiles of channel flow with $\mathrm{Re} = 10^3$.

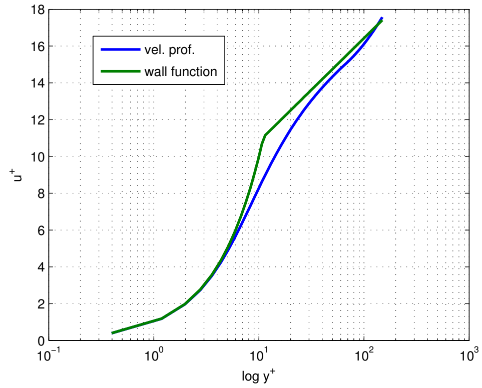

Figure 2: Comparison of turbulent channel flow result with wall function, for $\mathrm{Re} = 10^5$.

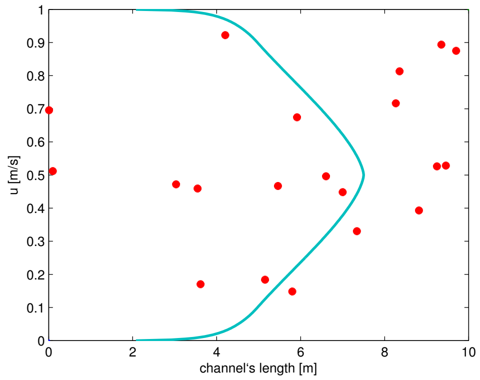

Figure 3: Particles transported by turbulent flow in the channel.

5 Conclusion

The numerical model implemented provides accurate results for the velocity profiles, but results for particle transport still need to be validated. However, the model covers several areas of CFD, such as basic turbulence modeling, finite difference method, numerical diffusion, particle transport, and serves as a complete learning tool.

References

[1] Clayton T. Crowe, John D. Schwarzkopf, Martin Sommerfeld, and Yutaka Tsuji. *Multiphase flows with droplets and particles*. CRC Press, 2012.