Finite Element Method for Low Froude Number Saint-Venant Equations

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ABSTRACT

The flow in shallow water bodies can be described by the Saint-Venant equations, including bottom friction, viscosity and Coriolis-Boussinesq factor. In this work a numerical method is proposed for the solution of the Saint-Venant system, for the case of negligeable Froude number, employing a rigid lid approximation. The problem is discretized employing the Finite Element Method in a triangular mesh. Spatial discretization of the diffusion and pressure terms is made through the Galerkin method. The *MINI* element is selected, among the Taylor-Hood family of conforming elements that satisfied the LBB condition. The substantial derivative is discretized through a semi-Lagrangean technique, using a first-order backward Euler implicit scheme. The linear system is solved employing the discrete projection method, based on LU decomposition. The code is developed using the object-oriented paradigm. This work is supported by Furnas Centrais Elétricas S.A..

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