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Finite Element Simulation of Fingering in Convective Dissolution in Porous Media

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Abstract

We address the problem of buoyancy-driven fingering generated in porous media by the instability of a partially miscible fluid layer dissolving in another denser fluid placed below it. A 2D time dependent numerical simulation is performed, assuming that the flow is governed by Darcy's law, along with the Boussinesq approximation to account for buoyancy effects introduced by concentration dependent densities. The velocity field is modeled by a vorticity-stream function formulation. The resulting equations are solved through the finite element method, with the material derivative of the concentrations numerically represented by a Semi-Lagrangian Scheme.

Keywords: fingering, instability, finite element method.

1. Introduction

Fingering refers to hydrodynamic instabilities of deforming interfaces into fingers during the displacement of fluids in porous media. These instabilities are closely linked to changes in viscosity or density between the different layers or within a single phase containing a solute invariable concentration that affects the fluid density or viscosity [1].

Fingering occurs in a variety of applications, including $\mathbf{CO_2}$ sequestration techniques, secondary and tertiary crude oil recovery, fixed bed regeneration chemical processing, hydrology, filtration, liquid chromatography, and medical applications, among others. In fact, the phenomena are expected to occur in different fields of science and technology, in which flows in porous media are present [1].

We consider the problem of buoyancy-driven fingering generated in porous media by the dissolution of a fluid layer initially placed is on the lower layer only where the convective dissolution dynamics takes place.

2. Finite Element Method

A 2D time dependent numerical simulation is performed, assuming that the flow is governed by Darcy's law, along with the Boussinesq approximation to account for buoyancy effects introduced by a concentration dependent density. The viscosity is assumed as constant. A vorticity-stream function formulation is adopted to solve the hydrodynamic field [2, 3]. The resulting equations are solved through the finite element method, with the material derivative of the concentrations numerically represented by a semi-lagrangian scheme [4].

Boundary conditions for the velocity field are prescribed as no slip at the upper and lower walls and periodic at the sidewalls. For the dimensionless concentration field, we prescribe as boundary conditions no flux at the lower wall, periodic boundary conditions at the sidewalls and at the top we set a constant

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value equal to 1. The upper boundary is assumed as flat and horizontal. The onset of fingering is induced by the numerical grid.

Darcy's law and the advective-diffusion equation of the transport of the relevant chemical species are solved in the weak form (also called variational form). The variational form of the governing equations are obtained by properly weighting the equations with weighting functions. The global system, with discrete representation of the derivatives and written in matrix form reads:

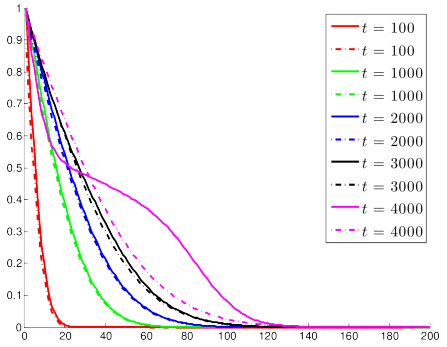
$$\begin{aligned} \mathbf{K}\psi + \mathbf{M}\omega &= 0 \\ -\mathbf{M}\omega &= \mathbf{D}_x c \\ (\mathbf{M} + \Delta t \mathbf{K})\, c^{n+1} - \mathbf{M} c_d^n &= 0. \end{aligned} \qquad (1)$$

where Eq. 1 are the stream function, vorticity equation (Darcy's Law) and concentration transport equation, respectively, $\mathbf{K}$ is the stiffness matrix, $\mathbf{M}$ is the mass matrix, $\mathbf{D}_x$ is the gradient matrix, c_d is the concentration at the departure points of the lagrangian trajectories.

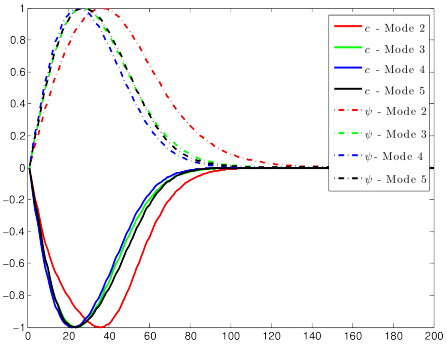
The system of equations is solved in two steps. In the first one we obtain the stream function and vorticity. Velocity components are then obtained and introduced in the transport equation of c, which is subsequently solved. An incomplete LU pre-conditioner is applied to the matrices and the linear systems are then solved with the use of GMRES (Generalized Minimal Residual) solver. Preconditioned conjugate gradients methods were also used to obtain the velocity field from the stream function solutions.

3. Numerical Results

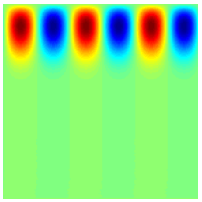
Figure 1(a) shows average concentration profiles c for different times, where the dashed curves refer to the analytical solution related to the numerical profile shown by the continuous curve of the same color. We can observe that for $t = 4000$ a sharp distortion due to the perturbation growth, can be observed in the nonlinear evolution regime of c (see Fig. 1(e)). Fig. 1(b) shows the amplitude of perturbation modes of concentration and stream function obtained from the solution of the governing equations using Fourier analysis, at $t = 3000$.



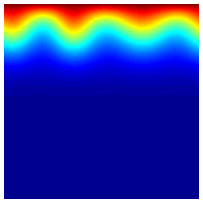
(a) Average concentration profile



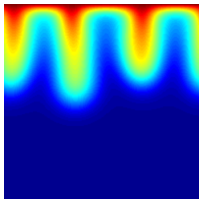
(b) ψ and c for different modes



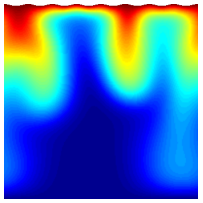
(c) Mode $n = 3$



(d) $t = 3000$



(e) $t = 4000$



(f) Forced

Figure 1 – Numerical results.

3. Conclusions

Solutions obtained up to now are in accordance with those found in the literature [5]. The evolution strongly depends on the initial condition. We are currently investigating the nonlinear evolution of instabilities developed with either a flat upper surface or forced by a deformed upper surface (Figs. 1(e) and 1(f)).

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