

Equações diferenciais ordinárias e álgebra linear

José Pontes

Programa de Pós-graduação em Engenharia Mecânica
PPG-EM/UERJ

jose.pontes@uerj.br
tel. 98858-2130

2º período, 2015

Sumário

Conteúdo

Avaliações

Bibliografia

Duas Aplicações à Ciência dos Materiais

Dinâmica de Discordâncias: Fadiga

O Mapa Logístico





Conteúdo

1. Equações diferenciais de evolução (ordinárias);
 - ▶ Classificação das eqs. diferenciais;
 - ▶ Equações de evolução: espaço de fases, subespaços invariantes, linearização em torno de pontos fixos;
 - ▶ Equações diferenciais **ordinárias lineares**, com coeficientes constantes:
 - ▶ Autovalores reais, não-repetidos;
 - ▶ Complexos, não-repetidos;
 - ▶ Exponencial de operadores;
 - ▶ Forma canônica de Jordan
 - ▶ Reais, repetidos;
 - ▶ Complexos, repetidos.
 - ▶ Equações diferenciais **ordinárias não-lineares**;
2. Métodos numéricos - diferenças finitas;
3. Sistemas com dependência espacial.

Avaliações

1. 2 provas + prova de reposição;
2. Frequência: não obrigatória.

Bibliografia recomendada I

-  Hirsch, M. W. & Smale, S.
Differential Equations, Dynamical Systems and Linear Algebra
Academic Press, 1974.
-  Friedman, B.
Principles and Techniques of Applied Mathematics
Dover, 1990.
-  Jordan, D. W. & Smith, P.
Non-linear Ordinary Differential Equations
Oxford, 1989.
-  Monteiro, Luiz Henrique Alves.
Sistemas Dinâmicos
Editora Livraria da Física, 2002.

Bibliografia recomendada II



Lipschutz, S.

Álgebra Linear

McGraw-Hill, 1972.



Doering, Claus I. & Lopes, Artur O.

Equações Diferenciais Ordinárias

Instituto Nacional de Matemática Pura e Aplicada, 2005.



Kreyszig, E.

Advanced Engineering Mathematics

Wiley, 1999.

Dinâmica de Discordâncias em Materiais Submetidos a Fadiga

O modelo Walgraef-Aifantis (WA)

Ref: *J. Appl. Phys.*, **58** (1985), 668

$$\begin{aligned}\frac{\partial \rho_s}{\partial t} &= D_s \nabla^2 \rho_s + \sigma - v_s d_c \rho_s^2 - \beta \rho_s + \gamma \rho_s^2 \rho_m \\ \frac{\partial \rho_m}{\partial t} &= \nabla_x \frac{v_g}{\gamma \rho_s^2} \nabla_x v_g \rho_m + \beta \rho_s - \gamma \rho_s^2 \rho_m\end{aligned}$$

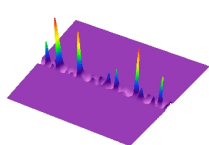
onde:

$\rho_s \longrightarrow$ Discordâncias estáticas/ μm^2

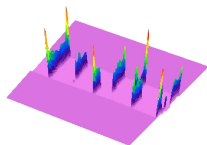
$\rho_m \longrightarrow$ Discordâncias móveis/ μm^2

O modelo WA: Integração numérica ($2D + t$)

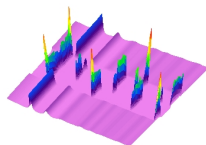
Evolução temporal de ρ_s



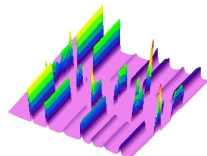
$t = 0.75$



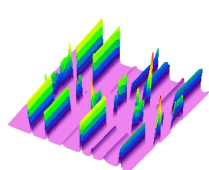
$t = 1.50$



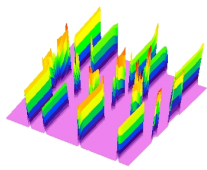
$t = 1.70$



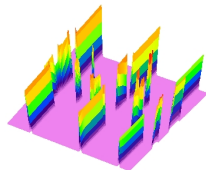
$t = 1.80$



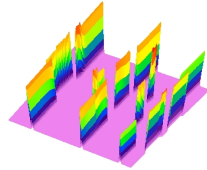
$t = 1.82$



$t = 2.00$



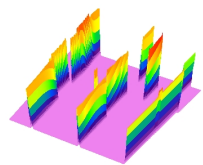
$t = 3.00$



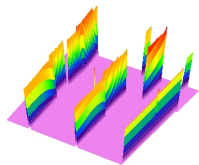
$t = 12.0$

O modelo WA: Integração numérica ($2D + t$)

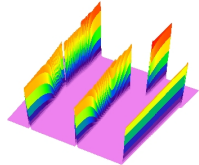
Evolução temporal de ρ_s



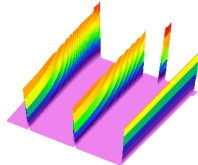
$t = 50.0$



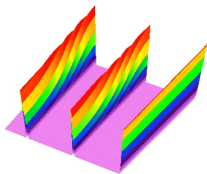
$t = 70.0$



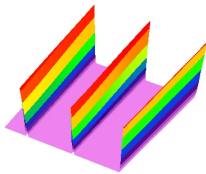
$t = 150.0$



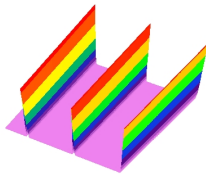
$t = 700.0$



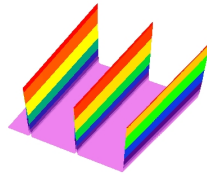
$t = 1000.0$



$t = 1500.0$



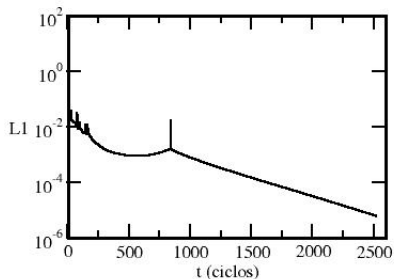
$t = 2000.0$



$t = 2524.0$

O modelo WA: Integração numérica ($2D + t$)

Velocidade de evolução $\times t$



O Mapa Logístico (Discreto)

$$X^{n+1} = 4\mu X^n (1 - X^n) \quad \left\{ \begin{array}{l} n \longrightarrow \text{Número da iteração} \\ 0 \leq X^0 \leq 1 \quad 0 \leq \mu \leq 1 \\ \mu \longrightarrow \text{Parâmetro de bifurcação} \end{array} \right.$$

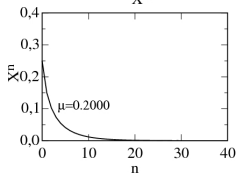
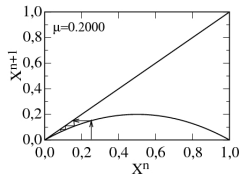
$$X^{n+1} = 4\mu X^n - 4\mu (X^n)^2$$

$$X^{n+1} - X^n = 4\mu X^n - X^n - 4\mu (X^n)^2$$

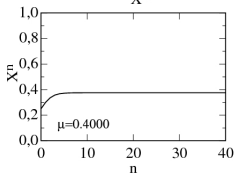
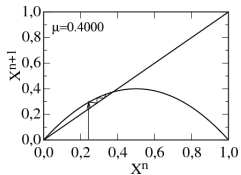
$$X^{n+1} - X^n = (4\mu - 1) X^n - 4\mu (X^n)^2$$

$$\frac{dX}{dt} = \lambda X - gX^2$$

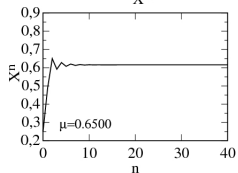
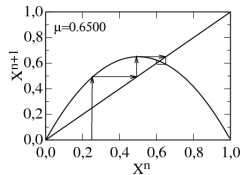
O Mapa Logístico (Discreto)



(a)

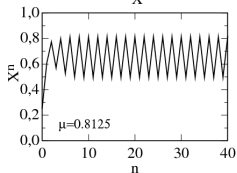
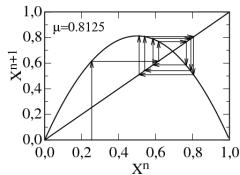


(b)

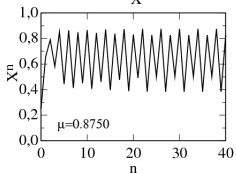
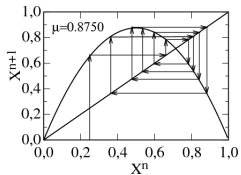


(c)

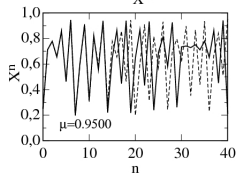
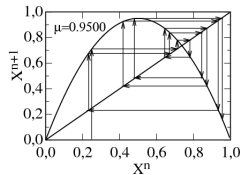
O Mapa Logístico (Discreto)



(d)



(e)



(f)

O Mapa Logístico (Discreto)

Resumo: $X^{n+1} = 4\mu X^n (1 - X^n)$

1. Um ponto fixo ($X^n = 0$);
2. Dois pontos fixos, um instável e outro estável:
 - ▶ Evolução monotônica;
 - ▶ Evolução oscilatória;
3. Dois pontos fixos instáveis – comportamento periódico;
4. Dobramento de período;
5. Comportamento aperiódico, com sensibilidade à condição inicial → **caos determinístico**.

Ref: Sir James Lighthill – The recently recognized failure of predictability in
Newtonian Mechanics – *Proc. R. Soc. Lond. A* **407**, 35-50, 1986

*... We collectively wish to apologize for having misled
the general educated people by spreading ideas about
determinism of systems satisfying Newton's laws of
motion that, after 1960, were to be proven incorrect.*