LINEAR STABILITY ANALYSIS OF FINGERING IN CONVECTIVE DISSOLUTION IN POROUS MEDIA

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1 Introduction

Fingering refers to hydrodynamic instabilities of deforming interfaces into fingers during the displacement of fluids in porous media. These instabilities are closely linked to changes in viscosity or density between the different layers or within a single phase containing a solute invariable concentration that affects the fluid density or viscosity (Homsy [3]). The phenomena occurs in a variety of applications, including $\mathbf{CO_2}$ sequestration techniques, secondary and tertiary crude oil recovery, fixed bed regeneration chemical processing, hydrology, filtration, liquid chromatography, and medical applications, among others. In fact, the phenomena are expected to occur in different fields of science and technology, in which flows in porous media are present.

We consider the problem of buoyancy-driven fingering generated in porous media by the dissolution of a fluid layer initially placed over a less dense one in which it is partially miscible. The focus is on the lower layer only where the convective dissolution dynamics takes place.

A 2D time dependent numerical simulation is performed, assuming that the flow is governed by Darcy's law, along with the Boussinesq approximation to account for buoyancy effects introduced by a concentration dependent density. The viscosity is assumed as constant. A vorticity-stream function formulation is adopted to solve the hydrodynamic field (Almarcha et al. [1], Budroni et al. [2]).

1.1 Model equations

The equations describing the dynamics in the flow field that governing the evolution of the concentration field are:

$$\begin{aligned} \nabla^2\psi &= -\omega_z && (1)\\ \omega_z &= R\frac{\partial c}{\partial x} && (2)\\ \frac{Dc}{Dt} &= \mathcal{D}\nabla^2 c, && (3) \end{aligned}$$

where: ψ is the stream function ($\mathbf{u} = (\partial\psi/\partial y, -\partial\psi/\partial x)$), ω_z is the vorticity ($\omega_z = \nabla^2\psi$), $R = -\beta g i_z$, c is the concentration field and $\mathcal{D}$ is the diffusion coefficient. Equation 1 is the vorticity equation, Eq. 2 is Darcy's Law for the vorticity and Eq. 3 is the concentration transport equation.

2 Linear Stability Analysis (LSA) of the Base State

The base state of the problem is the time dependent solution of Eq. 3 of the concentration field in absence of any flow:

$$\bar{c}(y,t) = 1 - \operatorname{erf}\left(\frac{y}{2\sqrt{t}}\right). \qquad (4)$$

On the basis of Eqs. 1-3, a LSA can be performed to obtain dispersion curves giving the growth rate of the perturbations as a function of the wavenumber.

The LSA consists in adding perturbations to the base state solution characterized by the concentration profile (4) and

$$\begin{pmatrix} c \\ \psi \end{pmatrix} = \begin{pmatrix} \bar{c} \\ 0 \end{pmatrix}(y,t) + \begin{pmatrix} \tilde{c} \\ \dfrac{i\tilde{\psi}}{k} \end{pmatrix}(y)\exp(\sigma t + ikx), \qquad (5)$$

where $i^2 = -1$, k is the wavenumber of the perturbation and σ is the growth rate. The linearised evolution equations for the disturbances $\tilde{c}$ and $\tilde{\psi}$ are thus:

$$\begin{aligned} \tilde{\psi}_{yy} - k^2\tilde{\psi} &= k^2\tilde{c} && (6)\\ \sigma\tilde{c} + \tilde{\psi}\bar{c}_y &= \tilde{c}_{yy} - k^2\tilde{c} && (7) \end{aligned}$$

Boundary conditions for the concentration and stream function perturbations $\tilde{c}$ and $\tilde{\psi}$ are thus:

$$\begin{aligned} y = 0: &\quad \tilde{c} = 0, \quad \tilde{\psi} = 0\\ y \to \infty: &\quad \tilde{c} \to 0, \quad \tilde{\psi} \to 0. \end{aligned}$$

Upon defining $D^n = d^n/dy^n$, we rewrite Eq. 6: $\left(D^2 - k^2\right)\tilde{\psi} = k^2\tilde{c}$ and inversely: $\tilde{\psi} = \left(D^2 - k^2\right)^{-1} k^2\tilde{c}$.

Upon replacing $\tilde{\psi}$ in Eq. 7 and rearranging terms we arrive at an eigenvalue-eigenfunction equation for the rate of growth σ and associated vertical concentration profile in the form:

$$\left[\left(D^2 - k^2\right) - \left(D^2 - k^2\right)^{-1} k^2 D\bar{c}\right] \tilde{c} = \sigma \tilde{c}.$$

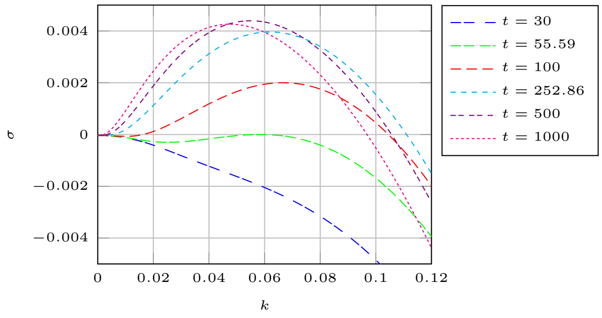

Figure 1: Dispersion curves.

Figure 1 shows the dispersion curves of normal mode perturbations of the base state, numerically obtained for several times. All perturbations are damped for $t < 55.59$. A bifurcation occurs at $t = 55.59$ when the first perturbation becomes marginally stable with a wavenumber $k = 0.06192$.

2.1 Deployment of instabilities with variable and frozen base state

This section reports the experiments conducted to evaluate the rate of growth of modes with the wavelength λ associated to $k_0 = 0.06192$. We denote this mode as "mode 4". Initial condition used in the experiments consisted of the base state at $t = 252$ plus perturbation with this wavenumber.

We investigate the deployment of instabilities with a time dependent base state by numerically integrating Eqs. 1-3.

We also investigate the rate of growth of perturbations with frozen base state. In order to extract the amplitude of mode 4 we adopted the following procedure:

1. The base state is subtracted from result of the numerical result of integration, both at the same time;
2. The result is integrated along the y direction, followed by evaluation of the Fourier transform of the result, giving the sought amplitude.

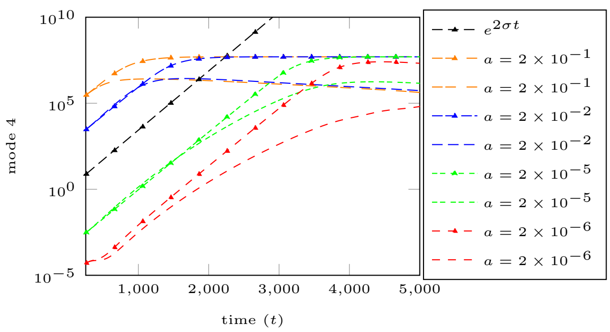

Figure 2: A comparison between the evolution of the amplitude of mode 4 with frozen base state (curves with marks) and evolving base state (curves without marks) for initial amplitudes as given in the figure.

Fig. 3 presents a plot of the rate of growth σ as a function of time with frozen base state (curves with marks) and evolving base state. This figure confirms that the rate of growth obtained from the numerical integration of the evolution equations matches the "exact" value from the linear stability analysis during the stages of linear growth.

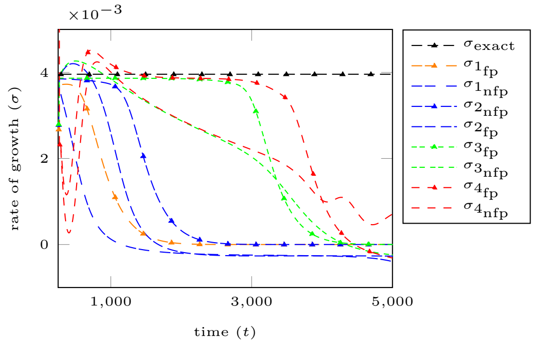

Figure 3: A comparison between the evolution of the rate of growth of mode 4 with frozen base state (curves with marks) and evolving base state (curves without marks) with the different initial amplitudes a of the perturbation (mode 4). $\sigma_{1_{\text{fp}}}$ and $\sigma_{1_{\text{nfp}}}$ refer to amplitude $a = 2 \times 10^{-1}$, $\sigma_{2_{\text{fp}}}$ and $\sigma_{2_{\text{nfp}}}$ to amplitude $a = 2 \times 10^{-2}$, $\sigma_{3_{\text{fp}}}$ and $\sigma_{3_{\text{nfp}}}$ to amplitude $a = 2 \times 10^{-5}$, $\sigma_{4_{\text{fp}}}$ and $\sigma_{4_{\text{nfp}}}$ to amplitude $a = 2 \times 10^{-6}$.

3 Conclusions

We observed that when integrating the evolution equation starting from the base state plus a perturbation a minimum initial level of this one is required to obtain the linear growth of the linear stability analysis in the first stages of evolution. If this minimum is not included in the initial condition a deviation occurs at the first stages of the linear growth, due to noise introduced by the grid.

As we allow the base state to evolve we observe a certain deviation in the rate of growth of the amplitude of modes when comparing with σ obtained from the linear analysis, due to nonlinear effects.

We conclude then that the increase in the interface gradient enhances dissolution of $\mathbf{CO_2}$.

References

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