ANALYSIS OF HEAT INTERACTION BETWEEN FINS AND VARIABLE THERMAL CONDUCTIVITY

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1 Introduction

The present work describes the thermal profile of a fin, which dissipates heat. Neumann and Dirichlet boundary conditions are established. Heat transfer analysis is performed by computational simulations. The Finite Differences Method is applied to the problem formulation. The thermal conductivity is as a function of temperature. Double fins are analyzed, where their surfaces interact thermally, generating mutual effects. Finally, the comparison of the results proves that the assumptions of variable thermal conductivity, heat dissipation by thermal radiation and mutual interaction are crucial to obtain results that are closer to reality.

2 Definitions and descriptions

The Fig. 1 presents a *Single Fin* which has only one surface extended to the primary surface and *Double Fin* which has two extended surfaces to the primary surface, where the distance between them is very small.

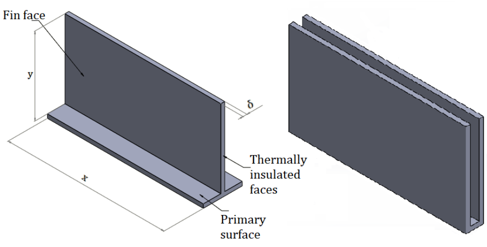

Figure 1: Fins

Some considerations should be made: The thermal transient is neglected; The fin is not a source of its own heat; and the double-fins have exactly the same geometric dimensions and are positioned symmetrically.

3 Mathematical approach

As one of the definitions of the problem is the condition of thermal steady state, the thermal distribution in the directions of each Cartesian axis of the fin, is [4]

$$\frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right)=0 \tag{1}$$

By the definitions of fins, only in the axis y the temperature differences between their points are considerable. This formulation suggests that the problem is analyzed in a one-dimensional approach.

3.1 Boundary conditions

Two mathematical boundary conditions (b.c.) were used. These conditions are very common in mathematical problems involving differential equations, as shown in Fig. 2.

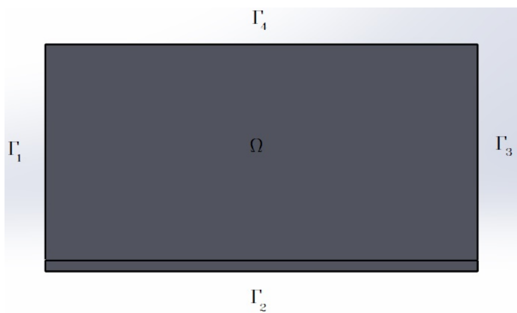

Figure 2: Boundary and domain

- The faces Γ_1 $(x = 0)$, Γ_3 $(x = L)$ and Γ_4 are thermally isolated. (Neumann b.c.)
- The face Γ_2 $(y = 0)$ is conditioned with a certain temperature imposed on it. (Dirichlet b.c.)
- In the faces $z = 0$ e $z = \delta$, heat dissipation is considered by thermal convection and radiation.

3.2 Application to modeling

Based on the assumptions and the boundary conditions presented, it follows that [3]:

$$\frac{d}{dz}\left(k\frac{d\overline{T}}{dz}\right) = -\frac{2}{\delta}[h\left(\overline{T} - T_\infty\right) + \varepsilon\sigma T^4] \tag{2}$$

Where k will be treated as a variable in function of y.

The mathematical result of the interaction between the fins in a radiation-only perspective is given by the following integral [5]:

$$E_{mut} = \int_0^b \sigma\overline{T}^4(\xi)\left(\frac{d^2}{2[(y-\xi)^2 + d^2]^{\frac{3}{2}}}\right) d\xi \tag{3}$$

3.3 Numerical methods

For the characterization of the thermal conductivity profile, it is necessary to trace parameters that relate the thermal conductivity (k) and temperature (T).

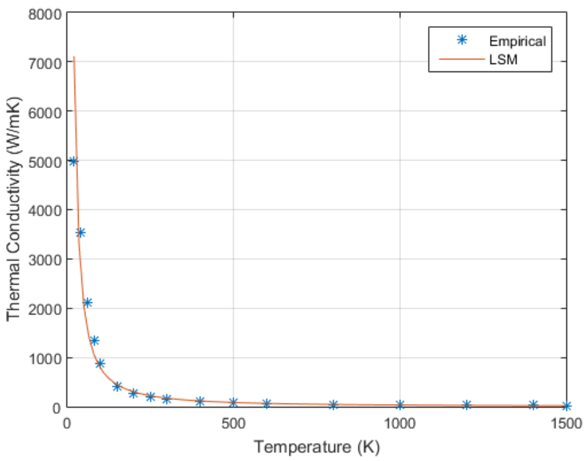

Figure 3: Curve fitting by LSM in exponential type

The method used was the *Least Squares Method* (LSM), where the empirical data were stipulated [1] and the approximation is illustrated in Fig.3.

In order to solve numerically the partial derivatives present in the problem studied, is used the Finite Differences Method(FDM). [2]

Given these relations, after algebraic adjustments takes the following form.

$$\begin{aligned} T_j = \frac{\delta}{2k_j + 2hl^2}&\left[T_{j+1}\left(\frac{k_j - k_{j-1}}{2} + k_j\right)\right. \\ &\left. -T_{j-1}\left(\frac{k_j - k_{j-1}}{2} - k_j\right)\right] + \\ &+\frac{2hl^2 T_\infty}{2k_j + 2hl^2} - \frac{2\varepsilon\sigma T_{rad}}{2k_j + 2hl^2} \end{aligned} \tag{4}$$

4 Results and conclusions

A graph in Fig.4 was generated that overlapped all the results in order to demonstrate the discrepancy between the results in all situations analyzed.

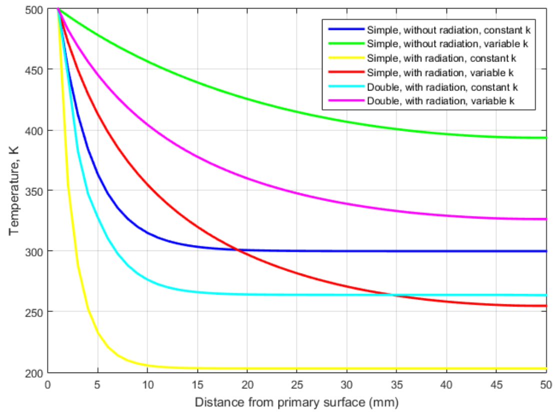

Figure 4: Comparison of generated thermal profiles

In this study, it was noted the importance of evaluating commonly overlooked phenomena, which generate very considerable discrepancies to the final results. The errors related to the expected result reached maximum values of 49.84%.

It can be concluded, therefore, that thermal dissipation analysis, in order to approach a real model, should never neglect the variation of thermal conductivity, the effects of thermal radiation and mutual interaction.

5 Acknowledgments

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