Numerical Modelling of Two-Phase Flows with moving Contact Lines

Erik Gros
erik.gros@epfl.ch

GESAR, Rio de Janeiro, August 19, 2015

Academic Background

- 02.2013 - B. Sc. Mechanical Engineering, RWTH

  Thesis: FEM simulation of viscoelastic flow

- 10.2014 - M. Sc. Aeronautical Engineering, RWTH

  Thesis: moving mesh implementation in FVM solver

- Since 03.2015 - Ph. D. student at LTCM, EPFL

  -Numerical simulation of two-phase flows

  -ALE, FEM, two-phase flow simulator

  -Contact lines, annular flow

Overview

- THERMAPOWER
- Two-Phase Flow Introduction
- Microchannel Simulation Results
- Contact Lines
- Conclusion & Further Work

Overview

- THERMAPOWER
- Two-Phase Flow Introduction
- Microchannel Simulation Results
- Contact Lines
- Conclusion & Further Work

THERMAPOWER

- Thermal Management of High Power Microsystems Using Multiphase Flows
- Funded by European Union
- Innovative cooling systems using two-phase flow for microelectronic devices
- Partner Universities:

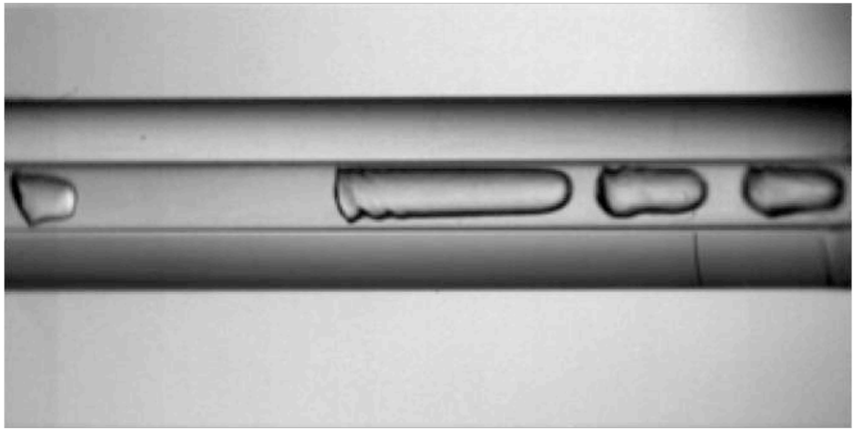
➢ University of Edinburgh, Shanghai Jiaotong, EPFL, University of Maryland, University of Nottingham and UERJ.

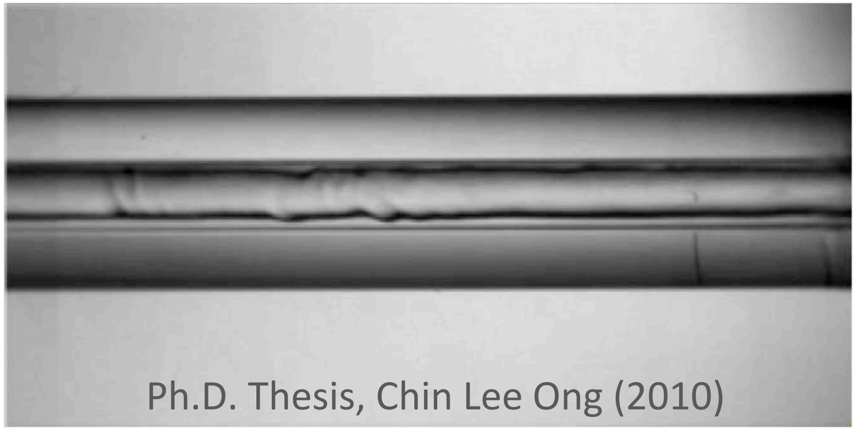
Overview

- THERMAPOWER
- Two-Phase Flow Introduction
- Microchannel Simulation Results
- Contact Lines
- Conclusion & Further Work

Two-phase flow in microchannels

- Higher heat fluxes compared to single-phase cooling
- Small scales make it difficult to perform quantitative measurements
- Numerical simulation well suited
- Typical flow conditions slug or annular flow:





Governing Equations

- One fluid formulation
- Incompressible Navier-Stokes equations:

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} - \vec{v}_{mesh}) \cdot \nabla \vec{v} = -\frac{\nabla p}{\rho} + \frac{\mu}{Re} \nabla \left(\nabla \vec{v} + (\nabla \vec{v})^T \right) + \frac{\vec{g}}{Fr^2} + \boxed{\frac{\sigma}{We} \kappa \vec{n} \delta}$$

surface tension

$$\nabla \cdot \vec{v} = 0$$

$$Re = \frac{\bar{\rho}\, \bar{v}_\infty\, \bar{L}}{\bar{\mu}}, \quad We = \frac{\bar{\rho}\, \bar{v}_\infty^2\, \bar{L}}{\bar{\sigma}}, \quad Fr = \frac{\bar{v}_\infty}{\sqrt{\bar{g}\, \bar{L}}}$$

Governing Equations

- One fluid formulation
- Incompressible Navier-Stokes equations:

inertia / viscous

inertia / surface tension

inertia / gravity

$$\boxed{Re = \frac{\bar{\rho}\,\bar{v}_\infty\,\bar{L}}{\bar{\mu}},} \quad \boxed{We = \frac{\bar{\rho}\,\bar{v}_\infty^2\,\bar{L}}{\bar{\sigma}},} \quad \boxed{Fr = \frac{\bar{v}_\infty}{\sqrt{\bar{g}\,\bar{L}}}}$$

Discretization

- Time derivative: Semi Lagrangian method

$$\frac{D\phi(\vec{x},t)}{Dt} \approx \frac{\phi(\vec{x},t+\Delta t) - \phi(\vec{x} - \vec{v}^n \Delta t, t)}{\Delta t}$$

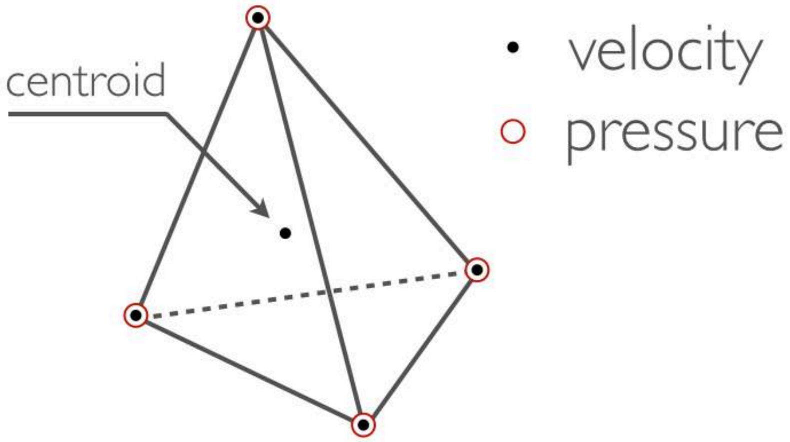
- Spatial derivatives:

➢ Finite Element Method (FEM)

➢ LBB -> Mini Element

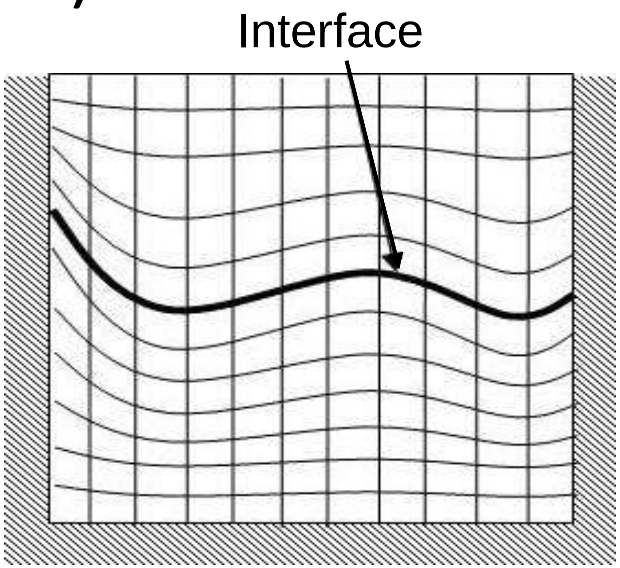
➢ Unstructured mesh

➢ 2d: triangular/ 3d: tetrahedral elements



Interface tracking

- Arbitrary Lagrangian Eulerian (ALE)
  - ➢ Interface: Lagrangian
  - ➢ Boundaries: Eulerian
  - ➢ Inside: “arbitrary”
  - ➢ Remeshing and smoothing operations to avoid mesh quality degradation



+ sharp interface, better resolution, no additional equations like VOF, Level-Set…

- expensive mesh operations, topology changes

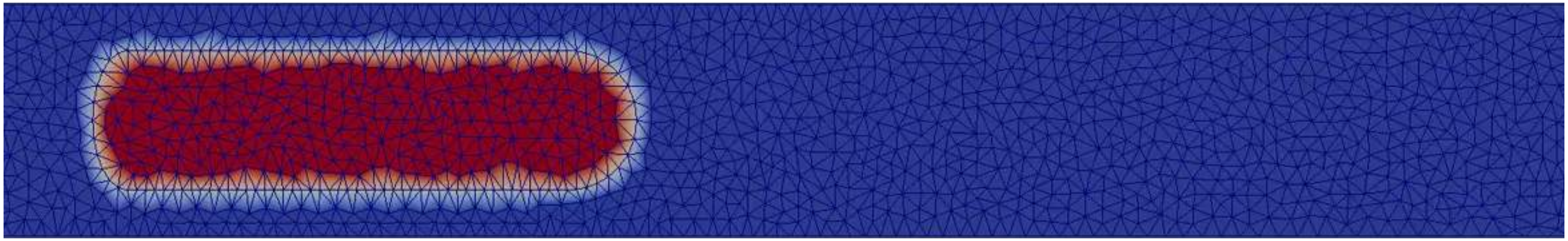
Overview

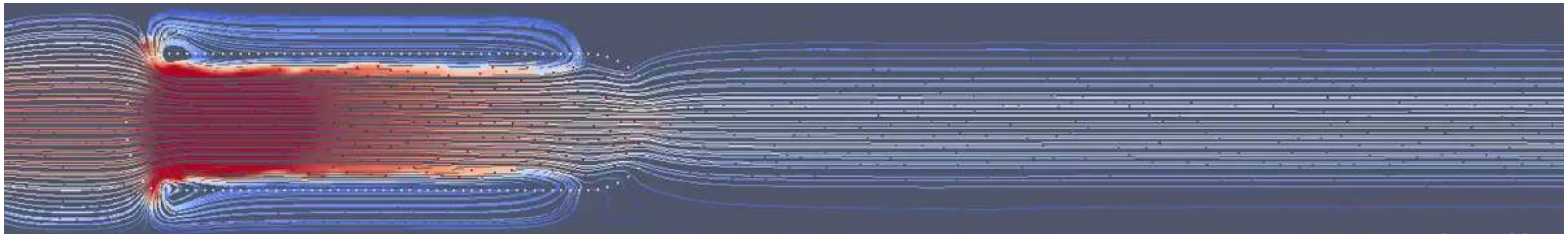
- THERMAPOWER
- Two-Phase Flow Introduction
- Microchannel Simulation Results
- Contact Lines
- Conclusion & Further Work

Microchannel Simulation

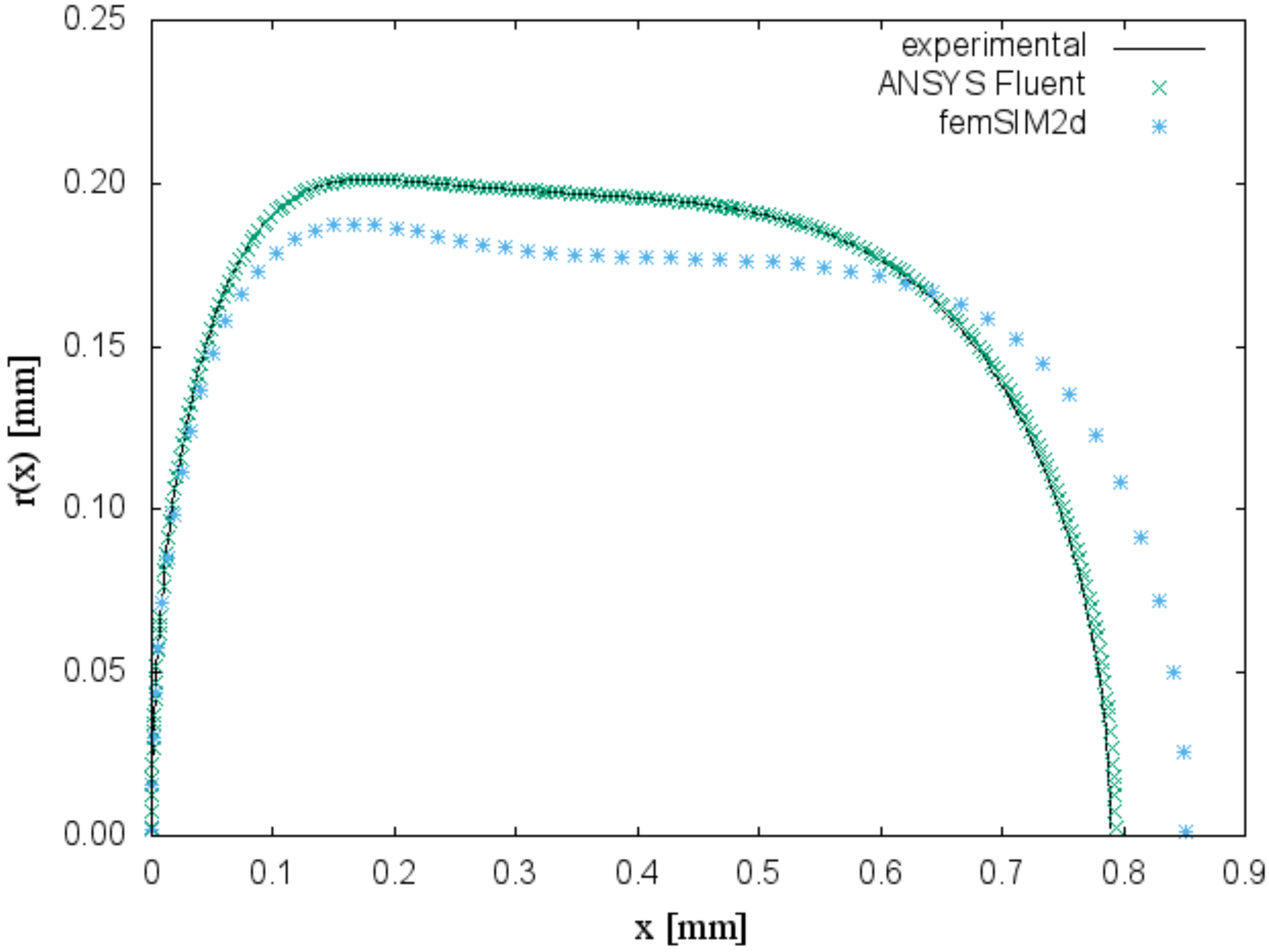
- Single elongated air bubble in glycerol solution:

$$Re = 0.0128552, \quad We = 0.00127691, \quad \frac{\rho_{in}}{\rho_{out}} = 9.632 \cdot 10^{-4}, \quad \frac{\mu_{in}}{\mu_{out}} = 3.455 \cdot 10^{-5}$$





Microchannel Simulation



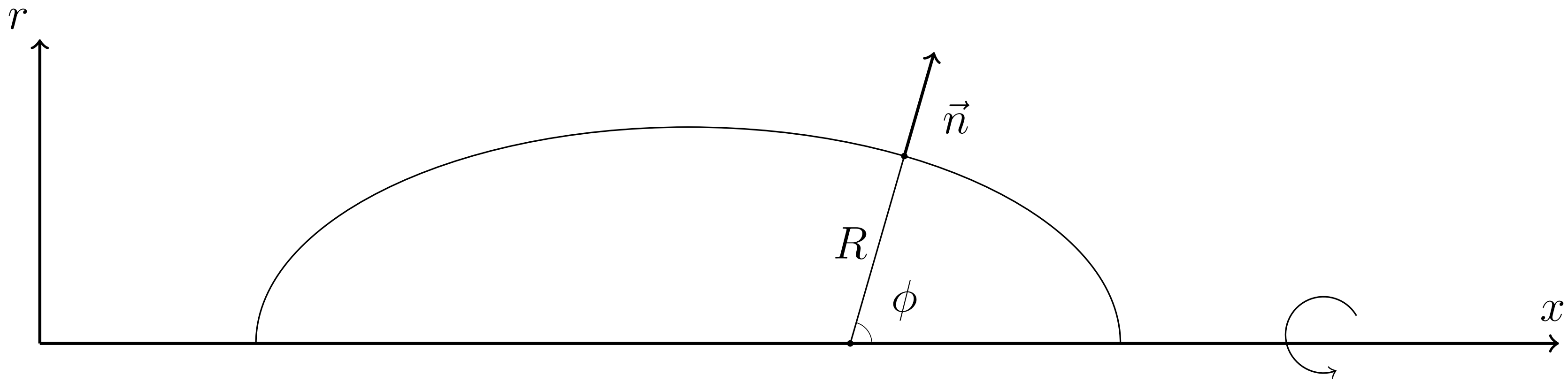
Axisymmetric Formulation

$$\rho \frac{Dv_x}{Dt} = -\frac{\partial p}{\partial x} + \mu \, \Delta v_x$$

$$\rho \frac{Dv_r}{Dt} = -\frac{\partial p}{\partial r} + \mu \left(\Delta v_r - \frac{v_r}{r^2} \right)$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$



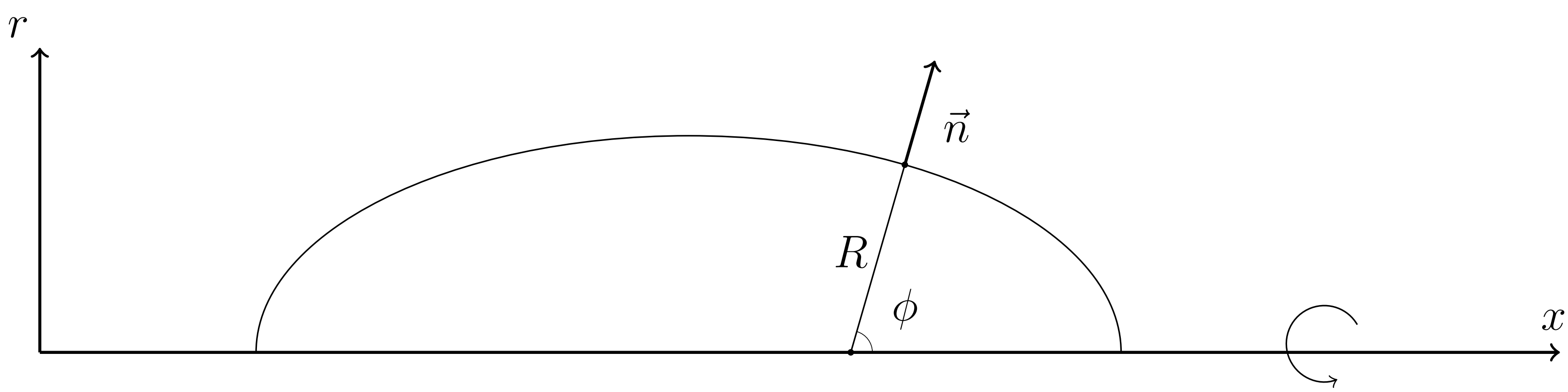
Axisymmetric Formulation

$$\rho \frac{Dv_x}{Dt} = -\frac{\partial p}{\partial x} + \mu\, \Delta v_x$$

$$\rho \frac{Dv_r}{Dt} = -\frac{\partial p}{\partial r} + \mu \left(\Delta v_r - \frac{v_r}{r^2} \right)$$

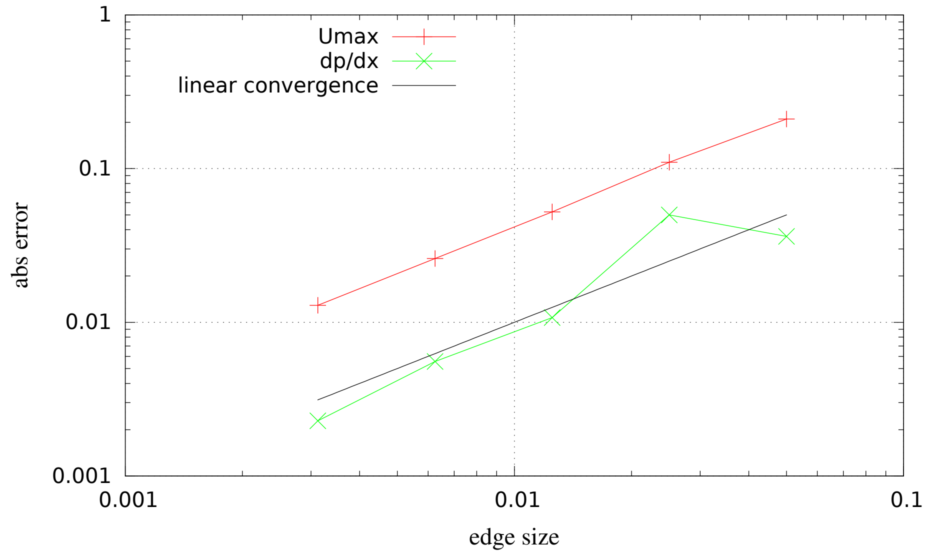
$$\frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0$$

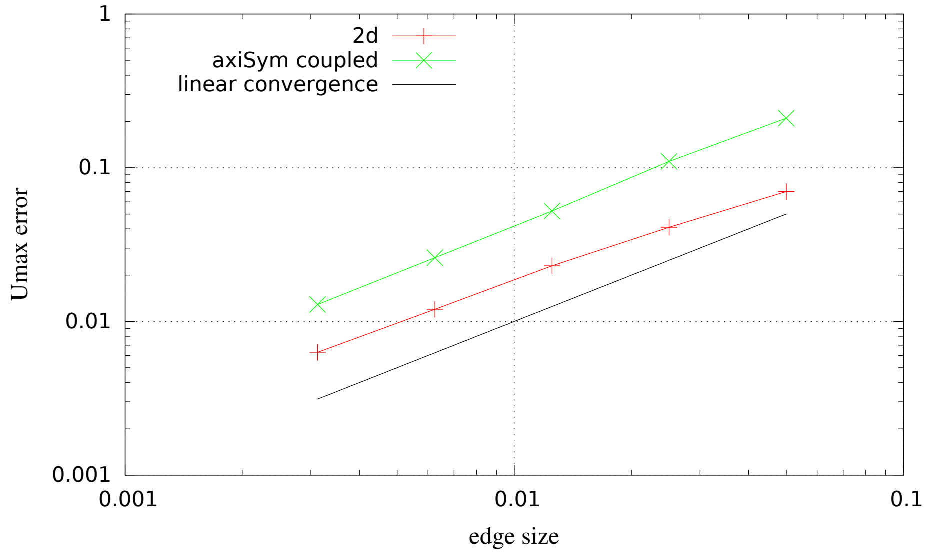
$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$



$$\kappa_{axi} = \kappa_{2d} + \frac{1}{R} = \kappa_{2d} + \frac{sin(\phi)}{r}$$

Convergence with Mesh Refinement





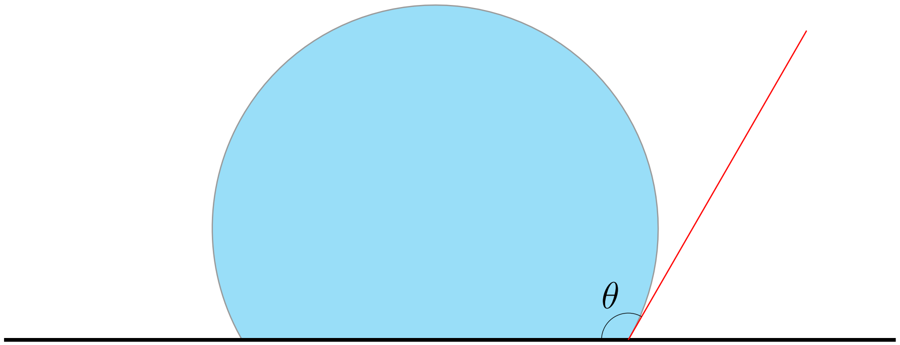
Overview

- THERMAPOWER
- Two-Phase Flow Introduction
- Microchannel Simulation Results
- Contact Lines
- Conclusion & Further Work

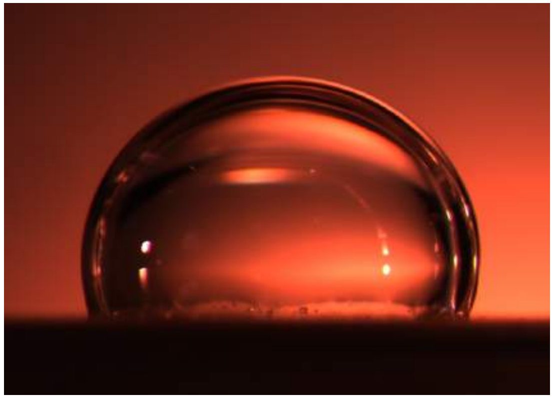
Contact Line

- Region of interaction of three different materials

Contact angle



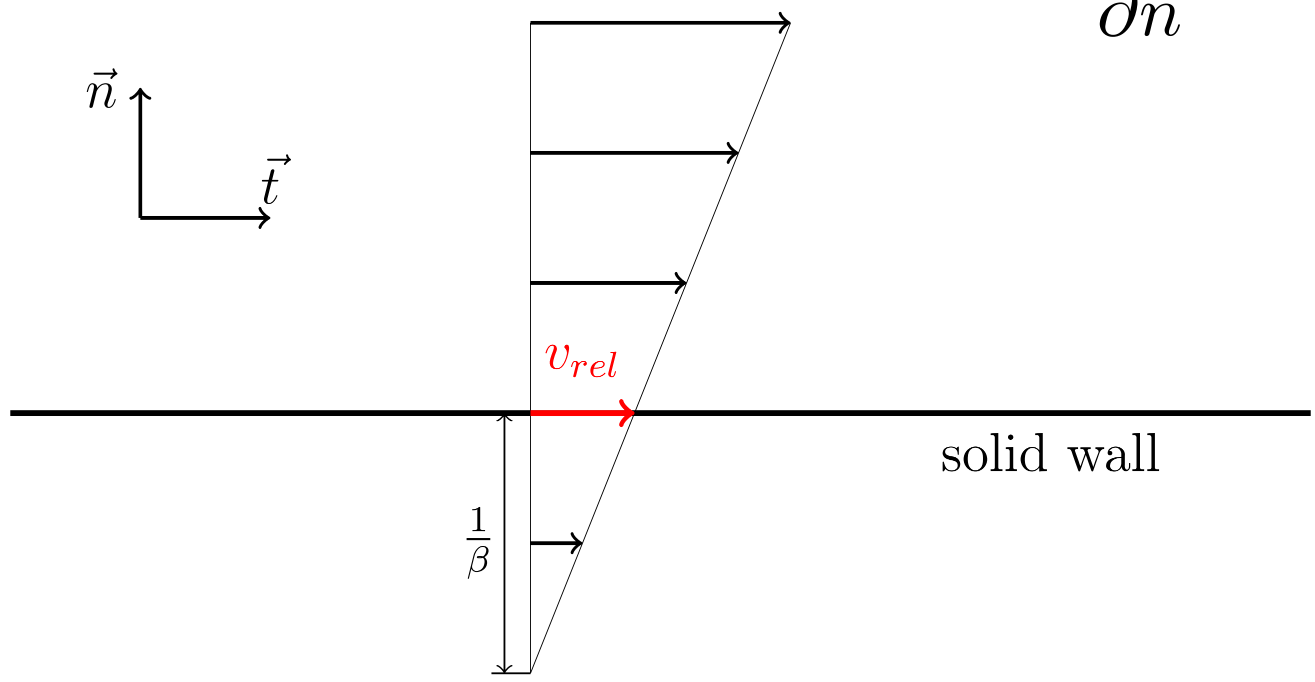
- Young's equation: $$\sigma_{gs} = \sigma_{ls} + \sigma_{lg}\, cos(\theta)$$



Moving Contact Lines

- The moving contact line problem
- Navier-Slip b.c.

$$\tau \propto \frac{\partial v_t}{\partial n} = \beta\, v_{rel}$$



Moving Contact Lines

- The moving contact line problem
- Navier-Slip b.c.

$$\vec{n} \cdot \vec{v}_{rel} = 0$$

$$\vec{t}\, \boldsymbol{\sigma}(\vec{v}, p)\, \vec{n} = \beta\, \vec{t} \cdot \vec{v}_{rel}$$

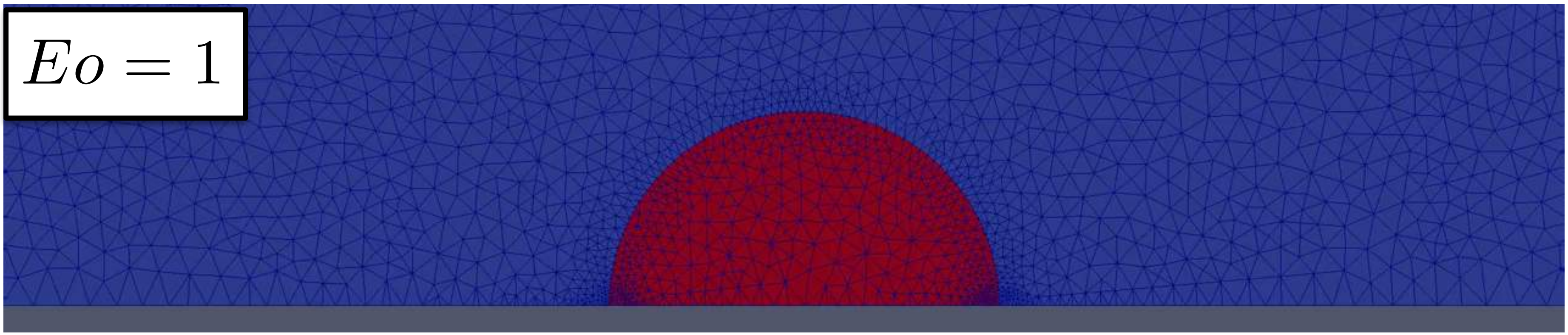
$$\vec{b}\, \boldsymbol{\sigma}(\vec{v}, p)\, \vec{n} = \beta\, \vec{b} \cdot \vec{v}_{rel}$$

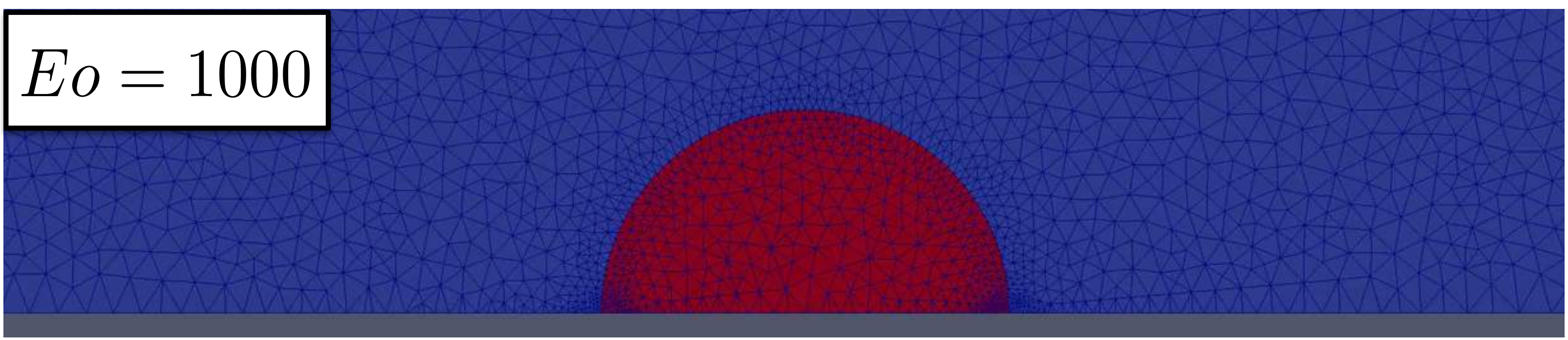
$$\vec{v}_{rel} = \vec{v} - \vec{v}_{wall}$$

Drop Spreading on a Surface

$$\theta = 50^\circ, \quad \frac{\rho_{in}}{\rho_{out}} = 100, \quad \frac{\mu_{in}}{\mu_{out}} = 1.111$$

$$\boxed{Eo = \frac{We}{Fr^2}}$$





Drop Spreading on a Surface

- Height of the bubble at steady-state

➢ $\mathbf{Eo >> 1:}$

$$h_\infty = 2\sqrt{\frac{\sigma}{\rho_l\, g}}\, sin\left(\frac{\theta}{2}\right)$$

➢ $\mathbf{Eo << 1:}$ (no gravity => spherical cap)

$$h_\infty = R_0\left(1 - cos(\theta)\right)\sqrt{\frac{\pi}{2\,\theta - sin(2\,\theta)}}$$

Overview

- THERMAPOWER
- Two-Phase Flow Introduction
- Microchannel Simulation Results
- Contact Lines
- Conclusion & Further Work

Conclusion & Further Work

- For problems with rotational symmetry axisymmetric formulation is very efficient
- Moving contact lines: imposing static contact angle can already give meaningful results

- Next steps:

➢ Navier-Slip boundary condition

➢ Dynamic contact angle

➢ Periodic boundary conditions

➢ Annular flow simulation

Thank you for your attention!