NUMERICAL SIMULATION OF FINS IN A CONTEXT OF HIGH TEMPERATURES

Author: Rodolfo Sobral[1,2] *rodolfo.sobral@cefet-rj.br*
Advisors: Rogério Martins Saldanha da Gama[1] ; Eduardo Dias Corrêa[1]

[1] State University of Rio de Janeiro
[2] Federal Center of Technological Education

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1 Introduction

The present work numerically investigates the coupled heat transfer performance for straight fin arrays including mutual irradiation between radiator elements. Numerical simulations are carried out through a sequence of linear problems, possessing an equivalent minimum principle, that has as its limit the solution of the original problem. This coupled conduction/radiation heat transfer process is an inherently nonlinear phenomenon, where this coupling on the body boundary is mathematically represented by a nonlinear relation between the absolute temperature and its exterior normal derivative, in which the unknown is the temperature distribution in the body. The solution to the problem is given by the limit of a sequence whose elements are obtained from the minimization of functionals. The problem is simulated with the aid of a finite element approximation.

2 Physycal model

Consider a heat sink with longitudinal fin of profile as shown in Fig. 1. The fin is attached to a primary surface at fixed temperature T_b and exchange heat by convection and radiation with the surrounding medium. The convective heat transfer coefficient over the exposed surface of the fin is assumed to be a constant. The heat loss from the tip of the fin compared to the lower part and finned side is assumed to be negligible. The heat conduction is assumed to occur solely in the cross-cut direction.

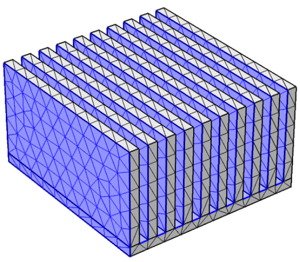

Figura 1: Longitudinal convecting-radiating copper fin

The material for the fin is chosen to be copper because of its high thermal conductivity. The investigation of heat transfer on finned surfaces has to be modeled by a bi-dimensional geometry as it is clear from Fig. 2.

2.1 Governing equations

The steady-state heat equation in a medium with temperature-dependent, inhomogeneous conductivity, such as straight fin in Fig. 2 is:

$$\frac{\partial}{\partial x}\left(k(T)\frac{\partial \overline{T}}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(T)\frac{\partial \overline{T}}{\partial y}\right) - \frac{2}{\delta}\left[h\left(\overline{T}-T_\infty\right)+\epsilon\sigma|\overline{T}|^3.\overline{T}\right]=0 \quad in \quad \Omega_1 \qquad (1)$$

where $k(T)$ is the thermal conductivity of regions Ω_i. The boundary conditions (b.c.) (see Fig. 2) are homogeneous Neumann b.c. on Γ_1, Γ_3 e Γ_4 (adiabatic boundary) and Dirichlet b.c. on Γ_2 (prescribed temperature).

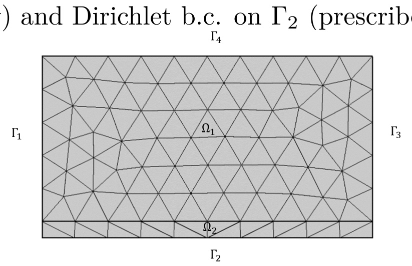

Figura 2: Two-region piecewise homogeneous fin

The coupling between the conduction heat transfer, convection and thermal radiant heat transfer is done on the body. For a convex fin surrounded by an atmosphere non-free space:

$$-k\nabla\overline{T}.\mathbf{n}=0 \;\; on \;\; \partial\Omega_1=\Gamma_1 \cup \Gamma_3 \cup \Gamma_4 \qquad (2)$$

and

$$T=T_{env}\left(\xi,\eta\right) \quad for \quad t=0; 1<\xi<H; 1<\eta<Z \qquad (3)$$

Equation (1) employed the term $|T|^3.T$ in place of the term T^4 in order to ensure the coerciveness of the operator in infinite dimension, preserving the physical structure of the phenomenon, analytical details in advisors works.

3 Kircchoff transformation

If k and q_{cond} are functions of space only, Eq. (1) becomes a linear differential equation with variable coefficients. The solution of such an equation requires no additional mathematics. But if k is dependent on temperature and independent of space, however, this equation becomes non-linear and difficult to solve. Usually, numerical methods have to be employed.

Equation(1) may be reduced to a linear differential equation by introducing a new temperature ω related to the temperature T of the problem by the Kirchhoff transformation:

$$\omega = f(T) = \int_{T_0}^{T} k(\epsilon)d\epsilon \tag{4}$$

For the copper empirical curve, there was obtained with the help of least squares method the adjusted curve $k(\epsilon)$ to be integrated in kirchhoff transformation.

The kirchhoff transformation is showed below

$$\omega = f(T) = \int_{T_0}^{T} k(\epsilon)d\epsilon = \frac{c}{(d+1)}.T^{(d+1)} \tag{5}$$

in which c and d were obtained with least square method in MatLab curve fitting.

According to

$$grad\omega = kgradT \tag{6}$$

The inverse of the above Kirchhoff transformation can be easily obtained as

$$T = f^{-1}(\omega) = e^{\frac{1}{d+1}\log\frac{\omega(d+1)}{c}} \tag{7}$$

with $\lambda = \frac{1}{d+1}\log\frac{\omega(d+1)}{c}$, the partial differential equation can be written.

$$div[grad\omega] - \frac{2}{\delta}\left[h.e^{\lambda} + \sigma|e^{\lambda}|^3.e^{\lambda}\right] = 0 \quad in \quad \Omega_1 \tag{8}$$

and the boundary condition becomes

$$-(grad\omega).\mathbf{n} = 0 \quad on \quad \partial\Omega_1. \tag{9}$$

The positiveness of the thermal conductivity ensures that ω is an increasing function of T, while T is an increasing function of ω.

4 Variational formulation

In order to ensure the existence of a minimum is necessary and sufficient to show that I, the functional associated to the physical problem is continuous, convex and coercive functional.

The solution of problem in Eq. (1) may be reached as the limit of a sequence whose elements are obtained form the minimization of a quadratic functional.

$$I[\nu] = \frac{1}{2}\int_0^H\int_0^Z\left[\left(\frac{\partial\nu}{\partial x}\right)^2 + \left(\frac{\partial\nu}{\partial y}\right)^2\right]dxdy + \int_0^H\int_0^Z\left[\frac{h}{\delta k}(\nu - T_\infty)^+ \frac{2\varepsilon\sigma}{5Dk}|\nu|^5\right]dxdy \tag{10}$$

The first variation of Eq. (10) describes the variational formulation of the problem addressed that may be mathematically represented by the minimization of a functional coercive. The existence of this minimum principle provides an easy and accurate tool for numerical simulations of such heat transfer phenomena.

In other words, the solution ω of problem given by Eq. (4) is given by

$$\omega = \lim_{i\to\infty}\Phi_i \tag{11}$$

in wich the elements of sequence $[\Phi_0, \Phi_1, \Phi_2, ..., \Phi_i]$ are obtained from

$$div\,(grad\Phi_{i+1}) = \alpha\Phi_{i+1} - \beta_i \quad in \quad \Omega_1 \tag{12}$$

and

$$-\,(grad\Phi_{i+1})\,\mathbf{n} = 0 \quad on \quad \partial\Omega_1 \tag{13}$$

The auxiliar term β:

$$\beta_i = \alpha\Phi_{i-1} - \Big(\sigma\,|\Phi_{i-1}|^3\,\Phi_{i-1} - h\,(\Phi_{i-1} - T_\infty)\Big) \quad for \quad i = 0, 1, 2, ... \tag{14}$$

where α is sufficiently large positive constant and $\Phi_0 \equiv 0$. This constant is evaluated from an priori estimate for the upper bound of the solution in Eq. (4) and ensures a bounded and nondecreasing sequence $[\Phi_0, \Phi_1, \Phi_2, ..., \Phi_i]$.

5 Results and discussion

This paper introduce the idea of Kirchoff Transformation and sequence of linear problems to find a solution of the steady-state nonlinear fins problem with temperature dependent thermal conductivity. For this, the mathematic analysis will be based on Murray–Gardner assumptions, for analysis come closer to the real-world situation.

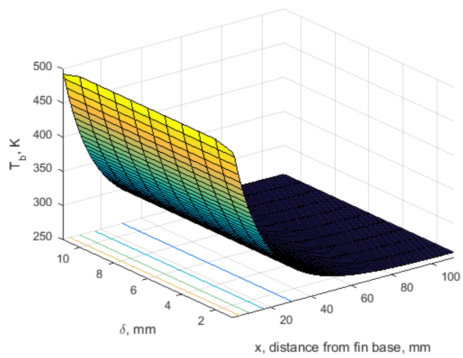

Figura 3: Temperature profile for a longitudinal convecting-radiating copper fins with mutual irradiation between radiator elements

6 Conclusions

Heat transfer analysis is an active and important engineering research field, as it increases the effectiveness of heat exchangers result in considerable technical advantages and especially in cost savings. It is noteworthy that correct and effective design of thermal load demand knowledge of heat transfer phenomena, so that erroneous considerations aren't taken compromising the integrity of all equipment. The present work shows the influence of the dependence between the temperature and the thermal conductivity on the effectiveness of finned surfaces. Results indicate that thermal conductivity can significantly impact the actual heat transfer response of a fin.