

# **FOZ2018**

## **PROGRAM**

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# Monday - July, 23

	Room Africa + Europe				
08:00h	FOZ 2018 - ICM Sattelite Event Opening Session				
	Room Africa + Europe				
	Chair: Clóvis Gonzaga				
08:30h	Alfredo Iusem				
09:15h	Sandra Augusta Santos				
10:00h	Break and BRAZIOPT Poster Session				
	Room Africa + Europe		Room Oceania	VIP Room	
	Chair: Ernesto Birgin		Chair: José Cuminato	10:20h Core Commitee Meeting	
10:30h	José Mario Martinez	10:30h	Felipe Pereira		
11:15h	Clóvis Caesar Gonzaga	11:15h	Jorge Lira		
12:00h	Lunch				
	Room Asia	Room Americas		Room Oceania	Room Mercosul
	Chair: Hugo Scolnik	Chair: Douglas S. Gonçalves		Chair: Justin Wan	Chair: Felipe Pereira
13:50h	Konstantin Khanin	Abel S. Siqueira	14:00h	Jiangong You	V.A. Vassiliev
14:15h	Joaquim Judice	John L. Gardenghi			
14:40h	Roberto Andreani	Graciela N. Sottosanto	14:45h	Qi-Man Shao	Weldon Lodwick
15:05h	Hugo Scolnik	Douglas S. Gonçalves			
15:30h	Break and BRAZIOPT Poster Session				
	Room Africa + Europe		Room Oceania	Room Mercosul	
	Chair: Alfredo Iusem		Chair: Pedro D. Damazio	Chair: Fernando Ávila	
16:00h	Benar F. Svaiter	16:00h	Qiang Li	Erick Moya	
		16:20h	Chao Wu	Maria Rojas	
		16:40h	André Jacomet Torii	Camille Poignard	
16:45h	Paulo J. S. Silva	17:00h	Akhlaq Husain	Mariana Kleina	
		17:20h	Pedro D. Damazio	Fernando Ávila	
			Room Oceania	Room Mercosul	
			Chair: Neela Nataraj	Chair: Weldon Lodwick	
		17:45h	Alexandre Madureira	Jacek Banasiak	



Communication Session



Plenary Session



Special Session



Break

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# Tuesday - July, 24

	Room Asia	Room Americas		Room Oceania	Room Mercosul
	Chair: Ademir A. Ribeiro	Chair: Hugo Lara Urdaneta		Chair: Messias Meneguetti	Chair: Cassio Oishi
08:20h	Damián Fernández	Gilson do N. Silva	08:20h	Ya-Feng Liu	Ting Wei
08:45h	Ellen H. Fukuda	Mauricio Romero Sicre	08:40h	Jian-Hua Wu	Elias Gudiño
09:10h	Juliano de B. Francisco	João C. de O. Souza	09:00h	Silas Abahia	Manuel Silvino
09:35h	Ademir A. Ribeiro	Hugo Lara Urdaneta	09:20h	Messias Meneguetti	Shuanping Du
			09:40h		Cassio Oishi
10:00h	Break and Ind Math and BRICS Poster Session				
	Room Africa + Europe			Room Oceania	Room Mercosul
	Chair: Geovani Grapiglia			Chair: Ya-Xiang Yuan	Chair: Li Weigang
10:30h	Yaxiang Yuan		10:30h	Pingwen Zhang	V.I. Lotov
11:15h	Mikhail Solodov		11:15h	Qiang Du	José Alberto Cuminato
12:00h	Lunch				
	Room Asia	Room Americas		Room Oceania	Room Mercosul
	Chair: Juan Pablo Luna	Chair: Marc Lassonde		Chair: Jorge Lira	Chair: Romes A. Borges
13:50h	Shuai Liu	Fernanda Raupp	14:00h	Tiago Pereira	S. Lubbe
14:15h	Jefferson G. Melo	Felipe Lara			
14:40h	Geovani N. Grapiglia	Marc Lassonde	14:45h	Jonathan D. Evans	Domingos Alves Rade
15:05h	Juan Pablo Luna				
15:30h	Break and Ind Math and BRICS Poster Session				
	Room Africa + Europe			Room Oceania	Room Mercosul
	Chair: Sandra A. Santos			Chair: Song Liu	Chair: XiangYun Zhang
16:00h	Philippe Toint		16:00h	Vinicius Albani	Huang Zhengda
			16:20h	Rawilson O. Araújo	Benito Pires
			16:40h	Shuqin Wang	Yu Chen
16:45h	Yu-Hong Dai		17:00h	Carlos E. Andrade	Xiaodong Zhang
			17:20h	Song Liu	XiangYun Zhang
				Room Oceania	Room Mercosul
				Chair: Ma To Fu	Chair: Domingos A. Rade
			17:45h	Jixiang Fu	João Luiz Azevedo
20:00h	Conference Dinner				

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# Thursday - July, 26

	Room Asia	Room Americas		Room Oceania	Room Mercosul
	Chair: Susana Scheimberg	Chair: Roger Behling		Chair: Elizabeth W. Karas	Chair: José dos Reis
08:20h	Vahid Mohebbi	Melissa W. Mendonça	08:20h	Mary Durojaye	Stanley Ferreira
08:45h	Pedro Jorge S. Santos	Maicon Marques Alves	08:40h	Cong Sun	Bruno Barela
09:10h	Reinier Díaz Millán	José Yunier Bello Cruz	09:00h	Luiz Carlos Matioli	Zhaofang Bai
09:35h	Susana Scheimberg	Roger Behling	09:20h	Daniela R. Cantane	Jianlong Chen
			09:40h	Elizabeth W. Karas	José dos Reis
10:00h	Break and Brazil-China Poster Session				
	Room Africa + Europe			Room Oceania	Room Mercosul
	Chair: Claudia Sagastizábal			Chair: Jixiang Fu	Chair: Natasa Krejic
10:30h	Mirjam Djur		10:30h	Marco Prate	Justin Wan
11:15h	Ernesto Birgin		11:15h	S. Kesavan	Luis Gustavo Nonato
12:00h	Lunch				
	Room Asia	Room Americas		Room Oceania	Room Mercosul
	Chair: Maria Cristina Maciel	Chair: Gabriel Haeser		Chair: Amiya Pani	Chair: Esdras P. Carvalho
13:50h	Ana Paula Chorobura	Leonardo D. Secchin	14:00h	Dipndra Prasad	Eban Mare
14:15h	Lauren K. S. Gonçalves	Luís Felipe Bueno			
14:40h	Maria de G. Mendonça	Alberto Ramos	14:45h	Ma To Fu	Mauro A. Ravagnani
15:05h	María Cristina Maciel	Gabriel Haeser			
15:30h	Break and Brazil-China Poster Session				
	Room Africa + Europe			Room Oceania	Room Mercosul
	Chair: Paulo J. S. Silva			Chair: Roberto Ribeiro S. Jr	Chair: Aleksandr A. Shanani
16:00h	Peter Richtarik		16:00h	Nanqing Ding	Jairo Rocha de Faria
			16:20h	Paulo N. S. Huertas	Felix Sadyrbaev
			16:40h	Hengling Hong	Flaviana M. Souza
16:45h	Claudia Sagastizábal		17:00h	Thelma P. B. Vecchi	Geraldo Brito Junior
			17:20h	Roberto Ribeiro S. Jr	Aleksandr A. Shanani
				Room Oceania	Room Mercosul
				Chair: Jacek Banasiak	Chair: Guoliang Chen
			17:45h	Andrey Vesnin	Mythily Ramaswamy



Communication Session



Plenary Session



Special Session



Break

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# Friday - July, 27

	Room Asia	Room Americas		Room Oceania	
	Chair: Max L. N. Gonçalves	Chair: Leandro Prudente		Chair: Paulo F. A. Mancera	
08:20h	Mael Sachine	Paulo S. M. dos Santos	08:20h	Ricardo Biloti	
08:45h	Luiz Rafael Santos	Joao Xavier da C. Neto	08:40h	Guo-Feng Zhang	
09:10h	Thiago P. da Silveira	Romulo Castillo	09:00h	Nader Jafari Rad	
09:35h	Max L. N. Gonçalves	Leandro Prudente	09:20h	Olivier Bokanowski	
			09:40h	Paulo F. A. Mancera	
10:00h	Break				
	Room Africa + Europe			Room Oceania	Room Mercosul
	Chair: Mikhail Solodov			Chair: Jonathan Evans	Chair: J. M. Martinez
10:30h	Hasnaa Zidani		10:30h	Shuhua Zhang	Natasa Krejic
11:15h	Aris Daniilidis		11:15h	Zhu Zuonong	Amiya Pani
12:00h	Lunch				
	Room Asia	Room Americas		Room Oceania	Room Mercosul
	Chair: Ovidiu Bagdasar	Chair: Orizon Pereira Ferreira		Chair: Elias Gudiño	Chair: Elias Gudiño
13:50h	Gislaine A. Perićaro	Pedro A. Soares Júnior	14:00h	Li Weigang	Neela Nataraj
14:15h	Daiana S. Viana	Adriano Delfino			
14:40h	Leonardo M. Mito	Teles Araújo Fernandes	14:45h	Hugo de la Cruz	
15:05h	Ovidiu Bagdasar	Orizon Pereira Ferreira			
15:30h	Break				
	Room Africa + Europe				
	Chair: José Mario Martínez				
16:00h	Jinyun Yuan				
	Room Africa + Europe				
16:45h	FOZ 2018 - ICM Sattelite Event Closing Session				



Communication Session



Plenary Session



Special Session



Break

**DAY 1**

**Plenarists**

# On complementary eigenvalue problems

*Alfredo Iusem*<sup>1</sup>

We introduce the Complementary Eigenvalue Problem (EiCP), and present basic results on existence and number of solutions. We discuss next some state-of-the-art methods for its solution; namely the spectral gradient method for the symmetric case and the hybrid method (combining a semismooth Newton method with a branch and bound search procedure) for the nonsymmetric case. Then we consider three extensions of the problem: The Conic Complementary Eigenvalue Problem (CEiCP), the Quadratic Complementary Eigenvalue Problem (QEiCP) and the Quadratic Conic Complementary Eigenvalue Problem (QCEiCP). We present some existence results for these problems, and describe a procedure which reduces a QEiCP to a higher dimensional EiCP (or a QCEiCP to CEiCP), so that these extensions can be efficiently solved with suitable adaptations of the above mentioned numerical methods for EiCP.

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<sup>1</sup> IMPA, Brazil;

# Optimization problems with noisy objective functions: an overview

*Sandra Augusta Santos<sup>1</sup>*

*Maria Aparecida Diniz-Ehrhardt<sup>1</sup>*

*Deise Gonçalves Ferreira<sup>1</sup>*

This presentation aims at methods for solving optimization problems with noisy objective functions. Besides presenting an overview of the literature, we focus on a recently proposed derivative-free algorithm for linearly constrained problems, combining some elements of the pattern search approach of Lewis and Torczon (2000) with ideas from the method of implicit filtering of Kelley (2011). The global convergence analysis is discussed, encompassing the degenerate case, under mild assumptions. Numerical experiments with linearly constrained problems from the literature and also with the feasible set defined by polyhedral 3D-cones with several degrees of degeneration at the solution were performed, including noisy functions that are not covered by the theoretical hypotheses. Furthermore, an instance of a parameter identification problem was considered as an application with inherent noise on the objective function, for which linear constraints are present in the model. To put PSIFA in perspective, comparative tests have been prepared, with the Pattern Search, the Implicit Filtering and the SID-PSM of Custódio and Vicente (2007). This is a joint work with Maria Aparecida Diniz-Ehrhardt and Deise Gonçalves Ferreira.

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<sup>1</sup> University of Campinas, Brazil;



# On the Levenberg-Marquardt path

*José Mario Martínez*<sup>1</sup>

This talk is mainly based on a joint work with Ernesto Birgin. Newton's method for unconstrained optimization, subject to proper regularization or special trust-region procedures, finds first-order stationary points with precision  $\varepsilon$  employing, at most,  $O(\varepsilon^{-3/2})$  functional and derivative evaluations. However, the computer work per iteration of the best-known implementations may need several factorizations per iteration or may use rather expensive matrix decompositions. We introduce a method that, preserving most features of the regularization approach, uses only one cheap factorization per iteration, as well as the same number of gradient and Hessian evaluations. We prove complexity and convergence results, even in the case in which the Hessians of the subproblems are far from being Hessians of the objective function.

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<sup>1</sup> University of Campinas, Brazil;

# Hydro-thermal power system operation: a sequential decisions stochastic problem

*Clóvis Caesar Gonzaga<sup>1</sup>*

Brazil power generation system is composed of over one hundred hydroelectric and fifty thermal plants distributed in five interconnected sub-systems and several hydrological basins. A long record of historical inflows provides data for statistical models. The optimal long term operation of such a system (say, fifty monthly decisions) is a huge problem that has to be solved every month to take short term decisions. In this talk we discuss the main features of the stochastic control model for this problem and summarize some methods that are used to tackle it, including Stochastic Dynamic Programming (primal and dual), deterministic optimization of chosen scenarios and non-anticipative scenario optimization. The main focus is not on algorithmic details: we discuss the main features of the methods and their limitations, and compare results for a small real life system.

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<sup>1</sup> UFSC, Brazil;

# The Multiscale Perturbation Method for Reservoir Simulation

*Luis Felipe Pereira*<sup>1</sup>

In the formulation of multiscale methods based on domain decomposition procedures for second order elliptic equations (see MMMFEM[1], MuMM[2], MRC[3], MHM[4] and references therein), the computational domain is divided into non-overlapping subdomains and for each subdomain a set of multiscale basis function is constructed. Consider the application of one of these procedures to the numerical solution of a multiphase porous media flow problem where, through an operator splitting algorithm, the velocity-pressure and transport equations are solved sequentially. From a time step to the next the multiscale basis functions should be recomputed because of the coupling of the underlying PDEs. Instead of recomputing all multiscale basis functions every time step of a numerical solution we propose the Multiscale Perturbation Method (MPM) to approximate velocity and pressure fields. In MPM an approximate solution for a new time is obtained by combining regular perturbation theory with multiscale basis functions computed at an earlier time. We focus the discussion on the MuMM, but the proposed method is also applicable to the other multiscale procedures mentioned above. The connection of MPM with state-of-the-art uncertainty quantification procedures will also be indicated. This is joint work with A. Alsadig, H. Mankad (UT Dallas, USA) and F. S. Sousa (USP, Brazil).

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<sup>1</sup> USA

# Mean curvature flow solitons in Riemannian products

*Jorge Lira*<sup>1</sup>

*Marco Rigoli*<sup>2</sup>

*Luis Alías*<sup>3</sup>

Mean curvature flow solitons are self-similar solutions of the mean curvature flow that describe the limits after rescaling of the singularities of the flow in Euclidean space. We will present a general definition of mean curvature flow solitons that provide a natural framework to their study in Riemannian manifolds endowed with infinitesimal symmetries or conformal transformations. Then we discuss some examples and their characterizations as well as some open problems.

This talk is based on recent papers in collaborations with Marco Rigoli (Milano) and Luis Alías (Murcia, Spain) and Francisco Martín (Granada, Spain).

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<sup>1</sup> Federal University of Ceará, Brazil;

<sup>2</sup> Università degli Studi di Milano, Italy;

<sup>3</sup> Universidad de Murcia, Spain;

# On Dry Ten Martini Problem

*Jiangong You*<sup>1</sup>

The spectrum and gaps of the almost Mathieu operator played an important role in Thouless' theoretical explanation of integer quantum Hall effect. The Almost Mathieu operator also attracted enormous attention of mathematicians. One of the biggest breakthrough is the solution of Ten Martini problem by Avila and Jitomirskaya. In this talk, I will briefly introduce this subject and emphasize the recent progress on Dry Ten Martini problem, which is the original problem raised by Mark Kac (Ten Martini problem is a weaker version of Dry Ten Martini problem). The talk is based on a joint work with Avila and Zhou.

<sup>1</sup> NJU, China;

# Title and Abstract to be announced

*Qi-Man Shao*<sup>1</sup>

To be announced;

<sup>1</sup> CUHK, Hong Kong;

# Ramified volumes and waves

*Victor A. Vassiliev*<sup>1</sup>

Some applications of the monodromy theory to the integral geometry and theory of PDEs will be demonstrated. In particular, we prove a multidimensional generalization of the Newton's Lemma XXVIII (the volumes cut off by hyperplanes from a bounded domain with smooth boundary in  $\mathbb{R}^{2k}$  never define an algebraic function on the space of hyperplanes), discuss an analogy of this problem with the lacuna problem in the theory of hyperbolic PDEs, and present a new series (in addition to the Archimedes' example of odd-dimensional balls) of domains in Euclidean spaces such that the volume functions are locally algebraic on the space of hyperplanes intersecting these domains.

<sup>1</sup> Steklov Math. Institute, Russia;

# Thirty-six Years of Mathematics Applied to Industrial and Public Institutional Problems – A Pedagogical Approach

*Weldon Lodwick<sup>1</sup>*

To be announced.

<sup>1</sup> University of Colorado, United States;



# A decomposition method for the sum of maximal monotone operators

*Benar Fux Svaiter*<sup>1</sup>

I will talk about a decomposition method for the sum of maximal monotone operators.

<sup>1</sup> IMPA, Brazil;

# Low order value support vector machines

*Paulo J. S. Silva<sup>1</sup>*

*R. Andreani,*

*G. Haeser,*

*L. Secchin*

In recent years, the theoretical convergence of iterative methods for solving nonlinear constrained optimization problems has been addressed using sequential optimality conditions, which are satisfied by minimizers independently of constraint qualifications (CQs). Even though there is a considerable literature devoted to sequential conditions for standard nonlinear optimization, the same is not true for Mathematical Problems with Complementarity Constraints (MPCCs). In this paper, we show that the established sequential optimality conditions are not suitable for the analysis of convergence of algorithms for MPCC. We then propose new sequential optimality conditions for usual stationarity concepts for MPCC, namely, weak, Clarke and Mordukhovich stationarity.

We call these conditions AW-, AC- and AM-stationarity, respectively. The weakest MPCC-tailored CQs associated with them are also provided. We show that some of the existing methods for MPCC reach AC-stationary points, extending previous convergence results. In particular, the new results include the linear case, not previously covered.

<sup>1</sup> University of Campinas, Brazil;

<sup>2</sup> Federal University of Ouro Preto, Brazil;

# LSD: a robust and efficient finite element method for solving elliptic PDEs

*Alexandre Madureira*<sup>1</sup>

The Localized Spectral Decomposition finite element method is based on a hybrid formulation of elliptic partial differential equations, that is then transformed via several space decompositions. Such decompositions make the formulation embarrassingly parallel and efficient, in particular in the presence of multiscale coefficients. It differs from the myriad of methods out there since it requires solution's minimum regularity. Also, it is robust with respect to high contrast coefficients.

<sup>1</sup> LNCC, Brazil;

# Dynamical systems on networks - a playground of discrete and continuous mathematics

*Jacek Banasiak*<sup>1</sup>

Recently there has been an interest in dynamical problems on graphs, where some evolution operators, such as transport or diffusion, act on the edges of a graph and interact through nodes. One can mention here quantum graphs, diffusion on graphs in probabilistic context, transport problems, both linear and nonlinear, or migrations. In this talk we shall focus on general linear transport problems posed on networks consisting of one dimensional domains, called edges, which are coupled through transmission conditions between an arbitrary selection of the endpoints of the edges. This allows for communication between the domains which not necessarily are physically connected, and makes it possible to consider, within the same framework, not only classical transport and diffusion problems on graphs but also models such as Lebovitz-Rubinov type models describing mutations in dividing cells. In this talk we address the following problems: Well-posedness of the problems; Conditions under which such generalized models have classical graph representation; Chaotic and asymptotically periodic dynamics of network transport. Asymptotic state lumping; that is, conditions under which such network problems can be approximated by appropriately constructed system of ordinary differential equations.

<sup>1</sup> University of Pretoria, South Africa;

# **DAY 1**

## **Communicators**

# On variational problems for random Lagrangian systems and KPZ universality

*Konstantin Khanin*<sup>1</sup>

We shall discuss the problem of global minimizers for random Lagrangian systems. While the situation in the compact setting is well understood by now, the case of unbounded space remains largely open. We shall also discuss a connection with the problem of KPZ (Kardar-Parisi-Zhang) universality.

<sup>1</sup> University of Toronto, Canada;

# Standard Fractional Quadratic Programming and Eigenvalue Complementarity Problem

*Joaquim Judice*<sup>1</sup>

*Alfredo Iusem*<sup>2</sup>

*Masao Fukushima*<sup>3</sup>

*Valentina Sessa*<sup>4</sup>

In this talk, we address the computation of a Stationary Point (SP) for the Standard Fractional Quadratic Program (SFQP). It is shown that this problem is equivalent to an Eigenvalue Complementarity Problem (EiCP) with symmetric matrices. EiCP is an extension of the traditional Eigenvalue Problem. We discuss iterative algorithms for the solution of symmetric and nonsymmetric EiCPs, namely an Alternative Direction Method of Multipliers (ADMM) and a splitting algorithm. Some results concerning the convergence of these algorithms are introduced. The splitting algorithm is shown to perform well for the symmetric EiCP (SP of SFQP). ADDM is in general robust but slow for symmetric and nonsymmetric EiCP. A hybrid method combining ADMM and the semi-smooth Newton (SN) method is introduced and is shown to be efficient for solving symmetric and nonsymmetric EiCP.

<sup>1</sup> Instituto de Telecomunicações, Portugal;

<sup>2</sup> IMPA, Rio do Janeiro, Brazil;

<sup>4</sup> Nanzan University, Japan

<sup>5</sup> University of Sannio, Benevento, Italy;

# A sequential optimality condition associated to quasinormality and its algorithmic consequences

*Roberto Andreani*<sup>1</sup>

*Nadia Fazzio*<sup>2</sup>

*Maria Laura Schuverdt*<sup>2</sup>

*Leonardo Secchin*<sup>3</sup>

In the present paper, we prove that the Augmented Lagrangian method converges to KKT points under the quasinormality constraint qualification, which is associated to the external penalty theory. For this purpose, a new sequential optimality condition, called PAKKT, for smooth constrained optimization is dened. The new condition takes into account the sign of the dual sequence, constituting an adequate sequential counterpart to the (extended) Fritz-John necessary optimality conditions popularized by Bertsekas and Hestenes. We also define the appropriate strict constraint qualification associated with the PAKKT sequential optimality condition and we prove that it is strictly weaker than both quasinormality and cone continuity property constraint qualifications. This generalizes all previous theoretical convergence results for the Augmented Lagrangian method in the literature.

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<sup>1</sup> University of Toronto, Canada;

<sup>2</sup> Universidad de la Plata, Argentina;

<sup>3</sup> Universidade Federal de Espírito Santo, Brazil;



# An approach for stabilizing simulation of stochastically perturbed systems

*Hugo de la Cruz*<sup>1</sup>

To be announced.

<sup>1</sup> FGV-Rio, Brazil;

# A Regularized Interior-Point Method for Constrained Nonlinear Least Squares

*Abel Soares Siqueira*<sup>1</sup>

*Dominique Orban*<sup>2</sup>

We propose an interior-point algorithm for constrained nonlinear least-squares problems based on the primal-dual regularization of Friedlander and Orban (2012). At each iteration, we solve a linear system with a symmetric quasi-definite matrix. This system can be solved via LDLT factorization or with the use of iterative methods for linear least squares. This last approach results in a factorization-free implementation, that is, one using only matrix-vector products, which is desirable for large-scale problems.

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<sup>1</sup>UFPR - Federal University of Parana, Brazil;

<sup>2</sup>GERAD/Polytechnique Montreal, Canada.

# On the use of third-order derivatives in regularization methods

*John L. Gardenghi<sup>1</sup>*

*Ernesto G. Birgin<sup>2</sup>*

*Jose M. Martínez<sup>1</sup>*

*Sandra A. Santos<sup>1</sup>*

In the context of complexity analysis in nonlinear optimization, a recent interest in regularization methods had a surge in the last years. In particular, it was shown in a recent paper that worst-case evaluation complexity  $O(\epsilon^{-(p+1)/p})$  may be obtained by means of algorithms that employ sequential approximate minimizations of  $p$ -th order Taylor models plus  $(p+1)$ -th order regularization terms. This result generalizes the case  $p=2$ , known since 2006 and successfully implemented afterwards. The natural question that we made was if there was a reasonable implementation for the case  $p=3$ , i.e., the case for which we apply third-order derivatives of the objective function and fourth-order regularization models. We present the algorithm and numerical results of such an implementation, with classic problems from the literature.

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<sup>1</sup> IMECC-Unicamp, Brazil;

<sup>2</sup> IME-USP, Brazil;

# A structured SQP algorithm for solving the constrained least squares problem

*Graciela Noemi Sottosanto<sup>1</sup>*

*Graciela Marta Croceri<sup>1</sup>*

*Gonzalo Pizarro<sup>1</sup>*

In this work, we propose a method that belongs to the class of sequential quadratic programming (SQP) for solving the nonlinear least squares problem with equality and inequality constraints. In order to exploit the structure present in the problem, a structured secant approach of the Hessian matrix, belonging to the BFGS family, is used. To enlarge its convergence region, techniques of trust region methods are employed. As a merit function, an augmented Lagrangian function is used to avoid the need of calculating second order correction steps. A feasibility restoration phase is introduced if inconsistency in the subproblem occurs. During the restoration phase the trial steps are determined in two phases. First, the minimum constraint violation that can be achieved within the trust region bound is determined. Then a second subproblem is solved where the violated constraints are relaxed. The quality of a calculated trial step is evaluated by means an update scheme for the penalty parameter. The presented algorithm is implemented in Scilab. Some numerical results are given to compare the proposed algorithm with some existing methods.

<sup>1</sup> Universidad Nacional del Comahue, Argentina;

# Local convergence of Levenberg-Marquardt methods for nonzero-residue nonlinear least-squares problems under an error bound condition

*Douglas S. Gonçalves<sup>1</sup>*

*Sandra A. Santos<sup>2</sup>*

*Roger Behling<sup>1</sup>*

The Levenberg-Marquardt method (LMM) is widely used for solving nonlinear equations and nonlinear least-squares problems. For consistent systems of nonlinear equations or zero-residue nonlinear least-squares problems, many recent papers have proved the local convergence of LMM for suitable choices of the regularization parameter and under error bound conditions, that are weaker than non-singularity assumptions. In this study, we consider the class of non-zero residue nonlinear least-squares problems and, by viewing the LM model as a Quasi-Newton model with quadratic regularization, we present a local convergence analysis for LMM under a different error bound condition.

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<sup>1</sup> UFSC - Federal University of Santa Catarina, Brazil;

<sup>2</sup> UNICAMP - University of Campinas, Brazil;

# The Thermistor Problem with Hyperbolic Electrical

*Mary Durojaye*<sup>1</sup>

*J. T. Agee*<sup>2</sup>

This paper presents the steady state solution of the one-dimensional, positive temperature coefficient (PTC) thermistor equation, using the hyperbolic-tangent function as an approximation to the electrical conductivity of the device. The hyperbolic-tangent function describes the qualitative behaviour of the evolving solution of the thermistor in the entire domain. The steady state solution using the new approximation yielded a distribution of device temperature over the spatial dimension and all the phases of temperature distribution of the device without having to look for a moving boundary which has been a major problem encountered in literature. The analysis of the steady state solution and the numerical solution of the unsteady state is presented in the paper.

<sup>1</sup> University of Abuja, Nigeria;

<sup>2</sup> University of KwaZulu - Natal, South Africa;

# Superconvergence of edge finite element solution for Maxwell's

*Chao Wu*<sup>1</sup>

*Jinyun Yuan*<sup>2</sup>

*Yunqing Huang*<sup>3</sup>

In this talk, we discuss the superconvergence of edge finite element solution for Maxwell's equations. First, we solve Maxwell's equations by linear edge finite element method on both uniform triangular mesh and strongly regular triangle mesh, we obtain superconvergence results at the interior edge by using the average technology. Second, we resolve Maxwell's equations by linear edge finite element method on both uniform tetrahedral mesh and strongly regular tetrahedral mesh, we obtain superconvergence results at the interior edge by using the average technology. Third, we use the second order and third order rectangular edge finite element method to solve the harmonic Maxwell's equations, we obtain the superconvergence results at Gauss point. Finally, numerical examples to testify our theories are presented.

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<sup>1</sup> School of Mathematics and Computational Science, Hunan University of Science and Technology, China;

<sup>2</sup> Universidade Federal do Paraná, Brazil;

<sup>3</sup> Hunan Key Laboratory for Computation and Simulation in Science, China;

# Topology Optimization in the context of heat equation

*André Jacomel Torii<sup>1</sup>*

*Diogo Pereira da Silva Santos<sup>2</sup>*

In this work we address the problem of Topology Optimization in the context of the stationary heat equation. In particular, we seek the optimum distribution of material inside a design domain subject to heat generation and/or heat external flow that minimizes the norm of the temperature field, while satisfying a prescribed amount of material to be employed. The main objective of this work is to present, in the simplest manner possible, the relation between theoretical and numerical aspects of the problem. Emphasis is given to sensitivity analysis, where both the variational problem and its Finite Element Method (FEM) approximation are presented. We also describe in details the Adjoint Method. Finally, numerical examples are presented in order to illustrate the main properties of the problem under study.

<sup>1</sup> UNILA, Brazil;

<sup>2</sup> LNCC, Brazil;



# Spectral element methods for three dimensional elliptic problems with smooth interfaces

*Akhlaq Husain<sup>1</sup>*

*Arbaz Khan<sup>2</sup>*

Many problems in engineering are characterized by elliptic partial differential equations with discontinuous coefficients, steady state heat diffusion, electro static, multi-phase and porous flow problems are the few examples. An interface problem is a special case of an elliptic partial differential equation with discontinuous coefficients. Such interface problems arise in different situations, for example, in heat conduction or in elasticity problems whose domain of definitions are composed of several different materials. In this talk we propose a least-squares spectral element method for elliptic interface problems in three dimensions with smooth interface. The solution is obtained by solving the normal equations using preconditioned conjugate gradient method. The method is essentially nonconforming and a diagonal matrix is constructed as a preconditioner based on the stability estimate and separation of variables technique. We prove that the proposed method gives exponential converges with respect to the number of elements. Numerical results for a number of test problems are presented to validate the theory and our estimates of computational complexity of the proposed method.

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<sup>1</sup> BML Munjal University Gurgaon, India;

<sup>2</sup> University of Manchester, United Kingdom;

# Existence and regularity of solutions of the magnetohydrodynamic system with mass diffusion

*Pedro Danizete Damazio*<sup>1</sup>

*Enrique Fernández-Caras*<sup>2</sup>

*Marko A. Rojas-Medar*<sup>3</sup>

In this talk we will present results on the existence and regularity of solutions for the model of the magnetohydrodynamic equations in presence of mass diffusion in regular enough bounded domains in 2 and 3 spatial dimension.

<sup>1</sup> Universidade Federal do Paraná, Brazil;

<sup>2</sup> Universidad de Sevilla. Spain;

<sup>3</sup> Universidad de Tarapacá, Chile;

# Stabilization for a Sub-quintic Wave Equation with Localized Nonlinear Damping

*Maria Rosario Astudillo Rojas*<sup>1</sup>

We consider the semilinear wave equation posed in an inhomogeneous medium with smooth boundary subject to a non linear damping distributed around a neighborhood of the boundary according to the Geometric Control Condition. We show that the energy of the wave equation goes uniformly to zero for all initial data of finite energy phase-space. We assume a nonlinearity which is subcritical in the sense that it grows as a power of at most  $p < 5$  in three dimensions. The method of proof combines Strichartz's estimates, results by P. Gerard on microlocal defect measures and ideas first introduced in the literature by Lasiecka and Tataru in order to deal with the nonlinear damping term.

<sup>1</sup> Universidade Federal do Paraná, Brazil;

# Impacts of Structural Perturbations on the dynamics of Networks

*Camille Poignard*<sup>1</sup>

*Jan Philipp Pade*<sup>2</sup>

*Tiago Pereira*<sup>1</sup>

We study the effects of structural perturbations on the dynamics of networks. We first show how the synchronizability of a diffusive network increases (or decreases) when we add some links in its underlying graph. This is of interest in the context of power grids where people want to prevent from having blackouts, or for neural networks where Synchronization is responsible of many diseases such as Parkinson. Based on spectral properties for Laplacian matrices, we show some classification results obtained (with Tiago Pereira and Philipp Pade) with respect to the effects of these links. Then I will show how we can desynchronize (i.e induce chaos) in a stable network by adding links to it.

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<sup>1</sup> Universidade de São Paulo, Brazil;

<sup>2</sup> Humboldt University, Brazil;

# A new algorithm for clustering based on kernel density estimation

Mariana Kleina<sup>1</sup>

Luiz Carlos Matioli<sup>1</sup>

Solange Regina dos Santos<sup>2</sup>

In this paper, we present an algorithm for clustering based on univariate kernel density estimation, named ClusterKDE. It consists of an iterative procedure that in each step a new cluster is obtained by minimizing a smooth kernel function. Although in our applications we have used the univariate Gaussian kernel, any smooth kernel function can be used. The proposed algorithm has the advantage of not requiring a priori the number of cluster. Furthermore, the ClusterKDE algorithm is very simple, easy to implement, well-defined and stops in a finite number of steps, namely, it always converges independently of the initial point. We also illustrate our findings by numerical experiments which are obtained when our algorithm is implemented in the software Matlab and applied to practical applications. The results indicate that the ClusterKDE algorithm is competitive and fast when compared with the well-known Clusterdata and K-means algorithms, used by Matlab to clustering data.

<sup>1</sup> Universidade Federal do Paraná, Brazil;

<sup>2</sup> Universidade Estadual do Paraná, Brazil;

# Global Hypoellipticity on Manifolds and Fourier Expansion of Elliptic Operators

*Fernando de Ávila Silva*<sup>1</sup>

*Alexandre Kirilov*<sup>1</sup>

In this work we present recent results on the investigation of the Global Hypoellipticity of operators in the class

and its corresponded perturbations

where  $T = \mathbb{R}/2\pi\mathbb{Z}$  stands for the flat torus,  $M$  is a closed smooth manifold and  $C(t, D_x)$  is an operator on  $M$  smoothly depending on the periodic variable  $t$ . We propose a novel approach, as far as we know, based on generalizations for parameter depending operators which were inspired by:

i) J. Hounie's abstract approach (Trans AMS, 1979) for the study of operator  $\partial_t + b(t, A)$ , with  $t$  in  $T$  and  $A$  being a linear self-adjoint operator, densely defined in a separable complex Hilbert space  $H$ , with suitable conditions;

ii) S. Greenfield's and N. Wallach's paper (Trans AMS, 1973) where the authors investigate the Global Hypoellipticity of invariant differential operators with respect to the eigenspaces of a fixed elliptic normal differential operator  $E$ ;

iii) the recent generalization of the notion of invariance for elliptic pseudo-differential operators on compact manifolds given by J. Delgado and M. Ruzhansky (C.R.Math. Acad. Sci., 2014) where the authors use a discretization approach based on the Fourier expansions for characterizing functional spaces defined by R.T. Seeley, (Proc. AMS, 1965 and 1969). In our approach we assume that  $C(t, D_x)$  belongs to the commutator

<sup>1</sup> Universidade Federal do Paraná, Brazil;

# **DAY 2**

## **Plenarists**

# Stochastic Proximal Quasi-Newton methods for Nonconvex Composite Optimization

*Ya-Xiang Yuan*<sup>1</sup>

In this paper, we propose a generic algorithmic framework for proximal stochastic quasi-Newton (SPQN) methods to solve nonconvex composite optimization problems. Stochastic second-order information is explored to construct proximal subproblem. Under mild conditions we show the non-asymptotic convergence of the proposed algorithm to stationary point of original problems and analyze its computational complexity. Besides, we extend the proximal form of Polyak-Lojasiewicz(PL) inequality to constrained settings and obtain the constrained proximal PL (CP-PL) inequality. Under CP-PL inequality linear convergence rate of the proposed algorithm is achieved. Moreover, we propose a modified self-scaling symmetric rank one (MSSR1) incorporated in the framework for SPQN method, which is called stochastic symmetric rank one (StSR1) method. Finally, we report some numerical experiments to reveal the effectiveness of the proposed algorithm (joint work with Xiaoyu Wang and Xiao Wang).

<sup>1</sup> Chinese Academy of Sciences, China;



# A globally convergent Linear-Programming-Newton method for piecewise smooth constrained equations

*Mikhail Solodov*<sup>1</sup>

The LP-Newton method for constrained equations, introduced some years ago, has powerful properties of local superlinear convergence, covering both possibly nonisolated solutions and possibly nonsmooth equation mappings. We develop a related globally convergent algorithm, based on the LP-Newton subproblems and linesearch for the equation's infinity-norm residual. In the case of smooth equations, global convergence of this algorithm to B-stationary points of the residual over the constraint set is shown, which is a natural result: nothing better should generally be expected in variational settings. However, for the piecewise smooth case only a property weaker than B-stationarity can be guaranteed in general. We then develop an additional procedure for piecewise smooth equations that avoids undesirable accumulation points, thus achieving the intended property of B-stationarity.

<sup>1</sup> IMPA, Brazil;

# Predictor Reconstitution from Data

*Pingwen Zhang*<sup>1</sup>

Predictor reconstitution method is a modeling based on the data concluded from the weather consultation of the forecasters. By means of machine learning, this method can reconstitute the predictors of European Centre for Medium-Range Weather Forecasts (ECMWF) model combined with the observations and give the weather forecast results. As an example, we used this method to forecast the temperature at the Yanqing weather station of Beijing (a station nearby the Yanqing Zone of 2022 Beijing Olympic Winter Games) and that in the whole Beijing area respectively. Compared with classical Model Output Statistic (MOS) prediction, predictor reconstitution method can dramatically improve the accuracy of temperature forecast and reduce the root-mean-square errors.

<sup>1</sup> Peking University, China;

# Title and Abstracts to be announced

*Qiang Du*<sup>1</sup>

TBA

<sup>1</sup> TBA

# Factorization method in boundary crossing problems for random walks

*Vladimir I. Lotov*<sup>1</sup>

It is well known that the study of many statistical tests (sequential analysis, change point problem, Kolmogorov-Smirnov test, etc.) leads to the boundary crossing problems for random walks. We demonstrate an analytical approach to a number of problems related to crossing linear boundaries by the trajectory of a random walk. Main results consist in finding explicit expressions and asymptotic expansions for distributions of various boundary functionals such as first exit time and overshoot, the crossing number of a strip, sojourn time, etc. The method includes several steps. We start with the identities containing Laplace transforms of joint distributions under study. The use of the Wiener-Hopf factorization is the main instrument to solve these identities. Thus we obtain explicit expressions for the Laplace transforms in terms of factorization components. It turns out that in many cases Laplace transforms are expressed through the special factorization operators which are of particular interest. We further discuss possibilities of exact expressions for these operators, analyze their analytic structure, and obtain asymptotic representations for them under the assumption that the boundaries tend to infinity. \\ After that we invert Laplace transforms asymptotically to get limit theorems and asymptotic expansions, including complete asymptotic expansions.

<sup>1</sup> Novosibirsk State University, Sobolev Institute of Mathematics, Russia;

# To be announced

*Jose Alberto Cuminato*<sup>1</sup>

TBA

<sup>1</sup> ICMC/USP, Brazil;

# Title and Abstract to be announced

*Tiago Pereira*<sup>1</sup>

TBA

<sup>1</sup> ICMC/USP, Brazil;

# Stick-slip singularity of PTT, Giesekus and Oldroyd-B

*Jonathan David Evans*<sup>1</sup>

Abstract to be announced.

<sup>1</sup> University of Bath, United Kingdom;

# Visualisation as a tool to understand the sights and sounds of data

*Sugnet Lubbe*<sup>1</sup>

The biplot was introduced by Gabriel in 1971 as a tool to visualise a two-way samples variables data matrix. A short review of the developments between the first biplot and Gower and Hand's 1996 monograph on biplots will be given. The Gower-philosophy, based on exactly the same underlying mathematical constructs, enhanced biplots by moulding it into a multivariate scatterplot. This idea forms the basis for extending biplot methodology to provide an array of tools for exploring data originating from a variety of applications visually.

The examples are chosen to show case different types of data: continuous, ordinal and nominal; different models: single population or groups of data; observations made at a single time point or longitudinally; sets of pairwise dissimilarities rather than individual observations; three-way data with samples variables occasions; parametric and nonparametric plots.

The basics of the underlying matrix and other algebraic results will be given, illustrating the relationships between methods. However, rather than being disorientated in a maze of equations, the focus will be on the application to data and what can be learnt by exploring data visually.

<sup>1</sup> TBA



# Mathematical and Computational Models in Structural Dynamics: Theory, Applications and Challenges

*Domingos Alves Rade<sup>1</sup>*

Structural dynamics is the field of Engineering devoted to study of vibration motion of mechanical systems, encompassing civil engineering structures, vehicles, industrial equipment, spacecraft, etc., in response to various types of excitations (earthquakes, ground roughness, wind, collisions with surrounding bodies, etc.). Mechanical vibrations can be understood as continuous exchanges between kinetic and potential (elastic) energies, part of which is dissipated owing to the inevitable presence of dissipation mechanisms (friction with surrounding fluids or solid bodies, for example). One of the main tasks of engineers and scientists devoted to structural dynamics problems is the derivation of mathematical and computational models capable of predicting, with satisfactory accuracy, the vibration motion of mechanical systems, accounting for their complex geometry and material behavior. In this sense, after introducing several problems of structural dynamics of scientific and technological interest, the author revisits the most common types of mathematical and computation models used to solve them, the foundations of which belong to various fields of Mathematics, including Linear Algebra, Functional Analysis, Ordinary Differential Equations, Partial Differential Equations and Probability Theory. Through a number of examples drawn from his research work, the author highlights the existing challenges for the improvement of the modeling procedures, especially in terms of accuracy and computational cost involved when dealing with high dimensional computational models.

<sup>1</sup> ITA, Brazil;

# Minimizing convex quadratics with variable precision Krylov methods

*Philippe Toint*<sup>1</sup>

Iterative algorithms for the solution of convex quadratic optimization problems are investigated, which exploit inaccurate matrix-vector products. Theoretical bounds on the performance of a Conjugate Gradients and a Full-Orthonormalization methods are derived, the necessary quantities occurring in the theoretical bounds estimated and new practical algorithms derived. Numerical experiments suggest that the new methods have significant potential, including in the steadily more important context of multi-precision computations.

<sup>1</sup> Université de Namur, Belgium;

# Steepest Descent Methods Revisited

*Yu-Hong Dai*<sup>1</sup>

The steepest decent method plays a special role in the development of nonlinear optimization and numerical analysis. The classical steepest decent method, which was dated back to Cauchy (1847), keeps a monotone decrease in the objective function at each iteration, but performs very slow even when the problem is ill-conditioning and possesses the notorious zigzagging phenomenon. By imposing a certain quasi-Newton property, Barzilai and Borwein (1988) proposed a novel gradient method, which has a heavy non-monotone behavior in the objective function, but greatly improves the numerical efficiency. In this talk, we shall revisit both monotone and non-monotone steepest descent methods and point out some possible research problems.

<sup>1</sup> Chinese Academy of Sciences, China;

# On Balanced metrics

*Jixiang Fu*<sup>1</sup>

A balanced metric on a complex manifold is a special Hermitian metric which is weaker than the Kahler condition. In this talk we present some topics on balanced metrics, including the existence of solutions to the Strominger system in the superstring theory, the existence of balanced metrics on a class of non-Kahler Calabi-Yau three-folds, and the balanced cone of a compact complex manifold.

<sup>1</sup> FUDAN, China;

# Simulation of High-Lift Flows over Airliner Configurations

*João Luiz F. de Azevedo*<sup>1</sup>

TBA

<sup>1</sup> ITA, Brazil;

**DAY 2**

**Communicators**

# A quasi-Newton modified linear-programming-Newton method

*Damián Fernández*<sup>1</sup>

*María Martínez*<sup>1</sup>

In this work we consider a method to solve constrained system of nonlinear equations based on a modification of the Linear-Programming-Newton method and replacing the first order information with a quasi-Newton secant update, providing a computationally simple method. The proposed strategy combines good properties of two methods: the least change secant update for unconstrained system of nonlinear equations with isolated solutions and the Linear-Programming-Newton for constrained nonlinear system of equations with possible nonisolated solutions. We analyze the local convergence of the proposed method under suitable conditions proving its linear/superlinear convergence to possible nonisolated solutions.

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<sup>1</sup> UFSC - Federal University of Santa Catarina, Brazil;

<sup>2</sup> FaMAF, Universidad Nacional de Córdoba, Argentina;

# Nonlinear symmetric cones problems: optimality conditions and an augmented Lagrangian method

*Ellen H. Fukuda<sup>1</sup>*

*Bruno F. Lourenço<sup>1</sup>*

*Masao Fukushima*

Nonlinear symmetric cone problems (NSCP) generalize nonlinear semidefinite programming, nonlinear second-order cone programming and nonlinear programming (NLP) problems. In this work, we discuss the reformulation of NSCPs as NLP problems, using squared slack variables. With this, we prove a criterion for membership in a symmetric cone, and discuss the equivalence between Karush-Kuhn-Tucker points of the original and the reformulated problems. As the main result, we observe that the reformulation allows us to obtain second-order optimality conditions for NSCPs in a easy manner. We also show that by employing the slack variables approach, we can use the results for NLP to prove convergence results of a simple augmented Lagrangian function for NSCPs.

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<sup>1</sup> Kyoto University, Japan;

<sup>2</sup> Nanzan University, Japan



# Non-monotone inexact restoration method for minimization with orthogonality constraints

*Juliano de Bem Francisco<sup>1</sup>*

*D. G. Gonçalves<sup>1</sup>*

*L. E. T. Paredes<sup>1</sup>*

*F. S. Viloche-Bazán<sup>1</sup>*

In this work we consider the problem of minimizing a differentiable functional restricted to the set of matrices (of order  $n \times p$ ) with orthogonal columns. This problem arises from different fields of applications, such as, statistical, signal processing, global positioning system, machine learning, physics, chemistry and others. The numerical framework behind our approach is a non-monotone variation of the inexact restoration method. We give a simple characterization of the set of tangent directions (with respect to the orthogonal constraints) in order to handle with the tangent phase. For the restoration phase we use the well-known Cayley transformation for bring the computed point (at the tangent phase) back to the feasible set (i.e., the restoration phase is exact). We prove that all limit points of the generated sequence is stationary and we compare numerically our method with a well established algorithm for solving this optimization problem.

<sup>1</sup> Federal University of Santa Catarina, Brazil;

<sup>2</sup>

# Accelerated primal-dual fixed point algorithms for ridge regression problems

*Ademir Alves Ribeiro<sup>1</sup>*

*Peter Richtárik<sup>2</sup>*

*Tatiane Cazarin da Silva<sup>3</sup>*

*Gislaine Aparecida Perićaro<sup>4</sup>*

In this work we study the primal and dual formulations of the regularized least squares problem, in the special norm  $L_2$ , named Ridge Regression. We observe that the optimality conditions describing the primal and dual optimal solutions can be formulated in several different but equivalent ways. The optimality conditions we identify form a linear system involving a structured matrix depending on a single relaxation parameter which we introduce for regularization purposes. This leads to the idea of studying and comparing, in theory and practice, the performance of the fixed point method applied to these reformulations. We compute the optimal relaxation parameters and uncover interesting connections between the complexity bounds of the variants of the fixed point scheme we consider. These connections follow from a close link between the spectral properties of the associated matrices. For instance, some reformulations involve purely imaginary eigenvalues; some involve real eigenvalues and others have all eigenvalues on the complex circle. We show that our main method - which is a special case of the randomized dual coordinate ascent method with arbitrary sampling developed by Qu, Richtárik and Zhang - achieves the best rate in theory and in numerical experiments among the fixed point methods we study. Remarkably, the method achieves an accelerated convergence rate. We also establish the convergence of a gradient memory-like strategy. Numerical experiments indicate that our main algorithm is competitive with the conjugate gradient method.

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<sup>1</sup>UFPR - Federal University of Parana, Brazil;

<sup>2</sup>University of Edinburgh, Scotland;

<sup>3</sup>UTFPR - Federal University of Technology of Parana, Brazil;

<sup>4</sup>State University of Parana, Brazil.

# Non-monotone inexact restoration method for minimization with orthogonality constraints

*Gilson do Nascimento Silva*<sup>1</sup>

*Ioannis Konstantinos Argyros*<sup>2</sup>

In this work we consider the problem of minimizing a differentiable functional restricted to the set of matrices (of order  $n \times p$ ) with orthogonal columns. This problem arises from different fields of applications, such as, statistical, signal processing, global positioning system, machine learning, physics, chemistry and others. The numerical framework behind our approach is a non-monotone variation of the inexact restoration method. We give a simple characterization of the set of tangent directions (with respect to the orthogonal constraints) in order to handle with the tangent phase. For the restoration phase we use the well-known Cayley transformation for bring the computed point (at the tangent phase) back to the feasible set (i.e., the restoration phase is exact). We prove that all limit points of the generated sequence is stationary and we compare numerically our method with a well established algorithm for solving this optimization problem.

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<sup>1</sup> Universidade Federal do Oeste da Bahia, Brazil;

<sup>2</sup> Cameron University, United States;

# On the complexity of an hybrid proximal extragradiente projection method for solving monotone inclusion problems

*Mauricio Romero Sicre<sup>1</sup>*

In this work we establish the iteration complexity of an under-relaxed Hybrid Proximal Extragradient Projection method (HPEP) for finding a zero of a maximal monotone operator. These results extend the complexity analysis of the Hybrid Proximal Extragradient method (HPE), due to Svaiter and Monteiro, to a more general framework.

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<sup>1</sup> Universidade Federal da Bahia, Brazil;

# Solving Convex Feasibility Problems in Hadamard Manifolds

*Joao Xavier da Cruz Neto*<sup>1</sup>

*Italo Dowel Lira*<sup>1</sup>

*Paulo Alexandre Sousa*<sup>1</sup>

*João Carlos Souza*<sup>1</sup>

In this talk, we study the convergence issue of the gradient method for solving a convex feasibility problem in Hadamard manifolds. Clearly, our results extend the corresponding ones in Euclidean spaces and solve the open problem proposed by Bento and Melo [J. Optimization. Theory Application., 152 (2012), pp. 773-785] which was partially solved by Wang et al. [J. Optimization. Theory Application., 164 (2015), pp. 202-217].

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<sup>1</sup> Universidade Federal do Piauí, Brazil;

# On Riemannian Conjugate Gradient and non monotone linear search algorithm with mixed direction on Stiefel

*Hugo José Lara Urdaneta<sup>1</sup>*

*Harry Oviedo Leon<sup>2</sup>*

*Oscar Dalmau<sup>2</sup>*

*João Carlos Souza<sup>1</sup>*

In this talk, we study the convergence issue of the gradient method for solving a convex feasibility problem in Hadamard manifolds. Clearly, our results extend the corresponding ones in Euclidean spaces and solve the open problem proposed by Bento and Melo [J. Optimization. Theory Application., 152 (2012), pp. 773-785] which was partially solved by Wang et al. [J. Optimization. Theory Application., 164 (2015), pp. 202-217].

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<sup>1</sup> Universidade Federal de Santa Catarina, Brazil;

<sup>2</sup> CIMAT, Mexico;

# Local stabilization of time periodic evolution equations

*Mythily Ramaswamy*<sup>1</sup>

Local stabilization at a prescribed rate around a periodic trajectory of parabolic systems, using boundary control is an interesting problem. The main motivating example is the incompressible Navier-Stokes system. I will discuss this example and the general framework and indicate some results in this direction.

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<sup>1</sup> TIFR-B, India;

# A New and Enhanced Semidefinite Relaxation for a Class of Nonconvex Complex Quadratic Problems with Applications in Wireless Communications

*Ya-Feng Liu<sup>1</sup>*

In this talk, we shall consider a special class of nonconvex Complex Quadratic Problems (CQP), which finds many important and interesting applications in wireless communications. In this talk, we shall first develop a new and Enhanced Complex SemiDefinite Program, called ECSDP, for the considered CQP and then apply the ECSDP to MIMO detection, a fundamental problem in modern wireless communications. As our main result, we show the tightness of the ECSDP for MIMO detection under an easily checkable condition. This result answers an open question posed by So in 2010. Based on the ECSDP, we can also develop a branch-and-bound algorithm for globally solving the MIMO detection problem (even though the above condition does not hold true).

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<sup>1</sup> CAS, China;



# Title and Abstract to be announced

*Jian-Hua Wu*<sup>1</sup>

TBA

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<sup>1</sup> Shaanxi Normal University, China;

# Title and Abstract to be announced

*Silas Abahia Ihedioha*<sup>1</sup>

TBA

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<sup>1</sup> Plateau State University Boko, Nigeria;

# Title and Abstract to be announced

*Messias Meneguette*

TBA

# Title and Abstract to be announced

*Ting Wei*<sup>1</sup>

TBA

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<sup>1</sup> Lanzhou University, China;

# Modeling of non-Fickian diffusion and dissolution from a thin polymeric coating: An application to drug-eluting stents

*Elias Gudiño*<sup>1</sup>

*C. M. Oishi*<sup>2</sup>

*A. Sequeira*<sup>3</sup>

In this talk, we present a general model for non-Fickian diffusion and drug dissolution from a controlled drug delivery device coated with a thin polymeric layer. We propose an approach to reduced the computational cost of performing numerical simulations in complex 3-dimensional geometries. The model for mass transport by a coronary drug-eluting stent is coupled with a non-Newtonian blood model flow. In order to show the effectiveness of the method, numerical experiments and a model validation with experimental data are also included. In particular, we investigate the influence of the non-Newtonian flow regime on the drug deposition in the arterial wall.

<sup>1</sup> Universidade Federal do Paraná, Brazil;

<sup>2</sup> UNESP, Brazil;

<sup>3</sup> IST Lisboa, Portugal;

# Title and Abstract to be announced

*Manoel Silvino Batalha de Araujo*<sup>1</sup>

*C. Fernandes*<sup>2</sup>

*L.L. Ferrás*<sup>2</sup>

*J. Miguel Nóbrega*<sup>2</sup>

TBA

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<sup>1</sup> Universidade Federal do Paraná, Brazil;

<sup>2</sup> Institute for Polymers and Composites/i3N, University of Minho, Portugal;

# Title and Abstract to be announced

*Shuanping Du*<sup>1</sup>

TBA

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<sup>1</sup> School of Mathematical Sciences, Xiamen University, China;

# Computational simulation of non-Newtonian drop impact

*Cassio M. Oishi<sup>1</sup>*

To be announced.

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<sup>1</sup> UNESP, Brazil;



# An epsilon-VU algorithm with superlinear convergence

Shuai Liu<sup>1</sup>

Claudia A. Sagastizábal<sup>1</sup>

Mikhail V. Solodov<sup>2</sup>

The theories of  $\mathcal{VU}$ -space decomposition and  $\mathcal{U}$ -Lagrangian have been applied to develop algorithms for solving problems with structural properties. We introduce an algorithm based on the  $\epsilon$ - $\mathcal{VU}$ -space decomposition, where the  $\mathcal{V}_\epsilon$ -subspace is defined by the span of some enlargement of the subdifferential. The algorithm has two steps: the  $\mathcal{V}$ -step, which we show can be replaced by an exact prox-step, and the  $\mathcal{U}$ -step, a quasi-Newton step in the  $\mathcal{U}_\epsilon$ -subspace. The  $\mathcal{U}$ -step requires a basis matrix of the  $\mathcal{U}_\epsilon$ -subspace and a matrix containing second order information of the objective function in the  $\mathcal{U}_\epsilon$ -subspace. If  $\epsilon$  is suitably driven to zero, the superlinear convergence of the algorithm can be proven if the Dennis-More condition holds in our context. We give an application of our algorithm on minimizing a function whose proximal point can be easily calculated.

<sup>1</sup> IMECC/UNICAMP, Brazil;

<sup>2</sup> IMPA, Brazil;

# An adaptive accelerated proximal point method for solving non-convex optimization problems

*Jefferson G. Melo*<sup>1</sup>

*Weiwei Kong*<sup>2</sup>

*Renato DC Monteiro*<sup>2</sup>

In this talk, we present an adaptive accelerated proximal point type method for solving non-convex optimization problems. We discuss how to compute approximate solutions of the subproblems accepting some relative error criteria. Iteration-complexity bounds for the proposed method is analyzed and some numerical experiments are presented.

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<sup>1</sup> Universidade Federal de Goiás, Brazil;

<sup>2</sup> Georgia Tech, United States;

# Accelerated Regularized Newton Methods for Minimizing Composite Convex Functions

*Geovani Nunes Grapiglia*<sup>1</sup>

*Yurii Nesterov*<sup>2</sup>

In this talk, we present accelerated Regularized Newton Methods for minimizing objectives formed as a sum of two functions: one is convex and twice differentiable with  $H''$ -continuous Hessian, and the other is a simple closed convex function. For the case in which the  $H''$ -parameter  $\nu \in [0, 1]$  is known we propose methods that take at most  $\mathcal{O}\left(\frac{1}{\epsilon^{1/(2+\nu)}}\right)$  iterations to reduce the functional residual below a given precision  $\epsilon > 0$ . For the general case, in which the  $\nu$  is not known, we propose a universal method that ensures the same precision in at most  $\mathcal{O}\left(\frac{1}{\epsilon^{2/3(1+\nu)}}\right)$

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<sup>1</sup> UFPR - Federal University of Parana, Brazil

<sup>2</sup> CORE

# Analysis of EPEC Models for Power Markets

*Juan Pablo Luna*<sup>1</sup>

*J. Filiberti*<sup>2</sup>

*S.A. Gabriel*<sup>2</sup>

*C. Sagastizábal*<sup>3</sup>

*M. Solodov*<sup>4</sup>

A usual equilibrium model in power markets is to consider a leader-follower problem in which the top level involves multiplier power producers bidding prices and generation levels. At the bottom level, common to each producer, there is an independent system operator (ISO) that takes all the bids from producers and minimizes the total operation costs, subject to capacity and other bounds on production. As such, the system being modeled is an equilibrium problem with equilibrium constraints (EPEC). We show that already in their simplest instances, such models suffer from two serious drawbacks, related to: the existence of many equilibria, which harm the algorithmic solution (cycles); and equilibrium prices that can take values above the bids, even for the most expensive dispatched producer. To address these issues, we propose a dual regularization for the ISO problem, that has an enlightening interpretation in economical terms.

<sup>1</sup> UFRJ, Brazil;

<sup>2</sup> University of Maryland, College Park, Maryland, United States;

<sup>3</sup> UNICAMP, Brazil;

<sup>4</sup> IMPA, Brazil;

# An algorithm for projecting a point onto a level set of a quadratic function

*Fernanda Raupp*<sup>1</sup>

*Wilfredo Sosa*<sup>2</sup>

We propose an iterative algorithm to project a point onto a level set of a quadratic function, based on the spectral decomposition of the Hessian, which is performed in a unique iteration. The proposed algorithm was tested on instances with distinct Hessian matrices and shows great potential in applications, such as in computer graphics

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<sup>1</sup> LNCC, Brazil;

<sup>2</sup> UCB, Brazil;

# A new quasiconvex asymptotic function with applications in optimization

*Felipe Lara*<sup>1</sup>

*N. Hadjisavvas*<sup>2</sup>

*J. E. Martínez-Legaz*<sup>3</sup>

We introduce a new asymptotic function which is mainly adapted to quasiconvex functions. We establish several properties and calculus rules for this concept and compare it to previous notions of generalized asymptotic functions. Finally, we apply our new definition to quasiconvex optimization problems: we characterize the boundedness of the function, the nonemptiness and compactness of the set of minimizers, and we provide a sufficient condition for the closedness of the image of a nonempty closed convex set via a vector-valued function.

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<sup>1</sup> IMPA - National Institute of Pure and Applied Mathematics, Brazil;

<sup>2</sup> University of the Aegean, Greece;

<sup>3</sup> Universidad Autónoma de Barcelona, Spain;

# Limits of sequences of maximal monotone operators

Marc Lassonde<sup>1</sup>

Yboon García<sup>2</sup>

We consider a sequence  $(T_n)$  of maximal monotone operators on a reflexive Banach space. In general, the (Kuratowski) lower limit  $\liminf T_n$  of such a sequence is not a maximal monotone operator. So, what can be said? In the first part of the talk, we show that  $\liminf T_n$  is a representable monotone operator while its Mosco limit  $M\text{-}\lim T_n$ , when it exists, is a maximal monotone operator. As an application of the former result, we obtain that the variational sum of two maximal monotone operators is a representable monotone operator. In the second part of the talk, we consider a sequence  $(f_n)$  of representative functions of  $(T_n)$ . We show that if  $(f_n)$  epi-converges to a function  $f$ , then  $\liminf T_n$  is representable by  $f$ ; moreover if  $(f_n)$  Mosco-converges to  $f$ , then  $\liminf T_n$  is maximal monotone. As an application, we recover Attouch's result: if a sequence of convex lower semicontinuous functions  $(f_n)$  Mosco-converges to  $f$ , then  $\partial f = \liminf \partial f_n$ .

<sup>1</sup> Antilles University, Guadeloupe, and LIMOS, Clermont-Ferrand, France;

<sup>2</sup> Universidad del Pacífico, Lima, Perú;

# On the simulation and calibration of jump-diffusion models in finance

*Vinicius Albani*<sup>1</sup>

We apply a splitting strategy to identify simultaneously the local volatility surface and the jump-size distribution of a jump-diffusion driven asset from quoted European option prices. This is done by means of a Tikhonov-type regularization technique. Proofs of the convergence of the corresponding algorithm as well as the stability of the solution are provided. We also present numerical examples with synthetic, as well as, real data illustrating the robustness of this method.

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<sup>1</sup> UFSC, Brazil;



# Pullback dynamics of a non-autonomous Bresse system

*Rawilson de Oliveira Araújo*<sup>1</sup>

The Bresse system is a model for vibrations of a circular arched beam. Here we discuss the existence of pullback attractors for a weakly dissipative non-autonomous semilinear Bresse.

<sup>1</sup> UNESP, Brazil;

# Title and Abstract to be announced (TBA)

*Shuqin Wang*<sup>1</sup>

TBA

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<sup>1</sup> Northwest Polytechnical University, China;

# Title and Abstract to be announced (TBA)

*Carlos Eduardo Andrades<sup>1</sup>*

*Débora Aline Kotz<sup>1</sup>*

*Rafael Berkenbrock<sup>1</sup>*

*Enio Roberto Galli<sup>1</sup>*

*Paulo Marcos Flores<sup>1</sup>*

*Luiz Antônio Rasia<sup>1</sup>*

*Antonio Carlos Valdiero<sup>1</sup>*

TBA

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<sup>1</sup> Universidade do Noroeste do Estado do Rio Grande do Sul – UNIJUÍ, Brazil;

# Title and Abstract to be announced (TBA)

*Song Liu*<sup>1</sup>

TBA

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<sup>1</sup> Xi'an Jiaotong University, China;

# On an SSOR-like method with four parameters for saddle point problems

*Huang Zhengda<sup>1</sup>*

*Huidi Wang*

Since 2001 when several SOR-like methods for the saddle point problems was proposed by Golub, G. H., Wu, X. and Yuan, J.-Y., many papers have been appeared to consider the generalized SOR, AOR and SSOR-like methods based on the different splitting ways of the coefficient matrix and accompanied by different number of parameters. This talk is an short report on an SSOR-like method with four parameters, which is one of our works for the saddle point problem. To our best knowledge, it can't be written in the same classical forms used by the existed SSOR-like methods. A condition to guarantee the convergence and the optimal convergence factor are obtained, and comparisons with other SSOR-like methods are discussed. This work is coauthored with Dr. Huidi Wang.

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<sup>1</sup> School of Mathematical Sciences, China;

# Title and Abstract to be annouced (TBA)

*Benito Pires*<sup>1</sup>

TBA

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<sup>1</sup> USP, Brazil;

# Numerical method for unique continuation of elliptic equations

*Yu Chen*<sup>1</sup>

*Jin Cheng*<sup>1</sup>

In this talk, the numerical method for unique continuation is investigated, which can be applied to inverse problems and optimal design problems. We mainly consider the line and domain unique continuation of two dimensional elliptic equations. A numerical form of Green's function is also proposed to deal with the cases with variable coefficients and complex geometries. We apply the Tikhonov regularization for finding the minimal norm solution. The stability and error estimation of the algorithm are discussed. Numerical examples are presented to illustrate the validity and effectiveness of the present method.

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<sup>1</sup> Fudan University China, China;

# Title and Abstract to be annouced (TBA)

*Xiaodong Zhang*<sup>1</sup>

TBA

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<sup>1</sup> Shangai Jiao Tong University, China;



# Shifted Gradient Method for Computing Tensor Eigenpairs

*Xiangyun Zhang*<sup>1</sup>

*Hao Liang*<sup>2</sup>

*Guoliang Chen*<sup>2</sup>

In this talk, we propose a shifted gradient method (S-GM) to calculate the Z-eigenpairs of the symmetric tensor. S-GM can be viewed as a generalization of shifted symmetric higher-order power method (SS-HOPM). The convergence analysis and the fixed-point analysis of this algorithm are given. Numerical examples show that S-GM needs fewer iterations than SS-HOPM when the appropriate parameters were selected.

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<sup>1</sup> East China Normal University, China;

<sup>2</sup> School of Mathematical Sciences, East China Normal University, Shanghai, P.R. China;

# **DAY 3**

## **Plenarists**

# Conic Optimization Approaches to Combinatorial Problems

*Mirjam Dür*<sup>1</sup>

Conic programming opens a way to treat combinatorial problems with techniques from continuous optimization. The talk will introduce these techniques and highlight recent progress on various classical combinatorial problems, including the stable set problem on finite or infinite graphs.

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<sup>1</sup> University of Augsburg, Germany;

# On regularization and active-set methods with complexity for constrained optimization

*Ernesto G. Birgin<sup>1</sup>*

*J. M. Martínez<sup>2</sup>*

The main objective of this research is to introduce a practical method for smooth bound-constrained optimization that possesses worst-case evaluation complexity  $O(\varepsilon^{-3/2})$  for finding an  $\varepsilon$ -approximate first-order stationary point when the Hessian of the objective function is Lipschitz-continuous. As other well established algorithms for optimization with box constraints, the algorithm proceeds visiting the different faces of the domain aiming to reduce the norm of an internal projected gradient and abandoning active constraints when no additional progress is expected in the current face. The introduced method emerges as a particular case of a method for minimization with linear constraints. Moreover, the linearly-constrained minimization algorithm is an instance of a minimization algorithm with general constraints whose implementation may be unaffordable when the constraints are complicated. As a procedure for leaving faces, it is employed a different method that may be regarded as an independent device for constrained optimization. Such independent algorithm may be employed to solve linearly-constrained optimization problem on its own, without relying on the active-set strategy. A careful implementation and numerical experiments shows that the algorithm that combines active sets with leaving-face iterations is more effective than the independent algorithm on which leaving-face iterations are based, although both exhibits similar complexities  $O(\varepsilon^{-3/2})$ .

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<sup>1</sup> University of São Paulo, Brazil;

<sup>2</sup> State University of Campinas, Brazil;

# Title and Abstract to be announced (TBA)

*Marcos Prates<sup>1</sup>*

*Zaida Quiroz<sup>1</sup>*

*Dipak Dey<sup>2</sup>*

TBA

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<sup>1</sup> Universidade Federal de Minas Gerais, Brazil;

<sup>2</sup> University of Connecticut, United States;

# Title and Abstract to be announced (TBA)

*S. Kesavan*<sup>1</sup>

TBA

<sup>1</sup> IIT Madras, India;

# Title and Abstract to be announced (TBA)

*Justin Wan<sup>1</sup>*

*Soo-Min Kang<sup>1</sup>*

*Khadeejah Mohiuddin<sup>1</sup>*

*Yuehuan Chen<sup>1</sup>*

TBA

<sup>1</sup> University of Waterloo, China;

# Graph Signal Processing and Visualization: a worthwhile partnership

*Luis Gustavo Nonato*<sup>1</sup>

Abstract to be announced.

<sup>1</sup> ICMC - USP, Brazil;



# Title and Abstract to be announced (TBA)

*Dipendra Prasad*<sup>1</sup>

Abstract to be announced.

<sup>1</sup> TIFR, India;

# On dynamics of evolution equations with moving boundary

*Ma To Fu*<sup>1</sup>

This talk is concerned with heat and wave equations defined in domains with moving boundary. We discuss the dynamics of such non-autonomous problems as two-parameter evolution processes defined on time-dependent phase spaces.

<sup>1</sup> University of Sao Paulo, Brazil;

# Quantitative guidelines for retiring (safely) in a BRICS economy

*Eben Maré<sup>1</sup>*

In this contribution we consider some aspects of retirement in a BRICS type economy - the case study used being South Africa. We consider aspects of longevity and typical defined contribution type retirement which creates the situation that retirees need to “optimise” between spending and saving. We discuss spending (or so-called safe withdrawal rates). The problem is considered in the light of asset allocation strategies as well as risk management to yield successful longer-term investments portfolios.

<sup>1</sup> TIFR, India;

# Heat Exchanger Network Synthesis Using Mathematical Programming - Deterministic and Stochastic Methods

*Mauro A. Ravagani<sup>1</sup>*

Abstract to be announced.

<sup>1</sup> UEM, Brazil;

# Stochastic Quasi-Gradient Methods: Variance Reduction via Jacobian Sketching

*Peter Richtárik*<sup>1</sup>

*Robert M. Gower*<sup>2</sup>

*Francis Bach*<sup>3</sup>

We develop a new family of variance reduced stochastic gradient descent methods for minimizing the average of a very large number of smooth functions. Our method---JacSketch---is motivated by novel developments in randomized numerical linear algebra, and operates by maintaining a stochastic estimate of a Jacobian matrix composed of the gradients of individual functions. In each iteration, JacSketch efficiently updates the Jacobian matrix by first obtaining a random linear measurement of the true Jacobian through (cheap) sketching, and then projecting the previous estimate onto the solution space of a linear matrix equation whose solutions are consistent with the measurement. The Jacobian estimate is then used to compute a variance-reduced unbiased estimator of the gradient, followed by a stochastic gradient descent step. Our strategy is analogous to the way quasi-Newton methods maintain an estimate of the Hessian, and hence our method can be seen as a *stochastic quasi-gradient method*. Indeed, quasi-Newton methods project the current Hessian estimate onto a solution space of a linear equation consistent with a certain linear (but non-random) measurement of the true Hessian. Our method can also be seen as stochastic gradient descent applied to a *controlled stochastic optimization reformulation* of the original problem, where the control comes from the Jacobian estimates. We prove that for smooth and strongly convex functions, JacSketch converges linearly with a meaningful rate dictated by a single convergence theorem which applies to general sketches. We also provide a refined convergence theorem which applies to a smaller class of sketches, featuring a novel proof technique based on a *stochastic Lyapunov function*. This enables us to obtain sharper complexity results for variants of JacSketch with importance sampling. By specializing our general approach to specific sketching strategies, JacSketch reduces to the celebrated stochastic average gradient (SAGA) method, and its several existing and many new minibatch, reduced memory, and importance sampling variants. Our rate for SAGA with importance sampling is the current best-known rate for this method, resolving a conjecture by Schmidt et al (2015). The rates we obtain for minibatch SAGA are also superior to existing rates. Moreover, we obtain the first minibatch SAGA method with importance sampling.

<sup>1</sup> University of Edinburgh, United Kingdom;

<sup>2</sup> Telecom ParisTech, France;

<sup>3</sup> INRIA, India;

# Price Signals in Stochastic Optimization

*Claudia Sagastizábal<sup>1</sup>*

For stochastic optimization problems arising in the management of a mix with thermal, hydro-and renewable power plants, we study how uncertainty impacts on marginal prices. Specifically, for two-stage linear programs with recourse, we perform a theoretical study of the sensitivity of the dual variables with respect to variations of the right hand side in the constraints. We propose a stabilizing device that, among all the possible marginal prices, provides the one with smallest 1-norm. Joint work with Mikhail Solodov (IMPA) and Clara Lage (ENGIE and IMPA).

<sup>1</sup> UNICAMP, Brazil;

# Pogorelov polyhedra and hyperbolic 3-manifolds: when hyperbolic geometry, algebraic topology and knot theory meet

*Andrey Vesnin*<sup>1</sup>

The class  $R$  of Pogorelov polyhedral consists of combinatorial 3-dimensional polyhedra of simple combinatorial type (each vertex incident to three edges), different from a tetrahedron, without 3- and 4-belts of faces. In particular,  $R$  contains the dodecahedron and fullerenes. We will give the 4-coloring method to construct hyperbolic 3-manifolds from Pogorelov polyhedra and discuss hyperbolic volumes, complexity, cohomological rigidity and covering properties of obtained manifolds.

<sup>1</sup> Sobolev Institute of Mathematics, SB RAS, Russia;

# Title and Abstract to be announced (TBA)

*Weiguo Wu*<sup>1</sup>

To be announced.

<sup>1</sup> Xi'an Jiaotong University, China;



**DAY 3**

**Communicators**

# Weak and strong convergence theorems for equilibrium problems in Banach Spaces

*Vahid Mohebbi*<sup>1</sup>

In this talk, we introduce and analyze some convergence methods for solving equilibrium problems in Banach spaces. We prove weak convergence of the generated sequence to a solution of the equilibrium problem, under standard assumptions on the bifunction. Then, we propose a regularization procedure which ensures strong convergence of the generated sequence to a solution of the problem.

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<sup>1</sup> IMPA, Brazil;

# A Newton-type method for Quasi-Equilibrium Problems and applications

*Pedro Jorge Sousa dos Santos<sup>1</sup>*

*Susana Scheimberg<sup>2</sup>*

*Paulo Sérgio Marques dos Santos<sup>1</sup>*

We present a local fast convergence method for solving Quasi-Equilibrium Problems (QEPs). Applications to generalized Nash equilibrium problems (GNEPs) and multiobjective optimization problems (MPOs) are considered. In the case of jointly convex GNEP, our algorithm allows finding any solutions of the problem, not only the normalized equilibrium solutions. Some numerical results are reported showing the performance of the algorithm.

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<sup>1</sup> Universidade Federal do Piauí, Brazil;

<sup>2</sup> Universidade Federal do Rio de Janeiro, Brazil;

# Douglas-Rachford Method: a View from Strongly Quasi- Nonexpansive Operators

*Reinier Díaz Millán*<sup>1</sup>

*Scott Lindstrom*<sup>2</sup>

*Vera Roshchina*<sup>3</sup>

We focus on the convergence analysis of Douglas-Rachford method for convex feasibility problems in the context of inexact projections. Standard convergence analysis of Douglas-Rachford algorithms is based on the firm nonexpansivity property of the relevant operator. However, if the true projections are replaced by cutters (projections onto separating hyperplanes), the firm nonexpansivity is lost. We provide a proof of convergence of the method under reasonable assumptions, foregoing the usual operator theory scaffolding and relying on a simple geometric argument. This allows us to clarify fine details related to the allowable choice of the relaxation parameters, highlighting the distinction between the exact (firmly nonexpansive) and approximate (strongly quasi-nonexpansive) settings. We provide illustrative examples and discuss practical implementations of the method.

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<sup>1</sup> Federal Institute of Goiás, Brazil;

<sup>2</sup> University of Newcastle, Australia;

<sup>3</sup> RMIT University, Australia;

# An existence result for quasi-equilibrium problems

*Susana Scheimberg*<sup>1</sup>

*Paulo S. M. Santos*<sup>2</sup>

*Leonardo A. Santos*<sup>1</sup>

In this work we study the existence of solutions for quasi-equilibrium problems in Banach spaces in the setting of generalized KKM theory. As a particular case, we provide the existence of solutions of generalized Nash equilibrium problems.

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<sup>1</sup> UFRJ/ COPPE/PESC, Brazil;

<sup>2</sup> UFPI/ Campus Ministro Reis Velloso, Brazil;

# A discussion of new implementation strategies for solving the Weighted Orthogonal Procrustes Problem

*Melissa Weber Mendonça<sup>1</sup>*

*Juliano de Bem Francisco<sup>1</sup>*

In this work, we discuss new refinements and implementation strategies for the solution of the large-scale Weighted Orthogonal Procrustes Problem (WOPP). Although the so called balanced Orthogonal Procrustes Problem has a closed form solution using the singular value decomposition, the unbalanced or Weighted Problem requires the application of iterative methods, which can be costly due to the presence of several local minima. Most existing methods for the solution of this problem involve solving a balanced subproblem at each iteration. In our case, we have previously proposed a block Lanczos bidiagonalization strategy for reducing the cost of the overall iteration by solving increasing-sized problems and, hopefully, converging to a solution to the WOPP before solving the full-sized subproblem. Here we propose a new stopping criteria for these iterations, which can improve convergence by avoiding unnecessary iterations in earlier stages of the block Lanczos iterations. Furthermore, we present improvements on the implementation, including an application of the Polar Decomposition algorithm for the computation of the solution of the subproblem and some preconditioning strategies for solving ill-conditioned problems. We present the theoretical and computational aspects of these developments, including numerical tests that show the competitiveness of this new approach against previously presented methods and implementations.

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<sup>1</sup> Universidade Federal de Santa Catarina, Brazil;

# A parallel forward-backward splitting method for multiterm composite convex optimization

*Maicon Marques Alves*<sup>1</sup>

*Samara C. Lima*<sup>1</sup>

We propose and study the iteration complexity of a parallel version of the forward-backward (proximal gradient) splitting method for minimizing a (possibly) large sum of convex functions with many smooth and nonsmooth terms. We obtain pointwise (nonergodic) as well as ergodic nonasymptotic convergence rates by embedding the proposed method within the frameworks of the partial inverse of Spingarn and the HPE method of Solodov and Svaiter.

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<sup>1</sup> Universidade Federal de Santa Catarina, Brazil;

# On the Q-linear convergence of the forward-backward splitting method and uniqueness of optimal solution to Lasso

*José Yunier Bello Cruz*<sup>1</sup>

In this talk, by using tools of second-order variational analysis, we present the popular forward-backward splitting method with Beck-Teboulle's line search for solving convex optimization problem where the objective function can be split into the sum of a differentiable function and a possible nonsmooth function. We first establish that this method exhibits global convergence to an optimal solution of problem (if it exists) without the usual assumption that the gradient of the differentiable function involved is global Lipschitz continuous. We also obtain the  $\mathcal{O}(k^{-1})$  complexity for the function value sequence when this usual assumption is weakened from globally Lipschitz continuity to locally Lipschitz continuity; improving the existing  $\mathcal{O}(k^{-1})$  complexity result. We then derive the local and global Q-linear convergence of the method in terms of both the function value sequence and the iterative sequence, under a general metric subregularity assumption which is automatically satisfied for convex piecewise-quadratic optimization problems. In particular, we provide verifiable sufficient conditions for metric subregularity assumptions, and so, local and global Q-linear convergence of the proposed method for broad structured optimization problems arise in machine learning and signal processing including the partly smooth optimization problems as well as the  $\ell_1$ -regularized optimization problems. Our results complement the current literature by providing Q-linear convergence result to the forward-backward splitting method under weaker assumptions. Moreover, via this approach, we obtain several full characterizations for the uniqueness of the optimal solution to Lasso problem, which covers some recent results in this direction.

<sup>1</sup> Northern Illinois University, United States;



# On the linear convergence of the circumcentered-reflection method

*Roger Behling*<sup>1</sup>

*Luiz Rafael dos Santos*<sup>1</sup>

*José Yunier Bello Cruz*<sup>2</sup>

In order to accelerate the Douglas-Rachford method we recently developed the circumcentered-reflection method, which provides the closest iterate to the solution among all points relying on successive reflections, for the best approximation problem related to two affine subspaces. We now prove that this is still the case when considering a family of finitely many affine subspaces. This property yields linear convergence and incites embedding of circumcenters within classical reflection and projection based methods for more general feasibility problems.

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<sup>1</sup> Universidade Federal de Santa Catarina, Brazil;

<sup>2</sup> Northern Illinois University, United States;

# Macroscopic and mesoscopic numerical model of melt filling and gas penetration processes in complex mold cavity

*Qiang Li<sup>1</sup>*

*Jinyun Yuan<sup>2</sup>*

In this talk, an improved simple coupled level-set and volume of fluid (S-CLSVOF) method is proposed to trace the moving interfaces in melt filling and gas penetration processes in complex mold cavities. Firstly, the shape level-set (LS) function is used to represent the complex mold cavity with R-function. Then the benchmark problem of two-dimensional deformation is carried out to verify the ability of S-CLSVOF to capture the moving interfaces. Meanwhile it is simulated that the non-isothermal melt filling process of the viscoelastic fluid based on the FENE-P model in the complex mold cavity. And the macroscopic temperature, stress and the behavior of mesoscopic molecular orientation represented by oriented ellipses are shown and analyzed in details. Finally, the proposed method is further applied to the gas penetration process in gas-assisted injection molding. The bubble appearance, temperature and the behavior of molecular orientation are shown and analyzed in details. The phenomenon of asymmetrical gas flow is observed in the gas penetration process clearly. The numerical results illustrate that the coupled method has high accuracy, and can be applied to the viscoelastic flows with complex free surfaces in melt filling and gas-assisted injection molding processes.

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<sup>1</sup> School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, China;

<sup>2</sup> Universidade Federal do Paraná, Brazil;

# On a special robust optimization problem

*Cong Sun*<sup>1</sup>

We consider a robust optimization problem arising from wireless communications. In a relay-aided wiretap network, we minimize the total relay transmit power, while requiring that the achieved rate at the supported users are above some thresholds, and that at the eavesdropper is below a standard. This problem is modeled as an optimization problem with one robust constraint. We propose an algorithm to solve the problem iteratively while preserving the feasibility during the iteration. The problem with tightened worst case constraint is solved as the algorithm initialization. We apply the linesearch technique to update the feasible iterative point. All the subproblems are solved optimally and the convergence of the objective function is proved. The optimality condition of the robust optimization problem is analyzed. Simulation results show that our algorithm outperforms the state of the art, and has little loss compared to the result with perfect channel state information.

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<sup>1</sup> Beijing University of Post and Telecommunications

# Algorithms based on augmented Lagrangian methods for equilibrium problems

*Luiz Carlos Matioli<sup>1</sup>*

*Elvis Manuel Rodrigues Torrealba<sup>1</sup>*

*Romulo Alberto Castillo Cardenas<sup>2</sup>*

Augmented Lagrangian methods have been shown to be very efficient in solving problems of mathematical.

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<sup>1</sup> Universidade Federal do Paraná, Brazil;

<sup>2</sup> UFSC/Joinville, Brazil;

# Optimization Model Applied to Radiotherapy Planning Problem with Dose Intensity and Beam Choice

*Daniela Renata Cantane<sup>1</sup>*

*Juliana Campos de Freitas<sup>1</sup>*

*Helenice de Oliveira Florentino<sup>1</sup>*

A radiotherapy planning consists in choose the right dose amount to be delivered in the tumor tissue. This tissue is surrounded by health tissue and tissues at risk, which have to be preserved. Is important to consider these tissues during the planning because high dose delivered into it can cause cell mutation, what can become a malign tumor (cancer) in future. To prevent future cancer disease developed by a radiation treatment, the planning has to be precise considering the beam disposal and certain dose amount. Optimization models have been developed and improved to facilitate and assure the radiotherapy planning. Such type of model can approach three different problems: dose intensity, beam choice and blades opening. However, is common to find only one or two problems in the model. In the current work we propose a optimization model which treats the dose intensity problem and the beam choice problem. In the new model the best beam set is selected by solving the model through matheuristics. A matheuristic consists in solve the beam choice by metaheuristics, applying in this case Tabu Search and Variable Neighborhood Search, coupled with an exact method, such as Interior Point Method and Simplex Method to solve the model.

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<sup>1</sup> UNESP, Brazil;

# Multiobjective programming via bundle methods

*Elizabeth Wegner Karas<sup>1</sup>*

*Claudia A. Sagastizábal<sup>2</sup>*

*Hasnaa Zidani<sup>3</sup>*

We present a method solving multiobjective optimization problems that combines achievement and improvement functions. The algorithm exploits the specific structure of the achievement function from a nonsmooth optimization perspective based on bundle methods that it is specially tailored for efficiently building the Pareto front. This is done by parsing attainable points for the objective functions, in a manner that allows for warm starts of the successive nonsmooth problems solved by the bundle algorithm. The methodology is illustrated with several examples that show the interest of the approach.

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<sup>1</sup> Universidade Federal do Paraná, Brazil;

<sup>2</sup> UNICAMP, Brazil;

<sup>3</sup> Paris Tech, France;

# Damage Identification of Vehicle Brake Disks by the Use of Impedance-Based SHM and K-Means Method

*Stanley Washington Ferreira de Rezende<sup>1</sup>*

*Bruno Pereira Barella<sup>1</sup>*

*João Paulo Moreira Bento<sup>1</sup>*

*José dos Reis Vieira de Moura Júnior<sup>1</sup>*

The maximum operational efficiency has been a continuous search by the automotive engineering in the last decades, aiming at obtaining greater performance and safety of its mechanical systems at low production and maintenance costs. In this context, emerge some predictive studies related to suffered damage or that may occur over the lifetime of structures. Therefore, focusing on analyzing and avoiding failures, new structural health monitoring methodologies have been developed and electromechanical impedance-based SHM method is one of them. The impedance-based technique uses the dielectric and mechanical properties of piezoelectric materials, inspecting any extension of a structure, and calculating an index among impedance signatures and then detecting the damage. Brake system is one of the most important mechanical systems in a passenger vehicle and it is composed by brake pads and brake disc. This system was designed to promote wear in brake pads which are exchanged periodically while the brake discs continue to have a useful life. Thus, in this contribution a common vehicle brake disc is studied in order to evaluate the sensitivity of the impedance-based SHM application to identify mechanical changes and propose a method of checking the integrity of the brake discs. The proposed experimental damage was the mass addition attached to the system in different positions (3 cubic magnets with 10mm). The frequency range of monitoring used was from 20.5 kHz to 30 kHz. A set of 30 signals of each state of the structure, baseline and damage conditions, were acquired by the acquisition system. Then, it was implemented the algorithm of K-Means for the damage identification (cluster analysis for grouping). Finally, in order to validate the proposed damage identification method, it was performed the construction of a linear regression model with the RMSD damage metrics of the different damage sets. Results show the applicability of the method in the identification of damages.

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<sup>1</sup> Universidade Federal de Goiás, Brazil;

# Data Acquisition Reduction in Impedance-based SHM Method

*Bruno Pereira Barella<sup>1</sup>*

*Stanley Washington Ferreira de Rezende<sup>1</sup>*

*João Paulo Moreira Bento<sup>1</sup>*

*José dos Reis Vieira de Moura Júnior<sup>1</sup>*

The main purpose of the electromechanical impedance-based SHM method is to identify incipient damages in structures. This method can prevent failures in critical mechanical systems such as the aerospace and naval industry or in large structures such as bridges and buildings. The electromechanical impedance-based SHM method usually uses a piezoelectric transducer as sensor/actuator to excite/gather the dynamic response of the mechanical structure under investigation in order to find incipient damages. In SHM methods, many samples of the signature is gathered and recorded in order to perform analysis of the system. Then, the present contribution proposes a method to generate signatures based on some measured ones. The signature generator is based on the Monte Carlo method. Thus, this approach proposes to reduce the number of measured/recorded samples in a SHM system. The system under investigation was an aluminum beam and was applied four levels of damage (mass addition). It was measured 5 impedance signatures for each level of damage. Then, it was used the Monte Carlo Method to generate 200 more virtual signatures. Finally, these generated signatures were compared with the acquired signatures in order to measure the error when generating signatures from this method. Concluding, this contribution can illustrate the efficiency to use only part of the signatures (properties of the signatures) instead of the big amount of data recorded. Then, it is possible to check when it is necessary to record more data in order to classify damages or there is no need of additional signatures. Once neural network techniques need a big amount of data and the previous step is able to check the need of new measurements.

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<sup>1</sup> Universidade Federal de Goiás, Brazil;



# Title and Abstract to be announced (TBA)

*Zhaofang Bai*<sup>1</sup>

TBA

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<sup>1</sup> School of Mathematical Sciences, Xiamen University, China;

# Title and Abstract to be announced (TBA)

*Jianlong Chen*<sup>1</sup>

TBA

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<sup>1</sup> Southeast University, China;

# Markov Chains and Impedance-based SHM Method for Failure Prediction

*Jose dos Reis Vieira de Moura Junior<sup>1</sup>*

*Joao Paulo Moreira Bento<sup>1</sup>*

*Bruno Pereira Barella<sup>1</sup>*

*Stanley Washington Ferreira Rezende<sup>1</sup>*

In the last years it has been developed methods to monitor the structural health of systems as such as electromechanical impedance-based SHM (Structural Health Monitoring). Also, several statistical techniques as such as the Markov chains have become more familiar in engineering applications. The purpose of this contribution is to present a case study that aims to apply the concepts of electromechanical impedance-based SHM, optimization and Markov Chains to the monitoring of the structural integrity of a system. It was used a low cost impedance analyser (Eval - AD5933EBZ) in the experiment to measure the impedance signatures of a simple system with small parts. The monitoring frequency range used was 40000Hz - 52775Hz, with 511 points for analysis. Also, 100 signatures were detected in a certain period of time, which 75 of them were in pristine state and 25 for the structure in a fault state. The BCA (Bee Colony Algorithm) optimization method was used to reduce the region in the frequency domain of monitoring in order to find the most sensitive changes imposed to the system (largest difference between the signals). It was applied the RMSD damage metric to obtain a numerical value of damage and thus be able to define the states of the Markov Chain based on the respective index level. By observing the temporal sequence of the states, the transition of them was identified. Then, there are two possibilities: the system can remain in the same state or can change from one state to another (pristine or failure). The quantification of the transition matrix was performed by the relative frequency of occurrence, respecting the property of stochastic matrices and the transition probabilities were calculated. Concluding, with the case study is possible to understand the applicability of the Markov Chains associated to the electromechanical impedance-based method for monitoring and predicting future states.

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<sup>1</sup> Federal University of Goias, Brazil;

# Pareto front characterization for finite horizon optimal control problems with two different objectives

*Ana Paula Chorobura*<sup>1</sup>

*Hasnaa Zidani*<sup>2</sup>

In this talk, we present a characterization of the weak and strong Pareto fronts for optimal control problems with two objective functions of different nature that need to be minimized simultaneously. One objective is in the classical Bolza form and the other one is defined as a maximum function. Our approach is based on the Hamilton-Jacobi-Bellman framework. First we define an auxiliary optimal control problem without state constraints and show that the weak Pareto front is a subset of the zero level set of the corresponding value function. Then with a geometrical approach we establish a characterization of the Pareto front. Some numerical examples will be considered to show the relevance of our approach.

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<sup>1</sup> Universidade Federal do Paraná, Brazil;

<sup>2</sup> ENSTA ParisTech, France;

# Multiobjective Optimization Techniques applied to Fatigue Analysis of Viscoelastically Damped Systems

*Lauren Karoline de Sousa Gonçalves<sup>1</sup>*

*Ulisses Lima Rosa<sup>1</sup>*

*Antônio Marcos Gonçalves de Lima<sup>1</sup>*

In complex engineering structures, in order to reduce the risk of fatigue failure induced by mechanical vibrations, numerical optimization techniques have been used to determine the effectiveness design of viscoelastic dampers. The aim of this work is determine the optimal regions to apply the viscoelastic treatment and to evaluate the robustness from random parameters of optimal solutions through robust optimization techniques. Among these, NSGA technique (Non-Dominated Sorting Genetic Algorithm) was employed to minimize objective functions of multiobjective problem in order to increase fatigue life. After the presentation of the theoretical foundations, numerical studies were performed with a sandwich plate system incorporating viscoelastic damping. The computational implementation was developed employing the discretization of Gaussian random fields by Karhunen-Loève expansion and estimating the fatigue indexes estimated by Sines' criterion. The system robustness was evaluated considering fluctuation of design variables such as thickness and temperature, and these samples are generated by means of the well-known Latin Hypercube Sampling (LHS). Thus, based on Pareto optimal solutions, numerical results are presented in terms of frequency responses functions (FRFs), stress responses (PSDs) and fatigue indexes estimated by Sines' criterion. The obtained results highlighted the effectiveness of the optimization strategy mainly to demonstrate the importance of considering the robust solution in the fatigue analysis.

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<sup>1</sup> UFU, Brazil;

# Nonlinear programming algorithm based on trust-region-filter method for unconstrained multiobjective optimization problem

*Maria de Gracia Mendonça<sup>1</sup>*

*María Cristina Maciel<sup>2</sup>*

In this work we consider the differentiable unconstrained multiobjective optimization problem. An algorithm will be presented that extends the scalar case of the sequential quadratic programming method using a trust region approach to guarantee global convergence to a weak Pareto optimal point. For the solution of each quadratic subproblem, a generalization of the projected spectral gradient method for scalar case will be presented. At each iteration the trial step will be first analyzed by a suitable filter and a non-monotone acceptance condition.

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<sup>1</sup> Universidad Nacional de la Patagonia San Juan Bosco, Argentina;

<sup>2</sup> Universidad Nacional del Sur, Argentina;

# A SQP-Trust-Region algorithm for Nonlinear Multiobjective Optimization

*María Cristina Maciel<sup>1</sup>*

*Sandra Augusta Santos<sup>2</sup>*

*Graciela Noemí Sottosanto<sup>3</sup>*

This contribution deals with the differentiable nonlinear multiobjective optimization problem with equality constraints. An algorithm which extends the well known Sequential Quadratic Programming method for the scalar case is introduced. The trust region constraint is added to the subproblem in order to guarantee global convergence to a weak Pareto point.

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<sup>1</sup> Department of Mathematics, Southern National University, Bahía Blanca, Argentina;

<sup>2</sup> Department of Applied Mathematics, State University of Campinas, Campinas, Brazil;

<sup>3</sup> Department of Mathematics, Comahue National University, Neuquén, Argentina;

# New sequential optimality conditions for mathematical problems with complementarity constraints and algorithmic consequences

*Leonardo Delarmelina Secchin<sup>1</sup>*

*Roberto Andreani<sup>1</sup>*

*Gabriel Haeser<sup>2</sup>*

*Paulo José da Silva e Silva<sup>1</sup>*

In recent years, the theoretical convergence of iterative methods for solving nonlinear constrained optimization problems has been addressed by means of the so-called sequential optimality conditions. These conditions are satisfied by local minimizers independently of the fulfilment of constraint qualifications, and may be used as stopping criteria of algorithms. In this sense, they provide a suitable framework for unifying and extending the convergence results of various methods. Although there is a considerable literature devoted to sequential conditions for standard nonlinear optimization problems, the same is not true for Mathematical Problems with Complementarity Constraints (MPCCs). MPCCs are difficult problems that do not satisfy the majority of the usual constraint qualifications (CQs). In this paper, we argue that, unfortunately, the established sequential optimality conditions do not provide an adequate tool for the convergence analysis of algorithms in the MPCC context. We then propose sequential optimality conditions for usual stationarity concepts for MPCC, namely, weak, Clarke and Mordukhovich stationarity. We call these conditions AW-, AC- and AM-stationarity, respectively. The weakest MPCC-tailored CQs associated with each of the new conditions are also provided. We show that some of the methods for MPCC in the literature reach AC-stationary points, extending previous theoretical convergence results. In particular, the new results include the linear case, not previously covered.

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<sup>1</sup> IMECC/Unicamp, Brazil;

<sup>2</sup> IME/USP, Brazil;



# Optimality Conditions and Constraint Qualifications for Generalized Nash Equilibrium Problems and their Practical Implications

*Luís Felipe Bueno<sup>1</sup>*

*Gabriel Haeser*

*Frank Navarro Rojas*

Generalized Nash Equilibrium Problems (GNEPs) are a generalization of the classic Nash Equilibrium Problems (NEPs), where each player's strategy set depends on the choices of the other players. In this work we study constraint qualifications and optimality conditions tailored for GNEPs and we discuss their relations and implications for global convergence of algorithms. Surprisingly, differently from the case of nonlinear programming, we show that, in general, the KKT residual can not be made arbitrarily small near a solution of a GNEP. We then discuss some important practical consequences of this fact. We also prove that this phenomenon is not present in an important class of GNEPs, including NEPs. Finally, under a weak constraint qualification introduced, we prove global convergence to a KKT point of an Augmented Lagrangian algorithm for GNEPs and under the quasinormality constraint qualification for GNEPs, we prove boundedness of the dual sequence.

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<sup>1</sup> UNIFESP, Brazil;

# Two New Weak Constraint Qualifications for Mathematical Programs with Equilibrium Constraints and Applications

*Alberto Ramos<sup>1</sup>*

We introduce two new weaker Constraint Qualifications (CQs) for mathematical programs with equilibrium (or complementarity) constraints, MPEC for short. One of them is a tailored version of the constant rank of subspace component (CRSC) and the other is a relaxed version of the MPEC No Nonzero Abnormal Multiplier Constraint Qualification (MPEC-NNAMCQ). Both have the local preservation property and imply the error bound property under mild assumption. Thus, they can be used to extend some results on perturbation analysis and sensitivity existing in the literature.

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<sup>1</sup>UFPR - Federal University of Parana, Brazil;

# An Extension of Yuan's Lemma and its Applications in Optimization

*Gabriel Haeser*<sup>1</sup>

We prove an extension of Yuan's lemma to more than two matrices, as long as the set of matrices has rank at most 2. This is used to generalize the main result of Baccari and Trad (SIAM J Optim 15(2):394–408, 2005), where the classical necessary second-order optimality condition is proved, under the assumption that the set of Lagrange multipliers is a bounded line segment. We prove the result under the more general assumption that the Hessian of the Lagrangian, evaluated at the vertices of the Lagrange multiplier set, is a matrix set with at most rank 2. We apply the results to prove the classical second-order optimality condition to problems with quadratic constraints and without constant rank of the Jacobian matrix. Some further recent results about this conjecture will also be discussed.

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<sup>1</sup> USP - University of São Paulo, Brazil;

# Coherent rings and absolutely pure covers

*Nanqing Ding*<sup>1</sup>

In this talk, we prove that a ring  $R$  is left coherent if and only if the class of absolutely pure left  $R$ -modules is a covering class. This talk is a report on joint work with G.C. Dai.

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<sup>1</sup> Nanjing University, China;

# Stability of a non Fourier plate equation with variable density

*Paulo Nicanor Seminario Huertas*<sup>1</sup>

In this talk, motivated by recent literature for viscoelastic problems with variable density, we consider a model of (non Fourier) thermoelastic plates with velocity dependent density. Our main result establishes the exponential stability of the system without additional mechanical damping.

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<sup>1</sup> ICMC - Universidade de São Paulo, Brazil;

# Title and Abstract to be announced (TBA)

*Hengling Hong*<sup>1</sup>

TBA.

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<sup>1</sup> College of Computer Science and Technology Jilin University, China;

# Polynomial models to predict thermodynamic properties in turbulent flow

*Thelma Pretel Brandão Vecchi<sup>1</sup>*

*F. A. R. Cardoso<sup>1</sup>*

*R. A. Almeida<sup>2</sup>*

*R. V. P. Rezende<sup>2</sup>*

*L. Cardozo-Filho<sup>2</sup>*

The highlight of production of micro- and nanoparticles, from the supercritical technology in the pharmaceutical and food industry, is a process which has been proposed in the scientific literature as an alternative because of its benefits over conventional processes. The SAS method (Supercritical Antisolvent Process) has emerged as an effective alternative, since in this process the solute is dissolved in a conventional organic solvent and the solution is expanded through a capillary chamber containing a supercritical fluid. This acts as an antisolvent, leading to reduced solubility of the solute in the organic solvent causing supersaturation, which leads to the precipitation of particles. The system studied was the precipitation of  $\beta$ -carotene in carbon dioxide as antisolvent, using dichloromethane as the solvent in the SAS process and employed turbulence models  $k$ -Realizable and  $k$ -Standard. The main difference between both models is precisely the fact that the Realizable model is showing error margins between 15 and 30% relative to the experimental data, while the Standard model ranges between 30 and 60%. Thus, so that the model  $k$ - is always achievable, two changes from the standard model were incorporated. In this work, considering the non-ideal behavior of CO<sub>2</sub> under supercritical conditions, the physical properties (density, thermal conductivity, viscosity and mass diffusivity) were also evaluated using polynomials adjusted based on the Peng-Robinson equation of state (EOS), on the Van der Waals mixing rule and on the methods of Chung and Riazi & Whitson, for thermodynamic properties. Simulations performed on a 90 bar pressure and a temperature of 308 K showed results where there were no marked differences when the dependent properties of  $p$ ,  $T$  and composition were used, relative to the cases where the adjusted polynomials were used, indicating a good strategy to use the methodology of the polynomials adjusted to the thermodynamic properties.

<sup>1</sup> UTFPR, Brazil;

<sup>1</sup> UEM, Brazil;

# How to dive in a mathematical way?

*Roberto Ribeiro Santos Junior*<sup>1</sup>

*André Nachbin*<sup>2</sup>

*Paul Milewski*<sup>3</sup>

*Marcelo Flamarion*<sup>2</sup>

The purpose of this talk is to show how to use mathematical techniques such as conformal maps to explore underwater. We are going to discuss some results concerning the flow structure beneath rotational water waves. Our numerical formulation allows us to compute the particle trajectories and calculate accurately the location of stagnation points. Moreover, we present a strategy to capture particle trajectories, due to irrotational waves, in the presence of topography.

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<sup>1</sup> Universidade Federal do Paraná, Brazil;

<sup>2</sup> IMPA, Brazil;

<sup>3</sup> University of Bath, United Kingdom;



# Title and Abstract to be announced;

*Jairo Rocha de Faria<sup>1</sup>*

*Thiago José Machado<sup>1</sup>*

*Raul Renner Martins de Sá<sup>1</sup>*

*Rômulo da Silva Lima<sup>1</sup>*

*L. Cardozo-Filho<sup>2</sup>*

TBA

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<sup>1</sup> UFPB, Brazil;

# Title and Abstract to be announced;

*Felix Sadyrbaev*<sup>1</sup>

*Eduard Brokan*<sup>2</sup>

*Svetlana Atslega*<sup>1</sup>

TBA

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<sup>1</sup> Institute of Mathematics of University of Latvia, Latvia;

<sup>2</sup> Daugavpils University, Latvia;

# Title and Abstract to be announced;

*Flaviana Moreira de Souza Amorim<sup>1</sup>*

*Magna Paulina de Souza Ferreira<sup>1</sup>*

*Márcio da Silva Arantes<sup>2</sup>*

*Claudio Fabiano Motta Toledo<sup>1</sup>*

TBA

<sup>1</sup> USP/ICMC, Brazil;

<sup>2</sup> SENAI-SC, Brazil;

# Mathematics in the daily life of a hydropower plant engineer

*Geraldo C. Brito Junior*

Abstract to be announced;

# Title and Abstract to be announced;

*Aleksandr A. Shananin*

TBA

No informations to show, yet.

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[Click here to see the schedule](#)

# **DAY 4**

## **Plenarists**

# Relationship Between Maximum Principle and Dynamic Programming for State-Constrained Control Problems

*Hasnaa Zidani<sup>1</sup>*

In this talk, we will consider some state-constrained deterministic optimal control problems. For such problems, the main tools that have been developed, in the literature, are based on the Pontryagin Maximum Principle (PMP) or the Hamilton-Jacobi (HJ) approach. Both approaches give necessary conditions for the optimal controls and the associated optimal trajectories. They can also give sufficient optimality conditions under some suitable conditions. Usually, each approach is analysed separately and without connection with the other method. It is known that a relationship between the two approaches exists and it allows to link the adjoint state with the sensitivity of the value function along the optimal trajectory. This relationship is well established when the control problem is without state constraints. In this talk, we will give a general result for the relation between the PMP and HJ approach for some control problems with state constraints (including the case of end-point state constraint) and without assuming any controllability assumption. We shall then present a new algorithm for computing the optimal trajectories combining both approaches (PMP and HJ). Several numerical simulations will be presented to discuss the relevance of the theoretical results.

<sup>1</sup> ENSTA ParisTech, France;

# Gradient flows and determination of convexity

*Aris Daniilidis*<sup>1</sup>

We disclose an interesting connection between the gradient flow of a  $C^2$ -function  $f$  and strongly evanescent orbits of the second order gradient system defined by the square-norm of  $\nabla f$ , under an adequate convexity assumption. As a consequence, we obtain the following surprising result for two  $C^2$ , convex and bounded from below functions  $f, g$  : if  $\|\nabla f\| = \|\nabla g\|$ , then  $f = g + k$ , for some  $k \in \mathbb{R}$ . (Joint work with T. Boulmezaoud, P. Cieutat)

<sup>1</sup> Universidad de Chile, Chile;



# Modeling and Computation of Energy Efficiency Management with Emission Permits Trading

*Shuhua Zhang*<sup>1</sup>

*Xinyu Wang*<sup>1</sup>

In this talk, we present an optimal feedback control model to deal with the problem of energy efficiency management. Especially, an emission permits trading scheme is considered in our model, in which the decision maker can trade the emission permits flexibly. We make use of the optimal control theory to derive a Hamilton-Jacobi-Bellman (HJB) equation satisfied by the value function, and then propose an upwind finite difference method to solve it. The stability of this method is demonstrated and the accuracy, as well as the usefulness, is shown by the numerical examples. The optimal management strategies, which maximize the discounted stream of the net revenue, together with the value functions, are obtained. The effects of the emission permits price and other parameters in the established model on the results have been also examined. We find that the influences of emission permits price on net revenue for the economic agents with different initial quotas are quite different. All the results demonstrate that the emission permits trading scheme plays an important role in the energy efficiency management.

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<sup>1</sup> Tianjin University of Finance and Economics, China;

# Title and Abstract to be announced (TBA)

*Zhu Zuonong*<sup>1</sup>

Abstract to be announced;

<sup>1</sup> Shanghai Jiao Tong University, China;

# Distributed methods for minimization of large sums

*Natasa Krejic*<sup>1</sup>

Abstract to be announced;

<sup>1</sup> Novi Sad, Serbia;

# Through eyes of viscoelastic fluids: How can an external stimulus influence research in theoretical and computational PDEs ?

*Amiya Kumar Pani<sup>1</sup>*

Mathematical research is mainly driven by internal questions (internal to Mathematics). But when it is influenced by questions posed externally, there is no liberty to change the original theme and couple of such questions which are bothering us for the last two decades will be the focus of this talk. In order to understand the context, when mathematical model is formulated to explain approximately certain physical reality, it is important to derive relevant properties mathematically. Say for example in some viscoelastic fluid flows, numerous experiments suggest that after an instantaneous cessation of the external force, the velocity of the fluid dies down with a certain rate. Translating and proving this kind of property pose a challenge in Mathematics research. Starting with a model in a layman's language, in this talk, relevant mathematical issues summarizing our efforts for the last 20-25 years will be briefly discussed. If time permits in the later part of this talk, the focus will be on the following question posed by the user community `How do we believe the numbers crunched by the machine using an algorithm? Finally, we conclude the talk with some mathematical issues

<sup>1</sup> Indian Institute of Technology Bombay, India;

# Seeking Computational Solutions for Air Traffic Management

*Li Weiagng*<sup>1</sup>

Abstract to be announced;

<sup>1</sup> UNB, Brazil;

# An approach for stabilizing simulation of stochastically perturbed systems

*Hugo de la Cruz*<sup>1</sup>

Many physical and engineering processes are often subject to noisy fluctuations which may have a high impact on their evolution. A usual and, typically, appropriate way of mathematically modelling these systems is through stochastic and random differential equations. In this work we consider a new approach for constructing stable integrators for the reliable simulation of stochastically perturbed systems driven by multidimensional linear multiplicative noise. We take advantage of the existing conjugacy between random and stochastic equations to devise stable numerical methods and adaptive algorithms, with a consequent significant reduction of the overall computation cost in simulations of the trajectories of the system. Computational experiments are presented to illustrate the practical performance of the proposed methods.

<sup>1</sup> FGV-Rio, Brazil;

# Finite element methods for the numerical approximation of a distributed optimal control problem governed by the von Karman equations

*Neela Nataraj<sup>1</sup>*

*Jean-Pierre Raymond*

*Gouranga Mallik*

In this talk, we discuss the numerical approximation of a distributed optimal control problem governed by the von Karman equations, defined in polygonal domains with point-wise control constraints. Conforming finite elements are employed to discretize the state and adjoint variables. The control is discretized using piece-wise constant approximations. A priori error estimates for the state, adjoint and control variables and numerical results that justify the theoretical results will be presented.

This work is co-authored with Jean-Pierre Raymond and Gouranga Mallik.

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<sup>1</sup> Indian Institute of Technology Bombay, India;

# Local stabilization of time periodic evolution equations

*Mythily Ramaswamy*<sup>1</sup>

Local stabilization at a prescribed rate around a periodic trajectory of parabolic systems, using boundary control is an interesting problem. The main motivating example is the incompressible Navier-Stokes system. I will discuss this example and the general framework and indicate some results in this direction.

<sup>1</sup> India;



# Improved Optimization Methods for Image Registration Problems

*Yuan Jinyun*<sup>1</sup>

Image registration problems attract much attention of mathematicians and medical researchers because of the special requirements from medicine. Based some mathematical model, we proposed two new methods for optimization problems arising from Image Registration problems. These two methods are based on simple subspace heuristics. We give some theoretical analysis and numerical tests. Numerical results illustrate the improvements of proposed methods for the problem.

<sup>1</sup> CAS, China;

# **DAY 4**

## **Communicators**

# On the Approximate Solutions of Augmented Subproblems within Sequential Methods

*Mael Sachine<sup>1</sup>*

*Ademir Alves Ribeiro<sup>1</sup>*

*Sandra A. Santos<sup>2</sup>*

Within the context of sequential methods for solving general nonlinear programming problems this study deals with the theoretical reasoning behind handling the original subproblems by an augmentation strategy. We do not assume feasibility of the original problem, nor the fulfillment of any constraint qualification. The analysis is made along two directions. First and foremost, the exact nature of the stationary points usually considered is alleviated under an approximate stationary perspective. Second, the analysis has been developed using general vector norms. Therefore, the present results have been obtained under mild hypotheses, and with a involved examination. We stress that we are not concerned with the sequential method itself, nor with computational results. We focus on the features about the original problem that can be inferred from the properties of the solution of the augmented problem, being the solutions analyzed in an approximate sense. Examples illustrating the obtained results are included.

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<sup>1</sup> UFPR, Brazil;

<sup>1</sup> UNICAMP, Brazil;

# Circumcentering the Douglas-Rachford Method

*Luiz-Rafael Santos<sup>1</sup>*

*Roger Behling<sup>1</sup>*

*José-Yunier Bello Cruz<sup>2</sup>*

We introduce and study a geometric modification of the Douglas–Rachford method called the Circumcentered–Douglas–Rachford method. This method iterates by taking the intersection of bisectors of reflection steps for solving certain classes of feasibility problems. The convergence analysis is established for best approximation problems involving two (affine) subspaces and both our theoretical and numerical results compare favorably to the original Douglas–Rachford method. Under suitable conditions, it is shown that the linear rate of convergence of the Circumcentered–Douglas–Rachford method is at least the cosine of the Friedrichs angle between the (affine) subspaces, which is known to be the sharp rate for the Douglas–Rachford method. We also present a preliminary discussion on the Circumcentered–Douglas–Rachford method applied to the many set case and to examples featuring non-affine convex sets.

<sup>1</sup> UFSC, Brazil;

<sup>1</sup> NIU/USA, Brazil;

# Studies on Mathematical Optimization Models using Functions with Interval Parameters

*Thiago Parente da Silveira<sup>1</sup>*

Initially we present a method to provide the generalized  $n$ -dimensional interval space with a vector space structure, through a bijection with Euclidean space. A more recent application of interval analysis is in solving optimization problems in which some coefficients (either objective function or constraints) result from rounded values and / or incomplete information. It is very natural to use interval analysis to work with this type of problem. The interval value function is defined in parametric form and its properties are studied. The advantage of considering in the parametric form happens when analyzing the positivity of the Hessian matrix. It is possible to define partial order relations through known bijections and with this an Interval Optimization Problem (IOP) is defined. The interval optimization problem can be converted to a general optimization problem in the parametric form, and with this it is possible to determine the existence of a solution for the IOP. Convexity plays an important role in proving the existence of a solution to a classical optimization problem. Hence the need to study the convex case of the interval optimization problem. Since the set of intervals is not fully ordered, the convexity has to be studied with respect to a partial order. The problem of quadratic optimization is also defined, and a characterization is presented for direction of descent for this problem.

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<sup>1</sup> Universidade de São Paulo, Brazil;

# On the pointwise iteration-complexity of a dynamic regularized ADMM with over-relaxation stepsize

*Max Leandro Nobre Gonçalves<sup>1</sup>*

In this paper, we extend the improved pointwise iteration-complexity estimation of adynamic regularized alternating direction method of multipliers (ADMM) for a new stepsize domain. In this complexity analysis, the stepsize parameter can be chosen in the interval  $(0,2)$  instead of interval  $(0,(1+\sqrt{5})/2)$ . We illustrate, by means of a numerical experiment, that the enlargement of this stepsize domain can lead to better performance of the method in some applications. Our complexity study is established by interpreting this ADMM variant as an instance of a hybrid proximal extragradient framework applied to a specific monotone inclusion problem.

<sup>1</sup> Federal University of Goiás, Brazil;

# The Proximal Point Method For Locally Lipschitz Functions In Multiobjective Optimization With Application To The Compromise Problem

*João Carlos de Oliveira Souza*<sup>1</sup>

*G. C. Bento*<sup>2</sup>

*J. X. Cruz Neto*<sup>1</sup>

*G. López*<sup>3</sup>

*A. Soubeyran*<sup>4</sup>

This paper studies the constrained multiobjective optimization problem of finding Pareto critical points of vector-valued functions. The proximal point method considered by Bonnel, Iusem, and Svaiter [SIAM J. Optim., 15 (2005), pp. 953-970] is extended to locally Lipschitz functions in the finite dimensional multiobjective setting. To this end, a new (scalarization-free) approach for convergence analysis of the method is proposed where the first-order optimality condition of the scalarized problem is replaced by a necessary condition for weak Pareto points of a multiobjective problem. As a consequence, this has allowed us to consider the method without any assumption of convexity over the constraint sets that determine the vectorial improvement steps. This is very important for applications; for example, to extend to a dynamic setting the famous compromise problem in management sciences and game theory.

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<sup>1</sup> Universidade Federal do Piauí, Brazil;

<sup>2</sup> Universidade Federal de Goiás, Brazil;

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# The Proximal Point Method For Locally Lipschitz Functions In Multiobjective Optimization With Application To The Compromise Problem

*Romulo Castillo<sup>1</sup>*

*Clavel Quintana<sup>2</sup>*

We consider the proximal point method for solving unconstrained multiobjective programming problems including two families of real convex functions, one of them defined on the positive orthant and used for modifying a variant of the logarithm-quadratic regularization introduced recently in [22] and the other for defining a family of scalar representations based on 0-coercive convex functions. We show convergent results, in particular, each limit point of the sequence generated by the method is a weak Pareto solution. Numerical results over fourteen test problems are shown.

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# Non-linear conjugate gradient methods for vector optimization

*Leandro Prudente<sup>1</sup>*

*L. R. Lucambio Pérez<sup>1</sup>*

We propose non-linear conjugate gradient methods for finding critical points of vector-valued functions with respect to the partial order induced by a closed, convex, and pointed cone with non-empty interior. No convexity assumption is made on the objectives. The concepts of Wolfe and Zoutendijk conditions are extended for the vector-valued optimization. In particular, we show that there exist intervals of stepsizes satisfying the Wolfe-type conditions. The convergence analysis covers the vector extensions of the Fletcher-Reeves, Conjugate Descent, Dai-Yuan, Polak-Ribière-Polyak, and Hestenes-Stiefel parameters that retrieve the classical ones in the scalar minimization case. Under inexact line searches and without regular restarts, we prove that the sequences generated by the proposed methods find points that satisfy the first-order necessary condition for Pareto-optimality. Numerical experiments illustrating the practical behavior of the methods are presented.

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# Non-linear conjugate gradient methods for vector optimization

*Leandro Prudente<sup>1</sup>*

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# Two generalizations of orthogonal transformations and their properties

*Ricardo Bilioti*<sup>1</sup>

*Jin-Yun Yuan*<sup>2</sup>

Givens' rotations and Householder's reflectors are powerful orthogonal transformations with plenty of applications in scientific computing, like least-squares problems, algorithms for QR-decomposition, and eigenvalue problems, to mention a few. In this talk, we present extensions of these two transformations, which can be employed, for instance, to deal with non-symmetric eigenvalue problem. Since both extensions are no longer orthogonal in general, their properties are studied and special choices for their parameters are provided to attain good numerical properties.

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# Title and Abstract to be announced (TBA)

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TBA

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# Title and Abstract to be announced (TBA)

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TBA

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# Non-monotone schemes for Hamilton-Jacobi-Bellman equations

Olivier Bokanowski<sup>1</sup>

We discuss the use of non-monotone schemes for the approximation of Hamilton-Jacobi-Bellman equations, a particular form of non-linear Partial Differential Equations related to optimal control problems. Because of Godunov's barrier Theorem, it is known that non-monotonicity of the scheme is necessary in order to obtain high-order accuracy. On the other hand in the classical framework of viscosity, such as the Theorem of Barles and Souganidis (1991), or the Finite Difference schemes first introduced by Crandall and Lions (1984) in the Hamilton-Jacobi framework, the monotonicity or the "almost monotonicity" property is strongly used in order to ensure convergence. Furthermore non-monotone schemes may converge to wrong solutions. Here we shall present different situations where non-monotone schemes can be used (such as when using Discontinuous Galerkin approaches, Finite difference "anti-diffusive" schemes, or Backward Difference Formula), give some numerical analysis of the schemes when possible, and show some numerical advantage of the proposed schemes.

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# A mathematical model for chemoimmunotherapy of chronic lymphocytic leukemia

*Paulo Fernando de Arruda Mancera<sup>1</sup>*

*Diego Samuel Rodrigues*

*Tiago de Carvalho*

Immunotherapy is currently regarded as the most promising therapy to treat chronic lymphocytic leukemia, an indolent neoplastic disease of B-lymphocytes which eventually causes the immune system's failure. In this and other areas of cancer research, mathematical modelling is pointed out as a prominent tool to analyze theoretical and practical issues. We propose an ordinary differential equation model for chemoimmunotherapy of chronic lymphocytic leukemia. The action of the immune system and the chemoimmunotherapeutic role in promoting cancer cure are investigated by means of numerical simulations and linear stability analysis. From a theoretical point of view, the role of adoptive cellular immunotherapy is also discussed. The main conclusion is that chemoimmunotherapeutic protocols can be effective in treating chronic lymphocytic leukemia provided that chemotherapy is not a limiting factor to the immunotherapy efficacy.

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# Optimal non-anticipative scenarios for nonlinear hydrothermal power systems

*Gislaine A. Perićaro*<sup>1</sup>

*Elizabeth W. Karas*<sup>2</sup>

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Electrical energy in Brazil is generated by an interconnected system, composed of over 100 hydro plants complemented by over 50 thermal power plants, distributed in 5 interconnected sub-systems. Monthly inflow scenarios are generated based on 80 years of historical data. Deterministic problem: for a given scenario (typically 60 months), a non-linear programming Filter algorithm with sequential quadratic iterates computes an optimal scenario, i.e, a solution to the operation planning problem with minimum cost of thermal generation and deficit. This is a large scale problem with non-linear constraints due to the effects of head variation on the productivity of hydro plants. Stochastic problem: Given a set of scenarios, a cloud of optimal solutions is the corresponding set of optimal scenarios. A non-anticipative solution for the first stage is obtained by minimizing the expected value of the scenario costs with the extra constraint that the first hydro decision for the reservoir plants must be identical for all scenarios. This is now a larger problem. For small problems, it may be solved directly by the Filter algorithm; for larger problems a decomposition algorithm like Progressive Hedging should be used. A non-anticipative optimal scenario associated with a given hydrological scenario is obtained by sequential iterations of this procedure. In this talk we show solutions for a small system and compare them to other approaches.

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# Optimality conditions and global convergence for nonlinear semidefinite programming

*Daiana Viana*<sup>1</sup>

Sequential optimality conditions have played a major role in unifying and extending global convergence results for several classes of algorithms for general nonlinear optimization. In this paper, we extend these concepts for nonlinear semidefinite programming. We define two sequential optimality conditions for nonlinear semidefinite programming. The first is a natural extension of the so-called Approximate-Karush-Kuhn-Tucker (AKKT), well known in nonlinear optimization. The second one, called Trace-AKKT (TAKKT), is more natural in the context of semidefinite programming as the computation of eigenvalues is avoided. We propose an Augmented Lagrangian algorithm that generates these types of sequences and new constraint qualifications are proposed, weaker than previously considered ones, which are sufficient for the global convergence of the algorithm to a stationary point.

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# Augmented Lagrangian for nonlinear SDPs applied to the covering problem

*Leonardo M. Mito<sup>1</sup>*

*Gabriel Haeser<sup>1</sup>*

*Ernesto G. Birgin*

*Walter G. Bofill<sup>2</sup>*

*Daiana S. Viana<sup>1</sup>*

In this work we present an Augmented Lagrangian algorithm for nonlinear semidefinite programs (NLSDPs), which is a natural extension of its consolidated counterpart in nonlinear programming. This method works with two levels of constraints, one that is penalized and other that is kept within the subproblems. This is done in order to allow exploiting the subproblem structure while solving it. The global convergence theory is based on recent results regarding approximate Karush-Kuhn-Tucker optimality conditions for NLSDP, which is stronger than Fritz John optimality conditions that are usually employed. Additionally, we approach the so-called sphere covering problem exploiting some convex algebraic geometry results, such as Stengle's positivstellensatz and its variations. The problem can be written in terms of a standard NLSDP using Gram representations for real polynomials that are sums of squares of other polynomials. Numerical experiments are presented.

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# Continuous methods for the Traffic Assignment Problem: On the interplay between Optimization and Equilibrium solutions

*Ovidiu Bagdasar*<sup>1</sup>

All drivers aim to realise the fastest (i.e., shortest) journey time between an origin and destination. They often have to choose between a direct main road (high capacity but easily congested), a shorter route through residential areas (often called ‘rat-runs’), or a nearby motorway or ring-road (longer but potentially faster). In this paper we examine a simplified traffic model, where a fixed number of vehicles is distributed along alternative routes connecting an origin and a destination. We formulate discrete and continuous optimisation and equilibrium-type problems and various techniques to investigate the link between the equilibrium and optimization solutions.

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# On maximal monotonicity of bifunctions on Hadamard manifolds

*Pedro Antonio Soares Júnior<sup>1</sup>*

*J.X. Cruz Neto<sup>3</sup>*

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We study some conditions for a monotone bifunction to be maximally monotone by using a corresponding vector field associated to the bifunction and vice versa. This approach allows us to establish existence of solutions to equilibrium problems in Hadamard manifolds obtained by perturbing the equilibrium bifunction.

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# Outer-approximation algorithms for nonsmooth convex MINLP problems

*Adriano Delfino<sup>1</sup>*

*Wellington de Oliveira<sup>2</sup>*

In this work, we combine outer-approximation (OA) and bundle method algorithms for dealing with mixed-integer nonlinear programming (MINLP) problems with nonsmooth convex objective and constraint functions. As the convergence analysis of OA methods relies strongly on the differentiability of the involved functions, OA algorithms may fail to solve general nonsmooth convex MINLP problems. In order to obtain OA algorithms that are convergent regardless the structure of the convex functions, we solve the underlying OA's nonlinear subproblems by a specialized bundle method that provides necessary information to cut off previously visited (non-optimal) integer points.

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# Damped Newton's Method on Riemannian Manifolds

*Teles Araújo Fernandes<sup>1</sup>*

*Orizon Pereira Ferreira<sup>2</sup>*

*Márcio Antônio de Andrade Bortoloti<sup>1</sup>*

*Yuan Jin Yun<sup>3</sup>*

Algorithms using the differential structure of nonlinear manifolds play an important role in optimization. Recent years have witnessed a growing interest in the development of numerical algorithms for nonlinear manifolds, as there are many numerical problems posed in manifolds arising in various natural contexts. In this work, instead of focusing on finding singularities of gradient vector fields, including local minimizers, on Riemannian manifolds, we consider the more general problem of finding singularities of vector fields. In this context, we present a damped Newton's method to find a singularity of a vector field in Riemannian manifolds with global convergence. It is ensured that the sequence generated by the proposed method reduces to a sequence generated by the Riemannian version of the classical Newton's method after a finite number of iterations, consequently its convergence rate is superlinear/quadratic. Moreover, numerical experiments on the cone of symmetric positive definite matrices illustrate that the damped Newton's method has better performance than the Newton's method in number of iteration and computational time.

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# Metrically Regular Vector Field and Iterative Processes for Generalized Equations in Hadamard Manifolds

*Orizon Pereira Ferreira*<sup>1</sup>

*Célia Jean-Alexis*<sup>2</sup>

*Alain Piétrus*<sup>2</sup>

This paper is focused on the problem of finding a singularity of the sum of two vector fields defined on a Hadamard manifold, or more precisely, the study of a generalized equation in a Riemannian setting. We extend the concept of metric regularity to the Riemannian setting and investigate its relationship with the generalized equation in this new context. In particular, a version of Graves's theorem is presented and we also define some concepts related to metric regularity, including the Aubin property and the strong metric regularity of set-valued vector fields. A conceptual method for finding a singularity of the sum of two vector fields is also considered. This method has as particular instances: the proximal point method, Newton's method, and Zincenko's method on Hadamard manifolds. Under the assumption of metric regularity at the singularity, we establish that the methods are well defined in a suitable neighborhood of the singularity. Moreover, we also show that each sequence generated by these methods converges to this singularity at a superlinear rate.

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