LISTA 1

1)

- $\underline{T(n) = 7 + T(n-1) \quad \text{com } T(0)=0}$

$$T(n) = 7 + 7 + T(n-2)$$
$$= 7K + T(n-K) \qquad n-K=0, \; K=n$$
$$= 7n + T(0)$$
$$= 7n$$

- $\underline{T(n) = c + T(n-3) \quad T(p)=1 \;\; p \leq 3}$

$$T(n) = c + c + T(n-6)$$
$$= Kc + T(n-3K) \qquad n-3K=1 \qquad K = \frac{n-1}{3}$$
$$= \frac{nc-c}{3} + 1$$

- $\underline{T(n) = n/2 + T(n-1) \;\; \text{com } T(1)=1}$

$$T(n) = n/2 + \frac{n-1}{2} + T(n-2)$$
$$= \frac{n}{2} + \frac{n-1}{2} + \frac{n-2}{2} + T(n-3)$$
$$= \sum_{i=0}^{K-1} \frac{n-i}{2} + T(n-K) \qquad n-K=1, \; K=n-1$$
$$= \sum_{i=0}^{n-2} \frac{n-i}{2} + 1 = \frac{n-1}{2} \cdot \left(\frac{n-(n-2)}{2} + \frac{n}{2} \right) = \frac{n-1}{2} + \frac{n^2-n}{2}$$

2) b)

i)

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void menores(int *v, int* lt, int N) {
    for (i = 0; i < N; i++) {
        for (j = 0; j <= i; j++) {
            if (v[j] < v[i])
                lt[i]++;
    }
```

if (i == N) c
return

$a + n(2c+a)$

menores (v, lt, N, i+1)

}

1ª chamada → i = 0

ultima ⟶ i = n - 1

invertendo na recorrência

$$f(n) = c + a + n(2c+a) + f(n-1)$$

$$f(0) = c$$

c) $f(n) = c + a + n(2c+a) +$
$c + a + (n-1)(2c+a) +$
$f(n-2)$

$$= k(c+a) + \sum_{i=0}^{k-1} n-i(2c+a) + f(n-k)$$

$n - k = 0, \ k = n$

$$= n(c+a) + \sum_{i=0}^{n-1} n-i(2c+a)$$

$$= n(c+a) + (n^2+n)(2c+a)\frac{1}{2}$$

$$\boxed{f(n) = cn^2 + \frac{an^2}{2} + 2cn + an + \frac{an}{2}}$$

2) d)

$$T(n) = \overset{\text{callee}}{\downarrow} n + \overset{\text{preenche}}{\downarrow} p(n) + f(n)$$

$$= n + \underbrace{a + c + n(c+2a)} + f(n)$$

$$= n + a + c + \cancel{cn} + \cancel{2an} + \cancel{cn^2} + \frac{an^2}{2} + 2\cancel{c}n + an + \cancel{\frac{an}{2}}$$

$$= cn^2 + \frac{an^2}{2} + 3cn + \frac{7a}{2} + n + a + c$$

d) Basta encontrar Θ

$T(n)$ é $\Theta(n^2)$

$$c_1 n^2 \leq cn^2 + \frac{an^2}{2} + 3cn + \frac{7a}{2} + n + a + c \leq c_2 n^2$$

$$c_1 \leq c + \frac{a}{2} + \cancel{\frac{3c}{n}} + \cancel{\frac{7a}{2n^2}} + \cancel{\frac{1}{n}} + \cancel{\frac{a}{n^2}} + \cancel{\frac{c}{n^2}}$$

$n_0 = 1$, $c_1 = c + \frac{a}{2}$

$$c + \frac{a}{2} + \frac{3c}{n} + \frac{7a}{2n^2} + \frac{1}{n} + \frac{a}{n^2} + \frac{c}{n^2} \leq c_2$$

maior termo p/ $n = 1$

$$c + \frac{a}{2} + 3c + \frac{7a}{2} + 1 + a + c \leq c_2$$

$$c_2 = 4c + 5a + 1$$

$T(n)$ é $\Theta(n^2)$ com $n_0 = 1$ $c_1 = c + \frac{a}{2}$
$c_2 = 4c + 5a + 1$

3) c)

$$\frac{n(n-2)}{3} - 5 \text{ é } \Theta(n^2)$$

$$\underbrace{c_1 n^2 \leq \frac{n^2 - n}{3} - 5}\ \leq c_2 n^2$$

$$c_1 \leq -\frac{\cancel{5}}{n^2} - \frac{1}{\cancel{3}n} + \frac{1}{3} \quad p/ n_0 = 1 \quad \underline{\text{menor termo}}$$

$$-5 - \frac{1}{3} + \frac{1}{3} = -5 \quad n_0 = 5$$

$$-\frac{5}{25} - \frac{1}{15} + \frac{1}{3} = \frac{1}{15}$$

$$c_1 = \frac{1}{15}$$

$$-\cancel{\frac{5}{n^2}}^{\to 0} - \cancel{\frac{1}{3n}}^{\to 0} + \frac{1}{3} \leq c_2 \quad p/ n_0 = 5 \quad \underline{\text{maior termo}}$$

$$c_2 = \frac{1}{3}$$

é $\Theta(n^2)$ p/ $n_0 = 5$ $c_1 = \frac{1}{15}$, $c_2 = \frac{1}{3}$

4) e) $100n^4 + n^3$ é $O(2^n)$?

$$100n^4 + n^3 \leq c \cdot 2^n$$ p/ $n_0 = 1$

maior termo

$$\frac{100n^4 + \cancel{n^3}^{\,0}}{2^n} \leq c$$

$$\frac{101}{2} = c$$

é $O(2^n)$

p/ $n_0 = 1$ $c = \frac{101}{2}$

f) n^n é $O(2^n)$

$$n^n \leq c\, 2^n$$

maior termo

$$\frac{n^n}{2^n} \leq c$$ p/ $n_0 = 1$ $\frac{1}{2} \leq c$

$$\frac{n^n}{2^n} \to \infty$$ portanto $\nexists\ c \geq \frac{n^n}{2^n}$

n^n não é $O(2^n)$

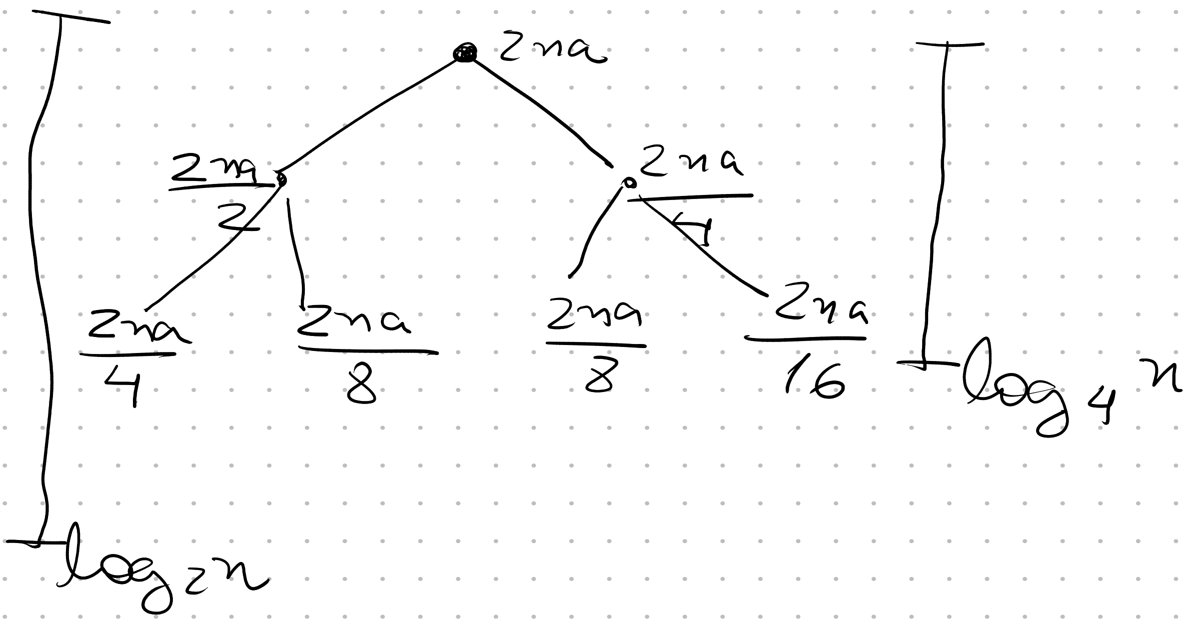
7) $\overset{\text{for}}{\widetilde{}}\quad \overset{\text{while}}{\widetilde{}}$

$$T(n) = f(n) + g(n)$$

$$f(n) = na + f(n/2), \quad f(1) = 0$$

$$g(n) = na + g(n/4), \quad g(1) = 0$$

$$T(n) = 2na + f(n/2) + g(n/4)$$



$$f(n) = na + f(n/2)$$

$$= Kna + f(n/2^k) \qquad \frac{n}{2^k} = 1$$

$$= an \cdot \log_2 n \qquad K = \log_2 n$$

$$g(n) = na + f(n/4)$$

$$= kna + f(n/4^k)$$

$$= an\log_4 n$$

$$T(n) = a \cdot n\log_2 n + a\, n\log_4 n$$

c) $\Theta(n \log_2 n)$

$$c_1 n \log_2 n \leq a\, n \log_2 n + a n \log_4 n \leq c_2 n \log_2 n$$

$$c_1 \leq a + \frac{a n \log_4 n}{n \log_2 n}$$

$$c_1 \leq a + a \cdot \frac{1}{2} \qquad c_1 = \frac{3a}{2}$$

$$\frac{3a}{2} \leq c_2 \qquad c_2 = \frac{3a}{2}$$

$T(n)$ é $\Theta(n \log_2 n)$ p/ $c_1, c_2 = \frac{3a}{2}$

$n_0 = 1$