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In[1]:= (* =====BOUNCING MASS WITH REALISTIC MASS-
DEPENDENT EFFECTS=====*)
Print["\n====="];
Print["BOUNCING MASS PHYSICS DEMONSTRATION"];
Print["REALISTIC MASS-DEPENDENT BEHAVIOR"];
Print["=====\\n"];

(* =====
IMPACT PHYSICS PARAMETERS=====*)
massObject1 = 0.7;      (*Mass of objects 1-3 (kg)*)
massObject4 = 1.5;      (*Mass of object 4-HEAVIER! (kg)*)
gBounce = 9.81;         (*Gravity (m/s^2)*)

(*VISUAL SIZES-Heavier mass looks bigger!*)
radiusMass1 = 0.15;     (*Radius for masses 1-3 (m)*)
radiusMass4 = 0.20;      (*Radius for mass 4-BIGGER! (m)*)

(*Impact Parameters*)
eBounce = 0.7;          (*Coefficient of restitution for all*)

(*MASS-DEPENDENT FRICTION:Heavier objects have more "grip"*)
muBounce1 = 0.09;       (*Friction for light masses*)
muBounce4 = 0.15;       (*Friction for heavy mass-MORE FRICTION!*)

(*Ground level*)
groundLevel = 0.0;

(*Throw parameters*)
throwHeight = 2.5;
throwVelocityX = 2.5;
throwVelocityY = 1.0;

Print["Physics Parameters:"];
Print[" Mass 1-3: ", massObject1, " kg, friction  $\mu$  = ", muBounce1];
Print[" Mass 4: ", massObject4, " kg (",
Round[(massObject4/massObject1 - 1) * 100], "% heavier), friction  $\mu$  = ",
muBounce4, " (", Round[(muBounce4/muBounce1 - 1) * 100], "% more friction!)"];
Print[" Gravity: ", gBounce, " m/s2"];
Print[" Restitution coefficient (e): ", eBounce];
Print[" Throw velocity: (", throwVelocityX, ", ", throwVelocityY, ") m/s"];
Print["\\nKey Difference: Heavier mass has MORE friction,"];
Print["so it loses horizontal velocity FASTER after bouncing!"];
Print[""];

(* =====
FLIGHT DYNAMICS=====*)

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xFlight[t_, x0_, vx0_] := x0 + vx0 * t
yFlight[t_, y0_, vy0_] := y0 + vy0 * t - 0.5 * gBounce * t^2
vxFlight[t_, vx0_] := vx0
vyFlight[t_, vy0_] := vy0 - gBounce * t

(* =====
IMPACT DETECTION AND PHYSICS=====*)

FindImpactTime[y0_, vy0_, radius_] :=
Module[{sol}, sol = Solve[y0 + vy0 * t - 0.5 * gBounce * t^2 == radius && t > 0, t, Reals];
If[Length[sol] > 0, t /. sol[[1]], Infinity]];

(*MODIFIED:ApplyImpact with mass-dependent friction*)
ApplyImpact[vxMinus_, vyMinus_, mass_, mu_] :=
Module[{vxPlus, vyPlus, Jn, Jt, deltaVx, status, energyRetained},
Print[" === IMPACT DETECTED ==="];
Print[" Mass: ", mass, " kg, μ = ", mu];
Print[" Pre-impact: vx⁻ = ", N[vxMinus, 4], " m/s, vy⁻ = ", N[vyMinus, 4], " m/s"];
(*Normal:Newton's Restitution*) vyPlus = -eBounce * vyMinus;
Jn = mass * Abs[vyMinus] * (1 + eBounce);
(*Tangential:Coulomb Friction-use provided mu*) Jt = mu * Jn;
deltaVx = Jt / mass;
If[Abs[vxMinus] > deltaVx, vxPlus = vxMinus - Sign[vxMinus] * deltaVx;
status = "SLIDING", vxPlus = 0;
status = "STICKING"];
energyRetained =
100 * 0.5 * mass * (vxPlus^2 + vyPlus^2) / (0.5 * mass * (vxMinus^2 + vyMinus^2));
Print[" ΔVx = ", N[deltaVx, 4], " m/s (friction effect)];
Print[" Status: ", status];
Print[" Post-impact: vx⁺ = ", N[vxPlus, 4], " m/s, vy⁺ = ", N[vyPlus, 4], " m/s"];
Print[" Energy retained: ", N[energyRetained, 4], "%"];
Print[""];
{vxPlus, vyPlus}];

(* =====
BUILD BOUNCING TRAJECTORY=====*)

BuildBouncingTrajectory[x0_, y0_, vx0_, vy0_, tMax_, massName_, mass_, radius_, mu_] :=
Module[{segments = {}, impactTimes = {}, impactLocations = {}, t = 0,
x = x0, y = y0, vx = vx0, vy = vy0, tImpact, xImpact, bounceCount = 0},
Print["== SIMULATING ", massName, " =="];
Print["Mass: ", mass, " kg, Radius: ", radius, " m, Friction: μ = ", mu];
Print["Initial: (", N[x0, 3], ", ",
N[y0, 3], ") m, velocity (", N[vx0, 3], ", ", N[vy0, 3], ") m/s"];
Print[""];
While[t < tMax && bounceCount < 20, tImpact = FindImpactTime[y, vy, radius];
If[tImpact == Infinity || t + tImpact > tMax, AppendTo[segments, {t, tMax, x, y, vx, vy}]];

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Break[]];
AppendTo[segments, {t, t + tImpact, x, y, vx, vy}];
t = t + tImpact;
xImpact = xFlight[tImpact, x, vx];
x = xImpact;
y = radius;
vx = vxFlight[tImpact, vx];
vy = vyFlight[tImpact, vy];
Print[" Bounce #", bounceCount + 1,
  " at t = ", N[t, 4], " s, location (", N[x, 4], ", ", N[y, 4], ") m"];
AppendTo[impactTimes, t];
AppendTo[impactLocations, {x, y}];
(*Apply impact with mass-specific friction*){vx, vy} = ApplyImpact[vx, vy, mass, mu];
bounceCount++;
If[Abs[vy] < 0.1, Print[" ", massName, " stopped bouncing"];
AppendTo[segments, {t, tMax, x, radius, vx, 0}];
Break[]];
Print[" Total bounces: ", bounceCount];
Print[" Final x position: ", N[x, 4], " m"];
Print[""];
{segments, impactTimes, impactLocations}];

(* =====
SIMULATE ALL 4 MASSES=====
*)

tSimMax = 8.0;

startX1 = -1.5;
startX2 = -0.5;
startX3 = 0.5;
startX4 = 1.5;

Print["=====];
Print["RUNNING BOUNCING SIMULATIONS"];
Print["=====\\n"];

{traj1, impacts1, locs1} = BuildBouncingTrajectory[startX1, throwHeight, throwVelocityX,
  throwVelocityY, tSimMax, "MASS 1 (Red)", massObject1, radiusMass1, muBounce1];

{traj2, impacts2, locs2} = BuildBouncingTrajectory[startX2, throwHeight, throwVelocityX,
  throwVelocityY, tSimMax, "MASS 2 (Green)", massObject1, radiusMass1, muBounce1];

{traj3, impacts3, locs3} = BuildBouncingTrajectory[startX3, throwHeight, throwVelocityX,
  throwVelocityY, tSimMax, "MASS 3 (Blue)", massObject1, radiusMass1, muBounce1];

{traj4, impacts4, locs4} =
BuildBouncingTrajectory[startX4, throwHeight, throwVelocityX, throwVelocityY,

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tSimMax, "MASS 4 (Magenta) - HEAVY", massObject4, radiusMass4, muBounce4];

Print["=====];
Print["ALL BOUNCING SIMULATIONS COMPLETE"];
Print["=====];
Print["\nCOMPARISON OF FINAL POSITIONS:"];
Print[" Mass 1 final x: ", N[Last[traj1][3], 3], " m"];
Print[" Mass 2 final x: ", N[Last[traj2][3], 3], " m"];
Print[" Mass 3 final x: ", N[Last[traj3][3], 3], " m"];
Print[" Mass 4 final x: ", N[Last[traj4][3], 3], " m"];
Print["\nMass 4 should travel LESS distance horizontally"];
Print["because it has MORE FRICTION (loses vx faster)! \n"];

(* =====
POSITION FUNCTION=====
*)

GetMassPosition[traj_, time_?NumericQ, radius_] :=
Module[{seg, tLocal, x, y}, seg = SelectFirst[traj, (#[[1]] <= time <= #[[2]]) &, None];
If[seg === None, seg = Last[traj]];
tLocal = If[seg === None, 0, time - seg[[1]]];
x = xFlight[tLocal, seg[[3]], seg[[5]]];
y = Max[radius, yFlight[tLocal, seg[[4]], seg[[6]]]];
{x, y}];

(* =====
VISUALIZATION=====
*)

Print["=====];
Print["CREATING PLOTS AND ANIMATION"];
Print["===== \n"];

allLocs = Flatten[{locs1, locs2, locs3, locs4}, 1];
xMaxPlot = If[Length[allLocs] > 0, Max[allLocs[[All, 1]]] + 2, startX4 + 5];
xMinPlot = Min[{startX1, startX2, startX3, startX4}] - 1;

ground = Graphics[
{Brown, Thickness[0.015], Line[{{xMinPlot, groundLevel}, {xMaxPlot, groundLevel}}]}];

allImpactMarkers =
Graphics[{If[Length[locs1] > 0, {Red, PointSize[0.012], Point[locs1]}, {}],
If[Length[locs2] > 0, {Green, PointSize[0.012], Point[locs2]}, {}],
If[Length[locs3] > 0, {Blue, PointSize[0.012], Point[locs3]}, {}],
If[Length[locs4] > 0, {Magenta, PointSize[0.012], Point[locs4]}, {}]}];

staticPlot = Show[ground, ParametricPlot[GetMassPosition[traj1, t, radiusMass1],
{t, 0, tSimMax}, PlotStyle -> {Thick, Red, Dashed}, PlotPoints -> 200],
ParametricPlot[GetMassPosition[traj2, t, radiusMass1], {t, 0, tSimMax},

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PlotStyle -> {Thick, Green, Dashed}, PlotPoints -> 200],
ParametricPlot[GetMassPosition[traj3, t, radiusMass1], {t, 0, tSimMax},
  PlotStyle -> {Thick, Blue, Dashed}, PlotPoints -> 200],
ParametricPlot[GetMassPosition[traj4, t, radiusMass4], {t, 0, tSimMax},
  PlotStyle -> {Thick, Magenta, Dashed}, PlotPoints -> 200], allImpactMarkers,
PlotRange -> {{xMinPlot, xMaxPlot}, {-0.5, throwHeight + 1}}, AspectRatio -> 1/3,
Frame -> True, FrameLabel -> {"x (m)", "y (m)"}, PlotLabel ->
  "Mass-Dependent Bouncing\nMass 4 (Magenta) has MORE FRICTION → stops sooner!",
ImageSize -> Large, GridLines -> Automatic];

staticPlot

bouncingAnimation = Animate[Module[{pos1, pos2, pos3, pos4, trail1, trail2, trail3, trail4},
  pos1 = GetMassPosition[traj1, t, radiusMass1];
  pos2 = GetMassPosition[traj2, t, radiusMass1];
  pos3 = GetMassPosition[traj3, t, radiusMass1];
  pos4 = GetMassPosition[traj4, t, radiusMass4];
  trail1 =
    If[t > 0.1, Table[GetMassPosition[traj1, tau, radiusMass1], {tau, 0, t, 0.1}], {}];
  trail2 =
    If[t > 0.1, Table[GetMassPosition[traj2, tau, radiusMass1], {tau, 0, t, 0.1}], {}];
  trail3 =
    If[t > 0.1, Table[GetMassPosition[traj3, tau, radiusMass1], {tau, 0, t, 0.1}], {}];
  trail4 =
    If[t > 0.1, Table[GetMassPosition[traj4, tau, radiusMass4], {tau, 0, t, 0.1}], {}];
  Show[ground,
    Graphics[{{If[Length[trail1] > 1, {Red, Dashed, Thickness[0.003], Line[trail1]}, {}],
      If[Length[trail2] > 1, {Green, Dashed, Thickness[0.003], Line[trail2]}, {}],
      If[Length[trail3] > 1, {Blue, Dashed, Thickness[0.003], Line[trail3]}, {}],
      If[Length[trail4] > 1, {Magenta, Dashed, Thickness[0.003], Line[trail4]}, {}]}],
    Graphics[{{Red, EdgeForm[{Black, Thick}], Disk[pos1, radiusMass1]},
      {Green, EdgeForm[{Black, Thick}], Disk[pos2, radiusMass1]},
      {Blue, EdgeForm[{Black, Thick}], Disk[pos3, radiusMass1]},
      {Magenta, EdgeForm[{Black, Thick}], Disk[pos4, radiusMass4]}}],
    allImpactMarkers, PlotRange -> {{xMinPlot, xMaxPlot}, {-0.5, throwHeight + 1}},
    AspectRatio -> 1/3, Frame -> True, FrameLabel -> {"x (m)", "y (m)"},
    PlotLabel -> "t = " <> ToString[NumberForm[t, {4, 2}]] <>
      " s\nMass 4 (BIGGER, MAGENTA) has MORE FRICTION → Stops sooner!",
    ImageSize -> Large}], {t, 0, tSimMax, 0.05}, AnimationRate -> 1, DefaultDuration -> 10];

Print["====="];
Print["MASS-DEPENDENT PHYSICS COMPLETE"];
Print["====="];
Print["\nKEY OBSERVATION:"];
Print["Watch Mass 4 (Magenta) - it should:"];
Print[" 1. Lose horizontal velocity FASTER"];
Print[" 2. Travel LESS DISTANCE overall"];

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Print["\nThis is because heavier objects have"];
Print["MORE friction force at impact!"];
Print["=====\\n"];

```

bouncingAnimation

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=====
BOUNCING MASS PHYSICS DEMONSTRATION
REALISTIC MASS-DEPENDENT BEHAVIOR
=====
```

Physics Parameters:

Mass 1-3: 0.7 kg, friction μ = 0.09

Mass 4: 1.5 kg (114% heavier), friction μ = 0.15 (67% more friction!)

Gravity: 9.81 m/s²

Restitution coefficient (e): 0.7

Throw velocity: (2.5, 1.) m/s

Key Difference: Heavier mass has MORE friction,
so it loses horizontal velocity FASTER after bouncing!

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=====
RUNNING BOUNCING SIMULATIONS
=====
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== SIMULATING MASS 1 (Red) ==

Mass: 0.7 kg, Radius: 0.15 m, Friction: μ = 0.09

Initial: (-1.5, 2.5) m, velocity (2.5, 1.) m/s

Bounce #1 at t = 0.801575 s, location (0.503938, 0.15) m

== IMPACT DETECTED ==

Mass: 0.7 kg, μ = 0.09

Pre-impact: vx⁻ = 2.5 m/s, vy⁻ = -6.86345 m/s

ΔV_x = 1.05011 m/s (friction effect)

Status: SLIDING

Post-impact: vx⁺ = 1.44989 m/s, vy⁺ = 4.80442 m/s

Energy retained: 47.2002%

Bounce #2 at t = 1.78107 s, location (1.9241, 0.15) m

```
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu$  = 0.09
Pre-impact:  $v_x^- = 1.44989$  m/s,  $v_y^- = -4.80442$  m/s
 $\Delta V_x = 0.735076$  m/s (friction effect)
Status: SLIDING
Post-impact:  $v_x^+ = 0.714816$  m/s,  $v_y^+ = 3.36309$  m/s
Energy retained: 46.9388%
```

```
Bounce #3 at t = 2.46672 s, location (2.41421, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu$  = 0.09
Pre-impact:  $v_x^- = 0.714816$  m/s,  $v_y^- = -3.36309$  m/s
 $\Delta V_x = 0.514553$  m/s (friction effect)
Status: SLIDING
Post-impact:  $v_x^+ = 0.200262$  m/s,  $v_y^+ = 2.35416$  m/s
Energy retained: 47.2213%
```

```
Bounce #4 at t = 2.94667 s, location (2.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu$  = 0.09
Pre-impact:  $v_x^- = 0.200262$  m/s,  $v_y^- = -2.35416$  m/s
 $\Delta V_x = 0.360187$  m/s (friction effect)
Status: STICKING
Post-impact:  $v_x^+ = 0$  m/s,  $v_y^+ = 1.64792$  m/s
Energy retained: 48.648%
```

```
Bounce #5 at t = 3.28263 s, location (2.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu$  = 0.09
Pre-impact:  $v_x^- = 0$  m/s,  $v_y^- = -1.64792$  m/s
 $\Delta V_x = 0.252131$  m/s (friction effect)
Status: STICKING
Post-impact:  $v_x^+ = 0$  m/s,  $v_y^+ = 1.15354$  m/s
Energy retained: 49.%
```

```
Bounce #6 at t = 3.51781 s, location (2.51032, 0.15) m
```

```
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -1.15354 m/s
ΔVx = 0.176492 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.807478 m/s
Energy retained: 49.%  
  
Bounce #7 at t = 3.68243 s, location (2.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.807478 m/s
ΔVx = 0.123544 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.565235 m/s
Energy retained: 49.%  
  
Bounce #8 at t = 3.79767 s, location (2.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.565235 m/s
ΔVx = 0.0864809 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.395664 m/s
Energy retained: 49.%  
  
Bounce #9 at t = 3.87834 s, location (2.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.395664 m/s
ΔVx = 0.0605367 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.276965 m/s
Energy retained: 49.%  
  
Bounce #10 at t = 3.9348 s, location (2.51032, 0.15) m
```

```

==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu$  = 0.09
Pre-impact:  $v_x^- = 0$  m/s,  $v_y^- = -0.276965$  m/s
 $\Delta V_x = 0.0423757$  m/s (friction effect)
Status: STICKING
Post-impact:  $v_x^+ = 0$  m/s,  $v_y^+ = 0.193876$  m/s
Energy retained: 49.%  
  

Bounce #11 at t = 3.97433 s, location (2.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu$  = 0.09
Pre-impact:  $v_x^- = 0$  m/s,  $v_y^- = -0.193876$  m/s
 $\Delta V_x = 0.029663$  m/s (friction effect)
Status: STICKING
Post-impact:  $v_x^+ = 0$  m/s,  $v_y^+ = 0.135713$  m/s
Energy retained: 49.%  
  

Bounce #12 at t = 4.002 s, location (2.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu$  = 0.09
Pre-impact:  $v_x^- = 0$  m/s,  $v_y^- = -0.135713$  m/s
 $\Delta V_x = 0.0207641$  m/s (friction effect)
Status: STICKING
Post-impact:  $v_x^+ = 0$  m/s,  $v_y^+ = 0.094999$  m/s
Energy retained: 49.%  
  

MASS 1 (Red) stopped bouncing
Total bounces: 12
Final x position: 2.51032 m  
  

==== SIMULATING MASS 2 (Green) ====
Mass: 0.7 kg, Radius: 0.15 m, Friction:  $\mu$  = 0.09
Initial: (-0.5, 2.5) m, velocity (2.5, 1.) m/s  
  

Bounce #1 at t = 0.801575 s, location (1.50394, 0.15) m
==== IMPACT DETECTED ====

```

Mass: 0.7 kg, μ = 0.09
Pre-impact: $v_x^- = 2.5$ m/s, $v_y^- = -6.86345$ m/s
 $\Delta V_x = 1.05011$ m/s (friction effect)
Status: SLIDING
Post-impact: $v_x^+ = 1.44989$ m/s, $v_y^+ = 4.80442$ m/s
Energy retained: 47.2002%

Bounce #2 at $t = 1.78107$ s, location (2.9241, 0.15) m

==== IMPACT DETECTED ===

Mass: 0.7 kg, μ = 0.09
Pre-impact: $v_x^- = 1.44989$ m/s, $v_y^- = -4.80442$ m/s
 $\Delta V_x = 0.735076$ m/s (friction effect)
Status: SLIDING
Post-impact: $v_x^+ = 0.714816$ m/s, $v_y^+ = 3.36309$ m/s
Energy retained: 46.9388%

Bounce #3 at $t = 2.46672$ s, location (3.41421, 0.15) m

==== IMPACT DETECTED ===

Mass: 0.7 kg, μ = 0.09
Pre-impact: $v_x^- = 0.714816$ m/s, $v_y^- = -3.36309$ m/s
 $\Delta V_x = 0.514553$ m/s (friction effect)
Status: SLIDING
Post-impact: $v_x^+ = 0.200262$ m/s, $v_y^+ = 2.35416$ m/s
Energy retained: 47.2213%

Bounce #4 at $t = 2.94667$ s, location (3.51032, 0.15) m

==== IMPACT DETECTED ===

Mass: 0.7 kg, μ = 0.09
Pre-impact: $v_x^- = 0.200262$ m/s, $v_y^- = -2.35416$ m/s
 $\Delta V_x = 0.360187$ m/s (friction effect)
Status: STICKING
Post-impact: $v_x^+ = 0$ m/s, $v_y^+ = 1.64792$ m/s
Energy retained: 48.648%

Bounce #5 at $t = 3.28263$ s, location (3.51032, 0.15) m

==== IMPACT DETECTED ===

```
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -1.64792 m/s
ΔVx = 0.252131 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 1.15354 m/s
Energy retained: 49.%  
  
Bounce #6 at t = 3.51781 s, location (3.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -1.15354 m/s
ΔVx = 0.176492 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.807478 m/s
Energy retained: 49.%  
  
Bounce #7 at t = 3.68243 s, location (3.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.807478 m/s
ΔVx = 0.123544 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.565235 m/s
Energy retained: 49.%  
  
Bounce #8 at t = 3.79767 s, location (3.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.565235 m/s
ΔVx = 0.0864809 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.395664 m/s
Energy retained: 49.%  
  
Bounce #9 at t = 3.87834 s, location (3.51032, 0.15) m
==== IMPACT DETECTED ====
```

```
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.395664 m/s
ΔVx = 0.0605367 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.276965 m/s
Energy retained: 49.%  
  
Bounce #10 at t = 3.9348 s, location (3.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.276965 m/s
ΔVx = 0.0423757 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.193876 m/s
Energy retained: 49.%  
  
Bounce #11 at t = 3.97433 s, location (3.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.193876 m/s
ΔVx = 0.029663 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.135713 m/s
Energy retained: 49.%  
  
Bounce #12 at t = 4.002 s, location (3.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.135713 m/s
ΔVx = 0.0207641 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.094999 m/s
Energy retained: 49.%  
  
MASS 2 (Green) stopped bouncing
Total bounces: 12
```

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Final x position: 3.51032 m

==== SIMULATING MASS 3 (Blue) ====
Mass: 0.7 kg, Radius: 0.15 m, Friction:  $\mu = 0.09$ 
Initial: (0.5, 2.5) m, velocity (2.5, 1.) m/s

Bounce #1 at t = 0.801575 s, location (2.50394, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu = 0.09$ 
Pre-impact: vx- = 2.5 m/s, vy- = -6.86345 m/s
 $\Delta V_x = 1.05011$  m/s (friction effect)
Status: SLIDING
Post-impact: vx+ = 1.44989 m/s, vy+ = 4.80442 m/s
Energy retained: 47.2002%

Bounce #2 at t = 1.78107 s, location (3.9241, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu = 0.09$ 
Pre-impact: vx- = 1.44989 m/s, vy- = -4.80442 m/s
 $\Delta V_x = 0.735076$  m/s (friction effect)
Status: SLIDING
Post-impact: vx+ = 0.714816 m/s, vy+ = 3.36309 m/s
Energy retained: 46.9388%

Bounce #3 at t = 2.46672 s, location (4.41421, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu = 0.09$ 
Pre-impact: vx- = 0.714816 m/s, vy- = -3.36309 m/s
 $\Delta V_x = 0.514553$  m/s (friction effect)
Status: SLIDING
Post-impact: vx+ = 0.200262 m/s, vy+ = 2.35416 m/s
Energy retained: 47.2213%

Bounce #4 at t = 2.94667 s, location (4.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg,  $\mu = 0.09$ 
```

```
Pre-impact: vx- = 0.200262 m/s, vy- = -2.35416 m/s
ΔVx = 0.360187 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 1.64792 m/s
Energy retained: 48.648%
```

```
Bounce #5 at t = 3.28263 s, location (4.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -1.64792 m/s
ΔVx = 0.252131 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 1.15354 m/s
Energy retained: 49.%
```

```
Bounce #6 at t = 3.51781 s, location (4.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -1.15354 m/s
ΔVx = 0.176492 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.807478 m/s
Energy retained: 49.%
```

```
Bounce #7 at t = 3.68243 s, location (4.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
Pre-impact: vx- = 0 m/s, vy- = -0.807478 m/s
ΔVx = 0.123544 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.565235 m/s
Energy retained: 49.%
```

```
Bounce #8 at t = 3.79767 s, location (4.51032, 0.15) m
==== IMPACT DETECTED ====
Mass: 0.7 kg, μ = 0.09
```

Pre-impact: $v_x^- = 0 \text{ m/s}$, $v_y^- = -0.565235 \text{ m/s}$

$\Delta V_x = 0.0864809 \text{ m/s}$ (friction effect)

Status: STICKING

Post-impact: $v_x^+ = 0 \text{ m/s}$, $v_y^+ = 0.395664 \text{ m/s}$

Energy retained: 49.%

Bounce #9 at $t = 3.87834 \text{ s}$, location (4.51032, 0.15) m

==== IMPACT DETECTED ===

Mass: 0.7 kg, $\mu = 0.09$

Pre-impact: $v_x^- = 0 \text{ m/s}$, $v_y^- = -0.395664 \text{ m/s}$

$\Delta V_x = 0.0605367 \text{ m/s}$ (friction effect)

Status: STICKING

Post-impact: $v_x^+ = 0 \text{ m/s}$, $v_y^+ = 0.276965 \text{ m/s}$

Energy retained: 49.%

Bounce #10 at $t = 3.9348 \text{ s}$, location (4.51032, 0.15) m

==== IMPACT DETECTED ===

Mass: 0.7 kg, $\mu = 0.09$

Pre-impact: $v_x^- = 0 \text{ m/s}$, $v_y^- = -0.276965 \text{ m/s}$

$\Delta V_x = 0.0423757 \text{ m/s}$ (friction effect)

Status: STICKING

Post-impact: $v_x^+ = 0 \text{ m/s}$, $v_y^+ = 0.193876 \text{ m/s}$

Energy retained: 49.%

Bounce #11 at $t = 3.97433 \text{ s}$, location (4.51032, 0.15) m

==== IMPACT DETECTED ===

Mass: 0.7 kg, $\mu = 0.09$

Pre-impact: $v_x^- = 0 \text{ m/s}$, $v_y^- = -0.193876 \text{ m/s}$

$\Delta V_x = 0.029663 \text{ m/s}$ (friction effect)

Status: STICKING

Post-impact: $v_x^+ = 0 \text{ m/s}$, $v_y^+ = 0.135713 \text{ m/s}$

Energy retained: 49.%

Bounce #12 at $t = 4.002 \text{ s}$, location (4.51032, 0.15) m

==== IMPACT DETECTED ===

Mass: 0.7 kg, $\mu = 0.09$

Pre-impact: $v_x^- = 0 \text{ m/s}$, $v_y^- = -0.135713 \text{ m/s}$
 $\Delta V_x = 0.0207641 \text{ m/s}$ (friction effect)
Status: STICKING
Post-impact: $v_x^+ = 0 \text{ m/s}$, $v_y^+ = 0.094999 \text{ m/s}$
Energy retained: 49.%

MASS 3 (Blue) stopped bouncing

Total bounces: 12

Final x position: 4.51032 m

==== SIMULATING MASS 4 (Magenta) - HEAVY ===

Mass: 1.5 kg, Radius: 0.2 m, Friction: $\mu = 0.15$

Initial: (1.5, 2.5) m, velocity (2.5, 1.) m/s

Bounce #1 at t = 0.794252 s, location (3.48563, 0.2) m

==== IMPACT DETECTED ===

Mass: 1.5 kg, $\mu = 0.15$

Pre-impact: $v_x^- = 2.5 \text{ m/s}$, $v_y^- = -6.79161 \text{ m/s}$

$\Delta V_x = 1.73186 \text{ m/s}$ (friction effect)

Status: SLIDING

Post-impact: $v_x^+ = 0.768139 \text{ m/s}$, $v_y^+ = 4.75413 \text{ m/s}$

Energy retained: 44.2794%

Bounce #2 at t = 1.76349 s, location (4.23014, 0.2) m

==== IMPACT DETECTED ===

Mass: 1.5 kg, $\mu = 0.15$

Pre-impact: $v_x^- = 0.768139 \text{ m/s}$, $v_y^- = -4.75413 \text{ m/s}$

$\Delta V_x = 1.2123 \text{ m/s}$ (friction effect)

Status: STICKING

Post-impact: $v_x^+ = 0 \text{ m/s}$, $v_y^+ = 3.32789 \text{ m/s}$

Energy retained: 47.7534%

Bounce #3 at t = 2.44196 s, location (4.23014, 0.2) m

==== IMPACT DETECTED ===

Mass: 1.5 kg, $\mu = 0.15$

Pre-impact: $v_x^- = 0 \text{ m/s}$, $v_y^- = -3.32789 \text{ m/s}$

```
ΔVx = 0.848612 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 2.32952 m/s
Energy retained: 49.%  
  
Bounce #4 at t = 2.91689 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -2.32952 m/s
ΔVx = 0.594028 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 1.63067 m/s
Energy retained: 49.%  
  
Bounce #5 at t = 3.24934 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -1.63067 m/s
ΔVx = 0.41582 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 1.14147 m/s
Energy retained: 49.%  
  
Bounce #6 at t = 3.48206 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -1.14147 m/s
ΔVx = 0.291074 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.799026 m/s
Energy retained: 49.%  
  
Bounce #7 at t = 3.64496 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -0.799026 m/s
```

```
ΔVx = 0.203752 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.559318 m/s
Energy retained: 49.%  
  
Bounce #8 at t = 3.75899 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -0.559318 m/s
ΔVx = 0.142626 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.391523 m/s
Energy retained: 49.%  
  
Bounce #9 at t = 3.83881 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -0.391523 m/s
ΔVx = 0.0998384 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.274066 m/s
Energy retained: 49.%  
  
Bounce #10 at t = 3.89468 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -0.274066 m/s
ΔVx = 0.0698868 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.191846 m/s
Energy retained: 49.%  
  
Bounce #11 at t = 3.93379 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -0.191846 m/s
```

```
ΔVx = 0.0489208 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.134292 m/s
Energy retained: 49.%  
  
Bounce #12 at t = 3.96117 s, location (4.23014, 0.2) m
==== IMPACT DETECTED ====
Mass: 1.5 kg, μ = 0.15
Pre-impact: vx- = 0 m/s, vy- = -0.134292 m/s
ΔVx = 0.0342446 m/s (friction effect)
Status: STICKING
Post-impact: vx+ = 0 m/s, vy+ = 0.0940047 m/s
Energy retained: 49.%  
  
MASS 4 (Magenta) - HEAVY stopped bouncing
Total bounces: 12
Final x position: 4.23014 m
```

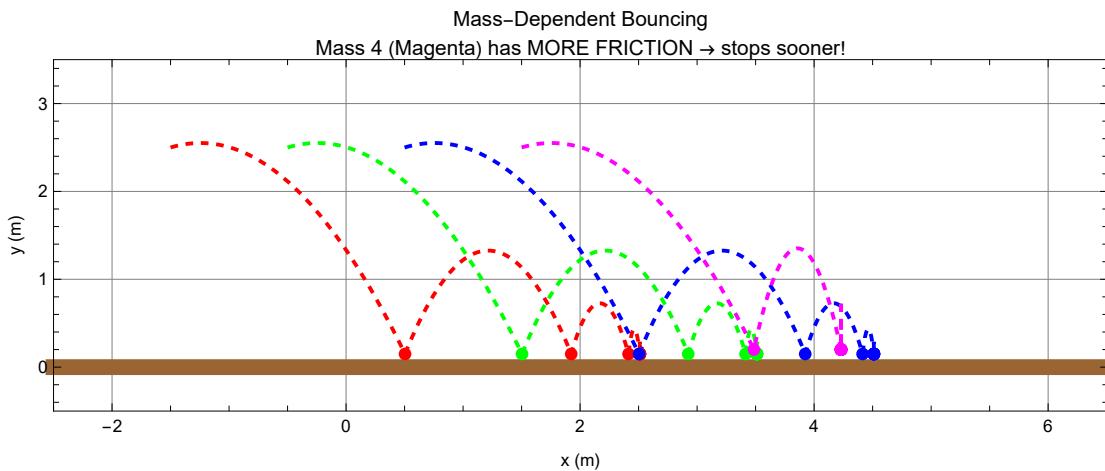
```
=====
ALL BOUNCING SIMULATIONS COMPLETE
=====
```

COMPARISON OF FINAL POSITIONS:

```
Mass 1 final x: 2.51032 m
Mass 2 final x: 3.51032 m
Mass 3 final x: 4.51032 m
Mass 4 final x: 4.23014 m
```

```
Mass 4 should travel LESS distance horizontally
because it has MORE FRICTION (loses vx faster)!  
  
=====
CREATING PLOTS AND ANIMATION
=====
```

Out[65]=



```
=====
MASS-DEPENDENT PHYSICS COMPLETE
=====
```

KEY OBSERVATION:

Watch Mass 4 (Magenta) – it should:

1. Lose horizontal velocity FASTER
2. Travel LESS DISTANCE overall

This is because heavier objects have
MORE friction force at impact!

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Out[77]=

