Adaptive Sliding Mode Control for a 4-DOF Serial Manipulator with Impact Dynamics

ME 7393: Robotics - Final Project Report

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1. Problem Definition

1.1 System Description

This project addresses the control of a 4-degree-of-freedom (4-DOF) planar serial manipulator tasked with performing sequential pick-and-place operations within a constrained diamond-shaped workspace. The manipulator consists of four rigid links, each with length $L = 1.10$ m, connected by revolute joints. The robot operates from a fixed base at the origin and must navigate through a narrow entry gap (width = 1.5 m) to reach target locations within the workspace.

1.2 Technical Challenges

The control problem presents several significant challenges:

Parametric Uncertainty: The true system parameters are unknown to the controller. Specifically:

- True link mass: $m_{true} = 1.0$ kg per link
- Controller estimate: $\hat{m} = 0.8$ kg (20% underestimate)
- True payload mass: $m_{payload} = 0.7$ kg
- Controller estimate: $\hat{m}_{payload} = 0.3$ kg (57% underestimate)

This parametric mismatch represents a realistic scenario where exact system identification is impractical or where payloads of varying mass must be manipulated.

Time-Varying Dynamics: The effective mass of the end-effector changes discontinuously when the manipulator grasps a payload. The controller must maintain trajectory tracking performance despite this sudden change in system dynamics.

Workspace Constraints: The diamond workspace (width = 5.0 m, height = 5.0 m, offset = 1.25 m) with a narrow entry gap requires precise trajectory planning to avoid collisions while reaching targets at heights up to 4.25 m.

Multiple Sequential Tasks: The system must complete four sequential pick-and-place operations, returning to a home configuration between each cycle, requiring robust performance over extended operation times (44 seconds total).

1.3 Impact Dynamics Consideration

An additional component of this project investigates the physics of impact when manipulated objects are released. Using Newton's coefficient of restitution ($e = 0.7$) and Coulomb friction ($\mu = 0.07$), the bouncing behavior of dropped masses is simulated to demonstrate:

- Energy dissipation through successive impacts
- Transition between sliding and sticking contact modes
- Trajectory prediction for dropped objects

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2. Objective

The primary objectives of this project are:

2.1 Control Objectives

1. **Trajectory Tracking:** Achieve accurate tracking of smooth reference trajectories connecting home configuration to pickup locations and back, with tracking errors remaining bounded despite parametric uncertainty.
2. **Robustness to Parameter Variation:** Maintain stability and acceptable performance when:
   - Link masses are incorrectly estimated by 20%
   - Payload mass is underestimated by 57%
   - System mass changes discontinuously at pickup
3. **Multi-Target Operation:** Successfully complete sequential pickup of four masses located at:
   - Mass 1: (-0.10, 4.10) m
   - Mass 2: (-0.10, 4.25) m
   - Mass 3: (-0.12, 4.22) m
   - Mass 4: (0.025, 4.00) m

2.2 Analysis Objectives

1. Demonstrate sliding mode controller convergence through sliding surface analysis
2. Quantify tracking error response to payload pickup disturbance

3. Analyze control torque requirements throughout operation
4. Visualize parameter estimation error and its effect on control

2.3 Impact Physics Objectives

1. Simulate realistic bouncing behavior using Newton-Euler impact mechanics
2. Demonstrate energy dissipation through coefficient of restitution
3. Model friction effects during impact (sliding vs. sticking)

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3. Solution

3.1 Dynamic Modeling

The system dynamics are derived using the Euler-Lagrange formulation. For the n-link planar manipulator:

Kinetic Energy:

$$T = \sum_{i=1}^{n} \left[\frac{1}{2} m_i (\dot{x}_{b_i}^2 + \dot{y}_{b_i}^2) + \frac{1}{2} J_i \dot{\theta}_i^2 \right]$$

where $J_i = \frac{1}{12} m_i L_i^2$ is the moment of inertia of link i about its center of mass.

Potential Energy:

$$V = \sum_{i=1}^{n} m_i g y_{b_i}$$

Equations of Motion:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$$

where:

- $M(q) \in \mathbb{R}^{4\times4}$ is the symmetric positive-definite mass matrix
- $C(q, \dot{q}) \in \mathbb{R}^{4\times4}$ is the Coriolis/centrifugal matrix (computed using Christoffel symbols)

- $G(q) \in \mathbb{R}^4$ is the gravity vector
- $\tau \in \mathbb{R}^4$ is the joint torque vector

3.2 Trajectory Generation

Smooth trajectories are generated using cosine interpolation to ensure continuous velocity profiles:

$$\sigma(t, t_0, t_1) = \frac{1 - \cos\left(\pi \frac{t - t_0}{t_1 - t_0}\right)}{2}$$

Each pick-and-place cycle consists of three phases:

- **Ingress (5.0 s):** Smooth motion from home to pickup configuration
- **Hold (1.0 s):** Stationary at pickup location (grasp payload)
- **Egress (5.0 s):** Return to home configuration while carrying payload

The desired trajectory for joint i is:

$$q_{d,i}(t) = q_{home,i} + (q_{pickup,i} - q_{home,i}) \cdot \sigma(t)$$

3.3 Inverse Kinematics

Target joint configurations are computed by solving the nonlinear optimization:

$$\min_{\theta_1, \theta_2, \theta_3, \theta_4} \left[(x_{tip} - x_{target})^2 + (y_{tip} - y_{target})^2 \right]$$

subject to: $-\frac{\pi}{2} < \theta_i < \frac{\pi}{2}$ for $i = 1, 2, 3, 4$

where the forward kinematics yield:

$$x_{tip} = -L \sum_{i=1}^{4} \sin(\theta_i), \quad y_{tip} = L \sum_{i=1}^{4} \cos(\theta_i)$$

3.4 Adaptive Sliding Mode Controller

The controller employs a regressor-based adaptive sliding mode approach that provides robustness to parametric uncertainty.

Tracking Error:

$$\tilde{q} = q - q_d$$

Sliding Surface:

$$s = \dot{\tilde{q}} + \Lambda\tilde{q}$$

where $\Lambda = \mathrm{diag}(4, 4, 4, 4)$ defines the convergence rate on the sliding surface.

Reference Trajectory:

$$\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}$$

Regressor Formulation: The dynamics can be written in regressor form:

$$M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \cdot a$$

where $Y \in \mathbb{R}^{4\times4}$ is the regressor matrix and $a \in \mathbb{R}^4$ contains the mass parameters.

Control Law:

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \cdot \hat{a} - K \cdot \mathrm{sat}(s)$$

where:

- $\hat{a} = [0.8, 0.8, 0.8, 0.8 + 0.3]^T$ kg are the estimated mass parameters
- $K = \mathrm{diag}(10, 10, 10, 10)$ is the switching gain matrix
- $\mathrm{sat}(s_i) = \begin{cases} s_i/\epsilon & |s_i| < \epsilon \\ \mathrm{sign}(s_i) & |s_i| \geq \epsilon \end{cases}$ with $\epsilon = 0.25$

The saturation function replaces the discontinuous sign function to reduce chattering while maintaining robustness.

3.5 Time-Varying Mass Handling

The true system mass changes when payload is grasped:

$$m_4(t) = \begin{cases} m_{link} = 1.0 \text{ kg} & t \leq t_{ingress} \\ m_{link} + m_{payload} = 1.7 \text{ kg} & t_{ingress} < t \leq t_{cycle} \end{cases}$$

The controller estimates this as:

$$\hat{m}_4(t) = \begin{cases} 0.8 \text{ kg} & t \leq t_{ingress} \\ 1.1 \text{ kg} & t_{ingress} < t \leq t_{cycle} \end{cases}$$

This creates a parameter error of 0.6 kg (35% of true mass) during the egress phase.

3.6 Impact Physics Model

For the bouncing mass simulation, Newton's impact law with Coulomb friction is applied:

Normal Velocity (Restitution):

$$v_n^+ = -e \cdot v_n^-$$

where $e = 0.7$ is the coefficient of restitution.

Tangential Velocity (Friction):

$$J_n = m|v_n^-|(1+e)$$

$$J_t = \mu \cdot J_n$$

$$v_t^+ = \begin{cases} v_t^- - \text{sign}(v_t^-) \cdot J_t/m & |v_t^-| > J_t/m \text{ (sliding)} \\ 0 & |v_t^-| \leq J_t/m \text{ (sticking)} \end{cases}$$

Energy Retained:

$$\eta = \frac{(v_x^+)^2 + (v_y^+)^2}{(v_x^-)^2 + (v_y^-)^2} \approx 49\%$$

3.7 Numerical Implementation

The equations of motion are solved using Mathematica's NDSolve with the following configuration:

- Method: Residual-based equation simplification for DAE handling
- Maximum steps: 10,000
- Automatic stiffness detection and step size control

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4. Results and Analysis

4.1 Simulation Results

All four pick-and-place operations completed successfully:

Mass	Target Location	IK Solution (degrees)	Status
1	(-0.10, 4.10)	[-26.7, -9.0, 11.3, 30.0]	✓ Success
2	(-0.10, 4.25)	[-15.6, -6.6, 3.4, 24.6]	✓ Success
3	(-0.12, 4.22)	[-20.0, -6.1, 9.2, 23.5]	✓ Success
4	(0.025, 4.00)	[-32.2, -12.7, 10.0, 33.9]	✓ Success

4.2 Controller Performance

Tracking Error: The tracking error exhibits a transient spike at $t = 5.0$ s when the payload is grasped, corresponding to the sudden change in system dynamics. Despite 35% parameter error, the sliding mode controller drives the error back toward zero within approximately 1-2 seconds.

Sliding Surface: The sliding surface $s(t)$ converges toward zero throughout operation, confirming that the system reaches and remains on the sliding manifold. Perturbations at payload pickup cause temporary deviation but rapid recovery.

Control Torques: Joint torques show a discontinuous jump at $t = 5.0$ s as the controller compensates for the increased load. The torque profiles remain bounded throughout operation, with peak values occurring during the initial acceleration phase.

4.3 Impact Simulation Results

The bouncing mass simulation demonstrated:

- 13 bounces before coming to rest
- Consistent ~49% energy retention per bounce
- Transition from sliding to near-sticking as velocity decreased
- Final rest position accurately predicted

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5. Conclusion

5.1 Summary of Achievements

This project successfully demonstrated:

1. **Robust Trajectory Tracking:** The adaptive sliding mode controller maintained trajectory tracking despite 20-57% parametric uncertainty in mass estimates.
2. **Handling of Time-Varying Dynamics:** The controller effectively compensated for sudden mass changes during payload grasping, with rapid recovery of tracking performance.
3. **Sequential Multi-Target Operation:** Four complete pick-and-place cycles were executed successfully over 44 seconds of operation.
4. **Physical Realism:** Impact dynamics simulation using Newton's restitution law and Coulomb friction accurately captured bouncing behavior with energy dissipation.

5.2 Key Findings

- The regressor-based adaptive control structure provides inherent robustness to parametric uncertainty without requiring explicit parameter adaptation laws.
- The saturation function boundary layer ($\epsilon = 0.25$) effectively balances chatter suppression against tracking accuracy.
- The sliding surface gain $\Lambda = 4$ provides sufficiently fast convergence without inducing excessive control effort.

- The switching gain $K = 10$ provides adequate robustness margin for the encountered parametric uncertainty.

5.3 Limitations and Future Work

Current Limitations:

- The controller does not include explicit parameter adaptation (estimates remain fixed)
- Workspace collision avoidance is handled through trajectory planning rather than real-time obstacle avoidance
- Joint limits are enforced through IK constraints rather than controller saturation

Future Extensions:

- Implement online parameter estimation using gradient-based or least-squares adaptation
- Add constraint-handling for joint limits and workspace boundaries within the controller
- Extend to 3D manipulation with 6-DOF kinematics
- Incorporate force/torque sensing for compliant manipulation
- Integrate impact dynamics for manipulation tasks involving contact

5.4 Broader Implications

This work demonstrates the effectiveness of sliding mode control for robotic manipulation under realistic conditions of parametric uncertainty. The approach is particularly relevant for:

- Surgical robotics where payload (tissue/instrument) mass varies
- Industrial pick-and-place with varying product weights
- Space robotics where gravity compensation must adapt to different environments
- Any application where accurate system identification is impractical

The combination of Lagrangian dynamics, adaptive sliding mode control, and impact mechanics provides a comprehensive framework for analyzing and controlling robotic manipulators in complex task scenarios.

References

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4. Craig, J.J. (2005). *Introduction to Robotics: Mechanics and Control*. Pearson.

Appendix: System Parameters

Parameter	**Symbol**	**Value**	**Units**
Number of links	n	4	-
Link length	L	1.10	m
True link mass	m_{link}	1.0	kg
Estimated link mass	$\hat{m}_{link}$	0.8	kg
True payload mass	$m_{payload}$	0.7	kg
Estimated payload mass	$\hat{m}_{payload}$	0.3	kg
Gravitational acceleration	g	9.81	m/s²
Sliding surface gain	Λ	4.0	1/s
Switching gain	K	10.0	N·m
Saturation boundary	ϵ	0.25	rad/s
Coefficient of restitution	e	0.7	-
Coefficient of friction	μ	0.07	-
Ingress time	$t_{ingress}$	5.0	s
Hold time	t_{hold}	1.0	s
Egress time	t_{egress}	5.0	s

Parameter	**Symbol**	**Value**	**Units**
Cycle time	t_{cycle}	11.0	s
Total simulation time	t_{total}	44.0	s