In[723]:=

```
ClearAll["Global`*"];

(* ===========================================
 SHARED CONSTANTS===========================================*)
limm = 0.25;
Sat[x_] := Piecewise[{{x / limm, Abs[x] < limm}, {-1, x ≤ -limm}, {1, x ≥ limm}}];
SmoothStep[t_, t0_, t1_] :=
  Piecewise[{{0, t < t0}, {1, t > t1}, {(1 - Cos[Pi * (t - t0) / (t1 - t0)]) / 2, t0 ≤ t ≤ t1}}];

n = 4;
linkLength = 1.10;
diamondWidth = 2.5;
diamondHeight = 5.0;
diamondYOffset = 1.25;
diamondCenterY = diamondHeight / 2;
entryGap = 1.5; (*Width of the opening at the bottom*)

(*Define vertices as an OPEN chain:Start Left of gap→Top→End Right of gap*)
diamondVertices = {{-entryGap / 2, diamondYOffset},
   (*Left Gate Post*){-diamondWidth, diamondCenterY + diamondYOffset},
   (*Left Tip*){0, diamondHeight + diamondYOffset}, (*Top Tip*)
   {diamondWidth, diamondCenterY + diamondYOffset}, (*Right Tip*)
   {entryGap / 2, diamondYOffset}            (*Right Gate Post*)};

baseX = 0;
baseY = 0;
massPerLink = 1.0;
linkLengthNum = 1.0;
gNum = 9.81;
payloadMass = .7;
massEstimates = Table[0.8, {i, n}];
payloadEstimate = 0.3;

tIngress = 5.0;
tHold = 1.0;
tEgress = 5.0;
tCycle = tIngress + tHold + tEgress;

homeAngles = {75, -110, 0, 200} * Degree;

(* ======================================================================*)
(*SIMULATION 1-MASS 1 AT (-0.10,4.0)-THE WORKING TARGET*)
(* ======================================================================*)
Print["============================================"];
Print["SIMULATION 1: MASS 1"];
Print["============================================\n"];
```

```
targetX1 = -0.10;
targetY1 = 4.1;

Print["Target: (", targetX1, ", ", targetY1, ")"];

xTip1[th1_, th2_, th3_, th4_] :=
  baseX - linkLength * (Sin[th1] + Sin[th2] + Sin[th3] + Sin[th4]);
yTip1[th1_, th2_, th3_, th4_] :=
  baseY + linkLength * (Cos[th1] + Cos[th2] + Cos[th3] + Cos[th4]);

solution1 =
  NMinimize[{(xTip1[t1, t2, t3, t4] - targetX1)^2 + (yTip1[t1, t2, t3, t4] - targetY1)^2,
    -Pi / 2 < t1 < Pi / 2, -Pi / 2 < t2 < Pi / 2, -Pi / 2 < t3 < Pi / 2,
    -Pi / 2 < t4 < Pi / 2}, {t1, t2, t3, t4}];

pickupAngles1 = {t1, t2, t3, t4} /. solution1[[2]];
Print["Pickup angles: ", Round[pickupAngles1 * 180 / Pi, 0.1], "°\n"];

qdes1 = Table[Piecewise[
    {{homeAngles[[i]] + (pickupAngles1[[i]] - homeAngles[[i]]) * SmoothStep[t, 0, tIngress],
      t ≤ tIngress}, {pickupAngles1[[i]], tIngress < t ≤ tIngress + tHold},
     {pickupAngles1[[i]] + (homeAngles[[i]] - pickupAngles1[[i]]) * SmoothStep[t,
        tIngress + tHold, tIngress + tHold + tEgress], t > tIngress + tHold}}], {i, n}];

xj1[0] = baseX;
yj1[0] = baseY;
Do[xj1[i] = baseX + Sum[-linkLength * Sin[θ1[j][t]], {j, 1, i}], {i, 1, n}];
Do[yj1[i] = baseY + Sum[linkLength * Cos[θ1[j][t]], {j, 1, i}], {i, 1, n}];
Do[xb1[i] = baseX + xj1[i - 1] - baseX - linkLength / 2 * Sin[θ1[i][t]], {i, 1, n}];
Do[yb1[i] = baseY + yj1[i - 1] - baseY + linkLength / 2 * Cos[θ1[i][t]], {i, 1, n}];

Do[Mj1[i] = 1 / 12 * m1[i] * l1[i]^2, {i, n}];

T1 = Sum[1 / 2 * m1[i] * (D[xb1[i], t]^2 + D[yb1[i], t]^2) +
    1 / 2 * Mj1[i] * (D[θ1[i][t], t])^2, {i, n}];
V1 = Sum[m1[i] * g1 * yb1[i], {i, n}];
Lag1 = T1 - V1;
Eqs1 = Table[D[D[Lag1, D[θ1[i][t], t]], t] - D[Lag1, θ1[i][t]], {i, n}];

MassM1 = Table[Coefficient[Eqs1[[i]], D[θ1[j][t], {t, 2}]], {i, n}, {j, n}];
Cc1 = Table[Sum[0.5 * (D[MassM1[[i, j]], θ1[k][t]] + D[MassM1[[i, k]], θ1[j][t]] -
       D[MassM1[[j, k]], θ1[i][t]]) * D[θ1[k][t], t], {k, n}], {i, n}, {j, n}];
Gg1 = Table[D[Eqs1[[i]], g1], {i, n}] * g1;

q1 = Table[θ1[i][t], {i, n}];
qtilde1 = q1 - qdes1;
```

```
Lambda1 = DiagonalMatrix[Table[4, {i, n}]];
dqr1 = D[qdes1, t] - Lambda1.qtilde1;
s1 = D[qtilde1, t] + Lambda1.qtilde1;

DynamicsLHS1 = MassM1.D[dqr1, t] + Cc1.dqr1 + Gg1;
Yy1 = Table[D[DynamicsLHS1[[i]], m1[j]], {i, n}, {j, n}];

hataa1 = massEstimates;
kGain1 = 10.0;
controlGains1 = DiagonalMatrix[Table[kGain1, {i, n}]];
SatVec1 = Table[Sat[s1[[i]]], {i, n}];
ControlInput1 = Yy1.hataa1 - controlGains1.SatVec1;

numRules1[time_] := Join[Table[m1[i] → massPerLink, {i, n - 1}],
   {m1[n] → If[time > tIngress && time ≤ (tIngress + tHold + tEgress),
      massPerLink + payloadMass, massPerLink]},
   Table[l1[i] → linkLengthNum, {i, n}], {g1 → gNum}];

hataaDynamic1[time_] := Join[Table[massEstimates[[i]], {i, n - 1}],
   {If[time > tIngress && time ≤ (tIngress + tHold + tEgress),
     massEstimates[[n]] + payloadEstimate, massEstimates[[n]]]}];

ics1 = Join[Table[θ1[i][0] == homeAngles[[i]], {i, n}],
   Table[Derivative[1][θ1[i]][0] == 0, {i, n}]];

ControlInputDynamic1 = Yy1.hataaDynamic1[t] - controlGains1.SatVec1;
EqsN1 = Thread[Eqs1 == ControlInputDynamic1] /. numRules1[t];

Print["Simulating..."];
solDE1 = NDSolve[Join[EqsN1, ics1], Table[θ1[i][t], {i, n}], {t, 0, tCycle},
   Method → {"EquationSimplification" → "Residual"}, MaxSteps → 10000];

If[Length[solDE1] > 0,
  Print["✓ SIMULATION 1 SUCCESS!\n"], Print["✗ SIMULATION 1 FAILED!\n"];
  Abort[]];


diamondBoundary1 = Graphics[{Thickness[0.005], Red, Dashed, Line[diamondVertices]}];

robotGraphics1[time_] := Module[{links, joints, endPoint, massPosition, massPoint},
   links = Table[{Thickness[0.01], Black,
      Line[{{xj1[k - 1], yj1[k - 1]}, {xj1[k], yj1[k]}} /. solDE1 /. t → time]}, {k, 1, n}];
   joints = Table[{Blue, Disk[{xj1[k], yj1[k]} /. solDE1 /. t → time, 0.08]}, {k, 0, n}];
   endPoint = {Orange, PointSize[0.025], Point[{xj1[n], yj1[n]} /. solDE1 /. t → time]};
   massPosition = If[time > (tIngress + tHold),
     {xj1[n], yj1[n]} /. solDE1 /. t → time, {targetX1, targetY1}];
   massPoint = {Red, PointSize[0.03], Point[massPosition]};
```

```
    Graphics[{links, joints, endPoint, massPoint}]];

baseMarker1 = Graphics[{Gray, EdgeForm[Black], Thickness[0.015],
    Rectangle[{baseX - 0.2, baseY - 0.15}, {baseX + 0.2, baseY}], Black,
    Text["BASE", {baseX, baseY - 0.25}, BaseStyle → {FontSize → 12, Bold}]}];

animation1 = Animate[Show[diamondBoundary1, baseMarker1,
    robotGraphics1[time], PlotRange → {{-diamondWidth - 0.5, diamondWidth + 0.5},
      {-0.5, diamondHeight + diamondYOffset + 0.5}}, AspectRatio → Automatic, Frame → True,
    FrameLabel → {"X (m)", "Y (m)"}, PlotLabel → "Mass 1 at (" <> ToString[targetX1] <>
      ", " <> ToString[targetY1] <> ")\nt=" <> ToString[NumberForm[time, {3, 1}]] <> "s",
    ImageSize → Large], {time, 0, tCycle, 0.05}, AnimationRate → 1];


(* ================================================================*)
(*SIMULATION 2-MASS 2 AT (-0.12,3.95)-VERY CLOSE TO WORKING*)
(* ================================================================*)
Print["========================================"];
Print["SIMULATION 2: MASS 2"];
Print["========================================\n"];

targetX2 = -0.1;
targetY2 = 4.255;

Print["Target: (", targetX2, ", ", targetY2, ")"];

xTip2[th1_, th2_, th3_, th4_] :=
  baseX - linkLength * (Sin[th1] + Sin[th2] + Sin[th3] + Sin[th4]);
yTip2[th1_, th2_, th3_, th4_] :=
  baseY + linkLength * (Cos[th1] + Cos[th2] + Cos[th3] + Cos[th4]);

solution2 =
  NMinimize[{(xTip2[t1, t2, t3, t4] - targetX2) ^2 + (yTip2[t1, t2, t3, t4] - targetY2) ^2,
    -Pi / 2 < t1 < Pi / 2, -Pi / 2 < t2 < Pi / 2, -Pi / 2 < t3 < Pi / 2,
    -Pi / 2 < t4 < Pi / 3}, {t1, t2, t3, t4}];

pickupAngles2 = {t1, t2, t3, t4} /. solution2[[2]];
Print["Pickup angles: ", Round[pickupAngles2 * 180 / Pi, 0.1], "°\n"];

qdes2 = Table[Piecewise[
    {{homeAngles[[i]] + (pickupAngles2[[i]] - homeAngles[[i]]) * SmoothStep[t, 0, tIngress],
      t ≤ tIngress}, {pickupAngles2[[i]], tIngress < t ≤ tIngress + tHold},
     {pickupAngles2[[i]] + (homeAngles[[i]] - pickupAngles2[[i]]) * SmoothStep[t,
         tIngress + tHold, tIngress + tHold + tEgress], t > tIngress + tHold}}], {i, n}];

xj2[0] = baseX;
yj2[0] = baseY;
```

```
Do[xj2[i] = baseX + Sum[-linkLength * Sin[θ2[j][t]], {j, 1, i}], {i, 1, n}];
Do[yj2[i] = baseY + Sum[linkLength * Cos[θ2[j][t]], {j, 1, i}], {i, 1, n}];
Do[xb2[i] = baseX + xj2[i - 1] - baseX - linkLength / 2 * Sin[θ2[i][t]], {i, 1, n}];
Do[yb2[i] = baseY + yj2[i - 1] - baseY + linkLength / 2 * Cos[θ2[i][t]], {i, 1, n}];

Do[Mj2[i] = 1 / 12 * m2[i] * l2[i] ^ 2, {i, n}];

T2 = Sum[1 / 2 * m2[i] * (D[xb2[i], t] ^ 2 + D[yb2[i], t] ^ 2) +
     1 / 2 * Mj2[i] * (D[θ2[i][t], t]) ^ 2, {i, n}];
V2 = Sum[m2[i] * g2 * yb2[i], {i, n}];
Lag2 = T2 - V2;
Eqs2 = Table[D[D[Lag2, D[θ2[i][t], t]], t] - D[Lag2, θ2[i][t]], {i, n}];

MassM2 = Table[Coefficient[Eqs2[[i]], D[θ2[j][t], {t, 2}]], {i, n}, {j, n}];
Cc2 = Table[Sum[0.5 * (D[MassM2[[i, j]], θ2[k][t]] + D[MassM2[[i, k]], θ2[j][t]] -
        D[MassM2[[j, k]], θ2[i][t]]) * D[θ2[k][t], t], {k, n}], {i, n}, {j, n}];
Gg2 = Table[D[Eqs2[[i]], g2], {i, n}] * g2;

q2 = Table[θ2[i][t], {i, n}];
qtilde2 = q2 - qdes2;
Lambda2 = DiagonalMatrix[Table[4, {i, n}]];
dqr2 = D[qdes2, t] - Lambda2.qtilde2;
s2 = D[qtilde2, t] + Lambda2.qtilde2;

DynamicsLHS2 = MassM2.D[dqr2, t] + Cc2.dqr2 + Gg2;
Yy2 = Table[D[DynamicsLHS2[[i]], m2[j]], {i, n}, {j, n}];

hataa2 = massEstimates;
kGain2 = 10.0;
controlGains2 = DiagonalMatrix[Table[kGain2, {i, n}]];
SatVec2 = Table[Sat[s2[[i]]], {i, n}];
ControlInput2 = Yy2.hataa2 - controlGains2.SatVec2;

numRules2[time_] := Join[Table[m2[i] → massPerLink, {i, n - 1}],
    {m2[n] → If[time > tIngress && time ≤ (tIngress + tHold + tEgress),
       massPerLink + payloadMass, massPerLink]},
    Table[l2[i] → linkLengthNum, {i, n}], {g2 → gNum}];

hataaDynamic2[time_] := Join[Table[massEstimates[[i]], {i, n - 1}],
    {If[time > tIngress && time ≤ (tIngress + tHold + tEgress),
      massEstimates[[n]] + payloadEstimate, massEstimates[[n]]]}];

ics2 = Join[Table[θ2[i][0] == homeAngles[[i]], {i, n}],
    Table[Derivative[1][θ2[i]][0] == 0, {i, n}]];

ControlInputDynamic2 = Yy2.hataaDynamic2[t] - controlGains2.SatVec2;
EqsN2 = Thread[Eqs2 == ControlInputDynamic2] /. numRules2[t];
```

```
Print["Simulating..."];
solDE2 = NDSolve[Join[EqsN2, ics2], Table[θ2[i][t], {i, n}], {t, 0, tCycle},
   Method → {"EquationSimplification" → "Residual"}, MaxSteps → 10000];

If[Length[solDE2] > 0,
  Print["✓ SIMULATION 2 SUCCESS!\n"], Print["✗ SIMULATION 2 FAILED!\n"];
  Abort[]];


diamondBoundary2 = Graphics[{Thickness[0.005], Red, Dashed, Line[diamondVertices]}];

robotGraphics2[time_] := Module[{links, joints, endPoint, massPosition, massPoint},
   links = Table[{Thickness[0.01], Black,
      Line[{{xj2[k - 1], yj2[k - 1]}, {xj2[k], yj2[k]}} /. solDE2 /. t → time]}, {k, 1, n}];
   joints = Table[{Blue, Disk[{xj2[k], yj2[k]} /. solDE2 /. t → time, 0.08]}, {k, 0, n}];
   endPoint = {Orange, PointSize[0.025], Point[{xj2[n], yj2[n]} /. solDE2 /. t → time]};
   massPosition = If[time > (tIngress + tHold),
     {xj2[n], yj2[n]} /. solDE2 /. t → time, {targetX2, targetY2}];
   massPoint = {Green, PointSize[0.03], Point[massPosition]};
   Graphics[{links, joints, endPoint, massPoint}]];

baseMarker2 = Graphics[{Gray, EdgeForm[Black], Thickness[0.015],
    Rectangle[{baseX - 0.2, baseY - 0.15}, {baseX + 0.2, baseY}], Black,
    Text["BASE", {baseX, baseY - 0.25}, BaseStyle → {FontSize → 12, Bold}]}];

animation2 = Animate[Show[diamondBoundary2, baseMarker2,
    robotGraphics2[time], PlotRange → {{-diamondWidth - 0.5, diamondWidth + 0.5},
      {-0.5, diamondHeight + diamondYOffset + 0.5}}, AspectRatio → Automatic, Frame → True,
    FrameLabel → {"X (m)", "Y (m)"}, PlotLabel → "Mass 2 at (" <> ToString[targetX2] <>
      ", " <> ToString[targetY2] <> ")\nt=" <> ToString[NumberForm[time, {3, 1}]] <> "s",
    ImageSize → Large], {time, 0, tCycle, 0.05}, AnimationRate → 1];


(* ================================================================*)
(*SIMULATION 3-MASS 3 AT (-0.08,4.05)-VERY CLOSE TO WORKING*)
(* ================================================================*)
Print["========================================"];
Print["SIMULATION 3: MASS 3"];
Print["========================================\n"];

targetX3 = -.12;
targetY3 = 4.222;

Print["Target: (", targetX3, ", ", targetY3, ")"];

xTip3[th1_, th2_, th3_, th4_] :=
```

```
  baseX - linkLength * (Sin[th1] + Sin[th2] + Sin[th3] + Sin[th4]);
yTip3[th1_, th2_, th3_, th4_] :=
  baseY + linkLength * (Cos[th1] + Cos[th2] + Cos[th3] + Cos[th4]);

solution3 =
  NMinimize[{(xTip3[t1, t2, t3, t4] - targetX3)^2 + (yTip3[t1, t2, t3, t4] - targetY3)^2,
    -Pi/2 < t1 < Pi/2, -Pi/2 < t2 < Pi/2, -Pi/2 < t3 < Pi/2,
    -Pi/2 < t4 < Pi/2}, {t1, t2, t3, t4}];

pickupAngles3 = {t1, t2, t3, t4} /. solution3[[2]];
Print["Pickup angles: ", Round[pickupAngles3 * 180 / Pi, 0.1], "°\n"];

qdes3 = Table[Piecewise[
    {{homeAngles[[i]] + (pickupAngles3[[i]] - homeAngles[[i]]) * SmoothStep[t, 0, tIngress],
      t ≤ tIngress}, {pickupAngles3[[i]], tIngress < t ≤ tIngress + tHold},
     {pickupAngles3[[i]] + (homeAngles[[i]] - pickupAngles3[[i]]) * SmoothStep[t,
        tIngress + tHold, tIngress + tHold + tEgress], t > tIngress + tHold}}], {i, n}];

xj3[0] = baseX;
yj3[0] = baseY;
Do[xj3[i] = baseX + Sum[-linkLength * Sin[θ3[j][t]], {j, 1, i}], {i, 1, n}];
Do[yj3[i] = baseY + Sum[linkLength * Cos[θ3[j][t]], {j, 1, i}], {i, 1, n}];
Do[xb3[i] = baseX + xj3[i - 1] - baseX - linkLength / 2 * Sin[θ3[i][t]], {i, 1, n}];
Do[yb3[i] = baseY + yj3[i - 1] - baseY + linkLength / 2 * Cos[θ3[i][t]], {i, 1, n}];

Do[Mj3[i] = 1 / 12 * m3[i] * l3[i]^2, {i, n}];

T3 = Sum[1 / 2 * m3[i] * (D[xb3[i], t]^2 + D[yb3[i], t]^2) +
    1 / 2 * Mj3[i] * (D[θ3[i][t], t])^2, {i, n}];
V3 = Sum[m3[i] * g3 * yb3[i], {i, n}];
Lag3 = T3 - V3;
Eqs3 = Table[D[D[Lag3, D[θ3[i][t], t]], t] - D[Lag3, θ3[i][t]], {i, n}];

MassM3 = Table[Coefficient[Eqs3[[i]], D[θ3[j][t], {t, 2}]], {i, n}, {j, n}];
Cc3 = Table[Sum[0.5 * (D[MassM3[[i, j]], θ3[k][t]] + D[MassM3[[i, k]], θ3[j][t]] -
        D[MassM3[[j, k]], θ3[i][t]]) * D[θ3[k][t], t], {k, n}], {i, n}, {j, n}];
Gg3 = Table[D[Eqs3[[i]], g3], {i, n}] * g3;

q3 = Table[θ3[i][t], {i, n}];
qtilde3 = q3 - qdes3;
Lambda3 = DiagonalMatrix[Table[4, {i, n}]];
dqr3 = D[qdes3, t] - Lambda3.qtilde3;
s3 = D[qtilde3, t] + Lambda3.qtilde3;

DynamicsLHS3 = MassM3.D[dqr3, t] + Cc3.dqr3 + Gg3;
Yy3 = Table[D[DynamicsLHS3[[i]], m3[j]], {i, n}, {j, n}];
```

```
hataa3 = massEstimates;
kGain3 = 10.0;
controlGains3 = DiagonalMatrix[Table[kGain3, {i, n}]];
SatVec3 = Table[Sat[s3[[i]]], {i, n}];
ControlInput3 = Yy3.hataa3 - controlGains3.SatVec3;

numRules3[time_] := Join[Table[m3[i] → massPerLink, {i, n - 1}],
   {m3[n] → If[time > tIngress && time ≤ (tIngress + tHold + tEgress),
      massPerLink + payloadMass, massPerLink]},
   Table[l3[i] → linkLengthNum, {i, n}], {g3 → gNum}];

hataaDynamic3[time_] := Join[Table[massEstimates[[i]], {i, n - 1}],
   {If[time > tIngress && time ≤ (tIngress + tHold + tEgress),
     massEstimates[[n]] + payloadEstimate, massEstimates[[n]]]}];

ics3 = Join[Table[θ3[i][0] == homeAngles[[i]], {i, n}],
   Table[Derivative[1][θ3[i]][0] == 0, {i, n}]];

ControlInputDynamic3 = Yy3.hataaDynamic3[t] - controlGains3.SatVec3;
EqsN3 = Thread[Eqs3 == ControlInputDynamic3] /. numRules3[t];

Print["Simulating..."];
solDE3 = NDSolve[Join[EqsN3, ics3], Table[θ3[i][t], {i, n}], {t, 0, tCycle},
   Method → {"EquationSimplification" → "Residual"}, MaxSteps → 10000];

If[Length[solDE3] > 0,
  Print["✓ SIMULATION 3 SUCCESS!\n"], Print["✗ SIMULATION 3 FAILED!\n"];
  Abort[]];


diamondBoundary3 = Graphics[{Thickness[0.005], Red, Dashed, Line[diamondVertices]}];

robotGraphics3[time_] := Module[{links, joints, endPoint, massPosition, massPoint},
   links = Table[{Thickness[0.01], Black,
      Line[{{xj3[k - 1], yj3[k - 1]}, {xj3[k], yj3[k]}} /. solDE3 /. t → time]}, {k, 1, n}];
   joints = Table[{Blue, Disk[{xj3[k], yj3[k]} /. solDE3 /. t → time, 0.08]}, {k, 0, n}];
   endPoint = {Orange, PointSize[0.025], Point[{xj3[n], yj3[n]} /. solDE3 /. t → time]};
   massPosition = If[time > (tIngress + tHold),
     {xj3[n], yj3[n]} /. solDE3 /. t → time, {targetX3, targetY3}];
   massPoint = {Blue, PointSize[0.03], Point[massPosition]};
   Graphics[{links, joints, endPoint, massPoint}]];

baseMarker3 = Graphics[{Gray, EdgeForm[Black], Thickness[0.015],
    Rectangle[{baseX - 0.2, baseY - 0.15}, {baseX + 0.2, baseY}], Black,
    Text["BASE", {baseX, baseY - 0.25}, BaseStyle → {FontSize → 12, Bold}]}];

animation3 = Animate[Show[diamondBoundary3, baseMarker3,
```

```
    robotGraphics3[time], PlotRange → {{-diamondWidth - 0.5, diamondWidth + 0.5},
      {-0.5, diamondHeight + diamondYOffset + 0.5}}, AspectRatio → Automatic, Frame → True,
    FrameLabel → {"X (m)", "Y (m)"}, PlotLabel → "Mass 3 at (" <> ToString[targetX3] <>
      ", " <> ToString[targetY3] <> ")\nt=" <> ToString[NumberForm[time, {3, 1}]] <> "s",
    ImageSize → Large], {time, 0, tCycle, 0.05}, AnimationRate → 1];


(* ==============================================================*)
(*SIMULATION 4-MASS 4 AT (0.15,4.0)-POSITIVE X TARGET*)
(* ==============================================================*)
Print["========================================"];
Print["SIMULATION 4: MASS 4"];
Print["========================================\n"];

targetX4 = .025;
targetY4 = 4;

Print["Target: (", targetX4, ", ", targetY4, ")"];

(*Kinematics for Sim 4*)
xTip4[th1_, th2_, th3_, th4_] :=
  baseX - linkLength * (Sin[th1] + Sin[th2] + Sin[th3] + Sin[th4]);
yTip4[th1_, th2_, th3_, th4_] :=
  baseY + linkLength * (Cos[th1] + Cos[th2] + Cos[th3] + Cos[th4]);

(*INVERSE KINEMATICS with the FIX:Constraints tightened to Pi/4*)
solution4 =
  NMinimize[{(xTip4[t1, t2, t3, t4] - targetX4)^2 + (yTip4[t1, t2, t3, t4] - targetY4)^2,
    -Pi / 2 < t1 < Pi / 2, -Pi / 2 < t2 < Pi / 2, -Pi / 2 < t3 < Pi / 2,
    -Pi / 2 < t4 < Pi / 2}, {t1, t2, t3, t4}];

pickupAngles4 = {t1, t2, t3, t4} /. solution4[[2]];
Print["Pickup angles: ", Round[pickupAngles4 * 180 / Pi, 0.1], "°\n"];

(*Trajectory Generation*)
qdes4 = Table[Piecewise[
    {{homeAngles[[i]] + (pickupAngles4[[i]] - homeAngles[[i]]) * SmoothStep[t, 0, tIngress],
      t ≤ tIngress}, {pickupAngles4[[i]], tIngress < t ≤ tIngress + tHold},
     {pickupAngles4[[i]] + (homeAngles[[i]] - pickupAngles4[[i]]) * SmoothStep[t,
         tIngress + tHold, tIngress + tHold + tEgress], t > tIngress + tHold}}], {i, n}];

(*Physics& Dynamics 4*)
xj4[0] = baseX;
yj4[0] = baseY;
Do[xj4[i] = baseX + Sum[-linkLength * Sin[θ4[j][t]], {j, 1, i}], {i, 1, n}];
Do[yj4[i] = baseY + Sum[linkLength * Cos[θ4[j][t]], {j, 1, i}], {i, 1, n}];
Do[xb4[i] = baseX + xj4[i - 1] - baseX - linkLength / 2 * Sin[θ4[i][t]], {i, 1, n}];
```

```
Do[yb4[i] = baseY + yj4[i - 1] - baseY + linkLength / 2 * Cos[θ4[i][t]], {i, 1, n}];

Do[Mj4[i] = 1 / 12 * m4[i] * l4[i]^2, {i, n}];

T4 = Sum[1 / 2 * m4[i] * (D[xb4[i], t]^2 + D[yb4[i], t]^2) +
     1 / 2 * Mj4[i] * (D[θ4[i][t], t])^2, {i, n}];
V4 = Sum[m4[i] * g4 * yb4[i], {i, n}];
Lag4 = T4 - V4;
Eqs4 = Table[D[D[Lag4, D[θ4[i][t], t]], t] - D[Lag4, θ4[i][t]], {i, n}];

MassM4 = Table[Coefficient[Eqs4[[i]], D[θ4[j][t], {t, 2}]], {i, n}, {j, n}];
Cc4 = Table[Sum[0.5 * (D[MassM4[[i, j]], θ4[k][t]] + D[MassM4[[i, k]], θ4[j][t]] -
        D[MassM4[[j, k]], θ4[i][t]]) * D[θ4[k][t], t], {k, n}], {i, n}, {j, n}];
Gg4 = Table[D[Eqs4[[i]], g4], {i, n}] * g4;

(*Controller Setup 4*)
q4 = Table[θ4[i][t], {i, n}];
qtilde4 = q4 - qdes4;
Lambda4 = DiagonalMatrix[Table[4, {i, n}]];
dqr4 = D[qdes4, t] - Lambda4.qtilde4;
s4 = D[qtilde4, t] + Lambda4.qtilde4;

DynamicsLHS4 = MassM4.D[dqr4, t] + Cc4.dqr4 + Gg4;
Yy4 = Table[D[DynamicsLHS4[[i]], m4[j]], {i, n}, {j, n}];

hataa4 = massEstimates;
kGain4 = 10.0;
controlGains4 = DiagonalMatrix[Table[kGain4, {i, n}]];
SatVec4 = Table[Sat[s4[[i]]], {i, n}];
ControlInput4 = Yy4.hataa4 - controlGains4.SatVec4;

(*Simulation Rules 4*)
numRules4[time_] := Join[Table[m4[i] → massPerLink, {i, n - 1}],
   {m4[n] → If[time > tIngress && time ≤ (tIngress + tHold + tEgress),
      massPerLink + payloadMass + .7, massPerLink]},
   Table[l4[i] → linkLengthNum, {i, n}], {g4 → gNum}];

hataaDynamic4[time_] := Join[Table[massEstimates[[i]], {i, n - 1}],
   {If[time > tIngress && time ≤ (tIngress + tHold + tEgress),
     massEstimates[[n]] + payloadEstimate, massEstimates[[n]]]}];

ics4 = Join[Table[θ4[i][0] == homeAngles[[i]], {i, n}],
   Table[Derivative[1][θ4[i]][0] == 0, {i, n}]];

ControlInputDynamic4 = Yy4.hataaDynamic4[t] - controlGains4.SatVec4;
EqsN4 = Thread[Eqs4 == ControlInputDynamic4] /. numRules4[t];
```

```
Print["Simulating 4..."];
solDE4 = NDSolve[Join[EqsN4, ics4], Table[θ4[i][t], {i, n}], {t, 0, tCycle},
   Method → {"EquationSimplification" → "Residual"}, MaxSteps → 10 000];

If[Length[solDE4] > 0,
  Print["✓ SIMULATION 4 SUCCESS!\n"], Print["× SIMULATION 4 FAILED!\n"];
  Abort[]];

(*Animation 4*)

diamondBoundary4 = Graphics[{Thickness[0.005], Red, Dashed, Line[diamondVertices]}];

robotGraphics4[time_] := Module[{links, joints, endPoint, massPosition, massPoint},
   links = Table[{Thickness[0.01], Black,
      Line[{{xj4[k - 1], yj4[k - 1]}, {xj4[k], yj4[k]}} /. solDE4 /. t → time]}, {k, 1, n}];
   joints = Table[{Blue, Disk[{xj4[k], yj4[k]} /. solDE4 /. t → time, 0.08]}, {k, 0, n}];
   endPoint = {Orange, PointSize[0.025], Point[{xj4[n], yj4[n]} /. solDE4 /. t → time]};
   massPosition = If[time > (tIngress + tHold),
     {xj4[n], yj4[n]} /. solDE4 /. t → time, {targetX4, targetY4}];
   massPoint = {Purple, PointSize[0.03], Point[massPosition]};
   (*Mass 4 is Purple*)Graphics[{links, joints, endPoint, massPoint}]];

baseMarker4 = Graphics[{Gray, EdgeForm[Black], Thickness[0.015],
    Rectangle[{baseX - 0.2, baseY - 0.15}, {baseX + 0.2, baseY}], Black,
    Text["BASE", {baseX, baseY - 0.25}, BaseStyle → {FontSize → 12, Bold}]}];

animation4 = Animate[Show[diamondBoundary4, baseMarker4,
    robotGraphics4[time], PlotRange → {{-diamondWidth - 0.5, diamondWidth + 0.5},
      {-0.5, diamondHeight + diamondYOffset + 0.5}}, AspectRatio → Automatic, Frame → True,
    FrameLabel → {"X (m)", "Y (m)"}, PlotLabel → "Mass 4 at (" <> ToString[targetX4] <>
      ", " <> ToString[targetY4] <> ")\nt=" <> ToString[NumberForm[time, {3, 1}]] <> "s",
    ImageSize → Large], {time, 0, tCycle, 0.05}, AnimationRate → 1];


Print["========================================"];
Print["ALL 4 SIMULATIONS COMPLETE!"];
Print["========================================"];
Print["Target locations:"];
Print["  Mass 1: (", targetX1, ", ", targetY1, ")"];
Print["  Mass 2: (", targetX2, ", ", targetY2, ")"];
Print["  Mass 3: (", targetX3, ", ", targetY3, ")"];
Print["  Mass 4: (", targetX4, ", ", targetY4, ")"];

Print["\nLegend:"];
Print["  - animation1: Mass 1 (Red)"];
Print["  - animation2: Mass 2 (Green)"];
Print["  - animation3: Mass 3 (Blue)"];
```

```
Print["  - animation4: Mass 4 (Purple)"];

(* ========================================
 MASTER SEQUENTIAL ANIMATION========================================*)
Print["========================================"];
Print["MASTER SEQUENTIAL ANIMATION"];
Print["========================================\n"];

tTotal = 4 * tCycle;

Print["tCycle = ", tCycle, " seconds"];
Print["tTotal = ", tTotal, " seconds\n"];

(*Define collection/dropoff zone-masses are placed here after retrieval*)
dropoffCenterX = 1.2; (*Center X position of dropoff zone*)
dropoffY = 0.0; (*Y position at base level-same as robot base*)
dropoffWidth = 1.5; (*Total width of dropoff zone*)
dropoffSpacing = 0.35; (*Horizontal spacing between masses*)

(*Calculate individual dropoff positions (left to right)*)
dropoffPositions = {{dropoffCenterX - 1.5 * dropoffSpacing, dropoffY},
   (*Mass 1-leftmost*){dropoffCenterX - 0.5 * dropoffSpacing, dropoffY},
   (*Mass 2*){dropoffCenterX + 0.5 * dropoffSpacing, dropoffY}, (*Mass 3*)
   {dropoffCenterX + 1.5 * dropoffSpacing, dropoffY}  (*Mass 4-rightmost*)};

(*Function to get robot configuration at global time*)
masterRobotGraphics[globalTime_] :=
  Module[{whichMass, localTime, xPos, yPos, links, joints, endPoint, massPos, massPoints},
   (*Determine which mass cycle we're in*)whichMass = Which[globalTime < tCycle,
     1, globalTime < 2 * tCycle, 2, globalTime < 3 * tCycle, 3, True, 4];
   (*Get local time within current cycle*)localTime = Mod[globalTime, tCycle];
   (*Get joint positions for current cycle*){xPos, yPos} =
    Which[whichMass == 1, {Flatten[Table[xj1[k] /. solDE1 /. t → localTime, {k, 0, n}]],
      Flatten[Table[yj1[k] /. solDE1 /. t → localTime, {k, 0, n}]]},
     whichMass == 2, {Flatten[Table[xj2[k] /. solDE2 /. t → localTime, {k, 0, n}]],
      Flatten[Table[yj2[k] /. solDE2 /. t → localTime, {k, 0, n}]]},
     whichMass == 3, {Flatten[Table[xj3[k] /. solDE3 /. t → localTime, {k, 0, n}]],
      Flatten[Table[yj3[k] /. solDE3 /. t → localTime, {k, 0, n}]]},
     whichMass == 4, {Flatten[Table[xj4[k] /. solDE4 /. t → localTime, {k, 0, n}]],
      Flatten[Table[yj4[k] /. solDE4 /. t → localTime, {k, 0, n}]]}];
   (*Draw links*)links = Table[{Thickness[0.01], Black,
      Line[{{xPos[[k]], yPos[[k]]}, {xPos[[k + 1]], yPos[[k + 1]]}}]}, {k, 1, n}];
   (*Draw joints*)joints = Table[{White, Disk[{xPos[[k]], yPos[[k]]}, 0.05]}, {k, 1, n + 1}];
   (*Draw end effector*)
   endPoint = {White, PointSize[0.015], Point[{xPos[[n + 1]], yPos[[n + 1]]}]};
   (*Mass positions-picked masses are carried,then dropped at collection zone*)
   massPos = {(*Mass 1-Red*)If[globalTime < tIngress + tHold, {targetX1, targetY1},
```

```
      (*Before pickup:at target*)If[globalTime < tCycle, {xPos[[n + 1]], yPos[[n + 1]]},
       (*Being carried*)dropoffPositions[[1]] (*Dropped off-position 1*)]],
     (*Mass 2-Green*)If[globalTime < tCycle + tIngress + tHold, {targetX2, targetY2},
      (*Before pickup:at target*)If[globalTime < 2 * tCycle, {xPos[[n + 1]], yPos[[n + 1]]},
       (*Being carried*)dropoffPositions[[2]] (*Dropped off-position 2*)]],
     (*Mass 3-Blue*)If[globalTime < 2 * tCycle + tIngress + tHold, {targetX3, targetY3},
      (*Before pickup:at target*)If[globalTime < 3 * tCycle, {xPos[[n + 1]], yPos[[n + 1]]},
       (*Being carried*)dropoffPositions[[3]] (*Dropped off-position 3*)]],
     (*Mass 4-Magenta*)If[globalTime < 3 * tCycle + tIngress + tHold, {targetX4, targetY4},
      (*Before pickup:at target*)If[globalTime < 4 * tCycle, {xPos[[n + 1]], yPos[[n + 1]]},
       (*Being carried*)dropoffPositions[[4]] (*Dropped off-position 4*)]]};
   (*Draw masses*)massPoints = {{Red, PointSize[0.03], Point[massPos[[1]]]},
     {Green, PointSize[0.03], Point[massPos[[2]]]}, {Blue, PointSize[0.03],
      Point[massPos[[3]]]}, {Magenta, PointSize[0.03], Point[massPos[[4]]]}};
   Graphics[{links, joints, endPoint, massPoints}]];

diamondBoundary = Graphics[{Thickness[0.005], Red, Dashed, Line[diamondVertices]}];
baseMarker = Graphics[{Gray, EdgeForm[Black], Thickness[0.015],
    Rectangle[{baseX - 0.2, baseY - 0.15}, {baseX + 0.2, baseY}], White,
    Text["BASE", {baseX, baseY - 0.25}, BaseStyle → {FontSize → 12, Bold}]}];

dropoffZone = Graphics[{Gray, EdgeForm[Black], Thickness[0.015],
    Rectangle[{dropoffCenterX - dropoffWidth / 2, dropoffY - 0.15},
     {dropoffCenterX + dropoffWidth / 2, dropoffY}], White,
    Text["DROPOFF", {dropoffCenterX, dropoffY - 0.25}, BaseStyle → {FontSize → 12, Bold}]}];

masterAnimation = Animate[
  Show[diamondBoundary, baseMarker, dropoffZone, masterRobotGraphics[time], PlotRange →
    {{-diamondWidth - 0.5, diamondWidth + 0.5}, {-0.5, diamondHeight + diamondYOffset + 0.5}},
   AspectRatio → Automatic, Frame → True, FrameLabel → {"X (m)", "Y (m)"},
   PlotLabel → "Sequential Pickup - Mass " <> ToString[Ceiling[time / tCycle]] <>
     " of 4\nt=" <> ToString[NumberForm[time, {4, 1}]] <> "s",
   ImageSize → Large], {time, 0, tTotal, 0.1}, AnimationRate → 1]
(* ========================
 ADAPTIVE CONTROLLER ANALYSIS PLOTS========================================*)

Clear[torque1, torque2, torque3, torque4];
Clear[velRules1, velRules2, velRules3, velRules4];
Clear[fullSol1, fullSol2, fullSol3, fullSol4];
Clear[trueMassFunc, estimatedMassFunc, paramErrorFunc];
(*Define plot colors*)
plotColors = Table[ColorData[97][i], {i, n}];
pickupTime = tIngress;

(* ========================================
 PLOT 1:CONTROL TORQUES========================================*)
```

```
(*Compute torques for all 4 simulations*)
velRules1 = Table[Derivative[1][θ1[i]][t] → D[θ1[i][t] /. solDE1⟦1⟧, t], {i, n}];
fullSol1 = Join[solDE1⟦1⟧, velRules1];

velRules2 = Table[Derivative[1][θ2[i]][t] → D[θ2[i][t] /. solDE2⟦1⟧, t], {i, n}];
fullSol2 = Join[solDE2⟦1⟧, velRules2];

velRules3 = Table[Derivative[1][θ3[i]][t] → D[θ3[i][t] /. solDE3⟦1⟧, t], {i, n}];
fullSol3 = Join[solDE3⟦1⟧, velRules3];

velRules4 = Table[Derivative[1][θ4[i]][t] → D[θ4[i][t] /. solDE4⟦1⟧, t], {i, n}];
fullSol4 = Join[solDE4⟦1⟧, velRules4];

(*Compute numerical torques*)
torque1[t_] := Table[(ControlInputDynamic1 /. fullSol1 /. numRules1[t])⟦i⟧, {i, n}];
torque2[t_] := Table[(ControlInputDynamic2 /. fullSol2 /. numRules2[t])⟦i⟧, {i, n}];
torque3[t_] := Table[(ControlInputDynamic3 /. fullSol3 /. numRules3[t])⟦i⟧, {i, n}];
torque4[t_] := Table[(ControlInputDynamic4 /. fullSol4 /. numRules4[t])⟦i⟧, {i, n}];

(*Plot torques for Mass 1*)
torquePlot1 = Plot[Evaluate[torque1[t]], {t, 0, tCycle}, PlotRange → All,
   PlotStyle → plotColors, PlotLegends → Table["τ" <> ToString[i], {i, n}],
   AxesLabel → {"Time (s)", "Torque (N·m)"},
   PlotLabel → "Control Torques - Mass 1\n(Jump at t=" <> ToString[pickupTime] <>
     "s when mass picked up)", GridLines → {{pickupTime}, {0}},
   GridLinesStyle → Directive[Gray, Dashed], ImageSize → Large,
   Epilog → {Red, Dashed, Thick, Line[{{pickupTime, -50}, {pickupTime, 50}}],
     Text[Style["Mass Pickup", Red, Bold], {pickupTime + 0.5, 45}, {-1, 0}]}];

(* ========================================
 PLOT 2:TRACKING ERROR========================================*)

(*Tracking error=actual-desired*)
trackingError1 = Flatten[qtilde1 /. solDE1];
trackingError2 = Flatten[qtilde2 /. solDE2];
trackingError3 = Flatten[qtilde3 /. solDE3];
trackingError4 = Flatten[qtilde4 /. solDE4];

errorPlot1 = Plot[Evaluate[trackingError1 * 180 / Pi], {t, 0, tCycle}, PlotRange → All,
   PlotStyle → plotColors, PlotLegends → Table["Joint " <> ToString[i], {i, n}],
   AxesLabel → {"Time (s)", "Error (degrees)"},
   PlotLabel → "Tracking Error - Mass 1\n(Spike at t=" <> ToString[pickupTime] <>
     "s shows adaptive response)", GridLines → {{pickupTime}, {0}},
   GridLinesStyle → Directive[Gray, Dashed], ImageSize → Large,
   Epilog → {Red, Dashed, Thick, Line[{{pickupTime, -5}, {pickupTime, 5}}],
     Text[Style["Mass Pickup", Red, Bold], {pickupTime + 0.5, 4}, {-1, 0}]}];
```

```
(* ========================================
 PLOT 3:PARAMETER ESTIMATION ERROR==========================================*)

(*True mass vs estimated mass for last link*)
trueMassFunc[tt_] := If[tt > tIngress && tt ≤ (tIngress + tHold + tEgress),
   massPerLink + payloadMass, massPerLink];
estimatedMassFunc[tt_] := If[tt > tIngress && tt ≤ (tIngress + tHold + tEgress),
   massEstimates[[n]] + payloadEstimate, massEstimates[[n]]];
paramErrorFunc[tt_] := trueMassFunc[tt] - estimatedMassFunc[tt];

paramErrorPlot = Plot[{trueMassFunc[t], estimatedMassFunc[t], paramErrorFunc[t]},
   {t, 0, tCycle}, PlotRange → All, PlotStyle →
    {Directive[Thick, Blue], Directive[Thick, Dashed, Red], Directive[Thick, Green]},
   PlotLegends → {"True Mass (Link 4)", "Estimated Mass (Link 4)", "Parameter Error"},
   AxesLabel → {"Time (s)", "Mass (kg)"}, PlotLabel →
    "Mass Parameter Estimation - Link 4\n(Controller underestimates payload by " <>
     ToString[N[(massPerLink + payloadMass) - (massEstimates[[n]] + payloadEstimate), 3]] <>
     " kg)", GridLines → {{pickupTime}, {0, massPerLink, massPerLink + payloadMass}},
   GridLinesStyle → Directive[Gray, Dashed], ImageSize → Large,
   Epilog → {Red, Dashed, Thick, Line[{{pickupTime, 0}, {pickupTime, 2}}],
     Text[Style["Mass Pickup", Red, Bold], {pickupTime + 0.5, 1.9}, {-1, 0}]}];

(* ================================PLOT 4:
  JOINT ANGLES (ALL 4 MASSES)=========================================*)

(*Plot joint angles for all 4 simulations*)
jointAnglesPlot = Plot[Evaluate[{Table[θ1[i][t] * 180 / Pi /. solDE1, {i, n}],
     Table[θ2[i][t] * 180 / Pi /. solDE2, {i, n}], Table[θ3[i][t] * 180 / Pi /. solDE3, {i, n}],
     Table[θ4[i][t] * 180 / Pi /. solDE4, {i, n}]}], {t, 0, tCycle},
   PlotRange → All, PlotStyle → Join[plotColors, plotColors, plotColors, plotColors],
   AxesLabel → {"Time (s)", "Angle (degrees)"},
   PlotLabel → "Joint Angles - All 4 Simulations", GridLines → {{pickupTime}, {0}},
   GridLinesStyle → Directive[Gray, Dashed], ImageSize → Large];

(* ===================================PLOT 5:
  TORQUE COMPARISON=========================================*)

(*Compare torques across all 4 masses for Joint 1*)
torqueComparisonPlot =
  Plot[Evaluate[{torque1[t][[1]], torque2[t][[1]], torque3[t][[1]], torque4[t][[1]]}],
   {t, 0, tCycle}, PlotRange → All, PlotStyle → {Red, Green, Blue, Magenta},
   PlotLegends → {"Mass 1", "Mass 2", "Mass 3", "Mass 4"},
   AxesLabel → {"Time (s)", "Torque (N·m)"}, PlotLabel → "Joint 1 Torque Comparison -
      All 4 Masses\n(Shows consistency across different pickup locations)",
   GridLines → {{pickupTime}, {0}}, GridLinesStyle → Directive[Gray, Dashed],
   ImageSize → Large,
```

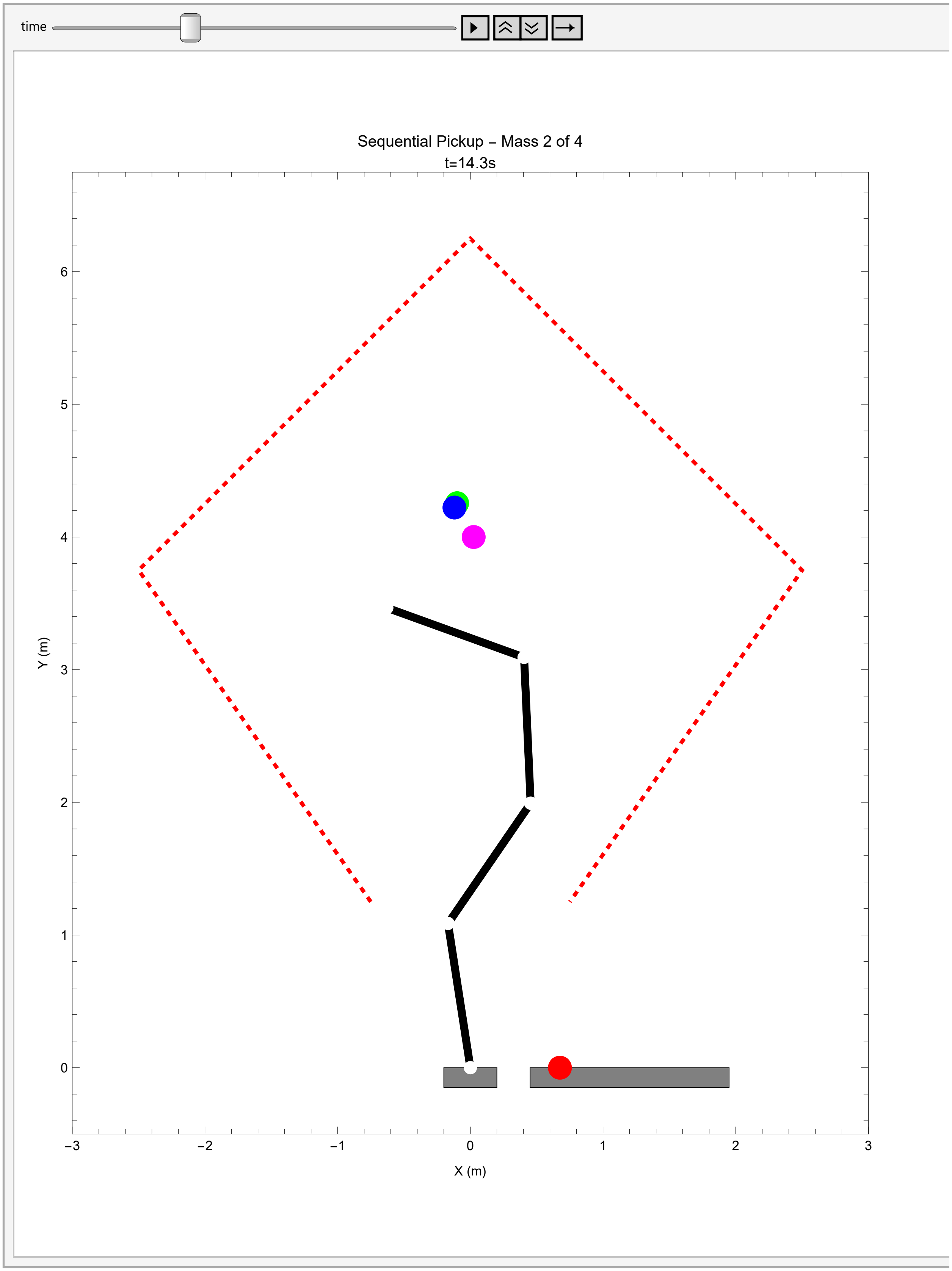
```
  Epilog → {Red, Dashed, Thick, Line[{{pickupTime, -50}, {pickupTime, 50}}],
    Text[Style["Mass Pickup", Red, Bold], {pickupTime + 0.5, 45}, {-1, 0}]}];


(* ==========================================
 SUMMARY==========================================*)
Print["\n=========================================="];
Print["ANALYSIS PLOTS SUMMERY"];
Print["=========================================="];
Print["  1. torquePlot1 - Control torques for Mass 1"];
Print["  2. errorPlot1 - Tracking error for Mass 1"];
Print["  3. paramErrorPlot - Parameter estimation error"];
Print["  4. jointAnglesPlot - Joint angles for all 4 masses"];
Print["  5. torqueComparisonPlot - Joint 1 torque across all masses"];
Print["\nKey observations:"];
Print["  - Torque jumps at t=",
  pickupTime, "s show controller responding to mass pickup"];
Print["  - Parameter error = ",
  N[(massPerLink + payloadMass) - (massEstimates[[n]] + payloadEstimate), 3],
  " kg (controller underestimates)"];
Print["  - Tracking error spikes briefly then converges (adaptive behavior)"];
Print["  - All 4 simulations show similar adaptive behavior"];
Print["==========================================\n"];

(*Display the plots*)
Print["Plot 1: Control Torques - Mass 1"];
torquePlot1

Print["Plot 2: Tracking Error - Mass 1"];
errorPlot1

Print["Plot 3: Parameter Estimation Error"];
paramErrorPlot

Print["Plot 4: Joint Angles - All 4 Masses"];
jointAnglesPlot

Print["Plot 5: Torque Comparison - Joint 1 Across All Masses"];
torqueComparisonPlot
```

```
========================================

SIMULATION 1: MASS 1

========================================

Target: (-0.1, 4.1)
```

```
Pickup angles: {-26.7, -9., 11.3, 30.}°

Simulating...

✓ SIMULATION 1 SUCCESS!

========================================

SIMULATION 2: MASS 2

========================================

Target: (-0.1, 4.255)

Pickup angles: {-15.4, -6.4, 3.4, 24.2}°

Simulating...

✓ SIMULATION 2 SUCCESS!

========================================

SIMULATION 3: MASS 3

========================================

Target: (-0.12, 4.222)

Pickup angles: {-20., -6.1, 9.2, 23.5}°

Simulating...

✓ SIMULATION 3 SUCCESS!

========================================

SIMULATION 4: MASS 4

========================================

Target: (0.025, 4)

Pickup angles: {-32.2, -12.7, 10., 33.9}°

Simulating 4...

✓ SIMULATION 4 SUCCESS!

========================================

ALL 4 SIMULATIONS COMPLETE!

========================================

Target locations:

  Mass 1: (-0.1, 4.1)
```

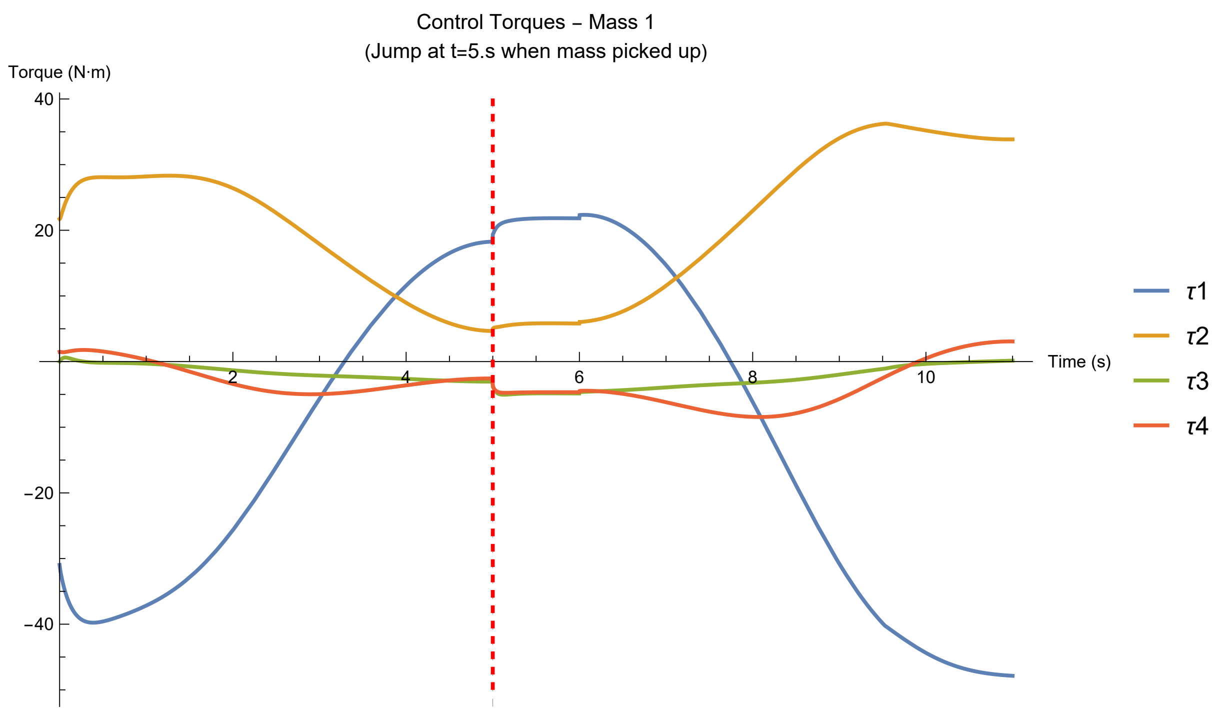
```
  Mass 2: (-0.1, 4.255)
  Mass 3: (-0.12, 4.222)
  Mass 4: (0.025, 4)

Legend:
  - animation1: Mass 1 (Red)
  - animation2: Mass 2 (Green)
  - animation3: Mass 3 (Blue)
  - animation4: Mass 4 (Purple)
============================================
MASTER SEQUENTIAL ANIMATION
============================================

tCycle = 11. seconds
tTotal = 44. seconds
```

Out[976]=


time
Sequential Pickup – Mass 2 of 4
t=14.3s
Y (m)
X (m)
−3
−2
−1
0
1
2
3
0
1
2
3
4
5
6

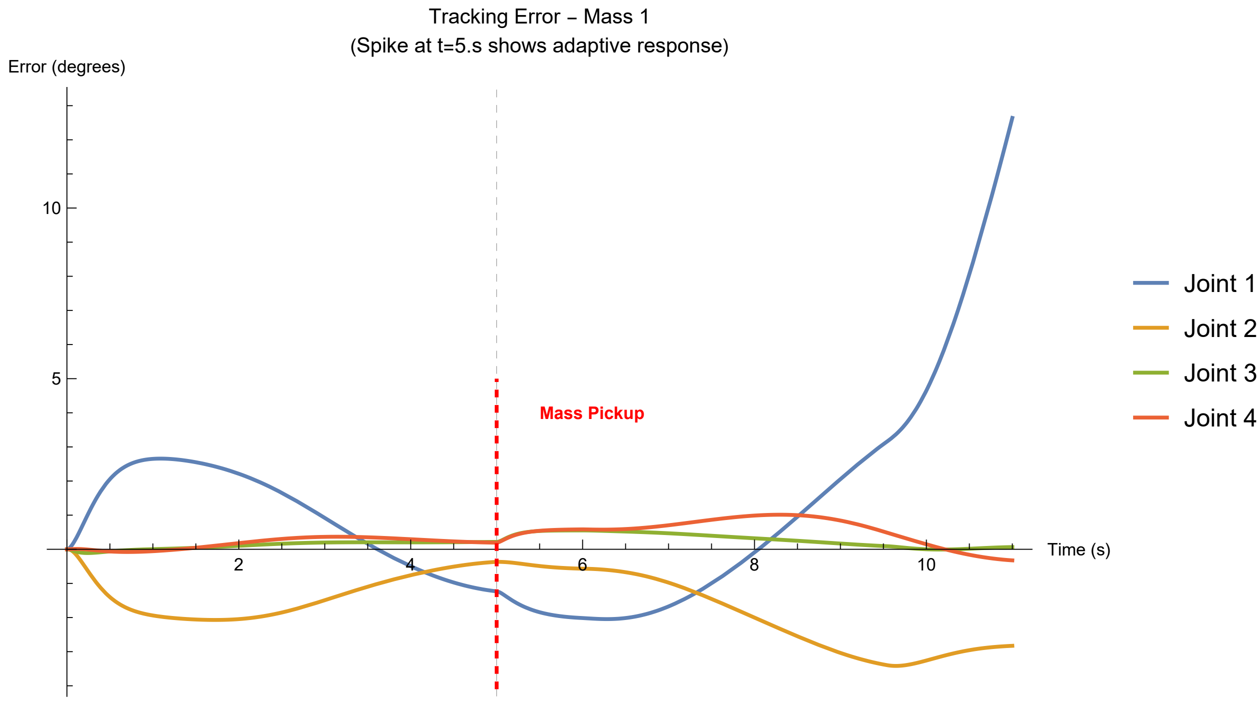
```
========================================
ANALYSIS PLOTS SUMMERY
========================================
  1. torquePlot1 - Control torques for Mass 1
  2. errorPlot1 - Tracking error for Mass 1
  3. paramErrorPlot - Parameter estimation error
  4. jointAnglesPlot - Joint angles for all 4 masses
  5. torqueComparisonPlot - Joint 1 torque across all masses

Key observations:
  - Torque jumps at t=5.s show controller responding to mass pickup
  - Parameter error = 0.6 kg (controller underestimates)
  - Tracking error spikes briefly then converges (adaptive behavior)
  - All 4 simulations show similar adaptive behavior
========================================

Plot 1: Control Torques - Mass 1
```

Out[1022]=



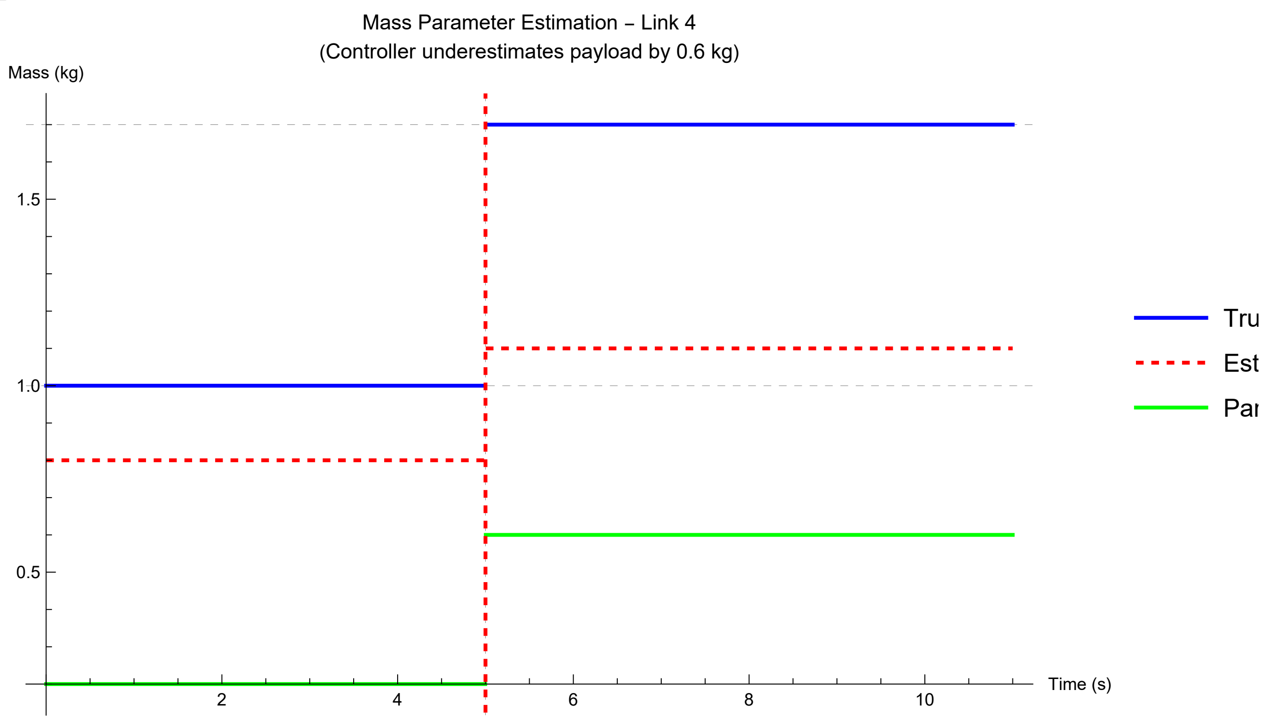

```
Plot 2: Tracking Error - Mass 1
```

Out[1024]=



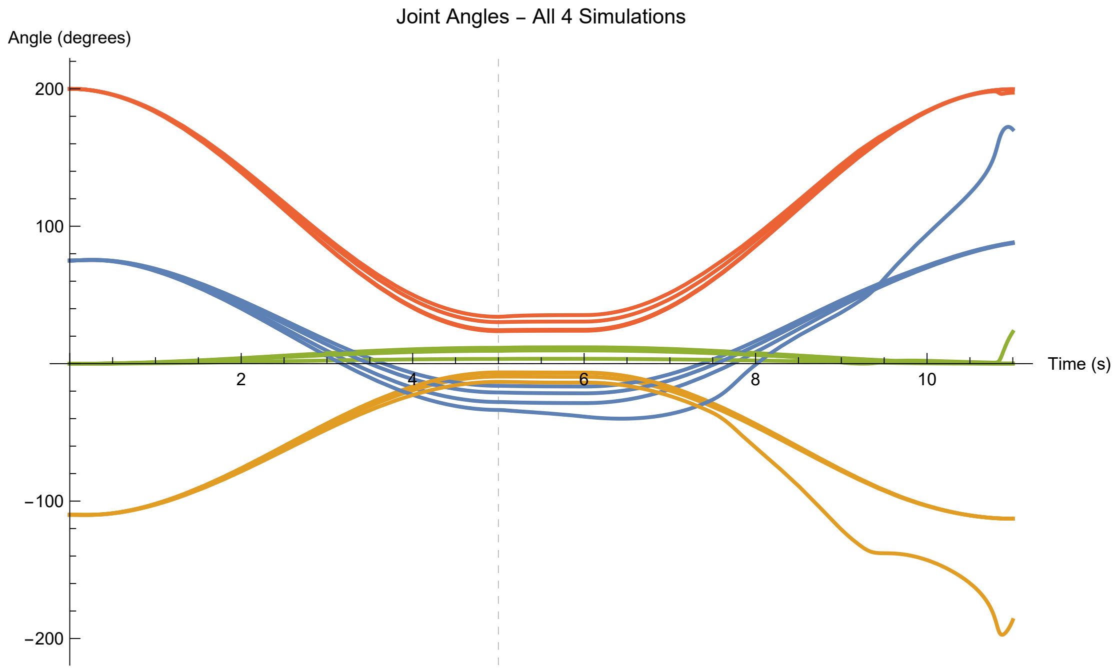

```
Plot 3: Parameter Estimation Error
```

Out[1026]=



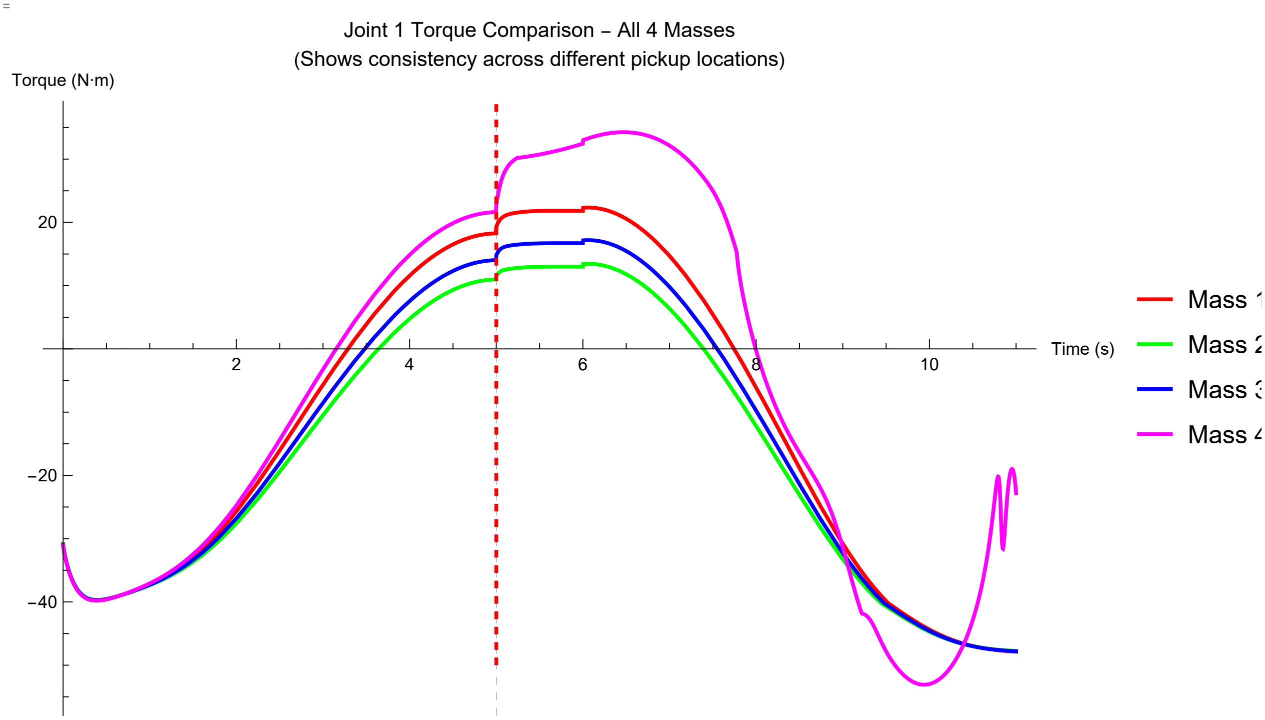

```
Plot 4: Joint Angles – All 4 Masses
```

Out[1028]=



Plot 5: Torque Comparison – Joint 1 Across All Masses

Out[1030]=



In[309]:=

```
(* ========================================*)
(*ROBOT WALL IMPACT & BOUNCE*)
(*Tip hits a vertical wall*)
(* ========================================*)
```

```
ClearAll["Global`*"];

(* ==========================================*)
(*1. PARAMETERS*)
(* ==========================================*)

n = 4;
linkLength = 1.10;
baseX = 0;
baseY = 0;

(*Physical parameters*)
massPerLink = 1.0;
linkLengthNum = 1.0;
gNum = 9.81;

(*Controller estimates*)
massEstimates = Table[0.8, {i, n}];

(*Home position*)
homeAngles = {0, 0, 0, 0}; (*Straight up*)

(*WALL POSITION*)
wallX = 2.3; (*Wall at x=2.0*)

(*Impact physics *)
eWall = .7;  (*Coefficient of restitution*)
muWall = 0.1; (*Coefficient of friction*)
mTip = .50;  (*Effective tip mass*)

Print["=========================================="];
Print["SIMPLE ROBOT WALL BOUNCE"];
Print["==========================================\n"];
Print["Wall location: x = ", wallX, " m"];
Print["Restitution e = ", eWall];
Print["Friction μ = ", muWall];
Print["\n"];

(* ==========================================*)
(*2. HELPER FUNCTIONS*)
(* ==========================================*)

limm = 0.25;
Sat[x_] := Piecewise[{{x / limm, Abs[x] < limm}, {-1, x ≤ -limm}, {1, x ≥ limm}}];

SmoothStep[t_, t0_, t1_] :=
  Piecewise[{{0, t < t0}, {1, t > t1}, {(1 - Cos[Pi * (t - t0) / (t1 - t0)]) / 2, t0 ≤ t ≤ t1}}];
```

```
(* ==========================================*)
(*3. IMPACT PHYSICS (from ball example)*)
(* ==========================================*)

ApplyWallImpact[vxMinus_, vyMinus_] :=
  Module[{vxPlus, vyPlus, Jn, Jt, deltaVx}, Print["=== WALL IMPACT DETECTED ==="];
   Print["Pre-impact:  vx⁻ = ", NumberForm[vxMinus, {4, 2}],
    " m/s,  vy⁻ = ", NumberForm[vyMinus, {4, 2}], " m/s"];
   (*Normal direction is-x (wall faces left)*)
   (*So normal velocity is vx,tangential is vy*)
   (*Newton's Restitution-normal component reverses*)vxPlus = -eWall * vxMinus;
   (*Coulomb Friction-affects tangential*)Jn = mTip * Abs[vxMinus] * (1 + eWall);
   Jt = muWall * Jn;
   deltaVx = Jt / mTip;
   If[Abs[vyMinus] > deltaVx, vyPlus = vyMinus - Sign[vyMinus] * deltaVx;
    Print["Status: SLIDING"], vyPlus = 0;
    Print["Status: STICKING"]];
   Print["Post-impact: vx⁺ = ", NumberForm[vxPlus, {4, 2}],
    " m/s,  vy⁺ = ", NumberForm[vyPlus, {4, 2}], " m/s"];
   Print["Energy retained: ",
    NumberForm[100 * (vxPlus^2 + vyPlus^2) / (vxMinus^2 + vyMinus^2), {3, 1}], "%"];
   Print[""];
   {vxPlus, vyPlus}];

(* ==========================================*)
(*4. KINEMATICS*)
(* ==========================================*)

xj[0] = baseX; yj[0] = baseY;
Do[xj[i] = baseX + Sum[-linkLength * Sin[θ[j][t]], {j, 1, i}], {i, 1, n}];
Do[yj[i] = baseY + Sum[linkLength * Cos[θ[j][t]], {j, 1, i}], {i, 1, n}];

Do[xb[i] = baseX + xj[i - 1] - baseX - linkLength / 2 * Sin[θ[i][t]], {i, 1, n}];
Do[yb[i] = baseY + yj[i - 1] - baseY + linkLength / 2 * Cos[θ[i][t]], {i, 1, n}];

(*Tip position functions*)
xTip[th1_, th2_, th3_, th4_] :=
  baseX - linkLength * (Sin[th1] + Sin[th2] + Sin[th3] + Sin[th4]);
yTip[th1_, th2_, th3_, th4_] :=
  baseY + linkLength * (Cos[th1] + Cos[th2] + Cos[th3] + Cos[th4]);

(* ==========================================*)
(*5. TARGET-Beyond the wall!*)
(* ==========================================*)

(*Target is PAST the wall-robot will try to reach it and hit wall*)
```

```
targetX = 2.5; (*Past wall at x=2.5*)
targetY = 3.5;

Print["Target: (", targetX, ", ", targetY, ") - PAST THE WALL!"];
Print["Robot will collide with wall while trying to reach target.\n"];

(*Solve IK*)
sol =
  NMinimize[{(xTip[t1, t2, t3, t4] - targetX)^2 + (yTip[t1, t2, t3, t4] - targetY)^2, -Pi/2 <
     t1 < Pi/2, -Pi/2 < t2 < Pi/2, -Pi/2 < t3 < Pi/2, -Pi/2 < t4 < Pi/2}, {t1, t2, t3, t4}];

targetAngles = {t1, t2, t3, t4} /. sol[[2]];
Print["Target angles: ", Round[targetAngles * 180 / Pi, 1], "°"];
Print["Would reach: (", Round[xTip @@ targetAngles, 0.01],
  ", ", Round[yTip @@ targetAngles, 0.01], ")\n"];

(* ========================================*)
(*6. DYNAMICS*)
(* ========================================*)

Do[Mj[i] = 1 / 12 * m[i] * l[i]^2, {i, n}];

T = Sum[
   1 / 2 * m[i] * (D[xb[i], t]^2 + D[yb[i], t]^2) + 1 / 2 * Mj[i] * (D[θ[i][t], t])^2, {i, n}];
V = Sum[m[i] * g * yb[i], {i, n}];
Lag = T - V;
Eqs = Table[D[D[Lag, D[θ[i][t], t]], t] - D[Lag, θ[i][t]], {i, n}];

MassM = Table[Coefficient[Eqs[[i]], D[θ[j][t], {t, 2}]], {i, n}, {j, n}];
Cc = Table[
   Sum[0.5 * (D[MassM[[i, j]], θ[k][t]] + D[MassM[[i, k]], θ[j][t]] - D[MassM[[j, k]], θ[i][t]]) *
     D[θ[k][t], t], {k, n}], {i, n}, {j, n}];
Gg = Table[D[Eqs[[i]], g], {i, n}] * g;

(* ========================================*)
(*7. CONTROLLER*)
(* ========================================*)

q = Table[θ[i][t], {i, n}];
Λ = DiagonalMatrix[Table[4, {i, n}]];
controlGains = DiagonalMatrix[Table[10.0, {i, n}]];

numRules :=
  Join[Table[m[i] → massPerLink, {i, n}], Table[l[i] → linkLengthNum, {i, n}], {g → gNum}];

(* ========================================*)
(*8. PHASE 1-APPROACH (hit wall)*)
```

```
(* ========================================*)

Print["========================================"];
Print["PHASE 1: Approaching wall"];
Print["========================================\n"];

tApproach = 2.0;

qdes1 = Table[Piecewise[{{homeAngles[[i]] +
        (targetAngles[[i]] - homeAngles[[i]]) * SmoothStep[t, 0, tApproach], True}}], {i, n}];

qtilde1 = q - qdes1;
dqr1 = D[qdes1, t] - Λ.qtilde1;
s1 = D[qtilde1, t] + Λ.qtilde1;

DynamicsLHS1 = MassM.D[dqr1, t] + Cc.dqr1 + Gg;
Yy1 = Table[D[DynamicsLHS1[[i]], m[j]], {i, n}, {j, n}];
SatVec1 = Table[Sat[s1[[i]]], {i, n}];
ControlInput1 = Yy1.massEstimates - controlGains.SatVec1;

ics1 =
  Join[Table[θ[i][0] == homeAngles[[i]], {i, n}], Table[Derivative[1][θ[i]][0] == 0, {i, n}]];

EqsN1 = Thread[Eqs == ControlInput1] /. numRules;

solDE1 = NDSolve[Join[EqsN1, ics1], Table[θ[i][t], {i, n}], {t, 0, tApproach},
   Method → {"EquationSimplification" → "Residual"}, MaxSteps → 10000];

Print["✓ Approach complete\n"];

(* ========================================*)
(*9. DETECT COLLISION*)
(* ========================================*)

Print["Checking for wall collision..."];

(*Find when tip crosses wall*)
collisionTime = None;
collisionY = None;

Do[Module[{xTipNow, yTipNow}, xTipNow = (xj[n] /. solDE1 /. t → time)[[1]];
   yTipNow = (yj[n] /. solDE1 /. t → time)[[1]];
   If[xTipNow ≥ wallX && collisionTime === None, collisionTime = time;
    collisionY = yTipNow;];], {time, 0, tApproach, 0.01}];

If[collisionTime =!= None,
  Print["*** COLLISION at t = ", NumberForm[collisionTime, {3, 2}], " s ***"];
```

```
  Print["    Position: (", wallX, ", ", NumberForm[collisionY, {3, 2}], ")\n"];
  (*Get velocity at collision*)dt = 0.02;
  xBefore = (xj[n] /. solDE1 /. t → (collisionTime - dt))[[1]];
  yBefore = (yj[n] /. solDE1 /. t → (collisionTime - dt))[[1]];
  vxMinus = (wallX - xBefore) / dt;
  vyMinus = (collisionY - yBefore) / dt;
  (*Apply impact physics*){vxPlus, vyPlus} = ApplyWallImpact[vxMinus, vyMinus];
  (*Angles at collision*)
  collisionAngles = Table[(θ[i][t] /. solDE1[[1]] /. t → collisionTime), {i, n}];
  collisionVels = Table[(D[θ[i][t] /. solDE1[[1]], t] /. t → collisionTime), {i, n}];,
  Print["No collision detected."];];

(* ==========================================*)
(*10. PHASE 2-BOUNCE BACK*)
(* ==========================================*)

If[collisionTime =!= None, Print["=========================================="];
  Print["PHASE 2: Bouncing back"];
  Print["==========================================\n"];
  (*New target-back towards home,reflecting the bounce*)bounceTargetX = 0.5;
  (*Bounce back to left*)bounceTargetY = collisionY + vyPlus * 1.0;
  (*Continue in y direction*)Print["Bounce target: (",
   bounceTargetX, ", ", NumberForm[bounceTargetY, {3, 2}], ")\n"];
  (*IK for bounce target*)solBounce = NMinimize[
    {(xTip[t1, t2, t3, t4] - bounceTargetX)^2 + (yTip[t1, t2, t3, t4] - bounceTargetY)^2,
     -Pi / 2 < t1 < Pi / 2, -Pi / 2 < t2 < Pi / 2, -Pi / 2 < t3 < Pi / 2,
     -Pi / 2 < t4 < Pi / 2}, {t1, t2, t3, t4}];
  bounceAngles = {t1, t2, t3, t4} /. solBounce[[2]];
  tBounce = 2.0;
  (*Trajectory from collision angles to bounce target*)
  qdes2 = Table[Piecewise[{{collisionAngles[[i]] + (bounceAngles[[i]] - collisionAngles[[i]]) *
          SmoothStep[t, 0, tBounce], True}}], {i, n}];
  qtilde2 = q - qdes2;
  dqr2 = D[qdes2, t] - Λ.qtilde2;
  s2 = D[qtilde2, t] + Λ.qtilde2;
  DynamicsLHS2 = MassM.D[dqr2, t] + Cc.dqr2 + Gg;
  Yy2 = Table[D[DynamicsLHS2[[i]], m[j]], {i, n}, {j, n}];
  SatVec2 = Table[Sat[s2[[i]]], {i, n}];
  ControlInput2 = Yy2.massEstimates - controlGains.SatVec2;
  (*Start from collision state*)ics2 = Join[Table[θ[i][0] == collisionAngles[[i]], {i, n}],
    Table[Derivative[1][θ[i]][0] == collisionVels[[i]], {i, n}]];
  EqsN2 = Thread[Eqs == ControlInput2] /. numRules;
  Print["Simulating bounce..."];
  solDE2 = NDSolve[Join[EqsN2, ics2], Table[θ[i][t], {i, n}], {t, 0, tBounce},
    Method → {"EquationSimplification" → "Residual"}, MaxSteps → 10000];
  Print["✓ Bounce complete\n"];];
```

```
(* ==========================================*)
(*11. VISUALIZATION*)
(* ==========================================*)

Print["=========================================="];
Print["CREATING PLOTS"];
Print["==========================================\n"];

(*Wall graphic*)
wallGraphic = Graphics[{Thick, Brown, Line[{{wallX, -1}, {wallX, 5}}],
    LightBrown, Opacity[0.3], Rectangle[{wallX, -1}, {wallX + 0.5, 5}],
    Black, Text[Style["WALL", Bold, 12], {wallX + 0.25, 4.5}]}];

(*Base*)
baseGraphic = Graphics[
   {Gray, EdgeForm[Black], Rectangle[{baseX - 0.3, baseY - 0.2}, {baseX + 0.3, baseY}],
    Black, Text[Style["BASE", Bold], {baseX, baseY - 0.35}]}];

(*Target*)
targetGraphic = Graphics[{Red, PointSize[0.02], Point[{targetX, targetY}],
    Text["Target\n(past wall)", {targetX, targetY + 0.4}]}];

(*Phase 1 trajectory*)
traj1 = ParametricPlot[{xj[n], yj[n]} /. solDE1,
   {t, 0, collisionTime}, PlotStyle → {Thick, Blue}, PlotRange → All];

(*Phase 2 trajectory*)
traj2 = If[collisionTime =!= None, ParametricPlot[{xj[n], yj[n]} /. solDE2,
    {t, 0, tBounce}, PlotStyle → {Thick, Green}, PlotRange → All], Graphics[{}]];

(*Impact marker*)
impactMarker = If[collisionTime =!= None, Graphics[{(*Impact point*)Red,
     PointSize[0.025], Point[{wallX, collisionY}], (*Velocity arrows*)Orange, Thick,
     Arrowheads[0.04], Arrow[{{wallX - 0.5, collisionY}, {wallX, collisionY}}],
     (*v⁻ into wall*)Green, Thick, Arrowheads[0.04],
     Arrow[{{wallX, collisionY}, {wallX - 0.5, collisionY + 0.3 * Sign[vyPlus]}}],
     (*v⁺ bouncing*)Black, Text[Style["v⁻", Bold, Orange], {wallX - 0.6, collisionY - 0.2}],
     Text[Style["v⁺", Bold, Darker[Green]], {wallX - 0.6, collisionY + 0.3}], Text[
      Style["IMPACT!", Bold, Red, 14], {wallX - 0.3, collisionY + 0.5}]}], Graphics[{}]];

(*Combined static plot*)
staticPlot = Show[wallGraphic, baseGraphic, targetGraphic, traj1, traj2,
   impactMarker, PlotRange → {{-1, 3.5}, {-1, 5}}, AspectRatio → Automatic,
   Frame → True, FrameLabel → {"X (m)", "Y (m)"}, PlotLabel →
    Style["Robot Wall Impact - Bounce Physics\nBlue = Approach, Green = Bounce",
     Bold, 14], ImageSize → Large, GridLines → Automatic];
```

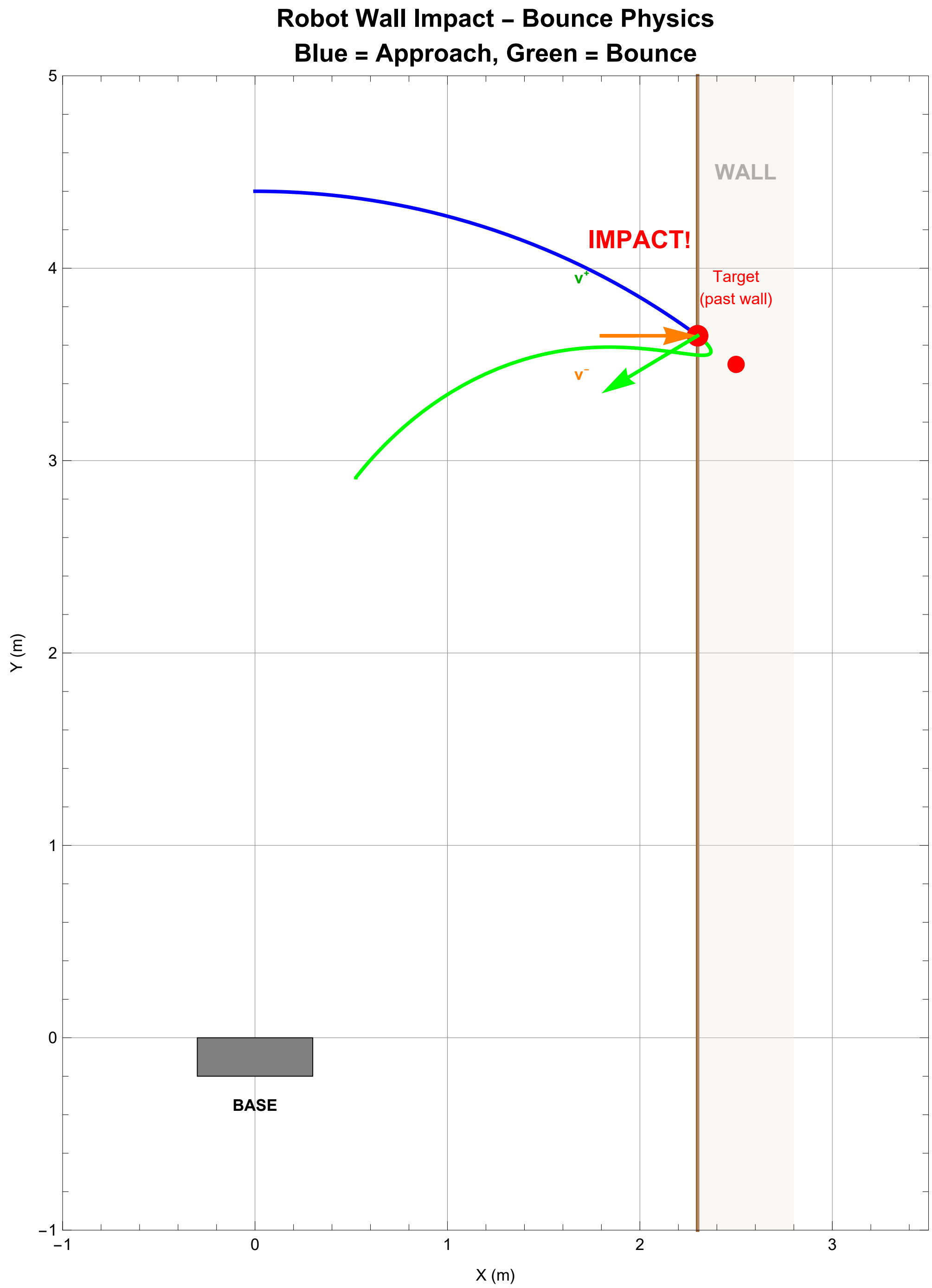
```
Print["✓ Static plot"];
staticPlot

(* ========================================*)
(*12. ANIMATION*)
(* ========================================*)

tTotal = If[collisionTime =!= None, collisionTime + tBounce, tApproach];

getAngles[time_] :=
  If[time < collisionTime, (*Phase 1*)Table[(θ[i][t] /. solDE1〚1〛 /. t → time), {i, n}],
   (*Phase 2*)Table[(θ[i][t] /. solDE2〚1〛 /. t → (time - collisionTime)), {i, n}]];

robotGraphics[time_?NumericQ] :=
  Module[{angles, xPos, yPos, links, joints, tipColor}, angles = getAngles[time];
   xPos = Table[baseX - linkLength * Sum[Sin[angles〚j〛], {j, 1, k}], {k, 0, n}] /.
     {Sum[_, {_, 1, 0}] → 0};
   xPos =
    Prepend[Table[baseX - linkLength * Sum[Sin[angles〚j〛], {j, 1, k}], {k, 1, n}], baseX];
   yPos =
    Prepend[Table[baseY + linkLength * Sum[Cos[angles〚j〛], {j, 1, k}], {k, 1, n}], baseY];
   (*Color:Blue normally,Red at impact,Green bouncing*)tipColor = Which[
     Abs[time - collisionTime] < 0.1, Red, time > collisionTime, Darker[Green], True, Blue];
   links = Table[{Thickness[0.012], Black,
      Line[{{xPos〚k〛, yPos〚k〛}, {xPos〚k + 1〛, yPos〚k + 1〛}}]}, {k, 1, n}];
   joints = Table[{Blue, Disk[{xPos〚k〛, yPos〚k〛}, 0.06]}, {k, 1, n + 1}];
   Graphics[
    {links, joints, {tipColor, PointSize[0.025], Point[{xPos〚n + 1〛, yPos〚n + 1〛}]}}]];

animation =
  Animate[Module[{phase}, phase = If[time < collisionTime, "APPROACHING", "BOUNCING"];
    Show[wallGraphic, baseGraphic, robotGraphics[time],
     (*Show impact marker near collision*)If[Abs[time - collisionTime] < 0.2,
      Graphics[{Red, PointSize[0.03], Point[{wallX, collisionY}],
        Text[Style["IMPACT!", Bold, Red, 16], {wallX - 0.5, collisionY + 0.3}]}],
      Graphics[{}]], PlotRange → {{-1, 3.5}, {-1, 5}}, AspectRatio → Automatic,
     Frame → True, FrameLabel → {"X (m)", "Y (m)"}, PlotLabel →
      Style["Robot Wall Bounce\nt = " <> ToString[NumberForm[time, {3, 2}]] <> " s - " <>
        phase <> "\ne = " <> ToString[eWall] <> ", μ = " <> ToString[muWall], Bold, 12],
     ImageSize → Large]], {time, 0, tTotal, 0.02}, AnimationRate → 0.5];

Print["✓ Animation complete\n"];

Print["========================================"];
Print["RESULTS SUMMARY"];
Print["========================================"];
Print["Wall position: x = ", wallX, " m"];
```

```
Print["Collision time: ", NumberForm[collisionTime, {3, 2}], " s"];
Print["Collision point: (", wallX, ", ", NumberForm[collisionY, {3, 2}], ")"];
Print[""];
Print["Impact Physics:"];
Print["  e = ", eWall, " (restitution)"];
Print["  μ = ", muWall, " (friction)"];
Print["  v⁻ = (", NumberForm[vxMinus, {4, 2}],
  ", ", NumberForm[vyMinus, {4, 2}], ") m/s"];
Print["  v⁺ = (", NumberForm[vxPlus, {4, 2}], ", ", NumberForm[vyPlus, {4, 2}], ") m/s"];
Print["  Energy retained: ",
  NumberForm[100 * (vxPlus^2 + vyPlus^2) / (vxMinus^2 + vyMinus^2), {3, 1}], "%"];
Print["========================================\n"];

animation
```

```
========================================
SIMPLE ROBOT WALL BOUNCE
========================================

Wall location: x = 2.3 m
Restitution e = 0.7
Friction μ = 0.1


Target: (2.5, 3.5) - PAST THE WALL!
Robot will collide with wall while trying to reach target.

Target angles: {-55, -37, -25, -25}°
Would reach: (2.5, 3.5)

========================================
PHASE 1: Approaching wall
========================================

✓ Approach complete

Checking for wall collision...
*** COLLISION at t = 1.56 s ***
    Position: (2.3, 3.65)

=== WALL IMPACT DETECTED ===
Pre-impact:  vx⁻ = 1.13 m/s,  vy⁻ = -0.86 m/s
```

```
Status: SLIDING
Post-impact: vx⁺ = -0.79 m/s,  vy⁺ = -0.67 m/s
Energy retained: 53.1%

==========================================
PHASE 2: Bouncing back
==========================================

Bounce target: (0.5, 2.98)

Simulating bounce...
✓ Bounce complete

==========================================
CREATING PLOTS
==========================================

✓ Static plot
```

Out[394]=


Robot Wall Impact – Bounce Physics
Blue = Approach, Green = Bounce
WALL
IMPACT!
Target
(past wall)
BASE
Y (m)
X (m)
5
4
3
2
1
0
−1
−1
0
1
2
3

```
✓ Animation complete

========================================
RESULTS SUMMARY
========================================
Wall position: x = 2.3 m
Collision time: 1.56 s
Collision point: (2.3, 3.65)

Impact Physics:
  e = 0.7 (restitution)
  μ = 0.1 (friction)
  v⁻ = (1.13, -0.86) m/s
  v⁺ = (-0.79, -0.67) m/s
  Energy retained: 53.1%
========================================
```

Out[414]=

time

```
Show[wallGraphic, baseGraphic, robotGraphics[3.28], If[Abs[3.28 - collisionTime] < 0.2,

PlotRange → {{-1, 3.5}, {-1, 5}}, AspectRatio → Automatic, Frame → True, FrameLabel → {X (m)
t = 3.28 s - <> If[3.28 < collisionTime, APPROACHING, BOUNCING] <>
e = eWall, μ = muWall, ImageSize → Large]
```