International Economic Analysis

ANLY 560 Project Report

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Introduction

With the rapid growth of global economic development, the economic conditions between countries significantly influence each other. Thus, it is essential to analyze the previous international economic data to forecast future trends and volatilities. In this project, three developed countries were selected: United States, Canada and the United Kingdom. The pairwise analysis across countries and the variation over time will be provided. This project is highly meaningful for investors in certain markets. Through the analysis of the indexes, investors could have an overall understanding of economic conditions of these countries and generated related investing strategies.

This project included 3 datasets. The first dataset is CPI (Customer Price Index). This index is the number to measure changes in the price of goods and services purchased or otherwise acquired by households directly or indirectly to satisfy their own needs and wants. CPIs are widely used to index pensions and social security benefits. The original huge data contains the overall and detailed CPI value of more than 200 countries. In the project, the chosen data contains the overall CPI data value of three selected countries: United States, Canada and United Kingdom from 01/01/1980 until 01/01/2020.

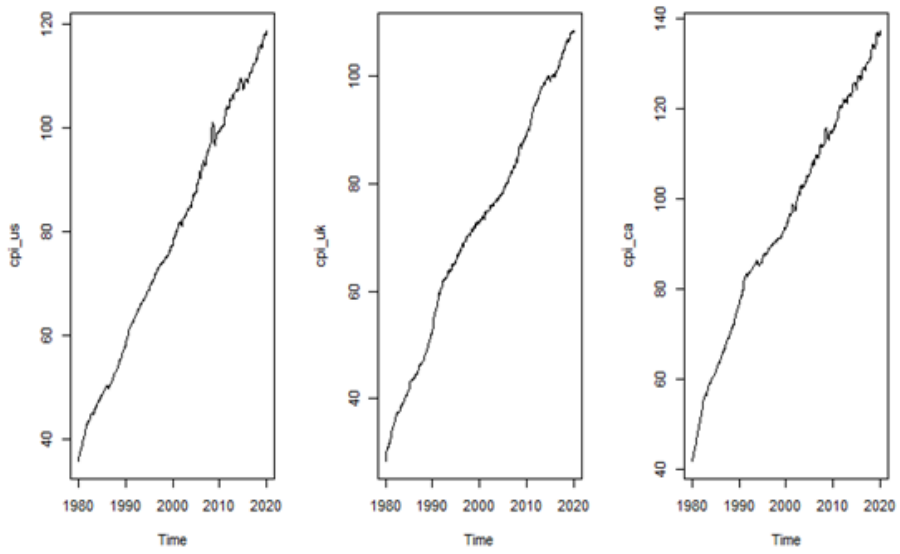
The second dataset is IFS (International Financial Statue). The original international financial database covered about 200 countries. The indicators in the dataset include the balance of payments, commodity prices, exchange rates, fund position, government finance, industrial production, interest rates, international investment position, international liquidity as well as other international financial status indicators. In this dataset, the first target index is the industrial production index (IPI). It measures the real output in the manufacturing, mining, electric and gas industries of a specific country. The higher industrial production index means higher production level that could be substantially maintained in the country. This index is critical for managers and investors within specific lines of business since it is an important macroeconomic indicator. The second target index is Number of Employments. This index measures the number of employments in each country. It is an important indicator the Federal uses to determine the health of the economy. The steady increase of the number of employments is an indicator of a healthy economy.

The third dataset is Commodity Net Export Price Index. This monthly data includes Canada, the United Kingdom, and the United States starting from 01/01/1980 until 02/01/2020. It measures the net changes in the price of goods and services in international trade. The data is used to deflate government trade statistics, predict future inflation and prices changes, set fiscal and monetary policy, measure exchange rates, negotiate trade contracts, and identify specific industry and global price trends.

Through the analysis of these three datasets, an overview of economic conditions of the three countries will be provided.

Modeling and Analysis

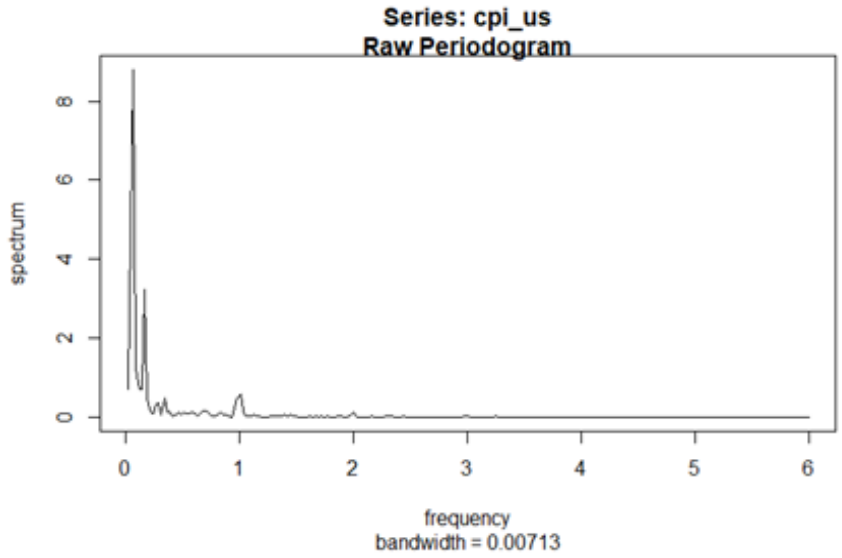
CPI (Customer Price Index)



(Figure 1-1 CPI Time-series Data plot of Three Countries)

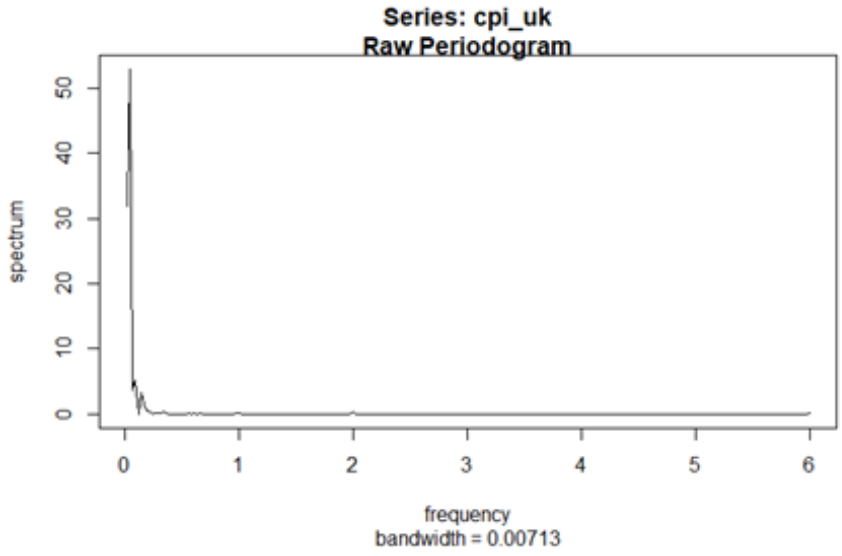
Figure1-1 shows the original data of the CPI index of the three countries, from left to right are the United States, the United Kingdom and Canada. All the three countries have an overall increasing trend of CPI, meaning that from 1980 to 2020, the consumption level of the three countries have witnessed an obvious improvement. But look at the plot closely, when it is nearly 2006-2008, the CPI suddenly dropped in the United States and Canada. This contrast is due to the outbreak of the terrible financial crisis in 2008. The financial crisis caused a huge influence on the global economy and made the consumption level drop a lot.

Spectral Analysis



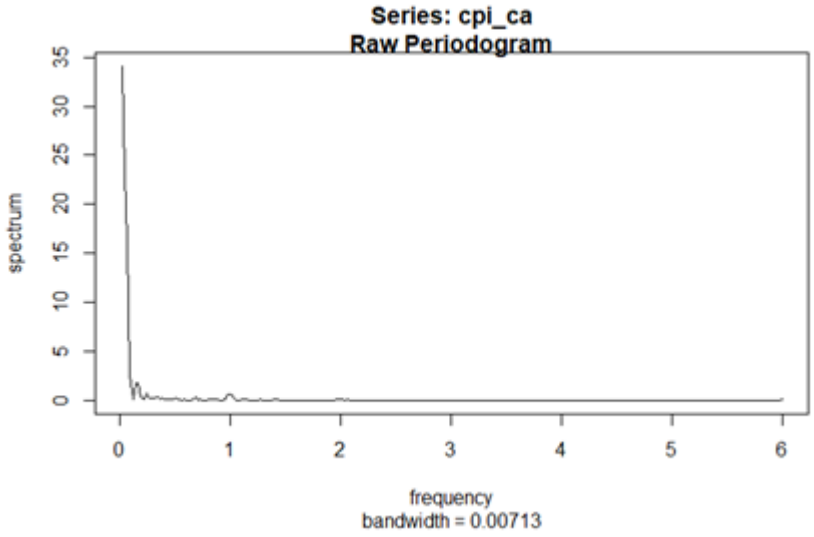
(Figure 1-2.1 Spectrum plot of United States)

Figure 1-2.1 is the spectrum plot of the United States' CPI, according to the graph, three obvious peaks with high power can be observed. The frequency with highest power is 0.074., means that the data may have a 13.5-month cycle.



(Figure 1-2.2 Spectrum plot of United Kingdom)

From Figure 1-2.2, one frequency with a high power can be found about this spectrum plot. The frequency is 0.049, and it points that the United Kingdom may have a 20.25-month cycle of the CPI data.

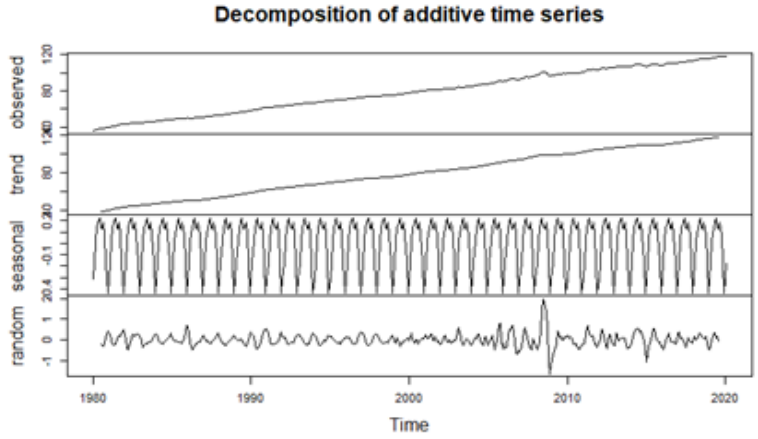


(Figure 1-2.3 Spectrum plot of Canada)

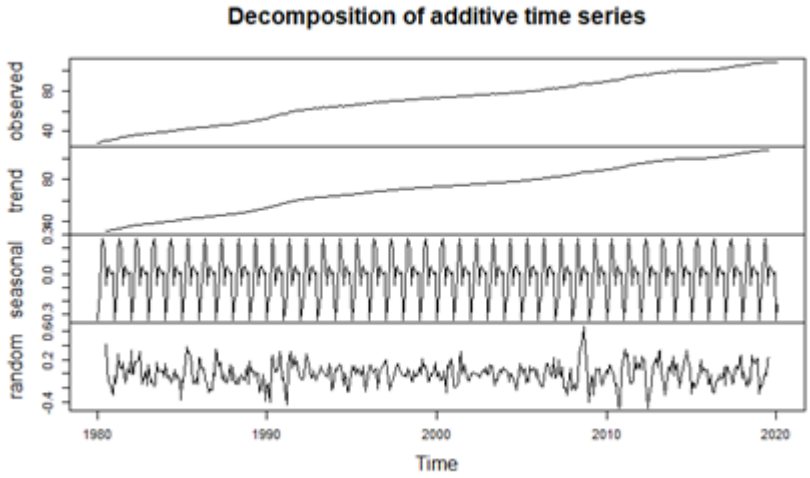
According to the figure 1-2.3 and the output, the frequency with highest power of Canada CPI data is nearly 0.025. A 40- month cycle may be observed of the Canada CPI.

Canada has the longest cycle among the three countries, it indicates that its change of CPI is not much fast and obvious. While the United States has the relatively shortest cycle, it may be the most unstable country with changing CPI value compared to Canada and the United Kingdom.

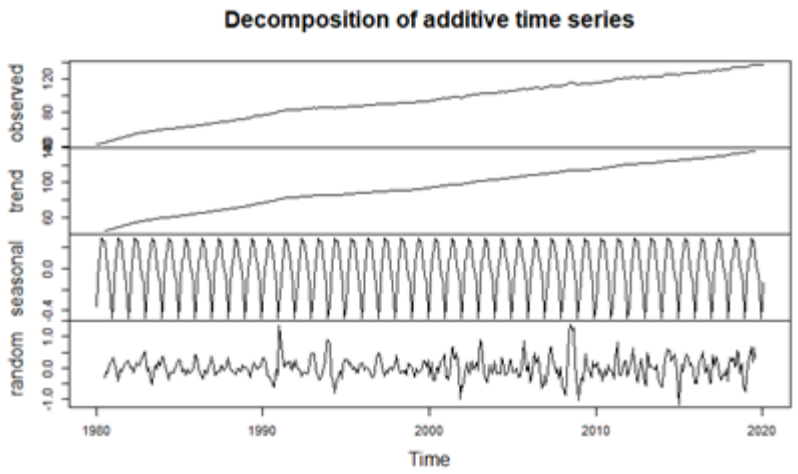
Decompose Analysis



(Figure 1-3.1 Decompose plot of United States)



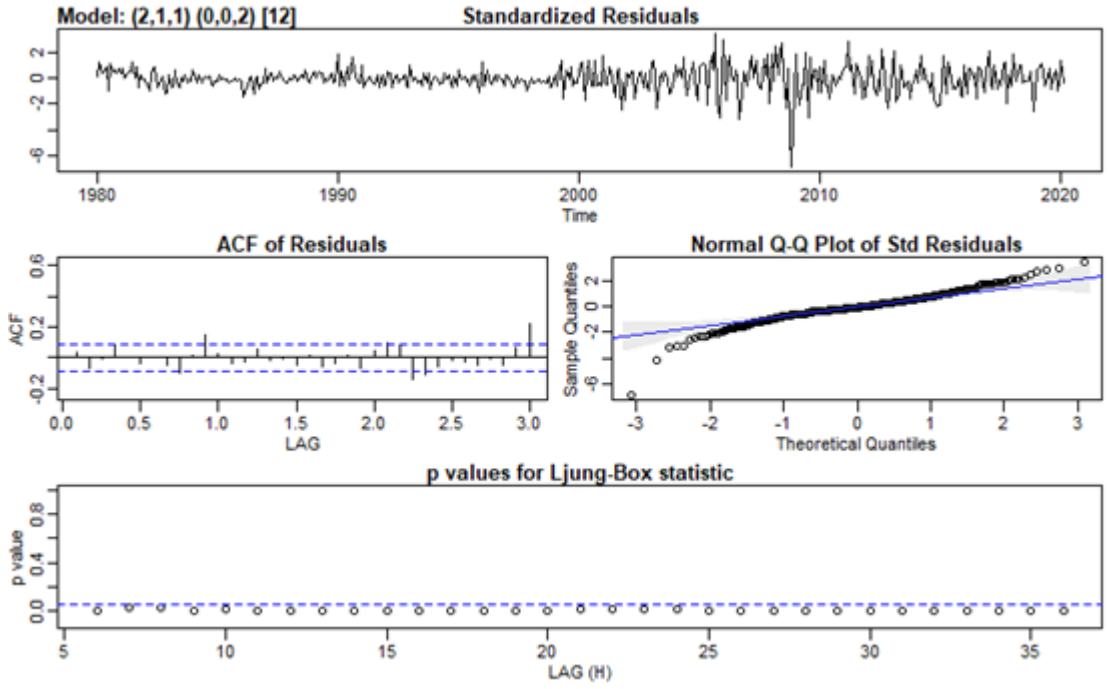
(Figure 1-3.2 Decompose plot of United Kingdom)



(Figure 1-3.3 Decompose plot of Canada)

Based on the figure 1-3.1 to 1.3.3, the decompose plot of the three countries, the increasing trend can be found more clearly. Besides, the decompose plot shows all of them have a seasonality, the data may fluctuate with the season or cycle.

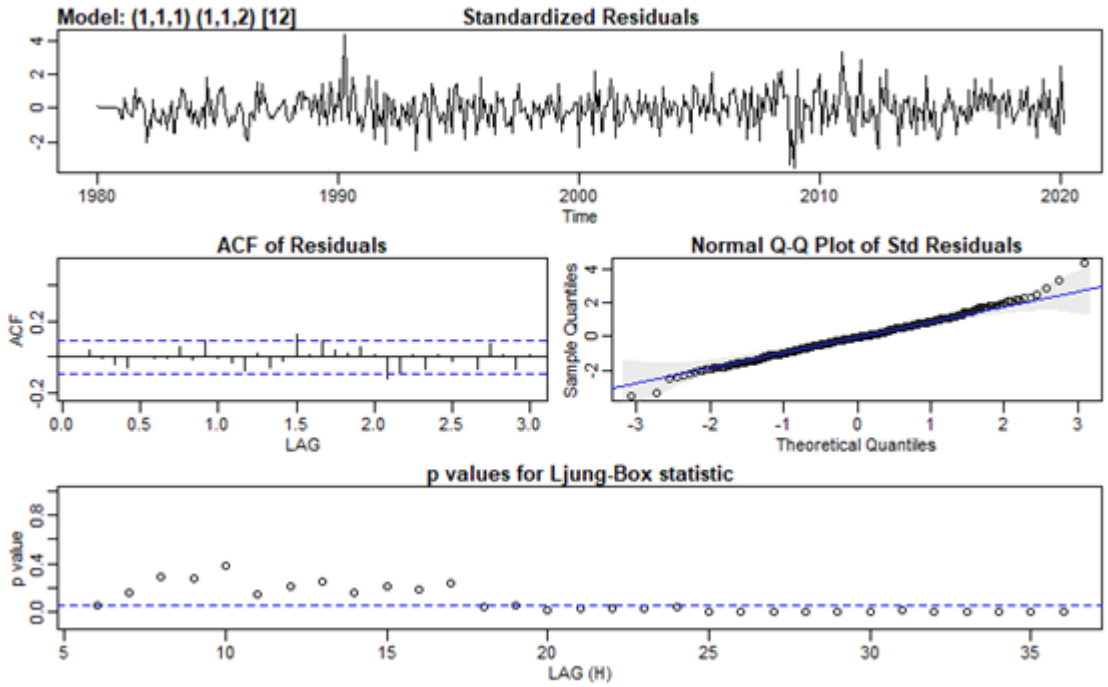
ARIMA Model



(Figure 1-4.1 Arima diagnosis plot of United States)

By comparing several Arima models, an seasonal Arima model of (2,1,1)(0,0,2)[12] is fitting the CPI of the United States were chosen based on the lowest AIC. The AIC value of the model is –4.2 and the p value of the coefficients are all small enough. Other models Arima(1,1,1) has an AIC value of 33.8 and (0,1,2) model has AIC value of 39.4.

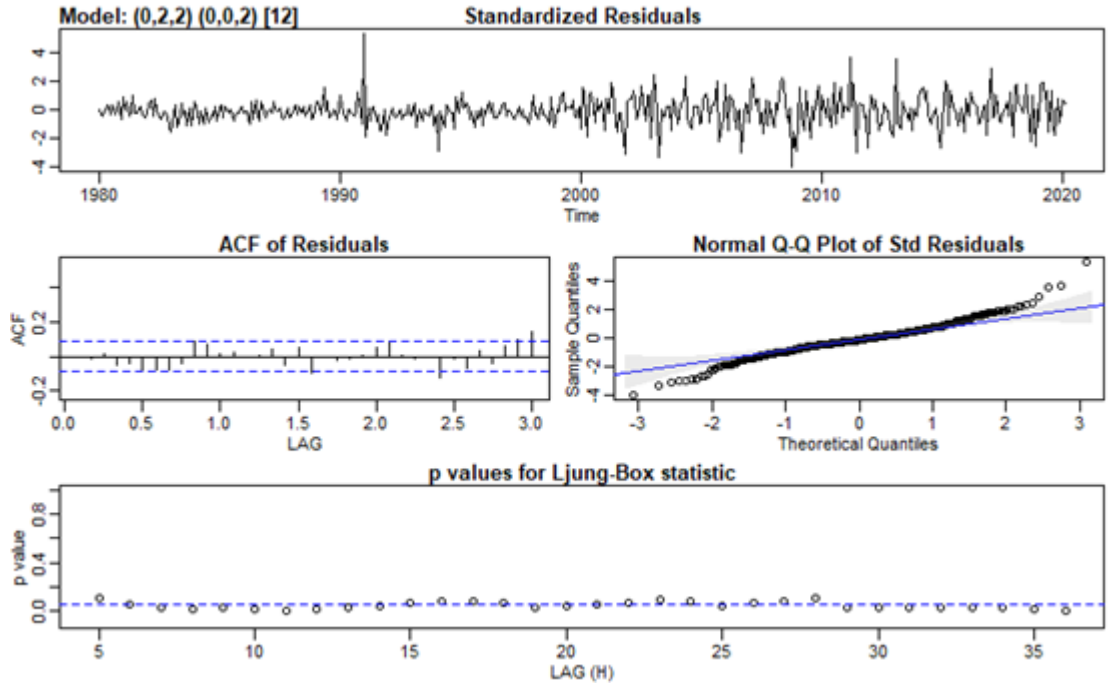
From figure 1-4.1, the diagnosis plot of the model, the Residuals is nearly normal distribution and stationery. But the Ljung-Box plot is not so ideal, maybe some residuals still can influence each other.



(Figure 1-4.2 Arima diagnosis plot of United Kingdom)

Comparing the AIC value of several Arima models. Model(1,2,1) has AIC of 19.6, and Model (1,2,2) has AIC value of 15.4. While the seasonal Arima model of (1,1,1)(1,1,2)[12] only has -440.7, so this model is fitting the CPI of the United Kingdom, and the p value of the coefficients are all small enough.

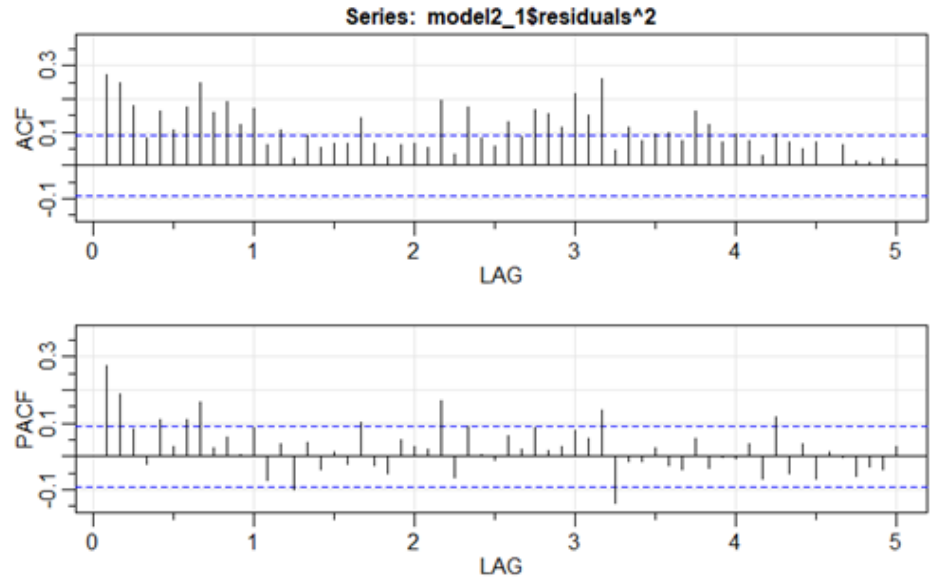
From figure 1-4.2, the diagnosis plot of the model, the Residuals is nearly normal distribution and stationery. The ACF also states the good effect of the model. For the Ljung-Box, most of the dots before lag 20 are away from the blue line, so the residuals are similar to the white noise.



(Figure 1-4.3 Arima diagnosis plot of Canada)

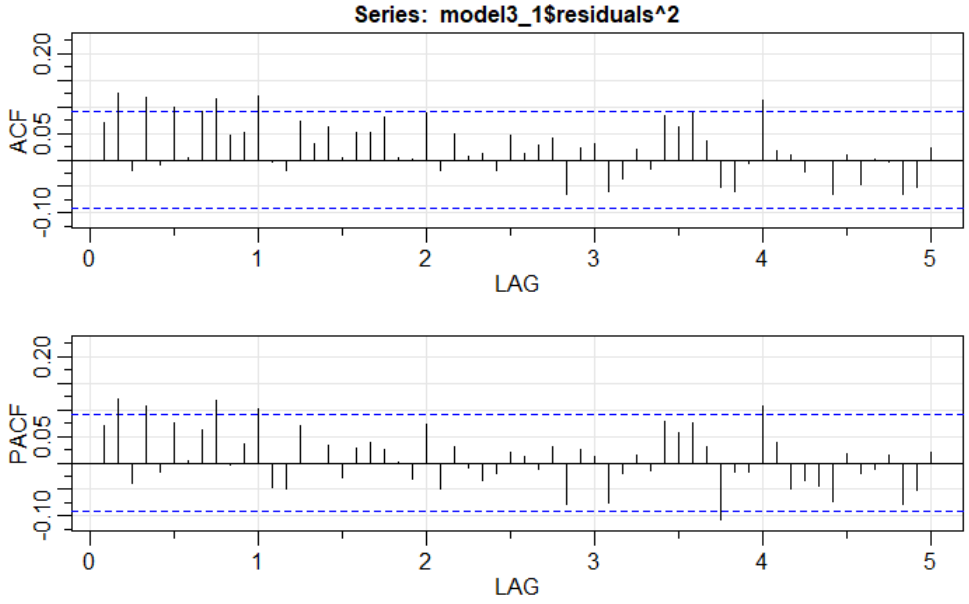
From figure 1-4.3, the diagnosis plot of the Arima model of Canada CPI, the Residuals is nearly normal distribution and stationary from the QQ plot and ACF plot. And the model with lowest AIC is a seasonal Arima model (0,2,2)(0,0,2)[12], has an AIC of 299.2, and all the coefficients are significant due to the small p values. The other models , like model(0,2,1) has AIC value of 358 and model (0,2,2) has AIC value of 349, so the model (0,2,2)(0,0,2)[12] has the best performance.

ARCH/GARCH Model



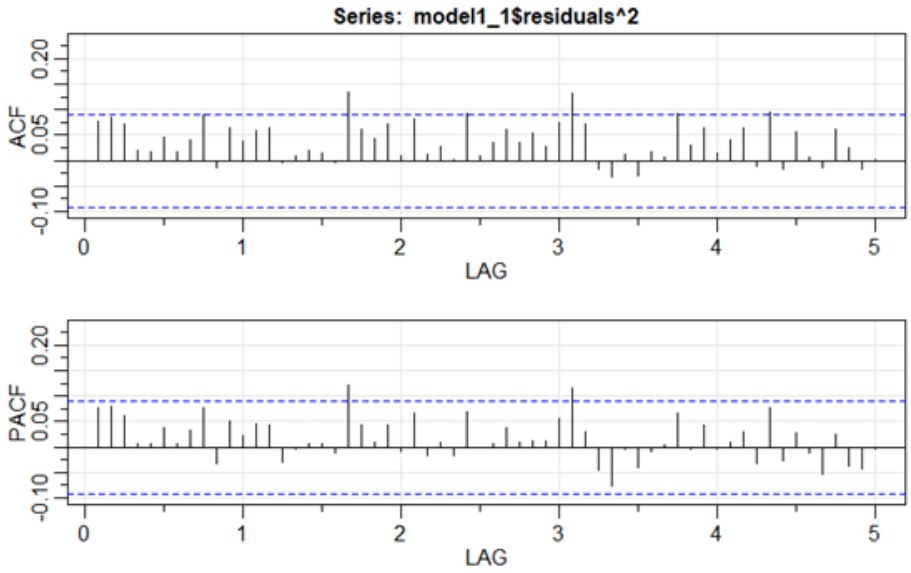
(Figure 1-4.1 Squared residuals PACF/ACF plot of the United States)

According to figure 1-4.1, the PACF and ACF plot of the squared residuals of the Arima model, the GARCH model of ARCH model can be used. By comparing the AIC value of a (3,3) model and a (0,3) model, the GARCH (3,3) model has a better performance. Ljung-Box test shows good effect and all the coefficients are significant.



(Figure 1-4.2 Squared residuals PACF/ACF plot of the United Kingdom)

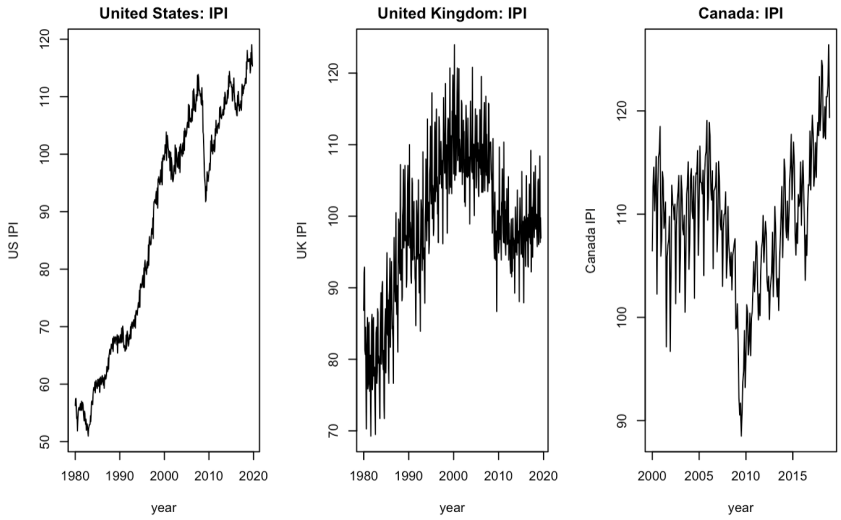
According to figure 1-4.2, the PACF and ACF plot of the squared residuals of the Arima model, the GARCH model of ARCH model can be used. By comparing the AIC value of a (4,4) model and a (0,4) model, the GARCH (4,4) model has a better performance. Ljung-Box test shows good effect and all the coefficients are significant.



(Figure 1-4.3 Squared residuals PACF/ACF plot of Canada)

For the CPI Canada, fitted a GARCH model as well. After comparing the AIC value of several models, the GARCH(3,3) model has the best performance.

Industrial Production Index

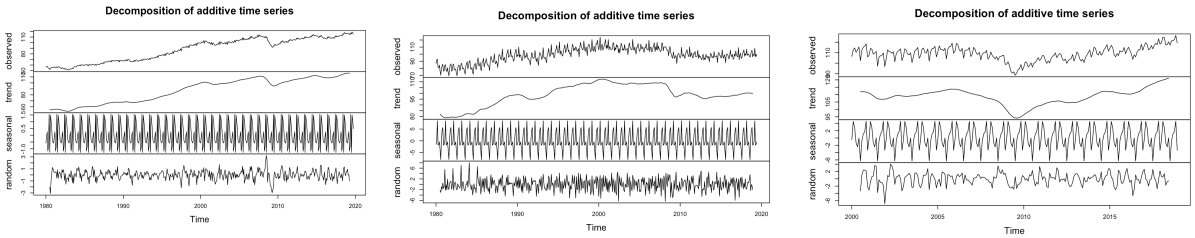


(Figure 2-1 IPI Time-series Data plot of Three Countries)

The plots shown above are the overview of the industrial production index of the United States, the United Kingdom and Canada. This index had an overall increasing trend with 2 significant decreasing terms. The first decreasing term is from 1980 to 1982. This is due to the stagflation in the early 1980s, the unemployment rate and inflation rate kept high during that recession time. The second decreasing term is from 2007 to 2010. From the year of 2007, the financial crisis brought a tremendous effect to the economic condition. This crisis was followed by a significant global economic downturn.

Decompose Analysis

To better analyze the component of this time series, a decomposition plot is provided. From the decomposition plot, the trend and seasonality are clearly observed.



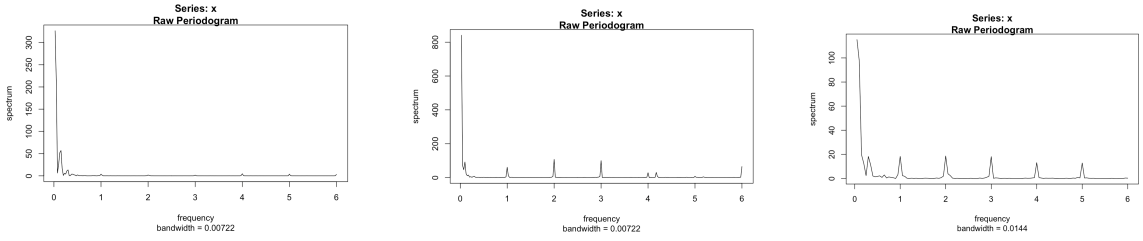
United States United Kingdom Canada

(Figure 2-2 Decompose plot of Three Countries)

Comparing between these three countries, the US has the strongest increasing trend, which means the US has the best macroeconomic condition and production power. Canada seems highly affected by the 2007 financial crisis and has a modest trend. Investors in the production industry may prefer investing in the US and the UK rather than Canada.

Spectral Analysis

The spectral analysis was performed to analyze the predominant frequency. From the results of this analysis, the predominant frequency for the United States and United Kingdom IPI is 0.025 and the cycle of this frequency is 40 months. This means every 40 months, there is a repeated pattern. The predominant frequency for Canada is 0.05 and the cycle of this frequency is 20 months. This means every 20 months, there is a repeated pattern.



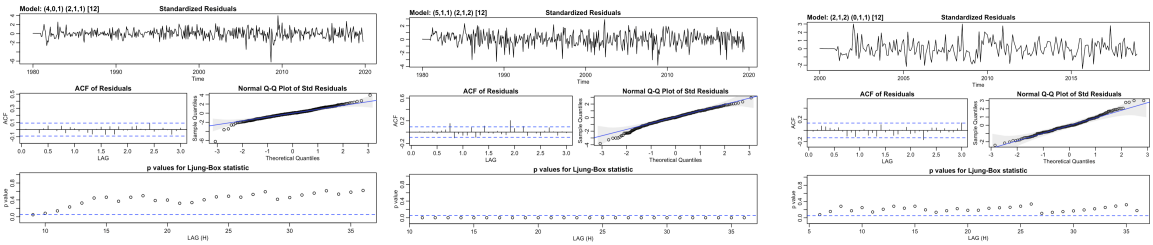
United States United Kingdom Canada

(Figure 2-3 Spectrum plot of Three Countries)

In practical meaning, the countries with a 40-month cycle will have more stable IPI status than the country with a 20-month cycle. Thus, the US and the UK have more stable macroeconomic conditions in industrial production. From the investment perspective, the US and the UK will have lower investment risks.

ARIMA Model

Fitting the arima model by hand and the auto.arima() function and were applied to fund the best fit ARIMA model. Then the diagnostic plots were created to compare the models. According to the results, the ARIMA(4,0,1)(2,1,1)[12] model, ARIMA(5,1,1)(2,1,2)[12] model and ARIMA(2,1,2)(0,1,1)[12] model are the best fits for IPI of the US, UK and Canada respectively, with the smallest AIC values and best residual acf results. From the diagnose plots, the model fits well: In the ACF plot, most of the lags are within the blue dashed lines which means the residuals are independent and close to white noise; The QQ plot indicates that the residuals are normally distributed.



United States United Kingdom Canada

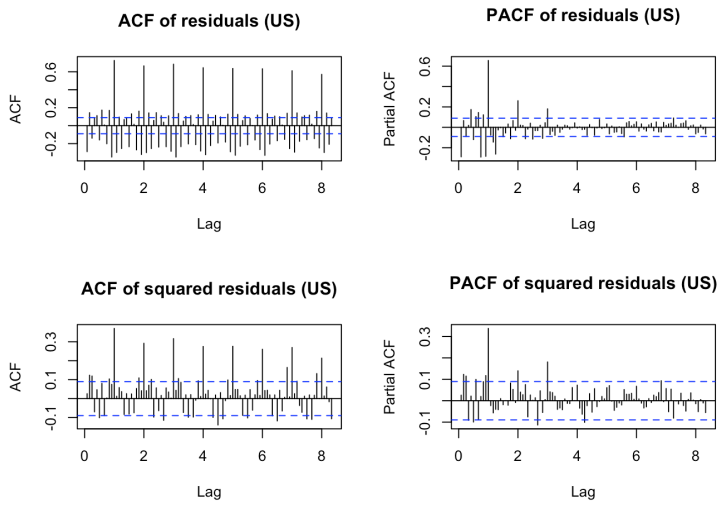
(Figure 2-4 Sarima model plot of Three Countries)

Compared with the 3 models, the US model has the best fit. This means that the US model may be the most accurate one. When using these models to predict the future trend, the US model will be the closest to the real situation.

ARCH/GARCH Model

To accurately forecast the variance of the index, the GARCH model and ARCH model were also applied to this dataset.

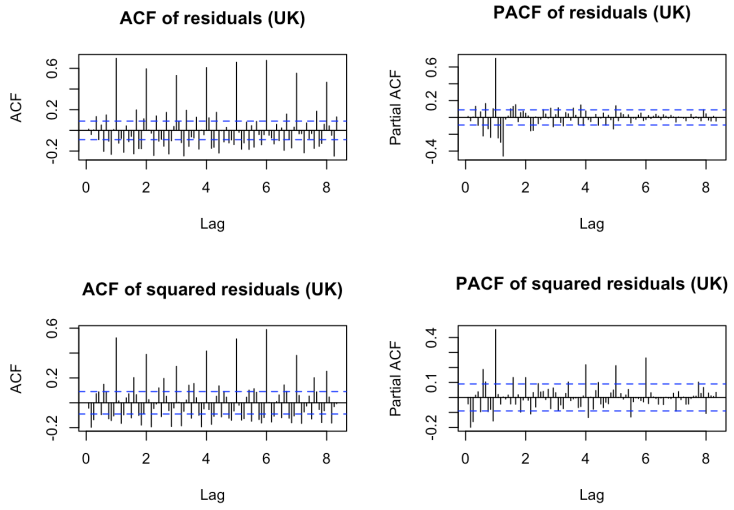
United States:



(Figure 2-5.1 ACF/PACF plot of the United States)

According to the ACF and PACF plot of residuals, a GARCH(3, 3) and ARCH(3) model were fitted. Compared to the AIC value of the two models, the ARCH(3) model was selected.

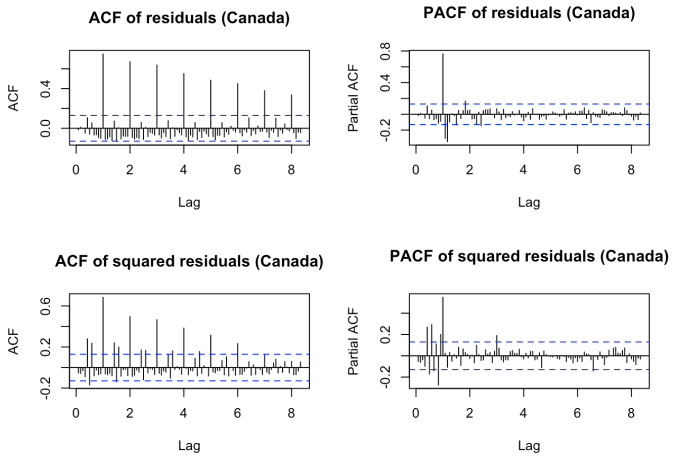
United Kingdom:



(Figure 2-5.2 ACF/PACF plot of the United Kingdom)

According to the ACF and PACF plot of residuals, a ARCH(7) and a GARCH(4,7) model were fitted. Compared to the AIC value of the two models, the ARCH(7) model was selected.

Canada:



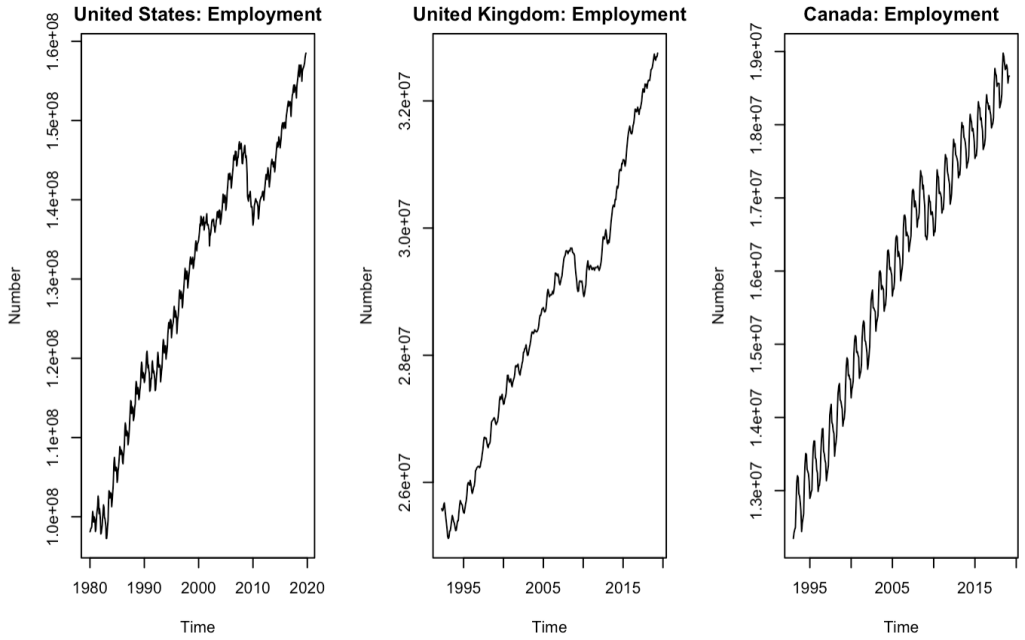
(Figure 2-5.3 ACF/PACF plot of Canada)

According to the ACF and PACF plot of residuals, a GARCH(6, 1) and a GARCH(6, 2) model were fitted. Compared to the AIC value of the two models, the GARCH(6, 2) model was selected.

However the GARCH model seems not a good fit for the IPI data. Although the relatively better models were selected by the smallest AIC values, most of the coefficients are not significant.

Number of Employment

The Original Data



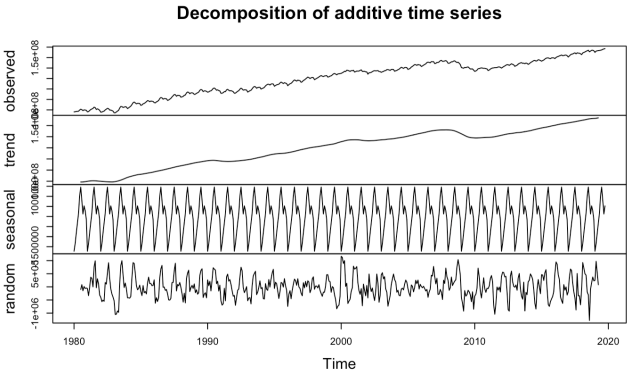
(Figure 3-1 # of Employment Time-series Data plot of Three Countries)

The plots shown above are the time series overview of the number of employment of the United States, the United Kingdom and Canada. By looking at the data, an increasing trend can be clearly observed. There is a decreasing pattern after 2008 in each of the three countries due to the financial crisis, which had a significant crash to the world's economy.

Decompose Analysis

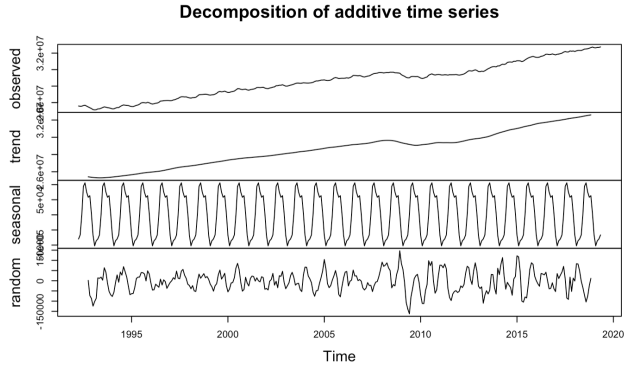
A decompose analysis can be helpful to further study the component of this time series data. The decompose plots below show a clear increasing trend. Besides, there is a obvious seasonal component in each of the three countries.

United States:



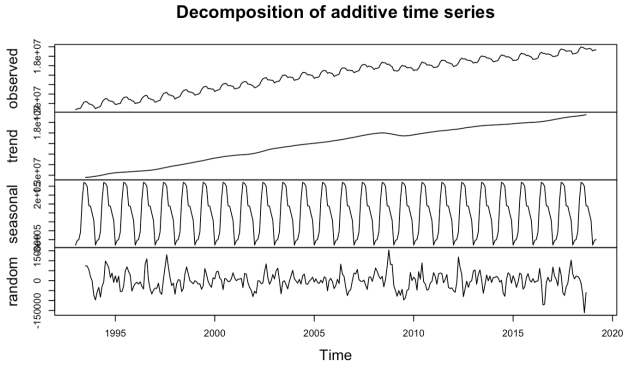
(Figure3-2.1 Decompose of the United States)

United Kingdom:



(Figure 3-2.2 Decompose of the United Kingdom)

Canada:



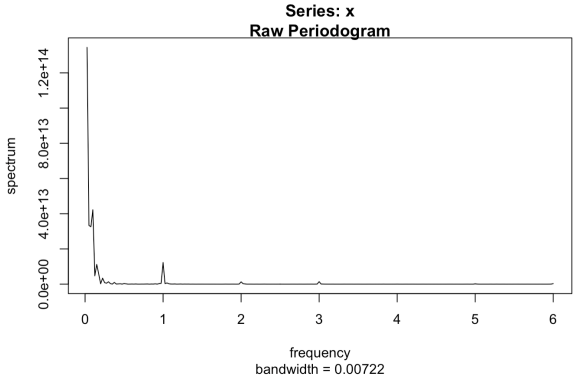
(Figure 3-2.3 Decompose of Canada)

Spectral Analysis

Spectral analysis was also performed on the data to study the predominant frequency. In the plot of the spectrum of the United States data, there are two obvious peaks

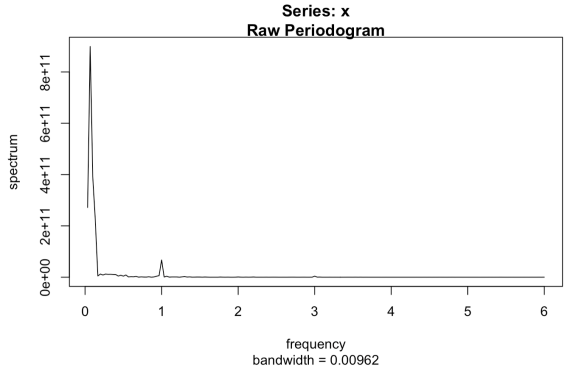
with strong power. The predominant frequency for the United States is 0.025 and it gives a cycle of 40 months. Every 40 months there is a repeated pattern. The predominant frequency for the United Kingdom is 0.066, which has a cycle of 15 months. There are two high peaks in the spectrum of Canada. The highest predominant frequency of Canada is at 0.0375, which gives a 26.7 month cycle. A larger repeat cycle means to have a more stable economy. The United States has the most stable economy among these three countries and the United Kingdom is the least stable one.

United States:



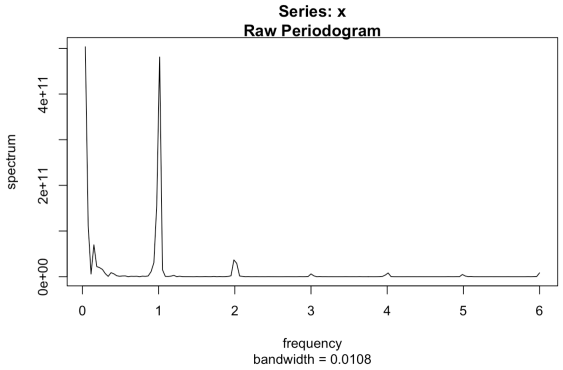
(Figure 3-3.1 Spectrum of the United States)

United Kingdom:



(Figure 3-3.2 Spectrum of the United Kingdom)

Canada:



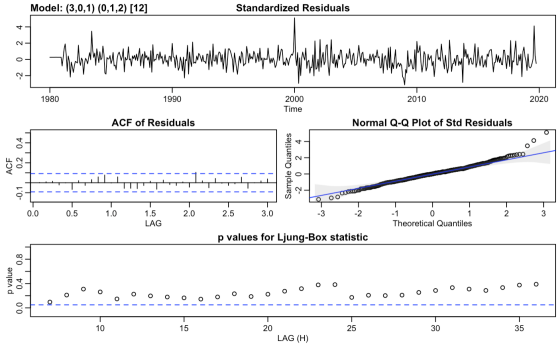
(Figure 3-3.3 Spectrum of Canada)

ARIMA Model

First order difference was made to all the three data to make them stationary. Arima models were fitted with different combinations of p and q by analyzing the acf and pacf of the first order difference and auto.arima was also used to generate the arima model. By comparing the AIC and BIC of the models, the best fitted model was selected to do residual diagnostic and prediction. The models selected were (3,0,1)(0,1,2)[12] for the United States, (2,1,3)(2,1,1)[12] for the United Kingdom and (0,1,0)(1,1,2)[12] for Canada.

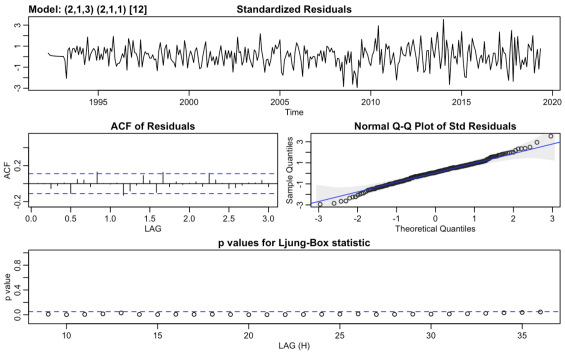
In the diagnose plots, almost all lags in the ACF of Residuals were within the blue dash line, meaning they are not correlated and like white noise. Almost all the dots in the QQ plot lies on the straight line and it tells us the residuals are normally distributed. The p value shows the model for the United States has a very good fit to the data and the models for the other two countries are not fitted so well.

United States:



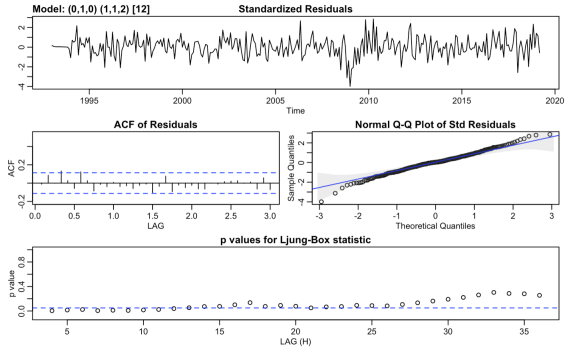
(Figure 3-4.1 sarima of the United States)

United Kingdom:



(Figure 3-4.2 sarima of the United Kingdom)

Canada:

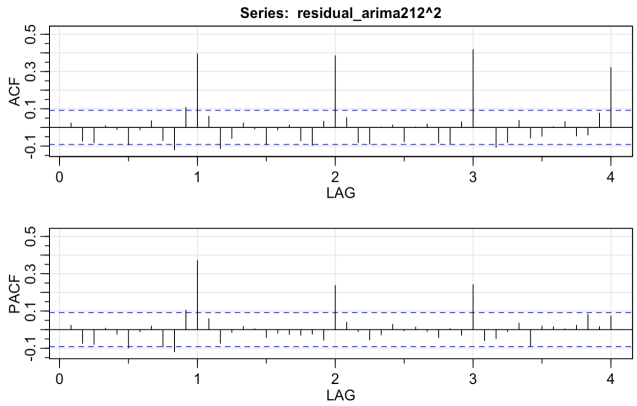


(Figure 3-4.3 sarima of Canada)

ARCH/GARCH Model

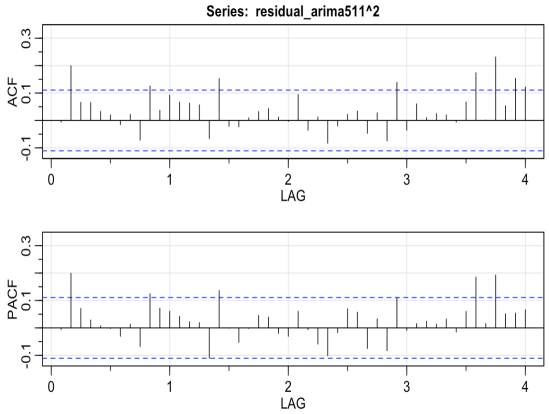
According to the information provided by the ACF and PACF of the squared residual, GARCH(3,3) and ARCH(3) were fitted to the United States data and ARCH(3) was selected by comparing their AIC. GARCH(1,1) and ARCH(1) were fitted to the United Kingdom data and GARCH(1,1) with smaller AIC was selected. GARCH(4,1) and ARCH(1) were fitted to the Canada data and GARCH(4,1) was selected

United States:



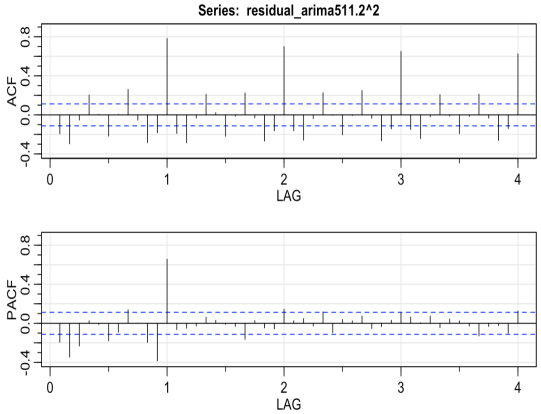
(Figure 3-5.1 ACF and PACF of squared residual of the United States)

United Kingdom:



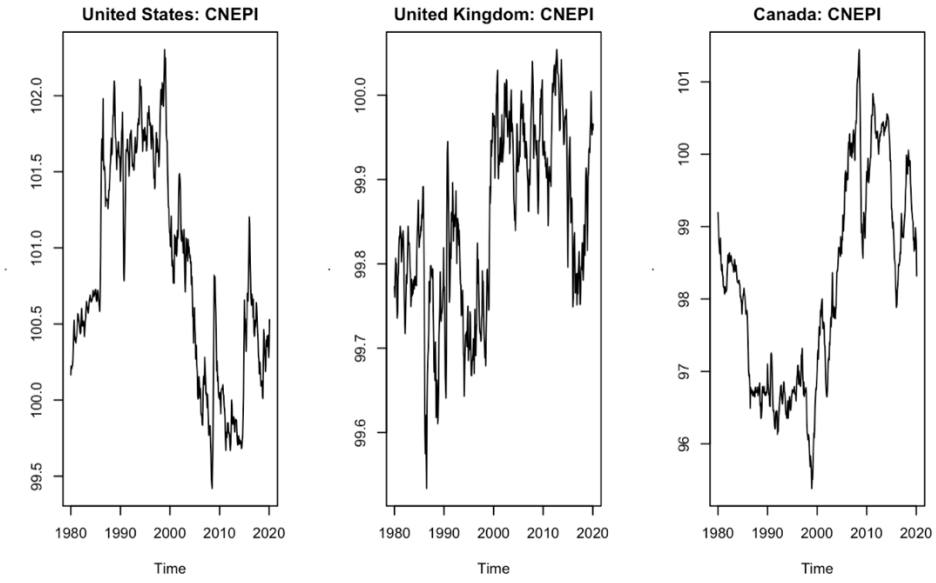
(Figure 3-5.2 ACF and PACF of squared residual of the United Kingdom)

Canada:



(Figure 3-5.3 ACF and PACF of squared residual of Canada)

Commodity Net Export Price Index

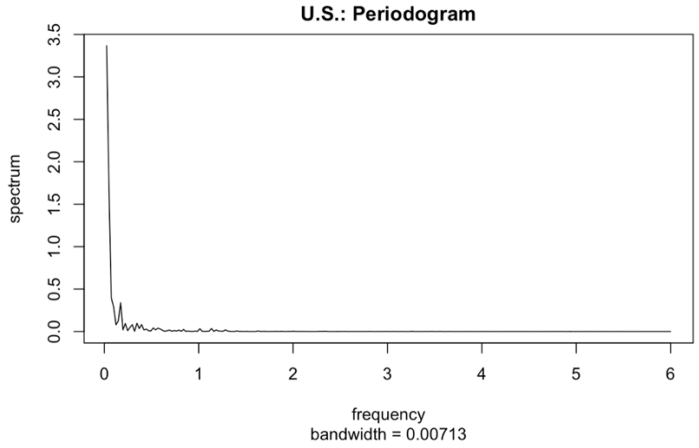


(Figure 4-1 CNEPI Time-series Data plot of Three Countries)

These pictures show monthly commodity net export price index for each country starting from January 1980 until February 2020. The left one is the U.S., which shows there was an increasing trend from 1980 to 2000, and there was a peak in 2000. Then it had a huge decreasing trend from 2000 to 2010. Around 2009, it had a valley of commodity net export price index. Then it increased again and decreased in 2018. Recently, it had an increasing trend. The middle one is the United Kingdom, which shows there was a valley of CNEPI in 1986, and then increased to more than 100 in 2001. Then it fluctuated from 2001 to 2010. Then it drops into a valley around 2017. Recently, it was in an increasing trend. The right one is Canada. Comparing with the other two graphs, it does not have that much fluctuations. The commodity net export price index dropped into a valley in 2009, and it increased to the peak around 2008. Recently, it was in a decreasing trend. By looking at these graphs, clearly, they are not stationary dataset. Therefore, in order to do further analysis, differencing and transformation should be done to make it stationary.

Spectral Analysis

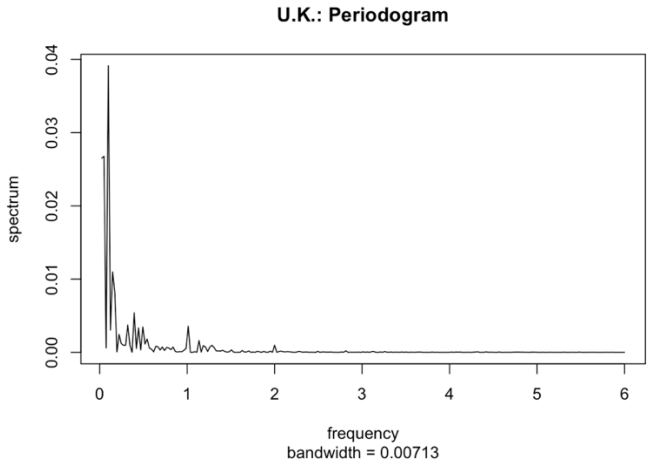
U.S.:



(Figure 4-2 U.S. Periodogram)

By creating the periodogram for the commodity net export price index in the U.S., it is easy to find that the predominant frequency is near 0.02. Therefore, the period of the cycle is 40.5 months. By calculating in R, the spectral density at predominant period is 3.37. The 95% confidence interval is [0.91, 132.97].

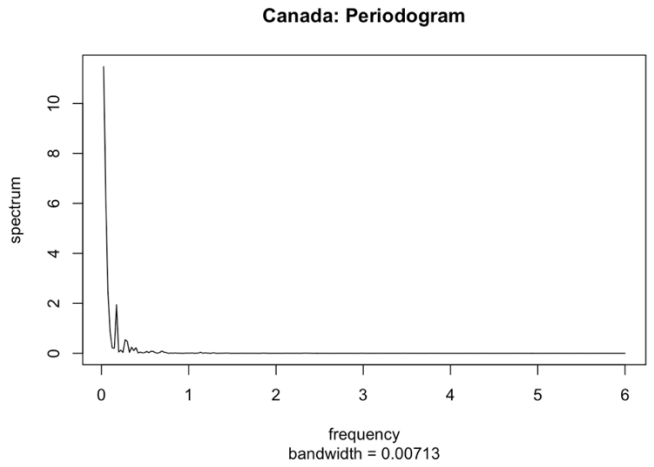
U.K.:



(Figure 4-3 U.K. Periodogram)

By creating the periodogram for the commodity net export price index in the U.K., it is easy to find that the predominant frequency is near 0.1. Therefore, the period of the cycle is 10 months. By calculating in R, the spectral density at predominant period is 0.04. The 95% confidence interval is [0.01, 1.55].

Canada:

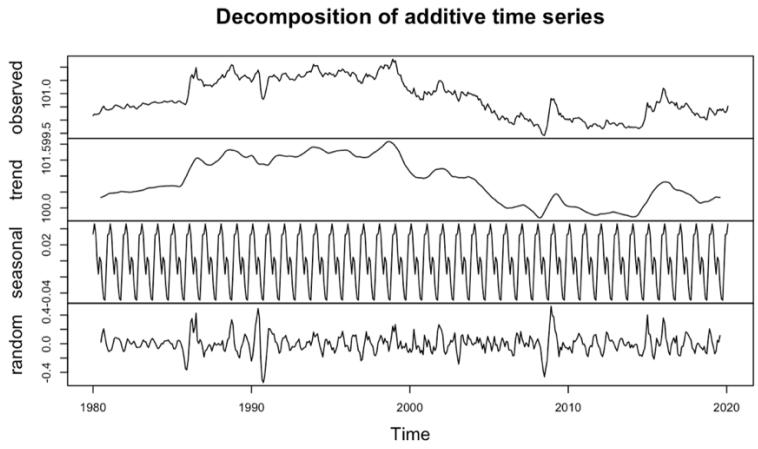


(Figure 4-4 Canada Periodogram)

By creating the periodogram for the commodity net export price index in Canada, it is easy to find that the predominant frequency is near 0.025. Therefore, the period of the cycle is 40.5 months. By calculating in R, the spectral density at predominant period is 11.47. The 95% confidence interval is [3.11, 453.16].

Decompose Analysis

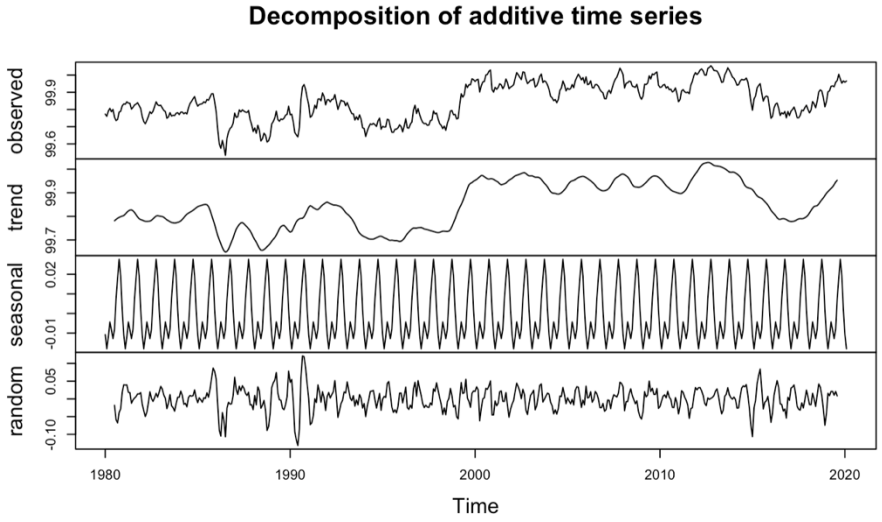
U.S.:



(Figure 4-5 U.S. Decomposition Time Series Plot)

Through decompose analysis, overall, the commodity net export price index in the U.S. has a negative trend, and some fluctuations.

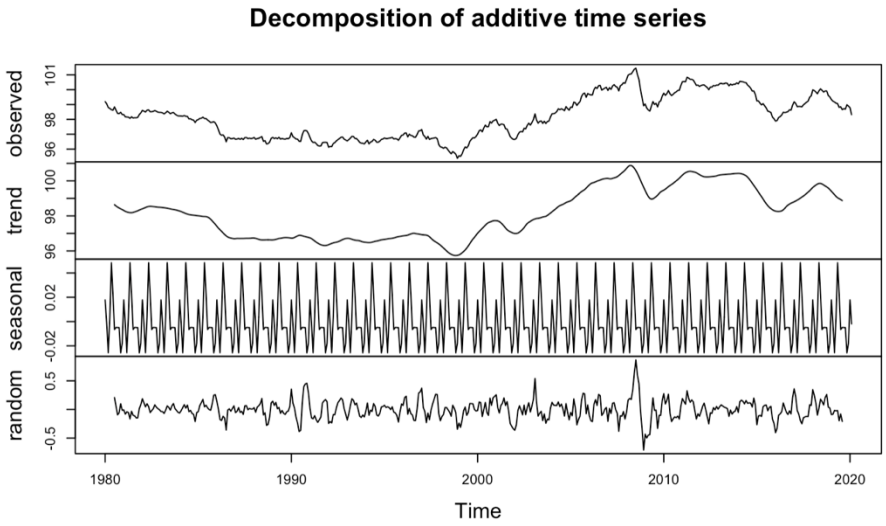
U.K.:



(Figure 4-6 U.K. Decomposition Time Series Plot)

Through decompose analysis, the commodity net export price index in the U.K. has an increasing trend.

Canada:



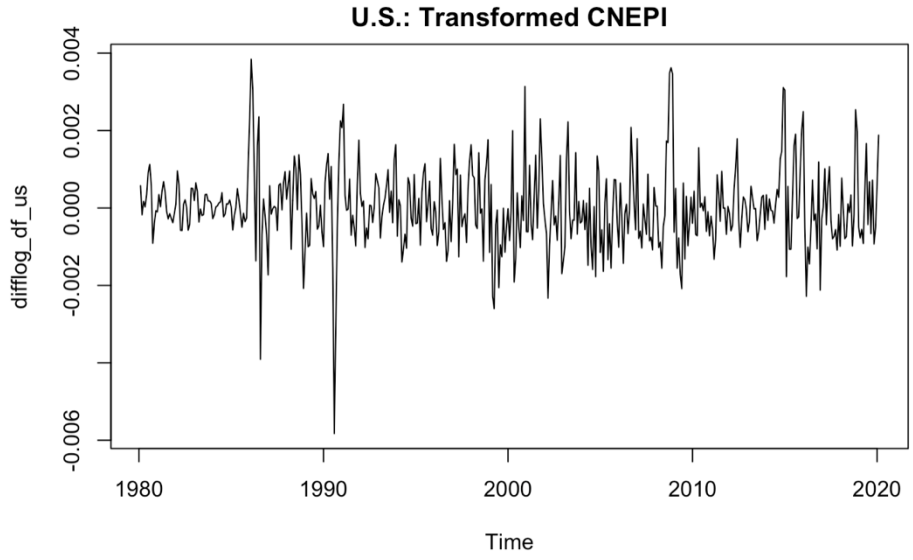
(Figure 4-7 Canada Decomposition Time Series Plot)

Through decompose analysis, overall, the commodity net export price index in Canada has a positive trend.

ARIMA Model

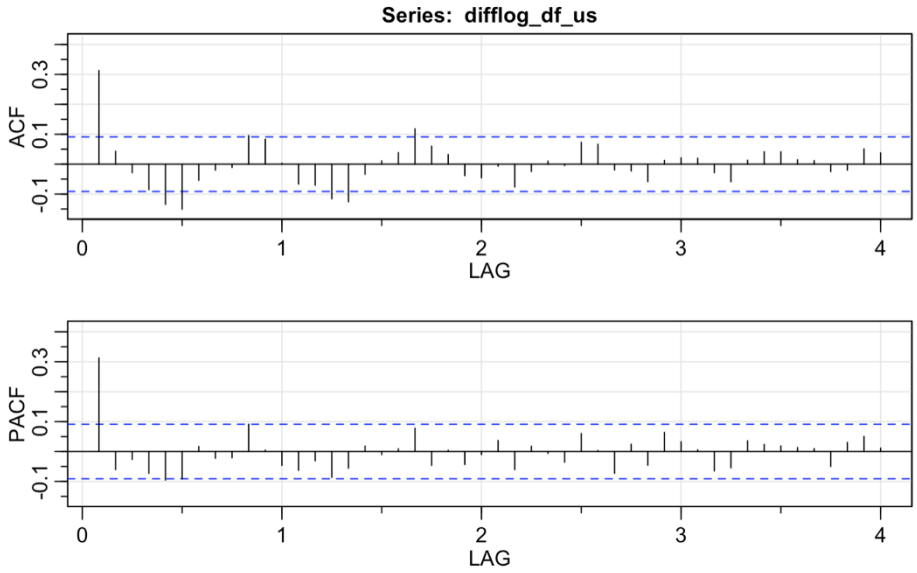
U.S.:

In order to make the dataset stationary, the first order difference and log transformation should be done. Then the transformed dataset looks like below.



(Figure 4-8 U.S. Transformed Time Series Plot)

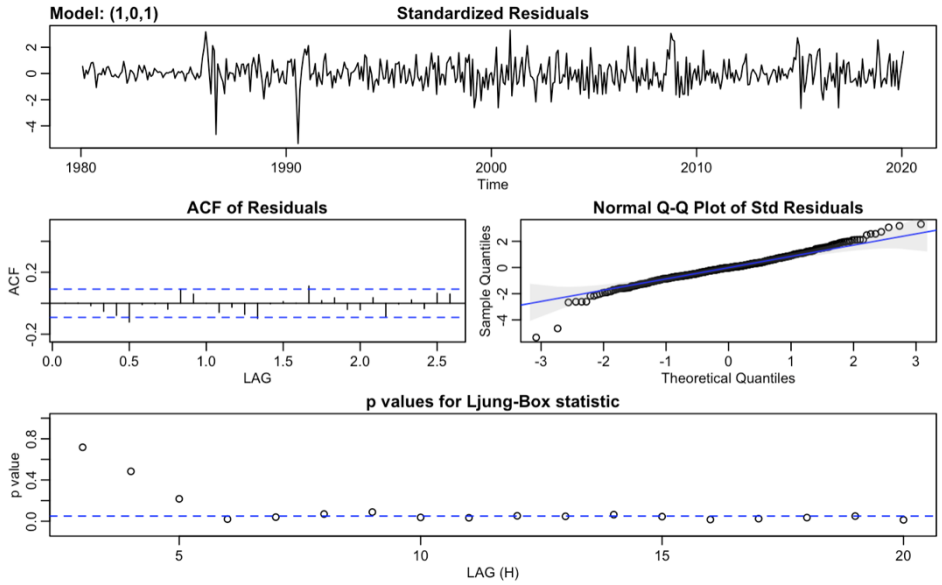
After transformation, it has constant variance and zero mean. But it still has some extreme values. Therefore, this should be weak stationary dataset.



(Figure 4-9 U.S. ACF and PACF of Transformed Time Series)

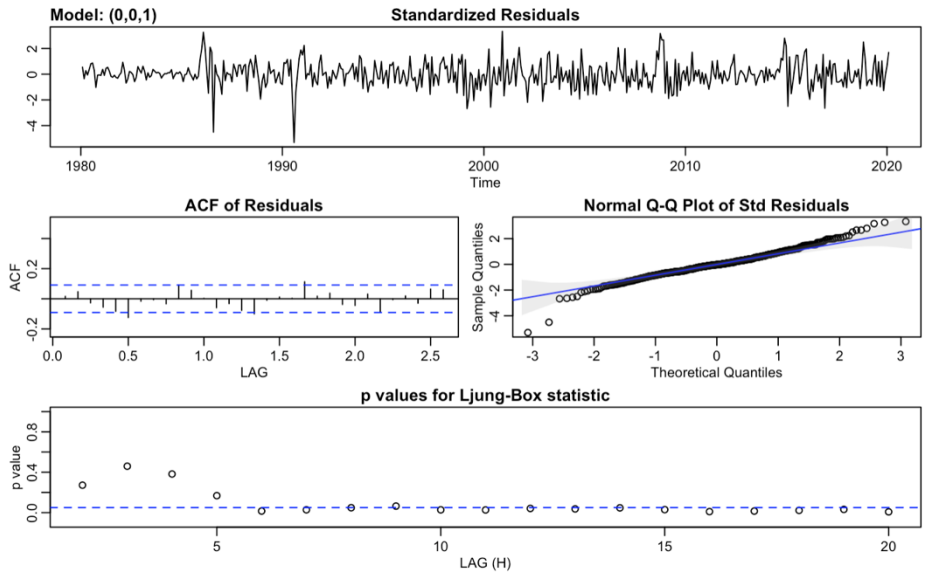
Based on the ACF and PACF above, it does not have significant seasonality component. And in ACF, lag 1 is significant; in PACF, lag 1 is significant as well.

Since first order difference has already taken, ARIMA(1,0,1) and ARIMA(0,0,1) should be tried to perform.



(Figure 4-10 U.S. residual diagnostics of ARIMA(1,0,1))

From the residual diagnostics of ARIMA(1,0,1), it is easy to find that the residuals series is more like a white noise, even if it has some outliers. Almost no lag is significant in ACF of residuals plot. Near all points are lined on the blue line except the head and the tail in the Normal Q-Q plot. Some points are on the blue dashed line in p values for Ljung-Box statistic. Therefore, this model is good, but from the Ljung-Box statistic plot, there may be some improvement.



(Figure 4-11 U.S. residual diagnostics of ARIMA(0,0,1))

The residual diagnostics of ARIMA(0,0,1) is similar to the ARIMA(1,0,1). Therefore, in order to decide which one is better, AIC and BIC should be considered.

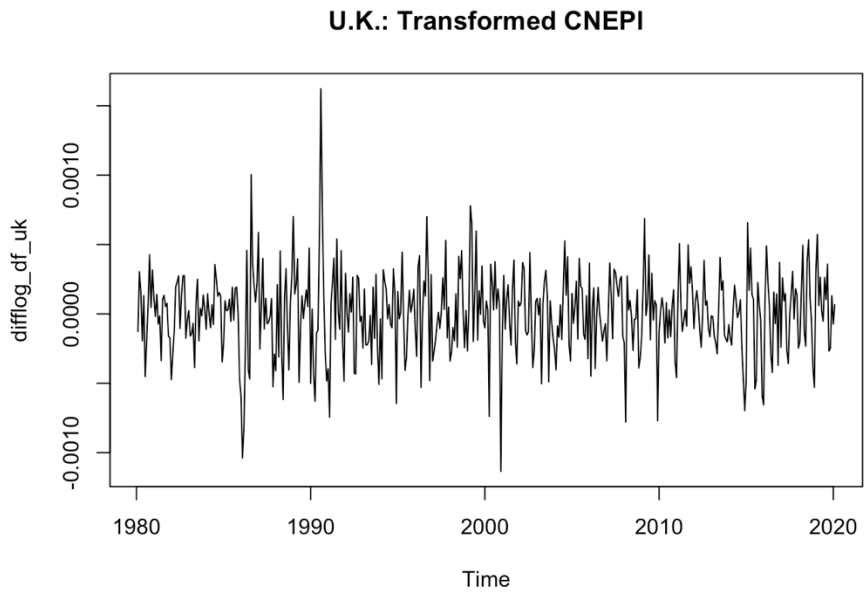
Model <fctr>	AIC <dbl>	BIC <dbl>	AICc <dbl>
ARIMA(1,0,1)	-10.98978	-10.95506	-10.98968
ARIMA(0,0,1)	-10.99121	-10.96517	-10.99116

(Figure 4-12 Comparison between ARIMA(1,0,1) and ARIMA(0,0,1))

Based on this table, since AIC, BIC, and AICc of ARIMA(0,0,1) are all smaller than ARIMA(1,0,1). Therefore, ARIMA(0,0,1) is slightly better than ARIMA(1,0,1), and it should be used to do prediction in the future. Auto.arima function can also be applied to do selection, and it proves that ARIMA(0,0,1) is the best model.

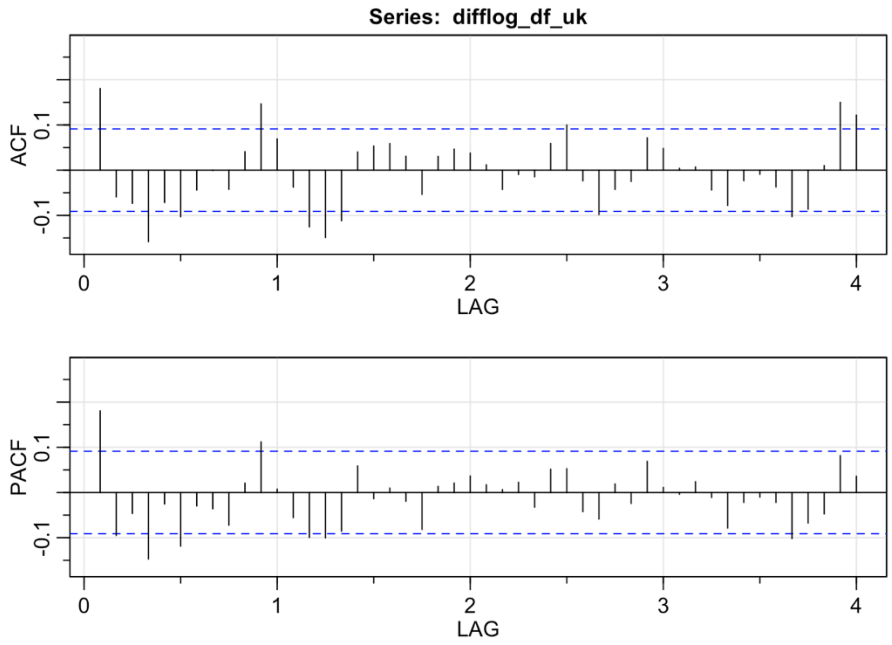
U.K.:

By taking the first order difference and log transformation, the dataset became weak stationary. The plot is below. It has constant variance and zero mean, but there are some extreme values.



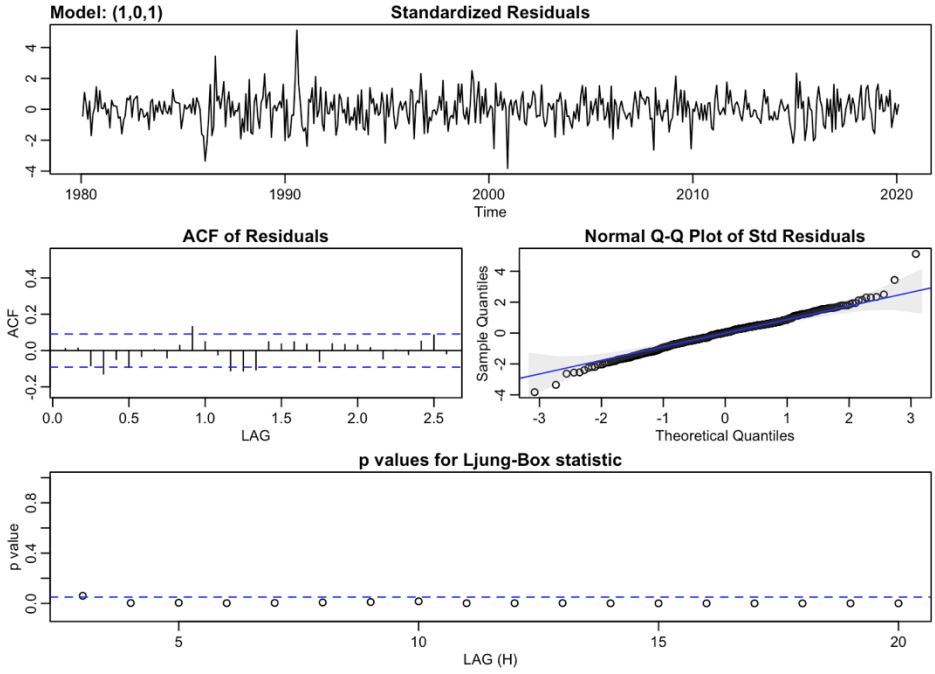
(Figure 4-13 U.K. Transformed Time Series Plot)

To decide the coefficients in ARIMA model, ACF and PACF should be analyzed. Below is the plot.



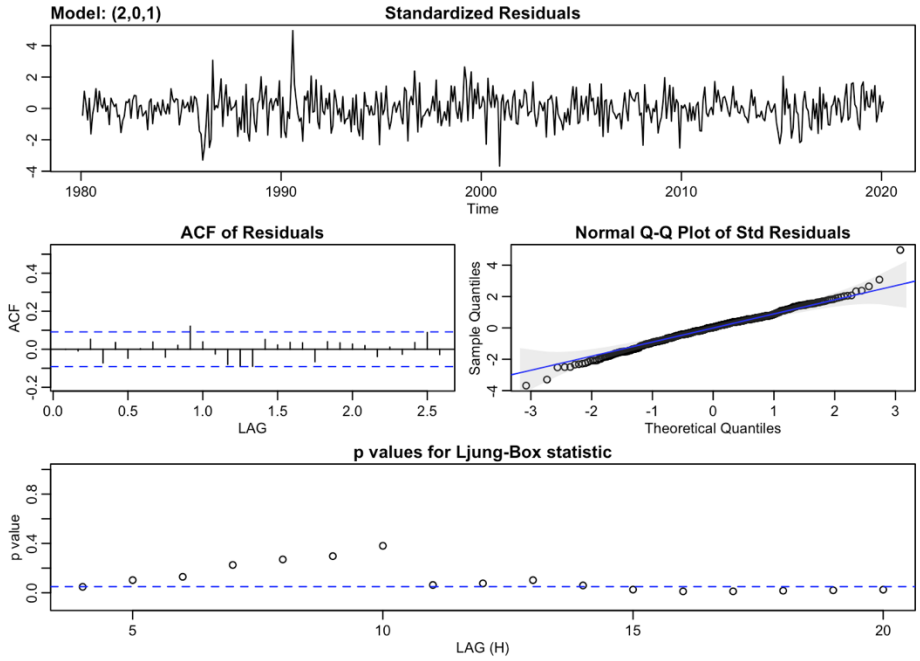
(Figure 4-14 U.K. ACF and PACF of Transformed Time Series)

This plot shows lag 1 is significant in ACF plot. And in PACF plot, lag 1 and lag 2 are significant. Therefore, ARIMA(2,0,1) and ARIMA(1,0,1) should be tested.



(Figure 4-15 Residual Diagnostics of ARIMA(1,0,1))

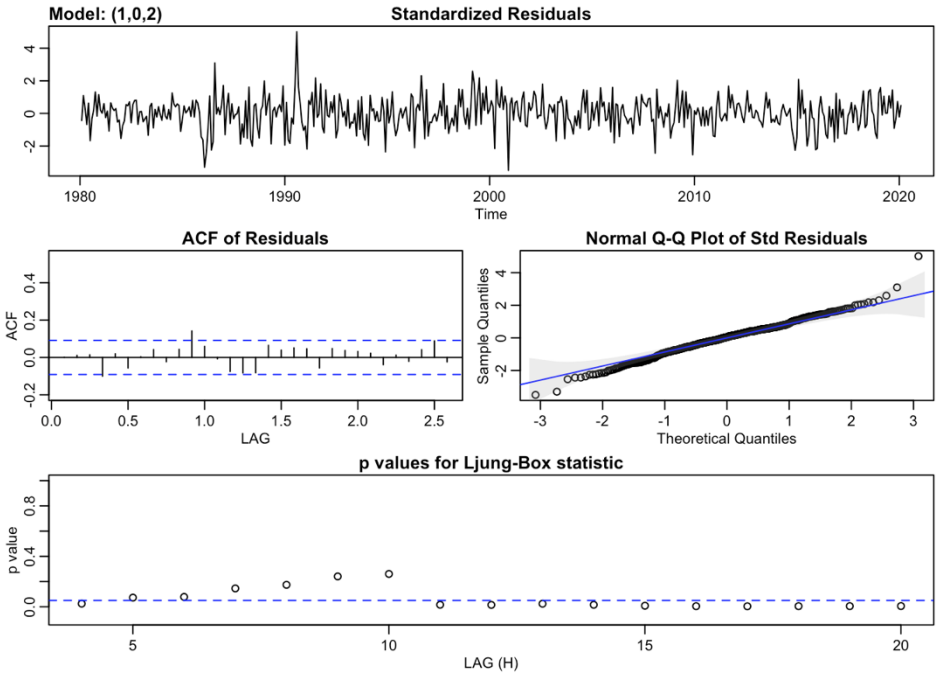
According to the residual diagnostics of ARIMA(1,0,1), it shows that the residuals series is more like a white noise, even if it has some outliers. It has the constant variance and zero mean value. Several lags are slightly significant in ACF of residuals plot. Near all points are lined on the blue line except the head and the tail in the Normal Q-Q plot. All points are on or below the blue dashed line in p values for Ljung-Box statistic. Therefore, this model is not good.



(Figure 4-16 Residual Diagnostics of ARIMA(2,0,1))

The residual diagnostics of ARIMA(2,0,1) is above. From the standardized residuals plot, it shows that the residuals series is more like a white noise, even if it has some outliers. Almost no lag is significant in ACF of residuals plot. Near all points are lined on the blue line except the head and the tail in the Normal Q-Q plot. Some points are above the blue dashed line in p values for Ljung-Box statistic, but some are on or below it. Therefore, this model is better than ARIMA(1,0,1).

Auto.arima is a function which can be applied to do model selection. It gives ARIMA(1,0,2) model. Therefore, residual diagnostics of ARIMA(1,0,2) should be considered.



(Figure 4-17 Residual Diagnostics of ARIMA(1,0,2))

It is very similar with the model ARIMA(2,0,1). Therefore, the model with the smallest AIC, BIC, and AICc should be selected.

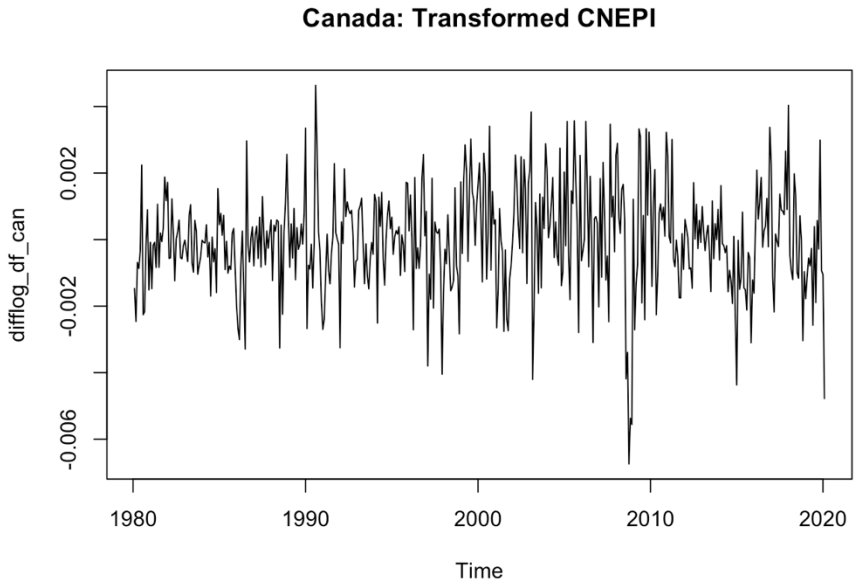
```
##          Model       AIC       BIC      AICc
## 1 ARIMA(1,0,1) -13.41393 -13.37920 -13.41383
## 2 ARIMA(2,0,1) -13.43946 -13.39606 -13.43929
## 3 ARIMA(1,0,2) -13.43820 -13.39479 -13.43802
```

(Figure 4-18 Comparison among ARIMA(1,0,1), ARIMA(2,0,1) and ARIMA(1,0,2))

Based on this table, ARIMA(2,0,1) should be selected, since it has the smallest. AIC, BIC, and AICc. And this model is going to be used in the prediction.

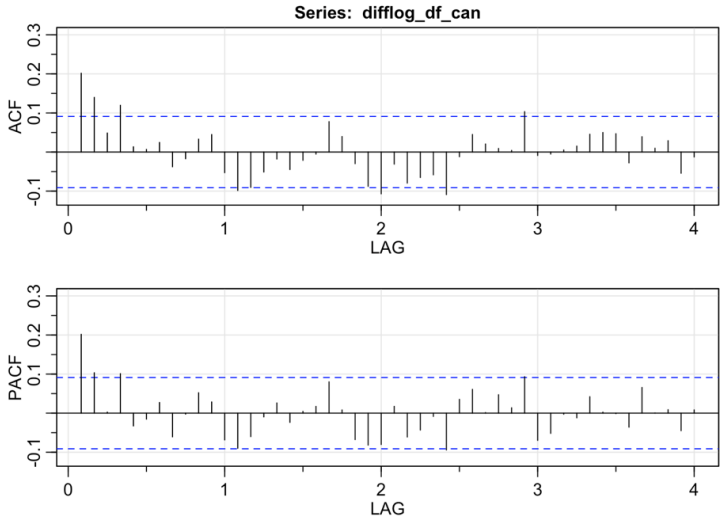
Canada:

In order to make the dataset stationary, the first order difference and log transformation should be done. Then the transformed dataset looks like below.



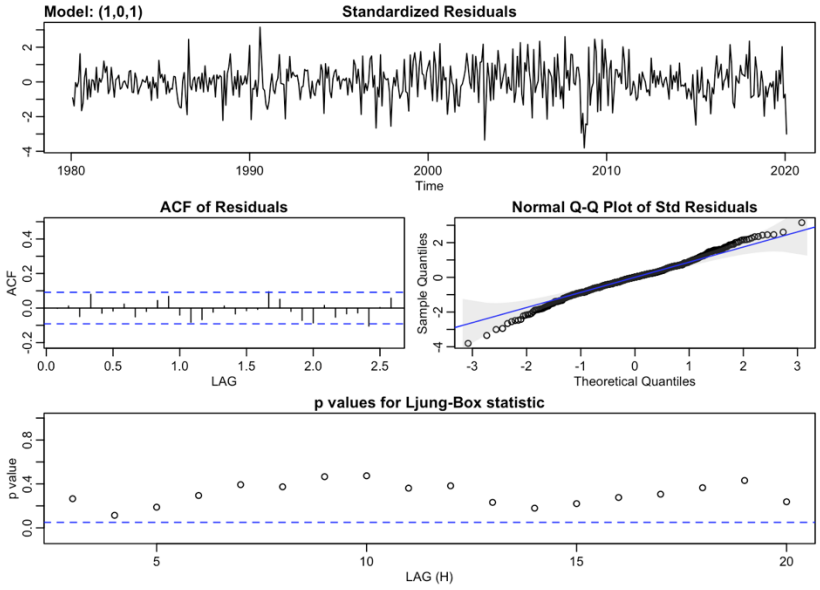
(Figure 4-19 Canada Transformed Time Series Plot)

After transformation, it has constant variance and zero mean. But it still has some extreme values. Therefore, this should be weak stationary dataset.



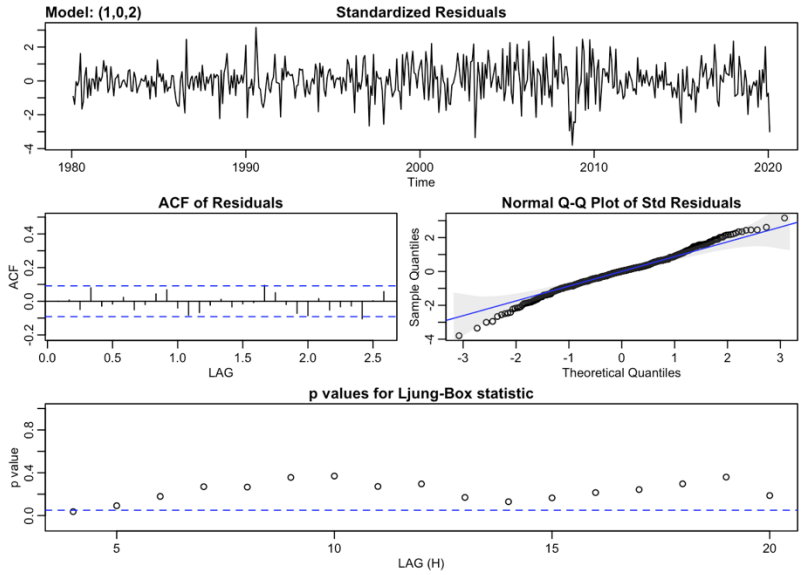
(Figure 4-20 Canada ACF and PACF of Transformed Time Series)

Based on the ACF and PACF above, it does not have significant seasonality component. And in ACF, lag 1, lag 2, and lag 4 are significant; in PACF, lag 1 is significant. Since first order difference has already taken, and to be conservative, ARIMA(1,0,1) and ARIMA(1,0,2) should be tried to perform.



(Figure 4-21 Residual Diagnostics of ARIMA(1,0,1))

From the residual diagnostics of ARIMA(1,0,1), it is easy to find that the residuals series is more like a white noise, even if it has some outliers. Almost no lag is significant in ACF of residuals plot. Near all points are lined on the blue line except the head and the tail in the Normal Q-Q plot. All points are above the blue dashed line in p values for Ljung-Box statistic. Therefore, this model is good.



(Figure 4-22 Residual Diagnostics of ARIMA(1,0,2))

The residual diagnostics of ARIMA(1,0,2) is similar to the ARIMA(1,0,1), except that in the p values for Ljung-Box statistic plot, ARIMA (1,0,2) has a point on the blue dashed line. Therefore, the ARIMA(1,0,1) is better than ARIMA(1,0,2). Additionally, AIC, BIC, and AICc can also be used to do selection.

```
##          Model       AIC       BIC      AICc
## 1 ARIMA(1,0,1) -10.10802 -10.07330 -10.10792
## 2 ARIMA(1,0,2) -10.10389 -10.06048 -10.10371
```

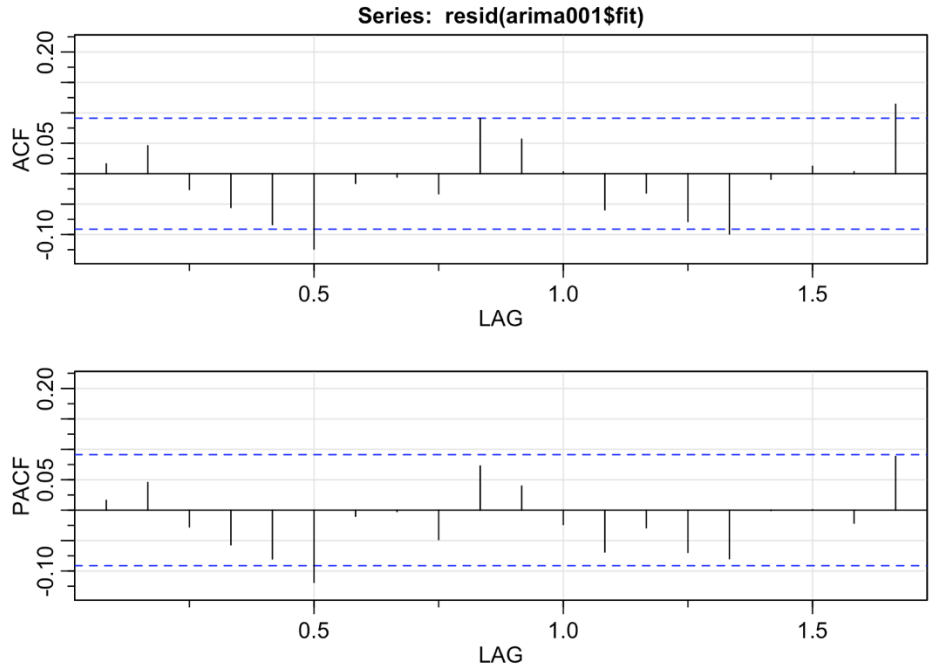
(Figure 4-23 Comparison between ARIMA(1,0,1) and ARIMA(1,0,2))

Based on this table, since AIC, BIC, and AICc of ARIMA(1,0,1) are all smaller than ARIMA(1,0,2). Therefore, ARIMA(1,0,1) should be used to do prediction in the future. Auto.arima function can also be applied to do selection, and it also proves that ARIMA(1,0,1) is the best model.

ARCH/GARCH Model

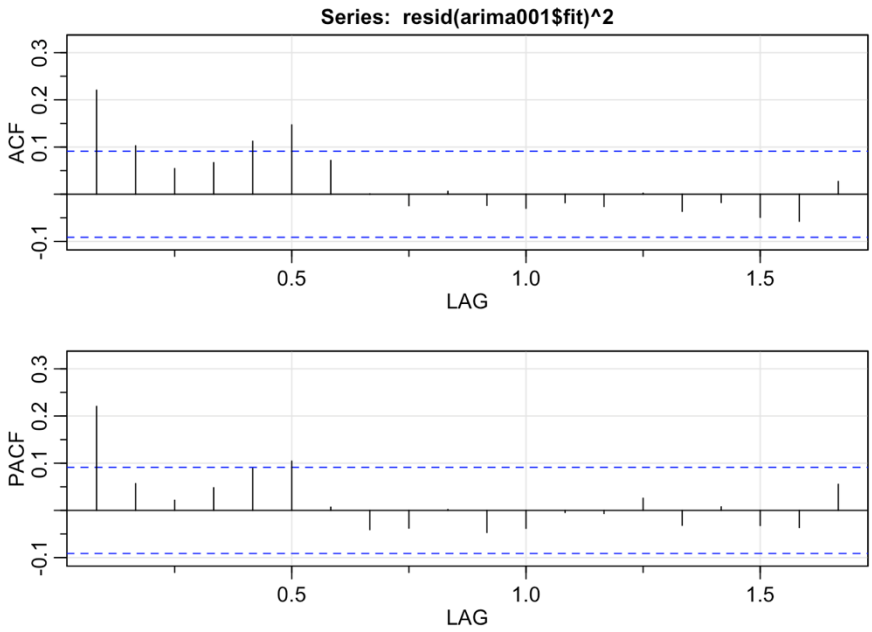
U.S.:

To improve the ARIMA model prediction accuracy, residuals may be able to be fitted by ARCH or GARCH model. The plot below is the ACF and PACF graph of residuals in ARIMA(0,0,1) model.



(Figure 4-24 U.S. ACF and PACF of Residuals)

The residuals series here is more like a white noise. The below plot is the ACF and PACF of residuals squared.



(Figure 4-25 U.S. ACF and PACF of Residuals Squared)

Based on this plot, GARCH(1,1) and GARCH(1,0) should be tried to fit the data combining with the ARIMA(0,0,1) model. Comparing the AIC and BIC between these two models, GARCH(1,1) with the smaller AIC and BIC should be selected. Therefore, the final model should be the combination of ARIMA(0,0,1) and GARCH(1,1).

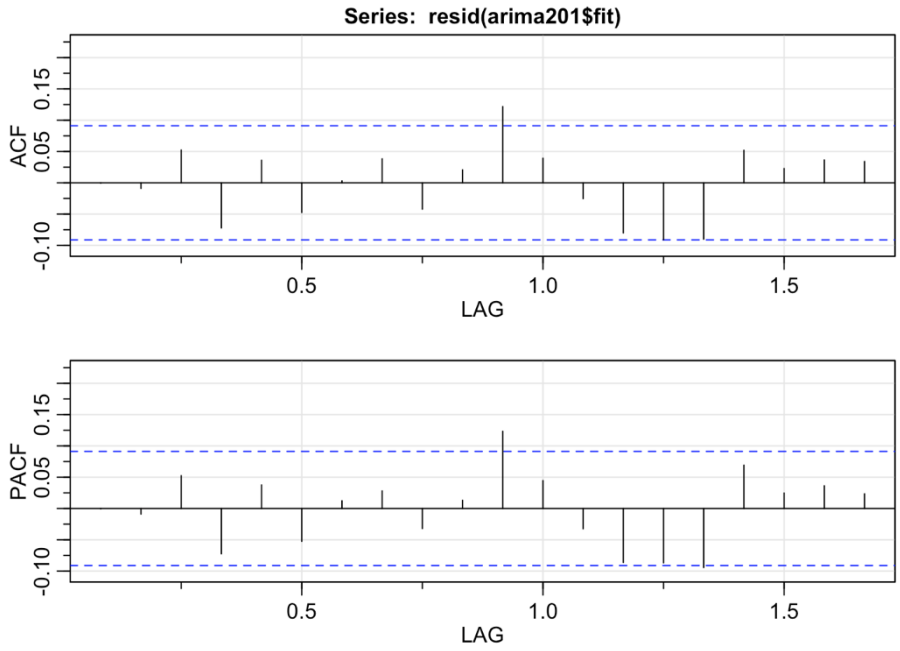
```
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(0, 1) + garch(1, 1), data = difflog_df_us)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(1, 1)
## <environment: 0x7fad01cdc740>
##  [data = difflog_df_us]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu         ma1       omega      alpha1       beta1
## 0.00271444  0.31102138  0.00030395  0.31141965  0.71037512
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0027144   0.0039512    0.687   0.4921
## ma1    0.3110214   0.0481527    6.459 1.05e-10 ***
## omega  0.0003040   0.0001669    1.821   0.0686 .
## alpha1 0.3114197   0.0587411    5.302 1.15e-07 ***
## beta1  0.7103751   0.0483309   14.698  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(Figure 4-26 U.S. Garch Model and ARIMA Result)

Based on this summary table, if the p-value is. 0.1, then almost all coefficients are significant. Therefore, this model is good to do forecasting.

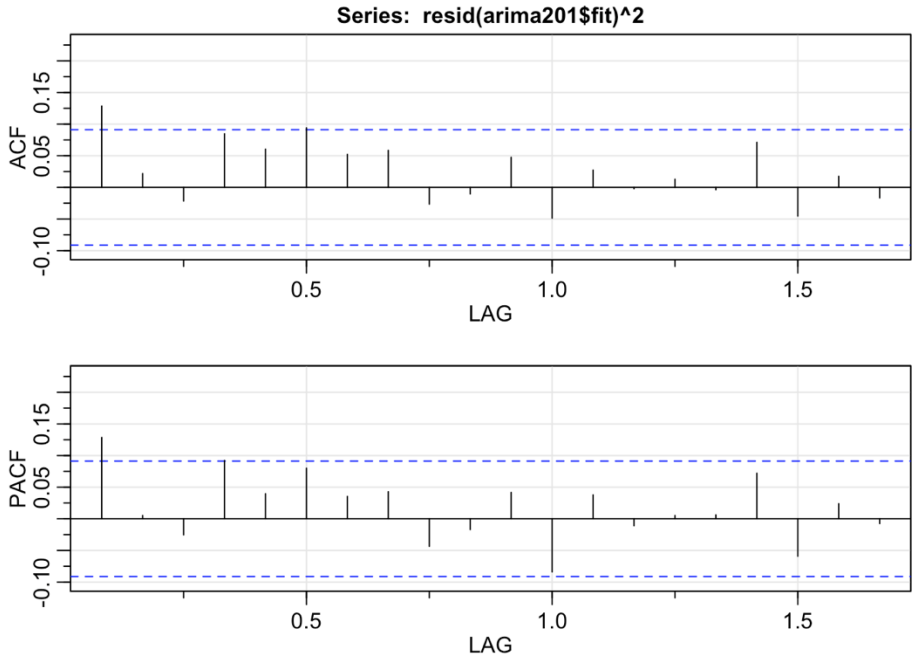
U.K.:

To fit the GARCH model, ACF and PACF of the residuals in ARIMA(2,0,1) and residuals squared should be analyzed.



(Figure 4-27 U.K. ACF and PACF of Residuals)

This is the ACF and PACF of residuals. It shows the residuals is more like the white noise. This is the prerequisite for doing GARCH model.



(Figure 4-28 U.K. ACF and PACF of Residuals Squared)

This is the ACF and PACF of the residuals squared. It seems like the lag 1 is both significant in ACF and PACF, and ACF is similar with PACF. Therefore, to be conservation, GARCH(1,0) should be fitted.

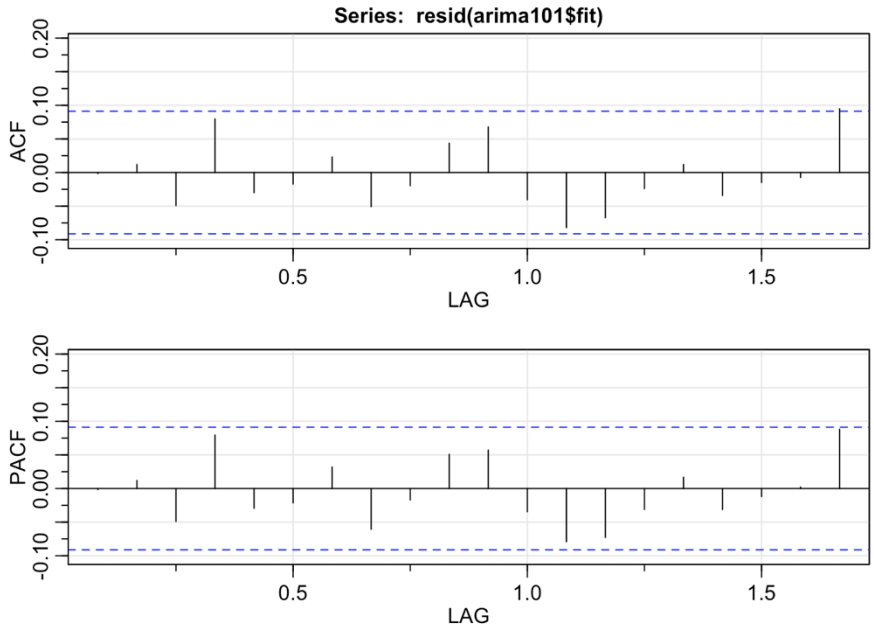
```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(2, 1) + garch(1, 0), data = difflog_df_uk)
##
## Mean and Variance Equation:
##  data ~ arma(2, 1) + garch(1, 0)
## <environment: 0x7fd9c92192f0>
##  [data = difflog_df_uk]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ma1        omega
##  6.0080e-07   9.9156e-01  -2.2727e-01  -8.5806e-01   6.9469e-08
##      alpha1
##  1.7600e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      6.008e-07   1.853e-06    0.324  0.74571
## ar1     9.916e-01   6.843e-02   14.491  < 2e-16 ***
## ar2    -2.273e-01   4.830e-02   -4.706 2.53e-06 ***
## ma1    -8.581e-01   5.626e-02  -15.253  < 2e-16 ***
## omega   6.947e-08   5.960e-09   11.655  < 2e-16 ***
## alpha1  1.760e-01   6.596e-02    2.668  0.00763 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

(Figure 4-29 U.K. Garch Model and ARIMA Result)

Based on the summary table, if the p-value is set to be 0.05, all coefficients except the mu are significant. Therefore, the combination model of ARIMA(2,0,1) and GARCH(1,0) is good.

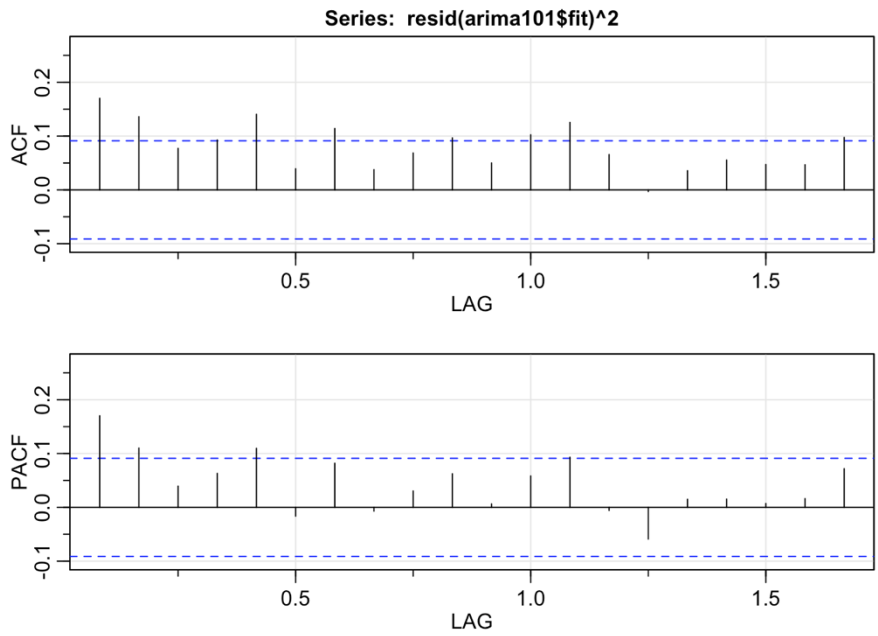
Canada:

To improve the ARIMA model prediction accuracy, residuals may be able to be fitted by ARCH or GARCH model. The plot below is the ACF and PACF graph of residuals in ARIMA(1,0,1) model.



(Figure 4-30 Canada ACF and PACF of Residuals)

The residuals series here is more like a white noise, since no lag is significant here. The below plot is the ACF and PACF of residuals squared.



(Figure 4-31 Canada ACF and PACF of Residuals. Squared)

Based on this plot, GARCH(1,1) and GARCH(1,2) should be tried to fit the data combining with the ARIMA(0,0,1) model. Comparing the AIC and BIC between these two models, GARCH(1,1) with the smaller AIC and BIC should be selected. Therefore, the final model should be the combination of ARIMA(1,0,1) and GARCH(1,1).

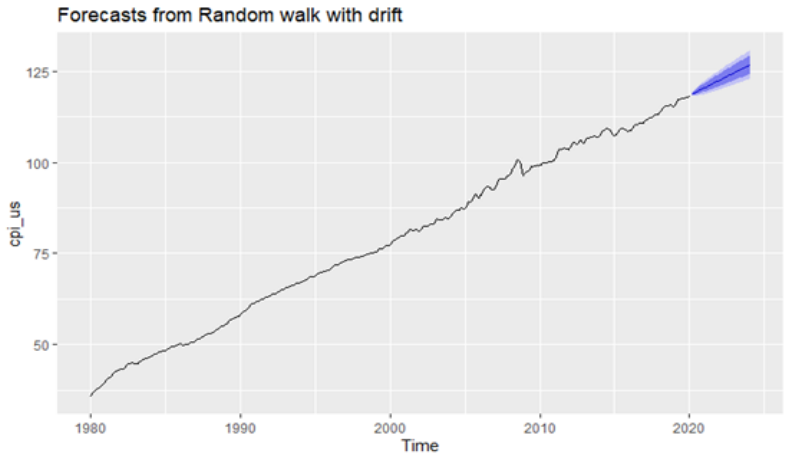
```
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 1) + garch(1, 1), data = difflog_df_can)
##
## Mean and Variance Equation:
##  data ~ arma(1, 1) + garch(1, 1)
## <environment: 0x7fd9ca78b068>
##  [data = difflog_df_can]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1        omega       alpha1
## -1.4606e-05   6.6631e-01  -5.2577e-01   8.3917e-08   8.9993e-02
##       beta1
##  8.7822e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu     -1.461e-05   3.082e-05   -0.474 0.635555
## ar1     6.663e-01   1.639e-01    4.065  4.8e-05 ***
## ma1    -5.258e-01   1.919e-01   -2.740 0.006145 **
## omega   8.392e-08   4.461e-08    1.881 0.059929 .
## alpha1  8.999e-02   2.649e-02    3.397 0.000681 ***
## beta1   8.782e-01   3.349e-02   26.227  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

(Figure 4-32 Canada Garch Model and ARIMA Result)

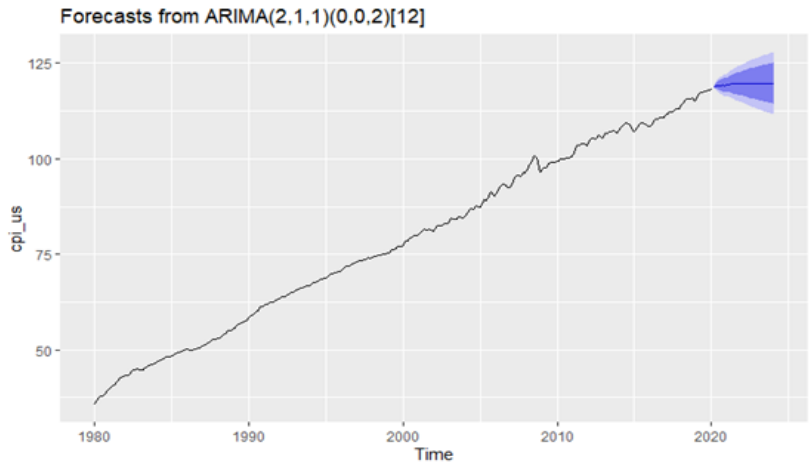
Based on this summary table, if the p-value is. 0.1, then all coefficients except the mu are significant. Therefore, this model is good to do forecasting.

Results

CPI

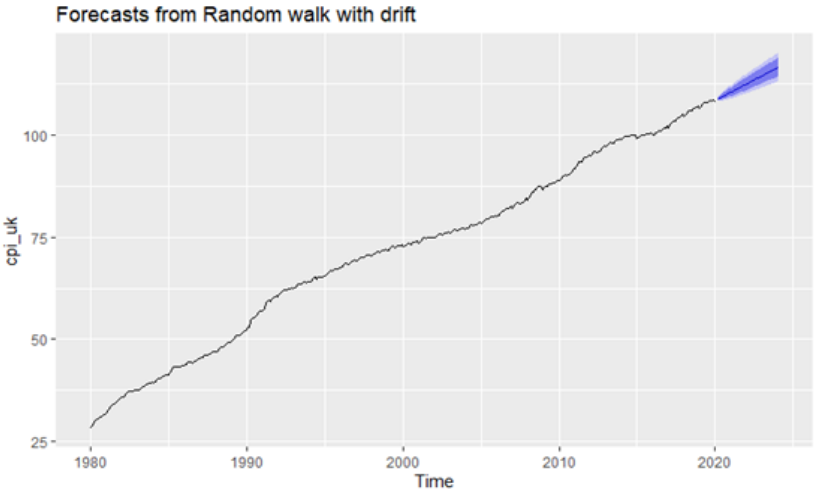


(Figure 3-1.1 Drift Predict of United States CPI)

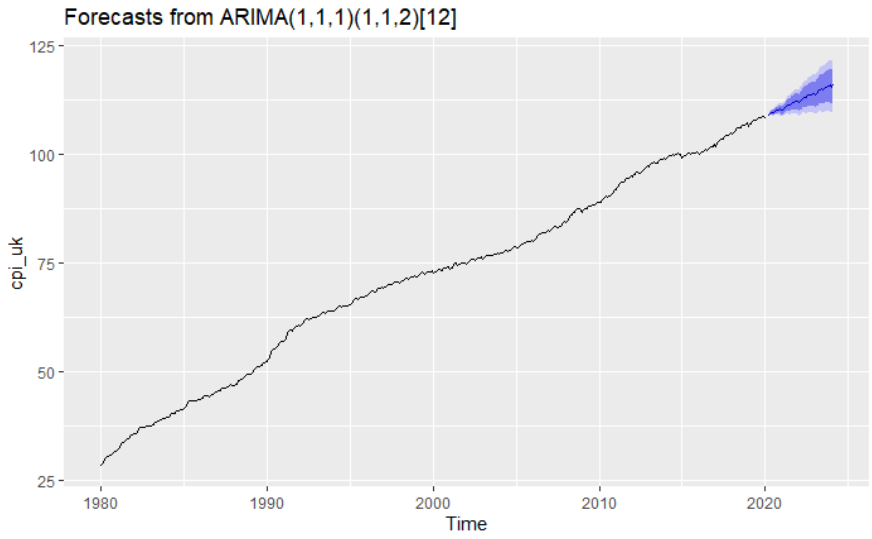


(Figure 3-1.2 Arima Predict of United States CPI)

Figure 3-1.1 and 3-1.2 shows two forecast models to predict the future 4 years' CPI value for the United States. The drift model shows it will have a steady increasing trend while the Arima model told us the future CPI will be approaching stable. The Arima model has a smaller error and wide confidence interval, so the prediction from the Arima model may be more accurate.

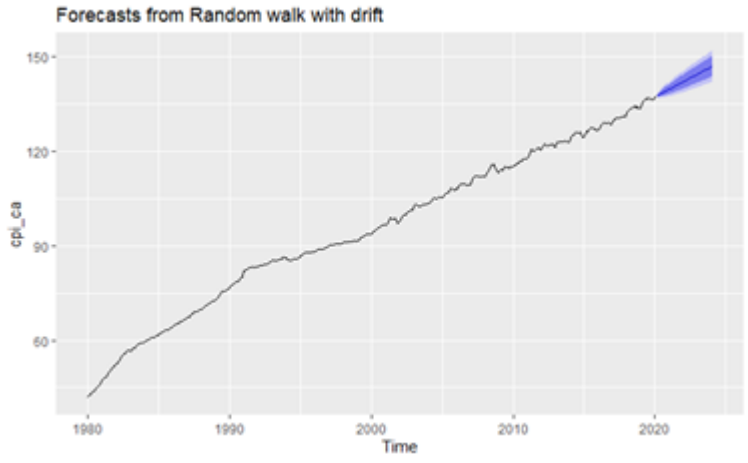


(Figure 3-2.1 Drift Predict of United Kingdom CPI)

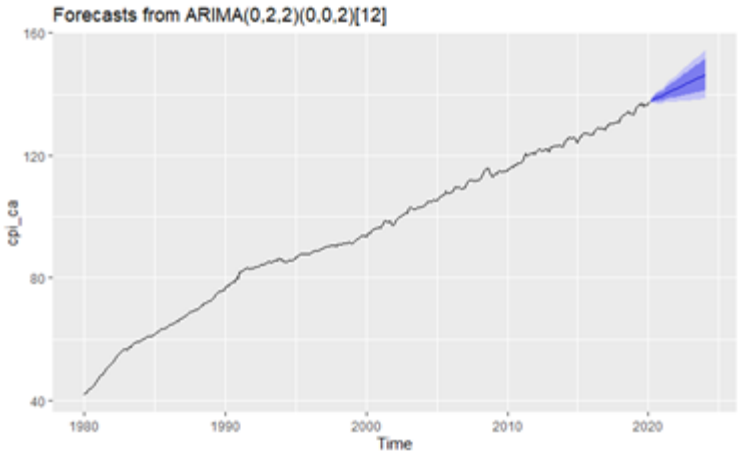


(Figure 3-2.2 Arima Predict of United Kingdom CPI)

Also, Using Drift model and the Arima model ,two ways to predict the future trend for 4 years (48 months) for the CPI of the United Kingdom. The seasonal Arima model has a wider confidence interval and has a smaller model error, too. The trend of the United Kingdom CPI seems to have a seasonal vibration. It has an overall increasing trend but will experience up and down often.



(Figure 3-3.1 Drift Predict of Canada CPI)



(Figure 3-3.2 Arima Predict of Canada CPI)

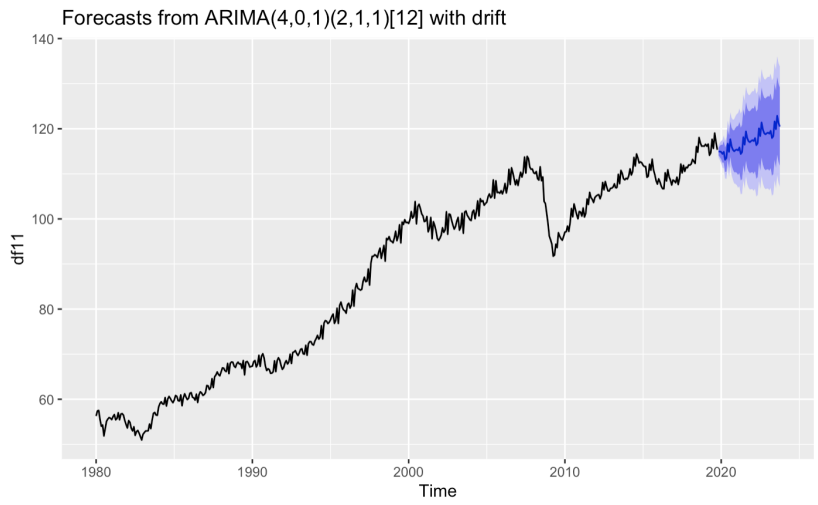
Using the Drift model and Arima model two ways predict the future trend of the CPI in Canada for 4 years (48 months). Both the models predict the CPI will have a steady increasing trend. The seasonal Arima model has a wider confidence interval and has a smaller model error. It has the best increasing trend among the three countries.

Industrial Production Index

Comparing the historical Industrial production Index trend of three countries, Canada was affected to the largest extent. The frequency of Canada IPI is higher than the other countries. This indicates that Canada has a shorter fluctuation period.

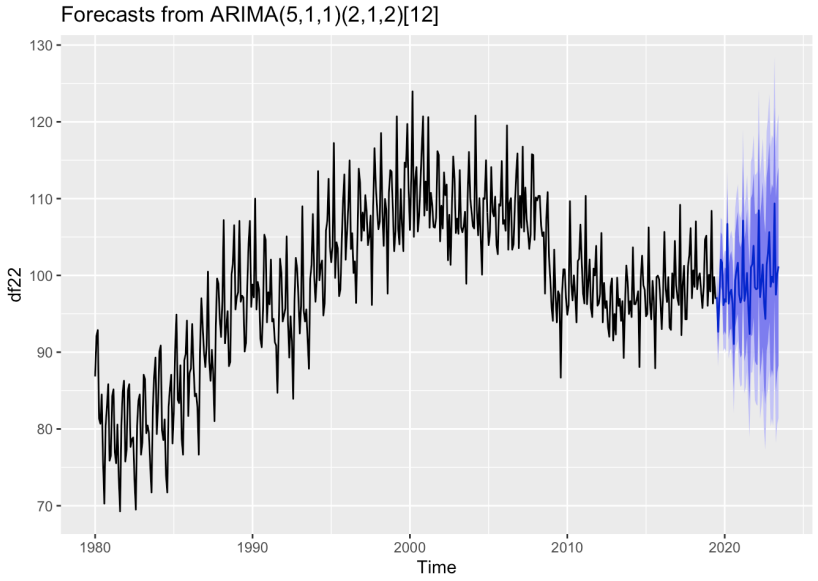
By applying this ARIMA model, the industrial production index of the future 48 months was predicted. Based on the historical data, the predicted IPI of the future 48 months will have an increasing trend with slight fluctuation. Among the three countries, the United States and Canada will have higher growth rates in IPI while the United Kingdom will have a relatively slow growth rate.

United States:



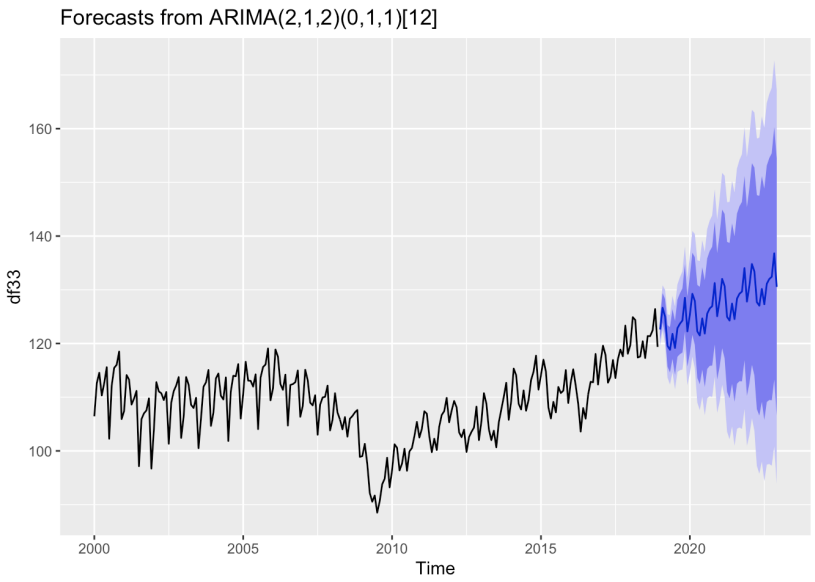
(Figure 4-1.1 Arima Predict of United States IPI)

United Kingdom:



(Figure 4-1.2 Arima Predict of United Kingdom IPI)

Canada:



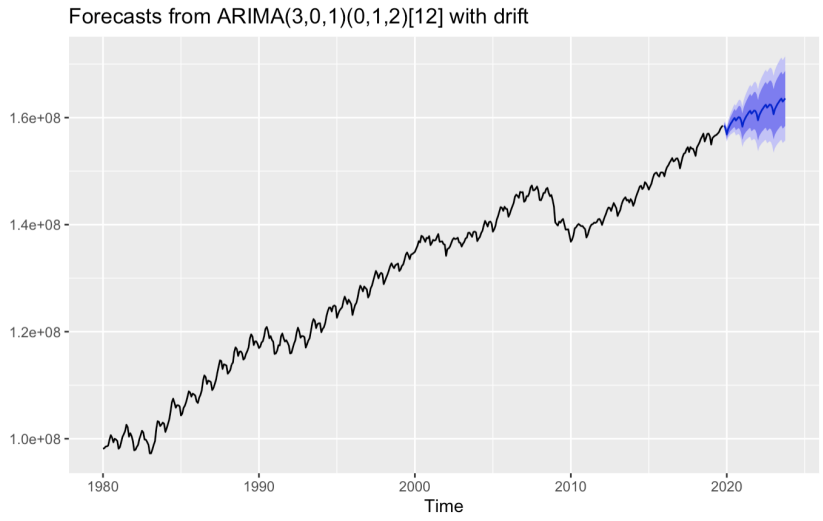
(Figure 4-1.3 Arima Predict of Canada IPI)

The GARCH model seems not a good fit for the IPI data. Although the relatively better models were selected by the smallest AIC values, most of the coefficients are not significant.

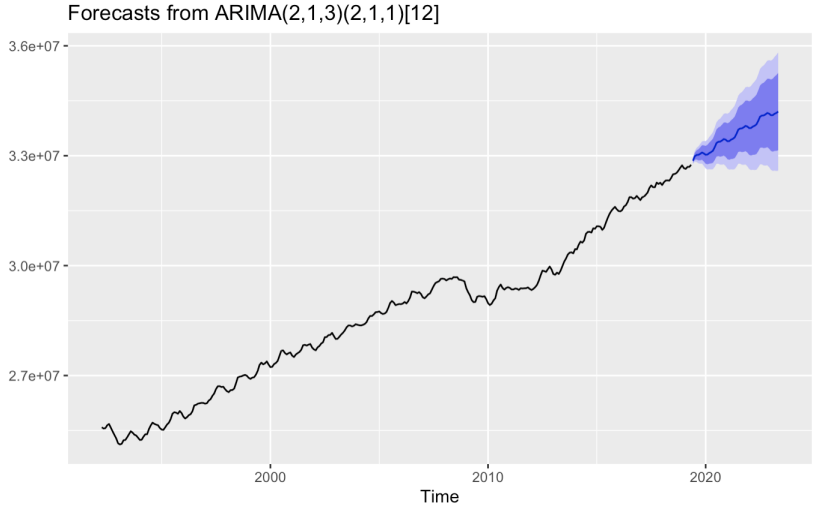
Number of Employment

Below are the prediction plots created by fitting the best Arima model selected. The number of employment of the future 48 months was predicted based on the historical data. All of the three predictions have an increasing trend. The prediction for the US growth rate has a smaller range of confidence interval which means to be more stable while the UK has a higher growth rate. There is a recurring seasonal component in the historical data of Canada and the prediction fitted really well. Its predicting confidence interval.

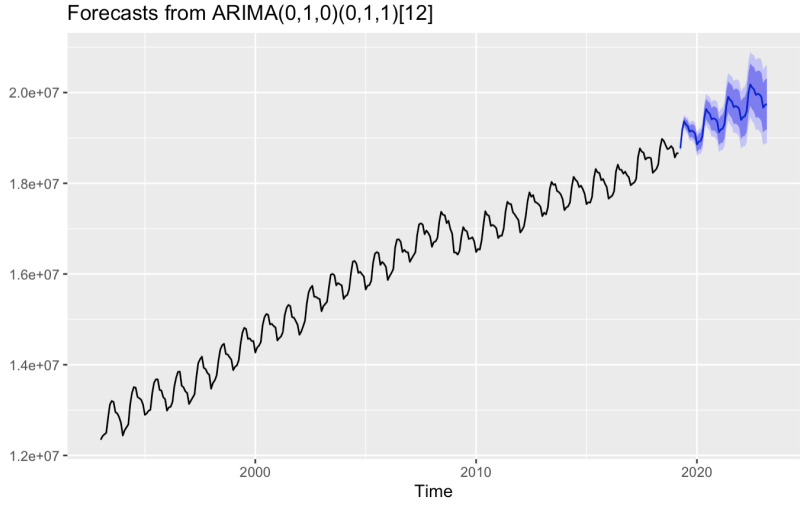
United States:



United Kingdom:



Canada:

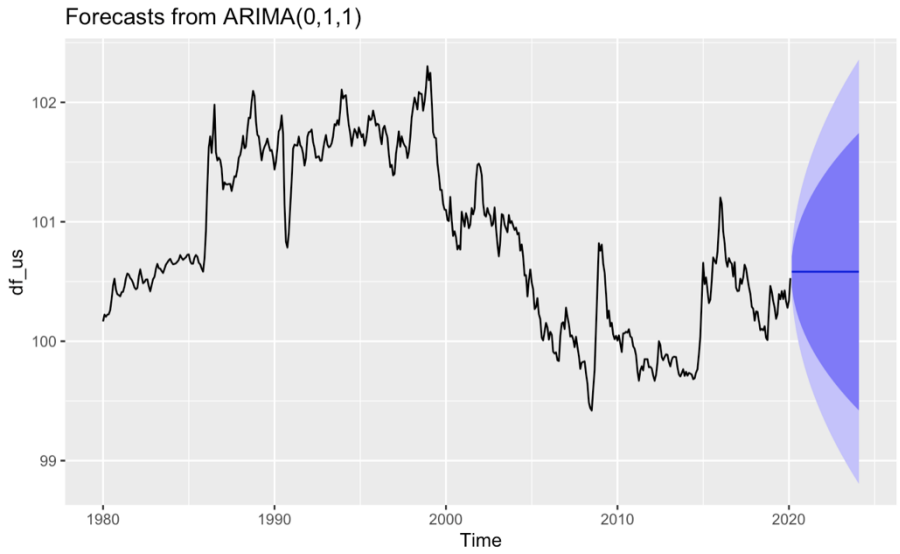


Commodity Net Export Price Index

In spectral analysis, the U.K. has the shortest cycle period which is 10 months; and the U.S. and Canada have the very similar cycle period. This is true, because by looking at the data plot, the U.K. has more fluctuations than the other two countries. In decompose analysis, the U.K. and Canada both have positive trend, but the. U.S. has a negative trend overall.

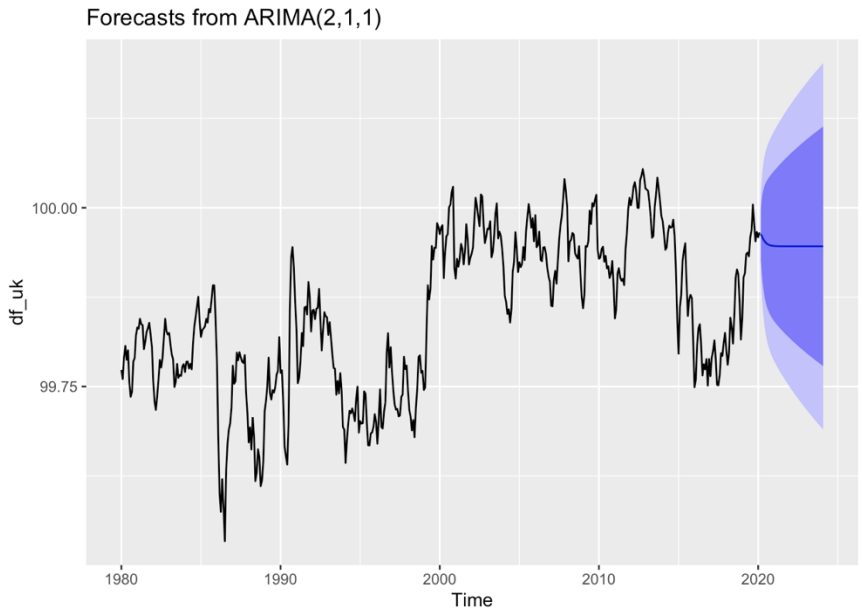
U.S. ARIMA prediction:

Since the ARIMA(0,1,1) is the best model from the previous analysis, then it is applied to predict the next 48 month of commodity net export prices index in the U.S.. It seems like this ARIMA model does not fit the data very well, and it is not informative.



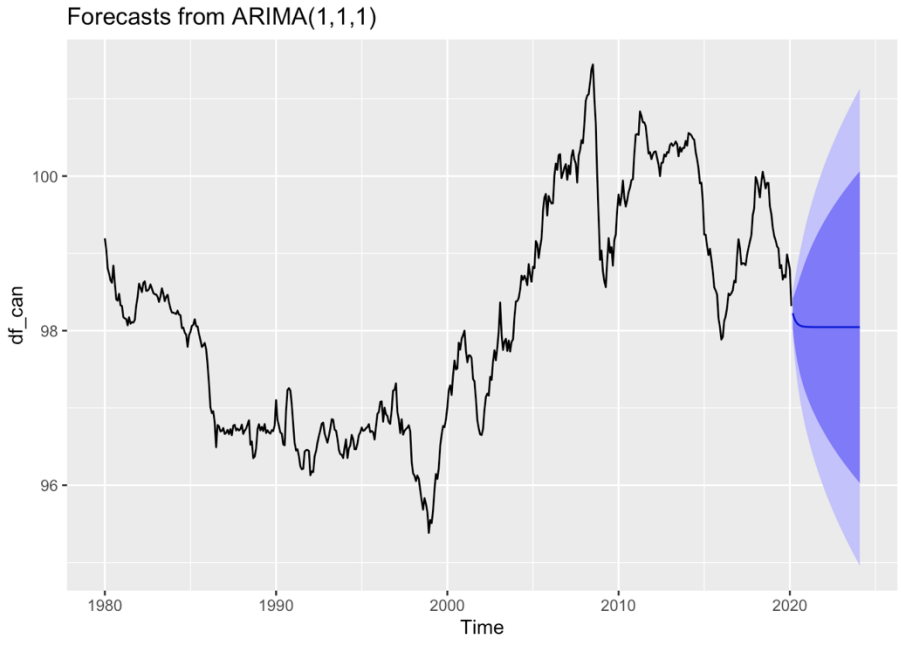
U.K. ARIMA prediction:

Based on the ARIMA model selection result, ARIMA(2,1,1) should be applied to predict the commodity net export prices index in the U.K. in next 48 months. In the prediction, it shows it is going to decrease at the very beginning, and then keep steady.



Canada ARIMA prediction:

Based on the ARIMA model selection result, ARIMA(1,1,1) should be applied to predict the commodity net export prices index in Canada in next 48 months. In the prediction, it shows it is going to decrease at the very beginning, and then keep steady.



Conclusion

Comparing the three countries' CPI trend. Canada may have the best promise of its CPI increase. Although the overall trend of the United Kingdom is increasing as well, its high volatility may cause many troubles to the economy. The United States' CPI may have little increase according to the prediction, it may violate the real rules, but the USA still needs to pay attention to its economic and consumption situation.

As for the industrial production index, the United States and Canada have higher growth rates in IPI while the UK has moderate growth trends. Investors of certain industries may be interested more in the US and Canada for the positive macroeconomic condition. However, Canada's IPI may not be as stable as the US and the UK. Risk averse investors should apply portfolio management strategy to leverage the risk and potential revenue return.

All the three countries have steady increases in the number of employments in the prediction, which means they are all expected to have a pretty healthy growth in their economy. The United Kingdom is expected to have the highest growth rate, but with a relatively higher confidence interval. The economy of the United Kingdom is expected to be strong, but it comes with a high risk. The other two countries have more stable economies as they were.

Based on Commodity Export Price Index dataset, U.S. and Canada have similar cycle period. In the real world, the U.S. and Canada have good trading relationship. Based on the ARIMA model, U.K. is going to have a decreasing trend, and then keep steady. For sure that being steady is not realistic, the reason for this result is that other external signals were not included. International trade can be affected by productivity, trade policy, exchange rates, and even weather. Currently, due to the

coronavirus, the international trade will definitely decrease. These are all external signals. Therefore, there are definitely some improvements can be made.

Reference

Dataset Reference Link:
http://data.imf.org/?sk=388DFA60-1D26-4ADE-B505-A05A558D9A42&sId=1479329132316