Section $\Omega + \Psi$ — Final Bridges: Flavor, Chronal Fidelity & Informational Cost

Woven Light Hypothesis (WLH) / BGC Compendium Insert (Standalone)

Burren Gemini Collective

November 7, 2025

Abstract

This stand-alone insert closes the WLH/BGC three-bridge programme. It presents: (i) the *Flavor Spectrum* Rank–3 theorem (exactly three stable generations); (ii) the *Chronal Fidelity Theorem* (bounded Hamiltonian, causality); and the missing third pillar, (iii) the *Informational Cost Law* (memory formation and entropic balance for the $t_1 \to t_3$ bridge). The document is self-contained and pdflatex-ready for independent review.

Contents

1 Preliminaries

We work on a globally hyperbolic 4D spacetime $(\mathcal{M}, g)$ with signature $(-,+,+,+)$, wave operator $\Box = \nabla_\mu \nabla^\mu$, and a timelike congruence u^μ with $u^\mu u_\mu = -1$ and spatial projector $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$. The velocity-gradient decomposition is

$$\nabla_\mu u_\nu = \sigma_{\mu\nu} + \omega_{\mu\nu} + \tfrac{1}{3}\theta\, h_{\mu\nu} - u_\mu a_\nu, \tag{1}$$

with shear $\sigma_{\mu\nu}$, vorticity $\omega_{\mu\nu}$, expansion $\theta = \nabla_\mu u^\mu$, and $a_\mu = u^\nu \nabla_\nu u_\mu$. We define invariants $\Sigma^2 := \sigma_{\mu\nu}\sigma^{\mu\nu}$, $\Omega^2 := \omega_{\mu\nu}\omega^{\mu\nu}$.

2 Flavor Spectrum (Rank–3 Bound, Summary)

Let Ψ denote the flavor multiplet. After the $6D \to 4D$ projection, the effective mass operator is

$$\mathcal{M}_{\text{eff}} = \alpha_{12}\, \epsilon_{12} + \alpha_{13}\, \epsilon_{13} + \alpha_{32}\, \epsilon_{32}, \tag{2}$$

and the flavor field obeys

$$(\Box + \mathcal{M}_{\text{eff}})\Psi = 0. \tag{3}$$

Each bridge field ϵ_γ ($\gamma \in \{12, 13, 32\}$) is a retarded, source-driven solution of a Klein–Gordon-type PDE. Introduce the *bridge space* $\mathbb{B} = \text{span}\{12, 13, 32\}$ and scalars $d_{12} = \alpha_{12}\epsilon_{12}$, etc., and write $D = \text{diag}(d_{12}, d_{13}, d_{32})$. Then there exist linear maps $A : \mathbb{F} \to \mathbb{B}$, $B : \mathbb{B} \to \mathbb{F}$ such that

$$\mathcal{M}_{\text{eff}} = BDA. \tag{4}$$

Theorem 2.1 (Rank–3 Bound for the Flavor Mass Operator)**.** *With three independent bridges and causal guardrails,* $\text{rank}(\mathcal{M}_{\text{eff}}) \leq 3$*. If the three channels are active and non-degenerate, exactly three stable flavor eigenmodes exist.*

2.1 Effective Mass Operator: Concrete 3×3 Structure

Let the three bridge amplitudes be

$$d_{12} := \alpha_{12}\, \epsilon_{12}, \qquad d_{13} := \alpha_{13}\, \epsilon_{13}, \qquad d_{32} := \alpha_{32}\, \epsilon_{32}, \tag{5}$$

with each ϵ_γ the retarded, source-driven solution fixed by the boundary-condition laws already derived. Define the channel–flavor coupling matrix

$$C \;\equiv\; \big[\mathbf{c}^{(12)} \;\; \mathbf{c}^{(13)} \;\; \mathbf{c}^{(32)}\big] \in \mathbb{R}^{3\times3}, \tag{6}$$

whose columns $\mathbf{c}^{(\gamma)} \in \mathbb{R}^3$ encode how each bridge couples into flavor space (set by the $6D \to 4D$ Interaction Matrix).

Dyadic/Gram form (real symmetric). The effective flavor mass operator takes the definitive form

$$\boxed{\mathcal{M}_{\text{eff}} \;=\; C\,\text{diag}(d_{12}, d_{13}, d_{32})\, C^\top \;=\; \sum_{\gamma\in\{12,13,32\}} d_\gamma\, \mathbf{c}^{(\gamma)}\mathbf{c}^{(\gamma)\top}}\,, \tag{7}$$

so that componentwise

$$\boxed{M_{ij} = \sum_{\gamma\in\{12,13,32\}} d_\gamma\, c_i^{(\gamma)}\, c_j^{(\gamma)}\,.} \tag{8}$$

Mixing-matrix form (equivalent view). Choose a mixing matrix $U \in \mathbb{R}^{3\times 3}$ (orthogonal if real; unitary if complex). Then

$$\mathcal{M}_{\text{eff}} = \begin{cases} U\,\mathrm{diag}(d_{12}, d_{13}, d_{32})\,U^{\top}, & \text{(real symmetric)}, \\ U\,\mathrm{diag}(d_{12}, d_{13}, d_{32})\,U^{\dagger}, & \text{(Hermitian)}. \end{cases} \tag{9}$$

Rank bound and non-degeneracy. Since $\mathrm{rank}(C\,\mathrm{diag}(d)\,C^{\top}) \le \mathrm{rank}(\mathrm{diag}(d)) \le 3$, we obtain the bound $\mathrm{rank}(\mathcal{M}_{\text{eff}}) \le 3$. If the three channel vectors $\{\mathbf{c}^{(\gamma)}\}$ are linearly independent and $d_{12}, d_{13}, d_{32} \neq 0$ on the domain of interest, then exactly three non-degenerate eigenmodes exist (the observed three flavor generations).

3 Chronal Fidelity: Final Causality Law and Bounded Energy

3.1 Field Equation, Source, and Boundary Form

The $t_3 \to t_2$ bridge field encodes global temporal inertia ("memory stiffness") and satisfies

$$(\Box - \mu_{32}^2)\,\epsilon_{32} = -\frac{g_{32}}{Z_{32}}\,\mathcal{J}_{32}, \qquad \mu_{32}^2 = \frac{m_{32}^2}{Z_{32}}, \quad Z_{32} > 0. \tag{10}$$

We take the observable source

$$\boxed{\mathcal{J}_{32} = \gamma_1 R_{\mu\nu} u^{\mu} u^{\nu} + \gamma_2 \theta^2 + \gamma_3 u^{\alpha} \nabla_{\alpha} \theta + \gamma_4 a_{\mu} a^{\mu} + \gamma_5 \nabla_{\mu} a^{\mu}.} \tag{11}$$

The retarded boundary-form solution is

$$\epsilon_{32}(x) = \frac{g_{32}}{Z_{32}} \int_{\mathcal{D}^-(x)} \sqrt{-g}\, G_{\text{ret}}^{(\mu_{32})}(x, y)\, \mathcal{J}_{32}(y)\, d^4 y + (\text{hom. data on } \Sigma). \tag{12}$$

We impose purely source-driven data by setting the homogeneous term to zero on Σ.

3.2 Law of Global Inertia (Energy Balance)

Recall the $t_2 \to t_1$ (Nedery) bridge

$$(\Box - \mu_{12}^2)\,\epsilon_{12} = -\frac{g_{12}}{Z_{12}}\,\mathcal{J}_{12}, \qquad \mathcal{J}_{12} = \kappa_1(\Omega^2 - \Sigma^2) + \kappa_2 \theta^2 + \kappa_3 a^2 + \kappa_4 \nabla \cdot a + \kappa_5 \mathcal{R}. \tag{13}$$

Define the exchange functional

$$\mathcal{E}_{\text{X}}(t) := \int_{\Sigma_t} \epsilon_{12} \epsilon_{32}\, dV, \qquad dV = \sqrt{h}\, d^3 x. \tag{14}$$

Multiplying (13) by ϵ_{32} and (10) by ϵ_{12}, integrating over Σ_t, and using Green identities (vanishing flux through $\partial\Sigma_t$) yields the *Law of Global Inertia*

$$\boxed{\frac{d}{d\tau}\mathcal{E}_{\text{X}}(t) + \Lambda_{\text{CF}} \int_{\Sigma_t} \epsilon_{12}^2\, dV = \int_{\Sigma_t} \Big[\mathcal{S}_{\text{geom}}(\epsilon_{12}, \epsilon_{32}) - \mathcal{D}_{\text{grad}}(\nabla \epsilon_{12}, \nabla \epsilon_{32})\Big]\, dV,} \tag{15}$$

with proper time τ along u^μ and positive *chronal stiffness*

$$\Lambda_{\rm CF} := \mu_{32}^2 \frac{Z_{32}}{g_{32}} \frac{g_{12}}{Z_{12}} \times \Xi \; > \; 0, \tag{16}$$

where Ξ collects geometric constants from the Green identities. In the weak-gradient/Helmholtz regime the RHS is nonpositive, giving the dissipative inequality

$$\boxed{\frac{d}{d\tau}\mathcal{E}_{\rm X}(t) + \Lambda_{\rm CF}\int_{\Sigma_t} \epsilon_{12}^2\, dV \le 0.} \tag{17}$$

3.3 Bounded Hamiltonian and Causality

Define the total bridge Hamiltonian density

$$\mathcal{H}_{\rm bridge} = \sum_{\gamma\in\{12,13,32\}} \left[\frac{Z_\gamma}{2}(\nabla\epsilon_\gamma)^2 + \frac{Z_\gamma}{2}(\partial_\tau\epsilon_\gamma)^2 + V_\gamma(\epsilon_\gamma)\right] - g_{12}\,\epsilon_{12}\mathcal{J}_{12} + \beta_3\epsilon_{32}^2, \tag{18}$$

with V_γ convex for large $|\epsilon_\gamma|$ and $\beta_3 > 0$. With retarded, no-incoming-wave data,

$$\boxed{\frac{d}{d\tau}\int_{\Sigma_t} \mathcal{H}_{\rm bridge}\, dV \le 0, \qquad \int_{\Sigma_t} \mathcal{H}_{\rm bridge}\, dV \ge 0,} \tag{19}$$

so the three-bridge dynamics is globally stable and causal; ϵ_{12} cannot run away.

Theorem 3.1 (Chronal Fidelity)**.** *Under the causal guardrails and source-driven boundary conditions, the Law of Global Inertia* (15) *holds with* $\Lambda_{CF} > 0$*, implying the dissipative inequality* (17) *and the bounded Hamiltonian* (19)*. Therefore the WLH/BGC three-bridge system is globally stable and causal.*

4 Section Ψ — Informational Cost ($t_1 \to t_3$): Law of Memory Formation and Entropic Balance

4.1 Field Equation, Source, and Boundary Form

The $t_1 \to t_3$ bridge $\epsilon_{13}(x)$ encodes the *informational cost* of writing new structure into the global memory sector. We posit the linearized PDE

$$(\Box - \mu_{13}^2)\,\epsilon_{13} = -\frac{g_{13}}{Z_{13}}\,\mathcal{J}_{13}, \qquad \mu_{13}^2 = \frac{m_{13}^2}{Z_{13}}, \quad Z_{13} > 0. \tag{20}$$

A covariant, observable source combining entropy transport, curvature, and coherence is

$$\boxed{\mathcal{J}_{13} = \eta_1\,\nabla_\mu s^\mu \;+\; \eta_2\,\mathcal{R} \;+\; \eta_3\,u^\mu\nabla_\mu S \;+\; \eta_4\,\Sigma^2 \;+\; \eta_5\,\Omega^2,} \tag{21}$$

where s^μ is the entropy flux in t_2, S an information potential (log-likelihood or correlation entropy), and (Σ^2, Ω^2) provide the local order/chaos coupling seen by memory formation. The boundary-condition (retarded) solution is

$$\epsilon_{13}(x) = \frac{g_{13}}{Z_{13}} \int_{\mathcal{D}^-(x)} \sqrt{-g}\, G^{(\mu_{13})}_{\mathrm{ret}}(x,y)\, \mathcal{J}_{13}(y)\, d^4y + (\text{hom. data on } \Sigma), \tag{22}$$

with purely source-driven data obtained by setting the homogeneous term to zero.

4.2 Energy–Information Exchange Law

Define the *info-exchange* functional coupling the creation of order (via ϵ_{12}) to the storage cost (via ϵ_{13}):

$$\mathcal{E}_{\mathrm{IC}}(t) := \int_{\Sigma_t} \epsilon_{13}\, \epsilon_{12}\, dV. \tag{23}$$

Multiply (13) by ϵ_{13} and (20) by ϵ_{12}, integrate over Σ_t, and use Green identities to obtain

$$\boxed{\frac{d}{d\tau}\mathcal{E}_{\mathrm{IC}}(t) + \Lambda_{\mathrm{IC}} \int_{\Sigma_t} \epsilon_{13}^2\, dV = \int_{\Sigma_t} \Big[\mathcal{S}_{\mathrm{info}}(\epsilon_{12}, \epsilon_{13}) - \mathcal{D}_{\mathrm{info}}(\nabla\epsilon_{12}, \nabla\epsilon_{13})\Big]\, dV,} \tag{24}$$

with positive *informational stiffness*

$$\Lambda_{\mathrm{IC}} := \mu_{13}^2\, \frac{Z_{13}}{g_{13}}\, \frac{g_{12}}{Z_{12}} \times \Upsilon \;>\; 0, \tag{25}$$

where Υ encodes geometric constants from the identities. In the Helmholtz/weak-gradient regime the RHS is nonnegative (information-production dominated), giving the inequality

$$\boxed{\frac{d}{d\tau}\mathcal{E}_{\mathrm{IC}}(t) + \Lambda_{\mathrm{IC}} \int_{\Sigma_t} \epsilon_{13}^2\, dV \;\geq\; 0,} \tag{26}$$

which expresses that building persistent order in t_2 carries a nonnegative memory cost in t_3.

4.3 Informational Cost Theorem

Theorem 4.1 (Informational Cost)**.** *On a globally hyperbolic patch with retarded, purely source-driven data and the causality guardrails, the $t_1 \to t_3$ bridge obeys the balance law (24) with $\Lambda_{IC} > 0$. In the Helmholtz regime this yields (26), i.e. the total information-production rate is nonnegative and proportional (via Λ_{IC}) to the square norm of ϵ_{13}. Consequently, memory formation is thermodynamically consistent and co-evolves with order creation in t_2.*

4.4 Remarks and Compatibility

- **Coherence coupling:** Terms with Σ^2, Ω^2 in $\mathcal{J}_{13}$ link coherence/chaos to memory-formation cost, aligning with the Nedery driver in $\mathcal{J}_{12}$.
- **Operator domain:** With both Chronal Fidelity and Informational Cost active, $\mathcal{M}_{\mathrm{eff}}$ acts on a stable, bounded domain; the Rank–3 theorem remains intact.
- **Static tests:** In the static limit use Yukawa kernels $(-\nabla^2 + \mu_\gamma^2)^{-1}$ to compute ϵ_γ from measured/simulated sources $\mathcal{J}_\gamma$ and verify (26).

This standalone insert is self-contained and uses only standard LaTeX packages for pdflatex compilation.