Ancient Indian’s Pulsating Epicycle Model

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Github: https://github.com/guswnd914/cs189_indian_astronomy

Abstract

This project uses the *Surya Siddhanta*, the first astronomical text in India, as its basis for the mathematical model. To stay true to the ancient Indian methodology, the mathematical model was core to the advancement of this project so understanding the astronomical measurements and specific parameters became key. Ancient Indian astronomy also depends on a geocentric solar system and epicyclic orbits, so the whole machine learning model combined understanding of various astronomer’s models into the best approximations from ancient Indian times. Once the mathematical models were completely understood, the machine learning model was built on top pulling from various methods to fit these astronomical events properly within a certain degree of accuracy (depended on factors like planet and number of generated samples). The eclipse prediction accuracy was rather high but only for predicting the number of eclipses; once visualization of the waveform came into play, the model was realized as inaccurate for specific dates. In conclusion, we were able to successfully come out with a machine learning model based (most) entirely off of the ancient Indian astronomical model in spite of any issues encountered.

1 Introduction

The era for ancient Indian astronomy began ten thousand years ago, traced back to the origin of the *Surya Siddhanta*, the first astronomical text from India. The *Surya Siddhanta* proposes a model of planetary motion based upon a geocentric understanding of the solar system in which the Earth remains centric and the planets, moon, and sun (the text’s namesake and mythological author) orbit around in epicycles. The planetary motion model uses a top-down view in 2D, *i.e.* only using an xy-coordinate system. The text evolved with India in its understanding of the cosmos, edited throughout the following centuries by various astronomers. To model eclipses, we turn to the 7th century CE astronomer Brahmagupta. Brahmagupta wrote the *Khandakhadyaka*, a text which combines millenia-old traditions of the *Surya Siddhanta* and early medieval breakthroughs in Indian astronomy. The motivation behind using the *Surya Siddhanta* and *Khandakhadyaka* is in their aspiration toward the most accurate models for astronomical events. However, the mathematical model falls short in some calculations as discussed from our findings.

2 Methods

Dataset Used:

When surveying the Project S Early projects for ancient Indian astronomy, we realized that neither repository contained the parameters we were looking for in our model. Thus, we decided to check their references and found the skyfield API. We realized almost immediately that this library contained all we needed for our model, some constants used in the model, astronomical units we required, and a full dataset to train and test on. Their dataset contains information from years -3000 to 3000, so we knew that if we used a portion of this timeframe to model off of, we could use API calls to check accuracy of our results.

Planetary Motion:

In this mathematical model, ancient Indians believed that the planets, sun, and moon were circularly orbiting around Earth in epicylical manner where angular velocity expands and contracts depending on each quadrant. Generally, the latitude of Earth is fixed, hence the model presents a 2-dimensional overview of the longitude with X and Y parameters.

$$X = Rcos(\alpha) + R_a + \left|(R_a + R_b)\, sin(\alpha)\right|$$
$$Y = Rsin(\alpha)$$

where :

$R \rightarrow$ distance between Earth and planet

$\alpha \rightarrow$ mean latitude

$R_a \rightarrow$ anomaly at apogee, V

$R_b \rightarrow$ anomaly at contracted, C

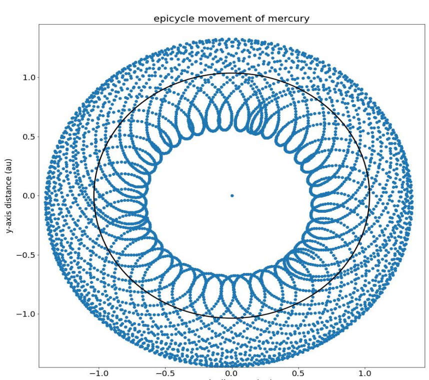


Fig1. Pulsating Epicycle Model (Mercury)

One thing to note is that when α reaches 0° and 180°, X approximately reaches its apogee, $max(X) = R + R_a$, and perigee, $min(X) = -R + R_a$. Calculating their mean cancels out the R value, hence leading to the value for R_a. Furthermore, when α reaches 90° and 270°, X approximately reaches $2R_a + R_b$, and given the value for R_a, we can reach the value of R_b.

Due to the nature of orbits, the plotting of planetary motions along a time-axis creates a sinusoidal-like wave. In the ancient Indian model, however, the epicycles add more variation to this waveform. To approximate this wave, we decided to use Fourier curve fitting for this part of our machine learning model.

Eclipses:

To model eclipses, we need to know several variables dependent on time. Specifically, these variables will help us understand the moon's latitude, angular diameter of bodies involved (sun, moon, earth's shadow), and finally the obscured portion on an eclipsed body.

True Angular Diameters:

In ancient Indian text, astronomers used a value called the "true daily motion" which took the daily motion of a planet given a time. In the API, this is referred to as the "mean daily motion" which is a misnomer compared to our mathematical model which uses mean daily motion to refer to an averaged daily motion over a year's time. Moving on, the "true diameter" meaning the body's true *angular* diameter is represented by equation (1) below.

$$true\ angular\ diameter = mean\ diameter\ * \ \frac{true\ daily\ motion}{mean\ daily\ motion}$$

Because the ancient Indians knew approximate mean diameter and mean daily motion, the model simplifies for us as shown below:

$$DM_m = true\ daily\ motion\ of\ moon$$
$$DM_s = true\ daily\ motion\ of\ sun$$
$$D_s = \frac{11}{20}DM_s$$
$$D_m = \frac{10}{247}DM_m$$
$$D_{shadow} = \frac{1}{60}(28DM_m - 25DM_s)$$

However, this model is inaccurate as we realized through testing a mathematical issue comes up with the angular diameter of the shadow: specifically that the equation is truly:

$$D_{shadow} = \frac{1}{60}(8DM_m - 25DM_s)$$

This issue came up due to some mistypings in a well-known book on ancient Indian astronomy. Once we derived the mathematical model ourselves and fixed this issue, our model gained much more accuracy in eclipse prediction.

In the skyfield API, we realized that there might be a discrepancy between values taken from various time periods. Specifically, there was an API call for the value "mean daily motion" but only through a class called Osculating Elements. This concept actually originates from Kepler elements which was discovered during the Keplerian age (early 1600s). However, these age-of-astronomy issues are unavoidable when calling on a modern API, so we used it initially with this issue in mind and saw positive results, so we mention it here to highlight a minor discrepancy.

The angular diameter of the Earth's shadow can be approximated well in this equation for the shadow onto the moon due to horizontal parallax. However, it should be noted that ancient Indian astronomers' approximation of horizontal parallax was simply dividing by 15. This assumption only proves accurate for the moon but not for the sun or other planets.

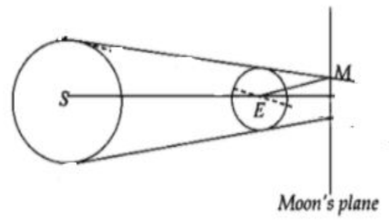


Fig2. Bodies Depiction used for Eclipses

Moon's Latitude:

Ancient Indian astronomers realized that because there are three bodies involved in eclipse events, we need a parameter to measure the latitude of the fastest-moving body: namely, the moon. Thus, we needed to find this beta value. However, due to the little literature and ill descriptions, being able to find this value became a behemoth task. We realized through trial and error that this beta value was truly called "elliptical latitude", but the API referenced this value as "ecliptical longitude". However, to make sure of the accuracy, we took the osculating elements as mentioned in the previous section and checked its argument of latitude which matched exactly with the "elliptical longitude" value. Another issue we ran into was understanding measurement units, specifically where latitude and diameters interacted in angle units rather than distance. However, once we reached this beta value we were able to move onto the modeling for the obscured portion of the eclipsed body.

Obscured Portion on Eclipsed Body:

Moving on, we can now approximate the obscured portion on the eclipsed body. According to Brahmagupta, we can assume the following:

$$\beta = moon's\ latitude$$
$$obscured = \frac{1}{2}(D_b + D_e) - \beta$$

where D_b is the diameter of the eclipsed body and D_e is the diameter of the eclipser

Thus, converting it into our system, we get the following model:

$$\text{Obscured Portion} \rightarrow \begin{Bmatrix} \text{Lunar Eclipse} = \frac{1}{2}(D_m + D_{shadow}) - \beta \\ \\ \text{Solar Eclipse} = \frac{1}{2}(D_s + D_m) - \beta \end{Bmatrix}$$

where :

$D_s \rightarrow$ diameter of the sun
$D_m \rightarrow$ diameter of the moon
$D_{shadow} \rightarrow$ diameter of the shadow
$DM_s \rightarrow$ true daily motion of the sun
$DM_m \rightarrow$ true daily motion of the moon
$\beta \rightarrow$ instant latitude at eclipse

if :

$Obscured\ Portion < 0 \Rightarrow$ No Eclipse
$Obscured\ Portion < D_m \Rightarrow$ Partial Eclipse
$Obscured\ Portion > D_m \Rightarrow$ Total Eclipse

For the lunar eclipse, if the obscured portion of the body is less than the diameter of the moon, then the eclipse is partial (else total). The same logically follows for the sun.

Due to the axial tilt of the Earth, calculating the exact times of the lunar eclipse is quite challenging, especially, given that the Earth, moon and sun are three orbital bodies that must precisely interact for an eclipse to show. For this reason, the ancient Indian model is unable to accurately predict the exact days of lunar eclipses; however, it is able to quite precisely calculate the maximum number of lunar eclipses in a given time frame. We decided to take two approaches to determining the number of lunar eclipses that had occurred: calculating the ancient Indian lunar eclipse formula and utilizing machine learning to generate a classifier.

We used a Stochastic Gradient Boosting Classifier, and as variables we used the diameter of the moon, diameter of the shadow, the moon phase and the instant latitude at eclipse. The purpose of utilizing machine learning methods was to apply modern science to the variables that the ancient Indian astronomers had themselves analyzed in reference to lunar eclipses. In this way, machine learning allows us to dig deeper to understand the relationship that these variables have to one another in regards to the appearance of a lunar eclipse. In tuning the hyperparameters of the Gradient Boosting Classifier, we reduced the subsample to 0.0099, as it was less than 1 this effectively turned our classifier into a Stochastic Gradient Boosting Classifier. The classifier was able to accurately predict the number of lunar eclipses that had occurred in a later time frame.

Moon Phases:

As the ancient Indian model lacked any sort of formulas to predict lunar phases, we relied on machine learning to produce such a model. We called on the skyfield API to collect the illuminated fractions of the moon. The phases of the moon are cyclic in nature, and as such was aptly predicted by a sinusoidal model. We used curve-fitting to produce a sinusoidal model that would accurately predict the future lunar phases.

3 Results

Planetary Motion:

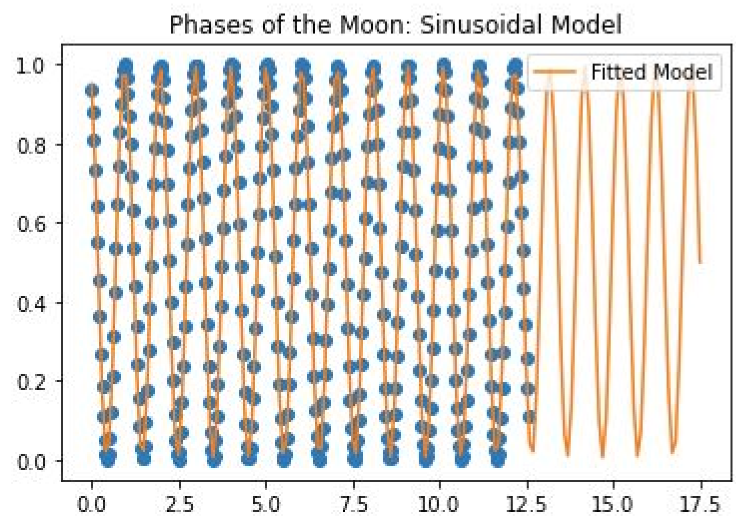
We found the R_a and R_b values for the planetary motion to approximate the epicyclic orbits but ran into some issues. When we visualized the sine function fitting from the Fourier curve fitting model, varying the number of samples led to more accuracy in some planetary motions but destroyed epicycles if too many were used. We received accurate results to a degree depending on the above restriction for planetary motion, plotted visualizations properly, and saw results we wanted to despite restrictions. It made us considerably look through the natural pattern of cyclical movement, and it led us to apply sine function on movement of planets in respect to time. Indeed, the visualization and graph provide the discrete movement of heavenly bodies. We expected it to find an accurate solution for planetary motion. However, as the amount of training data points reach beyond a certain limit, the Fourier series undergoes overfitting and it leads to poor accuracy, and when it was below the limit, the model again predicted inaccurately. For instance, when N (# of training points) was 50, the prediction for sun had mean absolute error of 0.02 au. But when it reaches beyond 70, the error exponentially increases upto 0.9 au, which is roughly 450% of the previous one. It shows that the sine function was not enough to clearly express the pattern of the universe.

Eclipses:

As the eclipse information in the skyfield API was limited to lunar eclipse data, we focused on lunar eclipses so that we could check for the accuracy of our predictions. Initially, we saw an 89% accuracy in our model. However, we soon realized that this high accuracy was due to the high frequency of no eclipses versus the infrequent partial and total eclipses. Thus, our eclipse readings were giving an inaccurate measure of the strength of our model. Then, we saw through visualization that our mathematical model was flawed. We saw that eclipse count matched very well (99% accuracy) with the true values, but when we plotted the predicted lunar eclipses against the true values, and we saw that the times of the eclipses were inaccurate. Looking more closely at the matter, we found a resource detailing this issue. "The rules given by Brahmegupta, Varah-Acharija, and Aryabhatta may be used with such precautions. Any person may compose a set of rules for the common purposes of astronomy, but with regard to the duties necessary in eclipses, the computation must be made by the books of the Munis and the Bija applied" [1]. Thus, though the theory of the model is sound, other variables make computation of said model not as accurate as desired. Some factors that make this problem more complex than the ancient Indian model include the three bodies and 3D nature of eclipses (we cannot ignore the Z dimension because the Earth's axis tilt changes angular diameter). Additionally, the difference between ancient Indian methods and modern methodology (values received from API) may create errors in approximations as well. In conclusion, we received accurate results for counting the eclipses but inaccurate for specific dates, plotted the occurrence wave properly, and saw results we wanted given the restrictions mentioned above. The ancient Indian model had an accuracy of 99.04% with a mean-squared error of 0.01698. Our Stochastic Gradient Boosting Classifier had an accuracy of 98.99% with a mean-squared error of 0.01423. The ancient Indian model had marginally better accuracy and the classifier model had marginally better mean-squared error. However, the ancient Indian model (the maximum number of eclipses) predicted an underestimate of the maximum eclipses providing an estimate that was lower than the true eclipses. In contrast, the Stochastic Gradient Boosting Classifier produced a suitable maximum which was just above the true number of eclipses—a greater proportion of eclipses by only 0.001.

Moon Phases:

The ancient Indian model did not predict phases of the moon, so we utilized machine learning to accurately predict the lunar moon. Our sinusoidal model, which we curve-fitted to the illuminated fraction of the moon date from the API, was used to predict future lunar eclipses. The model had an accuracy of mean-squared error of 0.2389. The lunar phases were naturally almost perfectly sinusoidal, allowing the sine function to closely fit the data and provide highly accurate predictions.



Compared to Real Data and Current Prediction Algorithms:

Because of the various parameters not used in the *Surya-Siddhanta* (*e.g.* right ascension, azimuth), even beginning with problems like planetary motion or eclipse prediction becomes exponentially more difficult. Ancient Indian models also were based on a geocentric solar system, so this theoretically does not stand to real data. However, our planetary motion predictions are pretty accurate to the epicyclic model as seen through the visualization. Our prediction for eclipses came out very well aligning with count of eclipses, but it does not stand well with time of eclipses, so other models may predict this value more accurately. Finally, since our model did not have mathematical standing in moon phases, these predictions were entirely accurate through the API calls and stand well to current prediction models.

4 Conclusion

Given the mathematical model of *Surya-Siddhanta*'s Heavenly Motion and the pulsating epicycle, we were able to model planetary motions, eclipse count over time, and moon phases well. Our results compared well with real data and current prediction algorithms to a limited extent as mentioned in our results. We've learned that in spite of being completely new to this field, we were able to piece together the right parts to our project and fit it all well with the intention of staying true to the ancient Indian astronomical model. Our unique approach (and contribution) was to look through a time-series approach. In all our sections, we tried understanding the astronomical events through a time series fitting to a sine wave whenever possible. Though our model fell short with predicting the timing of eclipses, this is completely understandable. The mathematical model itself could not support this end result, providing inaccurate calculations for predicted times of eclipses. Modern methods of estimating eclipses may be more accurate though sources vary on estimating the accuracy (total lunar eclipses and solar eclipses rarely occur). In conclusion, this project has been a good experience for us to struggle with an unknown topic that seems so easy to pattern but can be so difficult to estimate and predict with machine learning.

References

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[2] Burgess, E. (1858 - 1860) *Translation of the Sûrya-Siddhânta, A Text-Book of Hindu Astronomy; With Notes, and an Appendix. Vol. 6 (1858 - 1860), pp. 141-498.* American Oriental Society.

[3] Rao, S. Balachandra. (2010) *Classical Astronomy in India-An Overview, in Padmanabhan, (ed) Astronomy in India: A Historical Perspective*. Delhi: Springer.

Data Used:

[4] https://rhodesmill.org/skyfield/