Machine Learning

CSCI 567 Spring 2021

Discussion: HMM

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Recall a hidden Markov model is parameterized by:

- initial state distribution $P(Z_1 = s) = \pi_s$
- transition distribution $P(Z_{t+1} = s'|Z_t = s) = a_{s,s'}$
- emission distribution $P(X_t = o|Z_t = s) = b_{s,o}$

These parameters are assumed to be known for this problem. Now suppose we observe a sequence of outcomes $x_1, \ldots, x_{t-1}, x_{t+1}, \ldots, x_T$ with the outcome at time t missing $2 \leq t \leq T-1$.
(a) Derive the conditional probability of the state at time t being s, that is,

$$P(Z_t = s|X_{1:t-1} = x_{1:t-1}, X_{t+1,T} = x_{t+1:T}).$$

Express you answer in terms of the forward message at time $t-1$

$$\alpha_{s'}(t-1) = P(Z_{t-1} = s', X_{t1:t-1} = x_{1:t-1}),$$

and the backward message at time t

$$\beta_{s'}(t) = P(X_{t+1:T} = x_{t+1:T}|Z_t = s').$$

(b) Derive the conditional probability of the outcome at time t being o, that is,

$$P(X_t = o|X_{1:t-1} = x_{1:t-1}, X_{t+1:T} = x_{t+1:T}).$$

You can express you answer using the quantity $P(Z_t = s|X_{1:t-1} = x_{1:t-1}, X_{t+1:T} = x_{t+1:T})$ from the last question.