Machine Learning

CSCI 567 Spring 2021 (April 14, 2021)

Discussion: Constrained Optimization and EM

(1) Consider a mixture of two exponential distributions, with parameters λ_1, λ_2 respectively. Let $z \in \{0, 1\}$ be the hidden variable to indicate which exponential distribution the observed data x is drawn from ($z = 0$ if $\text{Exp}(\lambda_1)$ is sampled, else $z = 1$ for $\text{Exp}(\lambda_2)$). Let the prior probability of $z = 1$ be denoted using parameter $\pi \in (0, 1)$, i.e., $p(z = 1) = \pi$. **Note:** An exponential distribution with parameter λ has the following density function: $p(x) = \lambda e^{-\lambda x}, \forall x \geq 0$.

Each example in the dataset $\{x_i\}_{i=1}^N$ is drawn i.i.d. from the mixture model. Write the E-step and M-step to estimate parameters $\lambda_1, \lambda_2, \pi$. Solve the M-step to obtain the update equations for the parameters at each iteration of EM algorithm.

(2) Find the maxima and minima of $f(x, y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$.