NYU Center for Data Science

DS-UA 111
Data Science for Everyone

Week 11: Lecture 2
Comparing Two Samples

NYU Center for Data Science

Could we use hypothesis testing to assess the relationship between two samples?

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Data Science for Everyone

Week 11: Lecture 2
Comparing Two Samples

Adapted from Adhikari, DeNero, Wagner, Milner, Foulkes

Announcements

- Please check Week 11 agenda on NYU Classes
  - Homework 3/4
  - Lab 6
  - Project Milestone
- Refer to the Calendar linked to NYU Classes

Review

- Studies
  - Observational
  - Experimental
- Permutation Testing
  - Do two samples come from the same population?

References

- Comparing Samples:
  - Chapter 12.1

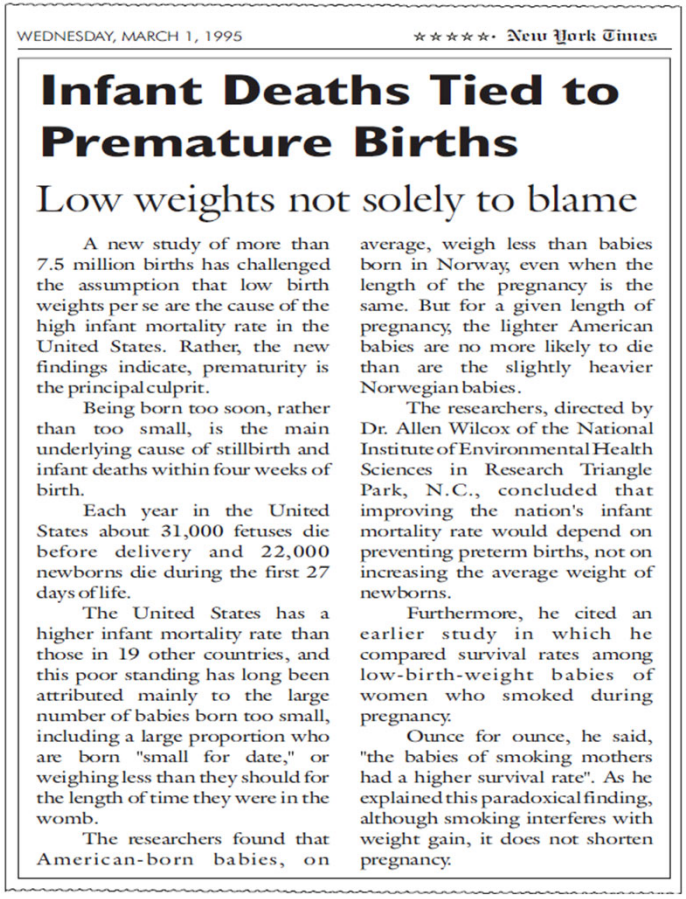
Review

- Researchers use experiments to study the association or causation among variables in situations where they control on values of these variables.
- Search engines and social media use different versions of their websites to experiment with user experience

Check out the optional reading on the use of permutation testing at Facebook

Review

- Researchers use observations to study association among variables in situations where they cannot control the values of these variables.
- Physician gathered data in the Child Health and Development Studies on links between maternal smoking and infant health



WEDNESDAY, MARCH 1, 1995 ★★★★★ New York Times

Infant Deaths Tied to Premature Births

Low weights not solely to blame

A new study of more than 7.5 million births has challenged the assumption that low birth weights per se are the cause of the high infant mortality rate in the United States. Rather, the new findings indicate, prematurity is the principal culprit.

Being born too soon, rather than too small, is the main underlying cause of stillbirth and infant deaths within four weeks of birth.

Each year in the United States about 31,000 fetuses die before delivery and 22,000 newborns die during the first 27 days of life.

The United States has a higher infant mortality rate than those in 19 other countries, and this poor standing has long been attributed mainly to the large number of babies born too small, including a large proportion who are born "small for date," or weighing less than they should for the length of time they were in the womb.

The researchers found that American-born babies, on average, weigh less than babies born in Norway, even when the length of the pregnancy is the same. But for a given length of pregnancy, the lighter American babies are no more likely to die than are the slightly heavier Norwegian babies.

The researchers, directed by Dr. Allen Wilcox of the National Institute of Environmental Health Sciences in Research Triangle Park, N.C., concluded that improving the nation's infant mortality rate would depend on preventing preterm births, not on increasing the average weight of newborns.

Furthermore, he cited an earlier study in which he compared survival rates among low-birth-weight babies of women who smoked during pregnancy.

Ounce for ounce, he said, "the babies of smoking mothers had a higher survival rate". As he explained this paradoxical finding, although smoking interferes with weight gain, it does not shorten pregnancy.

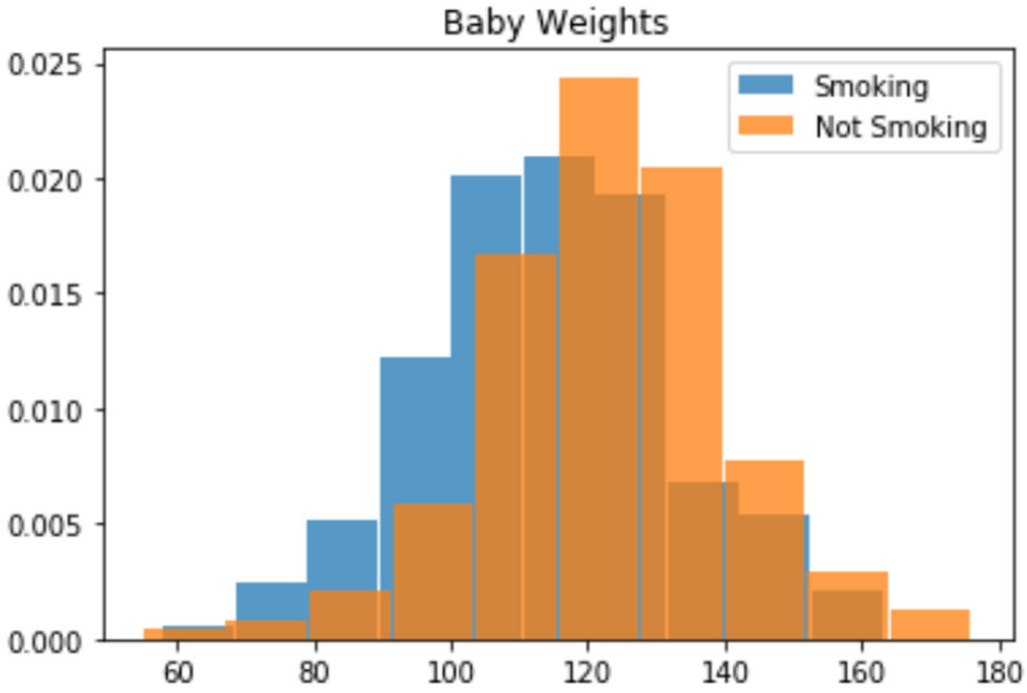
Review

- We can use permutation testing in both experimental studies and observational studies to compare samples
  - Do two samples come from the same population?
  - Does the distribution of a property in sample A match the distribution of a property in sample B
- Permutation means shuffling. We will use hypothesis testing with data generated from shuffling the records between the two group.

	Maternal Smoker	Birth Weight	Shuffled Labels
0	False	120	False
1	False	113	False
2	True	128	False
3	True	108	False
4	False	136	False

Exercise

- Commonly permutation testing will compare the means of the groups. We can use the difference in means for the test statistic.
- We use an hypothesis test of the form
  - Null Hypothesis : The samples come from the same population. Any difference in the properties of the samples owes to randomness.
  - Alternative Hypothesis : The samples come from different populations



Agenda

- Permutation Testing
  - Does the distribution of some feature match between two groups?
- Designing Experiments

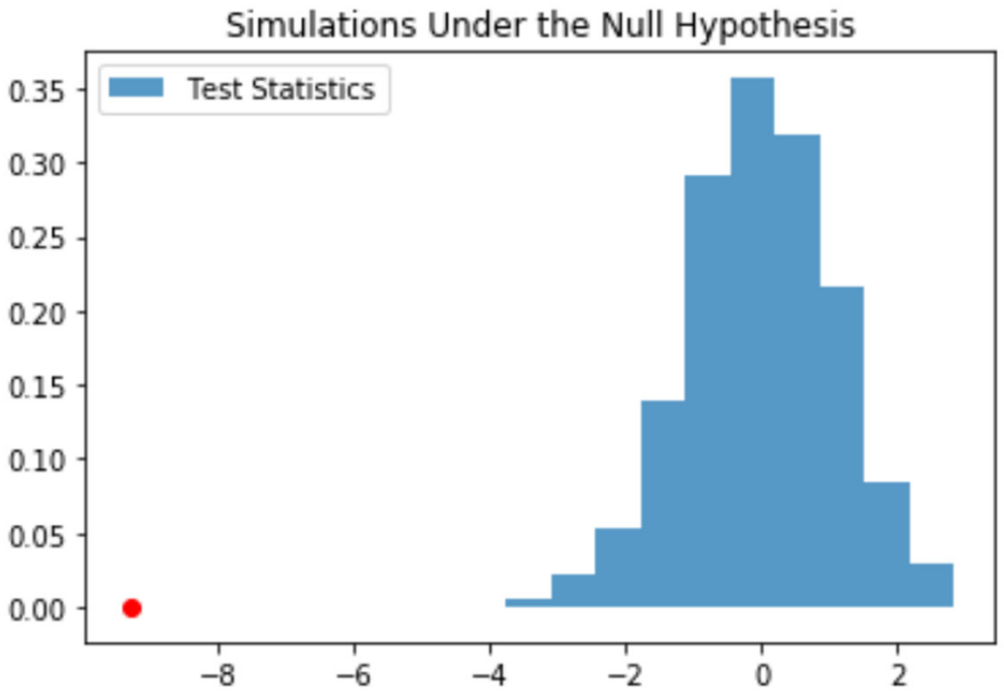
References

- Comparing Samples
  - Chapters 12.3

Permutation Testing

Steps for Permutation Testing

1. Fix a null hypothesis and alternative hypothesis
2. Determine a test statistic
3. Calculate the observed test statistic for the sample
4. Simulate test statistics under the null hypothesis with many trials
5. Calculate p-value for the observed test statistic with the empirical distribution



Hypotheses

- Null hypothesis:
  - The two samples are drawn from the same underlying population distribution; they look like random draws from the same set.
- Alternative hypothesis:
  - The samples are drawn from different distributions; they don't look like random draws from the same set.

- Suppose that the two samples are drawn randomly from the same underlying distribution.
- All rearrangements of the variable values among the two samples are equally likely.
- So we can
  - compute the observed test statistic
  - then shuffle the attribute values and recompute the statistic; repeat; compare with the observed statistic

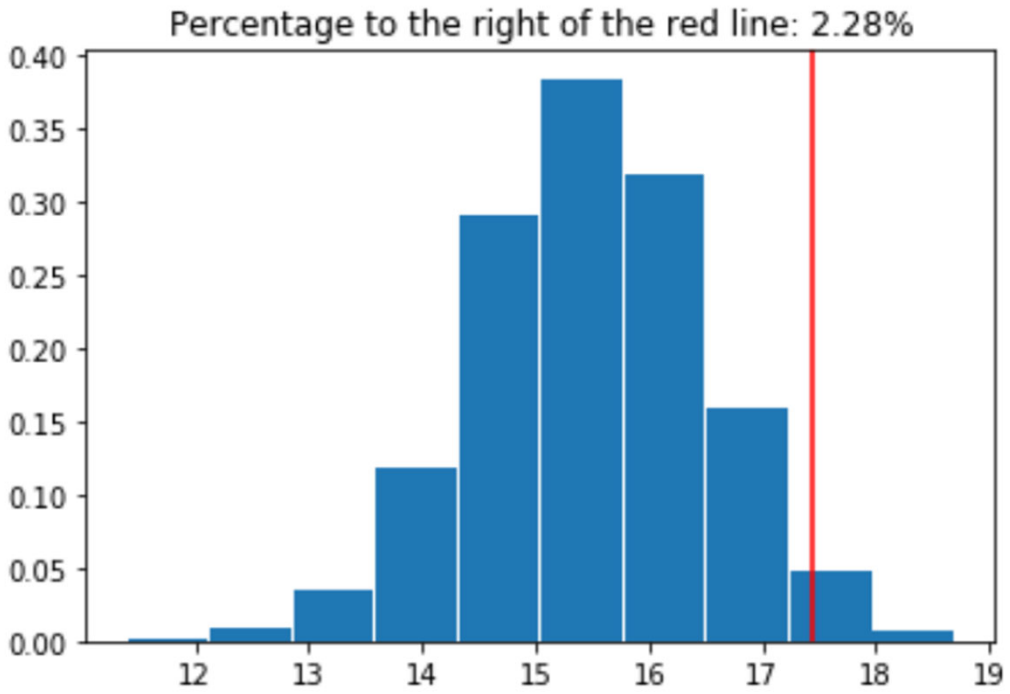
Example

- Suppose we have two samples consisting of numbers {1,9} and {3}
- Since we have quantitative data, we take the test statistic to be the absolute value of the difference between the averages of the groups
- We have three numbers across the two samples (1,9,3). So we have six permutations.
- The observed statistic is 2. So the p-value for the right tail is 1/6 + 1/6

permutation	value of T	probability
(1,9,3)	2	1/6
(9,1,3)	2	1/6
(1,3,9)	7	1/6
(3,1,9)	7	1/6
(3,9,1)	5	1/6
(9,3,1)	5	1/6

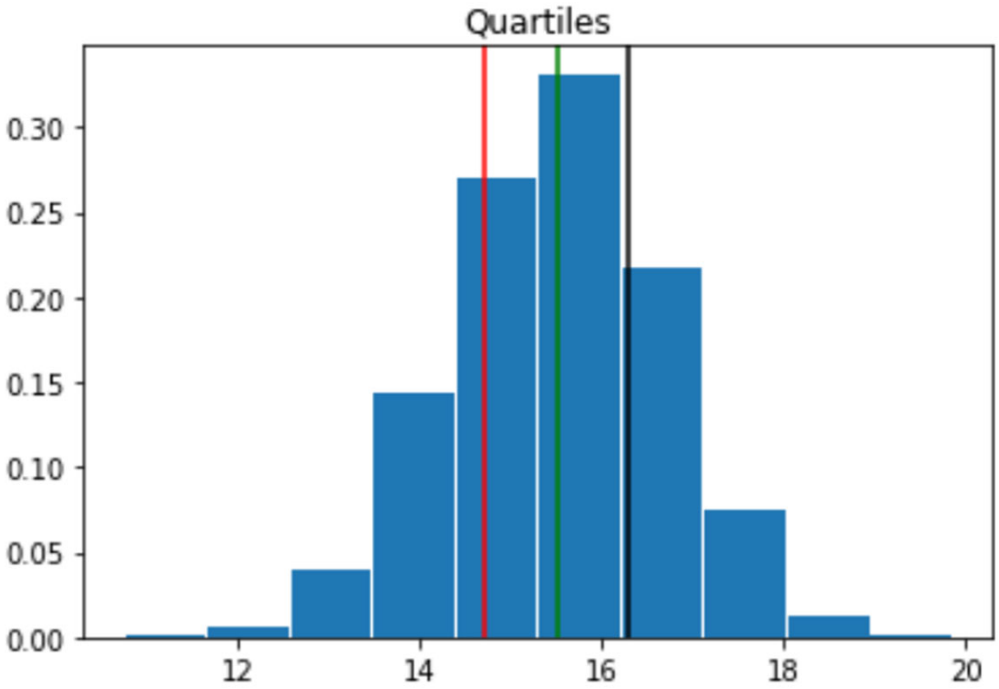
Observed Significance

- For hypothesis testing we need to compare the observed test statistic to the simulated test statistic under the null hypothesis. We check the left tail and right tail for outliers.
- We want to estimate the probability of a test statistic obtaining a value farther to the left in the left tail or farther to the right in the right tail.
- The number of simulated test statistics provides an estimate of the probability nicknamed p-value.



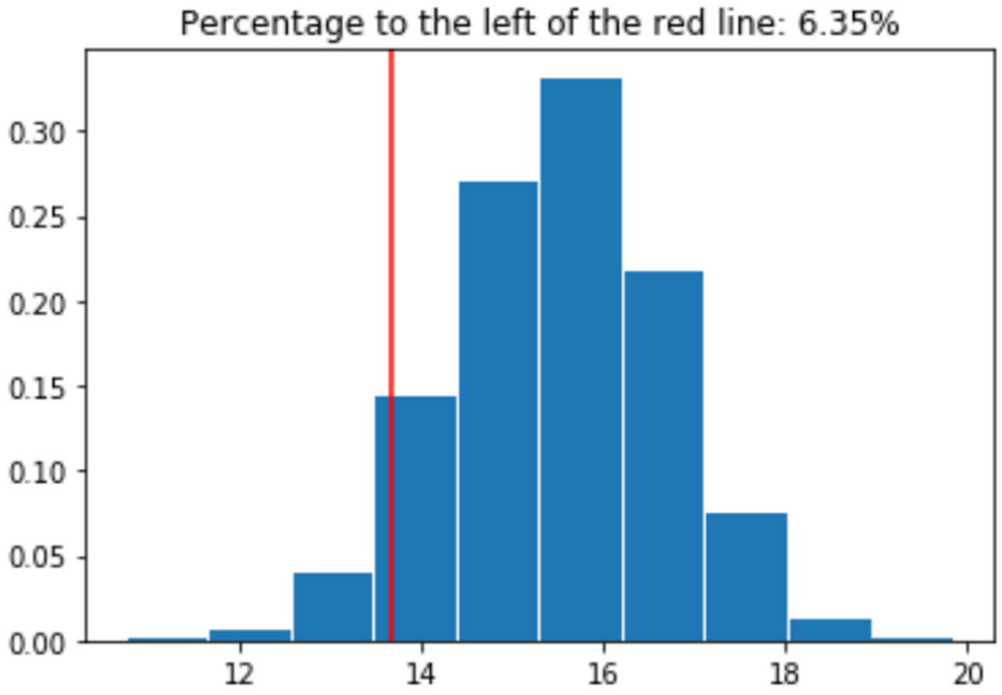
Observed Significance

- We can set thresholds for p-values with quantiles. Remember that quantiles are cut-points. These cut-points divide the data into subsets of equal size.
- We need to sort the data in increasing order to determine the cut-points. Commonly we use percentiles which divide the data into subsets of size 1/100.
- Just like we can compare a p-value to a level that sets a threshold for accepting or rejecting



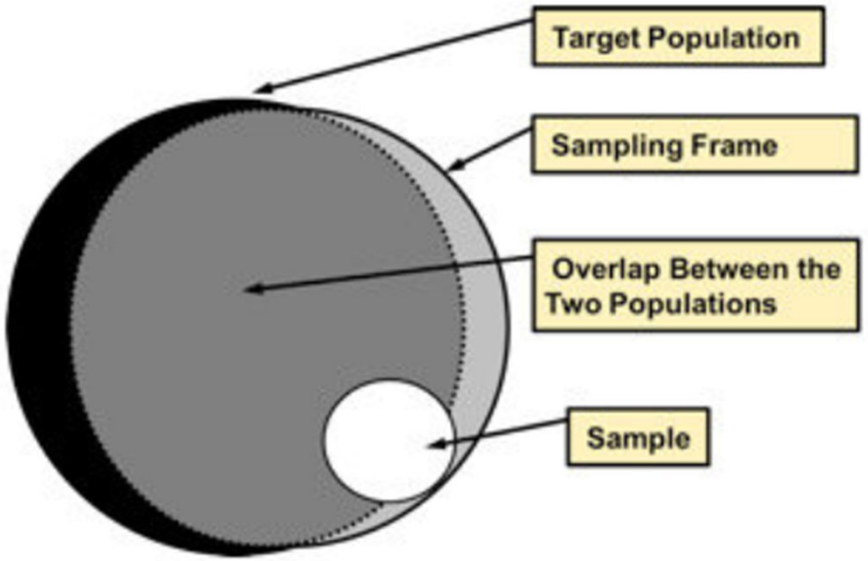
Observed Significance

- We can compare a p-value to an observed significance level that sets a threshold for accepting or rejecting the null hypothesis. Common levels are 1% or 5%.
- Instead of comparing the p-value to the level, we can compare the corresponding percentile to the observed test statistic.
- So the level sets a threshold for a p-value and the corresponding percentile determines rejection region in the left and / or right tails



Designing Experiments

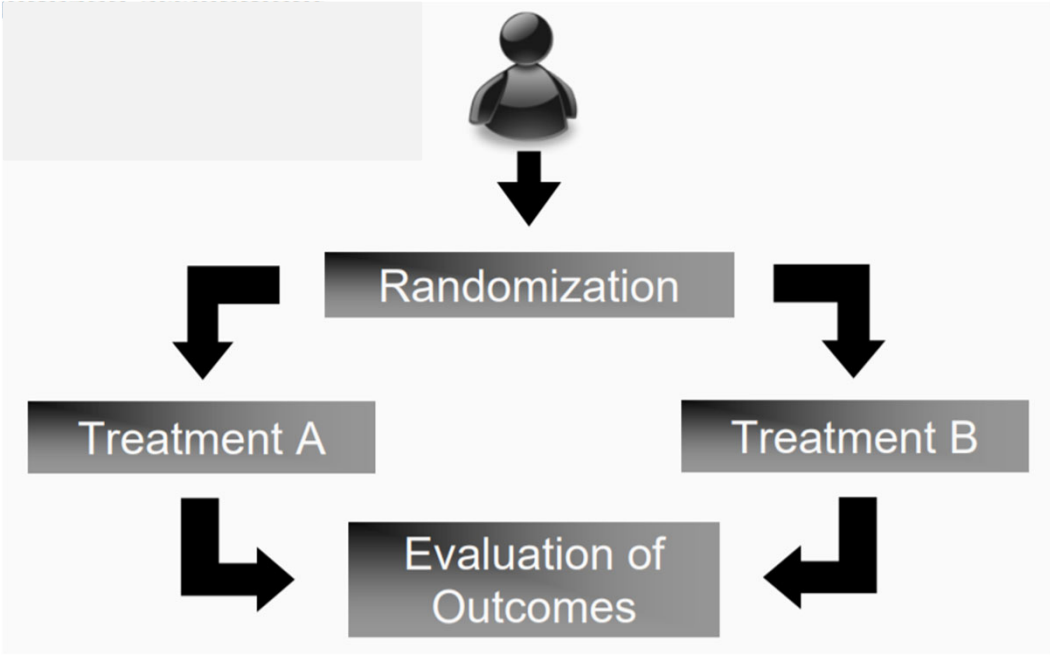
- Researchers perform studies on a target population
- The sampling frame is the subset of the population eligible for inclusion in the sample
- The sample contains randomly or deterministically selected participants or observations from the sampling frame



Designing Experiments

Randomized Controlled Trial

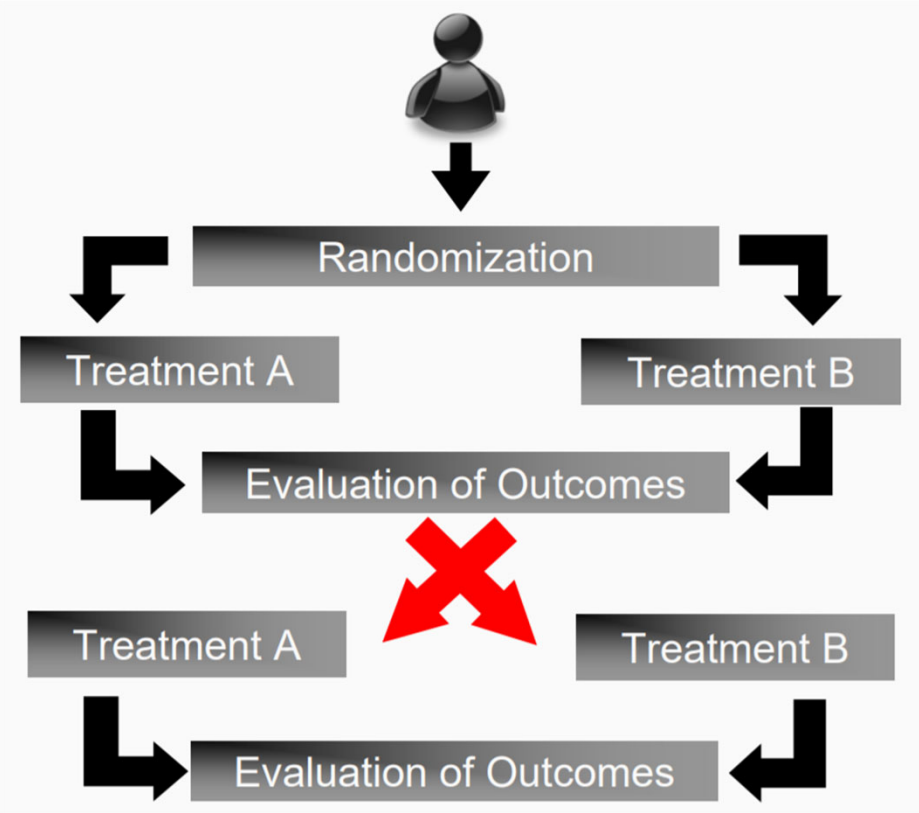
- Researchers randomly split the participants between two groups receiving either treatment A or treatment B
- Here neither the researchers nor the participants should know the division into groups



Designing Experiments

Cross-Over Design

- Researchers repeat the experiment switching the participants between the groups.
- Between the two rounds, each patient has both treatment A and treatment B



Summary

- Permutation Testing
  - Does the distribution of some feature match between two groups?
- Designing Experiments

Goals

- Use random sampling with replacement to shuffle the labels between two groups
- Understand the reasoning behind permutations for hypothesis testing on two samples