Applied Statistics and Data Analysis

Part I

Statistical Process and Quality Control

Introduction

Quality and Process Control

Definition of Quality
- Quality means fitness for use. and
- Quality is inversely proportional to variability.
- My definition: *Is* and *should* are the same.

Statistical Process Control (SPC)
- Statistical process control is, first and foremost, a way of thinking which happens to have some tools attached.

The Magnificent Seven
1. histogram 2. check sheet 3. Pareto chart 4. defect concentration diagram 5. cause-and-effect diagram 6. control chart 7. scatter diagram

Control Charts

Control Charts versus Hypothesis Testing

$$\bar{X} \approx \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right) \tag{1}$$

Arithmetic mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2}$$

Question: is $|\bar{x} - \mu_0|$ significant - μ_0 target value
- $\bar{x}$ Arithmetic mean of the measurements
Given: Level of significantce $\alpha = 0.0027$

Control Charts multiple Measurments per Sample

Mean values

$$\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij} \tag{3}$$

Standard deviations

$$s_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_i\right)^2} \tag{4}$$

Ranges

$$R_i = max\left\{x_{ij} | j \in \{1, ..., n\}\right\} - min\left\{x_{ij} | j \in \{1, ..., n\}\right\} \tag{5}$$

for all $i \in \{1, ..., k\}$

R Chart

Mean Range

$$\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i \tag{6}$$

Control limits

$$\begin{aligned} UCL =& D_4\bar{R} \\ LCL =& D_3\bar{R} \end{aligned} \tag{7}$$

$\bar{x}$ Chart

Mean Range

$$\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i \tag{8}$$

Control limits with sigma

$$\begin{aligned} UCL =& \mu + 3\frac{\sigma}{\sqrt{n}} \\ LCL =& \mu - 3\frac{\sigma}{\sqrt{n}} \end{aligned} \tag{9}$$

μ: Process mean
σ: Process standard deviation
We don't know mu and sigma so we have to estimate them:

$$E\left(\bar{x}\right) = \mu \tag{10}$$

and

$$Var\left(\bar{x}\right) = \frac{\sigma^2}{n} \tag{11}$$

The mean is an unbiased estimator with the standard error

$$SE\left(\bar{x}\right) = \frac{\sigma}{\sqrt{n}} \tag{12}$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \tag{13}$$

$$\bar{\bar{x}} = \frac{1}{k^*}\sum_{i=1}^{k^*} \bar{x}_i \tag{14}$$

Control limits

$$\begin{aligned} UCL =& \bar{\bar{x}} + 3\frac{\bar{R}}{d_2}\frac{1}{\sqrt{n}} \approx \bar{\bar{x}} + A_2\bar{R} \\ UCL =& \bar{\bar{x}} - 3\frac{\bar{R}}{d_2}\frac{1}{\sqrt{n}} \approx \bar{\bar{x}} - A_2\bar{R} \end{aligned} \tag{15}$$

Part II

Multiple Regression

Part III

Design of Experiment