Avaliação III de Álgebra Linear

Página 01 - (Valor 1,0)

19. The number of bacteria N_B measured at different times t is given in the following table. Determine an exponential function in the form $N_B = Ne^{\alpha t}$ that best fits the data. Use the equation to estimate the number of bacteria after 60 min. Make a plot of the points and the equation.

t (min)	10	20	30	40	50
N_B	15,000	215,000	335,000	480,000	770,000

30. [**M**] Let $A = \begin{bmatrix} -6 & 28 & 21 \\ 4 & -15 & -12 \\ -8 & a & 25 \end{bmatrix}$. For each value of a in the set $\{32, 31.9, 31.8, 32.1, 32.2\}$, compute the characteristic polynomial of A and the eigenvalues. In each case, create a graph of the characteristic polynomial $p(t) = \det(A - tI)$ for $0 \le t \le 3$. If possible, construct all graphs on one coordinate system. Describe how the graphs reveal the changes in the eigenvalues as a changes.

25. [**M**] Use a matrix program to diagonalize

$$A = \begin{bmatrix} -3 & -2 & 0 \\ 14 & 7 & -1 \\ -6 & -3 & 1 \end{bmatrix}$$

if possible. Use the eigenvalue command to create the diagonal matrix D. If the program has a command that produces eigenvectors, use it to create an invertible matrix P. Then compute $AP - PD$ and PDP^{-1}. Discuss your results.

26. [**M**] Repeat Exercise 25 for $A = \begin{bmatrix} -8 & 5 & -2 & 0 \\ -5 & 2 & 1 & -2 \\ 10 & -8 & 6 & -3 \\ 3 & -2 & 1 & 0 \end{bmatrix}$.

7. A certain experiment produces the data $(1, 1.8)$, $(2, 2.7)$, $(3, 3.4)$, $(4, 3.8)$, $(5, 3.9)$. Describe the model that produces a least-squares fit of these points by a function of the form

$$y = \beta_1 x + \beta_2 x^2$$

Such a function might arise, for example, as the revenue from the sale of x units of a product, when the amount offered for sale affects the price to be set for the product.

a. Give the design matrix, the observation vector, and the unknown parameter vector.

b. **[M]** Find the associated least-squares curve for the data.

11. **[M]** According to Kepler's first law, a comet should have an elliptic, parabolic, or hyperbolic orbit (with gravitational attractions from the planets ignored). In suitable polar coordinates, the position (r, ϑ) of a comet satisfies an equation of the form

$$r = \beta + e(r \cdot \cos \vartheta)$$

where β is a constant and e is the *eccentricity* of the orbit, with $0 \leq e < 1$ for an ellipse, $e = 1$ for a parabola, and $e > 1$ for a hyperbola. Suppose observations of a newly discovered comet provide the data below. Determine the type of orbit, and predict where the comet will be when $\vartheta = 4.6$ (radians).[3]

ϑ	.88	1.10	1.42	1.77	2.14
r	3.00	2.30	1.65	1.25	1.01

13. [**M**] To measure the takeoff performance of an airplane, the horizontal position of the plane was measured every second, from $t = 0$ to $t = 12$. The positions (in feet) were: 0, 8.8, 29.9, 62.0, 104.7, 159.1, 222.0, 294.5, 380.4, 471.1, 571.7, 686.8, and 809.2.

a. Find the least-squares cubic curve $y = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$ for these data.

b. Use the result of part (a) to estimate the velocity of the plane when $t = 4.5$ seconds.

4. A biologist is doing an experiment on the growth of a certain bacteria culture. After 4 hours the following data has been recorded:

t	0	1	2	3	4
p	1.0	1.8	3.3	6.0	11.0

where t is the number of hours and p the population in thousands. Determine the least-squares exponential that best fits these data. Use your results to predict the population of bacteria after 5 hours.

6. A parachutist jumps from a plane and the distance of his drop is measured. Suppose that the distance of descent d as a function of time t can be modeled by

$$d = \alpha t + \beta t^2 e^{-0.1t}.$$

Find values of α and β that best fit the data in the table below

t	5	10	15	20	25	30
d	30	83	126	157	169	190

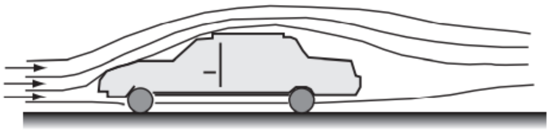
Página 09 - (Valor 1,0)

8. The population p of a small city during the period $[1960, 2000]$ is given by the table

t	1960	1970	1980	1990	2000
p	12600	14000	16100	19100	23200

Use the least-squares quadratic to predict the population in the year 2010.

25. The aerodynamic drag force F_D that is applied to a car is given by:



$$F_D = \frac{1}{2}\rho C_D A v^2$$

where $\rho = 1.2$ kg/m^3 is the air density, C_D is the drag coefficient, A is the projected front area of the car, and v is the speed of the car (in units of m/s) relative to the wind. The product $C_D A$ characterizes the air resistance of a car. (At speeds above 70 km/h the aerodynamic drag force is typically more than half of the total resistance to motion.) Data obtained in a wind tunnel test is displayed in the table. Use the data to determine the product $C_D A$ for the tested car using curve fitting. Make a plot of the data points and the curve-fitted equation.

v (km/h)	20	40	60	80	100	120	140	160
F_D (N)	10	50	109	180	300	420	565	771