

Trabalho 02 de Álgebra Linear

35. [M] Let $A = \begin{bmatrix} 7 & -9 & -4 & 5 & 3 & -3 & -7 \\ -4 & 6 & 7 & -2 & -6 & -5 & 5 \\ 5 & -7 & -6 & 5 & -6 & 2 & 8 \\ -3 & 5 & 8 & -1 & -7 & -4 & 8 \\ 6 & -8 & -5 & 4 & 4 & 9 & 3 \end{bmatrix}$.

- Construct matrices C and N whose columns are bases for $\text{Col } A$ and $\text{Nul } A$, respectively, and construct a matrix R whose rows form a basis for $\text{Row } A$.
 - Construct a matrix M whose columns form a basis for $\text{Nul } A^T$, form the matrices $S = [R^T \ N]$ and $T = [C \ M]$, and explain why S and T should be square. Verify that both S and T are invertible.
36. [M] Repeat Exercise 35 for a random integer-valued 6×7 matrix A whose rank is at most 4. One way to make A is to create a random integer-valued 6×4 matrix J and a random integer-valued 4×7 matrix K , and set $A = JK$. (See Supplementary Exercise 12 at the end of the chapter; and see the *Study Guide* for matrix-generating programs.)
37. [M] Let A be the matrix in Exercise 35. Construct a matrix C whose columns are the pivot columns of A , and construct a matrix R whose rows are the nonzero rows of the reduced echelon form of A . Compute CR , and discuss what you see.
38. [M] Repeat Exercise 37 for three random integer-valued 5×7 matrices A whose ranks are 5, 4, and 3. Make a conjecture about how CR is related to A for any matrix A . Prove your conjecture.

- 25.** A large apartment building is to be built using modular construction techniques. The arrangement of apartments on any particular floor is to be chosen from one of three basic floor plans. Plan A has 18 apartments on one floor, including 3 three-bedroom units, 7 two-bedroom units, and 8 one-bedroom units. Each floor of plan B includes 4 three-bedroom units, 4 two-bedroom units, and 8 one-bedroom units. Each floor of plan C includes 5 three-bedroom units, 3 two-bedroom units, and 9 one-bedroom units. Suppose the building contains a total of x_1 floors of plan A, x_2 floors of plan B, and x_3 floors of plan C.

- a. What interpretation can be given to the vector $x_1 \begin{bmatrix} 3 \\ 7 \\ 8 \end{bmatrix}$?
- b. Write a formal linear combination of vectors that expresses the total numbers of three-, two-, and one-bedroom apartments contained in the building.
- c. [M] Is it possible to design the building with exactly 66 three-bedroom units, 74 two-bedroom units, and 136 one-bedroom units? If so, is there more than one way to do it? Explain your answer.