Reading Quiz Section 2.2

1. Let $P(x)$ be the proposition $x^2 - 1 = 0$, with domain all real numbers. Which of the following statements are true?

   (a) $P(1)$ (b) $P(-1)$ (c) $P(3)$ (d) $P(x)$
   (e) $\forall x, P(x)$ (e) $\exists x, P(x)$ (f) $\neg(\forall x, P(x))$

2. A value x_0 in the domain of P for which $P(x_0)$ is *false* is known as a(n) ____________________

   (a) example (b) counterexample
   (c) realization (d) solution

3. Which of the following are equivalent to the given expression?

   $$\neg(\forall x, \exists y, P(x,y))$$

   (a) $\exists x, \forall y, P(x,y)$ (b) $\neg(\exists x, \forall y, P(x,y))$
   (c) $\exists x, \forall y, \neg P(x,y)$ (d) $\forall x, \exists y, \neg P(x,y)$

4. True or False: the order of quantifiers in an expression can always be switched without changing the meaning of the expression.

Practice Problems Section 2.2

1. Write each of the following using propositional functions and quantifiers. Make sure to define any propositional functions you are using.

   (a) Every class has an instructor.
   (b) For all real numbers x and y, if x and y are positive, then there exists a positive integer n such that $nx > y$.
   (c) For each positive integer n, there exists a real number which is positive and is less than $\frac{1}{n}$.

   Video Solution

2. Negate each proposition (a), (b), (c) in the previous problem.

   Video Solution

3. Which of the following propositions are true and which false? Justify your answers.

   (a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ y^4 = 4x$
   (b) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R},\ y^4 = 4x$
   (c) $\forall y \in \mathbb{R}, \exists x \in \mathbb{R},\ y^4 = 4x$
   (d) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R},\ y^4 = 4x$

   Video Solution