Reading Quiz Section 7.4

1. Which of the following are true and which false?

   (a) $[28] = [5]$ in $\mathbb{Z}_6$.

   (b) $[24] + \big([3] + [17]\big) = [-10]$ in $\mathbb{Z}_9$.

   (c) $[2]^3 + [3]^3 = [4]^3$ in $\mathbb{Z}_{29}$.

2. Is the following True or False?

   $$\text{In } \mathbb{Z}_n, \quad [x] + [y] = [z] \iff x + y = z$$

3. Let $\sim$ be an equivalence relation on X. What does it mean for a function $f : {}^{X}\!/_{\sim} \to B$ to be well-defined?

   (a) f is injective.

   (b) $[x] = [y]$ if and only if $f([x]) = f([y])$.

   (c) f is surjective.

   (d) $x \sim y$ implies $f([x]) = f([y])$.

4. True or False: the rule $[x] \mapsto x : \mathbb{Z}_n \to \mathbb{Z}$ is well-defined.

Practice Problems Section 7.4

1. Working in $\mathbb{Z}_n$, use Bézout's identity to prove that

   $$\big(\exists b \text{ such that } [a] \cdot [b] = [1]\big) \iff \gcd(a, n) = 1$$

2. Let k be a constant integer. If $f\big([x]_5\big) = [kx]_{18}$ is a well-defined function $f : \mathbb{Z}_5 \to \mathbb{Z}_{18}$, what can you say about the sequence of values

   $$f([0]_5), f([1]_5), f([2]_5), \ldots ?$$

   That is, when must they start repeating themselves?

3. In Exercise 7.4.13 show that F is surjective if and only if f is a bijection.