Reading Quiz Section 2.4

1. When proving a non-existence statement, i.e., proving that something *does not* exist, proof by contradiction is often useful because ________________________.

   (a) Contradiction is more powerful than a direct proof.

   (b) Direct and contrapositive proofs are too complicated.

   (c) It allows one to assume such an object exists, hence giving an object that can be manipulated.

   (d) It allows one to assume such an object does not exist, which is exactly what the problem is asking for.

2. In the proof that $\sqrt{2}$ is irrational, we started by assuming that $\sqrt{2} = \frac{m}{n}$ for integers m and n *with no common factors*. Why is this justified?

   (a) Because no pair of integers ever has a common factor.

   (b) Because any rational number $\frac{m}{n}$ can be seen, by canceling any common factors of m and n, to be equal to a rational $\frac{m'}{n'}$ where m' and n' have no common factors.

   (c) It is not justified, we have lost generality by making this assumption.

   (d) Because $\sqrt{2}$ is irrational.

Practice Problems Section 2.4

1. Let n be an integer. Prove: for n to be odd it is sufficient that its ones/units digit be odd.

   Video Solution

2. Critique the following proof. If the proof adequately demonstrates why the statement is true, explain why. Otherwise, identify any errors and explain how to correct them.

Theorem. *If x is a positive real number, then $x > 1$ if and only if $1/x < 1$.*

Proof. Suppose $1/x < 1$. Since x is positive, multiplying both sides of this inequality by x does not reverse the inequality and we obtain $1 < x$. ■

Video Solution