Reading Quiz Section 2.4

1. When proving a non-existence statement, proof by contradiction is often useful because:
   (a) Contradiction is more powerful than direct proof.
   (b) Direct and contrapositive proofs are too complicated.
   (c) It allows us to assume such an object exists, hence providing an object that may be manipulated.
   (d) It allows us to assume such an object does not exist, which is what the problem is asking for.

2. In the proof that $\sqrt{2}$ is irrational, we assumed that $\sqrt{2} = \frac{m}{n}$ for integers m and n *with no common factors*. Why is this justified?
   (a) Because no pair of integers ever has a common factor.
   (b) Because any rational number $\frac{m}{n}$ can be seen, by canceling any common factors of m and n, to be equal to a rational $\frac{m'}{n'}$ where m' and n' have no common factors.
   (c) It is not justified, we have lost generality by making this assumption.
   (d) Because $\sqrt{2}$ is irrational.

Questions 3 and 4 are to help revise this chapter. Can you answer these without writing anything down? Can you persuade a friend that you are correct?

3. We say that an integer y is *snake-like* if and only if there is some integer k such that $y = (6k)^2 + 9$.
   (a) Give three examples and three non-examples of snake-like integers.
   (b) Given $y \in \mathbb{Z}$, state the negation of the statement, "y is snake-like."
   (c) Show that every snake-like integer is a multiple of 9.
   (d) Show that the statements, "n is snake-like," and, "n is a multiple of 9," are not equivalent.

4. You meet three old men, Alain, Boris, and César, each of whom is a Truthteller or a Liar. Truthtellers speak only the truth; Liars speak only lies.

   You ask Alain whether he is a Truthteller or a Liar. Alain answers with his back turned, so you cannot hear what he says.

   > "What did he say?" you ask Boris.
   > Boris replies, "Alain says he is a Truthteller."
   > César says, "Boris is lying."

   Is César a Truthteller or a Liar? Explain your answer.

Practice Problems Section 2.4

1. Prove: For an integer n to be odd it is sufficient that its ones/units digit be odd.

   Video Solution

2. Consider the following claim and its "proof."

   Theorem. *If x is a positive real number, then $x > 1$ if and only if $\frac{1}{x} < 1$.*

   Proof. Suppose $\frac{1}{x} < 1$. Since x is positive, multiplying both sides of this inequality by x does not reverse the inequality and we obtain $1 < x$. ■

   If the proof adequately demonstrates why the statement is true, explain why. Otherwise, identify any errors and explain how to correct them.

   Video Solution