Reading Quiz Section 7.2

1. What does it mean for a *relation* $\mathcal{R} \subseteq A \times B$ to be a *function*? Select all that apply.
   (a) $\text{dom}(\mathcal{R}) = A$
   (b) $\text{range}(\mathcal{R}) = B$
   (c) For any $a \in A$, if $(a, b_1), (a, b_2) \in \mathcal{R}$, then $b_1 = b_2$
   (d) For any $b \in \text{range}(\mathcal{R})$, if $(a_1, b), (a_2, b) \in \mathcal{R}$, then $a_1 = a_2$

2. Let $f : A \to B$ be a function. If $f^{-1} : B \to A$ is a function, this means in particular that $\text{dom}(f^{-1}) = B$. This is equivalent to what property of f?
   (a) Injectivity
   (b) Surjectivity
   (c) $\text{dom}(f) = A$
   (d) That f is a symmetric relation.

3. True or False: a relation $\mathcal{R}$ has a domain and range if and only if it is a function.

Practice Problems Section 7.2

1. Let $f : A \to A$ be a function. Viewed as a relation, if f is symmetric, what can be said about f?

2. (a) Express the function $f : \mathbb{R} \to \mathbb{R} : x \mapsto x^2$ as a relation.
   (b) What is the inverse relation f^{-1}?
   (c) Use Definition 7.6 to prove that the relation f^{-1} is *not* a function.
   (d) Prove directly from Definition 4.18 that f is not injective and not surjective. Compare your arguments with your answer to part (c).