Reading Quiz Section 4.3

1. The range of a function $f : A \to B$ is (select all that apply):
   (a) A subset of the domain.
   (b) A subset of the codomain.
   (c) Always equal to the codomain.
   (d) Also called the image of the function.
   (e) Equal to $f(A)$.

2. Suppose $f : A \to B$ and $g : B \to C$ are functions. If $g \circ f$ is bijective, which of the following *must* be true?
   (a) f is injective.
   (b) g is injective.
   (c) f is surjective.
   (d) g is surjective.

3. True or False: We can always make a function surjective by making its domain smaller.

4. True or False: If $A \subseteq B$, there is an injective function $f : A \to B$.

Practice Problems Section 4.3

1. (a) Explain why the 'rule' $g : \{\text{all lines in the plane}\} \to \mathbb{R}$ which sends a line ℓ to the slope of ℓ does *not* define a function.
   Video Solution
   (b) Let L be the set of all *non-vertical* lines in the plane. The rule $f : L \to \mathbb{R}$ sending ℓ to its slope is a well-defined function.
      i. Find $f(Z)$ where Z is the set of lines intersecting $y = 2x + 5$ at exactly one point.
      Video Solution
      ii. Let $U = \{-2\}$. Describe the inverse image $f^{-1}(U)$.
      Video Solution
      iii. Explain why f is not bijective. Find a subset $B \subseteq L$ so that $f : B \to \mathbb{R}$ is a bijection.
      Video Solution 1   Video Solution 2

2. Suppose $f : A \to B$ and $g : B \to C$ are functions. For each of the following, either find an example or explain why no such example exists.
   (a) f surjective, g not surjective and $g \circ f$ surjective.
   (b) f not surjective, g surjective and $g \circ f$ surjective.
   (c) f surjective, g surjective and $g \circ f$ not surjective.
   (d) f injective, g not injective and $g \circ f$ injective.
   (e) f not injective, g injective and $g \circ f$ injective.
   (f) f injective, g injective and $g \circ f$ not injective.

   Video Solution (parts (a)-(c))

3. Suppose $f : A \to B$ is a function. Prove or disprove the following statements:
   (a) Let X and Y be subsets of A. If $X \cap Y = \emptyset$ then $f(X) \cap f(Y) = \emptyset$.
   (b) Let W and Z be subsets of B. If $W \cap Z = \emptyset$ then $f^{-1}(W) \cap f^{-1}(Z) = \emptyset$.

   Video Solution