

Reading Quiz Section 2.4

1. When proving a non-existence statement, proof by contradiction is often useful because:
 - (a) Contradiction is more powerful than direct proof.
 - (b) Direct and contrapositive proofs are too complicated.
 - (c) It allows us to assume such an object exists, hence providing an object that may be manipulated.
 - (d) It allows us to assume such an object does not exist, which is what the problem is asking for.
2. State the following non-existence result in five different ways (as in Example 2.32).

There are no real numbers whose square is -1 .

3. In the proof that $\sqrt{2}$ is irrational, we assumed that $\sqrt{2} = \frac{m}{n}$ for integers m and n with no common factors. Why is this justified?
 - (a) Because no pair of integers ever has a common factor.
 - (b) Because any rational number $\frac{m}{n}$ can be seen, by canceling any common factors of m and n , to be equal to a rational $\frac{m'}{n'}$ where m' and n' have no common factors.
 - (c) It is not justified, we have lost generality by making this assumption.
 - (d) Because $\sqrt{2}$ is irrational.

Questions 3 and 4 are to help revise this chapter. Can you answer these without writing anything down? Can you persuade a friend that you are correct?

3. We say that an integer y is *snake-like* if and only if there is some integer k such that $y = (6k)^2 + 9$.
 - (a) Give three examples and three non-examples of snake-like integers.
 - (b) Given $y \in \mathbb{Z}$, state the negation of the statement, “ y is snake-like.”
 - (c) Show that every snake-like integer is a multiple of 9.
 - (d) Show that the statements, “ n is snake-like,” and, “ n is a multiple of 9,” are not equivalent.
4. You meet three old men, Alain, Boris, and César, each of whom is a Truthteller or a Liar. Truthtellers speak only the truth; Liars speak only lies.

You ask Alain whether he is a Truthteller or a Liar. Alain answers with his back turned, so you cannot hear what he says.

“What did he say?” you ask Boris.

Boris replies, “Alain says he is a Truthteller.”

César says, “Boris is lying.”

Is César a Truthteller or a Liar? Explain your answer.

Practice Problems Section 2.4

1. Prove: For an integer n to be odd it is sufficient that its ones/units digit be odd.

Video Solution

2. Consider the following claim and its “proof.”

Theorem. *If x is a positive real number, then $x > 1$ if and only if $\frac{1}{x} < 1$.*

Proof. Suppose $\frac{1}{x} < 1$. Since x is positive, multiplying both sides of this inequality by x does not reverse the inequality and we obtain $1 < x$. ■

If the proof adequately demonstrates why the statement is true, explain why. Otherwise, identify any errors and explain how to correct them.

Video Solution