Reading Quiz Section 5.1

1. In an induction proof of the fact that $P(n)$ is true for all $n \in \mathbb{N}$, the base case consists of proving that

   (a) $P(1)$ is false.

   (b) $P(1)$ is true.

   (c) For all n, $P(n) \implies P(n+1)$.

   (d) $P(1) \implies P(2)$.

2. In an induction proof of the fact that $P(n)$ is true for all $n \in \mathbb{N}$, the induction hypothesis is the assumption that

   (a) $P(1)$ is true.

   (b) For all n, $P(n) \implies P(n+1)$.

   (c) $P(n)$ is true for some fixed $n \in \mathbb{N}$.

   (d) $P(n)$ is true for all $n \in \mathbb{N}$.

3. True or False: in formal proofs, it is acceptable to write

$$P(n) = \sum_{i=1}^{n} k = \frac{1}{2}n(n+1)$$

   as shorthand for "$P(n)$ is the proposition $\sum_{i=1}^{n} k = \frac{1}{2}n(n+1)$."

Practice Problems Section 5.1

1. (a) Prove by induction that $\forall n \in \mathbb{N}$ we have $3 \mid (2^n + 2^{n+1})$.

   (b) Give a direct proof that $3 \mid (2^n + 2^{n+1})$ for all integers $n \geq 1$ *and* for $n = 0$.

   (c) Look carefully at your proof for part (a). If you had started with the base case $n = 0$ instead of $n = 1$, would your proof still be valid?
   Video Solution