## Reading Quiz Section 5.2

1. Which of the following statements are true? Select all that apply.

   (a) Every well-ordered set of real numbers has a minimum element.

   (b) If a set of real numbers has a minimum element, then it is well-ordered.

   (c) Any finite set of real numbers is well-ordered.

   (d) Induction proofs must have a base case of 0 or 1.

2. True or false: the set $A = \{\frac{1}{n} : n \in \mathbb{N}\}$ is well-ordered.

3. For each $n \in \mathbb{N}$, let $P(n)$ and $Q(n)$ be propositions.

   (a) Let $s$ be the smallest natural number such that $P(s)$ is false. What can you say about the elements of the set $A = \{n \in \mathbb{N} : n < s\}$ with respect to the property $P$?

   (b) Let $a$ be minimal such that $P(a) \vee Q(a)$ is false. What can you say about the elements of the set $B = \{n \in \mathbb{N} : n < a\}$ with respect to the properties $P$ and $Q$?

   (c) Let $u$ be minimal such that $P(u) \wedge Q(u)$ is false. What can you say about the elements of the set $C = \{n \in \mathbb{N} : n < u\}$ with respect to the properties $P$ and $Q$?

   (d) Assume that $P(1)$ is true, but that "$\forall n \in \mathbb{N},\ P(n)$" is false. Explain why there exists a natural number $k$ such that the implication $P(k) \implies P(k+1)$ is *false.*

4. Here is an argument attempting to justify $\sum\limits_{i=1}^{n} i = \frac{1}{2}n(n+1) + 7$. What is wrong with it?

   Assume that the statement is true for some fixed $n$. Then

   $$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1) = \frac{1}{2}n(n+1) + 7 + (n+1) = \frac{1}{2}(n+1)[(n+1)+1] + 7$$

   hence the statement is true for $n+1$ and, by induction, for all $n \in \mathbb{N}$.

## Practice Problems Section 5.1

1. Prove that $n! > 2^n$ for all $n \geq 4$.

   Video Solution