Reading Quiz Section 8.1

1. A set A is *countably infinite* (or *denumerable*) if ________________. Select all that apply.

   (a) There exists a surjection from $\mathbb{N}$ onto A.

   (b) There exists an injection from $\mathbb{N}$ into A.

   (c) There exists a bijection between A and $\mathbb{Q}$.

   (d) There exists an injection from A into $\mathbb{N}$ and no injection from A into any finite set.

2. True or False: if A is a proper subset of B, then A has strictly smaller cardinality than B.

Practice Problems Section 8.1

1. Show that the set of *perfect squares* $A = \{n^2 : n \in \mathbb{N}\}$ is countably infinite.

2. Find an injective function $f : \mathbb{N} \to (0, 1)$.

3. Let $a, b \in \mathbb{R}$ with $a < b$. Find a bijective function $g : (0, 1) \to (a, b)$ and show that your function is bijective. Hence conclude that any two finite-length open intervals in $\mathbb{R}$ have the same cardinality.

4. Let B be a subset of A. Suppose B is countably infinite and let $a \in A \setminus B$. Show that $B \cup \{a\}$ is countably infinite.

   (*Hint: let* $h : B \to \mathbb{N}$ *be bijective: how might you construct* $k : B \cup \{a\} \to \mathbb{N}$ *bijective?*)

5. Let A be a countably infinite set.

   (a) Prove that the Cartesian product $A \times A$ is countably infinite.

   (b) By induction, prove that $\underbrace{A \times \cdots \times A}_{n \text{ times}}$ is countably infinite, for all $n \in \mathbb{N}$.