Reading Quiz Section 6.3

1. Let I be a set and $\{A_n : n \in I\}$ a family of sets indexed by I. The definition of $\bigcup_{n \in I} A_n$ uses the ____________ quantifier and the definition of $\bigcap_{n \in I} A_n$ uses the ____________ quantifier.

   (a) existential; existential
   (b) existential; universal
   (c) universal; existential
   (d) universal; universal

2. Let $\{A_n : n \in I\}$ be a nested collection of sets where $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots$. What can you conclude? Select all that apply.

   (a) $\bigcap_{n \in I} A_n \neq \emptyset$.
   (b) $\bigcup_{n \in I} A_n = A_1$.
   (c) The collection of sets is pairwise disjoint.
   (d) Each A_n must be an interval.

3. True or False:

$$B \subseteq \bigcup_{n \in I} A_n \iff \forall n \in I,\ B \subseteq A_n$$

Practice Problems Section 6.3

1. For each non-negative real number $r \geq 0$ let

$$A_r = \left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = r^2\right\}$$

   (a) Describe each of the sets A_r geometrically.
   (b) Prove that $\bigcup_{r \in \mathbb{R}_0^+} A_r = \mathbb{R}^2$.

   Video Solution

2. Let $\{A_n : n \in I\}$ be an indexed collection of sets and B a set. Prove:

   (a) $\left(\bigcup_{n \in I} A_n\right) \cap B = \bigcup_{n \in I} (A_n \cap B)$
   (b) $\left(\bigcap_{n \in I} A_n\right) \cup B = \bigcap_{n \in I} (A_n \cup B)$

   Video Solution