Reading Quiz Section 2.1

1. A *tautology* is a proposition which ____________________

   (a) is false no matter what the truth value of its component propositions.
   (b) is only true when all of its component propositions are true.
   (c) is never false, no matter what the truth value of its component propositions.
   (d) is built only using the connectives $\wedge, \vee$.

2. A *contradiction* is a proposition which ____________________

   (a) is false no matter what the truth value of its component propositions.
   (b) is only true when all of its component propositions are false.
   (c) is never false, no matter what the truth value of its component propositions.
   (d) is built only using the connective $\neg$.

3. The *contrapositive* of the conditional $P \implies Q$ is the conditional _________________

   (a) $\neg P \implies Q$
   (b) $\neg Q \implies \neg P$
   (c) $Q \implies P$
   (d) $P \implies \neg Q$

4. True or False: The *converse* of $P \implies Q$ is logically equivalent to $P \implies Q$.

5. The *negation* of the conditional "if I study at least 25 hours per week, then I will be successful" is the proposition _________________

   (a) "I study at least 25 hours per week, but I am not successful."
   (b) "Either I study less than 25 hours per week, or I am successful."
   (c) "Either I study at least 25 hours per week, or I am not successful."
   (d) 'If I am successful, then I will study at least 25 hours per week."

6. De Morgan's laws state that:

   $\neg(P \vee Q)$ is logically equivalent to (1)_______________
   $\neg(P \wedge Q)$ is logically equivalent to (2)_______________

   (a) (1) $\neg(P \implies Q)$, (2) $(\neg P \vee \neg Q)$
   (b) (1) $(\neg P \wedge \neg Q)$, (2) $(P \vee Q)$
   (c) (1) $(\neg P \wedge Q)$, (2) $(P \vee \neg Q)$
   (d) (1) $(\neg P \wedge \neg Q)$, (2) $(\neg P \vee \neg Q)$

Practice Problems Section 2.1

1. Suppose that "If Colin was early, then no-one was playing pool" is a true statement.

   (a) What is its contrapositive of this statement? Is it true?

   (b) What is the converse? Is it true?

   (c) What can we conclude (if anything?) if we discover each of the following? *Treat the two scenarios separately.*

      (i) Someone was playing pool.

      (ii) Colin was late.

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2. Prove that $P \vee \neg Q$ is logically equivalent to $\neg P \implies (\neg P \wedge \neg Q)$.

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3. Define the connective $\uparrow$ (called the *Sheffer stroke*, or *NAND*) by the following truth table:

P	Q	$P \uparrow Q$
T	T	F
T	F	T
F	T	T
F	F	T

   (a) Prove $P \uparrow Q$ is logically equivalent to $\neg(P \wedge Q)$.

   (b) Find an expression built using only P and the connective $\uparrow$ which is logically equivalent to $\neg P$.

   (c) Find an expression built using only P, Q, and the connective $\uparrow$ which is logically equivalent to $P \wedge Q$.

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