Reading Quiz Section 7.1

1. A relation $\mathcal{R} \subseteq A \times B$ is ________________

   (a) a nonempty subset of $A \times B$
   (b) a proper subset of $A \times B$
   (c) a function from A to B
   (d) a subset of $A \times B$

2. If $A \subseteq \mathbb{R}$, then the graph of a symmetric relation $\mathcal{R} \subseteq A \times A$ has what kind of symmetry?

   (a) reflection symmetry across the x-axis
   (b) reflection symmetry across the y-axis
   (c) reflection symmetry across the line $y = x$
   (d) symmetry across the origin

3. True or False: if $\mathcal{R}$ is symmetric, then it must contain an even number of elements.

Practice Problems Section 7.1

1. Given constants a, b, c, let $L_{a,b,c} = \{(x, y) : ax + by = c\} \subseteq \mathbb{R}^2$.

   (a) Describe $L_{a,b,c}$ geometrically.
   (b) Let $A = \mathbb{R}^2$ and $B = \{L_{a,b,c} : a, b, c \in \mathbb{R}\}$. Define $\mathcal{R} \subseteq A \times B$ by

   $$(x, y)\, \mathcal{R}\, L_{a,b,c} \iff ax + by = c$$

   Determine whether each of the following is true or false.

   i. $(1, 0)\, \mathcal{R}\, L_{1,1,1}$
   ii. $(3, -2) \not\mathcal{R}\, L_{1,1,1}$
   iii. If $(x, y)\, \mathcal{R}\, L_{a,b,c}$ and $(x, y)\, \mathcal{R}\, L_{d,e,f}$ for some (x, y) then $L_{a,b,c} = L_{d,e,f}$
   iv. Suppose $(x, y) \not\mathcal{R}\, L_{a,b,c}$. Then there exists $d, e, f \in \mathbb{R}$ such that

   $$(x, y)\, \mathcal{R}\, L_{d,e,f} \text{ and } L_{a,b,c} \cap L_{d,e,f} = \varnothing$$

2. Let X be a set. Let $\mathcal{R} \subseteq \mathcal{P}(X) \times \mathcal{P}(X)$ be the relation $A\, \mathcal{R}\, B \iff A \subseteq B$.

   (a) Show that $A\ (\mathcal{R} \cap \mathcal{R}^{-1})\ B$ implies $A = B$.
   (b) If $X = \{a, b\}$, compute $\mathcal{R}^{-1}$ explicitly as a set of ordered pairs.