Reading Quiz Section 2.3

1. In a *proof by contrapositive* of $P \implies Q$, we assume that (1) ________________ and deduce that (2) ________________.
   (a) (1) $\neg Q$ is true, (2) P is true
   (b) (1) Q is false, (2) P is true
   (c) (1) $\neg P$ is true, (2) $\neg Q$ is true
   (d) (1) $\neg Q$ is true, (2) $\neg P$ is true

2. A *proof by contradiction* of $P \implies Q$ begins by assuming that ____________________________.
   (a) $\neg P \vee Q$ is true
   (b) $P \wedge \neg Q$ is true
   (c) $P \implies Q$ is true
   (d) $Q \implies P$ is false

3. You wish to prove that $x^2 > 4 \implies x > -2$. How might a contradiction argument begin?
   (a) Suppose all real numbers x satisfy $x^2 > 4$ and $x > -2$.
   (b) Suppose all real numbers x satisfy $x^2 > 4$ and $x \leq -2$.
   (c) Suppose all real numbers x satisfy $x^2 \leq 4$ and $x > -2$.
   (d) Suppose x is a real number satisfying $x^2 > 4$ and $x > -2$.
   (e) Suppose x is a real number satisfying $x^2 > 4$ and $x \leq -2$.
   (f) Suppose x is a real number satisfying $x^2 \leq 4$ and $x > -2$.

4. In which of the following situations would it be correct to invoke *without loss of generality*? Select all that apply.
   (a) Suppose we are attempting to prove that for two integers m and n, if either one is even, then so is the product. Without loss of generality we can assume that n is even.
   (b) We are trying to prove that for two integers m and n, if both are odd, then so is the product. Without loss of generality we can assume that both m and n are equal to $2k + 1$ for some integer k.
   (c) Attempting to prove that if m is even and n is odd, then mn is even. Without loss of generality we assume that $m = 2$.
   (d) Attempting to prove that if three boxes are painted either green or gold, there must be two boxes which are painted the same color. Without loss of generality, we can assume that the first box is painted green.

Practice Problems Section 2.3

1. Let x and y be integers. Prove: For $x^2 + y^2$ to be even, it is necessary that x and y have the same parity (both even or both odd).
   Video Solution

2. Prove or disprove the following conjectures:
   (a) The sum of any 3 consecutive integers is divisible by 3.
   (b) The sum of any 4 consecutive integers is divisible by 4.
   (c) The product of any 3 consecutive integers is divisible by 6.

   Video Solution