Reading Quiz Section 3.1

1. Which of the following connectives makes the following true for any integers a, b and $n > 0$?

   $$a \equiv b \pmod{n} \quad \_\_\_\_\_ \quad a = b$$

   (a) $\implies$
   (b) $\impliedby$
   (c) $\iff$
   (d) $\wedge$

2. Let $m, n \in \mathbb{Z}$ where $n > 0$. Is it possible that there are multiple pairs of integers q and r such that $m = qn + r$ and $0 \leq r < n$?

   (a) It is never possible.
   (b) It is sometimes possible, depending on what m and n are.
   (c) It is always possible.

3. Which of the following are true statements for all integers a, b and $n > 0$? Select all that apply.

   (a) a is congruent to exactly one of $0, 1, \ldots, n-1$ modulo n.
   (b) a can be congruent to more than one of $0, 1, \ldots, n-1$ modulo n.
   (c) a is divisible by n if and only if $a \equiv 0 \pmod{n}$.
   (d) $n \equiv 0 \pmod{n}$.

Practice Problems Section 3.1

1. Use the Division algorithm to show that any prime number $p \geq 5$ must have remainder 1 or 5 on division by 6. Use this to show that $p^2 + 2$ is composite for all such primes p.

   Video Solution

2. Find the remainder of $57^{33} + 42^{100}$ upon division by 6.

   Video Solution

3. Prove that $n^2 \equiv 0 \pmod{4}$ or $n^2 \equiv 1 \pmod{4}$ for all $n \in \mathbb{Z}$.

   Video Solution