Deep Learning Midterm

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Problem Set 1 - Conv Layer and its gradient

Question 1

Design a 3×3 convolutional filter to detect the edges in an image.

Answer

There are many filters to detect the edges of a image. These are based on the gradient of the image. I will list the most common Sobel filter used in edge detection.

The Sobel filter is a 3×3 filter that is used to detect the edges in an image. It is based on the gradient of the image. The filter is defined as follows:

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

The G_x filter is used to detect the edges in the horizontal direction and the G_y filter is used to detect the edges in the vertical direction. We can also combine these two filters into a matrix G which is the magnitude of the gradient of the image.

$$G = \sqrt{G_x^2 + G_y^2}$$

Question 2

Denote the input 2D image as **I**(x, y), output image due to edge detector as **O**(x, y). Suppose we incurred a loss l on **O**. The gradient of l with respect to **O**(x, y) is $\frac{\partial l}{\partial O(x,y)}$. Derive the gradient of loss l with respect to the input image **I**(x, y).

Answer

The gradient of the loss l with respect to the input image **I**(x, y) can be derived using the chain rule. The chain rule states that the derivative of a composite function is the product of the derivatives of the individual functions. In this case, the loss l is a function of the output image **O**(x, y) and the output image **O**(x, y) is a function of the input image **I**(x, y). Therefore, the gradient of the loss l with respect to the input image **I**(x, y) can be derived using the chain rule as follows:

$$\frac{\partial l}{\partial I(x,y)} = \frac{\partial l}{\partial O(x,y)} \cdot \frac{\partial O(x,y)}{\partial I(x,y)}$$

Question 3

If we stack many convolution and ReLU layers after **O**, what would be a potential issue if we trained this network using gradient descent? How could we mitigate this issue?

Answer

There might be any number of potential issues when this kind of network is trained. I have listed some of them I could think of.

1. **Explosion of Feature Maps**: The number of feature maps can explode as we stack more and more convolutional layers. This can lead to overfitting and slow training times. To mitigate this issue, we can use techniques such as batch normalization and dropout to regularize the network and prevent overfitting. We can also use techniques like pooling to reduce the number of feature maps and speed up training.
2. **Vanishing Gradients**: As we stack more and more convolutional layers, the gradients can become very small and vanish. This can lead to slow training times and poor performance. To mitigate this issue, we can use techniques such as skip connections and residual connections to allow the gradients to flow more easily through the network. We can also use techniques such as gradient clipping to prevent the gradients from becoming too small.
3. **Overfitting**: As we stack more and more convolutional layers, the network can become very complex and overfit the training data. This can lead to poor generalization and poor performance on new data. To mitigate this issue, we can use techniques such as dropout and regularization to prevent overfitting. We can also use techniques such as data augmentation to increase the size of the training data and prevent overfitting.

Problem Set 2 - Backpropagation through Time

Consider a simple RNN model with scalar input x_t and scalar hidden state

$$h_t = a \cdot x_t + b \cdot h_{t-1}$$

where $1 \leq t \leq T$, we incur some loss l on the last hidden state h_T.

Question 1

Denote the gradient of l w.r.t h_T as $\frac{\partial l}{\partial h_T}$. Derive the gradient of l w.r.t a and b.

Answer

The gradient of the loss l with respect to the parameters a and b can be derived using the chain rule. The chain rule states that the derivative of a composite function is the product of the derivatives of the individual functions. In this case, the loss l is a function of the last hidden state h_T and the last hidden state h_T is a function of the parameters a and b. Therefore, the gradient of the loss l with respect to the parameters a and b can be derived using the chain rule as follows:

$$\frac{\partial l}{\partial a} = \frac{\partial l}{\partial h_T} \cdot \frac{\partial h_T}{\partial a}$$

But we can derive the second term as follows:

$$\frac{\partial h_T}{\partial a} = x_T$$

So substituting this back into the first equation we get:

$$\frac{\partial l}{\partial a} = \frac{\partial l}{\partial h_T} \cdot x_T$$

$$\frac{\partial l}{\partial b} = \frac{\partial l}{\partial h_T} \cdot \frac{\partial h_T}{\partial b}$$

Similarly, we can derive the second term as follows:

$$\frac{\partial h_T}{\partial b} = h_{T-1}$$

So substituting this back into the first equation we get:

$$\frac{\partial l}{\partial b} = \frac{\partial l}{\partial h_T} \cdot h_{T-1}$$

Question 2

What could be a potential issue if T is very large? How could we mitigate this issue?

Answer

If T is very large, then when we unroll the network and train using backpropagation through time, the gradients can become very small and vanish. This can lead to slow training times and poor performance. To mitigate this issue, we can use techniques such as gradient clipping to prevent the gradients from becoming too small. We can also use techniques such as skip connections and residual connections to allow the gradients to flow more easily through the network. We can also use techniques such as batch normalization and dropout to regularize the network and prevent overfitting.

Another solution is to use a different type of RNN cell like LSTM or GRU which are designed to mitigate the vanishing gradient problem. These cells have gating mechanisms that allow the gradients to flow more easily through the network and prevent them from becoming too small.

Problem Set 3 - Postional Encoding

Question 1

Describe when and why we need Positional Encoding in the context of Transformer model.

Answer

The Transformer model is a type of neural network architecture that is used for natural language processing tasks such as machine translation and language modeling. The Transformer model is based on the self-attention mechanism, which allows the model to attend to different parts of the input sequence at different positions. However, the self-attention mechanism does not have any notion of position, so it cannot distinguish between different positions in the input sequence. This can be a problem for tasks such as machine translation, where the position of a word in the input sequence can affect its meaning.

To address this issue, the Transformer model uses positional encoding to inject information about the position of each token in the input sequence. Positional encoding is added to the input embeddings of the tokens, so that the model can distinguish between different positions in the input sequence. This allows the model to attend to different parts of the input sequence at different positions, and helps the model to capture the positional information of the input sequence.

Question 2

Consider d-dimensional sinusoidal PE for the t-th position, defined as

$$p_t = [\ldots, \sin(w_k t), \cos(w_k t), \ldots], k = 0, \ldots, (d/2) - 1$$

where $w_k = \frac{1}{10000^{2k/d}}$. The PE is added to token embeddings x_t i.e. $y_t = x_t + p_t$.

Analyze the effect of PE. More specifically how does $y_t \dot{y}_\tau$ compare against $x_t \dot{x}_\tau$?

Discuss the impact of attention.

Answer

The positional encoding is added to the token embeddings to inject information about the position of each token in the input sequence. The positional encoding is a d-dimensional sinusoidal function that is added to the token embeddings, so that the model can distinguish between different positions in the input sequence. The positional encoding is defined as follows:

$$p_t = [\ldots, \sin(w_k t), \cos(w_k t), \ldots], k = 0, \ldots, (d/2) - 1$$

Where $w_k = \frac{1}{10000^{2k/d}}$. The positional encoding is added to the token embeddings as follows:

$$y_t = x_t + p_t$$

When we add these positional embeddings, the original token embeddings are modified to include information about the position of each token in the input sequence. So this essenstially creates a new embedding space where same words with different positions have different embeddings. This allows the model to distinguish between different positions in the input sequence and helps the model to capture the positional information of the input sequence.

When we multiply the positional embeddings y_t and y_{tau}, we are essentially comparing the similarity between the words and its position. When this is not done and we only use the token embeddings x_t and x_{tau}, the model cannot distinguish between different positions in the input sequence. This can be a problem for tasks such as machine translation, where the position of a word in the input sequence can affect its meaning. The positional encoding allows the model to attend to different parts of the input sequence at different positions, and helps the model to capture the positional information of the input sequence. This is especially important for the self-attention mechanism, which allows the model to attend to different parts of the input sequence at different positions.

Problem Set 4 - Model Design

You want to create a grammar error correction model.

Question 1

Sketch the overall model architecture, describe each building block and give as much detail as possible.

Answer

The overall model architecture consists of an encoder-decoder architecture with an attention mechanism. The encoder takes the input sentence and encodes it into a sequence of hidden states. The decoder takes the hidden states and generates the output sentence. The attention mechanism allows the decoder to focus on different parts of the input sentence at different time steps.

The encoder consists of an embedding layer, a stack of LSTM layers, and a self-attention mechanism. The embedding layer converts the input sentence into a sequence of word embeddings. The LSTM layers encode the word embeddings into a sequence of hidden states. The self-attention mechanism allows the encoder to focus on different parts of the input sentence at different time steps.

The decoder consists of an embedding layer, a stack of LSTM layers, and an attention mechanism. The embedding layer converts the output sentence into a sequence of word embeddings. The LSTM layers decode the word embeddings into a sequence of hidden states. The attention mechanism allows the decoder to focus on different parts of the input sentence at different time steps.

The attention mechanism consists of a query, key, and value matrix. The query matrix is the hidden state of the decoder at a given time step. The key matrix is the hidden states of the encoder. The value matrix is the hidden states of the encoder. The attention mechanism computes the attention weights by taking the dot product of the query and key matrices and then applying a softmax function. The attention weights are then used to compute the context vector, which is a weighted sum of the value matrix. The context vector is then concatenated with the hidden state of the decoder and used to generate the output word.

Question 2

What is the input and the output of the model? How would you train the model? Give as much detail as possible i.e. training loss, any tricks that can benefit the training process, etc.

Answer

The input of the model is a sequence of words representing a sentence with grammar errors. The output of the model is a sequence of words representing a corrected sentence.

The model is trained using a sequence-to-sequence loss function such as cross-entropy loss. The loss is computed between the predicted output sentence and the ground truth output sentence. The loss is then backpropagated through the model to update the parameters.

To train the model, we can use techniques such as teacher forcing, which involves using the ground truth output sentence as input to the decoder during training. This can help to stabilize the training process and prevent the model from getting stuck in local minima.

We can also use techniques such as scheduled sampling, which involves using the predicted output sentence as input to the decoder with a certain probability during training. This can help to improve the generalization of the model and prevent overfitting.

We can also use techniques such as beam search, which involves exploring multiple candidate output sentences during inference and selecting the one with the highest probability. This can help to improve the quality of the output sentences and prevent the model from getting stuck in local minima.

We can also use techniques such as attention regularization, which involves adding a regularization term to the loss function that encourages the attention weights to be sparse. This can help to improve the interpretability of the attention mechanism and prevent the model from overfitting.

Question 3

At test time, how would you generate the grammar corrected sentence? What metric would you use to evaluate the quality of the model?

Answer

At test time, we would generate the grammar corrected sentence using beam search. Beam search involves exploring multiple candidate output sentences and selecting the one with the highest probability. This can help to improve the quality of the output sentences and prevent the model from getting stuck in local minima.

We would use the BLEU score to evaluate the quality of the model. The BLEU score is a metric that measures the similarity between the predicted output sentence and the ground truth output sentence. It is based on the precision and recall of the n-grams in the predicted output sentence. A higher BLEU score indicates a higher quality of the model.

The formula for BLEU score is as follows:

$$BLEU = BP \cdot \exp(\sum_{n=1}^{N} w_n \cdot \log(p_n))$$

Where BP is the brevity penalty, w_n is the weight for n-grams, and p_n is the precision of n-grams. The brevity penalty is a term that penalizes the model for generating shorter output sentences. The weights for n-grams are used to give more importance to certain n-grams in the output sentence. The precision of n-grams is the ratio of the number of n-grams in the predicted output sentence that are also in the ground truth output sentence to the total number of n-grams in the predicted output sentence.

Another way of evaluating the model is to use this model for a downstream task like machine translation and evaluate the performance of the downstream task. This can give a better understanding of the model's performance in a real-world scenario.

Problem Set 5 - Optimization and Generalization

We use the MNIST dataset for this question. Please attach all your code. Details regarding the MNIST dataset can be found in HW-2. Again we use the following code to create training, validation and test splits.

```
import torch
from torchvision import datasets

train_all = datasets.MNIST('../data', train=True, download=True) #
60K images
train_data, val_data = torch.utils.data.random_split(train_all,
[50000, 10000], torch.Generator().manual_seed(0)) # train: 50K;
val: 10K
test_data = datasets.MNIST('../data', train=False) # 10K images
```

In [ ]:
```
import warnings

import matplotlib.pyplot as plt
import numpy as np
import torch
from PIL import Image
from torch import nn
from torchvision import datasets, transforms

# Filter warnings
warnings.filterwarnings("ignore")

# Set device for acceleration
DEVICE = (
    "cuda"
    if torch.cuda.is_available()
    else "mps"
    if torch.backends.mps.is_available()
    else "cpu"
)
```

```python
if DEVICE == "mps":
    torch.mps.empty_cache()
elif DEVICE == "cuda":
    torch.cuda.empty_cache()

print(
    "Using CPU for training and testing as no accelerator is available."
    if DEVICE == "cpu"
    else f"Using {DEVICE} for accleration."
)
```

```
Using mps for accleration.
```

In [ ]:
```python
HYPERPARAMETERS = {
    "batch_size": 128,
    "learning_rate": 0.01,
    "epochs": 15,
}
```

In [ ]:
```python
# Create one-hot encoding transformation for labels
one_hot_transform = transforms.Compose(
    [transforms.Lambda(lambda y: torch.zeros(10).scatter_(0, torch.tensor(y)
)

# Create a toTensor transformation for images
to_tensor_transform = transforms.Compose([transforms.ToTensor()])
```

In [ ]:
```python
train_all = datasets.MNIST(
    "../data",
    train=True,
    download=True,
    transform=to_tensor_transform,
    target_transform=one_hot_transform,
)
train_data, val_data = torch.utils.data.random_split(
    train_all, [50_000, 10_000], torch.Generator().manual_seed(0)
)
test_data = datasets.MNIST(
    "../data",
    train=False,
    download=True,
    transform=to_tensor_transform,
    target_transform=one_hot_transform,
)

# Create data loaders
train_loader = torch.utils.data.DataLoader(
    train_data, batch_size=HYPERPARAMETERS["batch_size"], shuffle=True
)
val_loader = torch.utils.data.DataLoader(
    val_data, batch_size=HYPERPARAMETERS["batch_size"], shuffle=True
)
test_loader = torch.utils.data.DataLoader(
```

```python
    test_data, batch_size=HYPERPARAMETERS["batch_size"], shuffle=True
)
```

In [ ]:
```python
def train(
    model, optimizer, criterion, train_loader, val_loader=None, epochs=15, c
):
    _epoch_wise_train_loss = []
    _epoch_wise_val_loss = []

    _epoch_wise_train_accuracy = []
    _epoch_wise_val_accuracy = []

    try:
        _n_batches = len(train_loader)
        _max_char_epoch, _max_char_batch = len(str(epochs)), len(str(_n_batc

        # BATCH TRAIN FORMAT STRING
        def _batch_train_message(i_epoch, i_batch):
            return f"Epoch {i_epoch:>{_max_char_epoch}}/{epochs} Batch: [{i_

        # CREATE EPOCH TRAIN FORMAT STRING
        def _epoch_train_message(
            i_epoch, i_batch, t_loss, t_accuracy, v_loss=None, v_accuracy=No
        ):
            return (
                f"Epoch {i_epoch:>{_max_char_epoch}}/{epochs} Batch: [{i_bat
                if v_loss is None or v_accuracy is None
                else f"Epoch {i_epoch:>{_max_char_epoch}}/{epochs} Batch: [{
            )

        # Move model to device
        model.to(device)
        print(f"Model moved to {device}.")

        # Train the model
        print("++++++++++ MODEL TRAINING STARTS ++++++++++")

        for epoch in range(1, epochs + 1):
            _batch_wise_train_loss = []
            _batch_wise_accuracy = []

            # Run batches
            for batch_idx, (data, labels) in enumerate(train_loader, 1):
                model.train()
                print(_batch_train_message(epoch, batch_idx), end="\r")

                # Move data to device
                data, labels = data.to(device), labels.to(device)

                # Zero the gradients
                optimizer.zero_grad()

                # Forward pass
```

```
            outputs = model(data)

            # Get accuracy
            batch_accuracy = torch.sum(
                torch.argmax(outputs, dim=1) == torch.argmax(labels, dim
            ).item() / len(labels)
            _batch_wise_accuracy.append(batch_accuracy)

            loss = criterion(outputs, labels)
            _batch_wise_train_loss.append(loss.item())

            loss.backward()
            optimizer.step()

            del data, labels, outputs, loss

        _t_loss = torch.mean(torch.tensor(_batch_wise_train_loss))
        _t_accuracy = torch.mean(torch.tensor(_batch_wise_accuracy))

        _epoch_wise_train_loss.append(_t_loss.item())
        _epoch_wise_train_accuracy.append(_t_accuracy)

        # Validation
        if val_loader:
            model.eval()

            with torch.no_grad():
                _batch_wise_val_loss = []
                _batch_wise_accuracy = []

                for data, labels in val_loader:
                    data, labels = data.to(device), labels.to(device)
                    outputs = model(data)

                    accuracy = torch.sum(
                        torch.argmax(outputs, dim=1) == torch.argmax(lab
                    ).item() / len(labels)
                    _batch_wise_accuracy.append(accuracy)

                    loss = criterion(outputs, labels)
                    _batch_wise_val_loss.append(loss.item())
                    _, predictions = torch.max(outputs, 1)

                    del data, labels, outputs, loss, predictions

                _v_loss = torch.mean(torch.tensor(_batch_wise_val_loss))
                _epoch_wise_val_loss.append(_v_loss.item())

                _v_accuracy = torch.mean(torch.tensor(_batch_wise_accura
                _epoch_wise_val_accuracy.append(_v_accuracy.item())

        print(
            _epoch_train_message(
```

```python
                    epoch, batch_idx, _t_loss, _t_accuracy, _v_loss, _v_accu
                )
            )

    except RuntimeError as re:
        print("++++++++++ MODEL TRAINING ENDS ++++++++++")
        print("Some error occurred. Training stopped.")
        print(re)
        return
    except KeyboardInterrupt:
        print("\n")
        print("++++++++++ MODEL TRAINING ENDS ++++++++++")
        print("Training interrupted.")

    print("++++++++++ MODEL TRAINING ENDS ++++++++++")
    print("Training completed.")
    return {
        "train_loss": _epoch_wise_train_loss,
        "val_loss": _epoch_wise_val_loss,
        "train_accuracy": _epoch_wise_train_accuracy,
        "val_accuracy": _epoch_wise_val_accuracy,
    }


def test(model, test_loader, criterion, device="cpu"):
    model.to(device)

    _n_test_batches = len(test_loader)
    _max_char_batch = len(str(_n_test_batches))

    _test_accuracy, _test_loss = [], []

    for batch_idx, (data, labels) in enumerate(test_loader, 1):
        # Set model to eval mode
        model.eval()

        # Log the batch testing
        print(f"Batch: [{batch_idx:>{_max_char_batch}}/{_n_test_batches}]",

        # Move data and label to device
        data, labels = data.to(device), labels.to(device)

        # Forward pass
        outputs = model(data)

        # Get accuracy
        _accuracy = torch.sum(
            torch.argmax(outputs) == torch.argmax(labels)
        ).item() / len(labels)

        _test_loss.append(criterion(outputs, labels).item())
        _test_accuracy.append(_accuracy)
```

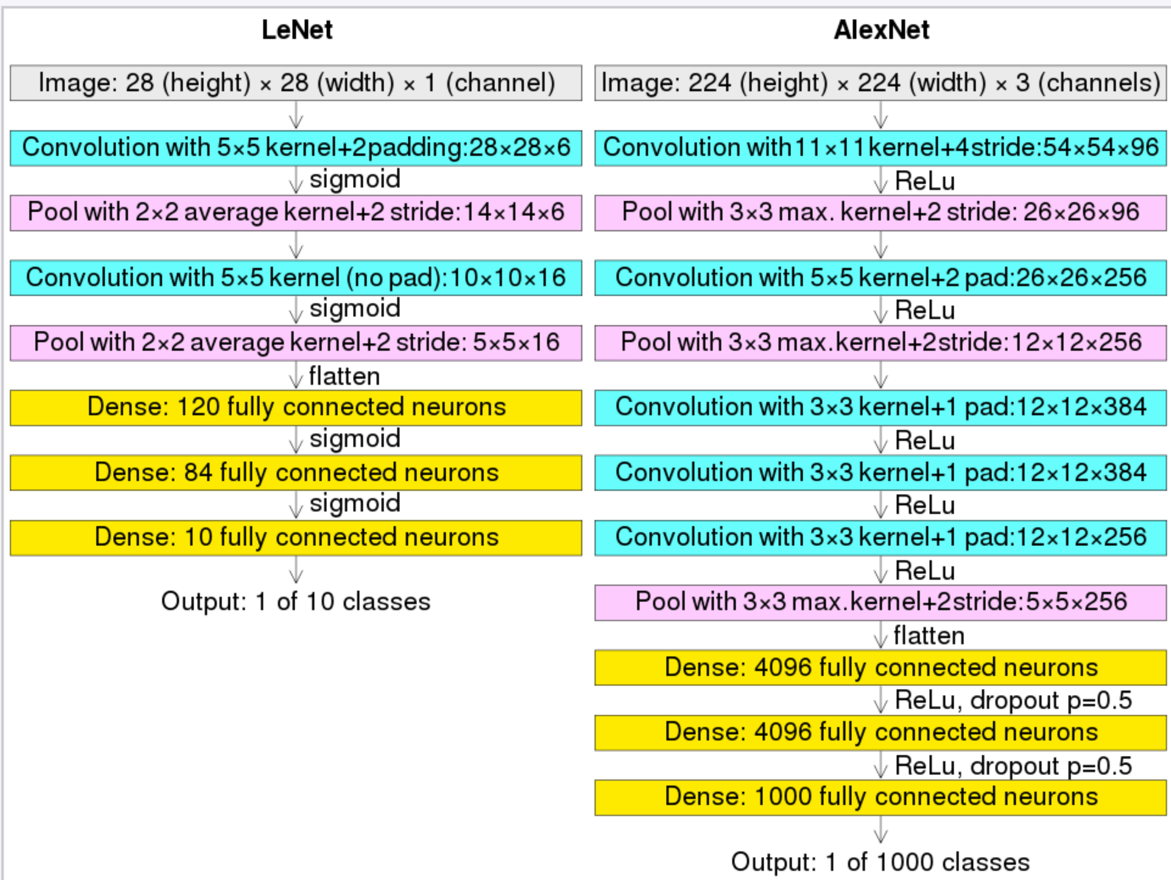
```
    # Finally move model back to CPU so that other models can use the GPU
    model.to("cpu")

    return {
        "test_loss": torch.mean(torch.tensor(_test_loss)).item(),
        "test_accuracy": torch.mean(torch.tensor(_test_accuracy)).item(),
    }
```

Question 1

Implement LeNet Architecture and train on all 50K training samples with vanilla SGD. Report the learning curves and the test accuracy.

Answer

The image of the LeNet architecture is as follows:



Comparison of the LeNet and AlexNet convolution, pooling, and dense layers (AlexNet image size should be 227×227×3, instead of 224×224×3, so the math will come out right. The original paper said different numbers, but Andrej Karpathy, the former head of computer vision at Tesla, said it should be 227×227×3 (he said Alex didn't describe why he put 224×224×3). The next convolution should be 11×11 with stride 4: 55×55×96 (instead of 54×54×96). It would be calculated, for example, as: [(input width 227 - kernel width 11) / stride 4] + 1 = [(227 - 11) / 4] + 1 = 55. Since the kernel output is the same length as width, its area is 55×55.)

In [ ]:

```
class LeNet(nn.Module):
    def __init__(self):
        super(LeNet, self).__init__()

        # Define the model architecture
        self.model = nn.Sequential(
            nn.Conv2d(1, 6, kernel_size=5, stride=1, padding=2),
            nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.Conv2d(6, 16, kernel_size=5, stride=1),
            nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.Flatten(),
            nn.Linear(in_features=16 * 5 * 5, out_features=120),
            nn.Sigmoid(),
            nn.Linear(in_features=120, out_features=84),
            nn.Linear(in_features=84, out_features=10),
            nn.Softmax(dim=1),
        )

    def forward(self, x):
        # Model returns the probability of each class
        return self.model(x)


le_net = LeNet()

# Define the loss function
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(le_net.parameters(), lr=HYPERPARAMETERS["learnir

q1_history = train(
    le_net,
    optimizer,
    criterion,
    train_loader,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)

q1_test = test(le_net, test_loader, criterion, device=DEVICE)
```

```
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [391/391] Train Loss: 2.3026 Train Accuracy: 0.1003 Val L
oss: 2.3024 Val Accuracy: 0.0981
Epoch  2/15 Batch: [391/391] Train Loss: 2.3024 Train Accuracy: 0.1028 Val L
oss: 2.3023 Val Accuracy: 0.0988
Epoch  3/15 Batch: [391/391] Train Loss: 2.3022 Train Accuracy: 0.1111 Val L
oss: 2.3022 Val Accuracy: 0.1108
Epoch  4/15 Batch: [391/391] Train Loss: 2.3021 Train Accuracy: 0.1129 Val L
oss: 2.3021 Val Accuracy: 0.1101
Epoch  5/15 Batch: [391/391] Train Loss: 2.3019 Train Accuracy: 0.1129 Val L
oss: 2.3020 Val Accuracy: 0.1101
Epoch  6/15 Batch: [391/391] Train Loss: 2.3018 Train Accuracy: 0.1129 Val L
oss: 2.3019 Val Accuracy: 0.1101
Epoch  7/15 Batch: [391/391] Train Loss: 2.3016 Train Accuracy: 0.1128 Val L
oss: 2.3019 Val Accuracy: 0.1101
Epoch  8/15 Batch: [391/391] Train Loss: 2.3015 Train Accuracy: 0.1128 Val L
oss: 2.3018 Val Accuracy: 0.1108
Epoch  9/15 Batch: [391/391] Train Loss: 2.3015 Train Accuracy: 0.1129 Val L
oss: 2.3019 Val Accuracy: 0.1101
Epoch 10/15 Batch: [391/391] Train Loss: 2.3014 Train Accuracy: 0.1128 Val L
oss: 2.3020 Val Accuracy: 0.1087
Epoch 11/15 Batch: [391/391] Train Loss: 2.3013 Train Accuracy: 0.1129 Val L
oss: 2.3017 Val Accuracy: 0.1108
Epoch 12/15 Batch: [391/391] Train Loss: 2.3013 Train Accuracy: 0.1128 Val L
oss: 2.3019 Val Accuracy: 0.1087
Epoch 13/15 Batch: [391/391] Train Loss: 2.3013 Train Accuracy: 0.1129 Val L
oss: 2.3018 Val Accuracy: 0.1101
Epoch 14/15 Batch: [391/391] Train Loss: 2.3013 Train Accuracy: 0.1129 Val L
oss: 2.3019 Val Accuracy: 0.1094
Epoch 15/15 Batch: [391/391] Train Loss: 2.3013 Train Accuracy: 0.1128 Val L
oss: 2.3019 Val Accuracy: 0.1094
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
Batch: [79/79]
```

In [ ]:

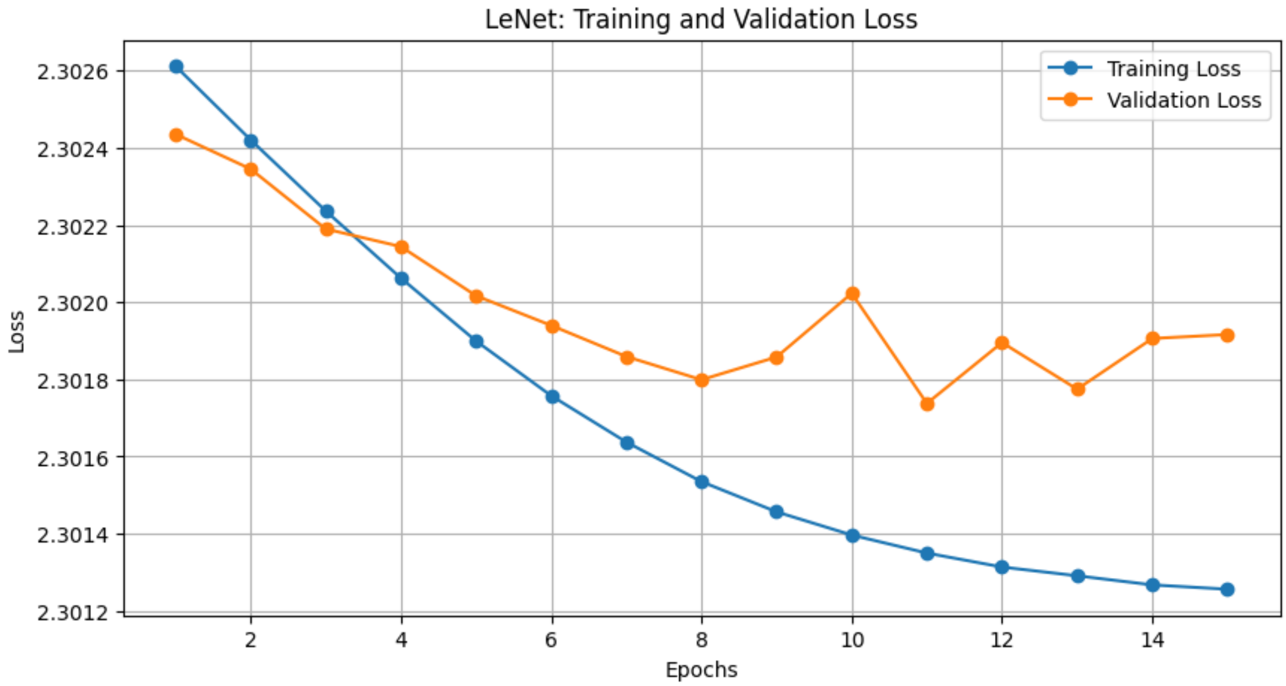
```python
# print test accuracy
print(f'The test accuracy is {q1_test["test_accuracy"]:.4f}.')
```

```
The test accuracy is 0.0000.
```

In [ ]:

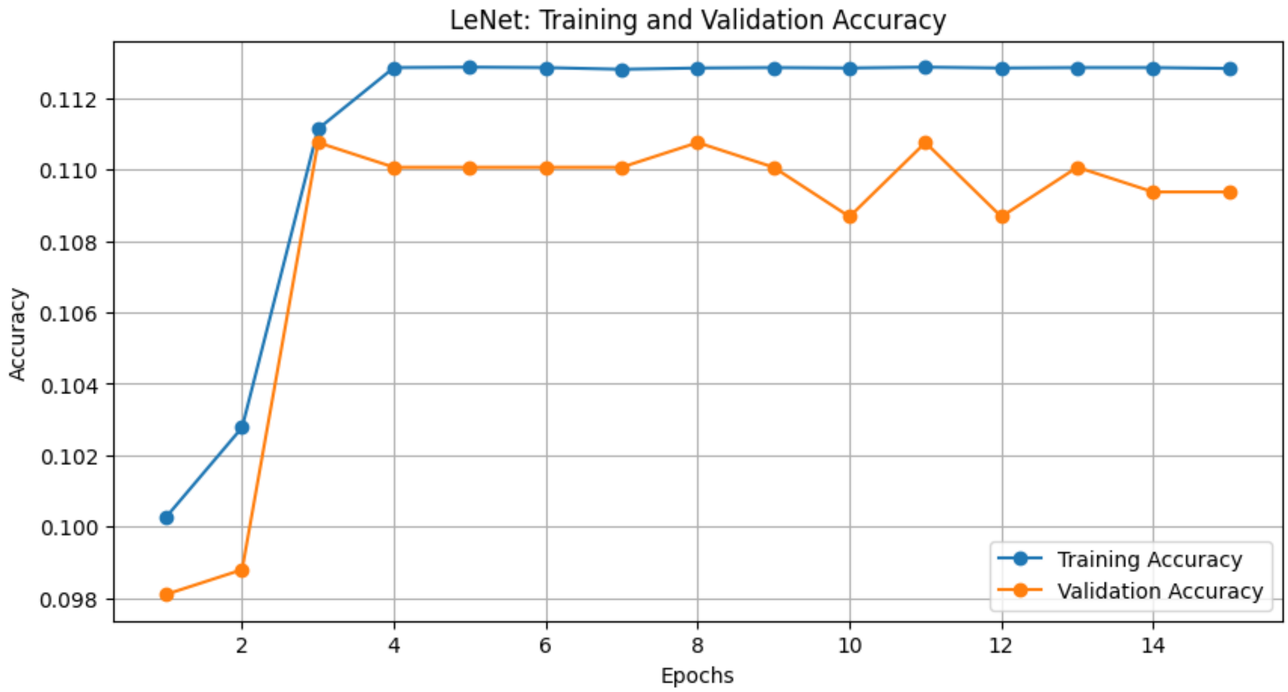
```python
# plot the training and validation loss
plt.figure(figsize=(10, 5))
plt.plot(
    list(range(1, HYPERPARAMETERS["epochs"] + 1)),
    q1_history["train_loss"],
    label="Training Loss",
    marker="o",
)
plt.plot(
    list(range(1, HYPERPARAMETERS["epochs"] + 1)),
    q1_history["val_loss"],
    label="Validation Loss",
    marker="o",
```

```
)
plt.xlabel("Epochs")
plt.ylabel("Loss")
plt.legend()
plt.title("LeNet: Training and Validation Loss")
plt.grid(True)
plt.show()
```



In [ ]:

```
# plot training and validation accuracy
plt.figure(figsize=(10, 5))
plt.plot(
    list(range(1, HYPERPARAMETERS["epochs"] + 1)),
    q1_history["train_accuracy"],
    label="Training Accuracy",
    marker="o",
)
plt.plot(
    list(range(1, HYPERPARAMETERS["epochs"] + 1)),
    q1_history["val_accuracy"],
    label="Validation Accuracy",
    marker="o",
)
plt.xlabel("Epochs")
plt.ylabel("Accuracy")
plt.legend()
plt.title("LeNet: Training and Validation Accuracy")
plt.grid(True)
plt.show()
```



Question 2

How can we make the training loss converge faster? Implement your idea and compare the new training curve against that in question 1.

Answer

We can use techniques such as batch normalization and weight initialization to make the training loss converge faster. Batch normalization normalizes the input to each layer so that the gradients do not become too large or too small. This can help to stabilize the training process and prevent the model from getting stuck in local minima. Weight initialization initializes the weights of the model to small random values so that the gradients do not become too large or too small. This can help to prevent the model from getting stuck in local minima and improve the generalization of the model.

The implementation of the LeNet architecture with batch normalization is as follows:

In [ ]:
```
class LeNet(nn.Module):
    def __init__(self):
        super(LeNet, self).__init__()

        # Define the model architecture with additional BatchNorm layers
        self.model = nn.Sequential(
            nn.Conv2d(1, 6, kernel_size=5, stride=1, padding=2),
            nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.BatchNorm2d(6),
            nn.Conv2d(6, 16, kernel_size=5, stride=1),
            nn.Sigmoid(),
```

```
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.BatchNorm2d(16),
            nn.Flatten(),
            nn.Linear(in_features=16 * 5 * 5, out_features=120),
            nn.Sigmoid(),
            nn.Linear(in_features=120, out_features=84),
            nn.Linear(in_features=84, out_features=10),
            nn.Softmax(dim=1),
        )

    def forward(self, x):
        # Model returns the probability of each class
        return self.model(x)


le_net = LeNet()

# Define the loss function
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(le_net.parameters(), lr=HYPERPARAMETERS["learnir

q2_history = train(
    le_net,
    optimizer,
    criterion,
    train_loader,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)

q2_test = test(le_net, test_loader, criterion, device=DEVICE)
```

```
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [391/391] Train Loss: 2.3007 Train Accuracy: 0.1002 Val L
oss: 2.2989 Val Accuracy: 0.0939
Epoch  2/15 Batch: [391/391] Train Loss: 2.2960 Train Accuracy: 0.1163 Val L
oss: 2.2937 Val Accuracy: 0.1518
Epoch  3/15 Batch: [391/391] Train Loss: 2.2901 Train Accuracy: 0.2123 Val L
oss: 2.2869 Val Accuracy: 0.2885
Epoch  4/15 Batch: [391/391] Train Loss: 2.2816 Train Accuracy: 0.3437 Val L
oss: 2.2759 Val Accuracy: 0.3663
Epoch  5/15 Batch: [391/391] Train Loss: 2.2660 Train Accuracy: 0.3675 Val L
oss: 2.2525 Val Accuracy: 0.3469
Epoch  6/15 Batch: [391/391] Train Loss: 2.2281 Train Accuracy: 0.3183 Val L
oss: 2.1966 Val Accuracy: 0.2935
Epoch  7/15 Batch: [391/391] Train Loss: 2.1661 Train Accuracy: 0.3271 Val L
oss: 2.1328 Val Accuracy: 0.3612
Epoch  8/15 Batch: [391/391] Train Loss: 2.1034 Train Accuracy: 0.3910 Val L
oss: 2.0764 Val Accuracy: 0.4231
Epoch  9/15 Batch: [391/391] Train Loss: 2.0446 Train Accuracy: 0.4559 Val L
oss: 2.0148 Val Accuracy: 0.4797
Epoch 10/15 Batch: [391/391] Train Loss: 1.9910 Train Accuracy: 0.5223 Val L
oss: 1.9638 Val Accuracy: 0.5612
Epoch 11/15 Batch: [391/391] Train Loss: 1.9466 Train Accuracy: 0.5612 Val L
oss: 1.9298 Val Accuracy: 0.5658
Epoch 12/15 Batch: [391/391] Train Loss: 1.9194 Train Accuracy: 0.5671 Val L
oss: 1.9078 Val Accuracy: 0.5722
Epoch 13/15 Batch: [391/391] Train Loss: 1.9015 Train Accuracy: 0.5714 Val L
oss: 1.8892 Val Accuracy: 0.5817
Epoch 14/15 Batch: [391/391] Train Loss: 1.8709 Train Accuracy: 0.6300 Val L
oss: 1.8522 Val Accuracy: 0.6583
Epoch 15/15 Batch: [391/391] Train Loss: 1.8438 Train Accuracy: 0.6575 Val L
oss: 1.8315 Val Accuracy: 0.6631
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
Batch: [79/79]
```

In [ ]:

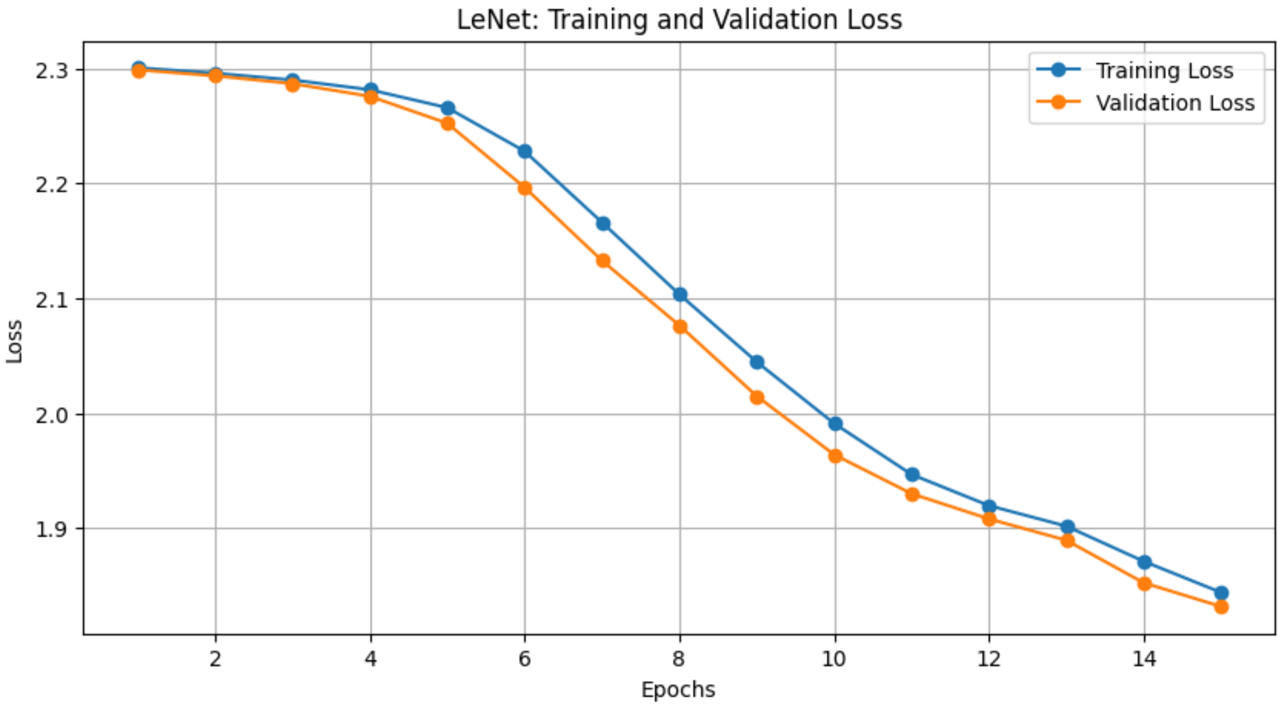
```
# print test accuracy
print(f'The test accuracy is {q2_test["test_accuracy"]:.4f}.')
```

```
The test accuracy is 0.0001.
```

In [ ]:

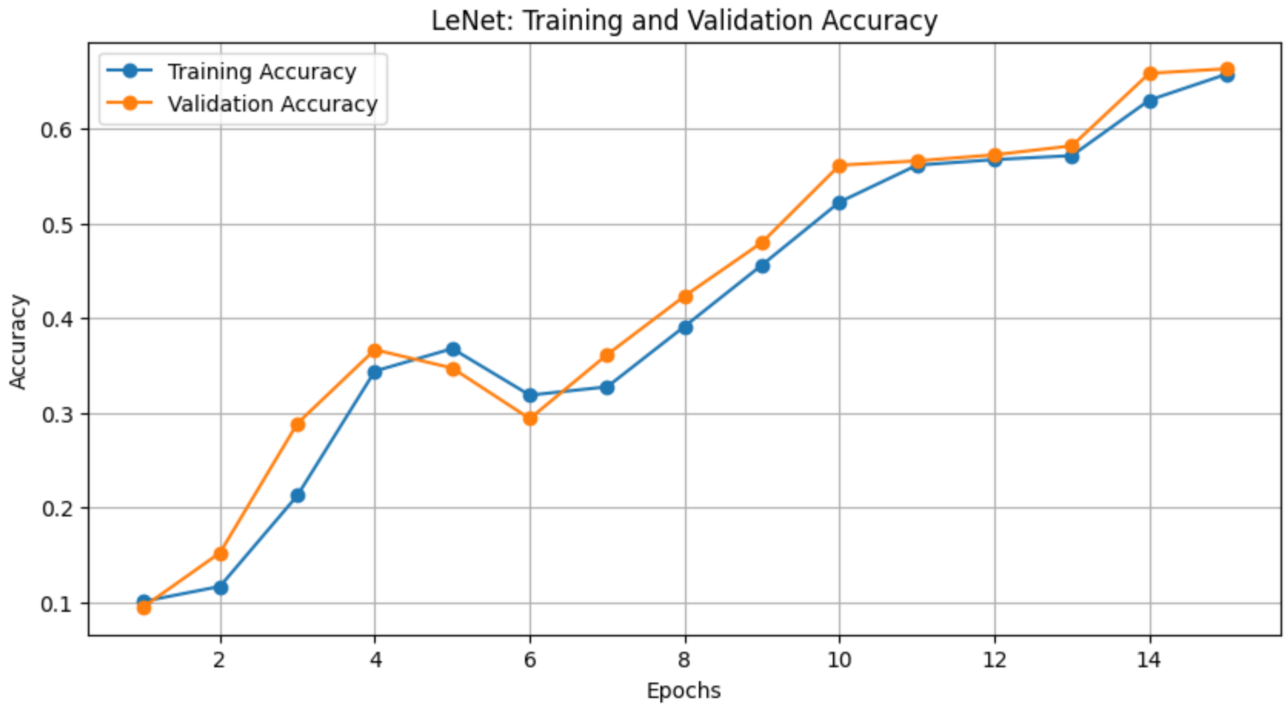
```
# plot the training and validation loss
plt.figure(figsize=(10, 5))
plt.plot(
    list(range(1, HYPERPARAMETERS["epochs"] + 1)),
    q2_history["train_loss"],
    label="Training Loss",
    marker="o",
)
plt.plot(
    list(range(1, HYPERPARAMETERS["epochs"] + 1)),
    q2_history["val_loss"],
    label="Validation Loss",
    marker="o",
```

```
)
plt.xlabel("Epochs")
plt.ylabel("Loss")
plt.legend()
plt.title("LeNet: Training and Validation Loss")
plt.grid(True)
plt.show()
```



In [ ]:

```
# plot training and validation accuracy
plt.figure(figsize=(10, 5))
plt.plot(
    list(range(1, HYPERPARAMETERS["epochs"] + 1)),
    q2_history["train_accuracy"],
    label="Training Accuracy",
    marker="o",
)
plt.plot(
    list(range(1, HYPERPARAMETERS["epochs"] + 1)),
    q2_history["val_accuracy"],
    label="Validation Accuracy",
    marker="o",
)
plt.xlabel("Epochs")
plt.ylabel("Accuracy")
plt.legend()
plt.title("LeNet: Training and Validation Accuracy")
plt.grid(True)
plt.show()
```



As we can see from both graphs, the highest val accuracy achieved for Q1 is **11.2%** whereas for Q2 it is around **65%**. This is a significant improvement in the val accuracy. So we can safely include batch normalization to make the training loss converge faster and improve the test accuracy.

Question 3

Train on 20%, 50%, 80% and 100% of the full 50K samples and report test accuracies. On a 2D coordinate axes (x-axis: training size, y-axis: test accuracy), plot the accuracies. Discuss the result.

In [ ]:
```
train_size = len(train_data)

# Create new data loaders with 20%, 50%, and 80% of the training data
train_sampler_20 = torch.utils.data.RandomSampler(
    train_data, replacement=True, num_samples=int(0.2 * train_size)
)
train_loader_20 = torch.utils.data.DataLoader(
    train_data, batch_size=HYPERPARAMETERS["batch_size"], sampler=train_samp
)

train_sampler_50 = torch.utils.data.RandomSampler(
    train_data, replacement=True, num_samples=int(0.5 * train_size)
)
train_loader_50 = torch.utils.data.DataLoader(
    train_data, batch_size=HYPERPARAMETERS["batch_size"], sampler=train_samp
)

train_sampler_80 = torch.utils.data.RandomSampler(
```

```
    train_data, replacement=True, num_samples=int(0.8 * train_size)
)
train_loader_80 = torch.utils.data.DataLoader(
    train_data, batch_size=HYPERPARAMETERS["batch_size"], sampler=train_samp
)

# Train the model with 20% of the data
le_net_20 = LeNet()
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(le_net_20.parameters(), lr=HYPERPARAMETERS["lear

le_net_20_history = train(
    le_net_20,
    optimizer,
    criterion,
    train_loader_20,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)

le_net_20_test = test(le_net_20, test_loader, criterion, device=DEVICE)

# Train the model with 50% of the data
le_net_50 = LeNet()
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(le_net_50.parameters(), lr=HYPERPARAMETERS["lear

le_net_50_history = train(
    le_net_50,
    optimizer,
    criterion,
    train_loader_50,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)

le_net_50_test = test(le_net_50, test_loader, criterion, device=DEVICE)

# Train the model with 80% of the data
le_net_80 = LeNet()
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(le_net_80.parameters(), lr=HYPERPARAMETERS["lear

le_net_80_history = train(
    le_net_80,
    optimizer,
    criterion,
    train_loader_80,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
```

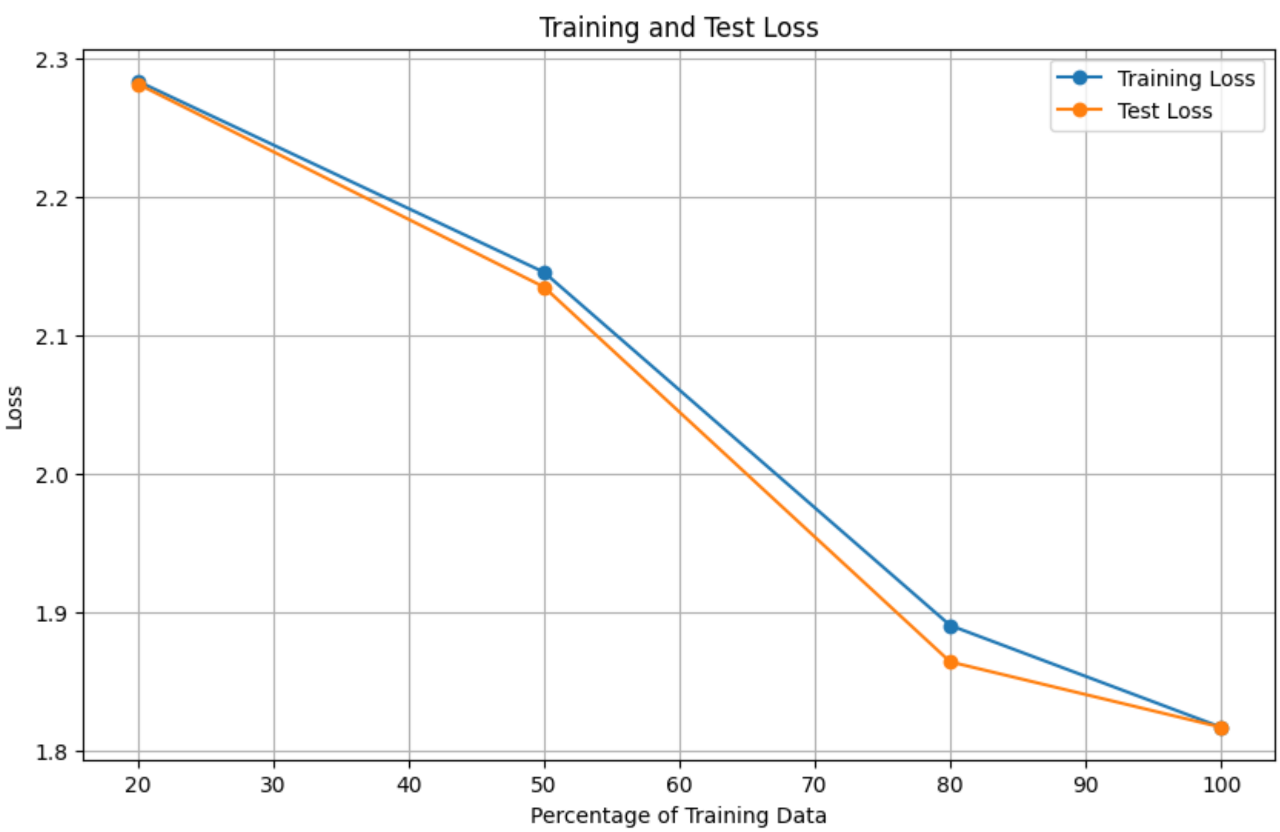
```python
)

le_net_80_test = test(le_net_80, test_loader, criterion, device=DEVICE)

# Train the model with 100% of the data
le_net_100 = LeNet()

criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(
    le_net_100.parameters(), lr=HYPERPARAMETERS["learning_rate"]
)

le_net_100_history = train(
    le_net_100,
    optimizer,
    criterion,
    train_loader,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)

# Plot the training loss
plt.figure(figsize=(10, 6))
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_history["train_loss"][-1],
        le_net_50_history["train_loss"][-1],
        le_net_80_history["train_loss"][-1],
        le_net_100_history["train_loss"][-1],
    ],
    marker="o",
    label="Training Loss",
)
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_test["test_loss"],
        le_net_50_test["test_loss"],
        le_net_80_test["test_loss"],
        le_net_100_history["train_loss"][-1],
    ],
    marker="o",
    label="Test Loss",
)
plt.xlabel("Percentage of Training Data")
plt.ylabel("Loss")
plt.title("Training and Test Loss")
plt.legend()
plt.grid(True)
plt.show()
```

```
Model moved to mps.
```

```
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [79/79] Train Loss: 2.3020 Train Accuracy: 0.1190 Val Los
s: 2.3018 Val Accuracy: 0.1159
Epoch  2/15 Batch: [79/79] Train Loss: 2.3014 Train Accuracy: 0.1197 Val Los
s: 2.3009 Val Accuracy: 0.1313
Epoch  3/15 Batch: [79/79] Train Loss: 2.3004 Train Accuracy: 0.1439 Val Los
s: 2.3002 Val Accuracy: 0.1497
Epoch  4/15 Batch: [79/79] Train Loss: 2.2996 Train Accuracy: 0.1506 Val Los
s: 2.2993 Val Accuracy: 0.1588
Epoch  5/15 Batch: [79/79] Train Loss: 2.2988 Train Accuracy: 0.1691 Val Los
s: 2.2984 Val Accuracy: 0.1654
Epoch  6/15 Batch: [79/79] Train Loss: 2.2978 Train Accuracy: 0.1777 Val Los
s: 2.2974 Val Accuracy: 0.1851
Epoch  7/15 Batch: [79/79] Train Loss: 2.2967 Train Accuracy: 0.2035 Val Los
s: 2.2965 Val Accuracy: 0.2076
Epoch  8/15 Batch: [79/79] Train Loss: 2.2957 Train Accuracy: 0.2437 Val Los
s: 2.2954 Val Accuracy: 0.2773
Epoch  9/15 Batch: [79/79] Train Loss: 2.2945 Train Accuracy: 0.3274 Val Los
s: 2.2941 Val Accuracy: 0.3412
Epoch 10/15 Batch: [79/79] Train Loss: 2.2933 Train Accuracy: 0.3431 Val Los
s: 2.2928 Val Accuracy: 0.3432
Epoch 11/15 Batch: [79/79] Train Loss: 2.2916 Train Accuracy: 0.3247 Val Los
s: 2.2911 Val Accuracy: 0.3190
Epoch 12/15 Batch: [79/79] Train Loss: 2.2898 Train Accuracy: 0.2896 Val Los
s: 2.2894 Val Accuracy: 0.2614
Epoch 13/15 Batch: [79/79] Train Loss: 2.2886 Train Accuracy: 0.2428 Val Los
s: 2.2875 Val Accuracy: 0.2344
Epoch 14/15 Batch: [79/79] Train Loss: 2.2863 Train Accuracy: 0.2163 Val Los
s: 2.2853 Val Accuracy: 0.2130
Epoch 15/15 Batch: [79/79] Train Loss: 2.2835 Train Accuracy: 0.2025 Val Los
s: 2.2825 Val Accuracy: 0.1991
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [196/196] Train Loss: 2.3016 Train Accuracy: 0.1039 Val L
oss: 2.3007 Val Accuracy: 0.0995
Epoch  2/15 Batch: [196/196] Train Loss: 2.2998 Train Accuracy: 0.1025 Val L
oss: 2.2987 Val Accuracy: 0.1156
Epoch  3/15 Batch: [196/196] Train Loss: 2.2976 Train Accuracy: 0.1460 Val L
oss: 2.2964 Val Accuracy: 0.1832
Epoch  4/15 Batch: [196/196] Train Loss: 2.2952 Train Accuracy: 0.2203 Val L
oss: 2.2938 Val Accuracy: 0.2536
Epoch  5/15 Batch: [196/196] Train Loss: 2.2923 Train Accuracy: 0.2806 Val L
oss: 2.2905 Val Accuracy: 0.2863
Epoch  6/15 Batch: [196/196] Train Loss: 2.2888 Train Accuracy: 0.2642 Val L
oss: 2.2863 Val Accuracy: 0.2573
Epoch  7/15 Batch: [196/196] Train Loss: 2.2836 Train Accuracy: 0.2364 Val L
oss: 2.2805 Val Accuracy: 0.2260
Epoch  8/15 Batch: [196/196] Train Loss: 2.2764 Train Accuracy: 0.2127 Val L
oss: 2.2702 Val Accuracy: 0.2143
Epoch  9/15 Batch: [196/196] Train Loss: 2.2628 Train Accuracy: 0.2104 Val L
oss: 2.2520 Val Accuracy: 0.2154
```

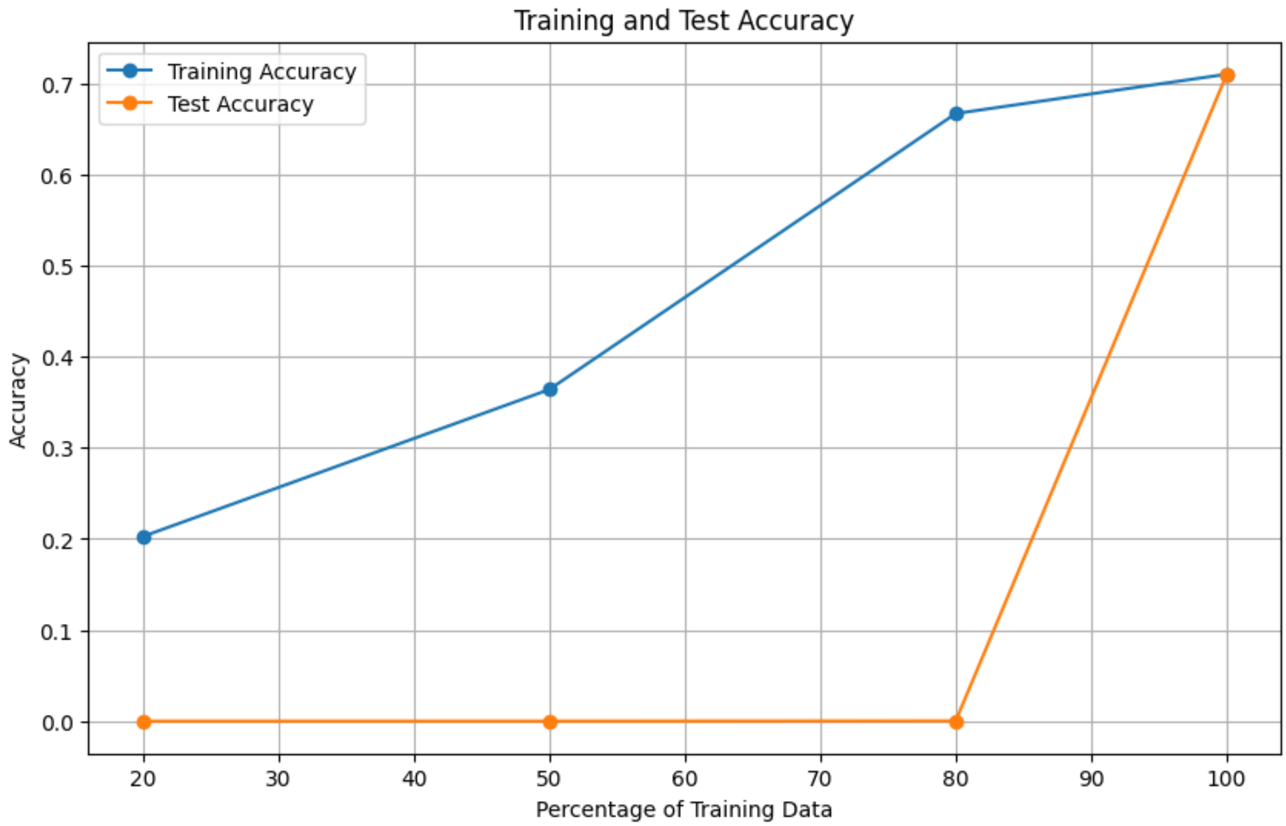
```
Epoch 10/15 Batch: [196/196] Train Loss: 2.2407 Train Accuracy: 0.2073 Val L
oss: 2.2236 Val Accuracy: 0.2155
Epoch 11/15 Batch: [196/196] Train Loss: 2.2099 Train Accuracy: 0.2142 Val L
oss: 2.1989 Val Accuracy: 0.2148
Epoch 12/15 Batch: [196/196] Train Loss: 2.1930 Train Accuracy: 0.2089 Val L
oss: 2.1831 Val Accuracy: 0.2136
Epoch 13/15 Batch: [196/196] Train Loss: 2.1748 Train Accuracy: 0.2192 Val L
oss: 2.1677 Val Accuracy: 0.2365
Epoch 14/15 Batch: [196/196] Train Loss: 2.1643 Train Accuracy: 0.2721 Val L
oss: 2.1522 Val Accuracy: 0.3257
Epoch 15/15 Batch: [196/196] Train Loss: 2.1454 Train Accuracy: 0.3639 Val L
oss: 2.1359 Val Accuracy: 0.4081
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [313/313] Train Loss: 2.3006 Train Accuracy: 0.1713 Val L
oss: 2.2989 Val Accuracy: 0.2094
Epoch  2/15 Batch: [313/313] Train Loss: 2.2963 Train Accuracy: 0.2087 Val L
oss: 2.2946 Val Accuracy: 0.1651
Epoch  3/15 Batch: [313/313] Train Loss: 2.2912 Train Accuracy: 0.1465 Val L
oss: 2.2887 Val Accuracy: 0.1273
Epoch  4/15 Batch: [313/313] Train Loss: 2.2839 Train Accuracy: 0.1267 Val L
oss: 2.2787 Val Accuracy: 0.1194
Epoch  5/15 Batch: [313/313] Train Loss: 2.2675 Train Accuracy: 0.1250 Val L
oss: 2.2591 Val Accuracy: 0.1244
Epoch  6/15 Batch: [313/313] Train Loss: 2.2455 Train Accuracy: 0.1499 Val L
oss: 2.2328 Val Accuracy: 0.1899
Epoch  7/15 Batch: [313/313] Train Loss: 2.2193 Train Accuracy: 0.2226 Val L
oss: 2.2073 Val Accuracy: 0.2544
Epoch  8/15 Batch: [313/313] Train Loss: 2.1914 Train Accuracy: 0.2817 Val L
oss: 2.1766 Val Accuracy: 0.3182
Epoch  9/15 Batch: [313/313] Train Loss: 2.1572 Train Accuracy: 0.3567 Val L
oss: 2.1415 Val Accuracy: 0.3930
Epoch 10/15 Batch: [313/313] Train Loss: 2.1205 Train Accuracy: 0.4099 Val L
oss: 2.0997 Val Accuracy: 0.4372
Epoch 11/15 Batch: [313/313] Train Loss: 2.0742 Train Accuracy: 0.4551 Val L
oss: 2.0477 Val Accuracy: 0.4804
Epoch 12/15 Batch: [313/313] Train Loss: 2.0241 Train Accuracy: 0.4824 Val L
oss: 1.9963 Val Accuracy: 0.4966
Epoch 13/15 Batch: [313/313] Train Loss: 1.9762 Train Accuracy: 0.5137 Val L
oss: 1.9528 Val Accuracy: 0.5576
Epoch 14/15 Batch: [313/313] Train Loss: 1.9319 Train Accuracy: 0.5986 Val L
oss: 1.9096 Val Accuracy: 0.6519
Epoch 15/15 Batch: [313/313] Train Loss: 1.8901 Train Accuracy: 0.6670 Val L
oss: 1.8665 Val Accuracy: 0.6868
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [391/391] Train Loss: 2.3004 Train Accuracy: 0.1045 Val L
oss: 2.2978 Val Accuracy: 0.1228
Epoch  2/15 Batch: [391/391] Train Loss: 2.2953 Train Accuracy: 0.2270 Val L
```

```
oss: 2.2919 Val Accuracy: 0.2991
Epoch  3/15 Batch: [391/391] Train Loss: 2.2879 Train Accuracy: 0.2873 Val L
oss: 2.2821 Val Accuracy: 0.2856
Epoch  4/15 Batch: [391/391] Train Loss: 2.2733 Train Accuracy: 0.2570 Val L
oss: 2.2586 Val Accuracy: 0.2436
Epoch  5/15 Batch: [391/391] Train Loss: 2.2367 Train Accuracy: 0.2246 Val L
oss: 2.2065 Val Accuracy: 0.2360
Epoch  6/15 Batch: [391/391] Train Loss: 2.1905 Train Accuracy: 0.2547 Val L
oss: 2.1655 Val Accuracy: 0.2929
Epoch  7/15 Batch: [391/391] Train Loss: 2.1535 Train Accuracy: 0.2938 Val L
oss: 2.1321 Val Accuracy: 0.3088
Epoch  8/15 Batch: [391/391] Train Loss: 2.1247 Train Accuracy: 0.3024 Val L
oss: 2.1057 Val Accuracy: 0.3171
Epoch  9/15 Batch: [391/391] Train Loss: 2.0959 Train Accuracy: 0.3387 Val L
oss: 2.0758 Val Accuracy: 0.3858
Epoch 10/15 Batch: [391/391] Train Loss: 2.0554 Train Accuracy: 0.4195 Val L
oss: 2.0264 Val Accuracy: 0.4690
Epoch 11/15 Batch: [391/391] Train Loss: 2.0037 Train Accuracy: 0.5223 Val L
oss: 1.9693 Val Accuracy: 0.5877
Epoch 12/15 Batch: [391/391] Train Loss: 1.9416 Train Accuracy: 0.6119 Val L
oss: 1.9100 Val Accuracy: 0.6338
Epoch 13/15 Batch: [391/391] Train Loss: 1.8855 Train Accuracy: 0.6409 Val L
oss: 1.8614 Val Accuracy: 0.6674
Epoch 14/15 Batch: [391/391] Train Loss: 1.8446 Train Accuracy: 0.6928 Val L
oss: 1.8279 Val Accuracy: 0.7069
Epoch 15/15 Batch: [391/391] Train Loss: 1.8165 Train Accuracy: 0.7102 Val L
oss: 1.8048 Val Accuracy: 0.7164
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
```



In [ ]:

```python
# Plot training and test accuracy
plt.figure(figsize=(10, 6))
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_history["train_accuracy"][-1],
        le_net_50_history["train_accuracy"][-1],
        le_net_80_history["train_accuracy"][-1],
        le_net_100_history["train_accuracy"][-1],
    ],
    marker="o",
    label="Training Accuracy",
)
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_test["test_accuracy"],
        le_net_50_test["test_accuracy"],
        le_net_80_test["test_accuracy"],
        le_net_100_history["train_accuracy"][-1],
    ],
    marker="o",
    label="Test Accuracy",
)
plt.xlabel("Percentage of Training Data")
plt.ylabel("Accuracy")
plt.title("Training and Test Accuracy")
plt.legend()
plt.grid(True)
plt.show()
```



We can observe that as the training size increases, the test accuracy also increases. This is because as the training size increases, the model has more data to learn from and can generalize better to new data. This is especially true for complex models like deep neural networks, which require a large amount of data to learn the parameters. As the training size increases, the model can learn more about the underlying distribution of the data and make better predictions on new data. This is why it is important to use as much data as possible when training deep neural networks.

Question 4

How can we improve the accuracies for cases with fewer training samples? Implement your idea and compare against what you have in question 3.

Answer

We can use techniques of data augmenation to improve the accuracies for cases with fewer training samples. Data augmentation involves applying random transformations to the input images such as rotation, scaling, and translation. This can help to increase the size of the training data and improve the generalization of the model.

The implementation of the LeNet architecture with data augmentation is as follows:

In [ ]:

```
# Create a new data loader with 100% of the training data and some data augm
aug_transform = transforms.Compose(
```

```
    [
        transforms.RandomRotation(20),
        transforms.RandomHorizontalFlip(),
        transforms.ToTensor(),
    ]
)

base_train_data = datasets.MNIST(
    "../data",
    train=True,
    download=True,
    transform=aug_transform,
    target_transform=one_hot_transform,
)

augmented_dataset = torch.utils.data.ConcatDataset([train_data, base_train_c

train_sampler_20_aug = torch.utils.data.RandomSampler(
    augmented_dataset, replacement=True, num_samples=int(0.2 * train_size)
)

train_loader_20_aug = torch.utils.data.DataLoader(
    augmented_dataset,
    batch_size=HYPERPARAMETERS["batch_size"],
    sampler=train_sampler_20_aug,
)

train_sampler_50_aug = torch.utils.data.RandomSampler(
    augmented_dataset, replacement=True, num_samples=int(0.5 * train_size)
)

train_loader_50_aug = torch.utils.data.DataLoader(
    augmented_dataset,
    batch_size=HYPERPARAMETERS["batch_size"],
    sampler=train_sampler_50_aug,
)

train_sampler_80_aug = torch.utils.data.RandomSampler(
    augmented_dataset, replacement=True, num_samples=int(0.8 * train_size)
)

train_loader_80_aug = torch.utils.data.DataLoader(
    augmented_dataset,
    batch_size=HYPERPARAMETERS["batch_size"],
    sampler=train_sampler_80_aug,
)

train_sampler_aug = torch.utils.data.RandomSampler(
    augmented_dataset, replacement=True, num_samples=train_size
)

train_loader_aug = torch.utils.data.DataLoader(
    augmented_dataset,
```

```
    batch_size=HYPERPARAMETERS["batch_size"],
    sampler=train_sampler_aug,
)

# Train the model with 20% of the data
le_net_20_aug = LeNet()
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(
    le_net_20_aug.parameters(), lr=HYPERPARAMETERS["learning_rate"]
)

le_net_20_aug_history = train(
    le_net_20_aug,
    optimizer,
    criterion,
    train_loader_20_aug,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)

le_net_20_aug_test = test(le_net_20_aug, test_loader, criterion, device=DEVI

# Train the model with 50% of the data
le_net_50_aug = LeNet()
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(
    le_net_50_aug.parameters(), lr=HYPERPARAMETERS["learning_rate"]
)

le_net_50_aug_history = train(
    le_net_50_aug,
    optimizer,
    criterion,
    train_loader_50_aug,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)

le_net_50_aug_test = test(le_net_50_aug, test_loader, criterion, device=DEVI

# Train the model with 80% of the data
le_net_80_aug = LeNet()
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(
    le_net_80_aug.parameters(), lr=HYPERPARAMETERS["learning_rate"]
)

le_net_80_aug_history = train(
    le_net_80_aug,
    optimizer,
    criterion,
```

```
    train_loader_80_aug,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)

le_net_80_aug_test = test(le_net_80_aug, test_loader, criterion, device=DEVI


# Train the model with 100% of the data
le_net_100_aug = LeNet()
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(
    le_net_100_aug.parameters(), lr=HYPERPARAMETERS["learning_rate"]
)

le_net_100_aug_history = train(
    le_net_100_aug,
    optimizer,
    criterion,
    train_loader_aug,
    val_loader,
    epochs=HYPERPARAMETERS["epochs"],
    device=DEVICE,
)
```

```
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [79/79] Train Loss: 2.3023 Train Accuracy: 0.1321 Val Los
s: 2.3019 Val Accuracy: 0.1651
Epoch  2/15 Batch: [79/79] Train Loss: 2.3016 Train Accuracy: 0.1642 Val Los
s: 2.3012 Val Accuracy: 0.1931
Epoch  3/15 Batch: [79/79] Train Loss: 2.3008 Train Accuracy: 0.1885 Val Los
s: 2.3004 Val Accuracy: 0.2183
Epoch  4/15 Batch: [79/79] Train Loss: 2.3000 Train Accuracy: 0.2159 Val Los
s: 2.2996 Val Accuracy: 0.2355
Epoch  5/15 Batch: [79/79] Train Loss: 2.2996 Train Accuracy: 0.2180 Val Los
s: 2.2988 Val Accuracy: 0.2529
Epoch  6/15 Batch: [79/79] Train Loss: 2.2988 Train Accuracy: 0.2419 Val Los
s: 2.2981 Val Accuracy: 0.2850
Epoch  7/15 Batch: [79/79] Train Loss: 2.2979 Train Accuracy: 0.2940 Val Los
s: 2.2973 Val Accuracy: 0.3353
Epoch  8/15 Batch: [79/79] Train Loss: 2.2975 Train Accuracy: 0.3112 Val Los
s: 2.2965 Val Accuracy: 0.3507
Epoch  9/15 Batch: [79/79] Train Loss: 2.2965 Train Accuracy: 0.3186 Val Los
s: 2.2955 Val Accuracy: 0.3485
Epoch 10/15 Batch: [79/79] Train Loss: 2.2957 Train Accuracy: 0.3217 Val Los
s: 2.2945 Val Accuracy: 0.3374
Epoch 11/15 Batch: [79/79] Train Loss: 2.2946 Train Accuracy: 0.3153 Val Los
s: 2.2933 Val Accuracy: 0.3294
Epoch 12/15 Batch: [79/79] Train Loss: 2.2938 Train Accuracy: 0.3049 Val Los
s: 2.2925 Val Accuracy: 0.3199
Epoch 13/15 Batch: [79/79] Train Loss: 2.2928 Train Accuracy: 0.2971 Val Los
s: 2.2913 Val Accuracy: 0.3120
```

```
Epoch 14/15 Batch: [79/79] Train Loss: 2.2913 Train Accuracy: 0.2946 Val Los
s: 2.2899 Val Accuracy: 0.3075
Epoch 15/15 Batch: [79/79] Train Loss: 2.2900 Train Accuracy: 0.2910 Val Los
s: 2.2886 Val Accuracy: 0.3038
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [196/196] Train Loss: 2.3017 Train Accuracy: 0.0978 Val L
oss: 2.3009 Val Accuracy: 0.1038
Epoch  2/15 Batch: [196/196] Train Loss: 2.3003 Train Accuracy: 0.1269 Val L
oss: 2.2991 Val Accuracy: 0.1973
Epoch  3/15 Batch: [196/196] Train Loss: 2.2985 Train Accuracy: 0.1913 Val L
oss: 2.2971 Val Accuracy: 0.2115
Epoch  4/15 Batch: [196/196] Train Loss: 2.2964 Train Accuracy: 0.1963 Val L
oss: 2.2947 Val Accuracy: 0.1827
Epoch  5/15 Batch: [196/196] Train Loss: 2.2945 Train Accuracy: 0.1591 Val L
oss: 2.2921 Val Accuracy: 0.1554
Epoch  6/15 Batch: [196/196] Train Loss: 2.2909 Train Accuracy: 0.1363 Val L
oss: 2.2881 Val Accuracy: 0.1227
Epoch  7/15 Batch: [196/196] Train Loss: 2.2873 Train Accuracy: 0.1174 Val L
oss: 2.2823 Val Accuracy: 0.1149
Epoch  8/15 Batch: [196/196] Train Loss: 2.2799 Train Accuracy: 0.1187 Val L
oss: 2.2731 Val Accuracy: 0.1224
Epoch  9/15 Batch: [196/196] Train Loss: 2.2690 Train Accuracy: 0.1351 Val L
oss: 2.2590 Val Accuracy: 0.1571
Epoch 10/15 Batch: [196/196] Train Loss: 2.2572 Train Accuracy: 0.1682 Val L
oss: 2.2442 Val Accuracy: 0.2051
Epoch 11/15 Batch: [196/196] Train Loss: 2.2400 Train Accuracy: 0.2024 Val L
oss: 2.2281 Val Accuracy: 0.2298
Epoch 12/15 Batch: [196/196] Train Loss: 2.2272 Train Accuracy: 0.2109 Val L
oss: 2.2122 Val Accuracy: 0.2233
Epoch 13/15 Batch: [196/196] Train Loss: 2.2127 Train Accuracy: 0.2108 Val L
oss: 2.1978 Val Accuracy: 0.2215
Epoch 14/15 Batch: [196/196] Train Loss: 2.1987 Train Accuracy: 0.2159 Val L
oss: 2.1853 Val Accuracy: 0.2263
Epoch 15/15 Batch: [196/196] Train Loss: 2.1868 Train Accuracy: 0.2286 Val L
oss: 2.1727 Val Accuracy: 0.2476
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [313/313] Train Loss: 2.3008 Train Accuracy: 0.1732 Val L
oss: 2.2994 Val Accuracy: 0.2092
Epoch  2/15 Batch: [313/313] Train Loss: 2.2983 Train Accuracy: 0.2060 Val L
oss: 2.2962 Val Accuracy: 0.2132
Epoch  3/15 Batch: [313/313] Train Loss: 2.2949 Train Accuracy: 0.2098 Val L
oss: 2.2920 Val Accuracy: 0.2144
Epoch  4/15 Batch: [313/313] Train Loss: 2.2905 Train Accuracy: 0.2082 Val L
oss: 2.2855 Val Accuracy: 0.2142
Epoch  5/15 Batch: [313/313] Train Loss: 2.2827 Train Accuracy: 0.2055 Val L
oss: 2.2730 Val Accuracy: 0.2135
Epoch  6/15 Batch: [313/313] Train Loss: 2.2651 Train Accuracy: 0.2105 Val L
```

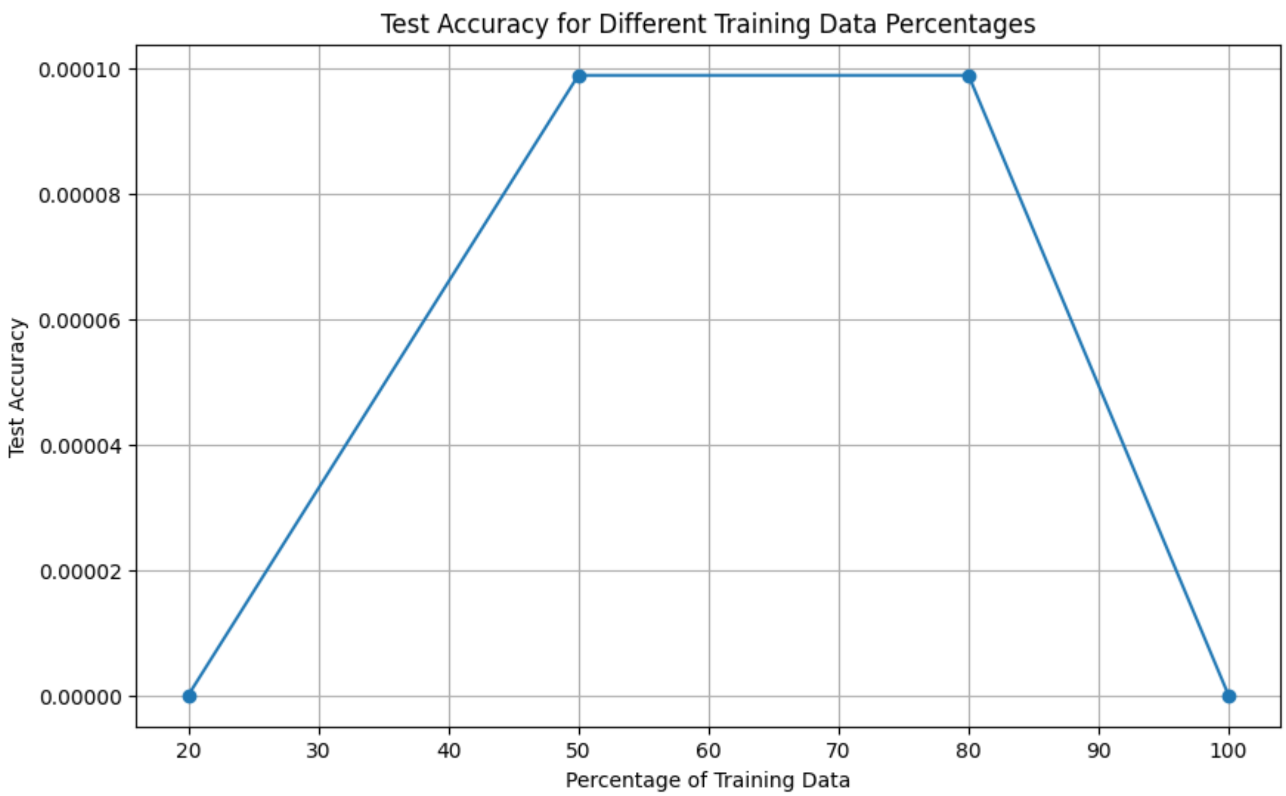
```
oss: 2.2485 Val Accuracy: 0.2124
Epoch  7/15 Batch: [313/313] Train Loss: 2.2369 Train Accuracy: 0.2099 Val L
oss: 2.2145 Val Accuracy: 0.2153
Epoch  8/15 Batch: [313/313] Train Loss: 2.2113 Train Accuracy: 0.2085 Val L
oss: 2.1920 Val Accuracy: 0.2154
Epoch  9/15 Batch: [313/313] Train Loss: 2.1944 Train Accuracy: 0.2082 Val L
oss: 2.1743 Val Accuracy: 0.2291
Epoch 10/15 Batch: [313/313] Train Loss: 2.1765 Train Accuracy: 0.2484 Val L
oss: 2.1588 Val Accuracy: 0.3016
Epoch 11/15 Batch: [313/313] Train Loss: 2.1591 Train Accuracy: 0.3121 Val L
oss: 2.1394 Val Accuracy: 0.3541
Epoch 12/15 Batch: [313/313] Train Loss: 2.1434 Train Accuracy: 0.3432 Val L
oss: 2.1210 Val Accuracy: 0.3767
Epoch 13/15 Batch: [313/313] Train Loss: 2.1262 Train Accuracy: 0.3622 Val L
oss: 2.0923 Val Accuracy: 0.3995
Epoch 14/15 Batch: [313/313] Train Loss: 2.0935 Train Accuracy: 0.3845 Val L
oss: 2.0653 Val Accuracy: 0.4042
Epoch 15/15 Batch: [313/313] Train Loss: 2.0668 Train Accuracy: 0.4010 Val L
oss: 2.0385 Val Accuracy: 0.4290
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
Model moved to mps.
++++++++++ MODEL TRAINING STARTS ++++++++++
Epoch  1/15 Batch: [391/391] Train Loss: 2.3021 Train Accuracy: 0.1035 Val L
oss: 2.2996 Val Accuracy: 0.1094
Epoch  2/15 Batch: [391/391] Train Loss: 2.2985 Train Accuracy: 0.1113 Val L
oss: 2.2950 Val Accuracy: 0.1494
Epoch  3/15 Batch: [391/391] Train Loss: 2.2934 Train Accuracy: 0.1922 Val L
oss: 2.2884 Val Accuracy: 0.2098
Epoch  4/15 Batch: [391/391] Train Loss: 2.2842 Train Accuracy: 0.1858 Val L
oss: 2.2737 Val Accuracy: 0.2009
Epoch  5/15 Batch: [391/391] Train Loss: 2.2632 Train Accuracy: 0.1966 Val L
oss: 2.2389 Val Accuracy: 0.2149
Epoch  6/15 Batch: [391/391] Train Loss: 2.2277 Train Accuracy: 0.2064 Val L
oss: 2.1990 Val Accuracy: 0.2160
Epoch  7/15 Batch: [391/391] Train Loss: 2.1974 Train Accuracy: 0.2205 Val L
oss: 2.1720 Val Accuracy: 0.2604
Epoch  8/15 Batch: [391/391] Train Loss: 2.1727 Train Accuracy: 0.2659 Val L
oss: 2.1444 Val Accuracy: 0.3053
Epoch  9/15 Batch: [391/391] Train Loss: 2.1408 Train Accuracy: 0.3161 Val L
oss: 2.1097 Val Accuracy: 0.3588
Epoch 10/15 Batch: [391/391] Train Loss: 2.1054 Train Accuracy: 0.3674 Val L
oss: 2.0710 Val Accuracy: 0.3904
Epoch 11/15 Batch: [391/391] Train Loss: 2.0726 Train Accuracy: 0.3863 Val L
oss: 2.0434 Val Accuracy: 0.4088
Epoch 12/15 Batch: [391/391] Train Loss: 2.0422 Train Accuracy: 0.4475 Val L
oss: 2.0022 Val Accuracy: 0.5228
Epoch 13/15 Batch: [391/391] Train Loss: 1.9997 Train Accuracy: 0.5314 Val L
oss: 1.9529 Val Accuracy: 0.5687
Epoch 14/15 Batch: [391/391] Train Loss: 1.9563 Train Accuracy: 0.5568 Val L
oss: 1.9236 Val Accuracy: 0.5760
Epoch 15/15 Batch: [391/391] Train Loss: 1.9318 Train Accuracy: 0.5636 Val L
oss: 1.9031 Val Accuracy: 0.5824
```

```
++++++++++ MODEL TRAINING ENDS ++++++++++
Training completed.
```

In [ ]:
```
le_net_100_aug_test = test(le_net_100_aug, test_loader, criterion, device=DE
```

```
Batch: [79/79]
```

In [ ]:
```
# Plot test accuracy for each model
plt.figure(figsize=(10, 6))
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_aug_test["test_accuracy"],
        le_net_50_aug_test["test_accuracy"],
        le_net_80_aug_test["test_accuracy"],
        le_net_100_aug_test["test_accuracy"],
    ],
    marker="o",
)
plt.xlabel("Percentage of Training Data")
plt.ylabel("Test Accuracy")
plt.title("Test Accuracy for Different Training Data Percentages")
plt.grid(True)
plt.show()
```



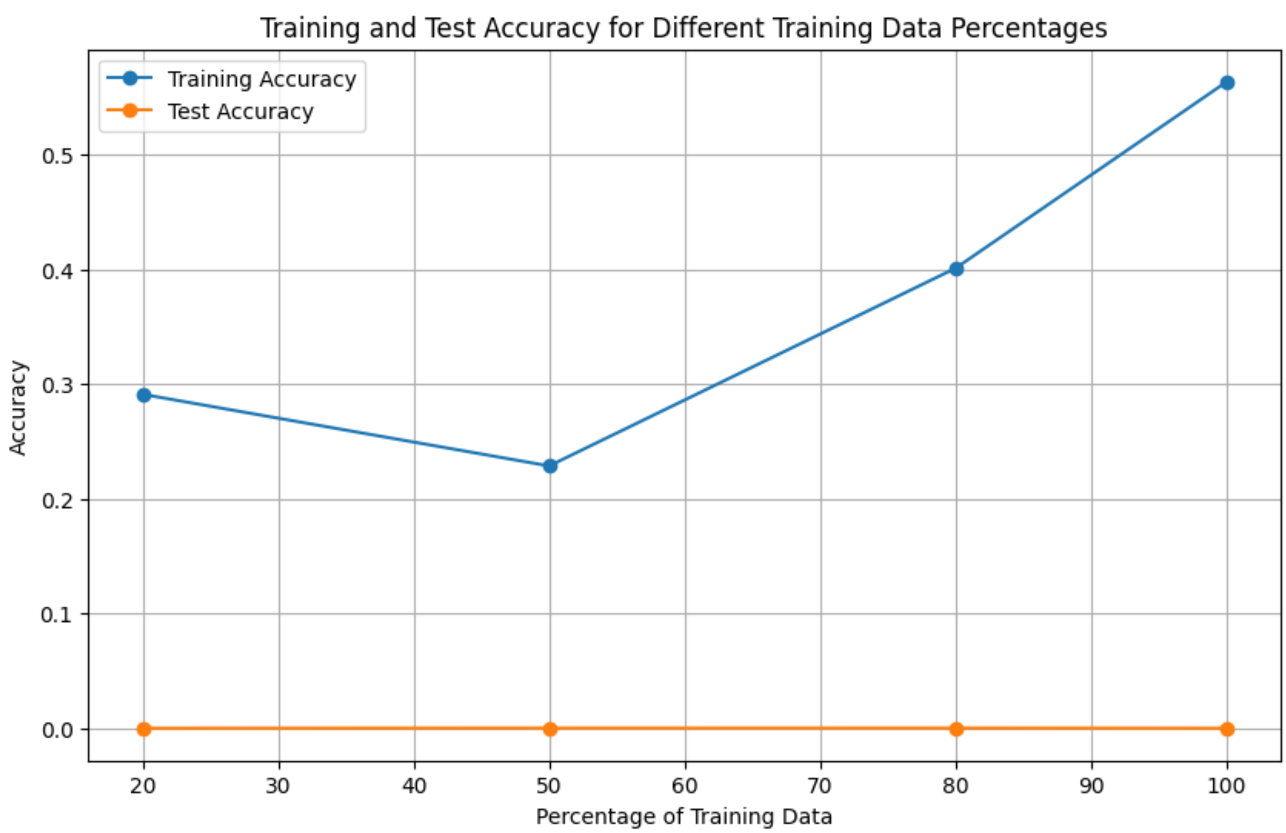
In [ ]:
```
# Plot training and test loss for each model
plt.figure(figsize=(10, 6))
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_aug_history["train_loss"][-1],
```

```
        le_net_50_aug_history["train_loss"][-1],
        le_net_80_aug_history["train_loss"][-1],
        le_net_100_aug_history["train_loss"][-1],
    ],
    marker="o",
    label="Training Loss",
)
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_aug_test["test_loss"],
        le_net_50_aug_test["test_loss"],
        le_net_80_aug_test["test_loss"],
        le_net_100_aug_test["test_loss"],
    ],
    marker="o",
    label="Test Loss",
)
plt.xlabel("Percentage of Training Data")
plt.ylabel("Loss")
plt.title("Training and Test Loss for Different Training Data Percentages")
plt.legend()
plt.grid(True)
plt.show()
```

In [ ]:

```
plt.figure(figsize=(10, 6))
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_aug_history["train_accuracy"][-1],
```

```
        le_net_50_aug_history["train_accuracy"][-1],
        le_net_80_aug_history["train_accuracy"][-1],
        le_net_100_aug_history["train_accuracy"][-1],
    ],
    marker="o",
    label="Training Accuracy",
)
plt.plot(
    [20, 50, 80, 100],
    [
        le_net_20_aug_test["test_accuracy"],
        le_net_50_aug_test["test_accuracy"],
        le_net_80_aug_test["test_accuracy"],
        le_net_100_aug_test["test_accuracy"],
    ],
    marker="o",
    label="Test Accuracy",
)
plt.xlabel("Percentage of Training Data")
plt.ylabel("Accuracy")
plt.title("Training and Test Accuracy for Different Training Data Percentage
plt.legend()
plt.grid(True)
plt.show()
```



For both Q3, Q4 we can see an increased test accuracy as the training size increases. But in Q4 this increase is more significant. This is because of the data augmentation which increases the size of the training data and improves the generalization of the model. This is especially important for cases with fewer training samples, where the model may not have enough data to learn the parameters. Data augmentation can help to increase the size of the training data and improve the generalization of the model.