Assignment 2.

Due date: 13/10/20

1. Code and output files are ~~attact~~ uploaded.

2. If computation of $f'(x)$ is numerically expensive, we will choose Secant ~~&~~ Method. Because →

1) ~~condition~~ It doesn't require a knowledge of $f'(x)$.

2) Though this method is slower than Newton's method, it's still quite rapid. It converges at faster than a linear rate, so that it is more rapidly convergent than bisection method.

3) It needs only one function evaluation per iteration, whereas Newton's method require both $f(x)$ and $f'(x)$.

3. If we ~~don't~~ want to compile 'C' codes without linking using gcc compiler, we will produce only the compiled code using following command →

```
gcc -c main.c
```

4. While studying linear regression, we minimize the least square error we use

$$\sum_i (f(x_i) - y_i)^2 \text{ and not } |f(x_i) - y_i|.$$

PTO.

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because,

- We can take the derivative or apply an integral on a squared function, whereas the derivatives are not well defined for absolute function.

Since data sets are cloudy, a lot of ~~stats~~ statistics is about minimizing the prediction error of a model. Absolute values of errors are not good for this purpose. The metric does not define a single well-defined center of the data that minimizes the sum of absolute values of the errors.

- ~~Also we don't use for 4th power of it.~~

~~Though~~ Also, quadratic terms have nice properties that the variance of total is equal to individual terms, which we want to minimize.

- Quadratic func. guarantees a unique global minima unlike quartic or quintic or hextic terms.

Also we don't take fourth order of difference, because, it will take large value if we have outlier points. Therefore fitting will be difficult.

5. Given Eqn $x^2-5=0$

we have to construct, $g(x)$

such that $x=g(x)$

(a) i> $x = \cancel{\frac{x+5}{x+1}}$

given, $x^2-5=0$

$\Rightarrow x^2+x-x-5=0$

$\Rightarrow x(x+1) = x+5$.

$\Rightarrow \boxed{x = \frac{x+5}{x+1}}$

(b) ii> $x^2-5=0$

$\Rightarrow 3x^2-2x^2-5=0$

$\Rightarrow \cancel{3x^2 = 2x^2+5}$.

$\Rightarrow 2x^2 = 3x^2-5$.

$\Rightarrow \boxed{x = \frac{3x^2-5}{2x}}$

hence both (i) and (ii) are obtainable.

(b) (i) $g(x) = \frac{x+5}{x+1}$

$g'(x) = \frac{(x+1)1-(x+5)1}{(x+1)^2}$

$\Rightarrow \cancel{\frac{x+x-x-5}{(x+1)^2}}$ $\frac{x+1-x-5}{(x+1)^2}$

$\Rightarrow g'(x) = \frac{-4}{(x+1)^2}$

at $x = \sqrt{5}$,

(i) $g'(\sqrt{5}) = \frac{-4}{(\sqrt{5}+1)^2}$

$= -0.381$

$|g'(\sqrt{5})| < 1$.

$\therefore$ this is convergent.

(ii) $g(x) = \frac{3x^2-5}{2x}$

$\Rightarrow g(x) = \frac{2x \times 6x - (3x^2-5) \cdot 2}{4x^2}$

$= \frac{12x^2 - 6x^2 + 10}{4x^2}$

$= \frac{6x^2+10}{4x^2}$

$\Rightarrow \frac{3}{2} + \frac{5}{2x^2}$

$g'(\sqrt{5}) = \frac{3}{2} + \frac{5.1}{2 \times 8}$

$|g'(\sqrt{5})| \Rightarrow 2 > 1$.

$\therefore$ So, it will be divergent.

6. $f(x) = x^3 - \cos x = 0$

We know, in Newton's method –

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = x^3 - \cos x$$

$$f'(x) = 3x^2 + \sin x$$

Given $\boxed{x_0 = 1}$

$$f(x_0) = \cancel{300}\ 1 - \cos(1)$$

$$f'(x_0) = 3 + \sin(1)$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow 1 - \frac{1 - \cos(1)}{3 + \sin(1)}$$

$$\Rightarrow \frac{3 + \sin(1) - 1 + \cos(1)}{3 + \sin(1)}$$

$$\boxed{x_1 \Rightarrow \frac{2 + \sin(1) + \cos(1)}{3 + \sin(1)}}$$

$\therefore \cancel{x_2} =$

$$f(x_1) = \left[\frac{2 + \sin(1) + \cos(1)}{3 + \sin(1)}\right]^3 - \cos\left[\frac{2 + \sin(1) + \cos(1)}{3 + \sin(1)}\right]$$

$$f'(x_1) = 3\left[\frac{2 + \sin(1) + \cos(1)}{3 + \sin(1)}\right]^2 - \sin\left[\frac{2 + \sin(1) + \cos(1)}{3 + \sin(1)}\right]$$

$$\left.\begin{array}{l}\sin 1 = 0.841 \\ \cos 1 = 0.54\end{array}\right\}$$

$$\therefore \boxed{x_1 = 0.8802}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

$$\Rightarrow x_2 = x_1 - \frac{x_1^3 - \cos x_1}{3x_1^2 + \sin x_1}$$

$$\Rightarrow x_2 = \frac{3x_1^3 + x_1 \sin x_1 - x_1^3 + \cos x_1}{3x_1^2 + \sin x_1}$$

$$\Rightarrow x_2 = \frac{2x_1^3 + x_1 \sin x_1 + \cos x_1}{3x_1^2 + \sin x_1}$$

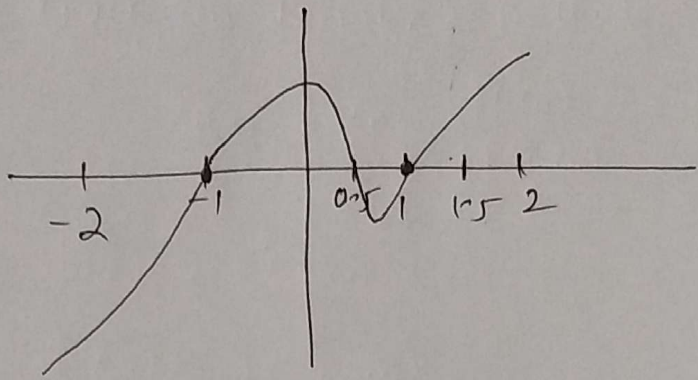
$$\Rightarrow \boxed{x_2 \approx 0.8653}$$

but we can't take $x_0 = 0$

because, $f'(x_0) = 0$

$\therefore$ the value of x_1 will diverge.

① $f(x) = (x+1)(x-1)(x-\frac{1}{2})$ | interval $\equiv [-2, 1.5]$



[Initially, I solved this using a code. So, I have attached the code and its output as well.]

$i=1 \Rightarrow \left.\begin{matrix} a = -2 \\ b = 1.5 \end{matrix}\right\}$ $c = -0.25$.

$i=2 \Rightarrow \left.\begin{matrix} a = -2 \\ b = -0.25 \end{matrix}\right\}$ $c = -1.125$.

$i=3 \Rightarrow \left.\begin{matrix} a = -1.125 \\ b = -0.25 \end{matrix}\right\}$ $c = -0.6875$.

$\therefore$ The soln. p_3 is $= -0.6875$.