

Home Assignment 1

HA1.1. Write down the equations of constraints in Cartesian coordinates for the following dynamical systems and categorize them according to the classification of the constraints:

- a) A small rigid rod of length l is allowed to move in any manner inside a balloon of fixed radius $R > l$, the end parts of the rod always touching the balloon's surface. What changes would it have, if R is now $R(t) = R + at$, say a being a small constant.
- b) A piece of flexible but nonextensible string of length l_0 is tied to the ceiling at a horizontal separation of $l < l_0$. A heavy bead is allowed to slide in any manner without any friction on along the string. Because of the weight of the bead the string is all the time stretched into a 'V' shape.

HA1.2. Consider the case of deformable body, a system of N particles, whose shape can change. Suppose that the deformation of the body is changing in time according to a certain prescribed function of time. Then the motion of such a body is constrained by the equation

$$|\mathbf{r}_i - \mathbf{r}_j| = f(t)$$

Where, \mathbf{r}_i and \mathbf{r}_j are position vectors of the pair of subscripts (i,k) runs over all the distinct pairs of particles in the body. Classify the constraint and calculate the mechanical work done by the constraint.

HA1.3. A particle of mass m is suspended by a massless wire of length $r = a + b \cos(\omega t)$, ($a > b > 0$) to form a spherical pendulum. Find the virtual displacement and the equations of motion using D'Alembert's principle.