Lumped Parameter Conversion

When the impedance matrix, $\mathbf{Z}$, or the admittance matrix, $\mathbf{Y}$, is available it is possible to calculate all other types of lumped parameter matrices from the relations below.

$$\mathbf{S} = \mathbf{G}_{\mathrm{ref}} \cdot (\mathbf{E} - (\mathbf{Z}_{\mathrm{ref}}{}^{*} \cdot \mathbf{Y})) \cdot (\mathbf{E} + \mathbf{Z}_{\mathrm{ref}} \cdot \mathbf{Y})^{-1} \cdot \mathbf{G}_{\mathrm{ref}}^{-1},$$

$$\mathbf{Z} = \mathbf{Y}^{-1}, \qquad \mathbf{L} = \frac{\mathrm{Im}(\mathbf{Z})}{\omega}, \qquad \mathbf{C} = \frac{\mathrm{Im}(\mathbf{Y})}{\omega},$$

$$\mathbf{R} = \mathrm{Re}(\mathbf{Z}), \qquad \mathbf{G} = \mathrm{Re}(\mathbf{Y})$$

where $\mathbf{L}$ is the inductance, $\mathbf{C}$ is the capacitance, $\mathbf{R}$ is the resistance, and $\mathbf{G}$ is the conductance. $\mathbf{S}$ is the S-parameter. The relations also include the following matrices

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{Z}_{\mathrm{ref}} = \mathbf{E} \cdot Z_0$$

$$\mathbf{G}_{\mathrm{ref}} = \mathbf{E} \cdot \frac{1}{2\sqrt{|\mathrm{Re}(Z_0)|}}$$

where Z_0 is the characteristic impedance.

For capacitance matrix calculations special transformations are available to obtain the mutual or SPICE capacitance matrix from the basic output from the solver. The latter is, for voltage excitation the so called Maxwell capacitance matrix and, for charge excitation the inverse Maxwell capacitance matrix. The Maxwell capacitance values, terminal charges, and terminal voltages are linked by the following matrix relation:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{3-9}$$

where Q is charge, V is voltage, and C is the Maxwell capacitance matrix.

In order to analyze the response of a device, it is more common to use the form of a mutual capacitance matrix. The mutual capacitance matrix can be converted from the Maxwell capacitance matrix:

$$\begin{bmatrix} C_{\mathrm{m}11} & C_{\mathrm{m}12} & C_{\mathrm{m}13} \\ C_{\mathrm{m}21} & C_{\mathrm{m}22} & C_{\mathrm{m}23} \\ C_{\mathrm{m}31} & C_{\mathrm{m}32} & C_{\mathrm{m}33} \end{bmatrix} = \begin{bmatrix} C_{\mathrm{M}11} + C_{\mathrm{M}12} + C_{\mathrm{M}13} & -C_{\mathrm{M}12} & -C_{\mathrm{M}13} \\ -C_{\mathrm{M}21} & C_{\mathrm{M}21} + C_{\mathrm{M}22} + C_{\mathrm{M}23} & -C_{\mathrm{M}23} \\ -C_{\mathrm{M}31} & -C_{\mathrm{M}32} & C_{\mathrm{M}31} + C_{\mathrm{M}32} + C_{\mathrm{M}33} \end{bmatrix} \tag{3-10}$$

where $C_{\mathrm{M}ij}$ are elements of the Maxwell capacitance matrix and $C_{\mathrm{m}ij}$ are elements of the mutual capacitance matrix.

The basic steps of this conversion are:

1. Set each diagonal element of the mutual capacitance matrix to the sum of the corresponding row elements of the Maxwell capacitance matrix (that is, to get $C_{\mathrm{m}11}$, sum $C_{\mathrm{M}1j}$ for all j).

2 Reverse the sign of all of the off-diagonal elements in the Maxwell capacitance matrix to get the corresponding mutual capacitance matrix elements.

This conversion can also be used to transform resistance and conductance matrices obtained from the **Electric Currents** physics. In that context the output resistance matrix corresponds to the inverse Maxwell capacitance matrix, electrode currents correspond to charges and the mutual or SPICE conductance matrix corresponds to the mutual or SPICE capacitance matrix.

You can compute conversions between the various matrices ($\mathbf{Z}$, $\mathbf{Y}$, $\mathbf{S}$, $\mathbf{C}_{\mathrm{Max}}$, $\mathbf{C}_{\mathrm{mut}}$) in a results table using the **Settings** window for the **Global Matrix Evaluation** node, which you can add under **Results>Derived Values**.

Global Matrix Evaluation in the *COMSOL Multiphysics Reference Manual*